Aakash
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Time : 3 hrs.

Answers & Solutions

for

JEE (MAIN)-2021 (Online) Phase-4

(Physics, Chemistry and Mathematics)

M.M. : 300

IMPORTANT INSTRUCTIONS :

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part has two sections.

(i) Section-I : This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) Section-II : This section contains 10 questions. In Section-II, attempt any **five questions out of 10.** There will be **no negative marking for Section-II**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and there is no negative marking for wrong answer.

PART–A : PHYSICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. If the length of the pendulum in pendulum clock increases by 0.1%, then the error in time per day is:

   (1) 43.2 s (2) 4.32 s

   (3) 86.4 s (4) 8.64 s

Answer (1)

Sol. $T = 2\pi\sqrt{\dfrac{l}{g}}$

$\Rightarrow \dfrac{\Delta T}{T} = \dfrac{1}{2}\dfrac{\Delta l}{l}$

$\Rightarrow$ Time lost in 1 day.

$\Delta t = \dfrac{1}{2}\left(\dfrac{\Delta l}{l}\right) \times t$

$= \dfrac{1}{2} \times \dfrac{0.1}{100} \times 86400$

$= 43.2$ s

2. In the given circuit the AC source has $\omega = 100$ rad s^{-1}. Considering the inductor and capacitor to be ideal, what will be the current I flowing through the circuit?

   (1) 4.24 A (2) 0.94 A

   (3) 5.9 A (4) 6 A

Answer (Bonus)

Sol. For upper elements

$R_1 = 100$

$X_C = \dfrac{10^6}{100 \times 100} = 100$

$Z_1 = \sqrt{100^2 + \dfrac{10^6}{100 \times 100}}$

$= 100\sqrt{2}$

$\tan\phi_1 = \dfrac{100}{100} = 1 \quad \Rightarrow \phi_1 = 45°$

So, $I_{10} = \dfrac{200\sqrt{2}}{100\sqrt{2}} = 2 \qquad \therefore I_1 = 2\sin(\omega t + 45°)$

for lower elements, $\qquad \therefore R_2 = 50\ \Omega$

$Z_2 = \sqrt{50^2 + 50^2} = 50\sqrt{2} \quad X_L = \omega L = 50\Omega$

$\tan\phi_2 = -1°,\ I_{20} = \dfrac{200\sqrt{2}}{50\sqrt{2}} = 4$

$\Rightarrow \phi_2 = -45°$

So, $I = I_1 + I_2$

So, $I_0 = \sqrt{20}$

So, $I_{rms} = \dfrac{I_0}{\sqrt{2}} = \sqrt{10}$ A

$= 3.16$ A

3. Four NOR gates are connected as shown in figure. The truth table for the given figure is:

   (1)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

   (2)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

(3)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(4)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

Answer (1)

Sol.

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

4. A refrigerator consumes an average 35 W power to operate between temperature −10°C to 25°C. If there is no loss of energy then how much average heat per second does it transfer?

(1) 263 J/s

(2) 350 J/s

(3) 298 J/s

(4) 35 J/s

Answer (1)

Sol. For refrigerator

$$COP = \frac{W}{Q_2} = \frac{T_H - T_L}{T_L}$$

$$\Rightarrow \frac{35}{Q_2} = \left(\frac{298}{263} - 1\right)$$

$$\Rightarrow Q_2 = \frac{35 \times 263}{35}$$

= 263 J/s

5. If you are provided a set of resistances 2 Ω, 4 Ω, 6 Ω and 8 Ω. Connect these resistances so as to obtain an equivalent resistance of $\frac{46}{3}$ Ω.

(1) 2 Ω and 6 Ω are in parallel with 4 Ω and 8 Ω in series

(2) 4 Ω and 6 Ω are in parallel with 2 Ω and 8 Ω in series

(3) 6 Ω and 8 Ω are in parallel with 2 Ω and 4 Ω in series

(4) 2 Ω and 4 Ω are in parallel with 6 Ω and 8 Ω in series

Answer (4)

Sol. Consider option (4)

$$\frac{2 \times 4}{2 + 4} + 6 + 8 = \frac{46}{3} \ \Omega$$

6. The two thin coaxial rings, each of radius 'a' and having charges +Q and −Q respectively are separated by a distance of 's'. The potential difference between the centres of the two rings is:

(1) $\dfrac{Q}{2\pi\varepsilon_0}\left[\dfrac{1}{a} - \dfrac{1}{\sqrt{s^2 + a^2}}\right]$

(2) $\dfrac{Q}{2\pi\varepsilon_0}\left[\dfrac{1}{a} + \dfrac{1}{\sqrt{s^2 + a^2}}\right]$

(3) $\dfrac{Q}{4\pi\varepsilon_0}\left[\dfrac{1}{a} + \dfrac{1}{\sqrt{s^2 + a^2}}\right]$

(4) $\dfrac{Q}{4\pi\varepsilon_0}\left[\dfrac{1}{a} - \dfrac{1}{\sqrt{s^2 + a^2}}\right]$
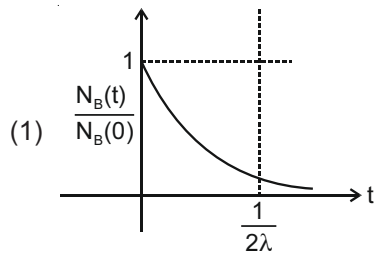
Answer (1)

Sol. $\Delta V = \left[\dfrac{Q}{4\pi\varepsilon_0 a} - \dfrac{Q}{4\pi\varepsilon_0 \sqrt{a^2 + s^2}}\right]$

$$-\left[\dfrac{-Q}{4\pi\varepsilon_0 a} - \dfrac{Q}{4\pi\varepsilon_0 \sqrt{a^2 + s^2}}\right]$$

$$= \dfrac{Q}{2\pi\varepsilon_0}\left[\dfrac{1}{a} - \dfrac{1}{\sqrt{s^2 + a^2}}\right]$$

7. At time t = 0, a material is composed of two radioactive atoms A and B, where $N_A(0) = 2N_B(0)$. The decay constant of both kind of radioactive atoms is λ. However, A disintegrates to B and B disintegrates to C. Which of the following figures represents the evolution of $N_B(t)/N_B(0)$ with respect to time t?

$$\begin{bmatrix} N_A(0) = \text{No. of A atoms at t = 0} \\ N_B(0) = \text{No. of B atoms at t = 0} \end{bmatrix}$$

(1)

$\dfrac{N_B(t)}{N_B(0)}$ vs t, with $\dfrac{1}{2\lambda}$ marked

(2)

$\dfrac{N_B(t)}{N_B(0)}$ vs t, with $\dfrac{1}{2\lambda}$ marked

(3)

$\dfrac{N_B(t)}{N_B(0)}$ vs t, with $\dfrac{1}{2\lambda}$ marked

(4)

$\dfrac{N_B(t)}{N_B(0)}$ vs t, with $\dfrac{1}{\lambda}$ marked

Answer (2)

Sol.
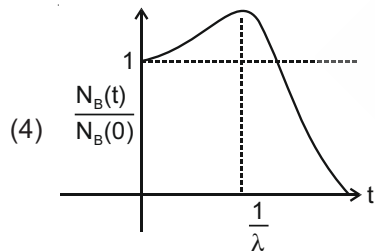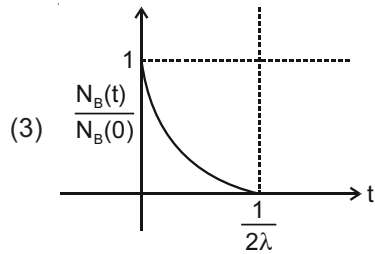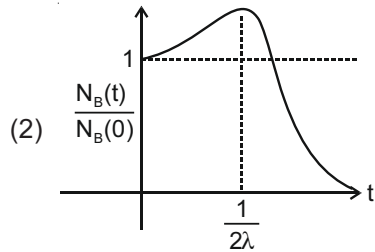
A $\longrightarrow$ B $\longrightarrow$ C
t = 0 2N$_0$ N$_0$

Initially, $N_B(t)$ will increase and after long time it will be tending to zero. Also N_B will be maximum at

$$t = \dfrac{1}{2\lambda}$$

8. A transmitting antenna at top a tower has a height of 50 m and the height of receiving antenna is 80 m. What is the range of communication for Line of Sight (LoS) mode?

[use radius of earth = 6400 km]

(1) 45.5 km   (2) 80.2 km

(3) 57.28 km   (4) 144.1 km

Answer (3)

Sol. $D_{max} = \sqrt{2h_T R_E} + \sqrt{2h_R R_E}$

$$= \sqrt{2\times 50 \times 6.4\times 10^6} + \sqrt{2\times 80 \times 6.4\times 10^6}$$

$$= 57.28 \text{ km}$$

9. The temperature of equal masses of three different liquids x, y and z are 10°C, 20°C and 30°C respectively. The temperature of mixture when x is mixed with y is 16°C and that when y is mixed with z is 26°C. The temperature of mixture when x and z are mixed will be:

(1) 25.62°C   (2) 28.32°C

(3) 23.84°C   (4) 20.28°C

Answer (3)

Sol. $10S_1 + 20S_2 = 16S_1 + 16S_2$

$\Rightarrow$ $4S_2 = 6S_1 \Rightarrow 2S_2 = 3S_1$ ...(1)

$20S_2 + 30S_3 = 26S_2 + 26S_3$

$\Rightarrow$ $6S_2 = 4S_3 \Rightarrow 3S_2 = 2S_3$ ...(2)

$10S_1 + 30S_3 = T(S_1 + S_3)$

$\Rightarrow$ $T = \dfrac{10S_1 + 30S_3}{S_1 + S_3} = 23.84°C$

10. Match **list-I** with **list-II**:

	List-I		List-II
(a)	Magnetic Induction	(i)	$ML^2T^{-2}A^{-1}$
(b)	Magnetic Flux	(ii)	$M^0L^{-1}A$
(c)	Magnetic Permeability	(iii)	$MT^{-2}A^{-1}$
(d)	Magnetization	(iv)	$MLT^{-2}A^{-2}$

Choose the **most appropriate** answer from the options given below:

(1) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

(2) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

(3) (a)-(iii), (b)-(ii), (c)-(iv), (d)-(i)

(4) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

Answer (4)

Sol. [Magnetic flux] $\rightarrow [M^1L^2T^{-2}I^{-1}]$,

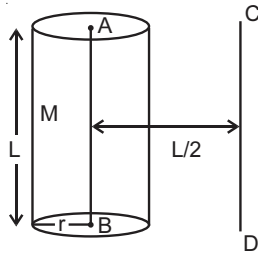$$[B] \rightarrow \dfrac{F}{IL} = \dfrac{MLT^{-2}}{AL} = \left[MT^{-2}A^{-1}\right]$$

$[\mu] \rightarrow [MLT^{-2}A^{-2}]$

[Magnetization] $\rightarrow [M^0L^{-1}A]$

(a)$\rightarrow$(iii), (b)$\rightarrow$(i), (c)$\rightarrow$(iv), (d)$\rightarrow$(ii)

11. The solid cylinder of length 80 cm and mass M has a radius of 20 cm. Calculate the density of the material used if the moment of inertia of the cylinder about an axis CD parallel to AB as shown in figure is 2.7 kg m^2.

(1) 7.5×10^1 kg/m^3 (2) 1.49×10^2 kg/m^3

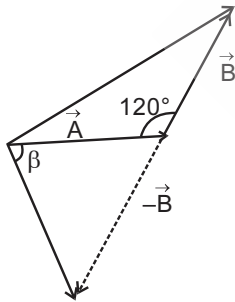(3) 7.5×10^2 kg/m^3 (4) 14.9 kg/m^3

Answer (2)

Sol. $I = \dfrac{Mr^2}{2} + \dfrac{ML^2}{4}$

$\Rightarrow$ M = 15 kg

$15 = \pi r^2 L \rho$

$\rho = 1.49 \times 10^2$ kg/m^3

12. The angle between vector $(\vec{A})$ and $(\vec{A} - \vec{B})$ is:

(1) $\tan^{-1}\left(\dfrac{-\dfrac{B}{2}}{A - B\dfrac{\sqrt{3}}{2}} \right)$

(2) $\tan^{-1}\left(\dfrac{\sqrt{3}B}{2A - B} \right)$

(3) $\tan^{-1}\left(\dfrac{B\cos\theta}{A - B\sin\theta} \right)$

(4) $\tan^{-1}\left(\dfrac{A}{0.7B} \right)$

Answer (2)

Sol. Angle between $\vec{A}$ and $\vec{B}$ = 60°

So angle between $\vec{A}$ and $-\vec{B}$ = 120°

If angle between $\vec{A}$ and $\vec{A} - \vec{B}$ is θ

then $\tan\theta = \dfrac{|-\vec{B}|\sin\theta}{\vec{A} + |-\vec{B}|\cos\theta}$

$= \dfrac{B\sin 120°}{A + B\cos 120°} = \dfrac{B\dfrac{\sqrt{3}}{2}}{A - \dfrac{B}{2}}$

$= \dfrac{\sqrt{3}\,B}{2A - B}$

$\theta = \tan^{-1}\left(\dfrac{B\sqrt{3}}{2A - B} \right)$

13. The de-Broglie wavelength of a particle having kinetic energy E is λ. How much extra energy must be given to this particle so that the de-Broglie wavelength reduces to 75% of the initial value?

(1) $\dfrac{7}{9}E$ (2) $\dfrac{1}{9}E$

(3) E   (4) $\dfrac{16}{9}E$

Answer (1)

Sol. $\lambda = \dfrac{h}{P} = \dfrac{h}{\sqrt{2Em}}$

Now, $\lambda_2 = \dfrac{h}{\sqrt{2E_2 m}}$

$\dfrac{3}{4}\lambda = \dfrac{h}{\sqrt{2E_2 m}}$

$\Rightarrow \dfrac{3}{4}\dfrac{h}{\sqrt{2Em}} = \dfrac{h}{\sqrt{2E_2 m}}$

$\dfrac{9}{16}\dfrac{1}{E} = \dfrac{1}{E_2}$

$E_2 = \dfrac{16}{9}E$

So, $E_2 - E = \dfrac{16}{9}E - E = \dfrac{7}{9}E$

14. A cylindrical container of volume 4.0×10^{-3} m^3 contains one mole of hydrogen and two moles of carbon dioxide. Assume the temperature of the mixture is 400 K. The pressure of the mixture of gases is:

[Take gas constant as 8.3 J mol^{-1} K^{-1}]

(1) 24.9×10^3 Pa

(2) 24.9×10^5 Pa

(3) 24.9 Pa

(4) 249×10^1 Pa

Answer (2)

Sol. No. of mole, n = 1 + 2 = 3

using PV = nRT

$$P = \frac{nRT}{V} = \frac{3 \times 8.3 \times 400}{4 \times 10^{-3}}$$

$= 24.9 \times 10^5$ Pa

15. Two blocks of masses 3 kg and 5 kg are connected by a metal wire going over a smooth pulley. The breaking stress of the metal is $\frac{24}{\pi} \times 10^2$ Nm^{-2}. What is the minimum radius of the wire?

(take g = 10 ms^{-2})
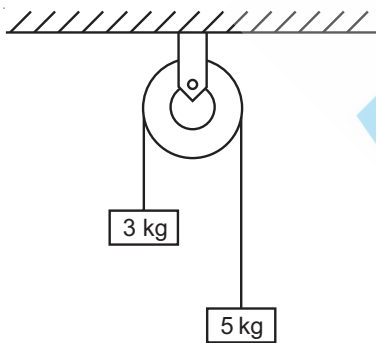
(1) 1250 cm   (2) 125 cm

(3) 1.25 cm   (4) 12.5 cm

Answer (4)

Sol. Acceleration, $a = \frac{5g - 3g}{8}$

$$= \frac{2g}{8} = \frac{g}{4}$$

FBD of 5 kg
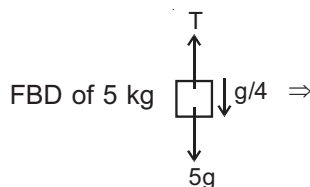
$$5g - T = \frac{5g}{4}$$

$$T = \frac{15g}{4}$$

Now, $\frac{T}{\pi r^2} \le \frac{24}{\pi} \times 10^2$

$$r^2 \ge \frac{15g}{4 \times 24 \times 10^2} \Rightarrow r \ge \frac{5}{4 \times 10} m$$

r_{min} = 12.5 cm

16. A particle of mass m is suspended from a ceiling through a string of length L. The particle moves in a horizontal circle of radius r such that $r = \frac{L}{\sqrt{2}}$. The speed of particle will be:
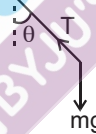
(1) $2\sqrt{rg}$

(2) $\sqrt{rg}$

(3) $\sqrt{2rg}$

(4) $\sqrt{\frac{rg}{2}}$

Answer (2)

Sol.

T cos θ = mg

$$T \sin \theta = \frac{mv^2}{r}$$

$$\frac{v^2}{rg} = \tan \theta$$

$v = \sqrt{rg}$

17. An electric bulb of 500 watt at 100 volt is used in a circuit having a 200 V supply. Calculate the resistance R to be connected in series with the bulb so that the power delivered by the bulb is 500 W.

(1) 5 Ω

(2) 30 Ω

(3) 20 Ω

(4) 10 Ω

Answer (3)

Sol. $R = \dfrac{V^2}{P}$

$R = 20 \ \Omega$

Let resistance to be connected in series be R_S

$I = \dfrac{200}{20 + R_S}$

$I^2 R = 500$

$I = 5 \ A$

$R_S = 20 \ \Omega$

18. A bomb is dropped by a fighter plane flying horizontally. To an observer sitting in the plane, the trajectory of the bomb is a :

    (1) Straight line vertically down the plane

    (2) Hyperbola

    (3) Parabola in the direction of motion of plane

    (4) Parabola in a direction opposite to the motion of plane

Answer (1)

Sol. Relative velocity of bomb w.r.t. observer in plane = 0

Bomb will fall down vertically. So, it will move in straight line w.r.t. observer.

19. A light beam is described by $E = 800 \sin\omega\left(t - \dfrac{x}{c}\right)$.

    An electron is allowed to move normal to the propagation of light beam with a speed of $3 \times 10^7 \ ms^{-1}$. What is the maximum magnetic force exerted on the electron ?

    (1) $1.28 \times 10^{-21} \ N$

    (2) $1.28 \times 10^{-18} \ N$

    (3) $12.8 \times 10^{-18} \ N$

    (4) $12.8 \times 10^{-17} \ N$

Answer (3)

Sol. $B = \dfrac{E}{c}$

$F = e(\vec{v} \times \vec{B})$

$= \dfrac{eE}{c} \cdot v$

$= \dfrac{1.6 \times 10^{-19} \times 800}{10}$

$= 1280 \times 10^{-20}$

$= 1.28 \times 10^{-17} \ N$

20. A parallel-plate capacitor with plate aera A has separation d between the plates. Two dielectric slabs of dielectric constant K_1 and K_2 of same area A/2 and thickness d/2 are inserted in the space between the plates. The capacitance of the capacitor will be given by :
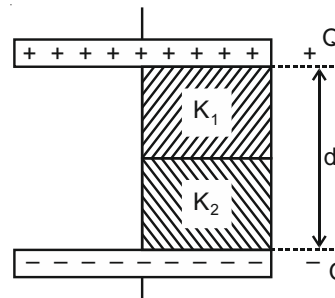
(1) $\dfrac{\varepsilon_0 A}{d}\left(\dfrac{1}{2} + \dfrac{K_1 + K_2}{K_1 K_2}\right)$
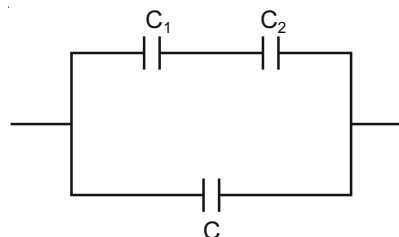
(2) $\dfrac{\varepsilon_0 A}{d}\left(\dfrac{1}{2} + \dfrac{K_1 K_2}{2(K_1 + K_2)}\right)$

(3) $\dfrac{\varepsilon_0 A}{d}\left(\dfrac{1}{2} + \dfrac{2(K_1 + K_2)}{K_1 K_2}\right)$

(4) $\dfrac{\varepsilon_0 A}{d}\left(\dfrac{1}{2} + \dfrac{K_1 K_2}{K_1 + K_2}\right)$

Answer (4)

Sol. Equivalent capacitor circuit is

$C_1 = \dfrac{K_1 \varepsilon_0 A}{d}$

$C_2 = \dfrac{K_2 \varepsilon_0 A}{d}$

$C = \dfrac{\varepsilon_0 A}{2d}$

$C_{eq} = \dfrac{K_1 K_2 \varepsilon_0 A}{(K_1 + K_2)d} + \dfrac{\varepsilon_0 A}{2d}$

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. For the given circuit, the power across zener diode is _____mW.

Answer (120)

Sol.
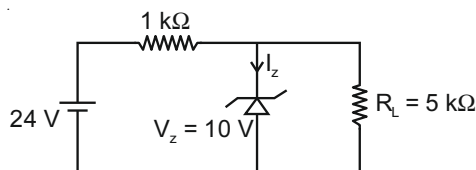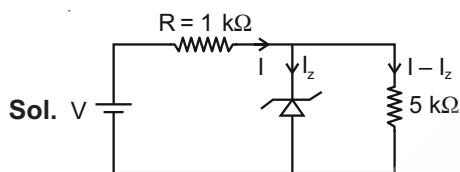
$$I - I_z = \frac{V_z}{R_L} = 2 \text{ mA}$$

$$I = \frac{V - V_z}{R} = 14 \text{ mA}$$

$$\Rightarrow I_z = 12 \text{ mA}$$

$$P = V_z I_z = 120 \text{ mW}$$

2. Two simple harmonic motions are represented by the equations $x_1 = 5\sin\left(2\pi t + \frac{\pi}{4}\right)$ and $x_2 = 5\sqrt{2}(\sin 2\pi t + \cos 2\pi t)$. The amplitude of second motion is _____ times the amplitude in first motion.

Answer (2)

Sol. $x_1 = 5\sin\left(2\pi t + \frac{\pi}{4}\right) \Rightarrow A_1 = 5$

$x_2 = 5\sqrt{2}(\sin 2\pi t + \cos 2\pi t)$

$= 10\sin\left(2\pi t + \frac{\pi}{4}\right) \Rightarrow A_2 = 10$

$$\frac{A_2}{A_1} = 2$$

3. A circular coil of radius 8.0 cm and 20 turns is rotated about its vertical diameter with an angular speed of 50 rad s^{-1} in a uniform horizontal magnetic field of 3.0×10^{-2} T. The maximum emf induced the coil will be _____ × 10^{-2} volt (rounded off to the nearest integer).

Answer (60)

Sol. $\varepsilon_{max} = NAB\omega$

$= 20 \times 3.14 \times (8 \times 10^{-2})^2 \times 3 \times 10^{-2} \times 50$

$= 60.29 \times 10^{-2}$ V

4. The acceleration due to gravity is found upto an accuracy of 4% on a planet. The energy supplied to a simple pendulum of known mass 'm' to undertake oscillations of time period T is being estimated. If time period is measured to an accuracy of 3%, the accuracy to which E is known as ____ %.

Answer (14)

Sol. $E = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 \ell^2 \theta_0^2$

$E = CT^2 g^2$

$$\frac{\Delta E}{E}\% = 2 \times (4 + 3) = 14$$

5. Two waves are simultaneously passing through a string and their equations are :

$y_1 = A_1 \sin k(x - vt)$, $y_2 = A_2 \sin k (x - vt + x_0)$. Given amplitudes $A_1 = 12$ mm and $A_2 = 5$ mm, $x_0 = 3.5$ cm and wave number $k = 6.28$ cm^{-1}. The amplitude of resulting wave will be ____ mm.

Answer (7)

Sol. $y_1 = A_1 \sin k(x - vt)$

$y_2 = A_2 \sin k (x - vt + x_0)$

$A^2 = A_1^2 + A_2^2 + 2A_1 A_2 \cos\phi$

where $\phi = kx_0 = 2\pi \times 3.5 = 7\pi$

$\Rightarrow A = |A_1 - A_2| = 7$ mm

6. A source of light is placed in front of a screen. Intensity of light on the screen is *I*. Two Polaroid P_1 and P_2 are so placed in between the source of light and screen that the intensity of light on screen is $I/2$. P_2 should be rotated by an angle of ___ (degrees) so that the intensity of light on the screen becomes $\dfrac{3I}{8}$.
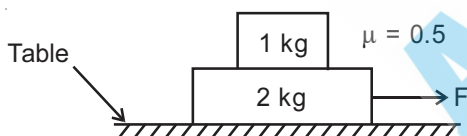
Answer (30)

Sol. Initially the polaroids are aligned i.e. angle between their axes is 0°.

Now if polaroid P_2 is rotated by angle θ to obtain required result then

$$\frac{I}{2}\times\cos^2\theta = \frac{3}{4}I$$

$$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2} \text{ or } \theta = 30°$$

7. The coefficient of static friction between two blocks is 0.5 and the table is smooth. The maximum horizontal force that can be applied to move the blocks together is ___ N. (take g = 10 ms^{-2})

Answer (15)

Sol. Maximum acceleration of block 1 is

$a_{max} = 5$ m/s^2

$\Rightarrow$ which is also maximum acceleration of the system without relative slipping in blocks.

$F_{max} = 3(a_{max}) = 15$ N

8. If the maximum value of accelerating potential provided by a radio frequency oscillator is 12 kV. The number of revolution made by a proton in a cyclotron to achieve one sixth of the speed of light is ____.

[$m_p = 1.67 \times 10^{-27}$ kg, $e = 1.6 \times 10^{-19}$ C, Speed of light = 3×10^8 m/s]

Answer (543)

Sol. $\dfrac{1}{2}m\left(\dfrac{c}{6}\right)^2 = n\times 2\times eV$

$$n = \frac{1.67\times 10^{-27}\times 9\times 10^{16}}{4\times 36\times 1.6\times 10^{-19}\times 12\times 10^3}$$

$$= 543.6197$$

9. A coil in the shape of an equilateral triangle of side 10 cm lies in a vertical plane between the pole pieces of permanent magnet producing a horizontal magnetic field 20 mT. The torque acting on the coil when a current of 0.2 A is passed through it and its plane becomes parallel to the magnetic field will be $\sqrt{x}\times 10^{-5}$ Nm. The value of *x* is _____.
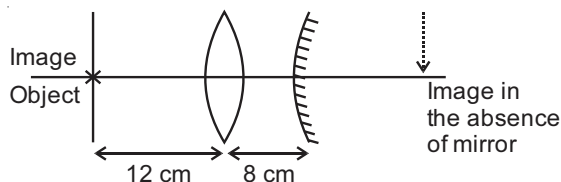
Answer (3)

Sol. $\vec{\tau} = \vec{M}\times\vec{B}$

$$\vec{\tau} = (0.2)\left(\frac{\sqrt{3}}{4}a^2\right)\times 20\times 10^{-3}\times\sin 90°$$

$$= \sqrt{3}\times 10^{-5} \text{ Nm}$$

10. An object is placed at a distance of 12 cm from a convex lens. A convex mirror of focal length 15 cm is placed on other side of lens at 8 cm as shown in the figure. Image of object coincides with the object.

When the convex mirror is removed, a real and inverted image is formed at a position. The distance of the image from the object will be _____ (cm).
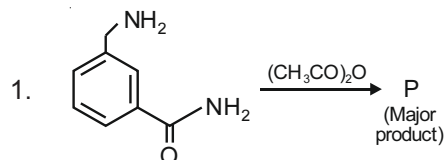
Answer (50)

Sol.
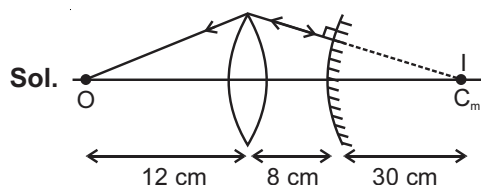
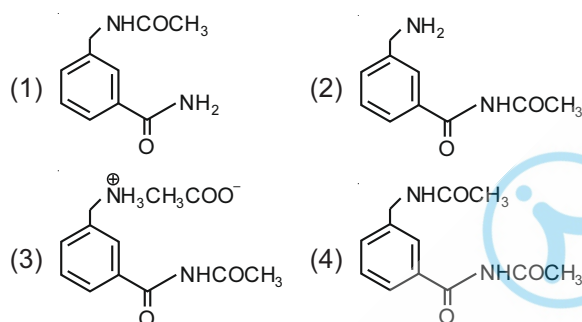Distance of image from object = 50 cm

PART–B : CHEMISTRY

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.
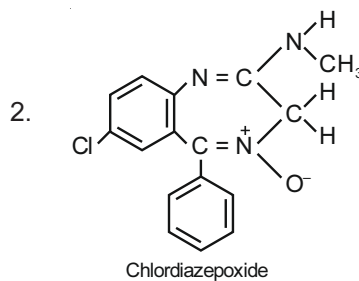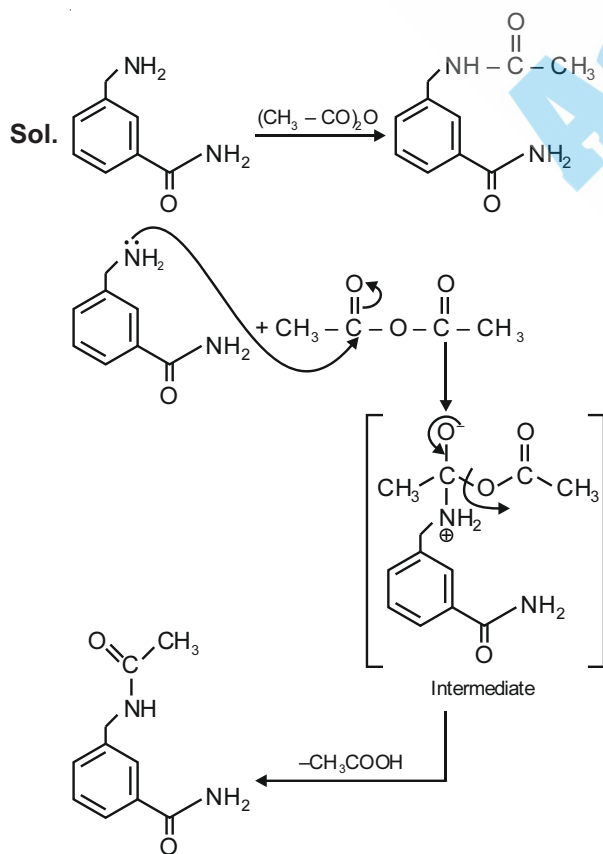
Choose the correct answer :

1.

The major product in the above reaction is :

(1)

(2)

(3)

(4)

Answer (1)

Sol.

2.

Chlordiazepoxide

The class of drug to which chlordiazepoxide with above structure belongs is :

(1) Antacid   (2) Analgesic

(3) Tranquilizer   (4) Antibiotic

Answer (3)
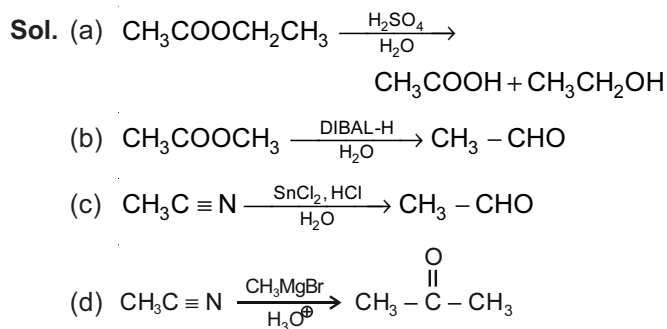
Sol. Given compound is an example of tranquilizer.

3. Match List-I with List-II.

List-I (Chemical reaction)		List-II (Reagent used)
(a) $CH_3COOCH_2CH_3$ $\rightarrow CH_3CH_2OH$	(i)	CH_3MgBr/H_3O^+ (1 equivalent)
(b) CH_3COOCH_3 $\rightarrow CH_3CHO$	(ii)	H_2SO_4/H_2O
(c) $CH_3C \equiv N$ $\rightarrow CH_3CHO$	(iii)	DIBAL-H/H_2O
(d) $CH_3C \equiv N$ $\rightarrow CH_3 \overset{\overset{\displaystyle O}{\|\|}}{C} CH_3$	(iv)	$SnCl_2$, HCl/H_2O

Choose the **most appropriate** match.

(1) (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)

(2) (a)-(iii), (b)-(ii), (c)-(i), (d)-(iv)

(3) (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)

(4) (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i)

Answer (1)

Sol. (a) $CH_3COOCH_2CH_3 \xrightarrow[\text{H}_2\text{O}]{\text{H}_2\text{SO}_4}$

       $CH_3COOH + CH_3CH_2OH$

(b) $CH_3COOCH_3 \xrightarrow[\text{H}_2\text{O}]{\text{DIBAL-H}} CH_3 - CHO$

(c) $CH_3C \equiv N \xrightarrow[\text{H}_2\text{O}]{\text{SnCl}_2, \text{HCl}} CH_3 - CHO$

(d) $CH_3C \equiv N \xrightarrow[\text{H}_3\text{O}^\oplus]{\text{CH}_3\text{MgBr}} CH_3 - \overset{\overset{\displaystyle O}{\|\|}}{C} - CH_3$

So, the correct match is

 (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)

Cyanides can be converted to aldehydes by DIBAL-H/H_2O as well.

4. Arrange the following cobalt complexes in the order of increasing Crystal Field Stabilisation Energy (CFSE) value.

   Complexes :

   A. $[CoF_6]^{3-}$ B. $[Co(H_2O)_6]^{2+}$

   C $[Co(NH_3)_6]^{3+}$ D. $[Co(en)_3]^{3+}$

   Choose the **correct** option :

   (1) C < D < B < A   (2) B < C < D < A

   (3) A < B < C < D   (4) B < A < C < D

Answer (4)

Sol. CFSE value increases as the strength of the ligand increases also with increase in positive charge of central atom. According to spectrochemical series, the order of ligand strength is en > NH_3 > H_2O > F^-. So, the CFSE value of the given complexes should be A < B < C < D. But as complex A contain cobalt in +3 OS. So A > B. Final order is B < A < C < D

5. The sol given below with negatively charged colloidal particles is :

   (1) KI added to $AgNO_3$ solution

   (2) $AgNO_3$ added to KI solution

   (3) $Al_2O_3 \cdot xH_2O$ in water

   (4) $FeCl_3$ added to hot water

Answer (2)

Sol. When highly diluted solution of silver nitrate is added to highly diluted potassium iodide solution, the precipitated AgI adsorbs iodide ions from the dispersion medium and negatively charged colloidal sol results.

$$AgI / I^-$$
Negatively charged colloidal
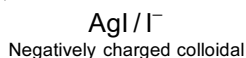
6. Indicate the complex/ complex ion which did not show any geometrical isomerism:

   (1) $\left[Co(NH_3)_4 Cl_2 \right]^+$

   (2) $\left[Co(CN)_5 (NC) \right]^{3-}$

   (3) $\left[CoCl_2 (en)_2 \right]$

   (4) $\left[Co(NH_3)_3 (NO_2)_3 \right]$

Answer (2)
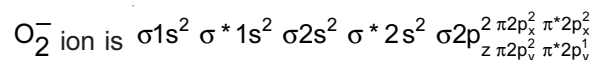
Sol. $\left[Co(CN)_5 (NC) \right]^{3-}$ can not show geometrical isomerism

7. The bond order and magnetic behaviour of O_2^- ion are, respectively:

   (1) 1.5 and diamagnetic.

   (2) 1.5 and paramagnetic.

   (3) 1 and paramagnetic.

   (4) 2 and diamagnetic.

Answer (2)

Sol. According to MOT the electronic configuration of

O_2^- ion is $\sigma 1s^2 \; \sigma^* 1s^2 \; \sigma 2s^2 \; \sigma^* 2s^2 \; \sigma 2p_z^2 \begin{matrix} \pi 2p_x^2 & \pi^* 2p_x^2 \\ \pi 2p_y^2 & \pi^* 2p_y^1 \end{matrix}$

Bond order $= \dfrac{1}{2}$ (Bonding electrons − anti bonding electrons)

Bond order $= \dfrac{1}{2}(10 - 7) = 1.5$

8. Given below are two statements: one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

   Assertion (A) : Barium carbonate is insoluble in water and is highly stable.

   Reason (R) : The thermal stability of the carbonates increases with increasing cationic size.

   Choose the **most appropriate** answer from the options given below:

   (1) Both **(A)** and **(R)** are true but **(R)** is not the true explanation of **(A)**

   (2) **(A)** is false but **(R)** is true.

   (3) **(A)** is true but **(R)** is false.

   (4) Both **(A)** and **(R)** are true and **(R)** is the true explanation of **(A)**.
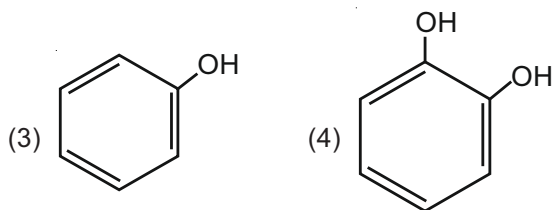
Answer (4)

Sol. Alkaline earth metal carbonate are insoluble in water.

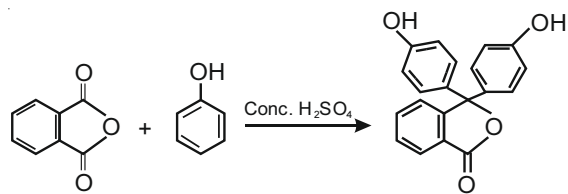   Thermal stability of carbonate increases with increasing cationic size.

9. Which one of the following phenols does not give colour when condensed with phthalic anhydride in presence of conc. H_2SO_4?
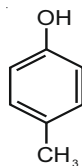

(1)

(2)

(3) ⟨phenol with OH⟩   (4) ⟨catechol with two OH⟩
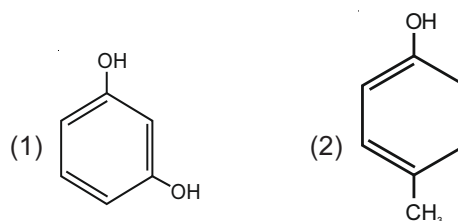
Answer (2)

Sol.

Phthalic anhydride + Phenol → Phenolphthalein, with Conc. H_2SO_4
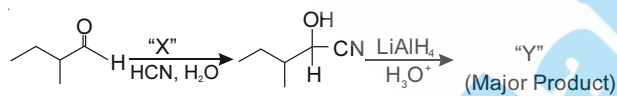
As phthalic anhydride being bulky, electrophilic substitution reaction occurs at para position in phenol or its derivatives
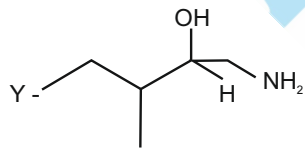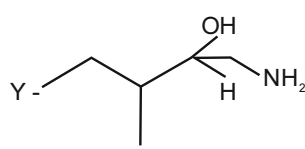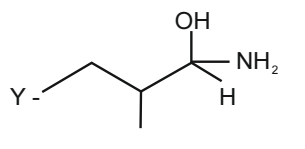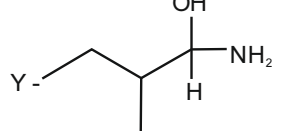

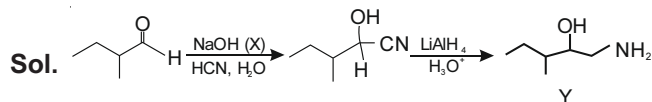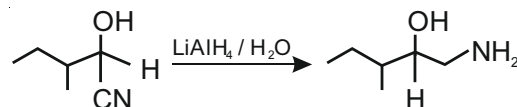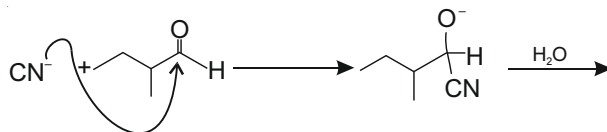Does not condense with phthalic anhydride because para position is blocked.

10.

Consider the given reaction, Identify "X" and "Y"

(1) X – NaOH

(2) X – HNO_3

(3) X – NaOH

(4) X – HNO_3

Answer (1)

Sol.

Addition of HCN to aldehydes and ketones occurs slowly because HCN is weak electrolyte and does not produce enough CN^- ion. A catalytic amount of base can fasten the reaction.

$$HCN + OH^- \rightarrow H_2O + CN^-$$
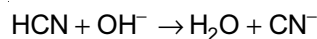
11. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Heavy water is used for the study of reaction mechanism.

Reason (R) : The rate of reaction for the cleavage of O – H bond is slower than that of O – D bond.

Choose the most appropriate answer from the options given below :

(1) (A) is true but (R) is false.

(2) Both (A) and (R) are true and (R) is the true explanation of (A).

(3) Both (A) and (R) are true but (R) is not the true explanation of (A)

(4) (A) is false but (R) is true

Answer (1)

Sol. • Heavy water (D_2O) is used for the study of reaction mechanism

• The rate of reaction for the cleavage of O-H bond should be faster than that of O–D bond because O–H bond is weaker than O–D bond

12. Chalcogen group elements are :

(1) O, Ti and Po   (2) S, Te and Pm

(3) Se, Tb and Pu   (4) Se, Te and Po

Answer (4)

Sol. Chalcogens are 16th group elements

–O, S, Se, Te, Po and Lv
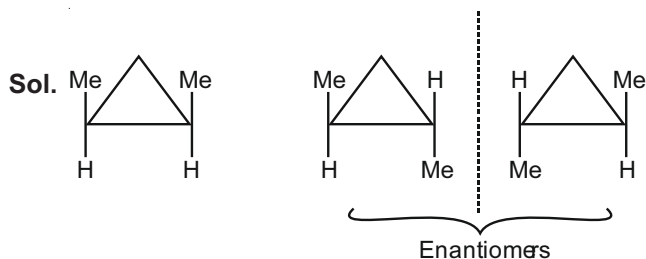
13. The number of stereoisomers possible for 1, 2-dimethyl cyclopropane is :

(1) Two (2) Three

(3) One (4) Four

Answer (2)

Sol.

Total three stereoisomers

14. The number of non-ionisable hydrogen atoms present in the final product obtained from the hydrolysis of PCl_5 is :

(1) 3 (2) 0

(3) 2 (4) 1

Answer (2)

Sol. $PCl_5 + 4H_2O \rightarrow H_3PO_4 + 5HCl$

All hydrogens are ionisable
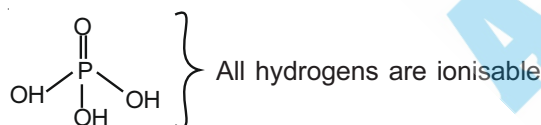
15. The interaction energy of London forces between two particles is proportional to r^x, where r is the distance between the particles. The value of x is :

(1) 3

(2) 6

(3) –6

(4) –3

Answer (3)

Sol. London forces or dispersion forces are always attractive and interaction energy is inversely proportional to the sixth power of the distance between two interacting particles

Interaction energy $\propto r^{-6}$

16.
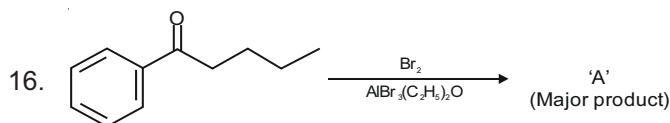
Consider the given reaction, the Product A is

(1)

(2)

(3)

(4)

Answer (2)

Sol.
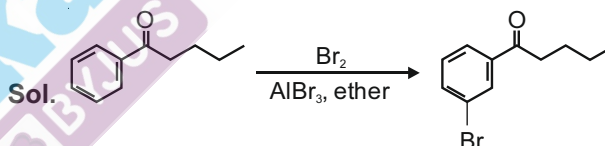
Ketones are meta directors.

$Br_2 + AlBr_3 \longrightarrow AlBr_4^- + Br^+$
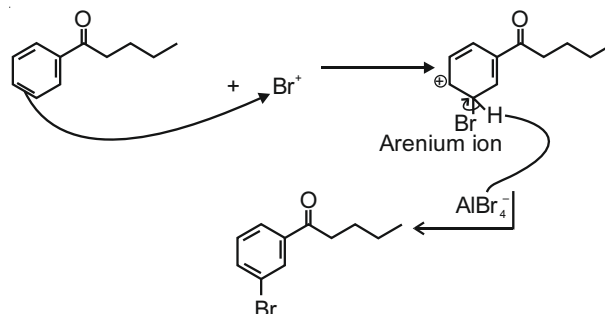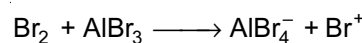
17. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Sucrose is a disaccharide and a non-reducing sugar.

Reason (R) : Sucrose involves glycosidic linkage between C_1 of β-glucose and C_2 of α-fructose.

the **most appropriate** answer from the options given below :

(1) Both (A) and (R) are true and (R) is the true explanation of (A)

(2) (A) is true but (R) is false

(3) Both (A) and (R) are true but (R) is not the true explanation of (A).

(4) (A) is false but (R) is true

Answer (2)

Sol. Sucrose is a disaccharides and a non-reducing sugar because it does not contain free hemiacetal linkage.

Sucrose involves glycosidic linkage between C_1 of α–D–glucose and C_2 of β–D– fructose.

18. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**

Assertion (A) : Photochemical smog causes cracking of rubber.

Reason (R) : Presence of ozone, nitric oxide, acrolein, formaldehyde and peroxyacetyl nitrate in photochemical smog makes it oxidizing

Choose the **most appropriate** answer from the options given below:

(1) Both (A) and (R) are true and (R) is the true explanation of (A)

(2) (A) is true but (R) is false

(3) (A) is false but (R) is true

(4) Both (A) and (R) are true but (R) is not the true explanation of (A).

Answer (1)

Sol. Photochemical smog causes cracking of rubber. It contains ozone, nitric oxide, acrolein, formaldehyde and peroxyacetyl nitrate. It contains high concentration of oxidizing agents and is, therefore, called as oxidizing smog.

19. Given below are two statements :

Statnement I : Sphalerite is a sulphide ore of zinc and copper glance is a sulphide ore of copper.

Statement II : It is possible to separate two sulphide ores by adjusting proportion of oil to water or by using depressants in a froth flotation method.

Choose the most appropriate answer from the options given below :

(1) **Statement I** is true but Stantement II is false
(2) **Statement I** is false but Statment II is true
(3) Both **Statement I** and Statement II are false
(4) Both **Statement I** and Statement II are true

Answer (4)

Sol. Sphalerite – ZnS

Copper glance – Cu_2S

Yes it is possible to separate two sulphide ores by adjusting proportion of oil to water or by using 'depressants' in a froth flotation process.

Eg : ZnS and PbS can separated by using NaCN.

$$ZnS + 4NaCN \longrightarrow Na_2\left[Zn(CN)_4\right] + Na_2S$$
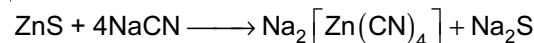
20. Which one of the following compounds is not aromatic?
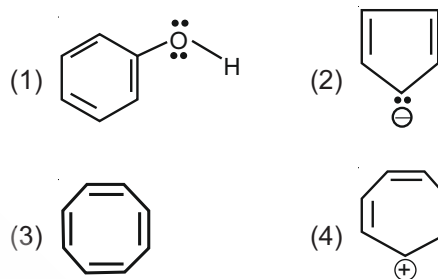
Answer (3)

Sol.

Non planar and number of π electrons are not equal to Huckel's rule [$(4n + 2)\pi$ electrons]. Hence it is non-aromatic

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SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, ?00.33, ?00.30, 30.27, ?27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

---

1. The overall stability constant of the complex ion $[Cu(NH_3)_4]^{2+}$ is 2.1×10^{13}. The overall dissociation constant is $y \times 10^{-14}$. Then y is _____. (Nearest integer)

Answer (5)

Sol. $Cu^{2+} + 4NH_3 \rightleftharpoons [Cu(NH_3)_4]^{2+}$ $K_S = 2.1 \times 10^{13}$

$[Cu(NH_3)_4]^{2+} \rightleftharpoons Cu^{2+} + 4NH_3$ $K_d = \dfrac{1}{K_s}$
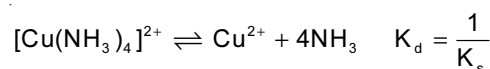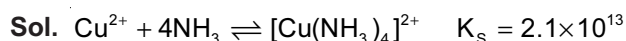
$K_d = \dfrac{1}{K_s} = \dfrac{10^{-13}}{2.1} = 4.76 \times 10^{-14}$

So nearest integer is 5.

2. For water $\Delta_{vap}H = 41\,kJ\,mol^{-1}$ at 373 K and 1 bar pressure. Assuming that water vapour is an ideal gas that occupies a much larger volume than liquid water, the internal energy change during evaporation of water is _____ kJ mol⁻¹.

[Use : R = 8.3 J mol⁻¹K⁻¹]

Answer (38)

Sol. $H_2O(l) \rightleftharpoons H_2O(g)$ $\quad\quad \Delta H_{vap} = 41\,kJ\,mol^{-1}$
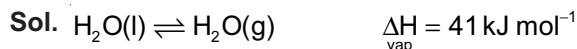
$\Delta H = \Delta E + \Delta n_g RT$

$41 = \Delta E + 1 \times 8.3 \times 10^{-3} \times 373$

$\Delta E \approx (41 - 3) = 38\,kJ\,mol^{-1}$

3. The equilibrium constant K_c at 298 K for the reaction

$A + B \rightleftharpoons C + D$

is 100. Starting with an equimolar solution with concentrations of A, B, C and D all equal to 1 M, the equilibrium concentration of D is _____ × 10⁻² M. (Nearest integer)

Answer (182)

Sol.

	A	+	B	$\rightleftharpoons$	C	+	D
Initial Concentration	1 M		1 M		1 M		1 M
At equilibrium	(1 − x)		(1 − x)		(1 + x)		(1 + x)
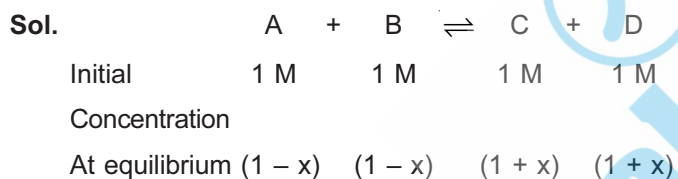
[$Q_C = 1$ and less than K_C indicates reaction moves forward]

$K_c = \dfrac{[C][D]}{[A][B]}$

$100 = \dfrac{(1+x)(1+x)}{(1-x)(1-x)} = \dfrac{(1+x)^2}{(1-x)^2}$

$\dfrac{(1+x)}{(1-x)} = 10$

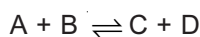$1 + x = 10 - 10x$

$\Rightarrow 11x = 9$

$\Rightarrow x = \dfrac{9}{11}$

$[D] = 1 + x = 1 + \dfrac{9}{11} = \dfrac{20}{11}$

$[D] = 1.8181\,M$

$[D] \approx 182 \times 10^{-2}\,M$

4. In the sulphur estimation, 0.471 g of an organic compound gave 1.44 g of barium sulfate. The percentage of sulphur in the compound is _____%. (Nearest integer)

(Atomic Mass of B = 137 u)

Answer (42)

Sol. Organic compound $\xrightarrow[\text{Heat}]{Na_2O_2} SO_4^{2-} \xrightarrow{BaCl_2} BaSO_4 \downarrow$
   (containing sulphur)
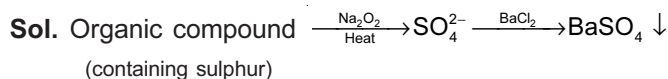
233 gram of $BaSO_4$ contains 32 gram of sulphur

1.44 gram of $BaSO_4$ contains $\dfrac{1.44 \times 32}{233}$ gram of sulphur

Percentage of sulphur in the organic compound

$= \dfrac{\text{weight of sulphur}}{\text{weight of organic compound}} \times 100$

$= \dfrac{1.44 \times 32}{233 \times 0.471} \times 100 \approx 42\%$

5. The reaction rate for the reaction

$[PtCl_4]^{2-} + H_2O \rightleftharpoons [Pt(H_2O)Cl_3]^- + Cl^-$
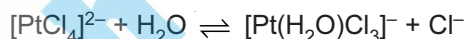
was measured as a function of concentrations of different species. It was observed that

$\dfrac{-d[PtCl_4]^{2-}}{dt} = 4.8 \times 10^{-5}[PtCl_4]^{2-}$
$\quad\quad - 2.4 \times 10^{-3}[Pt(H_2O)Cl_3]^-[Cl^-].$

where square brackets are used to denote molar concentrations. The equilibrium constant

$K_c = $ _____. (Nearest integer)

Answer (0)

Sol. $[PtCl_4]^{2-} + H_2O \underset{K_b}{\overset{K_f}{\rightleftharpoons}} [Pt(H_2O)Cl_3]^- + Cl^-$
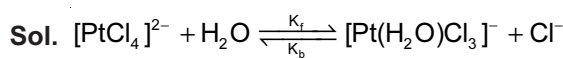
$\dfrac{-d[PtCl_4]^{2-}}{dt} = K_f[PtCl_4]^{2-} - K_b[Pt(H_2O)Cl_3]^-[Cl^-]$

$K_c = \dfrac{K_f}{K_b} = \dfrac{4.8 \times 10^{-5}}{2.4 \times 10^{-3}} = 2 \times 10^{-2}$

6. A chloro compound "A"

(i) forms aldehydes on ozonolysis followed by the hydrolysis.

(ii) when vaporized completely 1.53 g of A, gives 448 mL of vapour at STP.

The number of carbon atoms in a molecule of compound A is _____.

Answer (3)

Sol. Mole $= \dfrac{\text{Given mass}}{\text{Molar mass}}$

$= \dfrac{\text{Given volume (at STP in L)}}{22.4}$

$\dfrac{1.53}{\text{Molar mass}} = \dfrac{448 \times 10^{-3}}{22.4}$

Molar mass $= \dfrac{1.53 \times 22.4}{448 \times 10^{-3}} = 76.5$ g mol^{-1}

There can not be more than one chlorine atom per molecule because molar mass 76.5. One of the possible compounds is $Cl - CH_2 - CH = CH_2$.

$Cl - CH_2 - CH = CH_2$

$\xrightarrow[\text{(ii) Zn/ACOH}]{\text{(i) } O_3} ClCH_2 - CHO + HCHO$
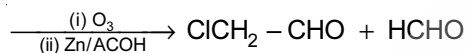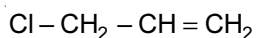
7. 100 mL of Na_3PO_4 solution contains 3.45 g of sodium. The molarity of the solution is _____ $\times 10^{-2}$ mol L^{-1}. (Nearest integer)

[Atomic Masses - Na : 23.0 u, O : 16.0 u, P : 31.0 u]

Answer (50)

Sol. Mole $= \dfrac{\text{Given mass}}{\text{Molar mass}}$

$= \dfrac{3.45}{23} = 0.15$ mol of Na$^+$

Each mole of Na_3PO_4 has 3 mole of Na$^+$. So 0.15 mole of Na$^+$ is present in $\dfrac{0.15}{3}$ mole of Na_3PO_4.

Molarity $= \dfrac{0.15 \times 1000}{3 \times 100} = 0.5$ mol L^{-1}

Molarity $= 50 \times 10^{-2}$ mol L^{-1}

8. A metal surface is exposed to 500 nm radiation. The threshold frequency of the metal for photoelectric current is 4.3×10^{14} Hz. The velocity of ejected electron is _____ $\times 10^5$ ms^{-1}. (Nearest integer)

[Use : h = 6.63×10^{-34} Js, m_e = 9.0×10^{-31} kg]

Answer (5)

Sol. $h\nu = h\nu_0 + \dfrac{1}{2}m_e v^2$

$\dfrac{6.63 \times 10^{-34} \times 3 \times 10^8}{500 \times 10^{-9}}$

$= 6.63 \times 10^{-34} \times 4.3 \times 10^{14} + \dfrac{1}{2} \times 9 \times 10^{-31} \times v^2$

$v \approx 5 \times 10^5$ ms^{-1}

9. For the galvanic cell,

$Zn(s) + Cu^{2+}(0.02\ M) \rightarrow Zn^{2+}(0.04\ M) + Cu(s)$,
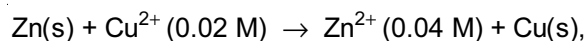
$E_{cell} = $ _____ $\times 10^{-2}$ V. (Nearest integer)

[Use : $E^o_{Cu/Cu^{2+}} = -0.34$ V, $E^o_{Zn/Zn^{2+}} = +0.76$ V,

$\dfrac{2.303\ RT}{F} = 0.059$ V]

Answer (109)

Sol. $Zn(s) + Cu^{2+}(0.02\ M) \rightarrow Zn^{2+}(0.04\ M) + Cu(s)$
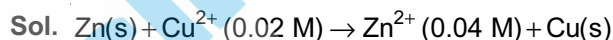
$Q = \dfrac{[Zn^{2+}]}{[Cu^{2+}]} = \dfrac{0.04}{0.02} = 2$

$E_{cell} = E^o_{cell} - \dfrac{0.059}{n}\log Q$

$E_{cell} = 1.1 - \dfrac{0.059}{2}\log 2$

$\approx 109 \times 10^{-2}$ V

10. 83 g of ethylene glycol dissolved in 625 g of water. The freezing point of the solution is _____ K. (Nearest integer)

[Use : Molal freezing point depression constant of water = 1.86 K kg mol^{-1},

Freezing point of water = 273 K,

Atomic masses : C : 12.0 u, O : 16.0 u, H : 1.0 u]

Answer (269)

Sol. $\Delta T_f = (T_0 - T_s) = i \times \text{Molality} \times k_f$

$273 - T_s = 1 \times \dfrac{83 \times 1000}{62 \times 625} \times 1.86$

$T_s = 269$

PART–C : MATHEMATICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The domain of the function $\operatorname{cosec}^{-1}\left(\dfrac{1+x}{x}\right)$ is :

   (1) $\left[-\dfrac{1}{2}, 0\right) \cup [1, \infty)$

   (2) $\left[-\dfrac{1}{2}, \infty\right) - \{0\}$

   (3) $\left(-1, -\dfrac{1}{2}\right] \cup (0, \infty)$

   (4) $\left(-\dfrac{1}{2}, \infty\right) - \{0\}$

Answer (2)

Sol. For domain

$$\frac{1+x}{x} \le -1 \text{ or } \frac{1+x}{x} \ge 1$$

$$\Rightarrow \frac{1+2x}{x} \le 0 \text{ or } \frac{1}{x} \ge 0$$

$$\Rightarrow x \in \left[-\frac{1}{2}, 0\right) \text{ or } x \in (0, \infty)$$

$$\therefore \text{ domian } x \in \left[-\frac{1}{2}, 0\right) \cup (0, \infty)$$

i.e $x \in \left[-\dfrac{1}{2}, \infty\right) - \{0\}$

2. A fair die is tossed until six is obtained on it. Let X be the number of required tosses, then the conditional probability P(X ≥ 5 | X > 2) is :

   (1) $\dfrac{5}{6}$

   (2) $\dfrac{125}{216}$

   (3) $\dfrac{11}{36}$

   (4) $\dfrac{25}{36}$

Answer (4)

Sol. $P(x \ge 5 / x > 2) = \dfrac{P(x \ge 5 \cap x > 2)}{P(x > 2)}$

$$= \frac{P(x \ge 5)}{P(x > 2)}$$

Now,

$$P(x \ge 5) = \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6} + \left(\frac{5}{6}\right)^4 \frac{5}{6}\frac{1}{6} + \left(\frac{5}{6}\right)^6 \frac{1}{6} + \ldots\infty$$

and $P(x > 2) = \dfrac{5}{6}\cdot\dfrac{5}{6}\cdot\dfrac{1}{6} + \left(\dfrac{5}{6}\right)^3 \dfrac{1}{6} + \left(\dfrac{5}{6}\right)^4 \dfrac{1}{6} + \ldots\infty$

$$\therefore \quad P(x \ge 5) = \left(\frac{5}{6}\right)^4 \frac{1}{6}\left(1 + \frac{5}{6} + \left(\frac{5}{6}\right)^2 + \ldots\infty\right)$$

$$= \frac{5^4}{6^5}\cdot 6 = \left(\frac{5}{6}\right)^4$$

and $P(x > 2) = \left(\dfrac{5}{6}\right)^2 \dfrac{1}{6}\cdot 6 = \left(\dfrac{5}{6}\right)^2$

$$\therefore \quad \text{Required probability} = \frac{(5/6)^4}{(5/6)^2} = \frac{25}{36}$$

3. Let [t] denote the greatest integer less than or equal to t. Let f(x) = x – [x], g(x) = 1 – x + [x], and h(x) = min{f(x), g(x)}, x∈ [–2, 2].

   (1) Not continuous at exactly four points in [–2, 2]

   (2) Continuous in [–2, 2] but not differentiable at more than four points in (-2, 2)

   (3) Not continuous at exactly three points in [–2, 2]

   (4) Continues in [–2, 2] but not differentiable at exactly three points in (–2, 2)

Answer (2)

Sol. $f(x) = \{x\}$ and $g(x) = 1 - \{x\}$

$h(x) = \min\{f(x), g(x)\}$
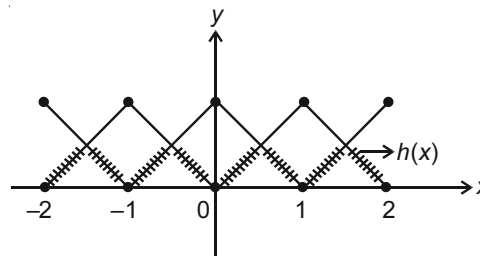
$\therefore$ h(x) is continuous everywhere and non-differentiable at 7 points

4. If the value of the integral $\int_0^5 \dfrac{x+[x]}{e^{x-[x]}}\,dx = \alpha e^{-1}+\beta$, where $\alpha, \beta \in \mathbf{R}$, $5\alpha + 6\beta = 0$, and $[x]$ denotes the greatest integer less than or equal to x; then the value of $(\alpha + \beta)^2$ is equal to :

(1) 25   (2) 16

(3) 36   (4) 100

Answer (1)

Sol. $I = \int_0^5 \dfrac{x+[x]}{e^{x-[x]}}\,dx$

$I = \int_0^1 \dfrac{x+0}{e^{x-0}}\,dx + \int_1^2 \dfrac{x+1}{e^{x-1}}\,dx + \ldots + \int_4^5 \dfrac{x+4}{e^{x-4}}\,dx$

$\therefore \quad I = \sum_{k=0}^4 \int_k^{k+1} \dfrac{x+k}{e^{x-k}}\,dx$

$= \sum_{k=0}^4 e^k \int_k^{k+1} (x+k)e^{-x}\,dx$

$= \sum_{k=0}^4 e^k \left| -(x+k)e^{-x} - e^{-x} \right|_k^{k+1}$

$= \sum_{k=0}^4 e^k \left(-(2k+1)e^{-k+1} - e^{-k+1} + 2ke^{-k} + e^{-k} \right)$

$= \sum_{k=0}^4 \left((-2k-1)e^{-1} - e^{-1} + (2k+1) \right)$

$\Rightarrow \quad -25e^{-1} - 5e^{-1} + 25 = 30e^{-1} + 25$

$\Rightarrow \quad \alpha = -30$ and $\beta = 25$

$\Rightarrow \quad (\alpha + \beta)^2 = 25$

5. $\displaystyle\lim_{x\to 2}\left(\sum_{n=1}^9 \dfrac{x}{n(n+1)x^2 + 2(2n+1)x+4} \right)$ is equal to :

(1) $\dfrac{7}{36}$ (2) $\dfrac{1}{5}$

(3) $\dfrac{5}{24}$ (4) $\dfrac{9}{44}$

Answer (4)

Sol. $\displaystyle\lim_{x\to 2}\left(\sum_{n=1}^9 \dfrac{x}{n(n+1)x^2 + 2(2n+1)x+4} \right)$

$\Rightarrow \quad \sum_{n=1}^9 \dfrac{2}{4n^2 + 12n + 8}$

$\Rightarrow \quad \dfrac{1}{2}\sum_{n=1}^9 \dfrac{1}{n^2 + 3n + 2}$

$\Rightarrow \quad \dfrac{1}{2}\sum_{n=1}^9 \dfrac{(n+2)-(n+1)}{(n+1)(n+2)}$

$\Rightarrow \quad \dfrac{1}{2}\sum_{n=1}^9 \left(\dfrac{1}{n+1} - \dfrac{1}{n+2} \right)$

$\Rightarrow \quad \dfrac{1}{2}\left(\dfrac{1}{2} - \dfrac{1}{3} + \dfrac{1}{3} - \dfrac{1}{4} + \ldots + \dfrac{1}{10} - \dfrac{1}{11} \right)$

$\Rightarrow \quad \dfrac{1}{2}\left(\dfrac{9}{22} \right) = \dfrac{9}{44}$

6. The local maximum value of the function $f(x) = \left(\dfrac{2}{x} \right)^{x^2}$, $x > 0$, is

(1) $\left(2\sqrt{e} \right)^{\frac{1}{e}}$ (2) $\left(\dfrac{4}{\sqrt{e}} \right)^{\frac{e}{4}}$

(3) 1   (4) $(e)^{\frac{2}{e}}$

Answer (4)

Sol. Let $y = \left(\dfrac{2}{x} \right)^{x^2}$ $(x > 0)$

$\ln y = x^2 \ln\left(\dfrac{2}{x} \right) = x^2 (\ln 2 - \ln x)$

Differentiate both sides

$\dfrac{1}{y}\cdot y' = 2x(\ln 2 - \ln x) + x^2\left(\dfrac{-1}{x} \right)$

$= x\left[(2\ln 2 - 1) = 2\ln x \right]$

$y' = \left(\dfrac{2}{x} \right)^{x^2} x\left(\ln\dfrac{4}{e} - 2\ln x \right)$

$y' = 0$

$\Rightarrow \ln\left(\dfrac{4}{e}\right) = 2\ln x = \ln x^2$

$\Rightarrow x^2 = \dfrac{4}{e}$

$\Rightarrow x = \dfrac{2}{\sqrt{e}}$

$\Rightarrow \dfrac{2}{x} = \sqrt{e}$

y is maximum at $x = \dfrac{2}{\sqrt{e}}$ as can be seen from sign change of y' across $x = \dfrac{2}{\sqrt{e}}$.

$y_{max} = y\left(\dfrac{2}{\sqrt{e}}\right) = \left(\sqrt{e}\right)^{\frac{4}{e}} = e^{\frac{1}{2} \times \frac{4}{e}} = (e)^{\frac{2}{e}}$

7. The value of $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\dfrac{1+\sin^2 x}{1+\pi^{\sin x}}\right)dx$ is

(1) $\dfrac{3\pi}{2}$

(2) $\dfrac{\pi}{2}$

(3) $\dfrac{5\pi}{4}$

(4) $\dfrac{3\pi}{4}$

Answer (4)

Sol. $I = \displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\dfrac{1+\sin^2 x}{1+\pi^{\sin x}}\right)dx = \int_0^{\frac{\pi}{2}}\left(\dfrac{1+\sin^2 x}{1+\pi^{\sin x}} + \dfrac{1+\sin^2 x}{1+\pi^{-\sin x}}\right)dx$

$= \displaystyle\int_0^{\frac{\pi}{2}}\left(\dfrac{1+\sin^2 x}{1+\pi^{\sin x}} + \dfrac{\pi^{\sin x}\left(1+\sin^2 x\right)}{1+\pi^{\sin x}}\right)dx$

$= \displaystyle\int_0^{\frac{\pi}{2}}\dfrac{\left(1+\sin^2 x\right)\left(1+\pi^{\sin x}\right)}{\left(1+\pi^{\sin x}\right)}dx$

$= \displaystyle\int_0^{\frac{\pi}{2}}\left(1+\sin^2 x\right)dx = \int_0^{\frac{\pi}{2}}1+\dfrac{1-\cos 2x}{2}dx$

$= \displaystyle\int_0^{\frac{\pi}{2}}\left(\dfrac{3}{2} - \dfrac{1}{2}\cos 2x\right)dx = \dfrac{3}{2}x - \dfrac{1}{4}\sin 2x\Big]_0^{\frac{\pi}{2}}$

$= \dfrac{3}{2}\left(\dfrac{\pi}{2}\right) - \dfrac{1}{4}(0)$

$= \dfrac{3\pi}{4}$

8. A 10 inches long pencil AB with mid point C and a small eraser P are placed on the horizontal top of a table such that PC = $\sqrt{5}$ inches and $\angle$PCB = $\tan^{-1}$ (2).

The acute angle through which the pencil must be rotated about C so that the perpendicular distance between eraser and pencil becomes exactly 1 inch is
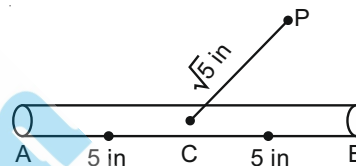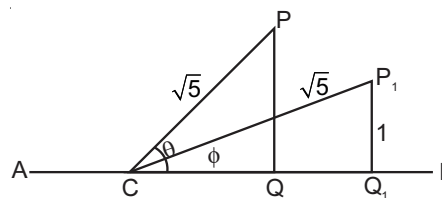

(1) $\tan^{-1}\left(\dfrac{1}{2}\right)$

(2) $\tan^{-1}\left(\dfrac{3}{4}\right)$

(3) $\tan^{-1}(1)$

(4) $\tan^{-1}\left(\dfrac{4}{3}\right)$

Answer (2)

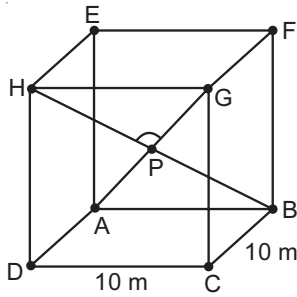**Sol.**

$\theta = \tan^{-1}\sqrt{2}$

$CQ_1 = 2$

$\tan\phi = \dfrac{1}{2}$

Required angle $= \theta - \phi = \tan^{-1}2 - \tan^{-1}\dfrac{1}{2}$

$= \tan^{-1}\left(\dfrac{2-\dfrac{1}{2}}{1+2\times\dfrac{1}{2}}\right) = \tan^{-1}\dfrac{3}{4}$
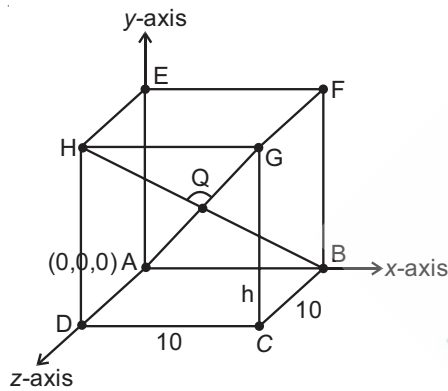
9. A hall has a square floor of dimension 10 m × 10 m (see the figure) and vertical walls. If the angle GPH between the diagonals AG and BH is $\cos^{-1}\dfrac{1}{5}$, then the height of the hall (in meters) is

(1) $2\sqrt{10}$

(2) $5\sqrt{2}$

(3) $5\sqrt{3}$

(4) 5

Answer (2)

Sol.

Let height be h.

$A \equiv (0, 0, 0)$

$G \equiv (10, h, 10)$

$B \equiv (10, 0, 0)$

$H \equiv (0, h, 10)$

DRs of AG $\equiv (10, h, 10)$

DRs of BH $\equiv (10, -h, -10)$

$\cos\theta = \left|\dfrac{10\times 10 + h(-h) + 10(-10)}{\sqrt{10^2 + h^2 + 10^2}\times\sqrt{10^2 + h^2 + 10^2}}\right|$

$= \dfrac{h^2}{200 + h^2} = \dfrac{1}{5}$

$\Rightarrow h = 5\sqrt{2}$

10. The value of

$2\sin\left(\dfrac{\pi}{8}\right)\sin\left(\dfrac{2\pi}{8}\right)\sin\left(\dfrac{3\pi}{8}\right)\sin\left(\dfrac{5\pi}{8}\right)\sin\left(\dfrac{6\pi}{8}\right)\sin\left(\dfrac{7\pi}{8}\right)$

is

(1) $\dfrac{1}{8\sqrt{2}}$

(2) $\dfrac{1}{4\sqrt{2}}$

(3) $\dfrac{1}{8}$

(4) $\dfrac{1}{4}$

Answer (3)

Sol. $\sin\dfrac{5\pi}{8} = \sin\left(\pi - \dfrac{3\pi}{8}\right) = \sin\left(\dfrac{3\pi}{8}\right)$

$\sin\left(\dfrac{7\pi}{8}\right) = \sin\left(\pi - \dfrac{\pi}{8}\right) = \sin\dfrac{\pi}{8}$

$2\sin\left(\dfrac{\pi}{8}\right)\sin\left(\dfrac{2\pi}{8}\right)\sin\left(\dfrac{3\pi}{8}\right)\sin\left(\dfrac{5\pi}{8}\right)\sin\left(\dfrac{6\pi}{8}\right)\sin\left(\dfrac{7\pi}{8}\right)$

$= 2\sin^2\dfrac{\pi}{8}\times\sin^2\left(\dfrac{3\pi}{8}\right)\times\left(\dfrac{1}{\sqrt{2}}\right)^2$

$= \left(\sin\dfrac{\pi}{8}\cdot\cos\dfrac{\pi}{8}\right)^2$

$\left[\because \sin\dfrac{3\pi}{8} = \sin\left(\dfrac{\pi}{2} - \dfrac{\pi}{8}\right) = \cos\dfrac{\pi}{8}\right]$

$= \left(\dfrac{1}{2}\sin\dfrac{\pi}{4}\right)^2 = \left(\dfrac{1}{2\sqrt{2}}\right)^2 = \dfrac{1}{8}$

11. Let $y(x)$ be the solution of the differential equation $2x^2dy + (e^y - 2x)dx = 0$, $x > 0$. If $y(e) = 1$, then $y(1)$ is equal to

(1) 2

(2) 0

(3) $\log_e(2e)$

(4) $\log_e 2$

Answer (4)

Sol. $\dfrac{dy}{dx} = \dfrac{2x - e^y}{2x^2}$

$\Rightarrow -e^{-y}\dfrac{dy}{dx} + \dfrac{e^{-y}}{x} = \dfrac{1}{2x^2}$ \quad Let $e^{-y} = \gamma$

$\Rightarrow \dfrac{d\gamma}{dx} + \dfrac{\gamma}{x} = \dfrac{1}{2x^2}$

$I\cdot f\cdot = e^{\int \frac{1}{x}dx} = x$

$\Rightarrow \gamma\cdot x = \int\dfrac{1}{2x}dx + C$

$\Rightarrow \quad xe^{-y} = \dfrac{1}{2}\ln x + C$

$\because \quad y(e) = 1 \Rightarrow e \cdot e^{-1} = \dfrac{1}{2} + C \Rightarrow C = \dfrac{1}{2}$

Now put $x = 1$, $e^{-y} = \dfrac{1}{2} \Rightarrow y = \ln 2$

12. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$. Then $A^{2025} - A^{2020}$ is equal to

(1) $A^6 - A$ (2) A^5

(3) $A^5 - A$ (4) A^6

Answer (1)

Sol. $|A - \lambda I| = \begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 1 \\ 1 & 0 & -\lambda \end{vmatrix} = 0$

$A^3 - 2A^2 + A = 0$

$\Rightarrow A^2 - A = A^3 - A^2 = A^4 - A^3 = A^5 - A^4 = A^6 - A^5$

$= A^7 - A^6$

So $A^7 - A^2 = A^6 - A$

$\Rightarrow A^8 - A^3 = A^7 - A^2 = A^6 - A$

And So on.

then $A^{2025} - A^{2020} = A^6 - A$

13. Two fair dice are thrown. The numbers on them are taken as λ and μ, and a system of linear equations

$x + y + z = 5$

$x + 2y + 3z = \mu$

$x + 3y + \lambda z = 1$

is constructed. If p is the probability that the system has a unique solution and q is the probability that the system has no solution, then

(1) $p = \dfrac{1}{6}$ and $q = \dfrac{1}{36}$ (2) $p = \dfrac{5}{6}$ and $q = \dfrac{5}{36}$

(3) $p = \dfrac{1}{6}$ and $q = \dfrac{5}{36}$ (4) $p = \dfrac{5}{6}$ and $q = \dfrac{1}{36}$

Answer (2)

Sol. $\begin{vmatrix} 1 & 3 & \lambda \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = 0 \Rightarrow 1 - 3(2) + \lambda(1) = 0 \Rightarrow \lambda = 5$

For $\lambda \neq 5$ there will be unique solution

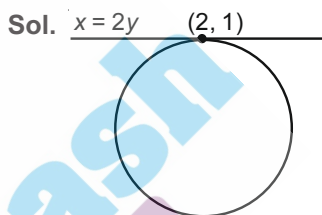$p = 1 - \dfrac{1}{6} = \dfrac{5}{6}$

For $\lambda = 5$ and $\mu = 3$ there will be infinitely many solutions and for $\lambda = 5$ and $\mu \neq 3$ there will be no solution.

$q = \dfrac{1}{6} \cdot \left(1 - \dfrac{1}{6}\right) = \dfrac{5}{36}$

14. A circle C touches the line $x = 2y$ at the point $(2, 1)$ and intersects the circle $C_1 : x^2 + y^2 + 2y - 5 = 0$ at two points P and Q such that PQ is a diameter of C_1. Then the diameter of C is

(1) $\sqrt{285}$ (2) 15

(3) $4\sqrt{15}$ (4) $7\sqrt{5}$

Answer (4)

Sol.

Equation of C,

$(x - 2)^2 + (y - 1)^2 + \lambda(x - 2y) = 0$

$C_1: x^2 + y^2 + 2y - 5 = 0$ has centre $(0, -1)$

PQ: $C - C_1 = 0$

$\Rightarrow$ PQ : $x(\lambda - 4) + y(-2\lambda - 4) + 10 = 0$

$\because$ $(0, -1)$ lies on PQ, then $\lambda = -7$

Diameter of $C = 2\sqrt{\dfrac{11^2 + 12^2}{4} - 5} = \sqrt{245} = 7\sqrt{5}$

15. The point $P\left(-2\sqrt{6}, \sqrt{3}\right)$ lies on the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ having eccentricity $\dfrac{\sqrt{5}}{2}$. If the tangent and normal at P to the hyperbola intersect its conjugate axis at the points Q and R respectively, then QR is equal to

(1) $3\sqrt{6}$ (2) 6

(3) $6\sqrt{3}$ (4) $4\sqrt{3}$

Answer (3)

Sol. Let P $(a\sec\theta, b\tan\theta)$

$$T: \frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1 \implies Q\left(0, -\frac{b}{\tan\theta}\right)$$

$$N: \frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a^2 + b^2 \implies R\left(0, \frac{a^2+b^2}{b}\tan\theta\right)$$

$$QR = \left|\left(\frac{a^2+b^2}{b}\right)\tan\theta + \frac{b}{\tan\theta}\right|$$

$$\because \quad \frac{b^2}{a^2} = e^2 - 1 = \frac{1}{4} \implies 4b^2 = a^2, \text{ so } b^2 = 3 \text{ and } a^2$$
$$= 12$$

$$\sec\theta = -\sqrt{2} \text{ and } \tan\theta = 1$$

Now, $QR = \left|\left(\frac{12+3}{\sqrt{3}}\right)1 + \frac{\sqrt{3}}{1}\right| = 6\sqrt{3}$

16. The locus of the mid points of the chords of the hyperbola $x^2 - y^2 = 4$, which touch the parabola $y^2 = 8x$, is

(1) $y^2(x - 2) = x^3$ (2) $y^3(x - 2) = x^2$

(3) $x^3(x - 2) = y^2$ (4) $x^2(x - 2) = y^3$

Answer (1)

Sol. Let mid point of chord of hyperbola $x^2 - y^2 = 4$ be (x_1, y_1)

$\therefore$ Equation of chord is :

$$xx_1 - yy_1 - 4 = x_1^2 - y_1^2 - 4$$

$$\therefore \quad yy_1 = xx_1 - x_1^2 + y_1^2$$

$$\therefore \quad y = \frac{x_1}{y_1}x + \frac{y_1^2 - x_1^2}{y_1} \quad \ldots(i)$$

$\because$ Equation (i) is tangent to parabola $y^2 = 8x$ then

$$\frac{y_1^2 - x_1^2}{y_1} = \frac{2}{\dfrac{x_1}{y_1}}$$

$$\therefore \quad (y_1^2 - x_1^2)x_1 = 2y_1^2$$

$$\therefore \quad y_1^2(x_1 - 2) = x_1^3$$

$\therefore$ Required locus is : $y^2(x - 2) = x^3$

17. If $(\sqrt{3} + i)^{100} = 2^{99}(p + iq)$, then p and q are roots of the equation

(1) $x^2 - (\sqrt{3} - 1)x - \sqrt{3} = 0$

(2) $x^2 + (\sqrt{3} - 1)x - \sqrt{3} = 0$

(3) $x^2 - (\sqrt{3} + 1)x + \sqrt{3} = 0$

(4) $x^2 + (\sqrt{3} + 1)x + \sqrt{3} = 0$

Answer (1)

Sol. $\because (\sqrt{3} + i)^{100} = 2^{99}(p + iq)$

$$\left(2e^{i\frac{\pi}{6}}\right)^{100} = 2^{99}(p + iq)$$

$$2e^{i\frac{50\pi}{3}} = p + iq$$

$$\implies 2e^{i\left(16\pi + \frac{2\pi}{3}\right)} = p + iq$$

$$= 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = p + iq$$

$$\therefore \quad p = -1, \quad q = \sqrt{3}$$

Equation with roots -1 and $\sqrt{3}$ is

$$x^2 - (\sqrt{3} - 1)x - \sqrt{3} = 0$$

18. If $\sum_{r=1}^{50}\tan^{-1}\frac{1}{2r^2} = p$, then the value of tan p is

(1) 100   (2) $\frac{50}{51}$

(3) $\frac{101}{102}$ (4) $\frac{51}{50}$

Answer (2)

Sol. $\because \tan^{-1}\frac{1}{2r^2} = \tan^{-1}\left(\frac{2}{1 + (4r^2 - 1)}\right)$

$$= \tan^{-1}\left(\frac{(2r + 1) - (2r - 1)}{1 + (2r + 1)(2r - 1)}\right)$$

$$= \tan^{-1}(2r + 1) - \tan^{-1}(2r - 1)$$

$$\therefore \quad \sum_{r=1}^{50} \tan^{-1}\left(\frac{1}{2r^2}\right) = \sum_{r=1}^{50}\left(\tan^{-1}(2r+1) - \tan^{-1}(2r-1)\right)$$

$$\therefore \quad p = \tan^{-1}(101) - \tan^{-1}1$$

$$= \tan^{-1}\left(\frac{101-1}{1+101\cdot 1}\right)$$

$$\therefore \quad \tan p = \frac{50}{51}$$

19. Let P be the plane passing through the point (1, 2, 3) and the line of intersection of the planes

$$\vec{r}\cdot\left(\hat{i}+\hat{j}+4\hat{k}\right) = 16 \text{ and } \vec{r}\cdot\left(-\hat{i}+\hat{j}+\hat{k}\right)=6.$$

Then which of the following points does NOT lie on P?

(1) (–8, 8, 6)

(2) (4, 2, 2)

(3) (3, 3, 2)

(4) (6, – 6, 2)

Answer (2)

Sol. Equation plane through point of intersection of planes

$$\vec{r}\cdot\left(\hat{i}+\hat{j}+4\hat{k}\right) = 16 \text{ and } \vec{r}\cdot(-\hat{i}+\hat{j}+\hat{k})=6 \text{ is}$$

$(x + y + 4z - 16) + \lambda(-x + y + z - 6) = 0$ ...(i)

This plane passes through point (1, 2, 3) then

$-1 - 2\lambda = 0$

$$\therefore \quad \lambda = -\frac{1}{2}$$

$\therefore$ Equation of plane is :

$2x + 2y + 8z - 32 + x - y - z + 6 = 0$

$\therefore \quad 3x + y + 7z - 26 = 0$

Clearly (4, 2, 2) does not lies on the plane.

20. Consider the two statements :

(S1) : $(p \to q) \vee (\sim q \to p)$ is a tautology.

(S2) : $(p \wedge \sim q) \wedge (\sim p \vee q)$ is a fallacy.

(1) Only (S2) is true

(2) Only (S1) is true

(3) Both (S1) and (S2) are true

(4) Both (S1) and (S2) are false

Answer (3)

Sol. For S1 and S2

p	q	~p	~q	p→q	~q→p	$(p \to q) \vee \binom{\sim q}{\to p}$	p∧~q	(~p∨q)	(p∧~q)∧ (~p∨q)
T	T	F	F	T	T	T	F	T	F
T	F	F	T	F	T	T	T	F	F
F	T	T	F	T	T	T	F	T	F
F	F	T	T	T	F	T	F	T	F
						tautology S1			fallacy S2

$\therefore$ Hence both S1 and S2 are true

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

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1. If the projection of the vector $\hat{i}+2\hat{j}+\hat{k}$ on the sum of the two vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $-\lambda\hat{i}+2\hat{j}+3\hat{k}$ is 1, then λ is equal to _____.

Answer (5)

Sol. $\bar{v}_1 = \hat{i}+2\hat{j}+\hat{k}, \ \bar{v}_2 = 2\hat{i}+4\hat{j}-5\hat{k}, \ \bar{v}_3 = -\lambda\hat{i}+2\hat{j}+3\hat{k}$

$\bar{v}_2 + \bar{v}_3 = (2-\lambda)\hat{i}+6\hat{j}-2\hat{k} = \bar{v}_4$

Projection of $\bar{v}_1$ on $\bar{v}_4 = \bar{v}_1 \cdot \dfrac{\bar{v}_4}{|\bar{v}_4|}$

$\Rightarrow \dfrac{1\times(2-\lambda)+2\times 6+1\times(-2)}{\sqrt{(2-\lambda)^2+6^2+(-2)^2}} = 1$

$\Rightarrow (12-\lambda)^2 = (2-\lambda)^2 + 40$

On solving

$\lambda = 5$

2. The sum of all 3-digit numbers less than or equal to 500, that are formed without using the digit "1" and they all are multiple of 11, is _____.

Answer (7744)

Sol. The required numbers are 209, 220, 231, ..., 495

This A.P. contains 27 terms

Sum of this A.P. $= \dfrac{27}{2}[209 + 495] = 9504$

Only 231, 319, 341, 418 and 451 are not allowed in this sum.

So required sum = 9504 – 1760 = 7744

3. Let a and b respectively be the points of local maximum and local minimum of the function

$$f(x) = 2x^3 - 3x^2 - 12x.$$

If A is the total area of the region bounded by $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$, then 4 A is equal to _____.

Answer (114)

Sol. $f(x) = 2x^3 - 3x^2 - 12x$

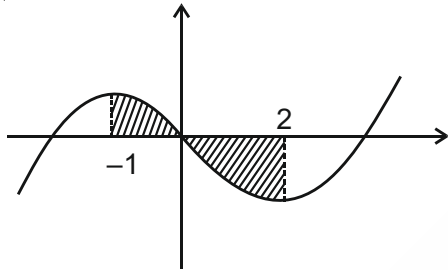$\Rightarrow f'(x) = 6x^2 - 6x - 12$

$\quad = 6(x - 2)(x + 1)$

$\Rightarrow f'(x) = 0 \quad \Rightarrow x = -1$ & $x = 2$

$x = -1$ is point of local maximum $\Rightarrow a = -1$

$x = 2$ is point of local minimum $\Rightarrow b = 2$

$f(-1) = 8$ and $f(2) = -20$

Required area is as shown in figure

Required area $= \int_{-1}^{0} (f(x) - 0)dx + \int_{0}^{2} (0 - f(x))dx$

$= \left(\frac{1}{2}x^4 - x^3 - 6x^2 \right)_{-1}^{0} - \left(\frac{x^4}{2} - x^3 - 6x^2 \right)_{0}^{2} = \frac{57}{2} = A$

$\Rightarrow 4A = 114$

4. Let A be a 3 × 3 real matrix. If det(2 Adj(2 Adj(Adj (2A)))) = 2^{41}, then the value of det (A²) equals _____.

Answer (4)

Sol. det(2 Adj(2 Adj(Adj·2A))) = 2^{41}

$\Rightarrow$ det(2 Adj(2 Adj(2²·Adj A))) = 2^{41}

$\Rightarrow$ det·(2 Adj(2⁵ Adj(Adj A))) = 2^{41}

$\Rightarrow$ det·(2¹¹ Adj(Adj(Adj A))) = 2^{41}

$\Rightarrow$ 2³³·det(Adj(Adj(Adj A))) = 2^{41}

$\Rightarrow |A|^8 = 2^8 \quad \Rightarrow |A| = 2$

$\Rightarrow |A|^2 = 4$

5. Let $\binom{n}{k}$ denote ^nC_k and

$$\begin{bmatrix} n \\ k \end{bmatrix} = \begin{cases} \binom{n}{k} & , \text{ if } 0 \le k \le n \\ 0 & , \text{ otherwise.} \end{cases}$$

If $A_k = \sum_{i=0}^{9} \binom{9}{i} \begin{bmatrix} 12 \\ 12 - k + i \end{bmatrix} + \sum_{i=0}^{8} \binom{8}{i} \begin{bmatrix} 13 \\ 13 - k + i \end{bmatrix}$ and $A_4 - A_3 = 190$ p, then p is equal to _____.

Answer (49)

Sol. $A_k = \sum_{k=0}^{9} {}^9C_i \, {}^{12}C_{12-k+i} + \sum_{k=0}^{8} {}^8C_i \, {}^{13}C_{13-k+i}$

$A_k = \sum_{k=0}^{9} {}^9C_i \cdot {}^{12}C_{k-i} + \sum_{i=0}^{8} {}^8C_i \cdot {}^{13}C_{k-i}$

A_k = Coeff of x^k in $(1 + x)^9 \cdot (1 + x)^{12} + (1 + x)^8 \cdot (1 + x)^{13}$

$A_k = 2 \cdot {}^{21}C_k$

$A_4 - A_3 = 2 \left[{}^{21}C_4 - {}^{21}C_3 \right]$

$= 2 \left[\frac{21 \times 20 \times 19 \times 18}{24} - \frac{21 \times 20 \times 19}{6} \right]$

$= 2 \times 21 \times 20 \times 19 \left[\frac{18}{24} - \frac{1}{6} \right] = 190 \times 49$

p = 49

6. Let $\lambda \ne 0$ be in **R**. If α and β are the roots of the equation $x^2 - x + 2\lambda = 0$, and α and γ are the roots of the equation $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to _____.

Answer (18)

Sol. $x^2 - x + 2\lambda = 0 \begin{cases} \alpha \\ \beta \end{cases} \Rightarrow \alpha \cdot \beta = 2\lambda$

$3x^2 - 10x + 27\lambda = 0 \begin{cases} \alpha \\ \gamma \end{cases} \Rightarrow \alpha \cdot \gamma = \frac{27}{3} = 9\lambda$

Both equations have a common root α.

$\frac{\alpha^2}{-27\lambda + 20\lambda} = \frac{\alpha}{6\lambda - 27\lambda} = \frac{1}{-10 + 3}$

$\frac{\alpha^2}{-7\lambda} = \frac{\alpha}{-19\lambda} = \frac{1}{-7}$

$\alpha^2 = \lambda$

Now, $(\alpha\beta) \cdot (\alpha \cdot \gamma) = (2\lambda)(9\lambda)$

$\frac{\beta \cdot \gamma}{\lambda} = 2 \times 9 \cdot \frac{\lambda}{\alpha^2} = 18$

7. Let the mean and variance of four numbers 3, 7, x and y (x > y) be 5 and 10 respectively. Then the mean of four numbers 3 + 2x, 7 + 2y, x + y and x − y is _____.

Answer (12)

Sol. Numbers 3, 7, x, y

$\bar{x} = 5$, $\sigma^2 = 10$

$5 = \dfrac{3 + 7 + x + y}{4} \Rightarrow x + y = 10$ ...(i)

$10 = \dfrac{1}{4}((3)^2 + (7)^2 + (x)^2 + (y)^2) - (5)^2$

$140 = 58 + x^2 + y^2 \Rightarrow x^2 + y^2 = 82$ ...(ii)

$(x + y)^2 = x^2 + y^2 + 2xy \Rightarrow 100 = 82 + 2xy$

$xy = 9$

$y = \dfrac{9}{x} \Rightarrow x + \dfrac{9}{x} = 10 \Rightarrow \begin{aligned} x &= 1 \text{ or } 9 \\ y &= 9 \text{ or } 1 \end{aligned}$

Given $x > y \Rightarrow x = 9$, $y = 1$

Now, $3 + 2x$, $7 + 2y$, $x + y$, $x - y = 21, 9, 10, 8$

$\bar{x} = \dfrac{21 + 9 + 10 + 8}{4} = \dfrac{48}{4} = 12$

8. Let Q be the foot of the perpendicular from the point P(7, –2, 13) on the plane containing the lines

$\dfrac{x + 1}{6} = \dfrac{y - 1}{7} = \dfrac{z - 3}{8}$ and $\dfrac{x - 1}{3} = \dfrac{y - 2}{5} = \dfrac{z - 3}{7}$.

Then $(PQ)^2$, is equal to _____.

Answer (96)

Sol. Normal vector for plane $\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 7 & 8 \\ 3 & 5 & 7 \end{vmatrix}$

$= 9\hat{i} - 18\hat{j} + 9\hat{k} = 9(\hat{i} - 2\hat{j} + \hat{k})$

$\Rightarrow$ Normal is parallel to $\hat{i} - 2\hat{j} + \hat{k}$

Plane passes through (1, 2, 3) as it is a point on L_2 so equation of plane

$1(x - 1) - 2(y - 2) + 1(z - 3) = 0$

$x - 2y + z = 0$

$PQ = \dfrac{7 - 2(-2) + 13}{\sqrt{6}} \Rightarrow PQ^2 = 96$

9. The least positive integers n such that

$\dfrac{(2i)^n}{(1 - i)^{n-2}}$, $i = \sqrt{-1}$, is a positive integer, is

_____.

Answer (16)

Sol. $\dfrac{(2i)^n}{(1 - i)^{n-2}} = \dfrac{\left(2e^{i\pi/2}\right)^n}{\left(\sqrt{2}e^{-i\pi/4}\right)^{n-2}}$

$= \left(\sqrt{2}\right)^{n+2} e^{i(3n-2)\frac{\pi}{4}}$

For positive integer n should be atleast 6

$= \left(\sqrt{2}\right)^8 e^{i \cdot 4\pi} = \left(\sqrt{2}\right)^8 = 16$

10. Let a_1, a_2, ..., a_{10} be an AP with common difference –3 and b_1, b_2, ..., b_{10} be a GP with common ratio 2. Let $c_k = a_k + b_k$, $k = 1, 2, ..., 10$. If $c_2 = 12$ and $c_3 = 13$, then $\displaystyle\sum_{k=1}^{10} c_k$ is equal to _____.

Answer (2021)

Sol. $c_2 = a_2 + b_2 = (a_1 - 3) + 2b_1 = 12 \Rightarrow a_1 = 11$

$c_3 = a_3 + b_3 = (a_1 - 6) + 4b_1 = 13 \Rightarrow b_1 = 2$

$c_k = a_k + b_k = (a_1 - 3(k - 1)) + (b_1 \cdot 2^{k-1})$

$= (11 - 3k + 3) + (2^k) = 14 - 3k + 2^k$

$\displaystyle\sum_{k=1}^{10} c_k = \sum_{k=1}^{10}(2^k - 3k + 14)$

$= \displaystyle\sum_{k=1}^{10} 2^k - 3\sum_{k=1}^{10} k + \sum_{k=1}^{10} 14$

$= 2(2^{10} - 1) - 3 \cdot \dfrac{10 \cdot 11}{2} + 140$

$= 2021$

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