24/06/2022
Evening

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Answers & Solutions

Time : 3 hrs. **for** M.M. : 300

JEE (Main)-2022 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Identify the pair of physical quantities that have same dimensions :

   (A) velocity gradient and decay constant

   (B) wien's constant and Stefan constant

   (C) angular frequency and angular momentum

   (D) wave number and Avogadro number

Answer (A)

Sol. Velocity gradient $= \dfrac{dv}{dx}$

$\Rightarrow$ Dimensions are $\dfrac{[LT^{-1}]}{[L]} = [T^{-1}]$

Decay constant λ has dimensions of $[T^{-1}]$ because of the relation $\dfrac{dN}{dt} = -\lambda N$

$\Rightarrow$ Velocity gradient and decay constant have same dimensions.

2. The distance between Sun and Earth is R. The duration of year if the distance between Sun and Earth becomes $3R$ will be :

   (A) $\sqrt{3}$ years (B) 3 years

   (C) 9 years (D) $3\sqrt{3}$ years

Answer (D)

Sol. We know that

$T^2 \propto R^3$

$\Rightarrow \left(\dfrac{T'}{T}\right)^2 = \left(\dfrac{3R}{R}\right)^3$

$\Rightarrow \dfrac{T'}{T} = 3\sqrt{3}$

$\Rightarrow T' = 3\sqrt{3}$ years

3. A stone of mass m tied to a string is being whirled in a vertical circle with a uniform speed. The tension in the string is

   (A) the same throughout the motion.

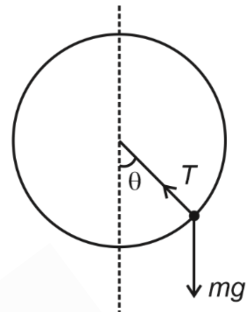
   (B) minimum at the highest position of the circular path.

   (C) minimum at the lowest position of the circular path.

   (D) minimum when the rope is in the horizontal position.

Answer (B)

Sol.



At any θ : $T - mg\cos\theta = \dfrac{mv^2}{R}$

$\Rightarrow T = mg\cos\theta + \dfrac{mv^2}{R}$

Since v is constant,

$\Rightarrow$ T will be minimum when $\cos\theta$ is minimum.

$\Rightarrow$ $\theta = 180°$ corresponds to $T_{minimum}$.

4. Two identical charged particles each having a mass 10 g and charge 2.0×10^{-7} C are placed on a horizontal table with a separation of L between them such that they stay in limited equilibrium. If the coefficient of friction between each particle and the table is 0.25, find the value of L. [Use g = 10 ms^{-2}]

   (A) 12 cm (B) 10 cm

   (C) 8 cm (D) 5 cm

Answer (A)

Sol. According to given information :

$\dfrac{kQ^2}{L^2} = \mu mg$

Putting the values, we get

L = 12 cm

5. A Carnot engine takes 5000 kcal of heat from a reservoir at 727°C and gives heat to a sink at 127°C. The work done by the engine is

   (A) 3×10^6 J (B) Zero

   (C) 12.6×10^6 J (D) 8.4×10^6 J

Answer (C)

Sol. Efficiency $\eta = 1 - \frac{T_L}{T_H}$

$= 1 - \frac{400}{1000}$

= 0.6

$\Rightarrow \quad 0.6 = \frac{W}{Q}$

$\Rightarrow$ $W = 0.6Q$ = 3000 kcal = 12.6×10^6 J

6. Two massless springs with spring constant 2 k and 9 k, carry 50 g and 100 g masses at their free ends. These two masses oscillate vertically such that their maximum velocities are equal. Then, the ratio of their respective amplitude will be

(A) 1 : 2 (B) 3 : 2

(C) 3 : 1 (D) 2 : 3

Answer (B)

Sol. $\omega_1 A_1 = \omega_2 A_2$

$\Rightarrow \frac{A_1}{A_2} = \frac{\omega_2}{\omega_1}$

$= \sqrt{\frac{k_2}{m_2}} \times \sqrt{\frac{m_1}{k_1}} = \sqrt{\frac{9k}{100} \times \frac{50}{2k}} = \frac{3}{2}$

7. What will be the most suitable combination of three resistors A = 2 Ω, B = 4 Ω, C = 6 Ω so that $\left(\frac{22}{3}\right)$ Ωis equivalent resistance of combination?

(A) Parallel combination of A and C connected in series with B.

(B) Parallel combination of A and B connected in series with C

(C) Series combination of A and C connected in parallel with B.

(D) Series combination of B and C connected in parallel with A.

Answer (B)

Sol. $R_{eq} = \frac{2 \times 4}{2+6} + 6 = \frac{22}{3}$

$\Rightarrow$ A and B are in parallel and C is in series.

8. The soft-iron is a suitable material for making an electromagnet. This is because soft-iron has

(A) Low coercivity and high retentivity

(B) Low coercivity and low permeability

(C) High permeability and low retentivity

(D) High permeability and high retentivity

Answer (C)

Sol. Theoretical.

Electromagnet requires high permeability and low retentivity.

9. A proton, a deuteron and an α-particle with same kinetic energy enter into a uniform magnetic field at right angle to magnetic field. The ratio of the radii of their respective circular paths is :

(A) $1 : \sqrt{2} : \sqrt{2}$

(B) $1 : 1 : \sqrt{2}$

(C) $\sqrt{2} : 1 : 1$

(D) $1 : \sqrt{2} : 1$

Answer (D)

Sol. $\therefore \quad r = \frac{mv}{qB} = \frac{\sqrt{2m(KE)}}{qB}$

$\Rightarrow \quad r_1 : r_2 : r_3 = \frac{\sqrt{m_1}}{q_1} : \frac{\sqrt{m_2}}{q_2} : \frac{\sqrt{m_3}}{q_3}$

$= \frac{\sqrt{1}}{1} : \frac{\sqrt{2}}{1} : \frac{\sqrt{4}}{2}$

$= 1 : \sqrt{2} : 1$

10. Given below are two statements:

Statement-I : The reactance of an ac circuit is zero. It is possible that the circuit contains a capacitor and an inductor.

Statement-II : In ac circuit, the average power delivered by the source never becomes zero.

In the light of the above statements, choose the correct answer from the options given below.

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Answer (C)

Sol. $X = |X_C - X_L|$

So, it can be zero if $X_C = X_L$

And, average power in ac circuit can be zero.

11. Potential energy as a function of r is given by $U=\frac{A}{r^{10}}-\frac{B}{r^5}$, where r is the interatomic distance, A and B are positive constants. The equilibrium distance between the two atoms will be:

(A) $\left(\frac{A}{B}\right)^{\frac{1}{5}}$ (B) $\left(\frac{B}{A}\right)^{\frac{1}{5}}$

(C) $\left(\frac{2A}{B}\right)^{\frac{1}{5}}$ (D) $\left(\frac{B}{2A}\right)^{\frac{1}{5}}$

Answer (C)

Sol. For equilibrium

$$-\frac{dU}{dr}=0=\frac{10A}{r^{11}}-\frac{5B}{r^6}$$

$$\Rightarrow r^5=\frac{2A}{B}$$

And $r=\left(\frac{2A}{B}\right)^{1/5}$

12. An object of mass 5 kg is thrown vertically upwards from the ground. The air resistance produces a constant retarding force of 10 N throughout the motion. The ratio of time of ascent to the time of descent will be equal to [Use g = 10 ms^{-2}].

(A) 1 : 1 (B) $\sqrt{2}:\sqrt{3}$

(C) $\sqrt{3}:\sqrt{2}$ (D) 2 : 3

Answer (B)

Sol. Let time taken to ascent is t_1 and that to descent is t_2. Height will be same so

$$H=\frac{1}{2}\times 12t_1^2=\frac{1}{2}\times 8t_2^2$$

$$\Rightarrow \frac{t_1}{t_2}=\frac{\sqrt{2}}{\sqrt{3}}$$

13. A fly wheel is accelerated uniformly from rest and rotates through 5 rad in the first second. The angle rotated by the fly wheel in the next second, will be:

(A) 7.5 rad (B) 15 rad

(C) 20 rad (D) 30 rad

Answer (B)

Sol. $\theta_1=\frac{1}{2}\alpha(2\times 1-1)=5$ rad

$\Rightarrow \alpha$ = 10 rad/sec^2

So $\theta_2=\frac{1}{2}\times\alpha\,(2\times 2-1)=15$ rad

14. A 100 g of iron nail is hit by a 1.5 kg hammer striking at a velocity of 60 ms^{-1}. What will be the rise in the temperature of the nail if one fourth of energy of the hammer goes into heating the nail?

[Specific heat capacity of iron = 0.42 $Jg^{-1}\,°C^{-1}$]

(A) 675°C (B) 1600°C

(C) 16.07°C (D) 6.75°C

Answer (C)

Sol. $\frac{1}{2}\times 1.5\times 60^2\times\frac{1}{4}=100\times 0.42\times\Delta T$

$$\Delta T=\frac{1.5\times 60^2}{8\times 100\times 0.42}=16.07°C$$

15. If the charge on a capacitor is increased by 2 C, the energy stored in it increases by 44%. The original charge on the capacitor is (in C)

(A) 10 (B) 20

(C) 30 (D) 40

Answer (A)

Sol. Let initially the charge is q so

$$\frac{1}{2}\frac{q^2}{C}=U_i$$

And $\frac{1}{2}\frac{(q+2)^2}{C}=U_f$

Given $\frac{U_f-U_i}{U_i}\times 100=44$

$$\frac{(q+2)^2-q^2}{q}=.44$$

$\Rightarrow q$ = 10C

16. A long cylindrical volume contains a uniformly distributed charge of density ρ. The radius of cylindrical volume is R. A charge particle (q) revolves around the cylinder in a circular path. The kinetic energy of the particle is:

(A) $\frac{\rho qR^2}{4\varepsilon_0}$ (B) $\frac{\rho qR^2}{2\varepsilon_0}$

(C) $\frac{q\rho}{4\varepsilon_0 R^2}$ (D) $\frac{4\varepsilon_0 R^2}{q\rho}$

Answer (A)

Sol. $\frac{mv^2}{r}=\frac{2k\rho\times\pi R^2 q}{r}$

$$\Rightarrow \frac{1}{2}mv^2=\frac{\rho R^2 q}{4\varepsilon_0}$$

17. An electric bulb is rated as 200 W. What will be the peak magnetic field at 4 m distance produced by the radiations coming from this bulb? Consider this bulb as a point source with 3.5% efficiency.

(A) 1.19×10^{-8}T (B) 1.71×10^{-8}T

(C) 0.84×10^{-8}T (D) 3.36×10^{-8}T

Answer (B)

Sol. $200 \times \frac{1}{4\pi \times 16} \times \frac{3.5}{100} = \frac{B_0^2}{2\mu_0} C$

$\mu_0 = 4\pi \times 10^{-7}$ Tm/A

$C = 3 \times 10^8$ m/sec

$\Rightarrow B_0 = 1.71 \times 10^{-8}$ T

18. The light of two different frequencies whose photons have energies 3.8 eV and 1.4 eV respectively, illuminate a metallic surface whose work function is 0.6 eV successively. The ratio of maximum speeds of emitted electrons for the two frequencies respectively will be

(A) 1 : 1 (B) 2 : 1

(C) 4 : 1 (D) 1 : 4

Answer (B)

Sol. $3.8 = 0.6 + \frac{1}{2} m v_1^2$

$1.4 = 0.6 + \frac{1}{2} m v_2^2$

$\Rightarrow \frac{v_1^2}{v_2^2} = \frac{3.2}{0.8} = \frac{4}{1}$

$\Rightarrow \frac{v_1}{v_2} = \frac{2}{1}$

19. Two light beams of intensities in the ratio of 9 : 4 are allowed to interfere. The ratio of the intensity of maxima and minima will be:

(A) 2 : 3

(B) 16 : 81

(C) 25 : 169

(D) 25 : 1

Answer (D)

Sol. $\frac{I_{\max}}{I_{\min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}} \right)^2 = \left(\frac{5}{1} \right)^2$

$= \frac{25}{1}$

20. In Bohr's atomic model of hydrogen, let *K*, *P* and *E* are the kinetic energy, potential energy and total energy of the electron respectively. Choose the correct option when the electron undergoes transitions to a higher level:

(A) All *K*, *P* and *E* increase

(B) *K* decreases, *P* and *E* increase

(C) *P* decreases, *K* and *E* increase

(D) *K* increases, *P* and *E* decrease

Answer (B)

Sol. $T.E. = \frac{-Z^2 me^4}{8(nh\varepsilon_0)^2}$

$P.E. = \frac{-Z^2 me^4}{4(nh\varepsilon_0)^2}$

$K.E. = \frac{Z^2 me^4}{8(nh\varepsilon_0)^2}$

As electron makes transition to higher level, total energy and potential energy increases (due to negative sign) while the kinetic energy reduces.

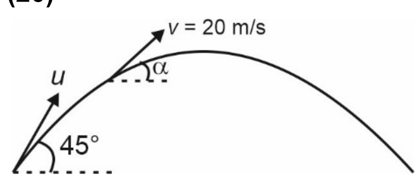
SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. A body is projected from the ground at an angle of 45° with the horizontal. Its velocity after 2 s is 20 ms^{-1}. The maximum height reached by the body during its motion is ________ m. (use g = 10 ms^{-2})

Answer (20)

Sol.



$\Rightarrow v \cos\alpha = u \cos 45°$ …(i)

& $v \sin\alpha = u \sin 45° - gt$ …(ii)

Solve for u we get

$u = 20\sqrt{2}$ m / s

$\Rightarrow H = \frac{u^2 \sin^2 45°}{20} = 20$ m

2. An antenna is placed in a dielectric medium of dielectric constant 6.25. If the maximum size of that antenna is 5.0 mm, it can radiate a signal of minimum frequency of __________GHz.

(Given μ_r = 1 for dielectric medium)

Answer (6)

Sol. We know that $v = f\lambda$

Putting the values,

$$\frac{3\times10^8}{\sqrt{6.25}} = f\times20\times10^{-3}$$

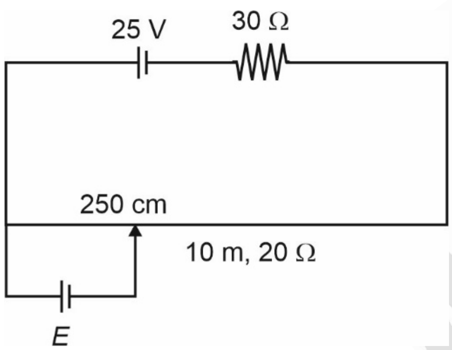
$\Rightarrow$ $f = 6 \times 10^9$ Hz

3. A potentiometer wire of length 10 m and resistance 20 Ω is connected in series with a 25 V battery and an external resistance 30 Ω. A cell of emf E in secondary circuit is balanced by 250 cm long potentiometer wire. The value of E (in volt) is $\frac{x}{10}$.

The value of x is ___.

Answer (25)

Sol.



$$\therefore E = I\times\left(\frac{20}{4}\right) = \frac{25}{(30+20)}\times\left(\frac{20}{4}\right)$$

$$= \frac{1}{2}\times5 = 2.5 \text{ volts}$$

$$= \frac{25}{10} \text{ volts}$$

4. Two travelling waves of equal amplitudes and equal frequencies move in opposite directions along a string. They interfere to produce a stationary wave whose equation is given by

$$y = \left(10\cos\pi x \sin\frac{2\pi t}{T}\right)\text{cm}$$

The amplitude of the particle at $x = \frac{4}{3}$ cm will be __________ cm.

Answer (5)

Sol. $A = |10\cos(\pi x)|$

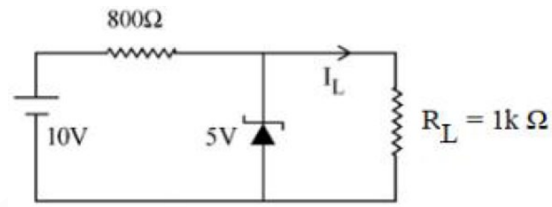
At $x = \frac{4}{3}$

$$A = \left|10\cos\left(\pi\times\frac{4}{3}\right)\right|$$

$= |-5 \text{ cm}|$

$\therefore$ Amp = 5 cm

5. In the given circuit, the value of current I_L will be __________ mA. (When R_L = 1 kΩ)



Answer (5)

Sol. V_L = 5 V as V_Z = 5 V

$$\therefore I_L = \frac{V_L}{R_L} = \frac{5}{10^3} = 5 \text{ mA}$$

6. A sample contains 10^{-2} kg each of two substances A and B with half lives 4 s and 8 s respectively. The ratio of their atomic weights is 1 : 2. The ratio of the amounts of A and B after 16 s is $\frac{x}{100}$. The value of x is ________.

Answer (25)

Sol. $N_1 = \dfrac{\left(\dfrac{10^{-2}}{1}\right)}{2^4}$

$$N_2 = \frac{\left(\frac{10^{-2}}{2}\right)}{2^2}$$

$$\Rightarrow \frac{N_1}{N_2} = \frac{1}{2}$$

$\therefore$ Mass ratio of A and B,

$$\frac{m_1}{m_2} = \frac{N_1}{N_2}\times\left(\frac{M_1}{M_2}\right)$$

$$= \frac{1}{2}\times\left(\frac{1}{2}\right)$$

$$= \frac{1}{4}$$

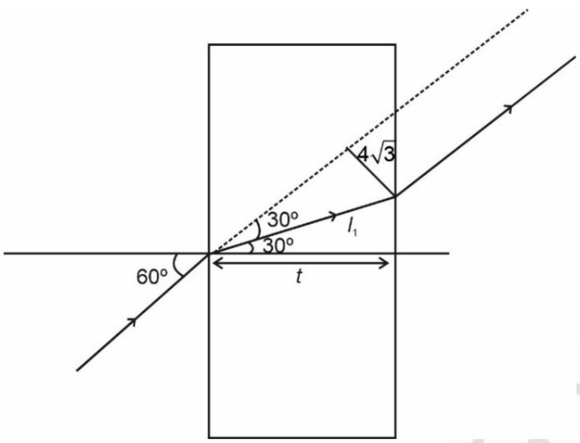
$$= \frac{25}{100}$$

$\therefore x = 25$

7. A ray of light is incident at an angle of incidence 60° on the glass slab of refractive index $\sqrt{3}$. After refraction, the light ray emerges out from other parallel faces and lateral shift between incident ray and emergent ray is $4\sqrt{3}$ cm. The thickness of the glass slab is ______ cm.

Answer (12)

Sol.



$1 \times \sin 60° = \sqrt{3} \times \sin r$

$\Rightarrow r = 30°$

$\therefore\ l_1 = 4\sqrt{3} \times 2$

$= 8\sqrt{3}$ cm

$\therefore$ Thickness, $t = l_1 \cos 30°$

$= 8\sqrt{3} \times \dfrac{\sqrt{3}}{2}$

$= 4 \times 3$

$= 12$ cm

8. A circular coil of 1000 turns each with area 1 m^2 is rotated about its vertical diameter at the rate of one revolution per second in a uniform horizontal magnetic field of 0.07 T. The maximum voltage generation will be _______ V.

Answer (440)

Sol. $V_{max} = NAB\omega$

$= 1000 \times 1 \times 0.07 \times (2\pi \times 1)$

$\simeq 440$ volts

9. A monoatomic gas performs a work of $\dfrac{Q}{4}$ where Q is the heat supplied to it. The molar heat capacity of the gas will be ____ R during this transformation. Where R is the gas constant.

Answer (2)

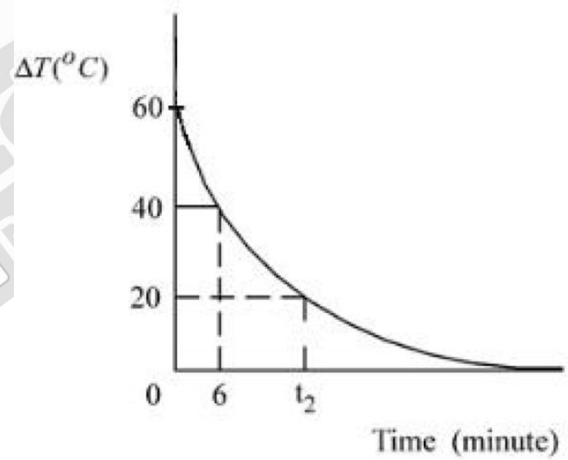
Sol. By 1st law,

$$\Delta U = \Delta Q - \frac{\Delta Q}{4} = \frac{3}{4}\Delta Q$$

$$\Rightarrow nC_v\Delta T = \frac{3}{4}nC\Delta T$$

$$\Rightarrow C = \frac{4C_v}{3} = 2R$$

10. In an experiment to verify Newton's law of cooling, a graph is plotted between, the temperature difference (ΔT) of the water and surroundings and time as shown in figure. The initial temperature of water is taken as 80°C. The value of t_2 as mentioned in the graph will be ______.



Answer (16)

Sol. Temperature of surrounding = 20°C

For 0 $\to$ 6 minutes, average temp. = 70°C

$\to$ Rate of cooling $\propto$ 70°C – 20°C = 50°C

For 6 $\to$ t_2 minutes, average temp. = 50°C

$\to$ Rate of cooling $\propto$ 30°C

$\Rightarrow t_2 - 6 = \dfrac{5}{3}$ (6 minutes)

$\Rightarrow t_2 = 16$ minutes

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. 120 g of an organic compound that contains only carbon and hydrogen gives 330 g of CO_2 and 270 g of water on complete combustion. The percentage of carbon and hydrogen, respectively are

(A) 25 and 75 (B) 40 and 60

(C) 60 and 40 (D) 75 and 25

Answer (D)

Sol. Mass of organic compound = 120 g

Mass of CO_2 = 330 g

Moles of CO_2 = $\frac{330}{44}$ = 7.5

Mass of carbon = 7.5 × 12 = 90 gm

Percentage of C = $\frac{90 \times 100}{120}$ = 75%

Mass of H_2O = 270 g

Moles of H_2O = $\frac{270}{18}$ = 15

Mass of hydrogen = 15 × 2 = 30 gm

Percentage of H = $\frac{30 \times 100}{120}$ = 25%

2. The energy of one mole of photons of radiation of wavelength 300 nm is (Given h = 6.63×10^{-34} Js, N_A = 6.02×10^{23} mol^{-1}, c = 3×10^8 ms^{-1})

(A) 235 kJ mol^{-1} (B) 325 kJ mol^{-1}

(C) 399 kJ mol^{-1} (D) 435 kJ mol^{-1}

Answer (C)

Sol. Wavelength of radiation = 300 nm

Photon energy = $\frac{hc}{\lambda}$

$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{300 \times 10^{-9}}$

$= 6.63 \times 10^{-19}$ J

Energy of 1 mole of photons

$= 6.63 \times 10^{-19} \times 6.02 \times 10^{23} \times 10^{-3}$

= 399 kJ

3. The correct order of bond orders of C_2^{2-}, N_2^{2-}, O_2^{2-} is, respectively

(A) $C_2^{2-} < N_2^{2-} < O_2^{2-}$ (B) $O_2^{2-} < N_2^{2-} < C_2^{2-}$

(C) $C_2^{2-} < O_2^{2-} < N_2^{2-}$ (D) $N_2^{2-} < C_2^{2-} < O_2^{2-}$

Answer (B)

Sol. C_2^{2-} : $\sigma_{1s}^2\ \sigma_{1s}^{*2}\ \sigma_{2s}^2\ \sigma_{2s}^{*2}\ \pi_{2p_x}^2 = \pi_{2p_y}^2\ \sigma_{2p_z}^2$

N_2^{2-} : $\sigma_{1s}^2\ \sigma_{1s}^{*2}\ \sigma_{2s}^2\ \sigma_{2s}^{*2}\ \sigma_{2p_z}^2\ \pi_{2p_x}^2 = \pi_{2p_y}^2\ \pi_{2p_x}^{*1} = \pi_{2p_y}^{*1}$

O_2^{2-} : $\sigma_{1s}^2\ \sigma_{1s}^{*2}\ \sigma_{2s}^2\ \sigma_{2s}^{*2}\ \sigma_{2p_z}^2\ \pi_{2p_x}^2 = \pi_{2p_y}^2\ \pi_{2p_x}^{*2} = \pi_{2p_y}^{*2}$

B.O. (C_2^{2-}) = 3; B.O. (N_2^{2-}) = 2; B.O. $(O_2^{2-}) = 1$

4. At 25°C and 1 atm pressure, the enthalpies of combustion are as given below :

Substance	H_2	C(graphite)	C_2H_6(g)
$\frac{\Delta_c H^\ominus}{kJmol^{-1}}$	–286.0	–394.0	–1560.0

The enthalpy of formation of ethane is

(A) +54.0 kJ mol^{-1} (B) –68.0 kJ mol^{-1}

(C) –86.0 kJ mol^{-1} (D) +97.0 kJ mol^{-1}

Answer (C)

Sol. $2C\ (\text{graphite}) + 3H_2(g) \rightarrow C_2H_6(g)$

$\Delta H_r = +1560 + 2(-394) + 3(-286)$

$= -86.0$ kJ mol^{-1}

Enthalpy of formation of C_2H_6(g) = –86.0 kJ mol^{-1}

5. For a first order reaction, the time required for completion of 90% reaction is 'x' times the half life of the reaction. The value of 'x' is (Given: ln 10 = 2.303 and log 2 = 0.3010)

(A) 1.12 (B) 2.43

(C) 3.32 (D) 33.31

Answer (C)

Sol. A $\rightarrow$ Products

For a first order reaction,

$$t_{1/2} = \frac{\ln 2}{k} = \frac{0.693}{k}$$

Time for 90% conversion,

$$t_{90\%} = \frac{1}{k}\ln\frac{100}{10} = \frac{\ln 10}{k} = \frac{2.303}{k}$$

$$t_{90\%} = \frac{2.303}{0.693}t_{1/2} = 3.32t_{1/2}$$

6. Metals generally melt at very high temperature. Amongst the following, the metal with the highest melting point will be

(1) Hg (2) Ag

(3) Ga (4) Cs

Answer (B)

Sol. Melting points of the given metals

Hg : –38.83° C

Ag : 961.8° C

Ga : 29.76° C

Cs : 28.44° C

∴ Metal having highest melting point is Ag.

7. Which of the following chemical reactions represents Hall-Heroult Process?

(1) $Cr_2O_3 + 2Al \rightarrow Al_2O_3 + 2Cr$

(2) $2Al_2O_3 + 3C \rightarrow 4Al + 3CO_2$

(3) $FeO + CO \rightarrow Fe + CO_2$

(4) $[2Au(CN)_2]^-_{(aq)} + Zn(s) \rightarrow 2Au(s) + [Zn(CN_4)]^{2-}$

Answer (B)

Sol. Hall-Herault process is used for the extraction of aluminium by electrolysis molten Al_2O_3

$2Al_2O_3 + 3C \rightarrow 4Al + 3CO_2$

8. In the industrial production of which of the following, molecular hydrogen is obtained as a byproduct?

(1) NaOH

(2) NaCl

(3) Na metal

(4) Na_2CO_3

Answer (A)

Sol. Molecular hydrogen is produced as a byproduct in the industrial production of NaOH by electrolysis of aq NaCl solution

$NaCl \rightarrow Na^+ + Cl^-$

$H_2O \rightleftharpoons H+ + OH^-$

Cathode : $2H_2O + 2e \rightarrow H_2 + 2OH^-$

Anode : $2Cl^- \rightarrow Cl_2 + 2e$

NaOH is crystallised from the remaining part of electrolyte.

9. Which one of the following compounds is used as a chemical in certain type of fire extinguishers?

(1) Baking soda

(2) Soda ash

(3) Washing soda

(4) Caustic Soda

Answer (A)

Sol. Baking soda ($NaHCO_3$) is used in certain type of fire extinguishers because it decomposes at high temperature to produce CO_2 which extinguishes fire

$$2NaHCO_3(s) \xrightarrow{\Delta} Na_2CO_3(s) + H_2O\uparrow + CO_2\uparrow$$

10. PCl_5 is well known, but NCl_5 is not. Because,

(1) nitrogen is less reactive than phosphorous

(2) nitrogen doesn't have d-orbitals in its valence shell.

(3) catenation tendency is weaker in nitrogen than phosphorous.

(4) size of phosphorous is larger than nitrogen

Answer (B)

Sol. PCl_5 is well known but NCl_5 is not because nitrogen does not have vacant d-orbitals in its valence shell. So, nitrogen cannot expand its octet. On the other hand phosphorus has vacant d-orbitals in its valence shell which enables it to expand its octet.

11. Transition metal complex with highest value of crystal field splitting (Δ_0) will be

(A) $[Cr(H_2O)_6]^{3+}$

(B) $[Mo(H_2O)_6]^{3+}$

(C) $[Fe(H_2O)_6]^{3+}$

(D) $[Os(H_2O)_6]^{3+}$

Answer (D)

Sol. Crystal field splitting (Δ_0) for octahedral complexes depends on oxidation state of the metal as well as to which transition series the metal belongs. For the same oxidation state, the crystal field splitting (Δ_0) increases as we move from 3*d* → 4*d* → 5*d*. Cr^{3+} and Fe^{3+} belong to 3*d* series, Mo^{3+} belongs to 4*d* series and Os^{3+} belongs to 5*d* series. Therefore crystal field splitting (Δ_0) is highest for $[Os(H_2O)_6]^{3+}$.

12. Some gases are responsible for heating of atmosphere (green house effect). Identify from the following the gaseous species which does not cause it.

(A) CH_4

(B) O_3

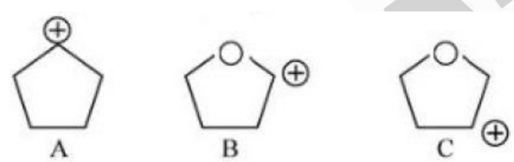
(C) H_2O

(D) N_2

Answer (D)

Sol. Among the given gases, the green house gases which are responsible for heating the atmosphere are CH_4, water vapour and ozone. Nitrogen is not a green house gas.

13. Arrange the following carbocations in decreasing order of stability.



(A) A > C > B

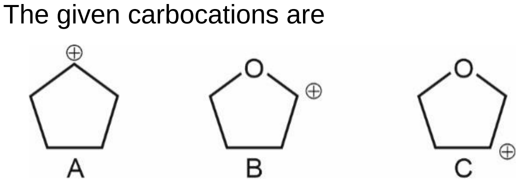
(B) A > B > C

(C) C > B > A

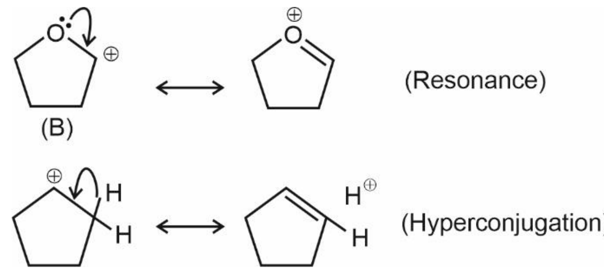
(D) C > A > B

Answer (Bonus)

Sol. The given carbocations are



Carbocation (A) is stabilised by hyperconjugation due to 4 α hydrogen atoms. Carbocation (C) is also stabilised by hyperconjugation due to 4 α hydrogen atoms but destabilised by –I effect of O-atom. Carbocation (B) is most stable as it is stabilised by resonance.



∴ Correct decreasing order of stability is

B > A > C

None of the given options is correct.

14. Given below are two statements.

Statement I: The presence of weaker π-bonds make alkenes less stable than alkanes.

Statement II: The strength of the double bond is greater than that of carbon-carbon single bond.

In the light of the above statements, choose the *correct* answer from the options given below.

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are incorrect.

(C) Statement I is correct but Statement II is incorrect.

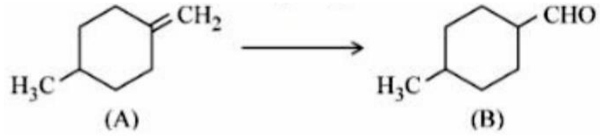
(D) Statement I is incorrect but Statement II is correct.

Answer (A)

Sol. The π-bond present is alkenes is weaker than σ-bond present in alkanes. That makes alkenes less stable than alkanes. Therefore, statement-I is correct.

Carbon-carbon double bond is stronger than Carbon-carbon single bond because more energy is required to break 1 sigma and 1 pi bond than to break 1 sigma bond only. Therefore, statement-II is also correct.

15. Which of the following reagents/reactions will convert 'A' to 'B'?



(A) PCC oxidation

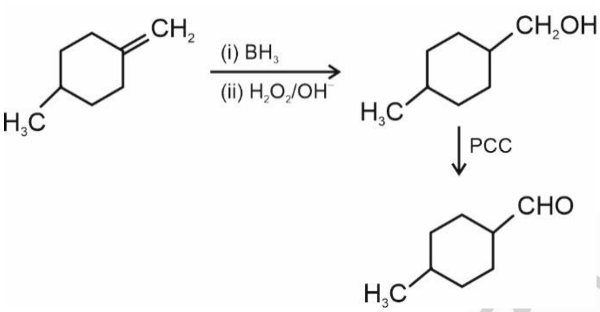
(B) Ozonolysis

(C) BH_3, H_2O_2/^{-}OH followed by PCC oxidation

(D) HBr, hydrolysis followed by oxidation by $K_2Cr_2O_7$.

Answer (C)

Sol.



The first step involves addition of H_2O to alkene according to anti-markownikoff's rule while the second step involves oxidation of 1° alcohol to aldehyde.

16. Hex-4-ene-2-ol on treatment with PCC gives 'A' on reaction with sodium hypoiodite gives 'B', which on further heating with soda lime gives 'C'. The compound 'C' is

(A) 2-pentene (B) Proponaldehyde

(C) 2-butene (D) 4-methylpent-2-ene

Answer (C)

Sol.

$$CH_3-\underset{}{\overset{OH}{\overset{|}{C}H}}-CH_2-CH=CH-CH_3 \xrightarrow{PCC}$$

$$CH_3-\overset{O}{\overset{||}{C}}-CH_2-CH=CH-CH_3$$

$$\xrightarrow[-CHI_3]{NaOI} {}^{-}O-\overset{O}{\overset{||}{C}}-CH_2-CH=CH-CH_3 \xrightarrow[\text{lime}]{\text{Soda}}$$

$$\underset{\text{2-butene}}{CH_3-CH=CH-CH_3}$$

17. The conversion of propan-1-ol to n-butylamine involves the sequential addition of reagents. The correct sequential order of reagents is

(A) (i) $SOCl_2$ (ii) KCN (iii) H_2/Ni, Na(Hg)/C_2H_5OH

(B) (i) HCl (ii) H_2/Ni, Na(Hg)/C_2H_5OH

(C) (i) SOCl2 (ii) KCN (iii) CH_3NH_2

(D) (i) HCl (ii) CH_3NH_2

Answer (A)

Sol. $\underset{\text{Propan-1-ol}}{CH_3-CH_2-CH_2-OH} \xrightarrow{SOCl_2}$

$$CH_3-CH_2-CH_2-Cl \xrightarrow{KCN}$$

$$CH_3-CH_2-CH_2-CN \xrightarrow[Na(Hg)/C_2H_5OH]{H_2/Ni}$$

$$\underset{\text{n-butylamine}}{CH_3-CH_2-CH_2-CH_2-NH_2}$$

18. Which of the following is not an example of a condensation polymer?

(A) Nylon 6,6 (B) Dacron

(C) Buna-N (D) Silicone

Answer (C)

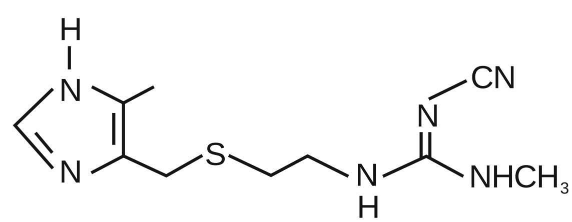
Sol. Nylon 6, 6 is a condensation polymer of hexamethylene diamine and adipic acid

Dacron is a condensation polymer of terephthalic acid and ethylene glycol.

Buna-N is an addition polymer of 1, 3-butadiene and acrylonitrile

Silicone is a condensation polymer of dialkyl silanediol.

19. The structure shown below is of which well-known drug molecule?



(A) Ranitidine (B) Seldane

(C) Cimetidine (D) Codeine

Answer (C)

Sol. The given structure is that of cimetidine which is well known antacid.

20. In the flame test of a mixture of salts, a green flame with blue centre was observed. Which one of the following cations may be present?

(A) Cu^{2+} (B) Sr^{2+}

(C) Ba^{2+} (D) Ca^{2+}

Answer (A)

Sol. Cupric salts give green flame with blue centre. The colour of other salts are

Sr^{2+}	Crimson red
Ca^{2+}	Brick red
Ba^{2+}	Green

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. At 300 K, a sample of 3.0 g of gas A occupies the same volume as 0.2 g of hydrogen at 200 K at the same pressure. The molar mass of gas A is ______ g mol^{-1}. (nearest integer) Assume that the behaviour of gases as ideal.

(Given: The molar mass of hydrogen (H_2) gas is 2.0 g mol^{-1}).

Answer (45)

Sol. V_1, Volume of 0.2 g H_2 at 200 K = $\frac{0.2\times R\times 200}{2\times P}$

V_2, Volume of 3.0 g of gas A at 300 K = $\frac{3.0\times R\times 300}{M\times P}$

$V_1 = V_2$ (Given)

$$\frac{0.2\times R\times 200}{2\times P} = \frac{3.0\times R\times 300}{M\times P}$$

$\therefore$ M = 45 g mol^{-1}

2. A company dissolves 'x' amount of CO_2 at 298 K in 1 litre of water to prepare soda water. X = _____ × 10^{-3} g. (nearest integer)

(Given: partial pressure of CO_2 at 298 K = 0.835 bar.

Henry's law constant for CO_2 at 298 K = 1.67 kbar.

Atomic mass of H, C and O is 1, 12, and 6 g mol^{-1}, respectively)

Answer (1221)

Sol. According to Henry's law, partial pressure of a gas is given by

$P_g = (K_H)\, X_g$

where X_g is mole fraction of gas in solution

$0.835 = 1.67 \times 10^3 \left(X_{CO_2}\right)$

$X_{CO_2} = 5\times 10^{-4}$

Mass of CO_2 in 1 L water = 1221 × 10^{-3} g

3. PCl_5 dissociates as

$PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$

5 moles of PCl_5 are placed in a 200 litre vessel which contains 2 moles of N_2 and is maintained at 600 K. The equilibrium pressure is 2.46 atm. The equilibrium constant K_p for the dissociation of PCl_5 is ______ × 10^{-3}. (nearest integer)

(Given: R = 0.082 L atm K^{-1} mol^{-1}; Assume ideal gas behaviour)

Answer (1107)

Sol. $PCl_5\,(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$

	PCl_5	PCl_3	Cl_2
Initial moles	5		
Equilibrium moles	5 – x	x	x

Number of moles of N_2 = 2

Equilibrium pressure = 2.46 atm

$$P_{eq} = \frac{(7+x)\times 0.082\times 600}{200} = 2.46$$

On solving, x = 3

$$\therefore \quad K_P = \frac{\left(\frac{3P}{10}\right)\left(\frac{3P}{10}\right)}{\left(\frac{2P}{10}\right)} = \frac{9\times 2.46}{20}$$

= 1107 × 10^{-3} atm

4. The resistance of a conductivity cell containing 0.01 M KCl solution at 298 K is 1750 Ω. If the conductivity of 0.01 M KCl solution at 298 K is 0.152×10^{-3} S cm^{-1}, then the cell constant of the conductivity cell is _______ $\times 10^{-3}$ cm^{-1}.

Answer (266)

Sol. Molarity of KCl solution = 0.1 M

Resistance = 1750 ohm

Conductivity = 0.152×10^{-3} S cm^{-1}

$$\text{Conductivity} = \frac{\text{Cell constant}}{\text{Resistance}}$$

∴ Cell constant = $0.152 \times 10^{-3} \times 1750$

= 266×10^{-3} cm^{-1}

5. When 200 mL of 0.2 M acetic acid is shaken with 0.6 g of wood charcoal, the final concentration of acetic acid after adsorption is 0.1 M. The mass of acetic acid adsorbed per gram of carbon is ________g.

Answer (2)

Sol. Mass of wood charcoal = 0.6 g

Initial moles of acetic acid = 0.2 × 0.2 = 0.04

Final moles of acetic acid = 0.1 × 0.2 = 0.02

Moles of acetic acid adsorbed = 0.04 – 0.02

= 0.02

Mass of acetic acid adsorbed per gm of charcoal

$$= \frac{0.02 \times 60}{0.6} = 2.0 \text{ g}$$

6. (a) Baryte, (b) Galena, (c) Zinc blende and (d) Copper pyrites. How many of these minerals are sulphide based?

Answer (3)

Sol.

Baryte	$BaSO_4$
Galena	PbS
Zinc blende	ZnS
Copper pyrites	$CuFeS_2$

Of the given minerals, only 3 are sulphide based.

7. Manganese (VI) has ability to disproportionate in acidic solution. The difference in oxidation states of two ions it forms in acidic solution is _____.

Answer (3)

Sol. Manganese (VI) disproportionates in acidic medium as

$$3MnO_4^{2-} + 4H^+ \longrightarrow 2MnO_4^- + MnO_2 + 2H_2O$$

Difference in oxidation states of Mn in the products formed = 7 – 4 = 3

8. 0.2 g of an organic compound was subjected to estimation of nitrogen by Duma's method in which volume of N_2 evolved (at STP) was found to be 22.400 mL. The percentage of nitrogen in the compound is _____. [nearest integer]

(Given : Molar mass of N_2 is 28 g mol^{-1}, Molar volume of N_2 at STP : 22.4L)

Answer (14)

Sol. Mass of organic compound = 0.2 g

Volume of N_2 gas evolved at STP = 22.4 mL

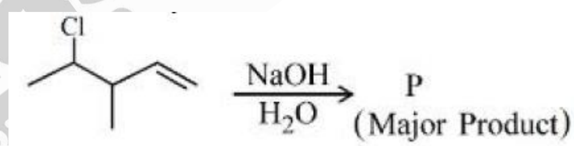
$$\text{Mass of } N_2 \text{ gas evolved} = \frac{22.4 \times 10^{-3} \times 28}{22.4}$$

= 0.028 g

Percentage of nitrogen in the compound

$$= \frac{0.028 \times 100}{0.2} = 14\%$$

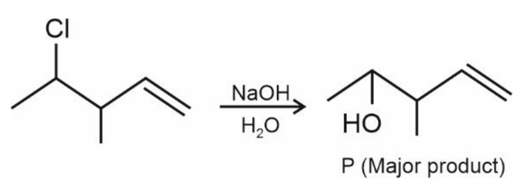
9.



Consider the above reaction. The number of π electrons present in the product 'P' is_________.

Answer (2)

Sol.



The given reaction undergoes nucleophilic substitution by SN2 mechanism at room temperature

∴ No. of π electrons present in P = 2

10. In alanylglycylleucylalanyvaline, the number of peptide linkages is __________.,

Answer (4)

Sol. The given pentapeptide is

ALA – GLY – LEU – ALA – VAL

It has 4 peptide linkages.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Let $x * y = x^2 + y^3$ and $(x * 1) * 1 = x * (1 * 1)$. Then a value of $2\sin^{-1}\left(\frac{x^4+x^2-2}{x^4+x^2+2}\right)$ is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$

Answer (B)

Sol. Given $x * y = x^2 + y^3$ and $(x * 1) * 1 = x * (1 * 1)$

So, $(x^2 + 1) * 1 = x * 2$

$\Rightarrow (x^2 + 1)^2 + 1 = x^2 + 8$

$\Rightarrow x^4 + 2x^2 + 2 = x^2 + 8$

$\Rightarrow (x^2)^2 + x^2 - 6 = 0$

$\therefore (x^2 + 3)(x^2 - 2) = 0$

$\therefore \boxed{x^2 = 2}$

Now, $2\sin^{-1}\left(\frac{x^4+x^2-2}{x^4+x^2+2}\right) = 2\sin^{-1}\left(\frac{4}{8}\right)$

$= 2\cdot\frac{\pi}{6} = \frac{\pi}{3}$

2. The sum of all the real roots of the equation $(e^{2x} - 4)(6e^{2x} - 5e^x + 1) = 0$ is

(A) $\log_e 3$ (B) $-\log_e 3$

(C) $\log_e 6$ (D) $-\log_e 6$

Answer (B)

Sol. Given equation : $(e^{2x} - 4)(6e^{2x} - 5e^x + 1) = 0$

$\Rightarrow e^{2x} - 4 = 0$ or $6e^{2x} - 5e^x + 1 = 0$

$\Rightarrow e^{2x} = 4$ or $6(e^x)^2 - 3e^x - 2e^x + 1 = 0$

$\Rightarrow 2x = \ln 4$ or $(3e^x - 1)(2e^x - 1) = 0$

$\Rightarrow \boxed{x = \ln 2}$ or $e^x = \frac{1}{3}$ or $e^x = \frac{1}{2}$

or $x = \ln\left(\frac{1}{3}\right), -\ln 2$

Sum of all real roots = ln2 – ln3 – ln2

= –ln3

3. Let the system of linear equations

$x + y + az = 2$

$3x + y + z = 4$

$x + 2z = 1$

have a unique solution (x^*, y^*, z^*). If (α, x^*), (y^*, α) and $(x^*, -y^*)$ are collinear points, then the sum of absolute values of all possible values of α is

(A) 4 (B) 3

(C) 2 (D) 1

Answer (C)

Sol. Given system of equations

$x + y + az = 2$ ...(i)

$3x + y + z = 4$ ...(ii)

$x + 2z = 1$ ...(iii)

Solving (i), (ii) and (iii), we get

$x = 1, y = 1, z = 0$ (and for unique solution $a \neq -3$)

Now, $(\alpha, 1)$, $(1, \alpha)$ and $(1, -1)$ are collinear

$\therefore \begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & -1 & 1 \end{vmatrix} = 0$

$\Rightarrow \alpha(\alpha + 1) - 1(0) + 1(-1 - \alpha) = 0$

$\Rightarrow \alpha^2 - 1 = 0$

$\therefore \alpha = \pm 1$

$\therefore$ Sum of absolute values of $\alpha = 1 + 1 = 2$

4. Let $x, y > 0$. If $x^3y^2 = 2^{15}$, then the least value of $3x + 2y$ is

(A) 30 (B) 32

(C) 36 (D) 40

Answer (D)

Sol. $x, y > 0$ and $x^3y^2 = 2^{15}$

Now, $3x + 2y = (x + x + x) + (y + y)$

So, by A.M $\geq$ G.M inequality

$\frac{3x+2y}{5} \geq \sqrt[5]{x^3 \cdot y^2}$

$\therefore 3x + 2y \geq 5\sqrt[5]{2^{15}}$

≥ 40

$\therefore$ Least value of $3x + 4y = 40$

5. Let $f(x)=\begin{cases}\dfrac{\sin(x-[x])}{x-[x]}, & x\in(-2,-1)\\ \max\{2x,\ 3[|x|]\}, & |x|<1\\ 1 & ,\ \text{otherwise}\end{cases}$

Where [t] denotes greatest integer $\le t$. If m is the number of points where f is not continuous and n is the number of points where f is not differentiable, then the ordered pair (m, n) is

(A) (3, 3)
(B) (2, 4)
(C) (2, 3)
(D) (3, 4)

Answer (C)

Sol. $f(x)=\begin{cases}\dfrac{\sin(x-[x])}{x[x]} & ,\ x\in(-2,-1)\\ \max\{2x,3[|x|]\} & ,\ |x|<1\\ 1 & ,\ \text{otherwise}\end{cases}$

$$f(x)=\begin{cases}\dfrac{\sin(x+2)}{x+2} & ,\ x\in(-2,-1)\\ 0 & ,\ x\in(-1,\ 0]\\ 2x & ,\ x\in(0,\ 1)\\ 1 & ,\ \text{othersiwe}\end{cases}$$

It clearly shows that $f(x)$ is discontinuous

At x = –1, 1 also non differentiable

and at x = 0, L.H.D = $\lim\limits_{h\to0}\dfrac{f(0-h)-f(0)}{-h}=0$

R.H.D = $\lim\limits_{h\to0}\dfrac{f(0+h)-f(0)}{h}=2$

$\therefore$ $f(x)$ is not differentiable at x = 0

$\therefore$ m = 2, n = 3

6. The value of the integral

$\displaystyle\int_{-\pi/2}^{\pi/2}\frac{dx}{(1+e^x)(\sin^6x+\cos^6x)}$ is equal to

(A) 2π
(B) 0
(C) π
(D) $\dfrac{\pi}{2}$

Answer (C)

Sol. $I=\displaystyle\int_{-\pi/2}^{\pi/2}\frac{dx}{(1+e^x)(\sin^6x+\cos^6x)}$ …(i)

$I=\displaystyle\int_{-\pi/2}^{\pi/2}\frac{dx}{(1+e^{-x})(\sin^6x+\cos^6x)}$ …(ii)

(i) and (ii)

From equation (i) & (ii)

$$2I=\int_{-\pi/2}^{\pi/2}\frac{dx}{\sin^6x+\cos^6x}$$

$$\Rightarrow\quad I=\int_0^{\pi/2}\frac{dx}{\sin^6x+\cos^6x}=\int_0^{\pi/2}\frac{dx}{1-\frac{3}{4}\sin^2 2x}$$

$$\Rightarrow\quad I=\int_0^{\pi/2}\frac{4\sec^2 2x\,dx}{4+\tan^2 2x}=2\int_0^{\pi/4}\frac{4\sec^2 2x}{4+\tan^2 2x}dx$$

Now, $\tan 2x=t$ — when $x=0,\ t=0$; when, $x=\dfrac{\pi}{4},\ t\to\infty$

$2\sec^2 2x\,dx=dt$

$$\therefore\quad I=2\int_0^{\infty}\frac{2dt}{4+t^2}=2\left(\tan^{-1}\frac{t}{2}\right)_0^{\infty}$$

$$=2\frac{\pi}{2}=\pi$$

7. $\displaystyle\lim_{n\to\infty}\left(\frac{n^2}{(n^2+1)(n+1)}+\frac{n^2}{(n^2+4)(n+2)}+\frac{n^2}{(n^2+9)(n+3)}+\dots+\frac{n^2}{(n^2+n^2)(n+n)}\right)$

is equal to

(A) $\dfrac{\pi}{8}+\dfrac{1}{4}\log_e 2$
(B) $\dfrac{\pi}{4}+\dfrac{1}{8}\log_e 2$
(C) $\dfrac{\pi}{4}-\dfrac{1}{8}\log_e 2$
(D) $\dfrac{\pi}{8}+\dfrac{1}{8}\log_e\sqrt{2}$

Answer (A)

Sol. $\lim_{n\to\infty}\left(\frac{n^2}{(n^2+1)(n+1)}+\frac{n^2}{(n^2+4)(n+2)}+....+\frac{n^2}{(n^2+n^2)(n+n)}\right)$

$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{n^2}{(n^2+r^2)(n+r)}$

$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\frac{1}{\left[1+\left(\frac{r}{n}\right)^2\right]\left[1+\left(\frac{r}{n}\right)\right]}$

$= \int_0^1\frac{1}{(1+x^2)(1+x)}dx$

$= \frac{1}{2}\int_0^1\left[\frac{1}{1+x}-\frac{(x-1)}{(1+x^2)}\right]dx$

$= \frac{1}{2}\left[\ln(1+x)-\frac{1}{2}\ln(1+x^2)+\tan^{-1}x\right]_0^1$

$= \frac{1}{2}\left[\frac{\pi}{4}+\frac{1}{2}\ln 2\right]=\frac{\pi}{8}+\frac{1}{4}\ln 2$

8. A particle is moving in the *xy*-plane along a curve *C* passing through the point (3, 3). The tangent to the curve *C* at the point *P* meets the *x*-axis at *Q*. If the *y*-axis bisects the segment *PQ*, then *C* is a parabola with

(A) Length of latus rectum 3

(B) Length of latus rectum 6

(C) Focus$\left(\frac{4}{3},0\right)$

(D) Focus$\left(0,\frac{3}{4}\right)$

Answer (A)

Sol. According to the question $(\text{Let } P(x,y))$

$2x-y\frac{dx}{dy}=0$ $\left(\because \text{ equation of tangent at } P: y-y=\frac{dy}{dx}(y-x)\right)$

$\therefore\ 2\frac{dy}{y}=\frac{dx}{x}$

$\Rightarrow\ 2\ln y=\ln x+\ln c$

$\Rightarrow\ y^2=cx$ $\quad\because$ this curve passes through (3, 3) $\therefore$ $\boxed{c=3}$ $\quad\therefore$ required parabola $y^2 = 3x$ and L.R = 3

9. Let the maximum area of the triangle that can be inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{4}=1,\ a>2,$ having one of its vertices at one end of the major axis of the ellipse and one of its sides parallel to the *y*-axis, be $6\sqrt{3}$. Then the eccentricity of the ellipse is

(A) $\frac{\sqrt{3}}{2}$

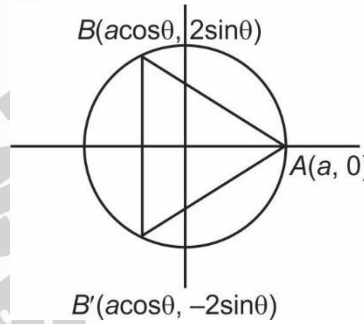
(B) $\frac{1}{2}$

(C) $\frac{1}{\sqrt{2}}$

(D) $\frac{\sqrt{3}}{4}$

Answer (A)

Sol. Given ellipse $\frac{x^2}{a^2}+\frac{y^2}{4}=1,\ a>2$



$\therefore$ Let $A(\theta)$ be the area of $\Delta ABB'$

Then $A(\theta)=\frac{1}{2}4\sin\theta(a+a\cos\theta)$

$A'(\theta)=a\left(2\cos\theta+2\cos^2\theta\right)$

For maxima $A'(\theta) = 0$

$\Rightarrow\ \cos\theta=-1,\ \cos\theta=\frac{1}{2}$

But for maximum area $\cos\theta=\frac{1}{2}$

$\therefore\ A(\theta)=6\sqrt{3}$

$\Rightarrow\ 2\frac{\sqrt{3}}{2}\left(a+\frac{a}{2}\right)=6\sqrt{3}$

$\Rightarrow\ a = 4$

$\therefore\ e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{4}{16}}=\frac{\sqrt{3}}{2}$

10. Let the area of the triangle with vertices $A(1, \alpha)$, $B(\alpha, 0)$ and $C(0, \alpha)$ be 4 sq. units. If the points $(\alpha, -\alpha)$, $(-\alpha, \alpha)$ and (α^2, β) are collinear, then β is equal to

(A) 64 (B) –8

(C) –64 (D) 512

Answer (C)

Sol. $\because$ $A(1, \alpha)$, $B(\alpha, 0)$ and $C(0, \alpha)$ are the vertices of ΔABC and area of $\Delta ABC = 4$

$$\therefore \left|\frac{1}{2}\begin{vmatrix}1 & \alpha & 1\\ \alpha & 0 & 1\\ 0 & \alpha & 1\end{vmatrix}\right| = 4$$

$$\Rightarrow \left|1(1-\alpha)-\alpha(\alpha)+\alpha^2\right| = 8$$

$$\Rightarrow \alpha = \pm 8$$

Now, $(\alpha, -\alpha)$, $(-\alpha, \alpha)$ and (α^2, β) are collinear

$$\therefore \begin{vmatrix}8 & -8 & 1\\ -8 & 8 & 1\\ 64 & \beta & 1\end{vmatrix} = 0 = \begin{vmatrix}-8 & 8 & 1\\ 8 & -8 & 1\\ 64 & \beta & 1\end{vmatrix}$$

$$\Rightarrow 8(8-\beta)+8(-8-64)+1(-8\beta-8\times 64) = 0$$

$$\Rightarrow 8-\beta-72-\beta-64 = 0$$

$$\Rightarrow \beta = -64$$

11. The number of distinct real roots of the equation $x^7 - 7x - 2 = 0$ is

(A) 5 (B) 7

(C) 1 (D) 3

Answer (D)

Sol. Given equation $x^7 - 7x - 2 = 0$

Let $f(x) = x^7 - 7x - 2$

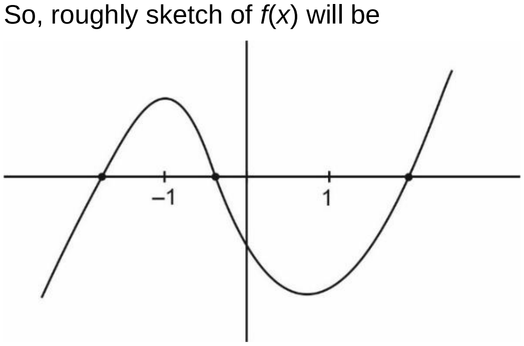
$f'(x) = 7x^6 - 7 = 7(x^6 - 1)$

and $f'(x) = 0 \Rightarrow x = +1$

and $f(-1) = -1 + 7 - 2 = 5 > 0$

$f(1) = 1 - 7 - 2 = -8 < 0$

So, roughly sketch of $f(x)$ will be



So, number of real roots of $f(x) = 0$ and 3

12. A random variable X has the following probability distribution :

X	0	1	2	3	4
$P(X)$	k	$2k$	$4k$	$6k$	$8k$

The value of $P(1 < X < 4 \mid x \le 2)$ is equal to

(A) $\frac{4}{7}$ (B) $\frac{2}{3}$

(C) $\frac{3}{7}$ (D) $\frac{4}{5}$

Answer (A)

Sol. $\because$ x is a random variable

$\therefore$ $k + 2k + 4k + 6k + 8k = 1$

$\therefore$ $k = \frac{1}{21}$

Now, $P(1 < x < 4 \mid x \le 2) = \frac{4k}{7k} = \frac{4}{7}$

13. The number of solutions of the equation $\cos\left(x+\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}-x\right) = \frac{1}{4}\cos^2 2x, \quad x \in [-3\pi, 3\pi]$ is:

(A) 8 (B) 5

(C) 6 (D) 7

Answer (D)

Sol. $\cos\left(x+\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}-x\right) = \frac{1}{4}\cos^2 2x,\ x \in [-3\pi, 3\pi]$

$$\Rightarrow \cos 2x + \cos\frac{2\pi}{3} = \frac{1}{2}\cos^2 2x$$

$$\Rightarrow \cos^2 2x - 2\cos 2x - 1 = 0$$

$\Rightarrow$ $\cos 2x = 1$

$\therefore$ $x = -3\pi, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi$

$\therefore$ Number of solutions = 7

14. If the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{\lambda}$ and $\frac{x-2}{1} = \frac{y-4}{4} = \frac{z-5}{5}$ is $\frac{1}{\sqrt{3}}$, then the sum of all possible values of λ is :

(A) 16

(B) 6

(C) 12

(D) 15

Answer (A)

Sol. Let $\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{a}_2 = 2\hat{i} + 4\hat{j} + 5\hat{k}$

$\vec{p} = 2\hat{i} + 3\hat{j} + \lambda\hat{k},\ \vec{q} = \hat{i} + 4\hat{j} + 5\hat{k}$

$\therefore\ \vec{p} \times \vec{q} = (15 - 4\lambda)\hat{i} - (10 - \lambda)\hat{j} + 5\hat{k}$

$\vec{a}_2 - \vec{a}_1 = \hat{i} + 2\hat{j} + 2\hat{k}$

$\therefore$ Shortest distance

$$= \left|\frac{(15 - 4\lambda) - 2(10 - \lambda) + 10}{\sqrt{(15 - 4\lambda)^2 + (10 - \lambda)^2 + 25}}\right| = \frac{1}{\sqrt{3}}$$

$\Rightarrow\ 3(5 - 2\lambda)^2 = (15 - 4\lambda)^2 + (10 - \lambda)^2 + 25$

$\Rightarrow\ 5\lambda^2 - 80\lambda + 275 = 0$

$\therefore$ Sum of values of $\lambda = \frac{80}{5} = 16$

15. Let the points on the plane P be equidistant from the points (–4, 2, 1) and (2, –2, 3). Then the acute angle between the plane P and the plane $2x + y + 3z = 1$ is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$ (D) $\frac{5\pi}{12}$

Answer (C)

Sol. Let $P(x, y, z)$ be any point on plane P_1

Then $(x + 4)^2 + (y - 2)^2 + (z - 1)^2 = (x - 2)^2 + (y + 2)^2 + (z - 3)^2$

$\Rightarrow\ 12x - 8y + 4z + 4 = 0$

$\Rightarrow\ 3x - 2y + z + 1 = 0$

And $P_2 : 2x + y + 3z = 1$

$\therefore$ angle between P_1 and P_2

$\cos\theta \left|\frac{6 - 2 + 3}{14}\right| \Rightarrow \theta = \frac{\pi}{3}$

16. Let $\hat{a}$ and $\hat{b}$ be two unit vectors such that $\left|(\hat{a} + \hat{b}) + 2(\hat{a} \times \hat{b})\right| = 2$. If $\theta \in (0, \pi)$ is the angle between $\hat{a}$ and $\hat{b}$, then among the statements:

(S1) : $2\left|\hat{a} \times \hat{b}\right| = \left|\hat{a} - \hat{b}\right|$

(S2) : The projection of $\hat{a}$ on $(\hat{a} + \hat{b})$ is $\frac{1}{2}$

(A) Only (S1) is true
(B) Only (S2) is true
(C) Both (S1) and (S2) are true
(D) Both (S1) and (S2) are false

Answer (C)

Sol. $\because\ \left|\hat{a} + \hat{b} + 2(\hat{a} \times \hat{b})\right| = 2,\ \theta \in (0, \pi)$

$\Rightarrow\ |\hat{a} + \hat{b} + 2(\hat{a} \times \hat{b})|^2 = 4$.

$\Rightarrow\ |\hat{a}|^2 + |\hat{b}|^2 + 4|\hat{a} \times \hat{b}|^2 + 2\hat{a} \cdot \hat{b} = 4$.

$\therefore\ \cos\theta = \cos 2\theta$

$\therefore\ \theta = \frac{2\pi}{3}$

where θ is angle between $\hat{a}$ and $\hat{b}$.

$\therefore\ 2|\hat{a} \times \hat{b}| = \sqrt{3} = |\hat{a} - \hat{b}|$

(S1) is correct

And projection of $\hat{a}$ on $(\hat{a} + \hat{b}) = \left|\frac{\hat{a} \cdot (\hat{a} + \hat{b})}{|\hat{a} + \hat{b}|}\right| = \frac{1}{2}$.

(S2) is correct.

17. If $y = \tan^{-1}\left(\sec x^3 - \tan x^3\right), \frac{\pi}{2} < x^3 < \frac{3\pi}{2}$, then

(A) $xy'' + 2y' = 0$ (B) $x^2y'' - 6y + \frac{3\pi}{2} = 0$

(C) $x^2y'' - 6y + 3\pi = 0$ (D) $xy'' - 4y' = 0$

Answer (B)

Sol. Let $x^3 = \theta \Rightarrow \frac{\theta}{2} \in \left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

$\therefore\ y = \tan^{-1}(\sec\theta - \tan\theta)$

$= \tan^{-1}\left(\frac{1 - \sin\theta}{\cos\theta}\right)$

$\therefore\ y = \frac{\pi}{4} - \frac{\theta}{2}$.

$y = \frac{\pi}{4} - \frac{x^3}{2}$

$\therefore\ y' = \frac{-3x^2}{2}$

$y'' = -3x$

$\therefore\ x^2y'' - 6y + \frac{3\pi}{2} = 0$.

18. Consider the following statements:

A : Rishi is a judge.

B : Rishi is honest.

C : Rishi is not arrogant.

The negation of the statement "if Rishi is a judge and he is not arrogant, then he is honest" is

(A) $B \to (A \vee C)$ (B) $(\sim B) \wedge (A \wedge C)$

(C) $B \to ((\sim A) \vee (\sim C))$ (D) $B \to (A \wedge C)$

Answer (B)

Sol. $\because$ given statement is

$(A \wedge C) \rightarrow B$

Then its negation is

$\sim \{(A \wedge C) \rightarrow B\}$

or $\sim \{\sim (A \wedge C) \vee B\}$

$\therefore$ $(A \wedge C) \wedge (\sim B)$

or $(\sim B) \wedge (A \wedge C)$

19. The slope of normal at any point (x, y), $x > 0$, $y > 0$ on the curve $y = y(x)$ is given by $\dfrac{x^2}{xy - x^2y^2 - 1}$. If the curve passes through the point (1, 1), then $e \cdot y(e)$ is equal to

(A) $\dfrac{1-\tan(1)}{1+\tan(1)}$ (B) $\tan(1)$

(C) 1 (D) $\dfrac{1+\tan(1)}{1-\tan(1)}$

Answer (D)

Sol. $\because$ $-\dfrac{dx}{dy} = \dfrac{x^2}{xy - x^2y^2 - 1}$

$\therefore$ $\dfrac{dy}{dx} = \dfrac{x^2y^2 - xy + 1}{x^2}$

Let $xy = v \Rightarrow y + x\dfrac{dy}{dx} = \dfrac{dv}{dx}$

$\therefore$ $\dfrac{dv}{dx} - y = \dfrac{(v^2 - v + 1)y}{v}$

$\therefore$ $\dfrac{dv}{dx} = \dfrac{v^2+1}{x}$

$\because$ $y(1) = 1 \Rightarrow \tan^{-1}(xy) = \ln x + \tan^{-1}(1)$

Put $x = e$ and $y = y(e)$ we get

$\tan^{-1}(e \cdot y(e)) = 1 + \tan^{-1} 1.$

$\tan^{-1}(e \cdot y(e)) - \tan^{-1} 1 = 1$

$\therefore$ $e(y(e)) = \dfrac{1+\tan(1)}{1-\tan(1)}$

20. Let λ^* be the largest value of λ for which the function $f_\lambda(x) = 4\lambda x^3 - 36\lambda x^2 + 36x + 48$ is increasing for all $x \in \mathbb{R}$. Then $f_\lambda^*(1) + f_\lambda^*(-1)$ is equal to :

(A) 36 (B) 48

(C) 64 (D) 72

Answer (D)

Sol. $\because$ $f_\lambda(x) = 4\lambda x^3 - 36\lambda x^2 + 36x + 48$

$\therefore$ $f'_\lambda(x) = 12(\lambda x^2 - 6\lambda x + 3)$

For $f_\lambda(x)$ increasing : $(6\lambda)^2 - 12\lambda \le 0$

$\therefore$ $\lambda \in \left[0, \dfrac{1}{3}\right]$

$\therefore$ $\lambda^* = \dfrac{1}{3}$

Now, $f_\lambda^*(x) = \dfrac{4}{3}x^3 - 12x^2 + 36x + 48$

$\therefore$ $f_\lambda^*(1) + f_\lambda^*(-1) = 73\dfrac{1}{2} - 1\dfrac{1}{2}$

$= 72.$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse andw the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Let $S = \{z \in \mathbb{C} : |z-3| \le 1$ and $z(4+3i) + \bar{z}(4-3i) \le 24\}$. If $\alpha + i\beta$ is the point in S which is closest to $4i$, then $25(\alpha + \beta)$ is equal to ______.

Answer (80)

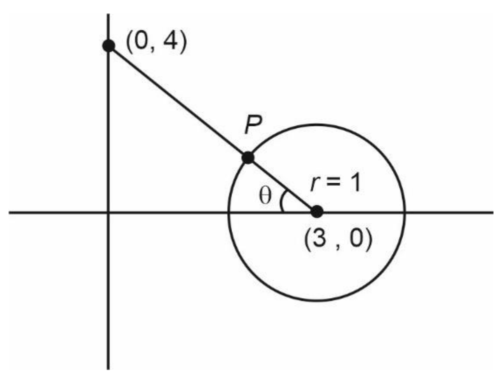
Sol. Here $|z - 3| < 1$

$\Rightarrow$ $(x - 3)^2 + y^2 < 1$

and $z = (4+3i) + \bar{z}(4-3i) \le 24$

$\Rightarrow$ $4x - 3y \le 12$

$\tan\theta = \dfrac{4}{3}$



$\therefore$ Coordinate of $P = (3 - \cos\theta, \sin\theta)$

$$= \left(3 - \frac{3}{5}, \frac{4}{5}\right)$$

$\therefore$ $\alpha + i\beta = \frac{12}{5} + \frac{4}{5}i$

$\therefore$ $25(\alpha + \beta) = 80$

2. Let $S = \left\{\begin{pmatrix} -1 & a \\ 0 & b \end{pmatrix}; a, b \in \{1, 2, 3, \ldots.. 100\}\right\}$ and let $T_n = \{A \in S : A^{n(n+1)} = I\}$. Then the number of elements in $\bigcap_{n=1}^{100} T_n$ is _____.

Answer (100)

Sol. $S = \left\{\begin{pmatrix} -1 & a \\ 0 & b \end{pmatrix} : a, b \in \{1, 2, 3, \ldots, 100\}\right\}$

$\because$ $A = \begin{pmatrix} -1 & a \\ 0 & b \end{pmatrix}$ then even powers of A as $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, if $b = 1$ and $a \in \{1, \ldots.., 100\}$

Here, $n(n + 1)$ is always even.

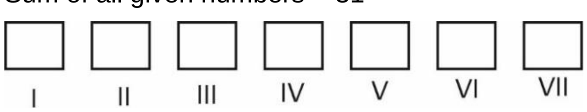
$\therefore$ $T_1, T_2, T_3, \ldots, T_n$ are all I for $b = 1$ and each value of a.

$\therefore$ $\bigcap_{n=1}^{100} T_n = 100$

3. The number of 7-digit numbers which are multiples of 11 and are formed using all the digits 1, 2, 3, 4, 5, 7 and 9 is _____.

Answer (576)

Sol. Sum of all given numbers = 31



Difference between odd and even positions must be 0, 11 or 22, but 0 and 22 are not possible.

$\therefore$ Only difference 11 is possible

This is possible only when either 1, 2, 3, 4 is filled in odd position in some order and remaining in other order. Similar arrangements of 2, 3, 5 or 7, 2, 1 or 4, 5, 1 at even positions.

$\therefore$ Total possible arrangements = $(4! \times 3!) \times 4$

$= 576$

4. The sum of all the elements of the set $\{\alpha \in \{1, 2, \ldots, 100\} : \text{HCF}(\alpha, 24) = 1\}$ is

Answer (1633)

Sol. The numbers upto 24 which gives g.c.d. with 24 equals to 1 are 1, 5, 7, 11, 13, 17, 19 and 23.

Sum of these numbers = 96

There are four such blocks and a number 97 is there upto 100.

$\therefore$ Complete sum

$= 96 + (24 \times 8 + 96) + (48 \times 8 + 96) + (72 \times 8 + 96) + 97$

$= 1633$

5. The remainder on dividing $1 + 3 + 3^2 + 3^3 + \ldots + 3^{2021}$ by 50 _____ is

Answer (4)

Sol. $1 + 3 + 3^2 + \ldots. + 3^{2021} = \frac{3^{2022} - 1}{2}$

$= \frac{1}{2}\left\{(10 - 1)^{1011} - 1\right\}$

$= \frac{1}{2}\{100k + 1011\,0 - 1\ - 1\}$

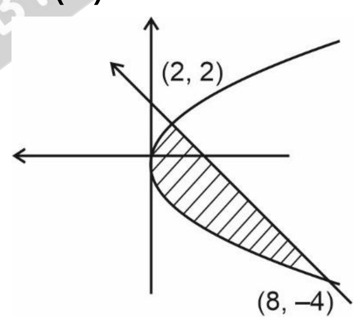
$= 50\,k_1 + 4$

$\therefore$ Remainder = 4

6. The area (in sq. units) of the region enclosed between the parabola $y^2 = 2x$ and the line $x + y = 4$ is _______.

Answer (18)

Sol.



The required area = $\int_{-4}^{2}\left(4 - y - \frac{y^2}{2}\right)dy$

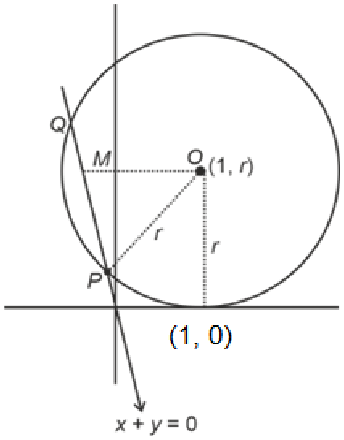
$= \left[4y - \frac{y^2}{2} - \frac{y^3}{6}\right]_{-4}^{2}$

= 18 square units

7. Let a circle $C : (x - h)^2 + (y - k)^2 = r^2$, $k > 0$, touch the x-axis at (1, 0). If the line $x + y = 0$ intersects the circle C at P and Q such that the length of the chord PQ is 2, then the value of $h + k + r$ is equal to _____.

Answer (7)

Sol.



Here, $OM^2 = OP^2 - PM^2$

$$\left(\frac{|1+r|}{\sqrt{2}}\right)^2 = r^2 - 1$$

$\therefore$ $r^2 - 2r - 3 = 0$

$\therefore$ $r = 3$

$\therefore$ Equation of circle is

$$(x-1)^2 + (y-3)^2 = 3^2$$

$\therefore$ $h = 1, k = 3, r = 3$

$\therefore$ $h + k + r = 7$

8. In an examination, there are 10 true-false type questions. Out of 10, a student can guess the answer of 4 questions correctly with probability $\frac{3}{4}$ and the remaining 6 questions correctly with probability $\frac{1}{4}$. If the probability that the student guesses the answers of exactly 8 questions correctly out of 10 is $\frac{27k}{4^{10}}$, then k is equal to

Answer (479)

Sol. Student guesses only two wrong. So there are three possibilities

(i) Student guesses both wrong from 1st section

(ii) Student guesses both wrong from 2nd section

(iii) Student guesses two wrong one from each section

Required probabilities = ${}^4C_2\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^2\left(\frac{1}{4}\right)^6 +$

$${}^6C_2\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^4 + {}^4C_1\cdot{}^6C_1\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^5$$

$$= \frac{1}{4^{10}}\left[6\times 9 + 15\times 9^4 + 24\times 9^2\right]$$

$$= \frac{27}{4^{10}}\left[2 + 27\times 15 + 72\right]$$

$$= \frac{27\times 479}{4^{10}}$$

9. Let the hyperbola $H : \frac{x^2}{a^2} - y^2 = 1$ and the ellipse $E : 3x^2 + 4y^2 = 12$ be such that the length of latus rectum of H is equal to the length of latus rectum of E. If e_H and e_E are the eccentricities of H and E respectively, then the value of $12\left(e_H^2 + e_E^2\right)$ is equal to ___.

Answer (42)

Sol. $\because$ $H : \frac{x^2}{a^2} - \frac{y^2}{1} = 1$

$\therefore$ Length of latus rectum = $\frac{2}{a}$

$E : \frac{x^2}{4} + \frac{y^2}{3} = 1$

Length of latus rectum = $\frac{6}{2} = 3$

$\because$ $\frac{2}{a} = 3 \Rightarrow a = \frac{2}{3}$

$\therefore$ $12\left(e_H^2 + e_E^2\right) = 12\left(1 + \frac{9}{4}\right) + \left(1 - \frac{3}{4}\right)$ = 42

10. Let P_1 be a parabola with vertex (3, 2) and focus (4, 4) and P_2 be its mirror image with respect to the line $x + 2y = 6$. Then the directrix of P_2 is $x + 2y =$ _______.

Answer (10)

Sol. Focus = (4, 4) and vertex = (3, 2)

$\therefore$ Point of intersection of directrix with axis of parabola = A = (2, 0)

Image of A(2, 0) with respect to line

$x + 2y = 6$ is $B(x_2, y_2)$

$$\therefore \frac{x_2 - 2}{1} = \frac{y_2 - 0}{2} = \frac{-2(2 + 0 - 6)}{5}$$

$\therefore$ $B(x_2, y_2) = \left(\frac{18}{5}, \frac{16}{5}\right)$.

Point B is point of intersection of direction with axes of parabola P_2.

$\therefore$ $x + 2y = \lambda$ must have point $\left(\frac{18}{5}, \frac{16}{5}\right)$

$\therefore$ $x + 2y = 10$

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