24/06/2022
Morning

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Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2022 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The bulk modulus of a liquid is 3×10^{10} Nm^{-2}. The pressure required to reduce the volume of liquid by 2% is

(A) 3×10^8 Nm^{-2}

(B) 9×10^8 Nm^{-2}

(C) 6×10^8 Nm^{-2}

(D) 12×10^8 Nm^{-2}

Answer (C)

Sol. $\because \; B = \dfrac{\Delta P}{\left(-\dfrac{\Delta V}{V}\right)}$

$\Rightarrow \; \Delta P = 3 \times 10^{10} \times (0.02)$

$= 6 \times 10^8$ N/m^2

2. Given below are two statements: One is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A): In an uniform magnetic field, speed and energy remains the same for a moving charged particle.

Reason (R): Moving charged particle experiences magnetic force perpendicular to its direction of motion.

(A) Both **(A)** and **(R)** true and **(R)** is the correct explanation of **(A)**.

(B) Both **(A)** and **(R)** are true but **(R)** is NOT the correct explanation of **(A)**.

(C) **(A)** is true but **(R)** is false.

(D) **(A)** is false but **(R)** is true.

Answer (A)

Sol. Magnetic force $\vec{F} \perp \vec{v}$

$\Rightarrow \; W_b = 0$

$\Rightarrow \; \Delta$KE = 0 and speed remains constant.

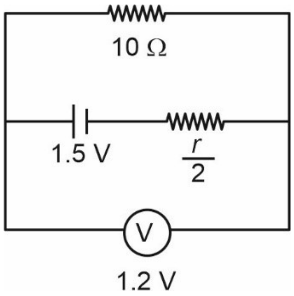
3. Two identical cells each of emf 1.5 V are connected in parallel across a parallel combination of two resistors each of resistance 20 Ω. A voltmeter connected in the circuit measures 1.2 V. The internal resistance of each cell is

(A) 2.5 Ω (B) 4 Ω

(C) 5 Ω (D) 10 Ω

Answer (C)

Sol.



$$\frac{1.5 \times 10}{10 + \frac{r}{2}} = 1.2$$

$\Rightarrow \; r = 5\,\Omega$

4. Identify the pair of physical quantities which have different dimensions.

(A) Wave number and Rydberg's constant

(B) Stress and Coefficient of elasticity

(C) Coercivity and Magnetisation

(D) Specific heat capacity and Latent heat

Answer (D)

Sol. $[S] = \dfrac{[C]}{[m] \times [\Delta T]}$

and, $[L] = \dfrac{[Q]}{[m]}$

$\Rightarrow$ They have different dimensions

5. A projectile is projected with velocity of 25 m/s at an angle θ with the horizontal. After t seconds its inclination with horizontal becomes zero. If R represents horizontal range of the projectile, the value of θ will be

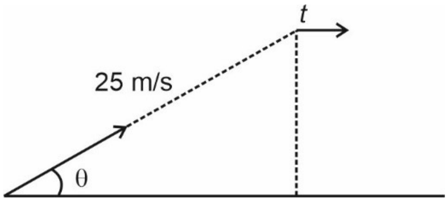
[use g = 10 m/s^2]

(A) $\dfrac{1}{2}\sin^{-1}\left[\dfrac{5t^2}{4R}\right]$ (B) $\dfrac{1}{2}\sin^{-1}\left[\dfrac{4R}{5t^2}\right]$

(C) $\tan^{-1}\left[\dfrac{4t^2}{5R}\right]$ (D) $\cot^{-1}\left[\dfrac{R}{20t^2}\right]$

Answer (D)

Sol.



$$t = \frac{25\sin\theta}{g}$$

and, $R = \dfrac{(25)^2(2\sin\theta\cos\theta)}{g}$

$\Rightarrow \quad R = \dfrac{25\times 25\times 2}{g}\times\dfrac{gt}{25}\times\cos\theta$

$\Rightarrow \quad R = 50t\cos\theta$

$\therefore \quad \tan\theta = \dfrac{gt}{25}\times\dfrac{50t}{R}$

$= \dfrac{20t^2}{R}$

$\Rightarrow \quad \theta = \cot^{-1}\left(\dfrac{R}{20t^2}\right)$

6. A block of mass 10 kg starts sliding on a surface with an initial velocity of 9.8 ms^{-1}. The coefficient of friction between the surface and block is 0.5. The distance covered by the block before coming to rest is

[use g = 9.8 ms^{-2}]

(A) 4.9 m (B) 9.8 m

(C) 12.5 m (D) 19.6 m

Answer (B)

Sol. $S = \dfrac{u^2}{2a} = \dfrac{u^2}{2(\mu g)}$

$= \dfrac{(9.8)^2}{2\times 0.5\times(9.8)}$

$= \dfrac{9.8}{1}$

= 9.8 m

7. A boy ties a stone of mass 100 g to the end of a 2 m long string and whirls it around in a horizontal plane. The string can withstand the maximum tension of 80 N. If the maximum speed with which the stone can revolve is $\dfrac{K}{\pi}$ rev./min. The value of K is

(Assume the string is massless and unstretchable)

(A) 400 (B) 300

(C) 600 (D) 800

Answer (C)

Sol. $T = m\omega^2 r$

$\Rightarrow \quad 80 = 0.1\times\left(2\pi\times\dfrac{K}{\pi}\times\dfrac{1}{60}\right)^2\times 2$

$\Rightarrow \quad \dfrac{800}{2} = \dfrac{K^2}{900}$

$\Rightarrow \quad K = 30 \times 20 = 600$

8. A vertical electric field of magnitude 4.9 × 10^5 N/C just prevents a water droplet of a mass 0.1 g from falling. The value charge on the droplet will be

(Given g = 9.8 m/s^2)

(A) 1.6 × 10^{-9} C

(B) 2.0 × 10^{-9} C

(C) 3.2 × 10^{-9} C

(D) 0.5 × 10^{-9} C

Answer (B)

Sol. Since the droplet is at rest

$\Rightarrow$ Net force = 0

$\Rightarrow \quad mg = qE$

$\Rightarrow \quad q = \dfrac{mg}{E} = 2\times 10^{-9}$ C

9. A particle experiences a variable force $\vec{F} = \left(4x\hat{i} + 3y^2\hat{j}\right)$ in a horizontal x-y plane. Assume distance in meters and force is newton. If the particle moves from point (1, 2) to point (2, 3) in the x-y plane; then Kinetic Energy changes by

(A) 50.0 J (B) 12.5 J

(C) 25.0 J (D) 0 J

Answer (C)

Sol. $W = \int \vec{F}\cdot d\vec{r}$

$= \int_1^2 4x\,dx + \int_2^3 3y^2\,dy$

$= [2x^2]_1^2 + \left[y^3\right]_2^3$

= 2 × 3 + (27 – 8)

= 25 J

10. The approximate height from the surface of earth at which the weight of the body becomes $\frac{1}{3}$ of its weight on the surface of earth is

[Radius of earth R = 6400 km and $\sqrt{3} = 1.732$]

(A) 3840 km (B) 4685 km

(C) 2133 km (D) 4267 km

Answer (B)

Sol. According to the given information

$$\frac{GM}{(R+h)^2} = \frac{1}{3}\times\frac{GM}{R^2}$$

$\Rightarrow$ $R + h = \sqrt{3}R$

$\Rightarrow$ $h = \left(\sqrt{3}-1\right)R \simeq 4685$ km

11. A resistance of 40 Ω is connected to a source of alternating current rated 220 V, 50 Hz. Find the time taken by the current to change from its maximum value to the rms value :

(A) 2.5 ms (B) 1.25 ms

(C) 2.5 s (D) 0.25 s

Answer (A)

Sol. $I = I_0\cos(\omega t)$ say

$\Rightarrow$ At maximum $\omega t_1 = 0$ or $t_1 = 0$

Then at rms value $I = I_0/\sqrt{2}$

$\Rightarrow$ $\omega t_2 = \pi/4$

$\Rightarrow$ $\omega(t_2 - t_1) = \pi/4$

$$\Delta t = \frac{\pi}{4\omega} = \frac{\pi T}{4\times 2\pi}$$

$$= \frac{1}{400}\text{ s or 2.5 ms}$$

$\Rightarrow$ Option A is right answer

12. The equations of two waves are given by :

$y_1 = 5\sin 2\pi(x - vt)$ cm

$y_2 = 3\sin 2\pi(x - vt + 1.5)$ cm

These waves are simultaneously passing through a string. The amplitude of the resulting wave is :

(A) 2 cm

(B) 4 cm

(C) 5.8 cm

(D) 8 cm

Answer (A)

Sol. $y_1 = 5\sin(2\pi x - 2\pi vt)$

$y_2 = 3\sin(2\pi x - 2\pi vt + 3\pi)$

$\Rightarrow$ Phase difference = 3π

$\Rightarrow$ $A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(3\pi)}$

$\Rightarrow$ A_{net} = 2 cm

13. A plane electromagnetic waves travels in a medium of relative permeability 1.61 and relative permittivity 6.44. If magnitude of magnetic intensity is 4.5×10^{-2} Am^{-1} at a point, what will be the approximate magnitude of electric field intensity at that point?

(Given : Permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ NA^{-2}, speed of light in vacuum $c = 3 \times 10^8$ ms^{-1})

(A) 16.96 Vm^{-1}

(B) 2.25×10^{-2} Vm^{-1}

(C) 8.48 Vm^{-1}

(D) 6.75×10^6 Vm^{-1}

Answer (C)

Sol. $H = 4.5 \times 10^{-2}$

So $B = \mu_0\mu H$

Thus $E = \frac{c}{n}B$ (where $n \Rightarrow$ refractive index)

$$\text{So } E = \frac{3\times10^8\times4\pi\times10^{-7}\times1.61\times4.5\times10^{-2}}{\sqrt{1.61\times6.44}}$$

$E = 8.48$

14. Choose the correct option from the following options given below :

(A) In the ground state of Rutherford's model electrons are in stable equilibrium. While in Thomson's model electrons always experience a net-force

(B) An atom has a nearly continuous mass distribution in a Rutherford's model but has a highly non-uniform mass distribution in Thomson's model

(C) A classical atom based on Rutherford's model is doomed to collapse.

(D) The positively charged part of the atom possesses most of the mass in Rutherford's model but not in Thomson's model.

Answer (C)

Sol. An atom based on classical theory of Rutherford's model should collapse as the electrons in continuous circular motion that is a continuously accelerated charge should emit EM waves and so should lose energy. These electrons losing energy should soon fall into heavy nucleus collapsing the whole atom.

15. Nucleus *A* is having mass number 220 and its binding energy per nucleon is 5.6 MeV. It splits in two fragments '*B*' and '*C*' of mass numbers 105 and 115. The binding energy of nucleons in '*B*' and '*C*' is 6.4 MeV per nucleon. The energy *Q* released per fission will be :

(A) 0.8 MeV (B) 275 MeV

(C) 220 MeV (D) 176 MeV

Answer (D)

Sol. $^{220}A \rightarrow {}^{105}B + {}^{115}C$

$\Rightarrow$ Q = [105 × 6.4 + 115×6.4] – [220 × 5.6] MeV

$\Rightarrow$ Q = 176 MeV

16. A baseband signal of 3.5 MHz frequency is modulated with a carrier signal of 3.5 GHz frequency using amplitude modulation method. What should be the minimum size of antenna required to transmit the modulated signal?

(A) 42.8 m (B) 42.8 mm

(C) 21.4 mm (D) 21.4 m

Answer (C)

Sol. $\nu_c = 3.5 \times 10^9$ Hz

$$\therefore \quad \lambda = \frac{c}{\nu_c} = \frac{3\times10^8}{3.5\times10^9}$$

$$\therefore \quad \text{Size of antenna} = \frac{\lambda}{4}$$

$$= \frac{8.57\times10^{-2}}{4}$$

= 21.4 mm

17. A Carnot engine whose heat sinks at 27°C, has an efficiency of 25%. By how many degrees should the temperature of the source be changed to increase the efficiency by 100% of the original efficiency?

(A) Increases by 18°C (B) Increases by 200°C

(C) Increases by 120°C (D) Increases by 73°C

Answer (B)

Sol. Initially : $\frac{1}{4} = 1 - \frac{300}{T_H}$

$\Rightarrow$ T_H = 400 K

Finally : Efficiency becomes $\frac{1}{2}$

$$\Rightarrow \quad \frac{1}{2} = 1 - \frac{300}{T_H'}$$

$$\Rightarrow \quad T_H' = 600 \text{ K}$$

$\Rightarrow$ Temperature of the source increases by 200°C.

18. A parallel plate capacitor is formed by two plates each of area 30π cm^2 separated by 1 mm. A material of dielectric strength 3.6×10^7 Vm^{-1} is filled between the plates. If the maximum charge that can be stored on the capacitor without causing any dielectric breakdown is 7×10^{-6} C, the value of dielectric constant of the material is :

[Use $\frac{1}{4\pi\varepsilon_0} = 9\times10^9$ Nm^2 C^{-2}]

(A) 1.66 (B) 1.75

(C) 2.25 (D) 2.33

Answer (D)

Sol. Field inside the dielectric $= \frac{\sigma}{k\varepsilon_0}$

According to the given information,

$$\frac{\sigma}{k\varepsilon_0} = 3.6\times10^7$$

$$\Rightarrow \quad \frac{\frac{Q}{A}}{k\varepsilon_0} = 3.6\times10^7$$

$\Rightarrow$ k = 2.33

19. The magnetic field at the centre of a circular coil of radius *r*, due to current *I* flowing through it, is *B*. The magnetic field at a point along the axis at a distance $\frac{r}{2}$ from the centre is :

(A) $\frac{B}{2}$ (B) $2B$

(C) $\left(\frac{2}{\sqrt{5}}\right)^3 B$ (D) $\left(\frac{2}{\sqrt{3}}\right)^3 B$

Answer (C)

Sol. $B = \frac{\mu_0 I}{2r}$

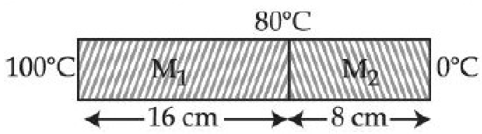
$$B_a = \frac{\mu_0 I r^2}{2\left(r^2 + \frac{r^2}{4}\right)}$$

$$\Rightarrow \frac{B_a}{B} = \left(\frac{2}{\sqrt{5}}\right)^3$$

$$\Rightarrow B_a = \left(\frac{2}{\sqrt{5}}\right)^3 B$$

20. Two metallic blocks M_1 and M_2 of same area of cross-section are connected to each other (as shown in figure). If the thermal conductivity of M_2 is K then the thermal conductivity of M_1 will be :

[Assume steady state heat conduction]



(A) $10K$

(B) $8K$

(C) $12.5K$

(D) $2K$

Answer (B)

Sol. Thermal current is same so

$$\frac{dQ}{dt} = \frac{\Delta T_1}{\frac{l_1}{K_1 A}} = \frac{\Delta T_2}{\frac{l_2}{K_2 A}}$$

$$\text{or } \frac{20}{16} \times K' = \frac{80}{8} \times K$$

$$\Rightarrow K' = 8\,K$$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. 0.056 kg of Nitrogen is enclosed in a vessel at a temperature of 127°C. The amount of heat required to double the speed of its molecules is ___ k cal.

(Take R = 2 cal mol^{-1} K^{-1})

Answer (12)

Sol. Because the vessel is closed, it will be an isochoric process.

To double the speed, temperature must be 4 times

$(v \,\alpha \sqrt{T})$

So T_f = 1600 K, T_i = 400 K

number of moles are $\frac{56}{28} = 2$

$$\text{so } Q = nCv\,\Delta T = 2 \times \frac{5}{2} \times 2 \times 1200$$

= 12000 cal = 12 K cal

2. Two identical thin biconvex lenses of focal length 15 cm and refractive index 1.5 are in contact with each other. The space between the lenses is filled with a liquid of refractive index 1.25. The focal length of the combination is ___ cm.

Answer (10)

Sol. $\frac{1}{f_l} = \left(\frac{\mu_e}{\mu_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

here $|R_1| = |R_2| = R$

$$\Rightarrow \frac{1}{f_{l_1}} = (1.5-1)\left(\frac{2}{R}\right) = \frac{1}{15}$$

$$\Rightarrow \frac{1}{R} = \frac{1}{15} \text{ or } R = 15 \text{ cm}$$

for the concave lens made up of liquid

$$\frac{1}{f_{l_2}} = (1.25-1)\left(-\frac{2}{R}\right) = -\frac{1}{30} \text{ cm}$$

now for equivalent lens

$$\frac{1}{f_e} = \frac{2}{f_{l_1}} + \frac{1}{f_{l_2}}$$

$$= \frac{2}{15} - \frac{1}{30} = \frac{3}{30} = \frac{1}{10}$$

or f_e = 10 cm

3. A transistor is used in common-emitter mode in an amplifier circuit. When a signal of 10 mV is added to the base-emitter voltage, the base current changes by 10 μA and the collector current changes by 1.5 mA. The load resistance is 5 kΩ. The voltage gain of the transistor will be ___.

Answer (750)

Sol. $R_B = \frac{10\times10^{-3}}{10\times10^{-6}}$

$= 10^3\ \Omega$

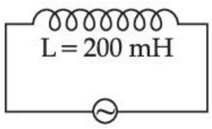
$\therefore \quad A_V = \left(\frac{\Delta I_C}{\Delta I_B}\right)\times\left(\frac{R_C}{R_B}\right)$

$= \frac{1.5\times10^{-3}}{10\times10^{-6}}\times\frac{5\times10^3}{1\times10^3}$

$= \frac{1.5\times5}{10}\times(1000)$

= 750

4. As shown in the figure an inductor of inductance 200 mH is connected to an AC source of emf 220 V and frequency 50 Hz. The instantaneous voltage of the source is 0 V when the peak value of current is $\frac{\sqrt{a}}{\pi}$ A. The value of a is ___.



Answer (242)

Sol. $I_{rms} = \frac{V_{rms}}{z}$

$z = X_2 = \omega_2$

$= 2\pi\times50\times\frac{200}{1000}$

$= 20\ \pi$

$\therefore \quad I_{rms} = \frac{220}{20\pi} = \frac{11}{\pi}$

$\therefore \quad I_{peak} = \sqrt{2}\times\frac{11}{\pi}$

$= \frac{\sqrt{2\times121}}{\pi}$

$= \frac{\sqrt{242}}{\pi}$

5. Sodium light of wavelengths 650 nm and 655 nm is used to study diffraction at a single slit of aperture 0.5 mm. The distance between the slit and the screen is 2.0 m. The separation between the positions of the first maxima of diffraction pattern obtained in the two cases is _____ × 10^{-5} m.

Answer (3)

Sol. Position of 1st maxima is $\frac{3}{2}\frac{\lambda D}{a}$

$\Rightarrow$ According to given values, required separation

$= \frac{3}{2}\times(655\text{ nm} - 650\text{ nm})\times\frac{2\text{m}}{0.5\text{ mm}}$

$\Rightarrow$ Required separation = 3 × 10^{-5} m.

6. When light of frequency twice the threshold frequency is incident on the metal plate, the maximum velocity of emitted electron is v_1. When the frequency of incident radiation is increased to five times the threshold value, the maximum velocity of emitted electron becomes v_2. If $v_2 = x\,v_1$, the value of x will be ________.

Answer (2)

Sol. Let us say the work function is ϕ

$\Rightarrow \quad 2\phi = \phi + \frac{1}{2}mv_1^2 \qquad \ldots (1)$

and $5\phi = \phi + \frac{1}{2}mv_2^2 \qquad \ldots (2)$

From (1) and (2)

$\frac{v_2^2}{v_1^2} = \frac{4}{1}$ or $\frac{v_2}{v_1} = 2$

7. From the top of a tower, a ball is thrown vertically upward which reaches the ground in 6 s. A second ball thrown vertically downward from the same position with the same speed reaches the ground in 1.5 s. A third ball released, from the rest from the same location, will reach the ground in ____ s.

Answer (3)

Sol. Based on the situation

$h = -ut_1 + \frac{1}{2}gt_1^2$ → throwing up …(i)

$h = ut_2 + \frac{1}{2}gt_2^2$ → throwing down …(ii)

$h = \frac{1}{2}gt^2$ → dropping … (iii)

and $0 = u(t_1 - t_2) - \frac{1}{2}g(t_1 - t_2)^2$ …(iv)

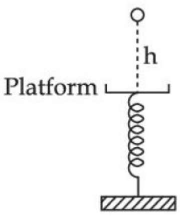
solving above equations

$t = \sqrt{t_1 t_2}$

$\Rightarrow t = \sqrt{6 \times 1.5} = 3\,\text{s}$

8. A ball of mass 100 g is dropped from a height h =10 cm on a platform fixed at the top of a vertical spring (as shown in figure). The ball stays on the platform and the platform is depressed by a distance $\frac{h}{2}$. The spring constant is ______ Nm^{-1}.

(Use g = 10 ms^{-2})



Answer (120)

Sol. $mg\left(h + \frac{h}{2}\right) = \frac{1}{2}k\left(\frac{h}{2}\right)^2$

$\Rightarrow 0.1 \times 10 \times (0.15) = \frac{1}{2}k(0.05)^2$

$\Rightarrow k$ =120 N/m

9. In a potentiometer arrangement, a cell gives a balancing point at 75 cm length of wire. This cell is now replaced by another cell of unknown emf. If the ratio of the emf's of two cells respectively is 3 : 2, the difference in the balancing length of the potentiometer wire in above two cases will be _____ cm.

Answer (25)

Sol. At balancing point, we know that emf is proportional to the balancing length. i.e.,

emf $\propto$ balancing length

Now, let the emf's be 3ε and 2ε.

$\Rightarrow 3\varepsilon = k(75)$ …(1)

and $2\varepsilon = k(l)$ …(2)

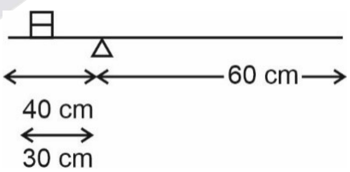
$\Rightarrow l$ = 50 cm

$\Rightarrow$ Difference is (75 – 50) cm = 25 cm.

10. A metre scale is balanced on a knife edge at its centre. When two coins, each of mass 10 g are put one on the top of the other at the 10.0 cm mark the scale is found to be balanced at 40.0 cm mark. The mass of the metre scale is found to be $x \times 10^{-2}$ kg. The value of x is _________.

Answer (6)

Sol.



If λ is the mass per unit length of the scale then

$0.02 \times (30) \times 10 + \lambda 40 \times 20 \times 10 = \lambda 60 \times 30 \times 10$

$0.006 = \lambda 10$

Or $100\,\lambda = 0.06$ kg

$= 6 \times 10^{-2}$ kg

$\Rightarrow x = 6$

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. If a rocket runs on a fuel ($C_{15}H_{30}$) and liquid oxygen, the weight of oxygen required and CO_2 released for every litre of fuel respectively are :

   (Given : density of the fuel is 0.756 g/mL)

   (A) 1188 g and 1296 g (B) 2376 g and 2592 g

   (C) 2592 g and 2376 g (D) 3429 g and 3142 g

Answer (C)

Sol. $C_{15}H_{30} + \frac{45}{2}O_2 \rightarrow 15CO_2 + 15H_2O$

One litre of fuel has a mass (0.756) × 1000 g.

$\therefore$ moles of $C_{15}H_{30} = \frac{756}{210}$

Moles of O_2 required $= \frac{45}{2} \times \frac{756}{210}$

Mass of O_2 required $= \frac{45}{2} \times \frac{756}{210} \times 32 \text{ g} = 2592 \text{ g}$

Mass of CO_2 formed $= 15 \times \frac{756}{210} \times 44 = 2376 \text{ g}$

2. Consider the following pairs of electrons

   (A) (a) $n = 3, l = 1, m_l = 1, m_s = +\frac{1}{2}$

   (b) $n = 3, l = 2, m_l = 1, m_s = +\frac{1}{2}$

   (B) (a) $n = 3, l = 2, m_l = -2, m_s = -\frac{1}{2}$

   (b) $n = 3, l = 2, m_l = -1, m_s = -\frac{1}{2}$

   (C) (a) $n = 4, l = 2, m_l = 2, m_s = +\frac{1}{2}$

   (b) $n = 3, l = 2, m_l = 2, m_s = +\frac{1}{2}$

   The pairs of electrons present in degenerate orbitals is /are:

   (A) Only (A) (B) Only (B)

   (C) Only (C) (D) (B) and (C)

Answer (B)

Sol. For degenerate orbitals, only the value of m must be different. The value of 'n' and 'l' must be the same.

Hence, the pair of electrons with quantum numbers given in (B) are degenerate.

3. Match **List-I** with **List-II** :

List-I	**List-II**
(A) $[PtCl_4]^{2-}$	(I) sp^3d
(B) BrF_5	(II) d^2sp^3
(C) PCl_5	(III) dsp^2
(D) $[Co(NH_3)_6]^{3+}$	(IV) sp^3d^2

   Choose the **most appropriate** answer from the options given below.

   (A) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

   (B) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

   (C) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

   (D) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

Answer (B)

Sol.

Complex/compound	Hybridisation of central atoms
(A) $[PtCl_4]^{-2}$	(III) dsp^2
(B) BrF_5	(IV) sp^3d^2
(C) PCl_5	(I) sp^3d
(D) $[Co(NH_3)_6]^{+3}$	(II) d^2sp^3

Hence, the most appropriate answer is given in option (B)

4. For a reaction at equilibrium

$$A(g) \rightleftharpoons B(g) + \frac{1}{2}C(g)$$

   the relation between dissociation constant (K), degree of dissociation (α) and equilibrium pressure (p) is given by :

(A) $K = \dfrac{\alpha^{\frac{1}{2}} p^{\frac{3}{2}}}{\left(1+\frac{3}{2}\alpha\right)^{\frac{1}{2}}(1-\alpha)}$

(B) $K = \dfrac{\alpha^{\frac{3}{2}} p^{\frac{1}{2}}}{(2+\alpha)^{\frac{1}{2}}(1-\alpha)}$

(C) $K = \dfrac{(\alpha p)^{\frac{3}{2}}}{\left(1+\frac{3}{2}\alpha\right)^{\frac{1}{2}}(1-\alpha)}$

(D) $K = \dfrac{(\alpha p)^{\frac{3}{2}}}{(1+\alpha)(1-\alpha)^{\frac{1}{2}}}$

Answer (B)

Sol.

	$A(g)$	$\rightleftharpoons$ $B(g)$	+ $\frac{1}{2}C(g)$
$t = 0$	p_i	–	–
$t = t$ (eq. m conditions)	$p_i - p_i\alpha$	$p_i\alpha$	$\frac{p_i\alpha}{2}$

$\therefore$ P (equilibrium pressure) $= p_i - p_i\alpha + p_i\alpha + \dfrac{p_i\alpha}{2}$

$$= p_i\left(1+\frac{\alpha}{2}\right)$$

$$\therefore \quad p_i = \frac{p}{\left(1+\frac{\alpha}{2}\right)}$$

$$K_P = \frac{\left(p_i\frac{\alpha}{2}\right)^{\frac{1}{2}} \times p_i\alpha}{p_i(1-\alpha)} = \frac{p^{\frac{1}{2}}\alpha^{\frac{3}{2}}}{\left(1+\frac{\alpha}{2}\right)^{\frac{1}{2}}(1-\alpha)} \times \frac{1}{2^{\frac{1}{2}}}$$

$$= \frac{p^{\frac{1}{2}}\alpha^{\frac{3}{2}}}{(2+\alpha)^{\frac{1}{2}}(1-\alpha)}$$

Hence the correct option is (B)

5. Given below are two statements:

Statement I : Emulsion of oil in water are unstable and sometimes they separate into two layers on standing.

Statement II : For stabilisation of an emulsion, excess of electrolyte is added.

In the light of the above statements, choose the most appropriate answer from the options given below:

(A) Both **Statement I** and **Statement II** are correct

(B) Both **Statement I** and **Statement II** are incorrect.

(C) **Statement I** is correct but **Statement II** is incorrect.

(D) **Statement I** is incorrect but **Statement II** is correct.

Answer (C)

Sol. Oil in water emulsions can sometimes separate into two layers on standing.

The most relevant example for the above case is milk, which can separate into two layers on standing for a longer time. Therefore, statement (I) is correct.

On adding excess of electrolyte, coagulation occurs and emulsion is further destabilised.

Therefore, statement (II) is incorrect.

6. Given below are the oxides:

Na_2O, As_2O_3, N_2O, NO and Cl_2O_7

Number of amphoteric oxides is:

(A) 0 (B) 1

(C) 2 (D) 3

Answer (B)

Sol. Oxides

$Na_2O \longrightarrow$ Basic

$As_2O_3 \longrightarrow$ Amphoteric

$N_2O \longrightarrow$ Neutral

$NO \longrightarrow$ Neutral

$Cl_2O_7 \longrightarrow$ Acidic

Hence, only one amphoteric oxide is present.

7. Match **List-I** with **List -II** :

List-I	**List-II**
(A) Sphalerite	(I) $FeCO_3$
(B) Calamine	(II) PbS
(C) Galena	(III) $ZnCO_3$
(D) Siderite	(IV) ZnS

Choose the **most appropriate** answer from the options given below:

(A) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(B) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

(C) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

(D) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Answer (A)

Sol.

Ores	Formula
(A) Sphalerite	(IV) ZnS
(B) Calamine	(III) $ZnCO_3$
(C) Galena	(II) PbS
(D) Siderite	(I) $FeCO_3$

Hence, the most appropriate option is (A).

8. The highest industrial consumption of molecular hydrogen is to produce compounds of element:

(A) Carbon (B) Nitrogen

(C) Oxygen (D) Chlorine

Answer (B)

Sol. Hydrogen combines with nitrogen to produce Ammonia in Haber's process.

$$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$$

In this process, iron oxide is used with small amounts of K_2O and Al_2O_3 to increase the rate of attainment of equilibrium.

Optimum conditions for the production of ammonia are a pressure of 200 atm and a temperature of 700K.

Earlier, iron was used as a catalyst with molybdenum as promoter in this reaction.

9. Which of the following statements are **correct**?

(A) Both LiCl and $MgCl_2$ are soluble in ethanol.

(B) The oxides Li_2O and MgO combine with excess of oxygen to give superoxide.

(C) LiF is less soluble in water than other alkali metal fluorides.

(D) Li_2O is more soluble in water than other alkali metal oxides.

Choose the **most appropriate** answer from the options given below:

(A) (A) and (C) only (B) (A), (C) and (D) only

(C) (B) and (C) only (D) (A) and (D) only

Answer (A)

Sol. (A) Both LiCl and $MgCl_2$ are covalent in nature due to high polarizing power of Li^+ and Mg^{+2} ions. Hence, they are soluble in ethanol.

(A) Oxides of Li_2O and MgO do not form superoxide

(B) LiF is least soluble among all other alkali metal fluorides due to high lattice energy of LiF

(C) Li_2O is least soluble among all other alkali metal oxides.

Hence, Statements (A) and (C) are correct.

10. Identify the correct statement for B_2H_6 from those given below:

(A) In B_2H_6, all B-H bonds are equivalent.

(B) In B_2H_6, there are four 3-centre-2-electron bonds.

(C) B_2H_6 is a Lewis acid.

(D) B_2H_6 can be synthesized from both BF_3 and $NaBH_4$.

(E) B_2H_6 is a planar molecule.

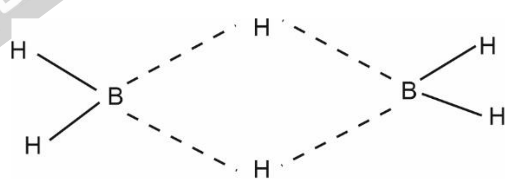
Choose the **most appropriate** answer from the options given below:

(A) (A) and (E) only (B) (B), (C) and (E) only

(C) (C) and (D) only (D) (C) and (E) only

Answer (C)

Sol. Structure of B_2H_6



It has two 3-centre-2-electron bonds and four 2-centre-2-electron bonds.

Hence, all B–H bonds are not equivalent.

It is an electron deficient compound as the octet of boron is incomplete.

Hence, it can behave as a Lewis acid.

It can be synthesized from both BF_3 and $NaBH_4$

$$2BF_3 + 6NaH \xrightarrow{450\,K} B_2H_6 + 6NaF$$

$$2NaBH_4 + I_2 \longrightarrow B_2H_6 + 2NaI + H_2$$

It is a non-planar molecule.

Hence, only Statements (C) and (D) are correct.

11. The most stable trihalide of nitrogen is:

(A) NF_3 (B) NCl_3

(C) NBr_3 (D) NI_3

Answer (A)

Sol. The most stable trihalide is NF_3

Order of stability: $NF_3 > NCl_3 > NBr_3 > NI_3$

NCl_3 is explosive is nature.

NBr_3 and NI_3 are known only as ammoniates. The stability of trihalides decreases down the group due to weakening of N – X bond and inability of N to accommodate large sized halogen atoms (Cl, Br, I) around it.

12. Which one of the following elemental forms is **not** present in the enamel of the teeth?

(A) Ca^{2+}

(B) P^{3+}

(C) F^-

(D) P^{5+}

Answer (B)

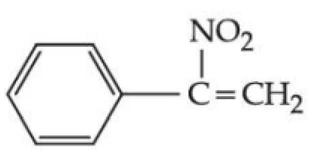
Sol. P^{+3} is not present is enamel of teeth.

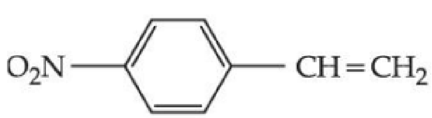
Th compound present is $[3Ca_3(PO_4)_2 \cdot CaF_2]$

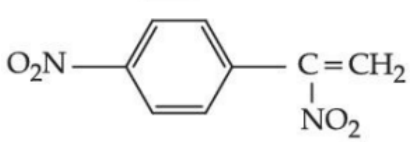
Which contains Ca^{+2}, P^{+5} & F^-

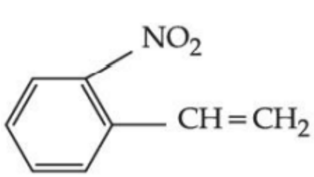
13. In the given reaction sequence, the major product 'C' is:

$$C_8H_{10} \xrightarrow[H_2SO_4]{HNO_3} A \xrightarrow[\Delta]{Br_2} B \xrightarrow[KOH]{alcoholic} C$$

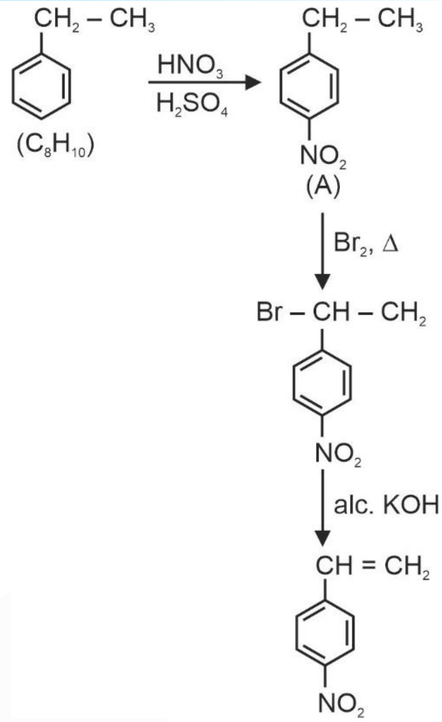
(A) 

(B) 

(C) 

(D) 

Answer (B)

Sol. 

14. Two statements are given below:

Statement I: The melting point of monocarboxylic acid with even number of carbon atoms is higher than that of with odd number of carbon atoms acid immediately below and above it in the series.

Statement II: The solubility of monocarboxylic acids in water decreases with increase in molar mass.

Choose the **most appropriate** option:

(A) Both **Statement I** and **Statement II** are correct.

(B) Both **Statement I** and **Statement II** are incorrect.

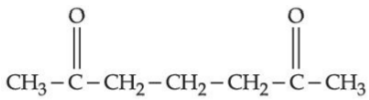
(C) **Statement I** is correct but **Statement II** is incorrect.

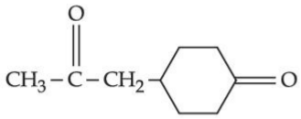
(D) **Statement I** is incorrect but **Statement II** is correct.

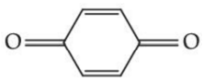
Answer (A)

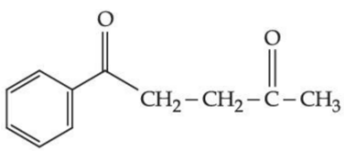
Sol. Statement (I) is correct as monocarboxylic acids with even number of carbon atoms show better packing efficiency in solid state, statement (II) is also correct as the solubility of carboxylic acids decreases with increase in molar mass due to increase in the hydrophobic portion with increase in the number of carbon atoms.

15. Which of the following is an example of conjugated diketone?

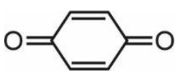
(A) 

(B) 

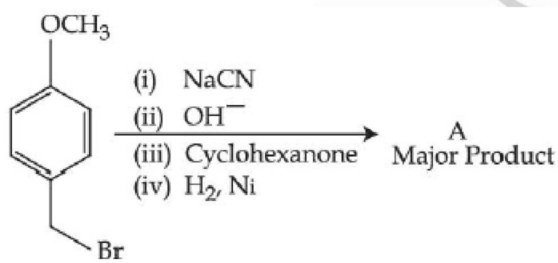
(C) 

(D) 

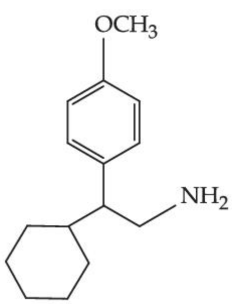
Answer (C)

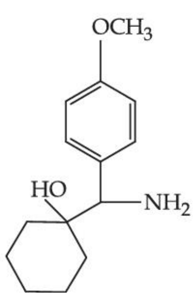
Sol.  is a conjugated diketone.

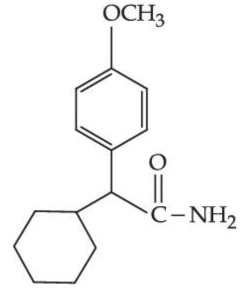
In rest of the diketones given in the question, the two (C = O) groups are not in conjugation with each other.

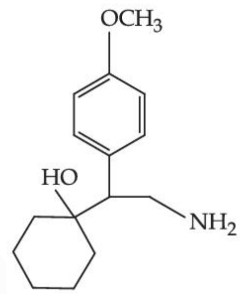
16. 

The major product of the above reactions is :

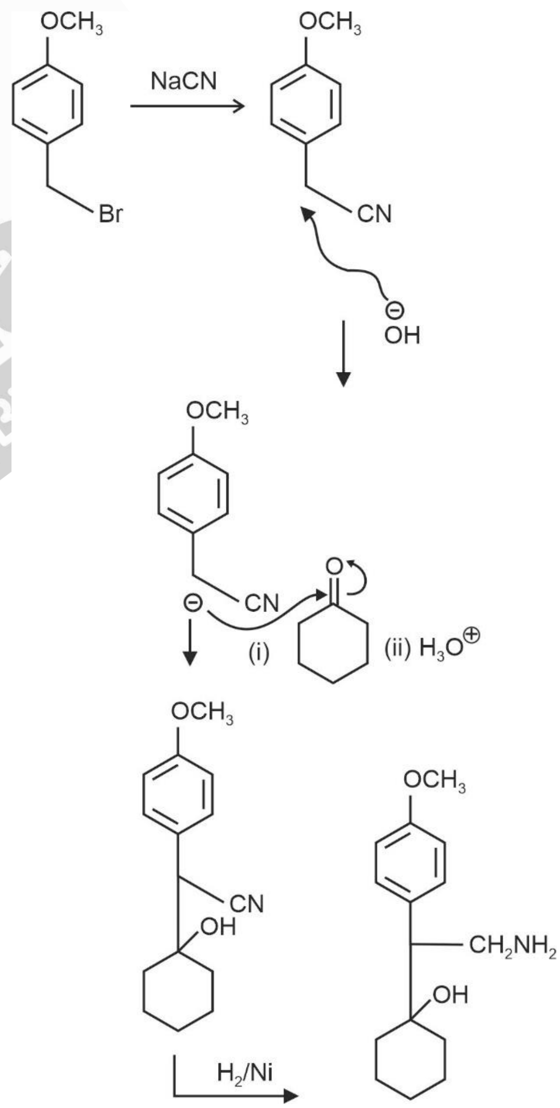
(A) 

(B) 

(C) 

(D) 

Answer (D)

Sol. 

Hence, the correct option is (D).

17. Which of the following is an example of polyester?

(A) Butadiene-styrene copolymer

(B) Melamine polymer

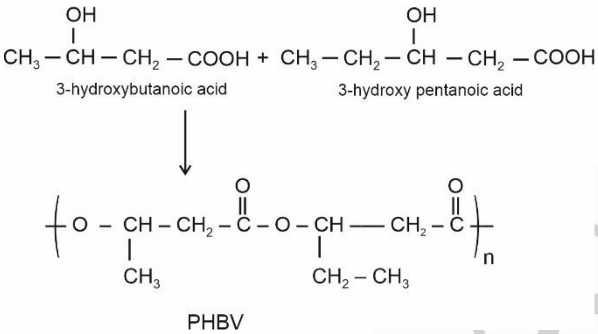
(C) Neoprene

(D) Poly-β-hydroxybutyrate-co-β-hydroxy valerate

Answer (D)

Sol. Polyesters are formed by condensation reaction between alcohols and carboxylic acid.

Poly-β-hydroxybutyrate-co-β-hydroxy valerate (PHBV) is a polymer obtained by condensation reaction of 3-hydroxybutanoic acid with 3-hydroxypentanoic acid.



Hence, PHBV is a polyester.

18. A polysaccharide 'X' on boiling with dil. H_2SO_4 at 393 K under 2-3 atm pressure yields 'Y'. 'Y' on treatment with bromine water gives gluconic acid. 'X' contains β-glycosidic linkages only. Compound 'X' is:

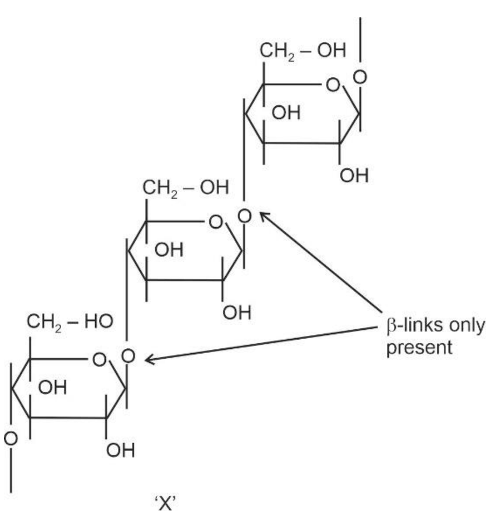
(A) starch (B) cellulose

(C) amylose (D) amylopectin

Answer (B)

Sol. Cellulose contains β-glycosidic linkages only.

Structure of cellulose



On boiling with dil. H_2SO_4 at 393 K under 2-3 atm, 'X' forms glucose, which given gluconic acid on treatment with bromine water.

19. Which of the following is not a broad-spectrum antibiotic?

(A) Vancomycin

(B) Ampicillin

(C) Ofloxacin

(D) Penicillin G

Answer (D)

Sol. Penicillin G is a narrow spectrum antibiotic. (Based on fact)

20. During the qualitative analysis of salt with cation y^{2+}, addition of a reagent (X) to alkaline solution of the salt gives a bright red precipitate. The reagent (X) and the cation (y^{2+}) present respectively are:

(A) Dimethylglyoxime and Ni^{2+}

(B) Dimethylglyoxime and Co^{2+}

(C) Nessler's reagent and Hg^{2+}

(D) Nessler's reagent and Ni^{2+}

Answer (A)

Sol. On addition of dimethylglyoxime to alkaline solution of Ni^{+2}, a bright red ppt. is obtained.

Ni^{+2} +2dmg → $[Ni(dmg)_2]^{+2}$ (Bright red ppt)

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Atoms of element X form hcp lattice and those of element Y occupy $\frac{2}{3}$ of its tetrahedral voids. The percentage of element X in the lattice is ________ . (Nearest integer)

Answer (43)

Sol. Since X occupies hcp lattice,

Number of particles of type X in a unit cell = 6

Number of particles of type Y = $\frac{2}{3} \times 12 = 8$

$\therefore$ Percentage of element X = $\frac{6}{14} \times 100$

$= \frac{300}{7}$

$= 42.85$

$\simeq 43\%$

2. $2O_3(g) \rightleftharpoons 3O_2(g)$

At 300 K, ozone is fifty percent dissociated. The standard free energy change at this temperature and 1 atm pressure is (–) ____ J mol^{-1}. (Nearest integer)

[Given: In 1.35 = 0.3 and R = 8.3 J K^{-1} mol^{-1}]

Answer (747)

Sol. $\underset{1-x}{2O_3(g)} \rightleftharpoons \underset{\frac{3x}{2}}{3O_2(g)}$

Given, x = 0.5

$\therefore\ k_p = \frac{[3(0.5)]^3 \times 1}{[2]^3 \times (0.5)^2 \times 1.25}$

$\therefore\ k_p = \frac{27}{8} \times \frac{0.5}{1.25} = 1.35$

$\Delta G^\circ = -2.303\ RT \log k_p$

$= -2.303 \times 8.3 \times 300 \log 1.35$

$= -8.3 \times 300 \ln(1.35)$

$= -747$ J mol^{-1}

3. The osmotic pressure of blood is 7.47 bar at 300 K. To inject glucose to a patient intravenously, it has to be isotonic with blood. The concentration of glucose solution in gL^{-1} is _______. (Molar mass of glucose = 180 g mol^{-1}

R = 0.083 L bar K^{-1} mol^{-1}) (Nearest integer)

Answer (54)

Sol. 7.47 = C × 0.083 × 300

(π = CRT)

(Where C represents the concentration of glucose solution and π represents osmotic pressure)

$C = \frac{7.47}{0.083 \times 300} (\text{mol L}^{-1})$

which in gm/L = $\frac{7.47}{0.083 \times 300} \times 180$

= 54 gm/l

4. The cell potential for the following cell

Pt $|H_2(g)|H^+$ (aq)|| Cu^{2+} (0.01 M)|Cu(s)

is 0.576 V at 298 K. The pH of the solution is ___ .

(Nearest integer)

(Given: $E^0_{Cu^{2+}/Cu} = 0.34$ V and $\frac{2.303\ RT}{F} = 0.06$ V)

Answer (5)

Sol. $E_{cell} = E^0_{cell} - \frac{0.06}{2} \log \frac{[H^{\oplus}]^2}{[Cu^{+2}]}$

$0.576 = 0.34 - 0.03 \log \frac{[H^{\oplus}]^2}{[0.01]}$

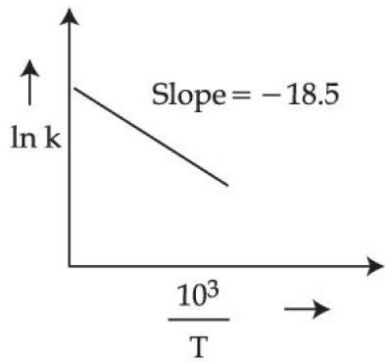
$0.576 - 0.34 = -0.03 \log [H^{\oplus}]^2 + 0.03 \log(0.01)$

$= 0.06\ pH - 0.06$

$pH \simeq 4.93 \simeq 5$

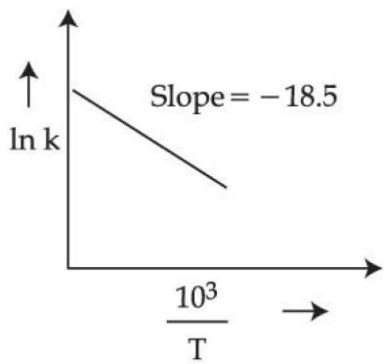
5. The rate constants for decomposition of acetaldehyde have been measured over the temperature range 700 – 1000 K. The data has been analysed by plotting In k vs $\frac{10^3}{T}$ graph. The value of activation energy for the reaction is _______ kJ mol^{-1}. (Nearest integer)

(Given : R = 8.31 J K^{-1} mol^{-1})



Answer (154)

Sol.



$\ln k = \ln A - \frac{E_a}{RT}$

$\therefore$ Slope of the graph $= -\frac{E_a}{R \times 10^3} = -18.5$

$\therefore \; E_a = 18.5 \times 8.31 \times 1000 \simeq 154 \text{ kJ mol}^{-1}$

6. The difference in oxidation state of chromium in *chromate* and *dichromate* salts is _______,

Answer (0)

Sol. Chromate ion $\rightarrow CrO_4^{2-}$, oxidation state of Cr = +6

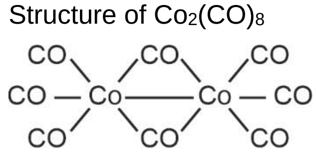
Dichromate ion $\rightarrow CrO_4^{2-}$, oxidation state of Cr = +6

$\therefore$ Difference in oxidation state = zero

7. In the cobalt-carbonyl complex: [$Co_2(CO)_8$], number of Co-Co bonds is "X" and terminal CO ligands is "Y". X + Y = ________.

Answer (7)

Sol. Structure of $Co_2(CO)_8$



Number of Co – Co bonds = 1 = X

Number of terminal CO ligands = 6 = Y

$\therefore$ X + Y = 1 + 6 = 7

8. A 0.166 g sample of an organic compound was digested with conc. H_2SO_4 and then distilled with NaOH. The ammonia gas evolved was passed through 50.0 mL of 0.5 N H_2SO_4. The used acid required 30.0 mL of 0.25 N NaOH for complete neutralisation. The mass percentage of nitrogen in the organic compound is _________.

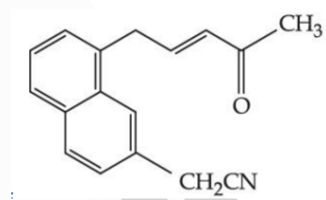
Answer (63)

Sol. Millimoles of used acid = $\frac{30 \times 0.25}{2}$

Millimoles of NH_3 = 30 × 0.25 = 7.5

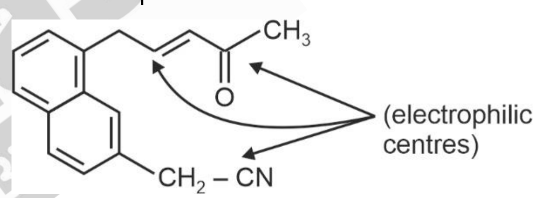
Mass% of nitrogen = $\frac{7.5}{0.166} \times 10^{-3} \times 14 \times 100 \simeq 63\%$

9. Number of electrophilic centres in the given compound is ______.



Answer (3)

Sol. Given compounds :



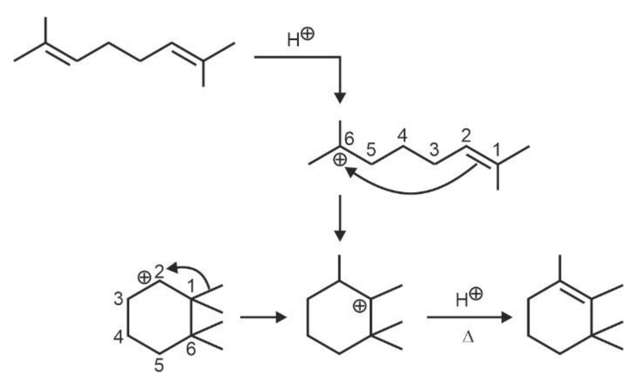
Number of electrophilic centres = 3

10. The major product 'A' of the following given reaction has _________ sp^2 hybridized carbon atoms.

2, 7-Dimethyl-2, 6-octadiene $\xrightarrow{H^+}$ $\underset{\text{Major Product}}{A}$

Answer (2)

Sol.



Number of sp^2 hybridised carbon atoms = 2

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Let $A = \{z \in \mathbf{C} : 1 \le |z - (1 + i)| \le 2\}$ and $B = \{z \in A : |z - (1 - i)| = 1\}$. Then, B :

   (A) Is an empty set

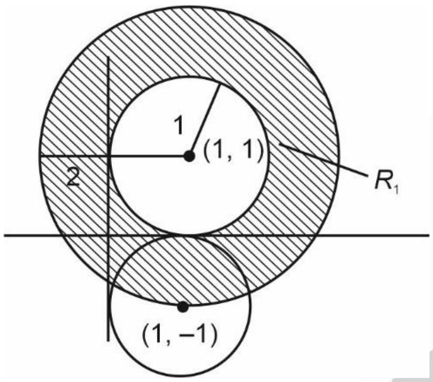
   (B) Contains exactly two elements

   (C) Contains exactly three elements

   (D) Is an infinite set

Answer (D)

Sol.



Set A represents region 1 *i.e.* R_1 and clearly set B has infinite points in it.

2. The remainder when 3^{2022} is divided by 5 is :

   (A) 1 (B) 2

   (C) 3 (D) 4

Answer (D)

Sol. $3^{2022} = (10 - 1)^{1011} = {}^{1011}C_0(10)^{1011}(-1)^0 + {}^{1011}C_1(10)^{1010}(-1)^1 + \ldots\ldots + {}^{1011}C_{1010}(10)^1(-1)^{1010} + {}^{1011}C_{1011}(10)^0(-1)^{1011}$

$= 5k - 1$, where $k \in I$

So when divided by 5, it leaves remainder 4.

3. The surface area of a balloon of spherical shape being inflated, increases at a constant rate. If initially, the radius of balloon is 3 units and after 5 seconds, it becomes 7 units, then its radius after 9 seconds is :

   (A) 9 (B) 10

   (C) 11 (D) 12

Answer (A)

Sol. $S = 4\pi r^2$

$$\frac{dS}{dt} = 8\pi r\frac{dr}{dt}$$

$$\frac{dS}{dt} = \text{constant so} \Rightarrow r\frac{dr}{dt} = k \text{ (Let)}$$

$$r\,dr = k\,dt \Rightarrow \frac{r^2}{2} = kt + C$$

at $t = 0$, $r = 3$

$$\frac{9}{2} = C$$

at $t = 5$,

$$\frac{49}{2} = k \cdot 5 + \frac{9}{2} \Rightarrow k = 4$$

At $t = 9$, $\dfrac{r^2}{2} = \dfrac{81}{2}$

So, $r = 9$

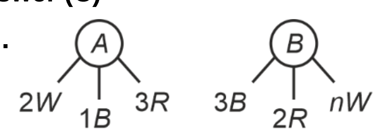
4. Bag A contains 2 white, 1 black and 3 red balls and bas B contains 3 black, 2 red and n white balls. One bag is chosen at random and 2 balls drawn from it at random, are found to be 1 red and 1 black. If the probability that both balls come from Bag A is $\frac{6}{11}$, then n is equal to ________.

   (A) 13 (B) 6

   (C) 4 (D) 3

Answer (C)

Sol.



$$P(1R \text{ and } 1B) = P(A)\cdot P\left(\frac{1R\ 1B}{A}\right) + P(B)\cdot P\left(\frac{1R\ 1B}{B}\right)$$

$$= \frac{1}{2}\cdot\frac{{}^3C_1 \cdot {}^1C_1}{{}^6C_2} + \frac{1}{2}\cdot\frac{{}^2C_1 \cdot {}^3C_1}{{}^{n+5}C_2}$$

$$P\left(\frac{1R\ 1B}{A}\right) = \frac{\frac{1}{2}\cdot\frac{3}{15}}{\frac{1}{2}\cdot\frac{3}{15} + \frac{1}{2}\cdot\frac{6\cdot 2}{(n+5)(n+4)}} = \frac{6}{11}$$

$$\Rightarrow \frac{\frac{1}{10}}{\frac{1}{10} + \frac{6}{(n+5)(n+4)}} = \frac{6}{11}$$

$\Rightarrow \quad \frac{11}{10} = \frac{6}{10} + \frac{36}{(n+5)(n+4)}$

$\Rightarrow \quad \frac{5}{10 \times 36} = \frac{1}{(n+5)(n+4)}$

$\Rightarrow \quad n^2 + 9n - 52 = 0$

$\Rightarrow \quad n = 4$ is only possible value

5. Let $x^2 + y^2 + Ax + By + C = 0$ be a circle passing through (0, 6) and touching the parabola $y = x^2$ at (2, 4). Then $A + C$ is equal to ________.

(A) 16

(B) $\frac{88}{5}$

(C) 72

(D) –8

Answer (A)

Sol. For tangent to parabola $y = x^2$ at (2, 4)

$\left.\frac{dy}{dx}\right|_{(2,\,4)} = 4$

Equation of tangent is

$y - 4 = 4(x - 2)$

$\Rightarrow \quad 4x - y - 4 = 0$

Family of circle can be given by

$(x - 2)^2 + (y - 4)^2 + \lambda(4x - y - 4) = 0$

As it passes through (0, 6)

$2^2 + 2^2 + \lambda(-10) = 0$

$\Rightarrow \quad \lambda = \frac{4}{5}$

Equation of circle is

$(x - 2)^2 + (y - 4)^2 + \frac{4}{5}(4x - y - 4) = 0$

$\Rightarrow \quad (x^2 + y^2 - 4x - 8y + 20) + \left(\frac{16}{5}x - \frac{4}{5}y - \frac{16}{5}\right) = 0$

$A = -4 + \frac{16}{5},\ C = 20 - \frac{16}{5}$

So, $A + C = 16$

6. The number of values of α for which the system of equations :

$x + y + z = \alpha$

$\alpha x + 2\alpha y + 3z = -1$

$x + 3\alpha y + 5z = 4$

is inconsistent, is

(A) 0 (B) 1

(C) 2 (D) 3

Answer (B)

Sol. $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ \alpha & 2\alpha & 3 \\ 1 & 3\alpha & 5 \end{vmatrix}$

$= 1(10\alpha - 9\alpha) - 1(5\alpha - 3) + 1(3\alpha^2 - 2\alpha)$

$= \alpha - 5\alpha + 3 + 3\alpha^2 - 2\alpha$

$= 3\alpha^2 - 6\alpha + 3$

For inconsistency $\Delta = 0$ i.e. $\alpha = 1$

Now check for $\alpha = 1$

$x + y + z = 1$ …(i)

$x + 2y + 3z = -1$ …(ii)

$x + 3y + 5z = 4$ …(iii)

By (ii) × 2 – (i) × 1

$x + 3y + 5z = -3$

so equations are

inconsistent for $\alpha = 1$

7. If the sum of the squares of the reciprocals of the roots α and β of the equation $3x^2 + \lambda x - 1 = 0$ is 15, then $6(\alpha^3 + \beta^3)^2$ is equal to :

(A) 18 (B) 24

(C) 36 (D) 96

Answer (B)

Sol. $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = 15 \Rightarrow \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha^2\beta^2} = 15$

$\Rightarrow \quad \frac{\frac{\lambda^2}{9} + \frac{2}{3}}{\frac{1}{9}} = 15$

$\Rightarrow \quad \frac{\lambda^2}{9} = 1 \Rightarrow \lambda^2 = 9$

$\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$

$= \left(\frac{-\lambda}{3}\right)\left(\frac{\lambda^2}{9} - 3\left(\frac{-1}{3}\right)\right) = \left(\frac{-\lambda}{3}\right)\left(\frac{\lambda^2}{9} + 1\right) = \frac{-2\lambda}{3}$

$6\left(\alpha^3 + \beta^3\right)^2 = 6 \cdot \frac{4\lambda^2}{9} = 24$

8. The set of all values of k for which $\left(\tan^{-1}x\right)^3 + \left(\cot^{-1}x\right)^3 = k\pi^3, x \in R$, is the interval:

(A) $\left[\frac{1}{32}, \frac{7}{8}\right)$ (B) $\left(\frac{1}{24}, \frac{13}{16}\right)$

(C) $\left[\frac{1}{48}, \frac{13}{16}\right]$ (D) $\left[\frac{1}{32}, \frac{9}{8}\right)$

Answer (A)

Sol. Let $\tan^{-1}x = t \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$$\cot^{-1}x = \frac{\pi}{2} - t$$

$$f(t) = t^3 + \left(\frac{\pi}{2} - t\right)^3 \Rightarrow f'(t) = 3t^2 - 3\left(\frac{\pi}{2} - t\right)^2$$

$f'(t) = 0$ at $t = \frac{\pi}{4}$

$$f(t)|_{\min} = \frac{\pi^3}{64} + \frac{\pi^3}{64} = \frac{\pi^3}{32}$$

Max will occur around $t = -\frac{\pi}{2}$

Range of $f(t) = \left[\frac{\pi^3}{32}, \frac{7\pi^3}{8}\right)$

$$k \in \left[\frac{1}{32}, \frac{7}{8}\right)$$

9. Let $S = \left\{\sqrt{n} : 1 \le n \le 50 \text{ and } n \text{ is odd}\right\}$.

Let $a \in S$ and $A = \begin{bmatrix} 1 & 0 & a \\ -1 & 1 & 0 \\ -a & 0 & 1 \end{bmatrix}$

If $\sum_{a \in S} \det \ (\text{adj } A) = 100\lambda$, then λ is equal to :

(A) 218

(B) 221

(C) 663

(D) 1717

Answer (B)

Sol. $|A| = a^2 + 1$

$|adj\ A| = (a^2 + 1)^2$

$$S = \left\{1, \sqrt{3}, \sqrt{5}, \sqrt{7}, \ldots, \sqrt{49}\right\}$$

$\sum_{a \in S} \det(adj\ A) = (1^2 + 1)^2 + (3 + 1)^2 + (5 + 1)^2 + \ldots + (49 + 1)^2$

$= 2^2 (1^2 + 2^2 + 3^2 + \ldots + 25^2)$

$$= 4.\frac{25 \cdot 26 \cdot 51}{6} = 100 \cdot 221$$

$\lambda = 221$

10. For the function

$f(x) = 4 \log_e(x - 1) - 2x^2 + 4x + 5, x > 1$, which one of the following is NOT correct?

(A) f is increasing in (1, 2) and decreasing in (2, ∞)

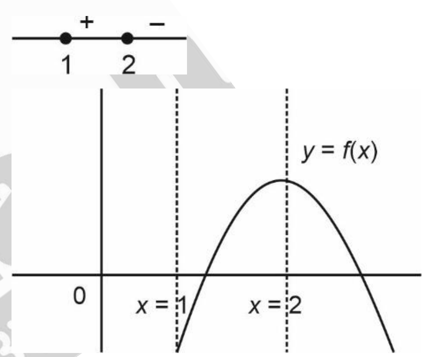
(B) $f(x) = -1$ has exactly two solutions

(C) $f'(e) - f''(2) < 0$

(D) $f(x) = 0$ has a root in the interval $(e, e + 1)$

Answer (C)

Sol. $f'(x) = \frac{4}{x-1} - 4x + 4 = \frac{4\left(2x - x^2\right)}{x-1}$



So maxima occurs at $x = 2$

$f(2) = 4 \cdot 0 - 2 \cdot 2^2 + 4 \cdot 2 + 5 = 5$

so clearly $f(x) = -1$ has

exactly 2 solutions

$$f''(x) = \frac{4(2-2x)(x-1)}{(x-1)^2} - \left(2x - x^2\right)$$

so $f'(e) - f''(2) > 0$

so option c is not correct

11. If the tangent at the point (x_1, y_1) on the curve $y = x^3 + 3x^2 + 5$ passes through the origin, then (x_1, y_1) does NOT lie on the curve :

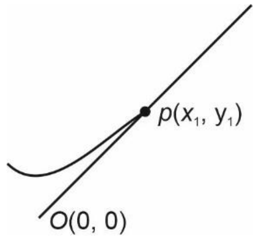
(A) $x^2 + \frac{y^2}{81} = 2$ (B) $\frac{y^2}{9} - x^2 = 8$

(C) $y = 4x^2 + 5$ (D) $\frac{x}{3} - y^2 = 2$

Answer (D)

Sol. $m_{op} - m_{Tangent}$

$$\frac{y_1}{x_1} = 3x_1^2 + 6x_1$$



$$\Rightarrow \frac{x_1^3 + 3x_1^2 + 5}{x_1} = 3x_1^2 + 6x_1$$

$$\Rightarrow x_1^3 + 3x_1^2 + 5 = 3x_1^3 + 6x_1^2$$

$$\Rightarrow 2x_1^3 + 3x_1^2 - 5 = 0$$

$$\Rightarrow (x_1 - 1)(2x_1^2 + 5x_1 + 5) = 0$$

So, $(x_1, y_1) = (1, 9)$

12. The sum of absolute maximum and absolute minimum values of the function $f(x) = |2x^2 + 3x - 2| + \sin x \cos x$ in the interval [0, 1] is :

(A) $3 + \dfrac{\sin(1)\cos^2\left(\frac{1}{2}\right)}{2}$

(B) $3 + \frac{1}{2}(1 + 2\cos(1))\sin(1)$

(C) $5 + \frac{1}{2}(\sin(1) + \sin(2))$

(D) $2 + \sin\left(\frac{1}{2}\right)\cos\left(\frac{1}{2}\right)$

Answer (B)

Sol. $f(x) = |(2x-1)(x+2)| + \dfrac{\sin 2x}{2}$

$\underline{0 \le x < \frac{1}{2}}$ $\quad f(x) = (1-2x)(x+2) + \dfrac{\sin 2x}{2}$

$f'(x) = -4x - 3 + \cos 2x < 0$

For $x \ge \frac{1}{2}$: $\quad f'(x) = 4x + 3 + \cos 2x > 0$

So, minima occurs at $x = \frac{1}{2}$

$$f(x)|_{min} = \left|2\left(\frac{1}{2}\right)^2 + \frac{3}{2} - 2\right| + \sin\left(\frac{1}{2}\right).\cos\left(\frac{1}{2}\right)$$

$$= \frac{1}{2}\sin 1$$

So, maxima is possible at $x = 0$ or $x = 1$

Now checking for $x = 0$ and $x = 1$, we can see it attains its maximum value at $x = 1$

$$f(x)|_{max} = |2 + 3 - 2| + \frac{\sin 2}{2}$$

$$= 3 + \frac{1}{2}\sin 2$$

Sum of absolute maximum and minimum value

$$= 3 + \frac{1}{2}(\sin 1 + \sin 2)$$

13. If $\{a_i\}_{i=1}^{n}$, where n is an even integer, is an arithmetic progression with common difference 1, and $\sum_{i=1}^{n} a_i = 192$, $\sum_{i=1}^{n/2} a_{2i} = 120$, then n is equal to :

(A) 48 (B) 96

(C) 92 (D) 104

Answer (B)

Sol. $a_1 + a_2 + \ldots + a_n = 192 \Rightarrow \frac{n}{2}(a_1 + a_n) = 192$ ...(1)

$a_2 + a_4 + a_6 + \ldots + a_n = 120$

$\Rightarrow \frac{n}{4}(a_1 + 1 + a_n) = 120$ ...(2)

From (2) & (1)

$$\frac{480}{n} - \frac{384}{n} = 1 \Rightarrow n = 96$$

14. If $x = x(y)$ is the solution of the differential equation $y\dfrac{dx}{dy} = 2x + y^3(y+1)e^y$, $x(1) = 0$; then $x(e)$ is equal to :

(A) $e^3(e^e - 1)$ (B) $e^e(e^3 - 1)$

(C) $e^2(e^e + 1)$ (D) $e^e(e^2 - 1)$

Answer (A)

Sol. $\dfrac{dx}{dy} - \dfrac{2x}{y} = y^2(y+1)e^y$

$$\text{If} = e^{\int -\frac{2}{y}dy} = e^{-2\ln y} = \frac{1}{y^2}$$

Solution is given by

$$x.\frac{1}{y^2} = \int y^2(y+1)e^y.\frac{1}{y^2}dy$$

$$\Rightarrow \frac{x}{y^2} = \int (y+1)e^y dy$$

$$\Rightarrow \frac{x}{y^2} = ye^y + c$$

$\Rightarrow$ $x = y^2 (ye^y + c)$

at, $y = 1$, $x = 0$

$\Rightarrow$ $0 = 1(1.e^1 + c) \Rightarrow c = -e$

at $y = e$,

$x = e^2(e.e^e - e)$

15. Let $\lambda x - 2y = \mu$ be a tangent to the hyperbola $a^2x^2 - y^2 = b^2$. Then $\left(\frac{\lambda}{a}\right)^2 - \left(\frac{\mu}{b}\right)^2$ is equal to :

(A) –2 (B) –4

(C) 2 (D) 4

Answer (D)

Sol. $\frac{x^2}{\left(\frac{b^2}{a^2}\right)} - \frac{y^2}{b^2} = 1$

Tangent in slope form $\Rightarrow$ $y = mx \pm \sqrt{\frac{b^2}{a^2}m^2 - b^2}$

i.e., same as $y = \frac{\lambda x}{2} - \frac{\mu}{2}$

Comparing coefficients,

$$m = \frac{\lambda}{2}, \frac{b^2}{a^2}m^2 - b^2 = \frac{\mu^2}{4}$$

Eliminating m, $\frac{b^2}{a^2}.\frac{\lambda^2}{4} - b^2 = \frac{\mu^2}{4}$

$$\Rightarrow \frac{\lambda^2}{a^2} - \frac{\mu^2}{b^2} = 4$$

16. Let $\hat{a}, \hat{b}$ be unit vectors. If $\vec{c}$ be a vector such that the angle between $\hat{a}$ and $\vec{c}$ is $\frac{\pi}{12}$, and $\hat{b} = \vec{c} + 2(\vec{c} \times \hat{a})$, then $|6\vec{c}|^2$ is equal to:

(A) $6(3 - \sqrt{3})$ (B) $3 + \sqrt{3}$

(C) $6(3 + \sqrt{3})$ (D) $6(\sqrt{3} + 1)$

Answer (C)

Sol. $\because$ $\hat{b} = \vec{c} + 2(\vec{c} \times \hat{a})$

$\Rightarrow$ $\hat{b} \cdot \vec{c} = |\vec{c}|^2$ ...(i)

$\therefore$ $\hat{b} - \vec{c} = 2(\vec{c} \times \vec{a})$

$\Rightarrow$ $|\hat{b}|^2 + |\vec{c}|^2 - 2\,\hat{b} \cdot \vec{c} = 4|\vec{c}|^2 |\vec{a}|^2 \sin^2 \frac{\pi}{12}$

$\Rightarrow$ $1 + |\vec{c}|^2 - 2|\vec{c}|^2 = 4|\vec{c}|^2 \left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2$

$\Rightarrow$ $1 = |\vec{c}|^2 (3 - \sqrt{3})$

$\Rightarrow$ $36|\vec{c}|^2 = \frac{36}{3 - \sqrt{3}} = 6(3 + \sqrt{3})$

17. If a random variable X follows the Binomial distribution $B(33, p)$ such that $3P(X = 0) = P(X = 1)$, then the value of $\frac{P(X=15)}{P(X=18)} - \frac{P(X=16)}{P(X=17)}$ is equal to:

(A) 1320 (B) 1088

(C) $\frac{120}{1331}$ (D) $\frac{1088}{1089}$

Answer (A)

Sol. $3P(X = 0) = P(X = 1)$

$3 \cdot {}^nC_0 P^0 (1-P)^n = {}^nC_1 P^1 (1-P)^{n-1}$

$\frac{3}{n} = \frac{P}{1-P} \Rightarrow \frac{1}{11} = \frac{P}{1-P}$

$\Rightarrow$ $1 - P = 11P$

$\Rightarrow$ $P = \frac{1}{12}$

$\frac{P(X=15)}{P(X=18)} - \frac{P(X=16)}{P(X=17)}$

$\Rightarrow$ $\frac{{}^{33}C_{15} P^{15} (1-P)^{18}}{{}^{33}C_{18} P^{18} (1-P)^{15}} - \frac{{}^{33}C_{16} P^{16} (1-P)^{17}}{{}^{33}C_{17} P^{17} (1-P)^{16}}$

$\Rightarrow$ $\left(\frac{1-P}{P}\right)^3 - \left(\frac{1-P}{P}\right)$

$\Rightarrow$ $11^3 - 11 = 1320$

18. The domain of the function $f(x) = \frac{\cos^{-1}\left(\frac{x^2 - 5x + 6}{x^2 - 9}\right)}{\log_e(x^2 - 3x + 2)}$ is:

(A) $(-\infty, 1) \cup (2, \infty)$

(B) $(2, \infty)$

(C) $\left[-\frac{1}{2}, 1\right) \cup (2, \infty)$

(D) $\left[-\frac{1}{2}, 1\right) \cup (2, \infty) - \left\{\frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}\right\}$

Answer (D)

Sol. $-1 \le \dfrac{x^2 - 5x + 6}{x^2 - 9} \le 1$ and $x^2 - 3x + 2 > 0, \ne 1$

$\dfrac{(x-3)(2x+1)}{x^2-9} \ge 0 \;\Big|\; \dfrac{5(x-3)}{x^2-9} \ge 0$

Solution to this inequality is

$x \in \left[\dfrac{-1}{2}, \infty\right) - \{3\}$

for $x^2 - 3x + 2 > 0$ and $\ne 1$

$x \in (-\infty, 1) \cup (2, \infty) - \left\{\dfrac{3-\sqrt{5}}{2}, \dfrac{3+\sqrt{5}}{2}\right\}$

Combining the two solution sets (taking intersection)

$x \in \left[-\dfrac{1}{2}, 1\right) \cup (2, \infty) - \left\{\dfrac{3-\sqrt{5}}{2}, \dfrac{3+\sqrt{5}}{2}\right\}$

19. Let $S = \left\{\theta \in [-\pi, \pi] - \left\{\pm\dfrac{\pi}{2}\right\} : \sin\theta\tan\theta + \tan\theta = \sin 2\theta\right\}$. If $T = \sum_{\theta \in S} \cos 2\theta$, then $T + n(S)$ is equal to:

(A) $7 + \sqrt{3}$

(B) 9

(C) $8 + \sqrt{3}$

(D) 10

Answer (B)

Sol. $\tan\theta\,(\sin\theta + 1) - \sin 2\theta = 0$

$\tan\theta(\sin\theta + 1 - 2\cos^2\theta) = 0$

$\Rightarrow$ $\tan\theta = 0$ or $2\sin^2\theta + \sin\theta - 1 = 0$

$\Rightarrow$ $(2\sin\theta + 1)(\sin\theta - 1) = 0$

$\Rightarrow$ $\sin\theta = \dfrac{-1}{2}$ or 1

But, $\sin\theta = 1$ not possible

$\theta = 0, \pi, -\pi, -\dfrac{\pi}{6}, \dfrac{-5\pi}{6}$

n(S) = 5

$T = \sum \cos 2\theta = \cos 0° + \cos 2\pi + \cos(-2\pi)$

$+ \cos\left(-\dfrac{5\pi}{3}\right) + \cos\left(-\dfrac{\pi}{3}\right)$

= 4

20. The number of choices for $\Delta \in \{\wedge, \vee, \Rightarrow, \Leftrightarrow\}$, such that $(p \,\Delta\, q) \Rightarrow ((p \,\Delta \sim q) \vee ((\sim p) \,\Delta\, q))$ is a tautology, is

(A) 1 (B) 2

(C) 3 (D) 4

Answer (B)

Sol. Let $x : (p\Delta q) \Rightarrow (p\Delta \sim q) \vee (\sim p\Delta q)$

Case-I

When Δ is same as $\vee$

Then $(p\Delta \sim q) \vee (\sim p\Delta q)$ becomes

$(p \vee \sim q) \vee (\sim p \vee q)$ which is always true, so x becomes a tautology.

Case-II

When Δ is same as $\wedge$

Then $(p \wedge q) \Rightarrow (p \wedge \sim q) \vee (\sim p \wedge q)$

If $p \wedge q$ is T, then $(p \wedge \sim q) \vee (\sim p \wedge q)$ is F

so x cannot be a tautology.

Case-III

When Δ is same as $\Rightarrow$

Then $(p \Rightarrow \sim q) \vee (\sim p \Rightarrow q)$ is same at $(\sim p \vee \sim q) \vee (p \vee q)$, which is always true, so x becomes a tautology.

Case-IV

When Δ is same as $\Leftrightarrow$

Then $(p \Leftrightarrow q) \Rightarrow (p \Leftrightarrow \sim q) \vee (\sim p \Leftrightarrow q)$

$p \Leftrightarrow q$ is true when p and q have same truth values, then $p \Leftrightarrow \sim q$ and $\sim p \Leftrightarrow q$ both are false. Hence x cannot be a tautology.

So finally x can be $\vee$ or $\Rightarrow$.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse andw the on-screen virtual numeric keypad in the place designated to enter the answer.

1. The number of one-one functions $f : \{a, b, c, d\} \to \{0, 1, 2, ..., 10\}$ such that $2f(a) - f(b) + 3f(c) + f(d) = 0$ is ________.

Answer (31)

Sol. $\because$ $3f(c) + 2f(a) + f(d) = f(b)$

Value of $f(c)$	Value of $f(a)$	Number of functions
0	1	7
	2	5
	3	3
	4	2
1	0	6
	2	2
	3	1
2	0	3
	1	1
3	0	1
	Total Number of functions =	**31**

2. In an examination, there are 5 multiple choice questions with 3 choices, out of which exactly one is correct. There are 3 marks for each correct answer, –2 marks for each wrong answer and 0 mark if the question is not attempted. Then, the number of ways a student appearing in the examination gets 5 marks is _____.

Answer (40)

Sol. Let student marks x correct answers and y incorrect. So

$3x - 2y = 5$ and $x + y \le 5$ where $x, y \in W$

Only possible solution is $(x, y) = (3, 2)$

Student can mark correct answer by only one choice but for incorrect answer, there are two choices. So total number of ways of scoring 5 marks $= {}^5C_3(1)^3.(2)^2 = 40$

3. Let $A\left(\frac{3}{\sqrt{a}}, \sqrt{a}\right), a > 0$, be a fixed point in the xy-plane, The image of A in y-axis be B and the image of B in x-axis be C. If $D(3\cos\theta, a\sin\theta)$ is a point in the fourth quadrant such that the maximum area of ΔACD is 12 square units, then a is equal to ______.

Answer (8)

Sol. Clearly B is $\left(-\frac{3}{\sqrt{a}}, +\sqrt{a}\right)$ and C is $\left(-\frac{3}{\sqrt{a}}, -\sqrt{a}\right)$

$$\text{Area of } \Delta ACD = \frac{1}{2}\begin{vmatrix} \frac{3}{\sqrt{a}} & \sqrt{a} & 1 \\ -\frac{3}{\sqrt{a}} & -\sqrt{a} & 1 \\ 3\cos\theta & a\sin\theta & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 0 & 0 & 1 \\ -\frac{3}{\sqrt{a}} & -\sqrt{a} & 1 \\ 3\cos\theta & a\sin\theta & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = \left|3\sqrt{a}\sin\theta + 3\sqrt{a}\cos\theta\right| = 3\sqrt{a}\left|\sin\theta + \cos\theta\right|$$

$$\Rightarrow \Delta_{\text{max}} = 3\sqrt{a}\cdot\sqrt{2} = 12 \Rightarrow a = \left(2\sqrt{2}\right)^2 = 8$$

4. Let a line having direction ratios 1, –4, 2 intersect the lines $\frac{x-7}{3} = \frac{y-1}{-1} = \frac{z+2}{1}$ and $\frac{x}{2} = \frac{y-7}{3} = \frac{z}{1}$ at the points A and B. Then $(AB)^2$ is equal to ________.

Answer (84)

Sol. Let $A(3\lambda + 7, -\lambda + 1, \lambda - 2)$ and $B(2\mu, 3\mu + 7, \mu)$

So, DR's of $AB \propto 3\lambda - 2\mu + 7, -(\lambda + 3\mu + 6), \lambda - \mu - 2$

Clearly $\frac{3\lambda - 2\mu + 7}{1} = \frac{\lambda + 3\mu + 6}{4} = \frac{\lambda - \mu - 2}{2}$

$\Rightarrow$ $5\lambda - 3\mu = -16$ …(i)

And $\lambda - 5\mu = 10$ …(ii)

From (i) and (ii) we get $\lambda = -5, \mu = -3$

So, A is $(-8, 6, -7)$ and B is $(-6, -2, -3)$

$AB = \sqrt{4 + 64 + 16} \Rightarrow (AB)^2 = 84$

5. The number of points where the function

$$f(x) = \begin{cases} |2x^2 - 3x - 7| & \text{if} \quad x \le -1 \\ \left[4x^2 - 1\right] & \text{if} \quad -1 < x < 1 \\ |x + 1| + |x - 2| & \text{if} \quad x \ge 1, \end{cases}$$

$[t]$ denotes the greatest integer $\le t$, is discontinuous is _____________.

Answer (7)

Sol. $\because$ $f(-1) = 2$ and $f(1) = 3$

For $x \in (-1, 1)$, $(4x^2 - 1) \in [-1, 3)$

hence $f(x)$ will be discontinuous at $x = 1$ and also whenever $4x^2 - 1 = 0$, 1 or 2

$$\Rightarrow \quad x = \pm\frac{1}{2},\ \pm\frac{1}{\sqrt{2}} \text{ and } \pm\frac{\sqrt{3}}{2}$$

So there are total 7 points of discontinuity.

6. Let $f(\theta) = \sin\theta + \int_{-\pi/2}^{\pi/2} (\sin\theta + t\cos\theta) f(t)dt$. Then the value of $\left|\int_0^{\pi/2} f(\theta)d\theta\right|$ is ________.

Answer (1)

Sol. $f(\theta) = \sin\theta\left(1 + \int_{-\pi/2}^{\pi/2} f(t)\,dt\right) + \cos\theta\left(\int_{-\pi/2}^{\pi/2} tf(t)dt\right)$

Clearly $f(\theta) = a\sin\theta + b\cos\theta$

Where $a = 1 + \int_{-\pi/2}^{\pi/2}(a\sin t + b\cos t)\,dt \Rightarrow a = 1 + 2b$ …(1)

and $b = \int_{-\pi/2}^{\pi/2}(at\sin t + bt\cos t)dt \Rightarrow b = 2a$ …(2)

from (1) and (2) we get

$a = -\frac{1}{3}$ and $b = -\frac{2}{3}$

So $f(\theta) = -\frac{1}{3}(\sin\theta + 2\cos\theta)$

$$\Rightarrow \quad \left|\int_0^{\pi/2} f(\theta)\,d\theta\right| = \frac{1}{3}(1 + 2\times 1) = 1$$

7. Let $\underset{0\le x\le 2}{\text{Max}}\left\{\frac{9-x^2}{5-x}\right\} = \alpha$ and $\underset{0\le x\le 2}{\text{Min}}\left\{\frac{9-x^2}{5-x}\right\} = \beta$.

If $\int_{\beta-\frac{8}{3}}^{2\alpha-1} \text{Max}\left\{\frac{9-x^2}{5-x}, x\right\}dx = \alpha_1 + \alpha_2 \log_e\left(\frac{8}{15}\right)$ then $\alpha_1 + \alpha_2$ is equal to ________.

Answer (34)

Sol. Let $f(x) = \frac{x^2-9}{x-5} \Rightarrow f'(x) = \frac{(x-1)(x-9)}{(x-5)^2}$

So, $\alpha = f(1) = 2$ and $\beta = \min(f(0), f(2)) = \frac{5}{3}$

Now, $\int_{-1}^{3} \max\left\{\frac{x^2-9}{x-5}, x\right\}dx = \int_{-1}^{9/5}\frac{x^2-9}{x-5}dx + \int_{9/5}^{3} x\,dx$

$$= \int_{-1}^{9/5}\left(x + 5 + \frac{16}{x-5}\right)dx + \left.\frac{x^2}{2}\right|_{9/5}^{3}$$

$$= \frac{28}{25} + 14 + 16\ln\left(\frac{8}{15}\right) + \frac{72}{25} = 18 + 16\ln\left(\frac{8}{15}\right)$$

Clearly $\alpha_1 = 18$ and $\alpha_2 = 16$, so $\alpha_1 + \alpha_2 = 34$.

8. If two tangents drawn from a point (α, β) lying on the ellipse $25x^2 + 4y^2 = 1$ to the parabola $y^2 = 4x$ are such that the slope of one tangent is four times the other, then the value of $(10\alpha + 5)^2 + (16\beta^2 + 50)^2$ equals ____________.

Answer (2929)

Sol. $\because$ (α, β) lies on the given ellipse, $25\alpha^2 + 4\beta^2 = 1$ …(1)

Tangent to the parabola, $y = mx + \frac{1}{m}$ passes through (α, β). So, $\alpha m^2 - \beta m + 1 = 0$ has roots m_1 and $4m_1$,

$m_1 + 4m_1 = \frac{\beta}{\alpha}$ and $m_1 \cdot 4m_1 = \frac{1}{\alpha}$

Gives that $4\beta^2 = 25\alpha$ …(2)

from (1) and (2)

$25(\alpha^2 + \alpha) = 1$ …(3)

Now, $(10\alpha + 5)^2 + (16\beta^2 + 50)^2$

$= 25(2\alpha + 1)^2 + 2500\,(2\alpha + 1)^2$

$= 2525\,(4\alpha^2 + 4\alpha + 1)$ from equation (3)

$= 2525\left(\frac{4}{25} + 1\right)$

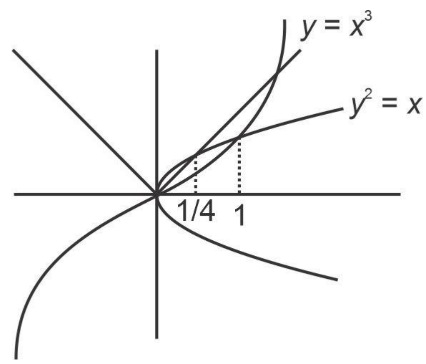
$= 2929$

9. Let S be the region bounded by the curves $y = x^3$ and $y^2 = x$. The curve $y = 2|x|$ divides S into two regions of areas R_1 and R_2.

If max $\{R_1, R_2\} = R_2$, then $\frac{R_2}{R_1}$ is equal to _____.

Answer (19)

Sol.



$C_1 : y = x^3$

$C_2 : y^2 = x$

and $C_3 = y = 2|x|$

C_1 and C_2 intersect at (1, 1)

C_2 and C_3 intersect at $\left(\frac{1}{4}, \frac{1}{2}\right)$

Clearly $R_1 = \int_0^{1/4}\left(\sqrt{x} - 2x\right) dx = \frac{2}{3}\left(\frac{1}{8}\right) - \frac{1}{16} = \frac{1}{48}$

and $R_1 + R_2 = \int_0^1\left(\sqrt{x} - x^3\right) dx = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$

So, $\frac{R_1 + R_2}{R_1} = \frac{5/12}{1/48} \Rightarrow 1 + \frac{R_2}{R_1} = 20$

$\Rightarrow \frac{R_2}{R_1} = 19$

10. If the shortest distance between the lines $\vec{r} = \left(-\hat{i} + 3\hat{k}\right) + \lambda\left(\hat{i} - a\hat{j}\right)$ and $\vec{r} = \left(-\hat{j} + 2\hat{k}\right) + \mu\left(\hat{i} - \hat{j} + \hat{k}\right)$ is $\sqrt{\frac{2}{3}}$, then the integral value of a is equal to ________________.

Answer (2)

Sol. $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -a & 0 \\ 1 & -1 & 1 \end{vmatrix} = -a\hat{i} - \hat{j} + (a-1)\hat{k}$

$\vec{a}_1 - \vec{a}_2 = -\hat{i} + \hat{j} + \hat{k}$

Shortest distance $= \left|\frac{(\vec{a}_1 - \vec{a}_2)\cdot(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$

$\Rightarrow \sqrt{\frac{2}{3}} = \frac{2(a-1)}{\sqrt{a^2 + 1 + (a-1)^2}}$

$\Rightarrow 6\left(a^2 - 2a + 1\right) = 2a^2 - 2a + 2$

$\Rightarrow (a-2)(2a-1) = 0 \quad \Rightarrow a = 2$ because $a \in z$.

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