25/06/2022
Evening

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Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2022 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Given below are two statements. One is labelled as **Assertion A** and the other is labelled as **Reason R**.

   Assertion A: Two identical balls *A* and *B* thrown with same velocity '*u*' at two different angles with horizontal attained the same range *R*. If *A* and *B* reached the maximum height h_1 and h_2 respectively, then $R = 4\sqrt{h_1 h_2}$.

   Reason R: Product of said heights.

   $$h_1 h_2 = \left(\frac{u^2 \sin^2\theta}{2g}\right)\cdot\left(\frac{u^2\cos^2\theta}{2g}\right)$$

   Choose the correct answer :

   (A) Both **A** and **R** are true and **R** is the correct explanation of **A**.

   (B) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**.

   (C) **A** is true but **R** is false.

   (D) **A** is false but **R** is true.

Answer (A)

Sol. $h_1 = \dfrac{u^2\sin^2\theta}{2g}$

$h_2 = \dfrac{u^2\cos^2\theta}{2g}$

$\therefore \quad \sqrt{h_1 h_2} = \dfrac{u^2 \sin\theta\cos\theta}{2g}$

$= \dfrac{R}{4}$

$\Rightarrow \quad R = 4\sqrt{h_1 h_2}$

2. Two buses *P* and *Q* start from a point at the same time and move in a straight line and their positions are represented by $X_P(t) = \alpha t + \beta t^2$ and $X_Q(t) = ft - t^2$. At what time, both the buses have same velocity?

   (A) $\dfrac{\alpha - f}{1+\beta}$

   (B) $\dfrac{\alpha + f}{2(\beta - 1)}$

   (C) $\dfrac{\alpha + f}{2(1+\beta)}$

   (D) $\dfrac{f - \alpha}{2(1+\beta)}$

Answer (D)

Sol. $X_P = \alpha t + \beta t^2$

$X_Q = ft - t^2$

$\therefore \quad V_P = \alpha + 2\beta t$

$V_Q = f - 2t$

$\because \quad V_P = V_Q$

$\Rightarrow \quad \alpha + 2\beta t = f - 2t$

$\Rightarrow \quad t = \dfrac{f - \alpha}{2(1+\beta)}$

3. A disc with a flat small bottom beaker placed on it at a distance *R* from its center is revolving about an axis passing through the center and perpendicular to its plane with an angular velocity ω. The coefficient of static friction between the bottom of the beaker and the surface of the disc is μ. The beaker will revolve with the disc if :

   (A) $R \le \dfrac{\mu g}{2\omega^2}$

   (B) $R \le \dfrac{\mu g}{\omega^2}$

   (C) $R \ge \dfrac{\mu g}{2\omega^2}$

   (D) $R \ge \dfrac{\mu g}{\omega^2}$

Answer (B)

Sol. To move together

$\omega^2 R \le \mu g$

$\Rightarrow \quad R \le \dfrac{\mu g}{\omega^2}$

4. A solid metallic cube having total surface area 24 m^2 is uniformly heated. If its temperature is increased by 10°C, calculate the increase in volume of the cube.

   (Given $\alpha = 5.0 \times 10^{-4}$ °C^{-1}).

   (A) 2.4×10^6 cm^3

   (B) 1.2×10^5 cm^3

   (C) 6.0×10^4 cm^3

   (D) 4.8×10^5 cm^3

Answer (B)

Sol. $6 \times l^2 = 24$

$\Rightarrow$ $l = 2$ m

$\therefore$ $\frac{\Delta V}{V} = 3 \times \frac{\Delta l}{l}$

$\Rightarrow$ $\Delta V = 3 \times (\alpha \Delta T) \times V$

$= 3 \times 5 \times 10^{-4} \times 10 \times (8)$

$= 120 \times 10^{-3}$ m^3

$= 120 \times 10^{-3} \times 10^6$ cm^3

$= 1.2 \times 10^5$ cm^3

5. A copper block of mass 5.0 kg is heated to a temperature of 500°C and is placed on a large ice block. What is the maximum amount of ice that can melt?

[Specific heat of copper : 0.39 J g^{-1} °C^{-1} and latent heat of fusion of water : 335 J g^{-1}]

(A) 1.5 kg (B) 5.8 kg

(C) 2.9 kg (D) 3.8 kg

Answer (C)

Sol. $mL = \Delta Q = ms\Delta T$

$\Rightarrow$ $m = \frac{5 \times 0.39 \times 10^3 \times 500}{335}$

$= 2.9$ kg

6. The ratio of specific heats $\left(\frac{C_p}{C_v}\right)$ in terms of degree of freedom (f) is given by:

(A) $\left(1 + \frac{f}{3}\right)$

(B) $\left(1 + \frac{2}{f}\right)$

(C) $\left(1 + \frac{f}{2}\right)$

(D) $\left(1 + \frac{1}{f}\right)$

Answer (B)

Sol. $\frac{C_P}{C_V} = \gamma$

$C_V = \left(\frac{f}{2}\right)R$ and $C_P - C_V = R$

$\Rightarrow \frac{C_P}{C} = \frac{1 + f/2}{f/2} = 1 + \frac{2}{f}$

7. For a particle in uniform circular motion, the acceleration $\vec{a}$ at any point $P(R, \theta)$ on the circular path of radius R is (when θ is measured from the positive x-axis and v is uniform speed):

(A) $-\frac{v^2}{R}\sin\theta\hat{i} + \frac{v^2}{R}\cos\theta\hat{j}$

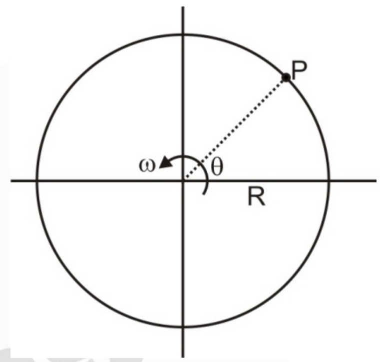
(B) $-\frac{v^2}{R}\cos\theta\hat{i} + \frac{v^2}{R}\sin\theta\hat{j}$

(C) $-\frac{v^2}{R}\cos\theta\hat{i} - \frac{v^2}{R}\sin\theta\hat{j}$

(D) $-\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$

Answer (C)

Sol.



As the particle in uniform circular motion experiences only centripetal acceleration of magnitude ω^2R or $\frac{v^2}{R}$ directed towards centre so from diagram.

$\vec{a} = \frac{v^2}{R}\cos\theta(-\hat{i}) + \frac{v^2}{R}\sin(-\hat{j})$

8. Two metallic plates form a parallel plate capacitor. The distance between the plates is 'd'. A metal sheet of thickness $\frac{d}{2}$ and of area equal to area of each plate is introduced between the plates. What will be the ratio of the new capacitance to the original capacitance of the capacitor?

(A) 2 : 1 (B) 1 : 2

(C) 1 : 4 (D) 4 : 1

Answer (A)

Sol. $C_{eq} = \frac{\varepsilon_0 A}{d - \frac{d}{2} + \frac{d}{2k}} = \frac{\varepsilon_0 A}{\frac{d}{2}} = \frac{2\varepsilon_0 A}{d}$

If $C = \frac{\varepsilon_0 A}{d}$

$\Rightarrow$ $C_{eq} = 2C$ or $\frac{C_{new}}{C_{old}} = \frac{2}{1}$

9. Two cells of same emf but different internal resistances r_1 and r_2 are connected in series with a resistance R. The value of resistance R, for which the potential difference across second cell is zero, is:

(A) $r_2 - r_1$

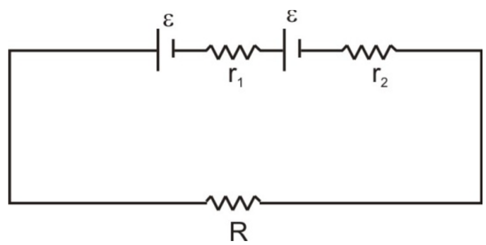
(B) $r_1 - r_2$

(C) r_1

(D) r_2

Answer (A)

Sol.



$$I = \frac{2\varepsilon}{R + r_1 + r_2}$$

As per the question,

$$\frac{2\varepsilon}{R + r_1 + r_2} \times r_2 - \varepsilon = 0$$

$$\Rightarrow \quad R = r_2 - r_1$$

10. Given below are two statements:

Statement-I : Susceptibilities of paramagnetic and ferromagnetic substances increase with decrease in temperature.

Statement-II : Diamagnetism is a result of orbital motions of electrons developing magnetic moments opposite to the applied magnetic field.

Choose the correct answer from the options given below:-

(A) Both Statement-I and Statement-II are true

(B) Both Statement-I and Statement-II are false

(C) Statement-I is true but Statement-II is false

(D) Statement-I is false but Statement-II is true

Answer (A)

Sol. Statement-I is true as susceptibility of ferromagnetic and paramagnetic materials is inversely related to temperature.

Statement-II is true as because of orbital motion of electrons the diamagnetic material is able to oppose external magnetic field.

11. A long solenoid carrying a current produces a magnetic field B along its axis. If the current is doubled and the number of turns per cm is halved, the new value of magnetic field will be equal to

(A) B

(B) $2B$

(C) $4B$

(D) $\frac{B}{2}$

Answer (A)

Sol. $B = \mu_0 n i$

Now $i \rightarrow 2i$

And $n \rightarrow \frac{n}{2}$

$B' = \mu_0 \frac{n}{2} \times 2i = \mu_0 n i = B$

12. A sinusoidal voltage $V(t) = 210 \sin 3000\, t$ volt is applied to a series LCR circuit in which $L = 10$ mH, $C = 25\ \mu$F and $R = 100\ \Omega$. The phase difference (Φ) between the applied voltage and resultant current will be:

(A) $\tan^{-1}(0.17)$

(B) $\tan^{-1}(9.46)$

(C) $\tan^{-1}(0.30)$

(D) $\tan^{-1}(13.33)$

Answer (A)

Sol. $X_L = 3000 \times 10 \times 10^{-3} = 30\Omega$

$$X_C = \frac{1}{3000 \times 25} \times 10^6 = \frac{40}{3}\Omega$$

$$\text{So } X_L - X_C = 30 - \frac{40}{3} = \frac{50}{3}\Omega$$

$$\tan\theta = \frac{X_L - X_C}{R} = \frac{50/3}{100} = \frac{1}{6}$$

So $\theta = \tan^{-1}(0.17)$

13. The electromagnetic waves travel in a medium at a speed of 2.0×10^8 m/s. The relative permeability of the medium is 1.0. The relative permittivity of the medium will be:

(A) 2.25

(B) 4.25

(C) 6.25

(D) 8.25

Answer (A)

Sol. $n = \frac{c}{v} = \frac{3}{2}$

$\sqrt{\in \mu} = n$

So $\in = \frac{9}{4} = 2.25$

14. The interference pattern is obtained with two coherent light sources of intensity ratio 4 : 1. And the ratio $\frac{I_{max}+I_{min}}{I_{max}-I_{min}}$ is $\frac{5}{x}$. Then, the value of x will be equal to:

(A) 3
(B) 4
(C) 2
(D) 1

Answer (B)

Sol. $\frac{I_{max}+I_{min}}{I_{max}-I_{min}} = \frac{I_1+I_2+2\sqrt{I_1I_2}+I_1+I_2-2\sqrt{I_1I_2}}{I_1+I_2+2\sqrt{I_1I_2}-I_1-I_2+2\sqrt{I_1I_2}}$

$$= \frac{2(I_1+I_2)}{4\sqrt{I_1I_2}}$$

$$= \frac{\left(\frac{I_1}{I_2}+1\right)}{2\sqrt{\frac{I_1}{I_2}}} = \frac{4+1}{2\times 2} = \frac{5}{4}$$

So $x = 4$

15. A light whose electric field vectors are completely removed by using a good polaroid, allowed to incident on the surface of the prism at Brewster's angle. Choose the most suitable option for the phenomenon related to the prism.

(A) Reflected and refracted rays will be perpendicular to each other.
(B) Wave will propagate along the surface of prism.
(C) No refraction, and there will be total reflection of light.
(D) No reflection, and there will be total transmission of light.

Answer (D)

Sol. When electric field vector is completely removed and incident on Brewster's angle then only refraction takes place.

16. A proton, a neutron, an electron and an α-particle have same energy. If λ_p, λ_n, λ_e and λ_α are the de Broglie's wavelengths of proton, neutron, electron and α particle respectively, then choose the correct relation from the following:

(A) $\lambda_p = \lambda_n > \lambda_e > \lambda_\alpha$
(B) $\lambda_\alpha < \lambda_n < \lambda_p < \lambda_e$
(C) $\lambda_e < \lambda_p = \lambda_n > \lambda_\alpha$
(D) $\lambda_e = \lambda_p = \lambda_n = \lambda_\alpha$

Answer (B)

Sol. de Broglie wavelength $\lambda = \frac{h}{p}$

$\Rightarrow \lambda = \frac{h}{\sqrt{2mK}}$

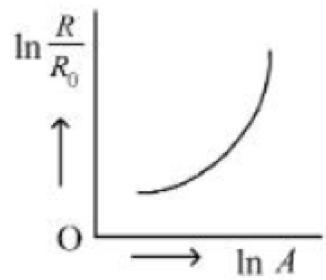
Where K: kinetic energy

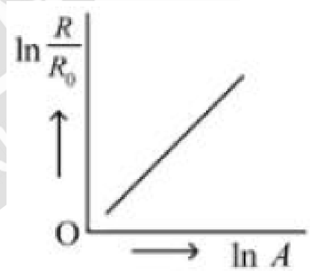
$\Rightarrow$ For some K, $\lambda \propto \frac{1}{\sqrt{m}}$

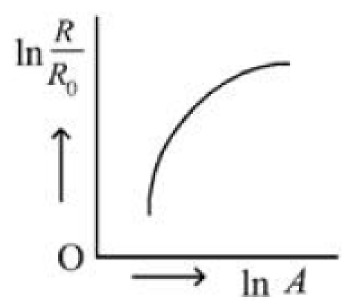
Since $m_\alpha > m_n > m_p > m_e$

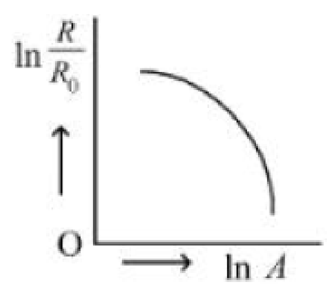
$\Rightarrow \lambda_\alpha < \lambda_n < \lambda_p < \lambda_e$

17. Which of the following figure represents the variation of $\ln\left(\frac{R}{R_0}\right)$ with $\ln A$ (if R = radius of a nucleus and A = its mass number)

(A)


(B)


(C)


(D)


Answer (B)

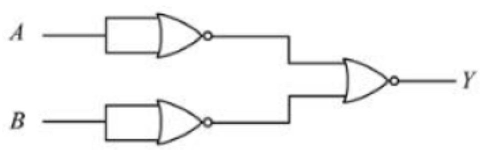
Sol. We know that

$R = R_0A^{1/3}$

$$\Rightarrow \underbrace{\ln\left(\frac{R}{R_0}\right)}_{y} = \underbrace{\frac{1}{3}}_{m}\underbrace{\ln(A)}_{x}$$

⇒ Straight line

18. Identify the logic operation performed by the given circuit:



(A) AND gate

(B) OR gate

(C) NOR gate

(D) NAND gate

Answer (A)

Sol. According to the circuit,

$Y = (A' + B')'$

$\Rightarrow Y = AB$

⇒ AND gate

19. Match List I with List II

	List I		List II
A.	Facsimile	I.	Static Document Image
B.	Guided media Channel	II.	Local Broadcast Radio
C.	Frequency Modulation	III.	Rectangular wave
D.	Digital Single	IV.	Optical Fiber

Choose the correct answer from the following options:

(A) A-IV, B-III, C-II, D-I

(B) A-I, B-IV, C-II, D-III

(C) A-IV, B-II, C-III, D-I

(D) A-I, B-II, C-III, D-IV

Answer (B)

Sol. The correct match is:

Facsimile	–	Static Document Image
Guided Media Channel	–	Optical Fiber
Frequency Modulation	–	Local Broadcast Radio
Digital single	–	Rectangular Wave

20. If n represents the actual number of deflections in a converted galvanometer of resistance G and shunt resistance S. Then the total current I when its figure of merit is K will be

(A) $\frac{KS}{(S+G)}$

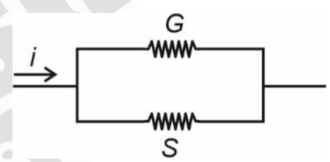
(B) $\frac{(G+S)}{nKS}$

(C) $\frac{nKS}{(G+S)}$

(D) $\frac{nK(G+S)}{S}$

Answer (D)

Sol. According to the information, current through galvanometer = nK



$$\Rightarrow \frac{S}{S+G}i = nK$$

$$\Rightarrow i = \frac{nK(S+G)}{S}$$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. For $z = a^2x^3y^{\frac{1}{2}}$, where '$a$' is a constant. If percentage error in measurement of 'x' and 'y' are 4% and 12%, respectively, then the percentage error for 'z' will be ________ %.

Answer (18)

Sol. % error in $z = 3\times 4+\frac{1}{2}\times 12$

$= 12 + 6 = 18\%$

2. A curved in a level road has a radius 75 m. The maximum speed of a car turning this curved road can be 30 m/s without skidding. If radius of curved road is changed to 48 m and the coefficient of friction between the tyres and the road remains same, then maximum allowed speed would be ______ m/s.

Answer (24)

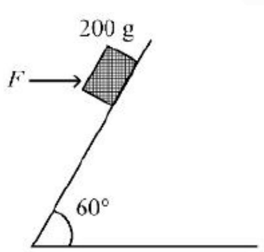
Sol. $\because \quad v=\sqrt{\mu g r}$

$\Rightarrow \frac{v_1}{v_2}=\sqrt{\frac{r_1}{r_2}}$

$\Rightarrow \frac{30}{v_2}=\sqrt{\frac{75}{48}}=\sqrt{\frac{25}{16}}=\frac{5}{4}$

$\Rightarrow V_2 = 24$ m/s

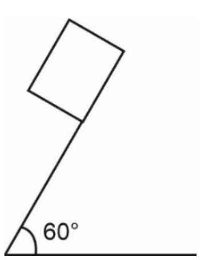
3. A block of mass 200 g is kept stationary on a smooth inclined plane by applying a minimum horizontal force $F=\sqrt{x}$ N as shown in figure.



The value of x = ______.

Answer (12)

Sol.



$F\cos 60° = mg\sin 60°$

$F\times\frac{1}{2}=0.2\times 10\times\frac{\sqrt{3}}{2}$

$\Rightarrow F=2\sqrt{3}$

$\Rightarrow \quad F=\sqrt{12}$ N

$\therefore \quad x = 12$

4. Moment of Inertia (M.I.) of four bodies having same mass 'M' and radius '$2R$' are as follows :

I_1 = M.I. of solid sphere about its diameter

I_2 = M.I. of solid cylinder about its axis

I_3 = M.I. of solid circular disc about its diameter.

I_4 = M.I. of thin circular ring about its diameter

If $2(I_2 + I_3) + I_4 = x\cdot I_1$ then the value of x will be ________.

Answer (5)

Sol. $2\left(\frac{1}{2}+\frac{1}{4}\right)\times M(2R)^2+\frac{1}{2}M(2R)^2=x\frac{2}{5}M(2R)^2$

$\Rightarrow \quad 1+\frac{1}{2}+\frac{1}{2}=x\times\frac{2}{5}$

$\Rightarrow \quad x = 5$

5. Two satellites S_1 and S_2 are revolving in circular orbits around a planet with radius R_1 = 3200 km and R_2 = 800 km respectively. The ratio of speed of satellite S_1 to the speed of satellite S_2 in their respective orbits would be $\frac{1}{x}$ where x =

Answer (2)

Sol. $v=\sqrt{\frac{GM}{R}}$

$\Rightarrow \frac{v_1}{v_2}=\sqrt{\frac{R_2}{R_1}}$

$\frac{v_2}{v_1}=\sqrt{\frac{3200}{800}}=2$

$\Rightarrow \frac{v_1}{v_2}=\frac{1}{2}$

$x = 2$

6. When a gas filled in a closed vessel is heated by raising the temperature by 1°C, its pressure increases by 0.4%. The initial temperature of the gas is _____ K.

Answer (250)

Sol. $PV = nRT$

So, $\frac{dP}{P}\times 100 = \frac{dT}{T}\times 100$

$0.4 = \frac{1}{T}\times 100$

$\Rightarrow T = 250$ K

7. 27 identical drops are charged at 22 V each. They combine to form a bigger drop. The potential of the bigger drop will be ______ V.

Answer (198)

Sol. Let the charge on one drop is q and its radius is r.

So for one drop $V = \frac{kq}{r}$

For 27 drops merged new charge will be $Q = 27q$

and new radius is $R = 3r$

So new potential is

$V' = \frac{kQ}{R} = 9\frac{kq}{r} = 9\times 22\,\text{V}$

$= 198$ V

8. The length of a given cylindrical wire is increased to double of its original length. The percentage increase in the resistance of the wire will be ______%.

Answer (300)

Sol. Volume is constant so on length doubled

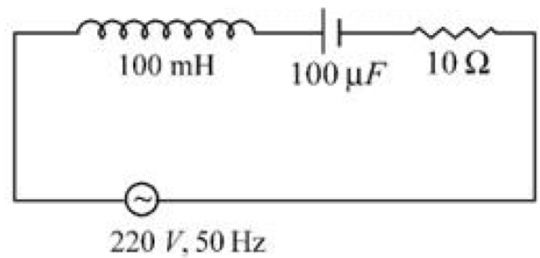
Area is halfed so

$R = \rho\frac{l}{A}$ and $R' = \rho\frac{2l}{\frac{A}{2}} = 4\rho\frac{l}{A} = 4R$

So percentage increase will be

$R\% = \frac{4R-R}{R}\times 100 = 300\%$

9. In a series LCR circuit, the inductance, capacitance and resistance are L = 100 mH, C = 100 μF and R = 10 Ω respectively. They are connected to an AC source of voltage 220 V and frequency of 50 Hz. The approximate value of current in the circuit will be ______ A.



Answer (22)

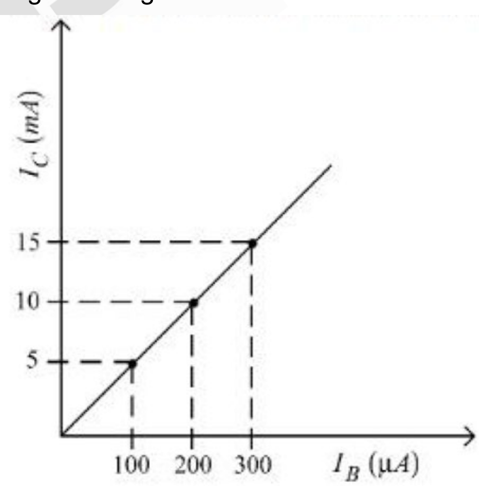
Sol. $Z = \sqrt{R^2 + (x_L - x_C)^2}$

$= \sqrt{10^2 + \left[10\pi - \frac{100}{\pi}\right]^2}\ \Omega$

$\simeq 10\ \Omega$

$\Rightarrow$ Current $= \frac{220}{10}$ A = 22 A

10. In an experiment of *CE* configuration of *n-p-n* transistor, the transfer characteristics are observed as given in figure.



If the input resistance is 200 Ω and output resistance is 60 Ω, the voltage gain in this experiment will be_________.

Answer (15)

Sol. Voltage gain $= \frac{I_C R_o}{I_B R_i}$

$= \frac{(10\text{ mA})(60\ \Omega)}{(200\ \mu\text{A})(200\ \Omega)}$

$\Rightarrow$ Voltage gain = 15

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The minimum energy that must be possessed by photons in order to produce the photoelectric effect with platinum metal is

[Given The threshold frequency of platinum is $1.3 \times 10^{15}\ s^{-1}$ and $h = 6.6 \times 10^{-34}$ Js.]

(A) 3.21×10^{-14} J
(B) 6.24×10^{-16} J
(C) 8.58×10^{-19} J
(D) 9.76×10^{-20} J

Answer (C)

Sol.: The minimum energy possessed by photons will be equal to the work function of the metal.

Hence,

$w_0 = h\nu_0$

$= 6.6 \times 10^{-34} \times 1.3 \times 10^{15}$

$= 8.58 \times 10^{-19}$ J

2. At 25°C and 1 atm pressure, the enthalpy of combustion of benzene (l) and acetylene (g) are –3268 kJ mol^{-1} and –1300 kJ mol^{-1}, respectively. The change in enthalpy for the reaction $3C_2H_2(g) \rightarrow C_6H_6(l)$, is

(A) +324 kJ mol^{-1}
(B) +632 kJ mol^{-1}
(C) –632 kJ mol^{-1}
(D) –732 kJ mol^{-1}

Answer (C)

Sol.: I. $C_6H_6(\ell) + \frac{15}{2}O_2(g) \rightarrow 6CO_2(g) + 3H_2O(g)$

$\Delta H_1 = -3268$ kJ/mol

II. $C_2H_2(g) + \frac{5}{2}O_2(g) \rightarrow 2CO_2(g) + H_2O(g)$

$\Delta H_2 = -1300$ kJ/mol

III. $3C_2H_2(g) \rightarrow C_6H_6(\ell) \quad \Delta H_3$

Applying Hess's law of constant heat summation

$\Delta H_3 = 3 \times \Delta H_2 - \Delta H_1$

$= 3 \times (-1300) - (-3268)$

$= -632$ kJ/mol

3. Solute A associates in water. When 0.7 g of solute A is dissolved in 42.0 gof water, it depresses the freezing point by 0.2°C. The percentage association of solute A in water is :

[Given : Molar mass of A = 93 g mol^{-1}. Molal depression constant of water is 1.86 K kg mol^{-1}.]

(A) 50%

(B) 60%

(C) 70%

(D) 80%

Answer (D)

Sol.: Since, $\Delta T_f = ik_f m$

$$m = \frac{0.7}{93} \times \frac{1000}{42}$$

$$0.2 = i \times 1.86 \times \frac{0.7 \times 1000}{93 \times 42}$$

$i = 0.6$

$$\alpha = \frac{i-1}{\frac{1}{n}-1} = \frac{0.6-1}{\frac{1}{2}-1} = 0.8$$

Hence, percentage association of solute A is 80%.

4. The Ksp for bismuth sulphide (Bi_2S_3) is $1.08 \times 10^{-73.}$ The solubility of Bi_2S_3 in mol L^{-1} at 298 K is

(A) 1.0×10^{-15}

(B) 2.7×10^{-12}

(C) 3.2×10^{-10}

(D) 4.2×10^{-8}

Answer (A)

Sol.: $Bi_2S_3 \rightleftharpoons \underset{2s}{2Bi^{3+}} + \underset{3s}{3S^{2-}}$

$K_{sp} = (2s)^2(3s)^3 = 108s^5$

$108s^5 = 108 \times 10^{-75}$

$s = 1.0 \times 10^{-15}$ mol/L

5. Match List I with List II.

List I		List II	
A	Zymase	I	Stomach
B	Diastase	II	Yeast
C	Urease	III	Malt
D	Pepsin	IV	Soyabean

Choose the correct answer from the options given below

(A) A-II, B-III, C-I, D-IV

(B) A-II, B-III, C-IV, D-I

(C) A-III, B-II, C-IV, D-I

(D) A-III, B-II, C-I, D-IV

Answer (B)

Sol.:

Enzyme		**Source**
Zymase	$\rightarrow$	Yeast
Diastase	$\rightarrow$	Malt
Urease	$\rightarrow$	Soyabean
Pepsin	$\rightarrow$	Stomach

Hence, A-II, B-III, C-IV, D-I

6. The correct order of electron gain enthalpies of Cl, F, Te and Po is

(1) F < Cl < Te < Po

(2) Po < Te < F < Cl

(3) Te < Po < Cl < F

(4) Cl < F < Te < Po

Answer (B)

Sol. Te $\rightarrow$ –190 kJ mol^{-1}

Po $\rightarrow$ –174 kJ mol^{-1}

F $\rightarrow$ –333 kJ mol^{-1}

Cl $\rightarrow$ –349 kJ mol^{-1}

Hence, correct order is Cl > F > Te > Po

7. Given below are two statements.

Statement-I: During electrolytic refining, blister copper deposits precious metals.

Statement-II: In the process of obtaining pure copper by electrolysis method, copper blister is used to make the anode.

In the light of the above statements, choose the correct answer from the options given below.

(1) Both Statement-I and Statement-II are **true**.

(2) Both Statement-I and Statement-II are **false**.

(3) Statement-I is **true** but Statement II is **false**.

(4) Statement-I is **false** but Statement-II is **true**

Answer (A)

Sol. Copper is refined using an electrolytic method. Anodes are of impure copper and pure copper strips are taken as cathode

Impurities from the blister copper deposit as anode mud which contains antimony, selenium, tellurium, silver, gold and platinum.

Hence both statements are true

8. Given below are two statements one is labelled as Assertion A and the other is labelled as Reason R:

Assertion A: The amphoteric nature of water is explained by using Lewis acid/base concept

Reason R: Water acts as an acid with NH3 and as a base with H_2S.

In the light of the above statements choose the *correct* answer from the options given below:

(1) Both A and R are true and R is the correct explanation of A.

(2) Both A and R are true but R is NOT the correct explanation of A.

(3) A is true but R is false.

(4) A is false but R is true.

Answer (D)

Sol. The amphoteric nature of water is explained by using Bronsted-Lowry acid base concept

$$\underset{\text{(acid)}}{H_2O} + NH_3 \rightleftharpoons OH^- + NH_4^+$$

$$\underset{\text{(base)}}{H_2O} + H_2S \rightleftharpoons H_3O^+ + HS^-$$

Hence, A is false but R is true

9. The correct order of reduction potentials of the following pairs is

(A) Cl_2/Cl^- (B) I_2/I^-

(C) Ag^+/Ag (D) Na^+/Na

(E) Li^+/Li

Choose the correct answer from the options given below:

(1) A > C > B > D > E

(2) A > B > C > D > E

(3) A > C > B > E > D

(4) A > B > C > E > D

Answer (A)

Sol. Standard Reduction Potential

(A) Cl_2/Cl	1.36 V
(B) I_2/I^-	0.54 V
(C) Ag^+/Ag	0.80 V
(D) Na^+/Na	–2.71 V
(E) Li^+/Li	–3.05 V

Hence, correct order is A > C > B > D > E

10. The number of bridged oxygen atoms present in compound B formed from the following reactions is

$$Pb(NO_3)_2 \xrightarrow{673\text{ K}} A + PbO + O_2$$

$$A \xrightarrow{\text{Dimerise}} B$$

(1) 0

(2) 1

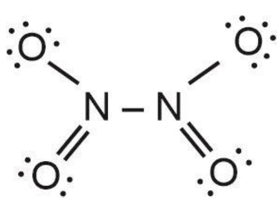
(3) 2

(4) 3

Answer (A)

Sol. $2Pb(NO_3)_2 \xrightarrow{673\text{ K}} \underset{A}{4NO_2} + 2PbO + O_2$

$$\underset{A}{NO_2} \xrightarrow{\text{Dimerise}} \underset{B}{N_2O_4}$$



Hence no bridged oxygen atom is present in N_2O_4.

11. The metal ion (in gaseous state) with lowest spin-only magnetic moment value is

(A) V^{2+}

(B) Ni^{2+}

(C) Cr^{2+}

(D) Fe^{2+}

Answer (B)

Sol.

		Valence shell Configuration	**Unpaired electrons**
V^{2+}	→	$3d^3 4s^0$	$n = 3$
Ni^{2+}	→	$3d^8 4s^0$	$n = 2$
Cr^{2+}	→	$3d^4 4s^0$	$n = 4$
Fe^{2+}	→	$3d^6 4s^0$	$n = 4$

Since Ni^{2+} has least number of unpaired electrons. Hence Ni^{2+} will have lowest spin only magnetic moment Value.

12. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A: Polluted water may have a value of BOD of the order of 17 ppm.

Reason R: BOD is a measure of oxygen required to oxidise both the bio-degradable and non-biodegradable organic material in water.

In the light of the above statements, choose the ***most appropriate*** answer from the options given below.

(A) Both **A** and **R** are correct and **R** is the correct explanation of **A**.

(B) Both **A** and **R** are correct but **R** is NOT the correct explanation of **A**.

(C) **A** is correct but **R** is not correct.

(D) **A** is not correct but **R** is correct.

Answer (C)

Sol. Highly polluted water could have a BOD value of 17 ppm or more.

The amount of oxygen required by bacteria to break down the organic matter present in a certain volume of a sample of water is called Biochemical Oxygen demand (BOD).

Hence **A** is correct but **R** is not correct.

13. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A: A mixture contains benzoic acid and naphthalene. The pure benzoic acid can be separated out by the use of benzene.

Reason R: Benzoic acid is soluble in hot water.

In the light of the above statements, choose the ***most appropriate*** answer from the options given below.

(A) Both **A** and **R** are true and **R** is the correct explanation of **A**.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) **A** is true but **R** is false.

(D) **A** is false but **R** is true.

Answer (D)

Sol. Since, both benzoic acid and naphthalene will dissolve in benzene. Hence assertion is wrong.

Benzoic acid is almost insoluble in cold water but soluble in hot water. Hence Reason is true

14. During halogen test, sodium fusion extract is boiled with concentrated HNO_3 to

(A) remove unreacted sodium

(B) decompose cyanide or sulphide of sodium

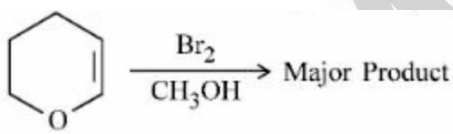
(C) extract halogen from organic compound

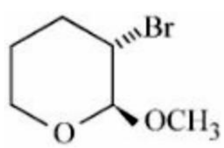
(D) maintain the pH of extract.

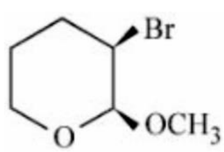
Answer (B)

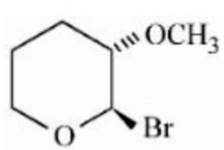
Sol. During test for halogen, if nitrogen or sulphur is also present in the compound, then sodium fusion extract is first boiled with concentrated nitric acid to decompose cyanide or sulphide of sodium formed during Lassaigne's test.

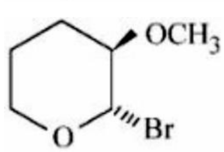
15. Amongst the following, the major product of the given chemical reaction is



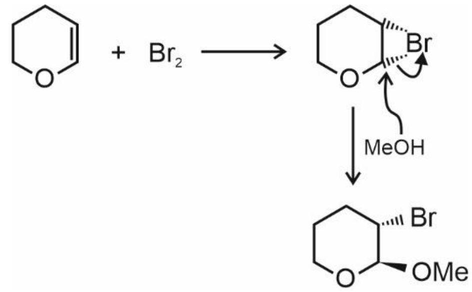
(A) 

(B) 

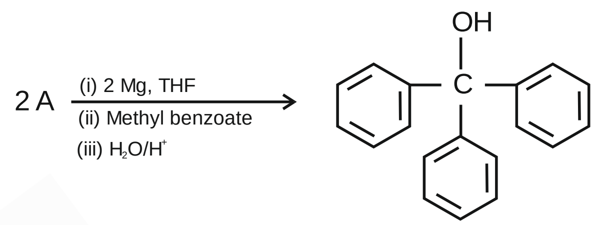
(C) 

(D) 

Answer (A)

Sol. 

16. In the given reaction



'A' can be

(A) Benzyl bromide

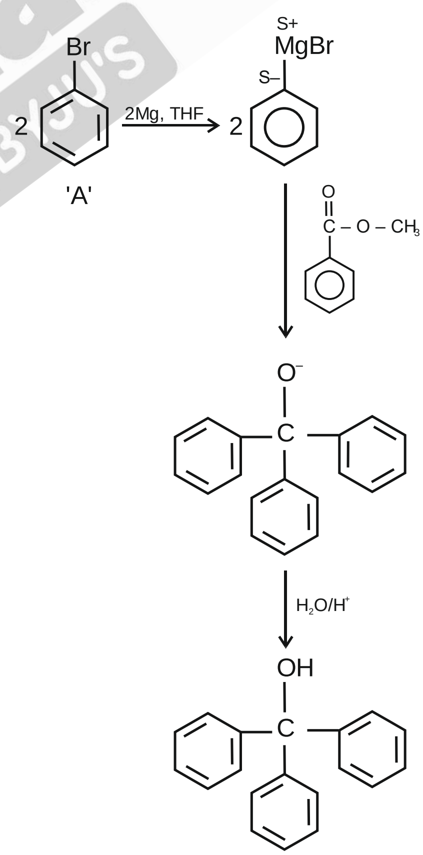
(B) Bromo benzene

(C) Cyclohexyl bromide

(D) Methyl bromide

Answer (B)

Sol.



Hence 'A' is bromobenzene.

17. Which of the following conditions or reaction sequence will NOT give acetophenone as the major product?

(A) (a) $C_6H_5-\overset{O}{\overset{||}{C}}-H + CH_3MgBr$

(b) $Na_2Cr_2O_7$, H+

(B) (a) $H_3C-\overset{O}{\overset{||}{C}}-H + C_6H_5MgBr$

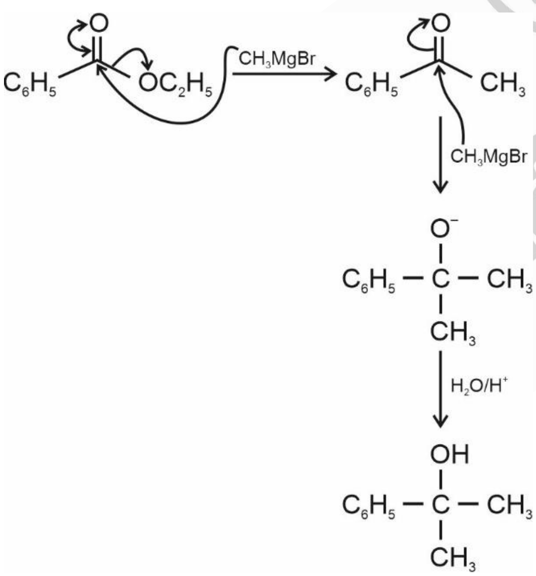
(b) PCC, DCM

(C) $C_6H_5-\overset{O}{\overset{||}{C}}-OC_2H_5 + 2CH_3MgBr$

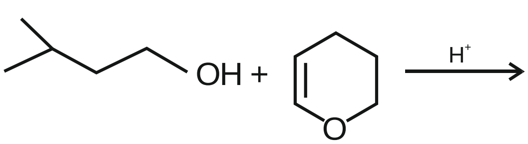
(D) $C_6H_5-\overset{O}{\overset{||}{C}}-Cl + CH_3MgBr + CdCl_2$

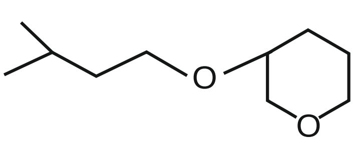
Answer (C)

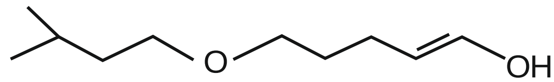
Sol. C will not give acetophenone

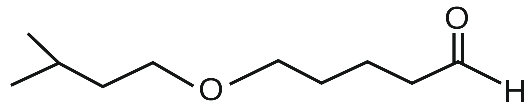


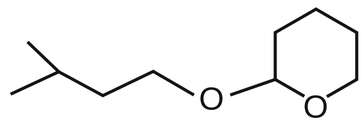
18. The major product formed in the following reaction, is



(A) 

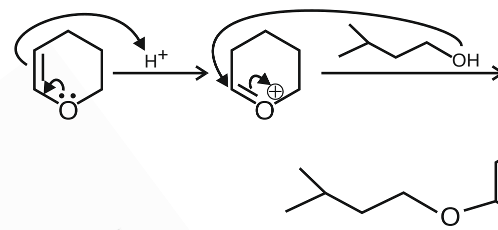
(B) 

(C) 

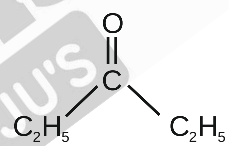
(D) 

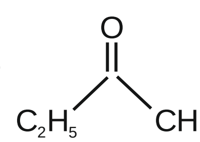
Answer (D)

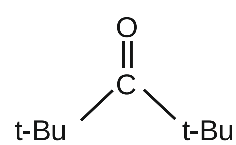
Sol.

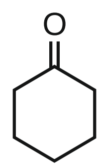


19. Which of the following ketone will NOT give enamine on treatment with secondary amines? [where t-Bu is –C(CH3)3]

(A) 

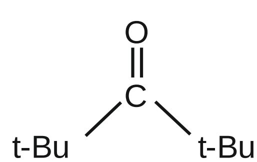
(B) 

(C) 

(D) 

Answer (C)

Sol. In order to form enamine from the reaction of carbonyl compound with 2º amine, the carbonyl compound must have α-hydrogen.

In  No α-hydrogen is present.

Along with this, due to steric **crowding** by t-Bu group, it is difficult for 2º amine to attack on this compound.

20. An antiseptic Dettol is a mixture of two compounds 'A' and 'B' where A has 6π electrons and B has 2π electrons. What is 'B'?

(A) Bithionol

(B) Terpineol

(C) Chloroxylenol

(D) Chloramphenicol

Answer (B)

Sol. Dettol is a mixture of chloroxylenol and terpineol. Chloroxylenol has 6π electrons and terpineol has 2π electrons.

Hence B is terpineol.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. A protein 'A' contains 0.30% of glycine (molecular weight 75). The minimum molar mass of the protein 'A' is ________ × 10^3 g mol^{-1} [nearest integer]

Answer (25)

Sol. 0.3% glycine means

100 g protein 'A' contains 0.3 g glycine.

Since, molar mass of glycine is 75

75 g glycine will be present in $\frac{100}{0.3}\times 75$ g protein

Minimum molar mass of protein A is 25 × 10^3 g/mol

2. A rigid nitrogen tank stored inside a laboratory has a pressure of 30 atm at 06:00 am when the temperature is 27°C. At 03:00 pm, when the temperature is 45°C, the pressure in the tank will be ________ atm. [nearest integer]

Answer (32)

Sol. Since

$P \propto T$

Hence, $\frac{P_1}{T_1} = \frac{P_2}{T_2}$ $\left(\begin{array}{l} P_1 \text{ is pressure at 6 am} \\ P_2 \text{ is pressure at 3 pm} \end{array}\right)$

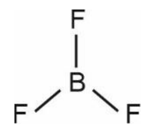
$$\frac{30}{300} = \frac{P_2}{318}$$

$P_2 \simeq 32$ atm

3. Amongst BeF_2, BF_3, H_2O, NH_3, CCl_4 and HCl, the number of molecules with non-zero net dipole moment is ______.

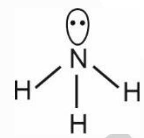
Answer (3)

Sol. F – Be – F $\mu = 0$

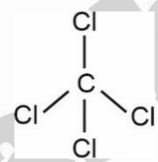


$\mu = 0$

$\mu \neq 0$



$\mu \neq 0$



$\mu = 0$

H – Cl $\mu \neq 0$

4. At 345 K, the half life for the decomposition of a sample of a gaseous compound initially at 55.5 kPa was 340 s. When the pressure was 27.8 kPa, the half life was found to be 170 s. The order of the reaction is ________. [integer answer]

Answer (0)

Sol. $t_{1/2} \propto \frac{1}{[P_0]^{n-1}}$

$$\frac{(t_{1/2})_1}{(t_{1/2})_2} = \frac{[P_0]_2^{n-1}}{[P_0]_1^{n-1}}$$

$$\frac{340}{170} = \left(\frac{27.8}{55.5}\right)^{n-1}$$

$$2 = \left(\frac{1}{2}\right)^{n-1}$$

$2 = (2)^{1-n}$

$1 - n = 1$

$n = 0$

5. A solution of $Fe_2(SO_4)_3$ is electrolyzed for 'x' min with a current of 1.5 A to deposit 0.3482 g of Fe. The value of x is _______. [nearest integer]

Given : 1 F = 96500 C mol^{-1}

Atomic mass of Fe = 56 g mol^{-1}

Answer (20)

Sol. $Fe^{3+} + 3e^- \rightarrow Fe$

Moles of Fe deposited = $\frac{0.3482}{56} = 6.2 \times 10^{-3}$

For 1 mole Fe, charge required is 3 F

For 6.2 × 10^{-3} mole Fe, charge required is 3 × 6.2 × 10^{-3} F

Since, charge required = 18.6 × 10^{-3} × 96500 C

= 1794.9 C

And,

1.5 × t = 1794.9

$t = \frac{1794.9}{1.5 \times 60}$ min

$t \simeq 20$ min

6. Consider the following reactions:

$PCl_3 + H_2O \rightarrow A + HCl$

$A + H_2O \rightarrow B + HCl$

The number of ionisable protons present in the product B is

Answer (2)

Sol. $PCl_3 + H_2O \longrightarrow \underset{A}{P(OH)Cl_2} + HCl$

$\underset{A}{P(OH)Cl_2} + H_2O \longrightarrow \underset{B}{P(OH)_2Cl} + HCl$

Hydrogen attached with oxygen are ionisable. Hence number of ionisable protons present in compound B are 2.

7. Amongst $FeCl_3.3H_2O$, $K_3[Fe(CN)_6)]$ and $[Co(NH_3)_6]Cl_3$, the spin-only magnetic moment value of the inner-orbital complex that absorbs light at shortest wavelength is_______B.M. [nearest integer]

Answer (2)

Sol. $[Fe(H_2O)_3Cl_3] \rightarrow$ Outer-orbital complex

$K_3[Fe(CN)_6] \rightarrow$ Inner-orbital complex

$[Co(NH_3)_6]Cl_3 \rightarrow$ Inner-orbital complex

Since CN^- is a strong field ligand than NH_3. Hence $K_3[Fe(CN)_6]$ is the inner-orbital complex that absorbs light at shortest wavelength.

Fe(III) $\rightarrow$ valence shell configuration $3d^5$

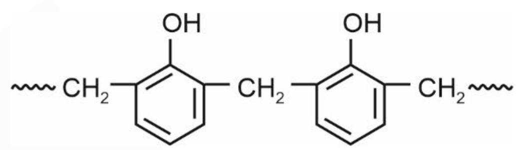
Since CN^- will do pairing, so unpaired electron = 1

$\mu = \sqrt{1(1+2)} = \sqrt{3}$ BM $\simeq 2$ BM

8. The Novolac polymer has mass of 963 g. The number of monomer units present in it are

Answer (9)

Sol. Novolac is



Molar mass of monomer is 107 g/mol

$n = \frac{963}{107} = 9$

Number of monomer units present in it are 9.

9. How many of the given compounds will give a positive Biuret test_________? Glycine, Glycylalanine, Tripeptide, Biuret.

Answer (2)

Sol. Since dipeptides and free amino acids do not give biuret test. Hence glycine and glycylalanine do not give this test.

10. The neutralization occurs when 10 mL of 0.1M acid 'A' is allowed to react with 30 mL of 0.05 M base $M(OH)_2$. The basicity of the acid 'A' is_________.

[M is a metal]

Answer (3)

Sol. Milieq of acid A = Milieq of base $M(OH)_2$

$(M \times V \times n - Factor)_A = (M \times V \times n - Factor)_{M(OH)_2}$

[n-Factor of $M(OH)_2$ = 2]

0.1 × 10 × n-Factor = 0.05 × 30 × 2

$(n\text{-Factor})_A = 3$

Hence basicity of acid A is 3.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Let $A = \{x \in R : |x + 1| < 2\}$ and $B = \{x \in R : |x - 1| \geq 2\}$. Then which one of the following statements is **NOT** true?

(A) $A - B = (-1, 1)$ (B) $B - A = R - (-3, 1)$

(C) $A \cap B = (-3, -1]$ (D) $A \cup B = R - [1, 3)$

Answer (B)

Sol. $A = (-3, 1)$ and $B = (-\infty, -1] \cup [3, \infty)$

So, $A - B = (-1, 1)$

$B - A = (-\infty, -3] \cup [3, \infty) = R - (-3, 3)$

$A \cap B = (-3, -1]$

and $A \cup B = (-\infty, 1) \cup [3, \infty) = R - [1, 3)$

2. Let $a, b \in R$ be such that the equation $ax^2 - 2bx + 15 = 0$ has a repeated root α. If α and β are the roots of the equation $x^2 - 2bx + 21 = 0$, then $\alpha^2 + \beta^2$ is equal to

(A) 37 (B) 58

(C) 68 (D) 92

Answer (B)

Sol. $ax^2 - 2bx + 15 = 0$ has repeated root so $b^2 = 15a$

and $\alpha = \frac{15}{b}$

$\because$ α is a root of $x^2 - 2bx + 21 = 0$

So $\frac{225}{b^2} = 9 \Rightarrow b^2 = 25$

Now $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 4b^2 - 42 = 100 - 42$
$= 58$

3. Let z_1 and z_2 be two complex numbers such that $\overline{z_1} = i\overline{z_2}$ and $\arg\left(\frac{z_1}{\overline{z_2}}\right) = \pi$. Then

(A) $\arg z_2 = \left(\frac{\pi}{4}\right)$ (B) $\arg z_2 = -\frac{3\pi}{4}$

(C) $\arg z_1 = \frac{\pi}{4}$ (D) $\arg z_1 = -\frac{3\pi}{4}$

Answer (C)

Sol. $\because \frac{z_1}{z_2} = -i \Rightarrow z_1 = -iz_2$

$\Rightarrow \arg(z_1) = -\frac{\pi}{2} + \arg(z_2)$ ...(i)

Also $\arg(z_1) - \arg(\bar{z}_2) = \pi$

$\Rightarrow \arg(z_1) + \arg(z_2) = \pi$ ...(ii)

From (i) and (ii), we get

$\arg(z_1) = \frac{\pi}{4}$ and $\arg(z_2) = \frac{3\pi}{4}$

4. The system of equations

$-kx + 3y - 14z = 25$

$-15x + 4y - kz = 3$

$-4x + y + 3z = 4$

is consistent for all k in the set

(A) R (B) $R - \{-11, 13\}$

(C) $R - \{13\}$ (D) $R - \{-11, 11\}$

Answer (D)

Sol. The system may be inconsistent if

$$\begin{vmatrix} -k & 3 & -14 \\ -15 & 4 & -k \\ -4 & 1 & 3 \end{vmatrix} = 0 \Rightarrow k = \pm 11$$

Hence if system is consistent then $k \in R - \{11, -11\}$

5. $\lim_{x \to \frac{\pi}{2}} \tan^2 x \left(\left(2\sin^2 x + 3\sin x + 4\right)^{\frac{1}{2}} - \left(\sin^2 x + 6\sin x + 2\right)^{\frac{1}{2}} \right)$ is equal to

(A) $\frac{1}{12}$

(B) $-\frac{1}{18}$

(C) $-\frac{1}{12}$

(D) $\frac{1}{6}$

Answer (A)

Sol. $\lim_{x\to\frac{\pi}{2}} \tan^2 x\left\{\sqrt{2\sin^2 x+3\sin x+4}-\sqrt{\sin^2 x+6\sin x+2}\right\}$

$$=\lim_{x\to\frac{\pi}{2}} \frac{\tan^2 x(\sin^2 x-3\sin x+2)}{\sqrt{2\sin^2 x+3\sin x+4}+\sqrt{\sin^2 x+6\sin x+2}}$$

$$=\frac{1}{6}\lim_{x\to\frac{\pi}{2}} \frac{(1-\sin x)(2-\sin x)}{\cos^2 x}\cdot\sin^2 x$$

$$=\frac{1}{6}\lim_{x\to\frac{\pi}{2}} \frac{(2-\sin x)\sin^2 x}{1+\sin x}$$

$$=\frac{1}{12}$$

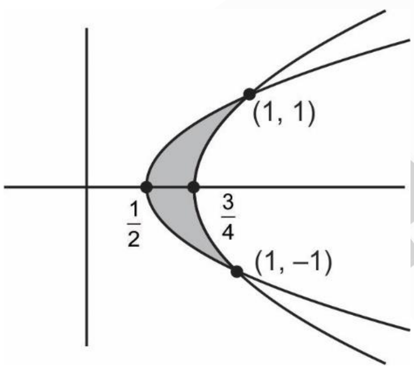
6. The area of the region enclosed between the parabolas $y^2 = 2x - 1$ and $y^2 = 4x - 3$ is

(A) $\frac{1}{3}$ (B) $\frac{1}{6}$

(C) $\frac{2}{3}$ (D) $\frac{3}{4}$

Answer (A)

Sol. Area of the shaded region



$$=2\int_0^1\left(\frac{y^2+3}{4}-\frac{y^2+1}{2}\right)dy$$

$$=2\int_0^1\left(\frac{1}{4}-\frac{y^2}{4}\right)dy$$

$$=2\left[\frac{1}{4}-\frac{1}{12}\right]=\frac{1}{3}$$

7. The coefficient of x^{101} in the expression $(5 + x)^{500} + x(5 + x)^{499} + x^2(5 + x)^{498} + \ldots\ldots + x^{500}$, $x > 0$, is

(A) ${}^{501}C_{101}\,(5)^{399}$ (B) ${}^{501}C_{101}\,(5)^{400}$

(C) ${}^{501}C_{100}\,(5)^{400}$ (D) ${}^{500}C_{101}\,(5)^{399}$

Answer (A)

Sol. Coeff. of x^{101} in $\dfrac{x^{500}\left[\left(\frac{x+5}{x}\right)^{501}-1\right]}{\frac{x+5}{x}-1}$

$$= \text{Coeff. of } x^{101} \text{ in } \frac{1}{5}\left[(x+5)^{501}-x^{501}\right]$$

$$=\frac{1}{5}\,{}^{501}C_{101}\cdot 5^{400}$$

$$={}^{501}C_{101}\cdot 5^{399}$$

8. The sum $1 + 2\cdot 3 + 3\cdot 3^2 + \ldots. + 10\cdot 3^9$ is equal to

(A) $\frac{2\cdot 3^{12}+10}{4}$ (B) $\frac{19\cdot 3^{10}+1}{4}$

(C) $5\cdot 3^{10}-2$ (D) $\frac{9\cdot 3^{10}+1}{2}$

Answer (B)

Sol. Let

$$S = 1.3^\circ+2.3^1+3.3^2+\ldots\ldots+10.3^9$$

$$3S = 1.3^1+2.3^2+\ldots\ldots\ldots\ldots+10.3^{10}$$

$$-2S = (1.3^\circ + 1.3^1 + 1.3^2 + \ldots\ldots + 1.3^9) - 10.3^{10}$$

$$\Rightarrow S=\frac{1}{2}\left[10.3^{10}-\frac{3^{10}-1}{3-1}\right]$$

$$\Rightarrow S=\frac{19.3^{10}+1}{4}$$

9. Let P be the plane passing through the intersection of the planes $\vec{r}\cdot(\hat{i}+3\hat{j}-\hat{k})=5$ and $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})=3$, and the point (2, 1, –2). Let the position vectors of the points X and Y be $\hat{i}-2\hat{j}+4\hat{k}$ and $5\hat{i}-\hat{j}+2\hat{k}$ respectively. Then the points

(A) X and $X + Y$ are on the same side of P

(B) Y and $Y – X$ are on the opposite sides of P

(C) X and Y are on the opposite sides of P

(D) $X + Y$ and $X – Y$ are on the same side of P

Answer (C)

Sol. Let the equation of required plane

$$\pi : (x+3y-z-5)+\lambda(2x-y+z-3)=0$$

$\because$ $(2, 1, -2)$ lies on it so, $2 + \lambda(-2) = 0$

$\Rightarrow$ $\lambda = 1$

Hence, $\pi : 3x + 2y - 8 = 0$

$\because$ $\pi_x = -9,\ \pi_y = 5,\ \pi_{x+y} = 4$

$\pi_{x-y} = -22$ and $\pi_{y-x} = 6$

Clearly X and Y are on opposite sides of plane π

10. A circle touches both the y-axis and the line $x + y = 0$. Then the locus of its center is

(A) $y = \sqrt{2}x$ (B) $x = \sqrt{2}y$

(C) $y^2 - x^2 = 2xy$ (D) $x^2 - y^2 = 2xy$

Answer (D)

Sol. Let the centre be (h, k)

So, $|h| = \left|\frac{h+k}{\sqrt{2}}\right|$

$\Rightarrow 2h^2 = h^2 + k^2 + 2hk$

Locus will be $x^2 - y^2 = 2xy$

11. Water is being filled at the rate of 1 cm³/sec in a right circular conical vessel (vertex downwards) of height 35 cm and diameter 14 cm. When the height of the water level is 10 cm, the rate (in cm²/sec) at which the wet conical surface area of the vessel increase, is

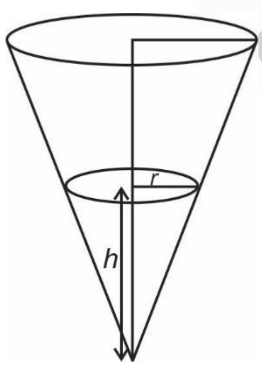
(A) 5 (B) $\frac{\sqrt{21}}{5}$

(C) $\frac{\sqrt{26}}{5}$ (D) $\frac{\sqrt{26}}{10}$

Answer (C)

Sol. $\because V = \frac{1}{3}\pi r^2 h$ and $\frac{r}{h} = \frac{7}{35} = \frac{1}{5}$

$\Rightarrow V = \frac{1}{75}\pi h^3$



$\frac{dV}{dt} = \frac{1}{25}\pi h^2 \frac{dh}{dt} = 1$

$\Rightarrow \frac{dh}{dt} = \frac{25}{\pi h^2}$

Now, $S = \pi r l = \pi\left(\frac{h}{5}\right)\sqrt{h^2 + \frac{h^2}{25}} = \frac{\pi}{25}\sqrt{26}h^2$

$\Rightarrow \frac{dS}{dt} = \frac{2\sqrt{26}\pi h}{25} \cdot \frac{dh}{dt} = \frac{2\sqrt{26}}{h}$

$\Rightarrow \left.\frac{dS}{dt}\right|_{(h=10)} = \frac{\sqrt{26}}{5}$

12. If $b_n = \int_0^{\frac{\pi}{2}} \frac{\cos^2 nx}{\sin x}dx, n \in \mathbb{N}$, then

(A) $b_3 - b_2, b_4 - b_3, b_5 - b_4$ are in an A.P. with common difference –2

(B) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in an A. P. with common difference 2

(C) $b_3 - b_2, b_4 - b_3, b_5 - b_4$ are in a G.P.

(D) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in an A.P. with common difference –2

Answer (D)

Sol. $b_n - b_{n-1} = \int_0^{\frac{\pi}{2}} \frac{\cos^2 nx - \cos^2(n-1)x}{\sin x}dx$

$= \int_0^{\pi/2} \frac{-\sin(2n-1)x \cdot \sin x}{\sin x}dx$

$= \left.\frac{\cos(2n-1)x}{2n-1}\right|_0^{\pi/2} = -\frac{1}{2n-1}$

So, $b_3 - b_2, b_4 - b_3, b_5 - b_4$ are in H.P.

$\Rightarrow \frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in A. P. with common difference –2.

13. If $y = y(x)$ is the solution of the differential equation $2x^2\frac{dy}{dx} - 2xy + 3y^2 = 0$ such that $y(e) = \frac{e}{3}$, then $y(1)$ is equal to

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $\frac{3}{2}$ (D) 3

Answer (B)

Sol. $2x^2\frac{dy}{dx} - 2xy + 3y^2 = 0$

$\Rightarrow 2x(xdy - ydx) + 3y^2 dx = 0$

$\Rightarrow 2\left(\frac{xdy - ydx}{y^2}\right) + 3\frac{dx}{x} = 0$

$\Rightarrow -\frac{2x}{y} + 3\ln x = C$

$\because y(e) = \frac{e}{3} \Rightarrow -6 + 3 = C \Rightarrow C = -3$

Now, at $x = 1$, $-\frac{2}{y} + 0 = -3$

$y = \frac{2}{3}$

14. If the angle made by the tangent at the point (x_0, y_0) on the curve $x = 12(t + \sin t \cos t)$,

$y = 12(1+\sin t)^2, 0 < t < \frac{\pi}{2}$, with the positive x-axis is $\frac{\pi}{3}$, then y_0 is equal to:

(A) $6\left(3+2\sqrt{2}\right)$ (B) $3\left(7+4\sqrt{3}\right)$

(C) 27 (D) 48

Answer (C)

Sol. $\because \frac{dy}{dx} = \frac{24(1+\sin t)\cos t}{12(1+\cos 2t)} = \frac{1+\sin t}{\cos t} = \tan\left(\frac{\pi}{4}+\frac{t}{2}\right)$

$\because \left.\frac{dy}{dx}\right|_{(x_0, y_0)} = \sqrt{3} = \tan\left(\frac{\pi}{4}+\frac{t}{2}\right)$

$\Rightarrow t = \frac{\pi}{6}$

So, $y_{0\left(\text{at } t=\frac{\pi}{6}\right)} = 12\left(1+\sin\frac{\pi}{6}\right)^2 = 27$

15. The value of 2sin(12°) – sin(72°) is :

(A) $\frac{\sqrt{5}\left(1-\sqrt{3}\right)}{4}$

(B) $\frac{1-\sqrt{5}}{8}$

(C) $\frac{\sqrt{3}\left(1-\sqrt{5}\right)}{2}$

(D) $\frac{\sqrt{3}\left(1-\sqrt{5}\right)}{4}$

Answer (D)

Sol. $2\sin 12° - \sin 72°$

$= \sin 12° + \left(-2\cos 42° \cdot \sin 30°\right)$

$= \sin 12° - \cos 42°$

$= \sin 12° - \sin 48°$

$= 2\sin 18° \cdot \cos 30°$

$= -2\left(\frac{\sqrt{5}-1}{4}\right)\cdot\frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}\left(1-\sqrt{5}\right)}{4}$

16. A biased die is marked with numbers 2, 4, 8, 16, 32, 32 on its faces and the probability of getting a face with mark n is $\frac{1}{n}$. If the die is thrown thrice, then the probability, that the sum of the numbers obtained is 48, is :

(A) $\frac{7}{2^{11}}$ (B) $\frac{7}{2^{12}}$

(C) $\frac{3}{2^{10}}$ (D) $\frac{13}{2^{12}}$

Answer (D)

Sol. There are only two ways to get sum 48, which are (32, 8, 8) and (16, 16, 16)

So, required probability

$= 3\left(\frac{2}{32}\cdot\frac{1}{8}\cdot\frac{1}{8}\right)+\left(\frac{1}{16}\cdot\frac{1}{16}\cdot\frac{1}{16}\right)$

$= \frac{3}{2^{10}} + \frac{1}{2^{12}}$

$= \frac{13}{2^{12}}$

17. The negation of the Boolean expression $((\sim q) \wedge p) \Rightarrow ((\sim p) \vee q)$ is logically equivalent to :

(A) $p \Rightarrow q$ (B) $q \Rightarrow p$

(C) $\sim (p \Rightarrow q)$ (D) $\sim (q \Rightarrow p)$

Answer (C)

Sol. Let $S : ((\sim q) \wedge p) \Rightarrow ((\sim p) \vee q)$

$\Rightarrow S : \sim ((\sim q) \wedge p) \vee ((\sim p) \vee q)$

$\Rightarrow S : (q \vee (\sim p)) \vee ((\sim p) \vee q)$

$\Rightarrow S : (\sim p) \vee q$

$\Rightarrow S : p \Rightarrow q$

So, negation of S will be $\sim (p \Rightarrow q)$

18. If the line $y = 4 + kx$, $k > 0$, is the tangent to the parabola $y = x - x^2$ at the point P and V is the vertex of the parabola, then the slope of the line through P and V is :

(A) $\frac{3}{2}$ (B) $\frac{26}{9}$

(C) $\frac{5}{2}$ (D) $\frac{23}{6}$

Answer (C)

Sol. $\because$ Line $y = kx + 4$ touches the parabola $y = x - x^2$.

So, $kx + 4 = x - x^2 \Rightarrow x^2 + (k - 1)x + 4 = 0$ has only one root

$(k - 1)^2 = 16 \Rightarrow k = 5$ or -3 but $k > 0$

So, $k = 5$.

And hence $x^2 + 4x + 4 = 0 \Rightarrow x = -2$

So, $P(-2, -6)$ and V is $\left(\frac{1}{2}, \frac{1}{4}\right)$

Slope of $PV = \dfrac{\frac{1}{4}+6}{\frac{1}{2}+2} = \dfrac{5}{2}$

19. The value of $\tan^{-1}\left(\dfrac{\cos\left(\frac{15\pi}{4}\right)-1}{\sin\left(\frac{\pi}{4}\right)}\right)$ is equal to :

(A) $-\frac{\pi}{4}$ (B) $-\frac{\pi}{8}$

(C) $-\frac{5\pi}{12}$ (D) $-\frac{4\pi}{9}$

Answer (B)

Sol. $\tan^{-1}\left(\dfrac{\cos\left(\frac{15\pi}{4}\right)-1}{\sin\frac{\pi}{4}}\right)$

$= \tan^{-1}\left(\dfrac{\frac{1}{\sqrt{2}}-1}{\frac{1}{\sqrt{2}}}\right)$

$= \tan^{-1}\left(1-\sqrt{2}\right) = -\tan^{-1}\left(\sqrt{2}-1\right)$

$= -\frac{\pi}{8}$

20. The line $y = x + 1$ meets the ellipse $\frac{x^2}{4}+\frac{y^2}{2}=1$ at two points P and Q. If r is the radius of the circle with PQ as diameter then $(3r)^2$ is equal to :

(A) 20 (B) 12

(C) 11 (D) 8

Answer (A)

Sol. Let point $(a, a + 1)$ as the point of intersection of line and ellipse.

So, $\frac{a^2}{4}+\frac{(a+1)^2}{2}=1 \Rightarrow a^2+2(a^2+2a+1)=4$

$\Rightarrow 3a^2 + 4a - 2 = 0$

If roots of this equation are α and β.

So, $P(\alpha, \alpha + 1)$ and $Q(\beta, \beta + 1)$

$PQ^2 = 4r^2 = (\alpha - \beta)^2 + (\alpha - \beta)^2$

$\Rightarrow 9r^2 = \frac{9}{4}(2(\alpha-\beta)^2)$

$= \frac{9}{2}\left[(\alpha+\beta)^2-4\alpha\beta\right]$

$= \frac{9}{2}\left[\left(-\frac{4}{3}\right)^2+\frac{8}{3}\right]$

$= \frac{1}{2}[16+24] = 20$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Let $A = \begin{pmatrix} 2 & -2 \\ 1 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 2 \\ -1 & 2 \end{pmatrix}$. Then the number of elements in the set $\{(n, m) : n, m \in \{1, 2, \ldots, 10\}$ and $nA^n + mB^m = I\}$ is ____________.

Answer (1)

Sol. $A^2 = \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}$

$= \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix} = A \quad \Rightarrow A^K = A,\ K \in I$

$B^2 = \begin{bmatrix} -1 & 2 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ -1 & 2 \end{bmatrix} = B$

So, $B^K = B,\ K \in I$

$nA^n + mB^m = nA + mB$

$= \begin{bmatrix} 2n & -2n \\ n & -n \end{bmatrix} + \begin{bmatrix} -m & 2m \\ -m & 2m \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

So, $2n - m = 1$, $-n + m = 0$, $2m - n = 1$

So, $(m, n) = (1, 1)$

2. Let $f(x) = [2x^2 + 1]$ and $g(x) = \begin{cases} 2x-3, & x<0 \\ 2x+3, & x\ge 0 \end{cases}$, where $[t]$ is the greatest integer $\le t$. Then, in the open interval $(-1, 1)$, the number of points where fog is discontinuous is equal to ________.

Answer (62)

Sol. $f(g(x)) = \begin{cases} [2(2x-3)^2]+1, & x<0 \\ [2(2x+3)^2]+1, & x \ge 0 \end{cases}$

The possible points where $fog(x)$ may be discontinuous are

$2(2x-3)^2 \in I$ & $x \in (-1, 0)$

$2(2x+3)^2 \in I$ & $x \in [0, 1)$

$x \in (-1, 0)$	$x \in [0, 1)$
$2x - 3 \in (-5, -3)$	$2x + 3 \in [3, 5)$
$2(2x-3)^2 \in (18, 50)$	$2(2x+3)^2 \in [18, 50)$
So, no. of points = 31	It is discontinuous at all points except $x = 0$ of no. points = 31

So, total = 62

3. The value of $b > 3$ for which $12\int_3^b \frac{1}{(x^2-1)(x^2-4)}dx$

$= \log_e\left(\frac{49}{40}\right)$, is equal to

Answer (6)

Sol. $I = \int \frac{1}{(x^2-1)(x^2-4)}dx = \frac{1}{3}\int\left(\frac{1}{x^2-4} - \frac{1}{x^2-1}\right)dx$

$= \frac{1}{3}\left(\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right| - \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|\right) + C$

$12I = \ln\left|\frac{x-2}{x+2}\right| - 2\ln\left|\frac{x-1}{x+1}\right| + C$

$12\int_3^b \frac{dx}{(x^2-4)(x^2-1)}$

$= \ln\left(\frac{b-2}{b+2}\right) - 2\ln\left(\frac{b-1}{b+1}\right) - \left(\ln\left(\frac{1}{5}\right) - 2\ln\left(\frac{1}{2}\right)\right)$

$= \ln\left(\left(\frac{b-2}{b+2}\right)\cdot\frac{(b+1)^2}{(b-1)^2}\right) - \left(\ln\frac{4}{5}\right)$

So, $\frac{49}{40} = \frac{(b-2)}{(b+2)}\frac{(b+1)^2}{(b-1)^2}\cdot\frac{5}{4}$

$\Rightarrow$ $b = 6$

4. If the sum of the co-efficients of all the positive even powers of x in the binomial expansion of $\left(2x^3 + \frac{3}{x}\right)^{10}$ is $5^{10} - \beta\cdot 3^9$, the β is equal to ____.

Answer (83)

Sol. $T_{r+1} = {}^{10}C_r\ (2x^3)^{10-r}\left(\frac{3}{x}\right)^r$

$= {}^{10}C_r\ 2^{10-r}\ 3^r\ x^{30-4r}$

So, $r \ne 8, 9, 10$

Sum of required Coeff. = $\left(2.1^3 + \frac{3}{1}\right)^{10}$

$\left({}^{10}C_8 2^2 3^8 + {}^{10}C_9\ 2^1 3^9 + {}^{10}C_{10} 2^0 3^{10}\right)$

$= 5^{10} - 3^9\left(\frac{{}^{10}C_8 \cdot 2^2}{3} + {}^{10}C_9 \cdot 2^1 + {}^{10}C_{10}\cdot 3\right)$

$\beta = \frac{4}{3}\cdot {}^{10}C_8 + 20 + 3 = 83$

5. If the mean deviation about the mean of the numbers 1, 2, 3, …. n, where n is odd, is $\frac{5(n+1)}{n}$, then n is equal to ____________.

Answer (21)

Sol. Mean = $\frac{n\frac{(n+1)}{2}}{n} = \frac{n+1}{2}$

M.D. = $\frac{2\left(\frac{n-1}{2} + \frac{n-3}{2} + \frac{n-5}{2} + ...0\right)}{n} = \frac{5(n+1)}{n}$

$\Rightarrow$ $((n-1) + (n-3) + (n-5) + \ldots 0) = 5(n+1)$

$\Rightarrow$ $\left(\frac{n+1}{4}\right)\cdot(n-1) = 5(n+1)$

So, $n = 21$

6. Let $\vec{b} = \hat{i} + \hat{j} + \lambda\hat{k}, \lambda \in \mathbb{R}$. If $\vec{a}$ is a vector such that $\vec{a}\times\vec{b} = 13\hat{i} - \hat{j} - 4\hat{k}$ and $\vec{a}\cdot\vec{b} + 21 = 0$, then $(\vec{b}-\vec{a})\cdot(\hat{k}-\hat{j}) + (\vec{b}+\vec{a})\cdot(\hat{i}-\hat{k})$ is equal to

Answer (14)

Sol. Let $\vec{a} = x\hat{i} = y\hat{j} + z\hat{k}$

So, $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 1 & 1 & \lambda \end{vmatrix} = \hat{i}(\lambda y - z) + \hat{j}(z - \lambda x) + \hat{k}(x - y)$

$\Rightarrow$ $\lambda y - z = 13$, $z - \lambda x = -1$, $x - y = -4$

and $x + y + \lambda z = -21$

$\Rightarrow$ clearly, $\lambda = 3$, $x = -2$, $y = 2$ and $z = -7$

So, $\vec{b}-\vec{a}=3\hat{i}-\hat{j}+10\hat{k}$

and $\vec{b}+\vec{a}=-\hat{i}+3\hat{j}-4\hat{k}$

$\Rightarrow \left(\vec{b}-\vec{a}\right)\cdot\left(\hat{k}-\hat{j}\right)+\left(\vec{b}+\vec{a}\right)\cdot\left(\hat{i}-\hat{k}\right)=11+3=14$

7. The total number of three-digit numbers, with one digit repeated exactly two times, is _______.

Answer (243)

Sol. C-1 : All digits are non-zero

$^9C_2\cdot 2\cdot\frac{3!}{2}=216$

C-2 : One digit is 0

0, 0, $x \Rightarrow {}^9C_1\cdot 1=9$

0, x, $x \Rightarrow {}^9C_1\cdot 2=18$

Total = 216 + 27 = 243

8. Let $f(x)=\left|(x-1)\left(x^2-2x-3\right)\right|+x-3, x\in R$. If m and M are respectively the number of points of local minimum and local maximum of f in the interval (0, 4), then $m + M$ is equal to

Answer (3)

Sol. $f(x) = |(x-1)(x+1)(x-3)|+(x-3)$

$$f(x)=\begin{cases}(x-3)(x^2) & 3\le x\le 4\\ (x-3)(2-x^2) & 1\le x<3\\ (x-3)(x^2) & 0<x<1\end{cases}$$

$$f'(x)=\begin{cases}3x^2-6x & 3<x<4\\ -3x^2+6x+2 & 1<x<3\\ 3x^2-6x & 0<x<1\end{cases}$$

$f'(3^+)>0 \quad f'(3^-)<0 \rightarrow$ Minimum

$f'(1^+)>0 \quad f'(1^-)<0 \rightarrow$ Minimum

$x\in(1,3)$ $f'(x)=0$ at one point $\rightarrow$ Maximum

$x\in(3,4)$ $f'(x)\neq 0$

$x\in(0,1)$ $f'(x)\neq 0$

So, 3 points

9. Let the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be $\frac{5}{4}$. If the equation of the normal at the point $\left(\frac{8}{\sqrt{5}},\frac{12}{5}\right)$ on the hyperbola is $8\sqrt{5}x+\beta y=\lambda$, then $\lambda-\beta$ is equal to ____.

Answer (85)

Sol. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \left(e=\frac{5}{4}\right)$

So, $b^2=a^2\left(\frac{25}{16}-1\right) \Rightarrow b=\frac{3}{4}a$

Also $\left(\frac{8}{\sqrt{5}},\frac{12}{5}\right)$ lies on the given hyperbola

So, $\frac{64}{5a^2}-\frac{144}{25\left(\frac{9a^2}{16}\right)}=1 \Rightarrow a=\frac{8}{5}$ and $b=\frac{6}{5}$

Equation of normal

$$\frac{64}{25}\left(\frac{x}{8/\sqrt{5}}\right)+\frac{36}{25}\left(\frac{y}{12/5}\right)=4$$

$\Rightarrow \frac{8}{5\sqrt{5}}x+\frac{3}{5}y=4$

$\Rightarrow 8\sqrt{5}x+15y=100$

So, $\beta=15$ and $\lambda=100$

Gives $\lambda-\beta=85$

10. Let l_1 be the line in xy-plane with x and y intercepts $\frac{1}{8}$ and $\frac{1}{4\sqrt{2}}$ respectively and l_2 be the line in zx-plane with x and z intercepts $-\frac{1}{8}$ and $-\frac{1}{6\sqrt{3}}$ respectively. If d is the shortest distance between the line l_1 and l_2, then d^{-2} is equal to ___________ .

Answer (51)

Sol. $\frac{x-\frac{1}{8}}{\frac{1}{8}}=\frac{y}{-\frac{1}{4\sqrt{2}}}=\frac{z}{0}$ ____L_1

or $\frac{x-\frac{1}{8}}{1}=\frac{y}{-\sqrt{2}}=\frac{z}{0}$ ...(i)

Equation of L_2

$\frac{x+\frac{1}{8}}{-6\sqrt{3}}=\frac{y}{0}=\frac{z}{8}$ ...(ii)

$$d=\left|\frac{(\vec{c}-\vec{a})\cdot(\vec{b}\times\vec{d})}{|\vec{b}\times\vec{d}|}\right|$$

$$=\frac{\left(\frac{1}{4}\hat{i}\right)\cdot\left(4\sqrt{2}\,\hat{i}+4\hat{j}+3\sqrt{6}\,\hat{k}\right)}{\sqrt{\left(4\sqrt{2}\right)^2+4^2+\left(3\sqrt{6}\right)^2}}$$

$$=\frac{\sqrt{2}}{\sqrt{32+16+54}}=\frac{1}{\sqrt{51}}$$

$d^{-2} = 51$

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