27/08/2021
Evening

Corporate Office: Aakash Tower, 8, Pusa Road, New Delhi-110005 Ph. 011-47623456

Time : 3 hrs. M.M. : 300

Answers & Solutions

for

JEE (MAIN)-2021 (Online) Phase-4

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS :

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part has two sections.

(i) Section-I : This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) Section-II : This section contains 10 questions. In Section-II, attempt any **five questions out of 10.** There will be **no negative marking for Section-II**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and there is no negative marking for wrong answer.

PART–A : PHYSICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. A coaxial cable consists of an inner wire of radius 'a' surrounded by an outer shell of inner and outer radii 'b' and 'c' respectively. The inner wire carries an electric current i_0, which is distributed uniformly across cross-sectional area. The outer shell carries an equal current in opposite direction and distributed uniformly. What will be the ratio of the magnetic field at a distance x from the axis when (i) x < a and (ii) a < x < b ?

(1) $\frac{x^2}{b^2-a^2}$ (2) $\frac{b^2-a^2}{x^2}$

(3) $\frac{a^2}{x^2}$ (4) $\frac{x^2}{a^2}$

Answer (4)

Sol. (i) $x < a$

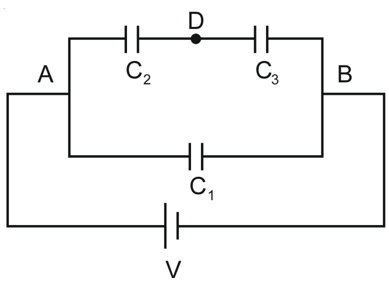
$$B_1 = \frac{\mu_0 I_0 x}{2\pi a^2}$$

(ii) $a < x < b$

$$B_2 = \frac{\mu_0 I_0}{2\pi x}$$

$$\therefore \frac{B_1}{B_2} = \frac{x^2}{a^2}$$

2. Three capacitors C_1 = 2 μF, C_2 = 6 μF and C_3 = 12 μF are connected as shown in figure. Find the ratio of the charges on capacitors C_1, C_2 and C_3 respectively:



(1) 2 : 3 : 3 (2) 1 : 2 : 2

(3) 2 : 1 : 1 (4) 3 : 4 : 4

Answer (2)

Sol. Charge on C_1, $q_1 = C_1V = 2V\mu C$

Charge on C_2 and C_3 are same

$\therefore q_2 = q_3 = C_{eq}V$

$= 4V\mu C$

$\therefore q_1 : q_2 : q_3 = 1 : 2 : 2$

3. An antenna is mounted on a 400 m tall building. What will be the wavelength of signal that can be radiated effectively by the transmission tower upto a range of 44 km?

(1) 75.6 m (2) 37.8 m

(3) 605 m (4) 302 m

Answer (3)

Sol. $d = \sqrt{2Rh_T}$

$$\Rightarrow 44\times10^3 = \sqrt{2\times6400\times10^3\times h_T}$$

$$\Rightarrow h_T = 151.25 \text{ m}$$

$$\because l_{antenna} \geq \frac{\lambda}{4}$$

$$\Rightarrow \lambda \leq 4l$$

considering, h_T = l = length of antenna

λ = 4 × 151.25 = 605 m

4. Match List-I with List-II.

	List-I		List-II
(a)	R_H(Rydberg constant)	(i)	kg $m^{-1}s^{-1}$
(b)	h(Planck's constant)	(ii)	kg m^2s^{-1}
(c)	μ_B(Magnetic field energy density)	(iii)	m^{-1}
(d)	η(coefficient of viscocity)	(iv)	kg $m^{-1}s^{-2}$

Choose the **most appropriate** answer from the options given below:

(1) (a)-(iii), (b)-(ii), (c)-(iv), (d)-(i)

(2) (a)-(iv), (b)-(ii), (c)-(i), (d)-(iii)

(3) (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)

(4) (a)-(iii), (b)-(ii), (c)-(i), (d)-(iv)

Answer (1)

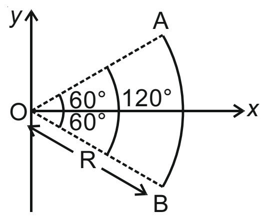
Sol. Unit of R_h = m^{-1}

Unit of h = ET = kg m^2s^{-1}

Unit of μ_B = kg $m^{-1}s^{-2}$

Unit of η = kg $m^{-1}s^{-1}$

5. Figure shows a rod AB, which is bent in a 120° circular arc of radius R. A charge (–Q) is uniformly distributed over rod AB. What is the electric field $\vec{E}$ at the centre of curvature O?



(1) $\frac{3\sqrt{3}\,Q}{8\pi^2\varepsilon_0 R^2}(\hat{i})$

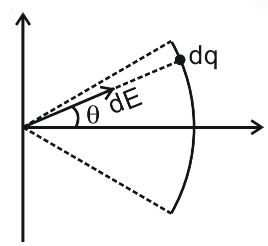
(2) $\frac{3\sqrt{3}\,Q}{8\pi^2\varepsilon_0 R^2}(-\hat{i})$

(3) $\frac{3\sqrt{3}\,Q}{8\pi\varepsilon_0 R^2}(\hat{i})$

(4) $\frac{3\sqrt{3}\,Q}{16\pi^2\varepsilon_0 R^2}(\hat{i})$

Answer (1)

Sol.



$$E = \int dE\cos\theta$$

$$= \int_{-\pi/3}^{\pi/3} \frac{K\times(+Q)}{\frac{2\pi}{3}R}\times\frac{Rd\theta}{R^2}\times\cos\theta$$

$$= +\frac{3}{2\pi}\frac{KQ}{R^2}[\sin\theta]_{-\pi/3}^{\pi/3}$$

$$= +\frac{3}{2\theta}\frac{KQ}{R}\times\frac{2\sqrt{3}}{2}$$

$$\therefore\ \vec{E} = \frac{3\sqrt{3}}{8\pi^2\varepsilon_0 R^2}(\hat{i})$$

6. Water drops are falling from a nozzle of a shower onto the floor, from a height of 9.8 m. The drops fall at a regular interval of time. When the first drop strikes the floor, at that instant, the third drop begins to fall. Locate the position of second drop from the floor when the first drop strikes the floor.

(1) 2.45 m

(2) 7.35 m

(3) 2.94 m

(4) 4.18 m

Answer (2)

Sol. $T = \sqrt{\frac{2H}{g}} \Rightarrow T = \sqrt{2}$ sec

at t = 0 → 1^{st} drop

at t = Δt → 2^{nd} drop

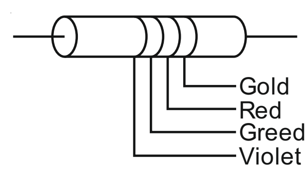
at t = $2\Delta t$ → 3^{rd} drop

$$\Rightarrow 2\Delta t = \sqrt{2} \Rightarrow \Delta t = \frac{1}{\sqrt{2}}$$

$$h = \frac{1}{2}\times g\times(\Delta t)^2 = \left(\frac{1}{2}\right)\times 9.8\times\frac{1}{2}$$

$$\Rightarrow H_F = 9.8 - \frac{9.8}{4} = \frac{3}{4}\times 9.8 = 7.35\text{ m}$$

7. The colour coding on a carbon resistor is shown in the given figure. The resistance value of the given resistor is:



(1) (7500 ± 750) Ω

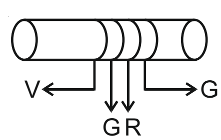
(2) (5700 ± 375) Ω

(3) (5700 ± 285) Ω

(4) (7500 ± 375) Ω

Answer (4)

Sol.

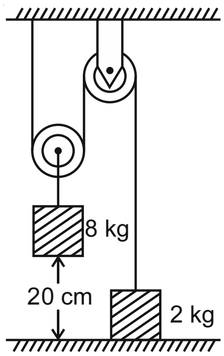


R = 75 × 100 ± 5%

⇒R = 7.5 kΩ ± 5%

⇒R = (7500 ± 375) Ω

8. The boxes of masses 2 kg and 8 kg are connected by a massless string passing over smooth pulleys. Calculate the time taken by box of mass 8 kg to strike the ground from rest. (use g = 10 m/s^2):



(1) 0.4 s

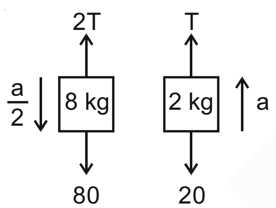
(2) 0.25 s

(3) 0.34 s

(4) 0.2 s

Answer (1)

Sol.



$80 - 2T = 4a$ ...(1)

$T - 20 = 2a$ ...(2)

From (1) and (2)

$a = 5\ m/s^2$

$$t = \sqrt{\frac{2H}{a/2}} = \sqrt{\frac{2\times 20\times 2}{100\times 5}} = 0.4\ s$$

9. A mass of 50 kg is placed at the centre of a uniform spherical shell of mass 100 kg and radius 50 m. If the gravitational potential at a point, 25 m from the centre is V kg/m. The value of V is:

(1) +2G (2) –60G

(3) –20G (4) –4G

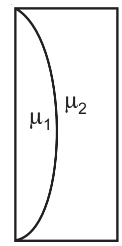
Answer (4)

Sol. $V = -\frac{GM}{R} - \frac{Gm}{r}$

$$= \frac{(-G)50}{25} - \frac{(G)(100)}{50}$$

$= -4G$

10. Curved surfaces of a plano-convex lens of refractive index μ_1 and a plano-concave lens of refractive index μ_2 have equal radius of curvature as shown in figure. Find the ratio of radius of curvature to the focal length of the combined lenses.



(1) $\mu_2 - \mu_1$

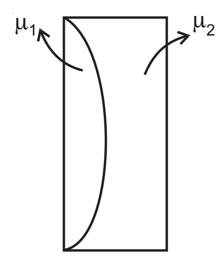
(2) $\mu_1 - \mu_2$

(3) $\frac{1}{\mu_2 - \mu_1}$

(4) $\frac{1}{\mu_1 - \mu_2}$

Answer (2)

Sol. $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$



$$= (\mu_1 - 1)\left(\frac{1}{R}\right) + (\mu_2 - 1)\left(-\frac{1}{R}\right)$$

$$\frac{R}{f} = (\mu_1 - 1) + (1 - \mu_2) = (\mu_1 - \mu_2)$$

11. A player kicks a football with an initial speed of 25 ms^{-1} at an angle of 45° from the ground. What are the maximum height and the time taken by the football to reach at the highest point during motion? (Take g = 10 ms^{-2})

(1) h_{max} = 10 m, T = 2.5 s

(2) h_{max} = 15.625 m, T = 1.77 s

(3) h_{max} = 3.54 m, T = 0.125 s

(4) h_{max} = 15.625 m, T = 3.54 s

Answer (2)

Sol. $$h_{max} = \frac{u^2 \sin^2\theta}{2g} = \frac{25^2 \times \frac{1}{2}}{2\times 10} = 15.625\ m$$

$$T = \frac{u\sin\theta}{g} = \frac{25\times\frac{1}{\sqrt{2}}}{10} = 1.77\ s$$

12. Two discs have moments of intertia I_1 and I_2 about their respective axes perpendicular to the plane and passing through the centre. They are rotating with angular speeds, ω_1 and ω_2 respectively and are brought into contact face to face with their axes of rotation coaxial. The loss in kinetic energy of the system in the process is given by:

(1) $\frac{I_1I_2}{(I_1+I_2)}(\omega_1-\omega_2)^2$

(2) $\frac{(\omega_1-\omega_2)^2}{2(I_1+I_2)}$

(3) $\frac{(I_1-I_2)^2\omega_1\omega_2}{2(I_1+I_2)}$

(4) $\frac{I_1I_2}{2(I_1+I_2)}(\omega_1-\omega_2)^2$

Answer (4)

Sol. Using conservation of angular momentum

$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1+I_2}$$

Loss in KE. $= \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2$

$$-\frac{1}{2}(I_1+I_2)\left(\frac{I_1\omega_1+I_2\omega_2}{I_1+I_2}\right)^2$$

$$=\frac{1}{2}(I_1\omega_1^2+I_2\omega_2^2)-\frac{(I_1\omega_1+I_2\omega_2)^2}{2(I_1+I_2)}$$

$$=\frac{I_1I_2}{2(I_1+I_2)}(\omega_1-\omega_2)^2$$

13. For full scale deflection of total 50 divisions, 50 mV voltage is required in galvanometer. The resistance of galvanometer if its current sensitivity is 2 div/mA will be

(1) 4 Ω

(2) 2 Ω

(3) 5 Ω

(4) 1 Ω

Answer (2)

Sol. Current sensitivity = 2 div/mA

So full scale current $= \frac{50}{2}$ mA

= 25 mA

Full scale voltage = 50 mV

So, Resistance $= \frac{V}{I} = 2\Omega$

14. The light waves from two coherent sources have same intensity $I_1 = I_2 = I_0$. In interference pattern the intensity of light at minima is zero. What will be the intensity of light of maxima?

(1) 5 I_0

(2) I_0

(3) 4 I_0

(4) 2 I_0

Answer (3)

Sol. $I_{max} = \left(\sqrt{I_1}+\sqrt{I_2}\right)^2$

$= \left(2\sqrt{I_0}\right)^2 = 4\ I_0$

15. For a transistor α and β are given as $\alpha = \frac{I_C}{I_E}$ and $\beta = \frac{I_C}{I_B}$. Then the correct relation between α and β will be:

(1) $\alpha = \frac{1-\beta}{\beta}$

(2) $\beta = \frac{\alpha}{1-\alpha}$

(3) $\alpha\beta = 1$

(4) $\alpha = \frac{\beta}{1-\beta}$

Answer (2)

Sol. We know, $I_E = I_C + I_B$

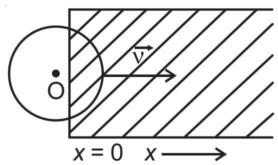
$$\frac{I_C}{\alpha} = I_C + \frac{I_C}{\beta}$$

$$\Rightarrow \frac{1}{\alpha} = 1 + \frac{1}{\beta}$$

$$\frac{1}{\beta} = \frac{1}{\alpha} - 1 \quad = \frac{1-\alpha}{\alpha}$$

$$\beta = \frac{\alpha}{1-\alpha}$$

16. A constant magnetic of 1 T is applied in the $x > 0$ region. A metallic circular ring of radius 1 m is moving with a constant velocity 1 m/s along the x-axis. At t = 0 s, the centre O of the ring is at $x = -1$ m. What will be the value of the induced emf in the ring at t = 1 s? (Assume the velocity of the ring does not change)



(1) 2 V (2) 2 πV

(3) 1 V (4) 0 V

Answer (1)

Sol. At t = 1 s, centre of ring is on verge of boundary

$\varepsilon = Bvl$

$l = 2R$

$\varepsilon = 1 \times 1 \times 2$

17. If force (F), length (L) and time (T) are taken as the fundamental quantities. Then what will be dimension of density:

(1) $[FL^{-3}T^{2}]$ (2) $[FL^{-3}T^{3}]$

(3) $[FL^{-4}T^{2}]$ (4) $[FL^{-5}T^{2}]$

Answer (3)

Sol. $F = M^1L^1T^{-2}$

$\rho = M^1L^{-3}$

$\rho = [F]^a[L]^b[T]^c$

$M^1L^{-3} = [M^1L^1T^{-2}]^a[L]^b[T]^c$

$\Rightarrow$ a = 1, b = –4, c = 2

$\rho = F^1L^{-4}T^2$

18. The height of victoria falls is 63 m. What is the difference in temperature of water at the top and at the bottom of fall?

[Given 1 cal = 4.2 J and specific heat of water = 1 cal $g^{-1°}$ C^{-1}]

(1) 0.147°C (2) 1.476°C

(3) 14.76°C (4) 0.014°C

Answer (1)

Sol. By principle of calorimetry

$mgh = mc\Delta T$

$10^3 \times 4.2 \times \Delta T = 630$

$\Delta T = 0.147°C$

19. A monochromatic neon lamp with wavelength of 670.5 nm illuminates a photo-sensitive material which has a stopping voltage of 0.48 V. What will be the stopping voltage if the source light is changed with another source of wavelength of 474.6 nm?

(1) 0.96 V

(2) 1.25 V

(3) 1.5 V

(4) 0.24 V

Answer (2)

Sol. $\frac{hc}{\lambda_1} = \phi + eV_1$

$\frac{hc}{\lambda_2} = \phi + eV_2$

$\frac{hc}{\lambda_2} - \frac{hc}{\lambda_1} = e(V_2 - V_1)$

$V_2 = \frac{hc}{e}\left[\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right] + V_1$

$V_2 = 1.25$ V

20. If the rms speed of oxygen molecules at 0°C is 160 m/s, find the rms speed of the hydrogen molecules at 0°C.

(1) 40 m/s

(2) 80 m/s

(3) 640 m/s

(4) 332 m/s

Answer (3)

Sol. $V_{rms} = \sqrt{\frac{3RT}{M}}$

$V_{O_2} = 160$

$V_{H_2} = V_{O_2} \times \sqrt{\frac{M_{O_2}}{M_{H_2}}}$

= 640 m/s

Aakash BYJU'S

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. A heat engine operates between a cold reservoir at temperature T_2 = 400 K and a hot reservoir at temperature T_1. It takes 300 J of heat from the hot reservoir and delivers 240 J of heat to the cold reservoir in a cycle. The minimum temperature of the hot reservoir has to be _______ K.

Answer (500)

Sol. $\eta = \frac{W}{Q} = \frac{300-240}{300} = 1 - \frac{T_2}{T_1}$

$\Rightarrow$ T_1 = 500 K.

2. A tuning fork is vibrating at 250 Hz. The length of the shortest closed organ pipe that will resonate with the tuning fork will be _______ cm.

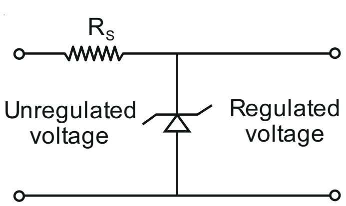
(Take speed of sound in air as 340 ms^{-1})

Answer (34)

Sol. $f = \frac{(2n-1)\,v}{4l}$

$l_{min} = \frac{v}{4f} = 34$ cm

3. A zener diode of power rating 2 W is to be used as a voltage regulator. If the zener diode has a breakdown of 10 V and it has to regulate voltage fluctuated between 6 V and 14 V, the value of R_S for safe operation should be _____ Ω.



Answer (20)

Sol. P_z = 2 W

V_z = 10 V

I_z = 0.2 A (max)

$I_{zmax} \times R_S = (14 - 10)$

$R_S = 20\ \Omega$

4. An ac circuit has an inductor and a resistor of resistance R in series, such that X_L = 3R. Now, a capacitor is added in series such that X_C = 2R. The ratio of new power factor with the old power factor of the circuit is $\sqrt{5}$: *x*. The value of *x* is ____

Answer (1)

Sol. $\cos\phi_2 = \frac{R}{Z_2} = \frac{R}{\sqrt{R^2 + (x_L - x_C)^2}} = \frac{1}{\sqrt{2}}$

$\cos\phi_1 = \frac{R}{Z_1} = \frac{R}{\sqrt{10}R} = \frac{1}{\sqrt{10}}$

$\frac{\cos\phi_2}{\cos\phi_1} = \frac{\sqrt{5}}{1}$

5. Two simple harmonic motion, are represented by the equations

$y_1 = 10\sin\left(3\pi t + \frac{\pi}{3}\right)$

$y_2 = 5\left(\sin 3\pi t + \sqrt{3}\cos 3\pi t\right)$

Ratio of amplitude of y_1 to y_2 = x : 1. The value of *x* is ____

Answer (1)

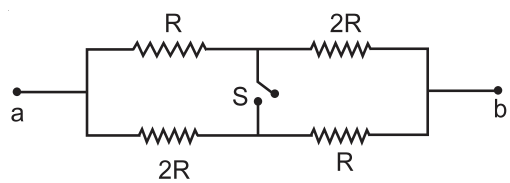
Sol. $y_1 = 10\sin\left(3\pi t + \frac{\pi}{3}\right)$

$y_2 = 5\left(\sin 3\pi t + \sqrt{3}\cos 3\pi t\right)$

$= 10\sin\left(3\pi t + \frac{\pi}{3}\right)$

$\frac{A_1}{A_2} = 1$

6. The ratio of the equivalent resistance of the network (shown in figure) between the points a and b when switch is open and switch is closed is x : 8. The value of x is ____.



Answer (9)

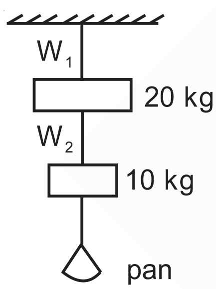
Sol. R_{eq} when switch is closed = $\frac{4R}{3}$

R_{eq} when switch is open = $\frac{3R}{2}$

$$\frac{R_{open}}{R_{close}} = \frac{\frac{3R}{2}}{\frac{4R}{3}} = \frac{9}{8}$$

7. Wires W_1 and W_2 are made of same material having the breaking stress of 1.25×10^9 N/m^2. W_1 and W_2 have cross-sectional area of 8×10^{-7} m^2 and 4×10^{-7} m^2, respectively. Masses of 20 kg and 10 kg hang from them as shown in the figure. The maximum mass that can be placed in the pan without breaking the wires is ___ kg.

(Use g = 10 m/s^2)



Answer (40)

Sol. $\sigma_1 = \frac{(m+30)g}{8\times10^{-7}} = 1.25\times10^9$

$\Rightarrow$ m + 30 = 100

m = 70

$\sigma_2 = \frac{(m+10)g}{4\times10^{-7}} = 1.25\times10^9$

m + 10 = 50

m = 40

$\Rightarrow$ 40 kg is safest maximum mass

8. X different wavelengths may be observed in the spectrum from a hydrogen sample if the atoms are exited to states with principal quantum number n = 6? The value of X is _______.

Answer (15)

Sol. Possible number of wavelength $= {}^nC_2$

= 15

9. A plane electromagnetic wave with frequency of 30 MHz travels in free space. At particular point in space and time, electric field is 6 V/m. The magnetic field at this point will be $x \times 10^{-8}$ T. The value of x is _______.

Answer (2)

Sol. $|E| = E_0 \sin(\omega t + \phi)$

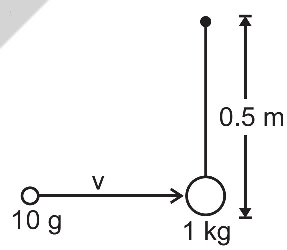
$|B| = B_0 \sin(\omega t + \phi)$

$$\frac{|B|}{|E|} = \frac{B_0}{E_0} = \frac{1}{C}$$

$|B| = 2 \times 10^{-8}$ T

10. A bullet of 10 g, moving with velocity v, collides head-on with the stationary bob of a pendulum and recoils with velocity 100 m/s. The length of the pendulum is 0.5 m and mass of the bob is 1 kg. The minimum value of v = _______ m/s so that the pendulum describes a circle.

(Assume the string to be inextensible and g = 10 m/s^2)



Answer (400)

Sol. For pendulum to describe circle

$v_{B/min} = \sqrt{5\times10\times0.5} = 5$ m/s

$p_{bullet\ initial} = 0.01 \times v$

$p_{system\ final} = 0.01\ (-100) + 1 \times 5$

$$\Rightarrow (0.01)(-100)+5 = \frac{v}{100}$$

$$\frac{v}{100} = 4$$

v = 400

PART–B : CHEMISTRY

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Choose the **correct** statement from the following :

   (1) The standard enthalpy of formation for alkali metal bromides becomes less negative on descending the group

   (2) The low solubility of CsI in water is due to its high lattice enthalpy

   (3) Among the alkali metal halides, LiF is least soluble in water

   (4) LiF has least negative standard enthalpy of formation among alkali metal fluorides

Answer (3)

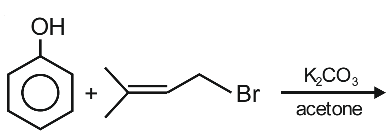
Sol.

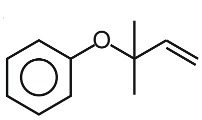
Alkali metal	Enthalpies of formation	($\Delta_f H°$) kJ/mol
	MF	MBr
Li	–612	–350
Na	–569	–360
K	–563	–392
Rb	–549	–389
Cs	–531	–395

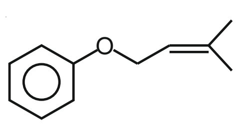
LiF has most negative standard enthalpy of formation among alkali metal fluorides.

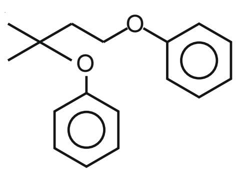
LiF is least soluble because of high lattice energy.

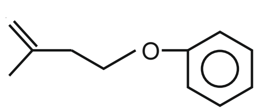
2. The major product of the following reaction, if it occurs by S_N2 mechanism is :



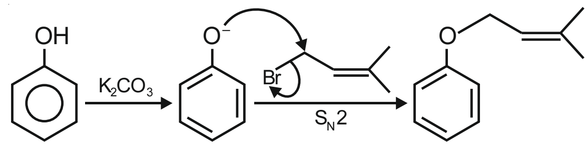
(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. 

(Attack is on least hindered position)

3. In stratosphere most of the ozone formation is assisted by :

   (1) Visible radiations
   (2) Ultraviolet radiation
   (3) γ-rays
   (4) Cosmic rays

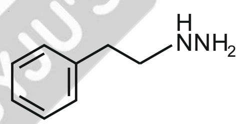
Answer (2)

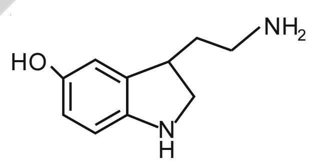
Sol. Ozone in the stratosphere is a product of UV radiations acting on dioxygen (O_2) molecules.

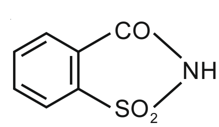
$$O_2(g) \xrightarrow{UV} \underset{\text{(free oxygen atoms)}}{O(g) + O(g)}$$

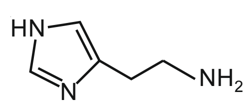
$$O(g) + O_2(g) \underset{}{\overset{UV}{\rightleftharpoons}} O_3(g)$$

4. Which one of the following chemicals is responsible for the production of HCl in the stomach leading to irritation and pain?

(1) 

(2) 

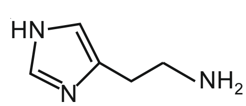
(3) 

(4) 

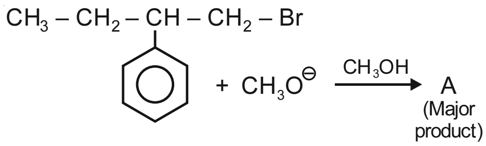
Answer (4)

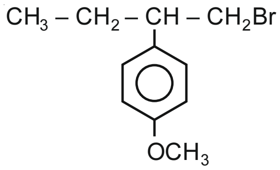
Sol. Histamine stimulates the secretion of pepsin and hydrochloric acid in the stomach.

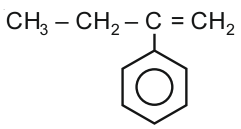
Structure of histamine is :

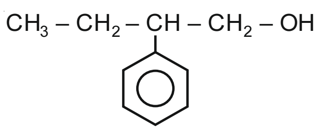


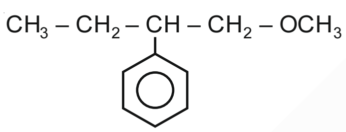
5. The major product (A) formed in the reaction given below is :



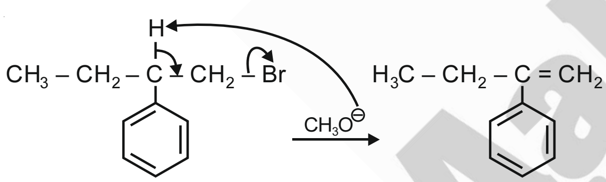
(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. 

6. Which one of the following reactions will not yield propionic acid?

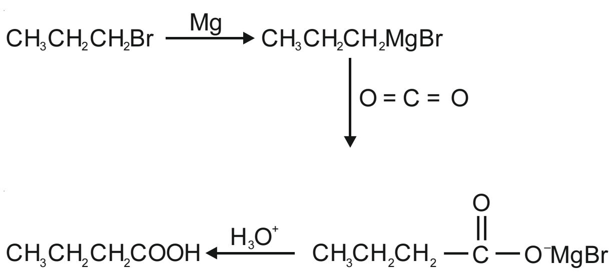
(1) $CH_3CH_2CH_3 + KMnO_4$(Heat), OH^-/H_3O^+

(2) $CH_3CH_2COCH_3 + OI^-/H_3O^+$

(3) $CH_3CH_2CH_2Br + Mg, CO_2$ dry ether/H_3O^+

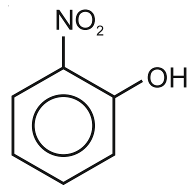
(4) $CH_3CH_2CCl_3 + OH^-/H_3O^+$

Answer (3)

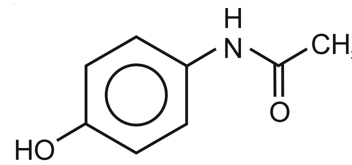
Sol. 

Butanoic Acid

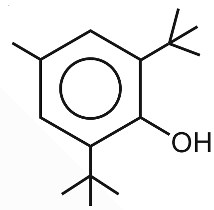
7. The compound/s which will show significant intermolecular H-bonding is/are



(a)



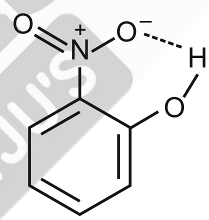
(b)



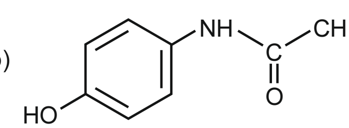
(c)

(1) (a) and (b) only
(2) (c) only
(3) (a), (b) and (c)
(4) (b) only

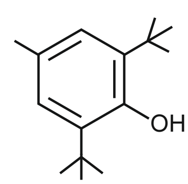
Answer (4)

Sol. (a) 

Intramolecular H-bonding

(b) 

Intramolecular H-bonding is not possible here. So, intermolecular H-bonding is prominent

(c) 

Because of large steric crowding, intermolecular H-bonding is difficult

8. The oxide that gives H_2O_2 most readily on treatment with H_2O is

(1) SnO_2
(2) PbO_2
(3) $BaO_2 \cdot 8H_2O$
(4) Na_2O_2

Answer (4)

Sol. Na_2O_2 gives H_2O_2 readily. (This is Merck's method of preparation of H_2O_2.)

SnO_2, PbO_2 are oxides and can't give peroxide on hydrolysis.

9. The correct order of ionic radii for the ions, P^{3-}, S^{2-}, Ca^{2+}, K^+, Cl^- is

(1) $K^+ > Ca^{2+} > P^{3-} > S^{2-} > Cl^-$

(2) $P^{3-} > S^{2-} > Cl^- > Ca^{2+} > K^+$

(3) $P^{3-} > S^{2-} > Cl^- > K^+ > Ca^{2+}$

(4) $Cl^- > S^{2-} > P^{3-} > Ca^{2+} > K^+$

Answer (3)

Sol. For isoelectronic species, as nuclear charge increases radius decreases.

Greater the positive charge, lesser the size of ion. Greater the negative charge, larger the size of ion.

$\therefore P^{3-} > S^{2-} > Cl^- > K^+ > Ca^{2+}$

10. Which one of the following is formed (mainly) when red phosphorus is heated in a sealed tube at 803 K?

(1) Yellow phosphorus (2) β-Black phosphorus

(3) α-Black phosphorus (4) White phosphorus

Answer (3)

Sol. Black phosphorus has two forms : α-black and β-black.

α-black phosphorus is formed when red phosphorus is heated in a sealed tube at 803 K.

11. Given below are two statements :

Statement I : Ethyl pent-4-yn-oate on reaction with CH_3MgBr gives a 3°-alcohol.

Statement II : In this reaction one mole of ethyl pent-4-yn-oate utilizes two moles of CH_3MgBr.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both Statement I and Statement II are true

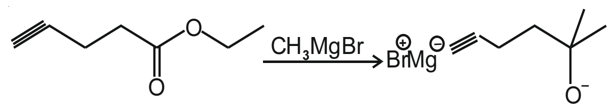
(2) Statement I is false but Statement II is true

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are false

Answer (3)

Sol.



For 1 mole → 3 moles of CH_3MgBr is used.

(2 moles required for ester and 1 mole for acidic H of ethyne)

12. Lyophilic sols are more stable than lyophobic sols because,

(1) The colloidal particles are solvated

(2) The colloidal particles have positive charge

(3) The colloidal particles have no charge

(4) There is a strong electrostatic repulsion between the negatively charged colloidal particles

Answer (1)

Sol. Lyophilic sols are more stable because they are solvated in solution. They are also called as solvent loving.

13. Which one of the following tests used for the identification of functional groups in organic compounds does not use copper reagent?

(1) Seliwanoff's test

(2) Barfoed's test

(3) Benedict's test

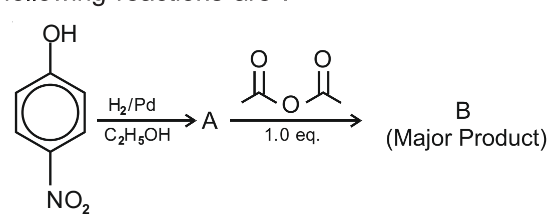
(4) Biuret test for peptide bond

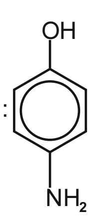
Answer (1)

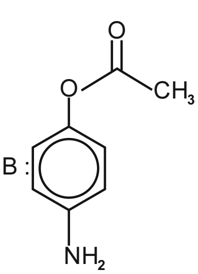
Sol. Seliwanoff's test → Resorcinol dissolved in conc HCl.

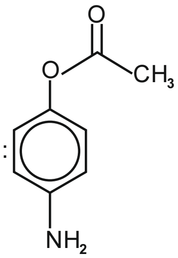
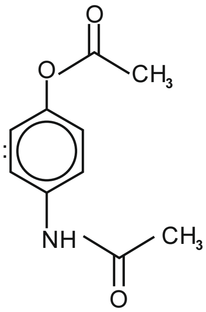
All other test use copper based reagent.

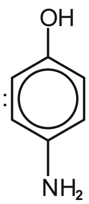
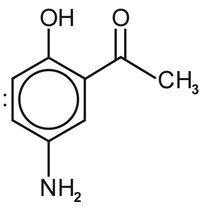
14. The correct structures of A and B formed in the following reactions are :

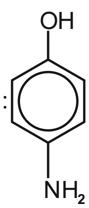
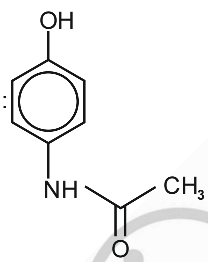


(1) A : 

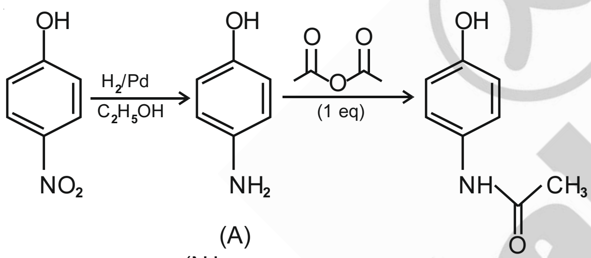
, B : 

(2) A :  , B : 

(3) A :  , B : 

(4) A :  , B : 

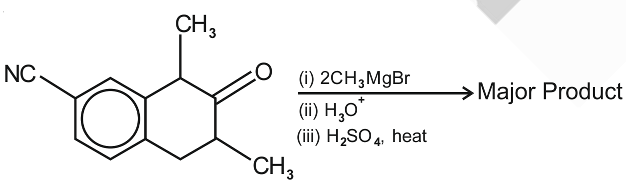
Answer (4)

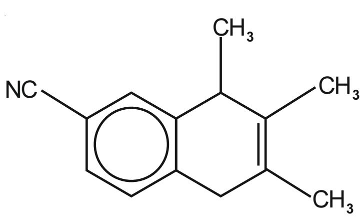
Sol. 

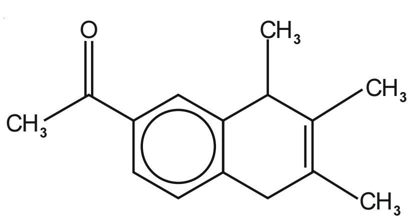
(A)
(N is more basic than O)

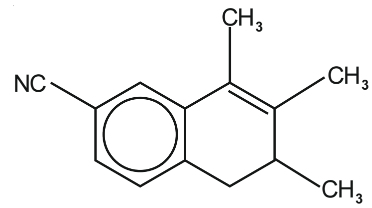
(B)
(Major)

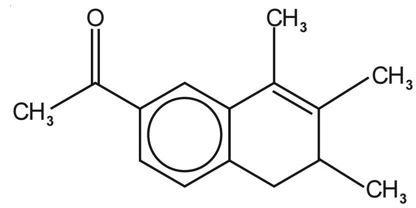
15. Which one of the following is the major product of the given reaction?



(1) 

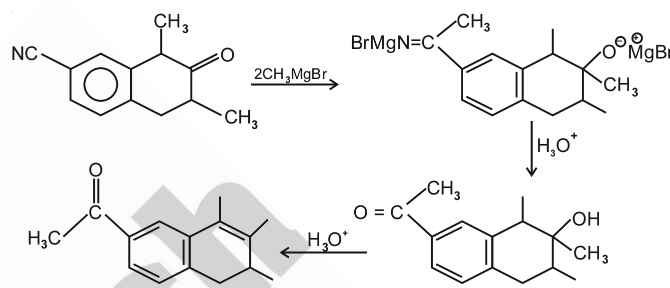
(2) 

(3) 

(4) 

Answer (4)

Sol.



16. The addition of dilute NaOH to Cr^{3+} salt solution will give :

(1) A solution of $[Cr(OH)_4]^-$

(2) Precipitate of $[Cr(OH)_6]^{3-}$

(3) Precipitate of $Cr_2O_3(H_2O)_n$

(4) Precipitate of $Cr(OH)_3$

Answer (4)

Sol. $Cr^{3+} + OH^- \longrightarrow \underset{\text{(green ppt)}}{Cr(OH)_3}$

If NaOH is present in excess, then

$$Cr(OH)_3 + OH^- \longrightarrow \underset{\text{(green solution)}}{\left[Cr(OH)_4\right]^-}$$

17. Potassium permaganate on heating at 513 K gives a product which is

(1) Paramagnetic and green

(2) Paramagnetic and colourless

(3) Diamagnetic and colourless

(4) Diamagnetic and green

Answer (1, 2)

Sol. $2KMnO_4 \xrightarrow{513K} \underset{\substack{\text{Paramagnetic}\\\text{(Green)}}}{K_2MnO_4} + MnO_2 + \underset{\substack{\text{Paramagnetic}\\\text{(Colorless)}}}{O_2}$

NTA Answer → (1)

Probable Answer → 1, 2 both

18. Which one of the following is used to remove most of plutonium from spent nuclear fuel?

(1) I_2O_5 (2) BrO_3

(3) O_2F_2 (4) ClF_3

Answer (3)

Sol. O_2F_2 oxidises plutonium to PuF_6 and the reaction is used in removing plutonium as PuF_6 from spent nuclear fuel.

19. Match List -I with List -II :

List-I (Name of ore/mineral)	List-II (Chemical formula)
a. Calamine	(i) ZnS
b. Malachite	(ii) $FeCO_3$
c. Siderite	(iii) $ZnCO_3$
d. Sphalerite	(iv) $CuCO_3.Cu(OH)_2$

(1) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

(2) (a)-(iii), (b)-(ii), (c)-(iv), (d)-(i)

(3) (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii)

(4) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

Answer (4)

Sol. Calamine — $ZnCO_3$

Malachite — $Cu(OH)_2.CuCO_3$

Siderite — $FeCO_3$

Sphalerite — ZnS

20. Hydrolysis of sucrose gives:

(1) α-D-(–)-Glucose and β-D-(–)-Fructose

(2) α-D-(+)-Glucose and α-D-(–)-Fructose

(3) α-D-(–)-Glucose and α-D-(–)-Fructose

(4) α-D-(+)-Glucose and β-D-(–)-Fructose

Answer (4)

Sol. $$\underset{\text{Sucrose}}{C_{12}H_{22}O_{11}} \xrightarrow{H_2O} \underset{\alpha\text{-D-(+)-Glucose}}{C_6H_{12}O_6} + \underset{\beta\text{-D-(–)-Fructose}}{C_6H_{12}O_6}$$

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. When 5.1 g of solid NH_4HS is introduced into a two litre evacuated flask at 27°C, 20% of the solid decomposes into gaseous ammonia and hydrogen sulphide. The K_p for the reaction at 27°C is x × 10^{-2}. The value of x is ______. (Integer answer)

[Given R = 0.082 L atm K^{-1} mol^{-1}]

Answer (6)

Sol.

	$NH_4HS(s)$ ⇌	$NH_3(g)$ +	$H_2S(g)$
Initially :	0.1 mole	—	—
At equil.:	0.1 – 0.02 = 0.08	0.02 mole	0.02 mole

$n_T = 0.02 + 0.02 = 0.04$

$$P_T = \frac{n_T RT}{V} = \frac{0.04 \times 0.082 \times 300}{2} = 0.492 \text{ atm}$$

$$K_P = \left(\frac{0.492}{2}\right)\left(\frac{0.492}{2}\right) = 6.05 \times 10^{-2}$$

∴ x = 6 (nearest integer)

2. Data given for the following reaction is as follows:

$$FeO_{(s)} + C_{(graphite)} \longrightarrow Fe_{(s)} + CO_{(g)}$$

Substance	$\Delta_f H^o$ (kJ mol^{-1})	ΔS^o (J $mol^{-1}K^{-1}$)
$FeO_{(s)}$	–266.3	57.49
$C_{(graphite)}$	0	5.74
$Fe_{(s)}$	0	27.28
$CO_{(g)}$	–110.5	197.6

The minimum temperature in K at which the reaction becomes spontaneous is________. (Integer answer)

Answer (964)

Sol. $FeO_{(s)} + C_{(graphite)} \longrightarrow Fe_{(s)} + CO_{(g)}$

$$\Delta_f H^o\text{(reaction)} = (0 + (-110.5)) - (-266.3)$$
$$= 155.8 \text{ kJ/mol}$$

$$\Delta S^o\text{(reaction)} = 27.28 + 197.6 - (57.49 + 5.74)$$
$$= 224.88 - 63.23$$
$$= 161.65 \text{ J mol}^{-1} \text{ K}^{-1}$$

For spontaneity

$\Delta G = \Delta H - T\Delta S$ $(\Delta G = 0)$

$\Delta H = T\Delta S$

$$T = \frac{\Delta H}{\Delta S} = \frac{155.8 \times 1000}{161.65} = 963.8$$

≈ 964 (nearest integer)

3. The first order rate constant for the decomposition of $CaCO_3$ at 700 K is 6.36×10^{-3} s^{-1} and activation energy is 209 kJ mol^{-1}. Its rate constant (in s^{-1}) at 600 K is $x \times 10^{-6}$. The value of x is _____. (Nearest integer)

[Given R = 8.31 J K^{-1} mol^{-1}; log 6.36×10^{-3} = –2.19, $10^{-4.79} = 1.62 \times 10^{-5}$]

Answer (16)

Sol. $k_1 = 6.36 \times 10^{-3}$ s^{-1} $\quad T_1 = 700$ k

$E_a = 209$ kJ/mol

$k_2 = x \times 10^{-6}$ s^{-1} $\quad T_2 = 600$ k

$$\ln\frac{k_2}{k_1} = \frac{E_a}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$\log\left(\frac{x \times 10^{-6}}{6.36 \times 10^{-3}}\right) = \frac{209 \times 10^3}{8.31 \times 2.303}\left(\frac{1}{700} - \frac{1}{600}\right)$$

$$\log(x \times 10^{-6}) = -4.79$$

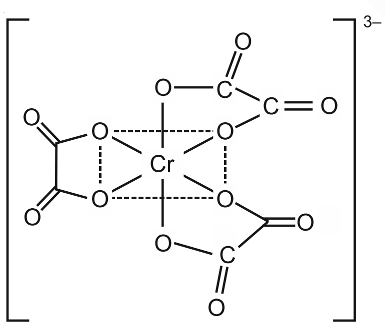
$$x \times 10^{-6} = 1.62 \times 10^{-5}$$

$$x = 16.2 \quad \simeq 16 \text{ (Nearest integer)}$$

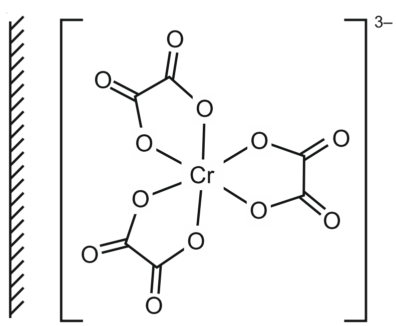
4. The number of optical isomers possible for $[Cr(C_2O_4)_3]^{3-}$ is _______.

Answer (2)

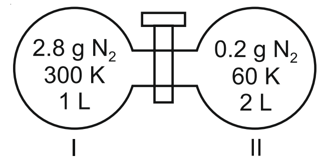
Sol.



Complex is optically active



5. Two flasks I and II shown below are connected by a valve of negligible volume.



When the valve is opened, the final pressure of the system in bar is $x \times 10^{-2}$. The value of x is _____. (Integer answer)

[Assume - Ideal gas; 1 bar = 10^5 Pa; Molar mass of N_2 = 28.0 g mol^{-1}, R = 8.31 J mol^{-1} K^{-1}]

Answer (84)

Sol. Number of moles of flask I = $\frac{2.8}{28} = 0.1$

Number of moles of flask II = $\frac{0.2}{28} = \frac{1}{140}$

$$0.1(300 - T) = \frac{1}{140}(T - 60)$$

T = 284 K

$$P_T = \frac{n_T RT}{V_T} = \frac{\left(\frac{1}{10} + \frac{1}{140}\right) \times 8.31 \times 284}{3} = 84.2$$

≈ 84 (nearest integer)

6. The number of photons emitted by a monochromatic (single frequency) infrared range finder of power 1 mW and wavelength of 1000 nm, in 0.1 second is $x \times 10^{13}$. The value of x is _____. (Nearest integer)

(h = 6.63×10^{-34} Js, c = 3.00×10^8 ms^{-1})

Answer (50)

Sol. Power = 1 mW

= 10^{-3} J in 1 sec.

= 10^{-4} J in 0.1 sec.

$$\therefore \quad \text{Energy} = \frac{nhc}{\lambda}$$

$$10^{-4} = \frac{n \times 6.63 \times 10^{-34} \times 3 \times 10^8}{1000 \times 10^{-9}}$$

n = 50.2×10^{13}

$\therefore \quad x \approx 50$

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7. 40 g of glucose (Molar mass = 180) is mixed with 200 mL of water. The freezing point of solution is _____ K. (Nearest integer)

[Given : K_f = 1.86 K kg mol^{-1}; Density of water = 1.00 g cm^{-3}; Freezing point of water = 273.15 K]

Answer (271)

Sol. Moles of glucose = $\frac{40}{180}$

$\Delta T_f = iK_f m$ (i = 1 for glucose)

$= 1 \times 1.86 \times \frac{40 \times 1000}{180 \times 200}$ $\begin{pmatrix} \text{Mass of water = 200 g} \\ \text{as d = 1 g/mL} \end{pmatrix}$

= 2.06

$\therefore$ Freezing point $= T_f - \Delta T_f$

= 273.15 – 2.06

= 271.09

$\simeq$ 271

8. 100 g of propane is completely reacted with 1000 g of oxygen. The mole fraction of carbon dioxide in the resuting mixture is x × 10^{-2}. The value of x is _____. (Nearest integer)

[Atomic weight : H = 1.008; C = 12.00; O = 16.00]

Answer (19)

Sol.

	C_3H_8	+	$5O_2$	$\longrightarrow$	$3CO_2$	+	$4H_2O$
Moles initial	$\frac{100}{44}$		$\frac{1000}{32}$		–		–
Moles final	–		$\frac{1000}{32} - \frac{100 \times 5}{44}$		$\frac{300}{44}$		$\frac{400}{44}$

Mole fraction of CO_2 = $\frac{\frac{300}{44}}{19.89 + 6.81 + 9.09}$

= 19.02

$\therefore$ x = 19

9. The number of species having non-pyramidal shape among the following is _____.

(A) SO_3

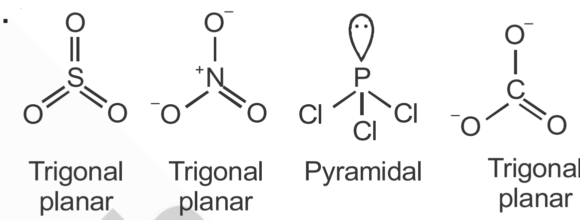
(B) NO_3^-

(C) PCl_3

(D) CO_3^{2-}

Answer (3)

Sol.



10. The resistance of a conductivity cell with cell constant 1.14 cm^{-1}, containing 0.001 M KCl at 298 K is 1500 Ω. The molar conductivity of 0.001 M KCl solution at 298 K in S cm^2 mol^{-1} is _____. (Integer answer)

Answer (760)

Sol. Cell constant = $\frac{l}{a} = 1.14\ cm^{-1}$

C = 0.001 M

T = 298 K

R = 1500 Ω

$k = \frac{1}{R}\left(\frac{l}{a}\right) = \frac{1}{1500} \times 1.14$

$\Lambda_m = \frac{k \times 1000}{C}$

$= \frac{\frac{1}{1500} \times 1.14 \times 1000}{0.001}$

= 760 S cm^2 mol^{-1}

Aakash BYJU'S

PART–C : MATHEMATICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. A box open from top is made from a rectangular sheet of dimension $a \times b$ by cutting squares each of side x from each of the four corners and folding up the flaps. If the volume of the box is maximum, then x is equal to

(1) $\dfrac{a+b-\sqrt{a^2+b^2+ab}}{6}$

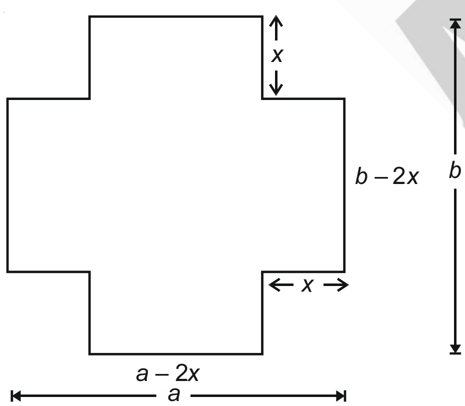
(2) $\dfrac{a+b-\sqrt{a^2+b^2-ab}}{6}$

(3) $\dfrac{a+b-\sqrt{a^2+b^2-ab}}{12}$

(4) $\dfrac{a+b+\sqrt{a^2+b^2-ab}}{6}$

Answer (2)

Sol.



$V = (a - 2x)(b - 2x)x$

for maximum volume

$\dfrac{dV}{dx} = 0$

$\Rightarrow -2(b - 2x)x + (a - 2x)(-2)x + (a - 2x)(b - 2x) = 0$

$\Rightarrow 12x^2 + x(-2a - 2b - 2b - 2a) + ab = 0$

$\Rightarrow 12x^2 - 4(a + b)x + ab = 0$

$\Rightarrow x = \dfrac{4(a+b) \pm \sqrt{16(a+b)^2 - 48ab}}{24}$

$= \dfrac{(a+b) \pm \sqrt{a^2+b^2-ab}}{6}$

$\dfrac{d^2V}{dx^2} = 24x - 4(a+b)$

$= 4(6x - (a + b)) < 0$

For $\dfrac{a+b-\sqrt{a^2+b^2-ab}}{6}$

$\therefore$ Hence V_{max} at $x = \dfrac{a+b-\sqrt{a^2+b^2-ab}}{6}$

2. If the solution curve of the differential equation $(2x - 10y^3)dy + ydx = 0$, passes through the points (0, 1) and (2, β), then β is a root of the equation

(1) $2y^5 - y^2 - 2 = 0$ (2) $y^5 - y^2 - 1 = 0$

(3) $y^5 - 2y - 2 = 0$ (4) $2y^5 - 2y - 1 = 0$

Answer (2)

Sol. $2xdy - 10y^3\,dy + ydx = 0$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{y}{10y^3 - 2x}$

$\Rightarrow \dfrac{dx}{dy} = \dfrac{10y^3 - 2x}{y}$

$\Rightarrow \dfrac{dx}{dy} + \dfrac{2x}{y} = 10y^2$ (Linear D.E.)

I.F. $= e^{\int \frac{2}{y} dy} = y^2$

$\Rightarrow \int d(xy^2) = \dfrac{10y^5}{5}$

$\Rightarrow xy^2 = 2y^5 + C$

$\downarrow (0, 1)$

$C = -2$

$\Rightarrow 2y^5 - xy^2 - 2 = 0$

(Put $x = 2$ gives equation whose root is β)

i.e. $y^5 - y^2 - 1 = 0$

3. Let $A(a, 0)$, $B(b, 2b + 1)$ and $C(0, b)$, $b \neq 0$, $|b| \neq 1$, be points such that the area of triangle ABC is 1 sq. unit, then the sum of all possible values of a is

(1) $\frac{2b}{b+1}$ (2) $\frac{-2b^2}{b+1}$

(3) $\frac{2b^2}{b+1}$ (4) $\frac{-2b}{b+1}$

Answer (2)

Sol. Area = $\left|\frac{1}{2}\begin{vmatrix} a & 0 & 1 \\ b & 2b+1 & 1 \\ 0 & b & 1 \end{vmatrix}\right| = 1$

$\Rightarrow$ $(a(2b + 1 - b) - b(-b) = \pm 2$

$\Rightarrow$ $a(b + 1) = \pm 2 - b^2$

$\Rightarrow$ $a = \frac{2-b^2}{b+1}$ or $\frac{-2-b^2}{b+1}$

Sum of values of a = $\frac{-2-b^2+2-b^2}{b+1} = \frac{-2b^2}{b+1}$

4. Two poles, AB of length a metres and CD of length $a + b$ $(b \neq a)$ metres are erected at the same horizontal level with bases at B and D. If $BD = x$ and $\tan\underline{|ACB} = \frac{1}{2}$, then

(1) $x^2 - 2ax + a(a + b) = 0$

(2) $x^2 + 2(a + 2b)x - b(a + b) = 0$

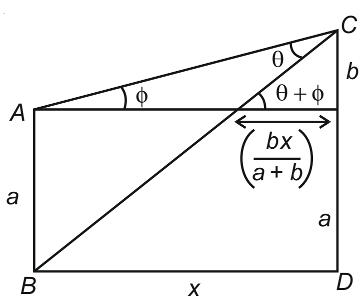
(3) $x^2 + 2(a + 2b)x + a(a + b) = 0$

(4) $x^2 - 2ax + b(a + b) = 0$

Answer (4)

Sol. $\because \tan\theta = \frac{1}{2}$

$\tan\phi = \frac{b}{x}$



and $\tan(\theta+\phi) = \frac{a+b}{x}$

$\Rightarrow \frac{\frac{1}{2}+\frac{b}{x}}{1-\frac{b}{2x}} = \frac{a+b}{x}$

$\Rightarrow 2bx = x^2 = (a+b)2x - b(a+b)$

$\Rightarrow x^2 - 2ax + b(a + b) = 0$

5. Let $[\lambda]$ be the greatest integer less than or equal to λ. The set of all values of λ for which the system of linear equations $x + y + z = 4$, $3x + 2y + 5z = 3$, $9x + 4y + (28 + [\lambda])z = [\lambda]$ has a solution is

(1) $(-\infty, -9) \cup [-8, \infty)$ (2) $[-9, -8)$

(3) R (4) $(-\infty, -9) \cup (-9, \infty)$

Answer (3)

Sol. $x + y + z = 4$

$3x + 2y + 5z = 3$

$9x + 4y + (28 + [\lambda])z = [\lambda]$

For unique solution $\Delta \neq 0$

$\begin{vmatrix} 1 & 1 & 1 \\ 3 & 2 & 5 \\ 9 & 4 & 28+[\lambda] \end{vmatrix} \neq 0$

$\Rightarrow (56 + 2[\lambda] - 20) - 1(84 + 3[\lambda] - 45) + 1(-6) \neq 0$

$\Rightarrow 36 + 2[\lambda] - 39 - 3[\lambda] - 6 \neq 0$

$\Rightarrow [\lambda] \neq -9$

$\Rightarrow \lambda \in (-\infty, -9) \cup [-8, \infty)$

and if $[\lambda] = -9$, $\Delta_x = \Delta_y = \Delta_z = 0$ gives infinite solution.

$\therefore$ for $\lambda \in R$ set of equations have solution.

6. If $\lim_{x\to\infty}\left(\sqrt{x^2-x+1}-ax\right) = b$, then the ordered pair (a, b) is

(1) $\left(1, \frac{1}{2}\right)$ (2) $\left(-1, -\frac{1}{2}\right)$

(3) $\left(-1, \frac{1}{2}\right)$ (4) $\left(1, -\frac{1}{2}\right)$

Answer (4)

Sol. $L = \lim_{x\to\infty} \sqrt{x^2-x+1} - ax$

$= \lim_{x\to\infty} \frac{(x^2-x+1)-(ax)^2}{\sqrt{x^2-x+1}+ax}$

$$L=\lim_{x\to\infty}\frac{(1-a^2)x^2-x+1}{\sqrt{x^2-x+1}+ax}$$

For limit to exist finitely $1-a^2=0$

$$\Rightarrow\quad L=\lim_{x\to\infty}\frac{-x+1}{\sqrt{x^2-x+1}+ax}=\lim_{x\to\infty}\frac{-1+\frac{1}{x}}{\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}+a}$$

$$L=\frac{-1}{1+a}=b$$

For b to be finite, $a\neq-1$

$$\therefore\quad a=1,\ b=\frac{-1}{2}$$

7. If $0<x<1$ and $y=\frac{1}{2}x^2+\frac{2}{3}x^3+\frac{3}{4}x^4+\ldots..$, then the value of e^{1+y} at $x=\frac{1}{2}$ is

(1) 2e

(2) $\frac{1}{2}e^2$

(3) $2e^2$

(4) $\frac{1}{2}\sqrt{e}$

Answer (2)

Sol. $y=\frac{1}{2}x^2+\frac{2}{3}x^3+\frac{3}{4}x^4+\ldots..$

$$=\left(1-\frac{1}{2}\right)x^2+\left(1-\frac{1}{3}\right)x^3+\left(1-\frac{1}{4}\right)x^4+\ldots..$$

$$=\left(x^2+x^3+x^4+\ldots..\right)+\left(-\frac{x^2}{2}-\frac{x^3}{3}-\frac{x^4}{4}+\ldots..\right)$$

$$=\frac{x^2}{1-x}+x+\left(-x-\frac{x^2}{2}-\frac{x^3}{3}-\frac{x^4}{4}+\ldots..\right)$$

$$y=\frac{x}{1-x}+\ln(1-x)$$

$$y+1=\frac{1}{1-x}+\ln(1-x)$$

$$e^{y+1}=e^{\frac{1}{1-x}+\ln(1-x)}=e^{\frac{1}{1-x}}\times e^{\ln(1-x)}=(1-x)e^{\frac{1}{1-x}}$$

$$\therefore\quad \text{at } x=\frac{1}{2}\quad y=\frac{1}{2}e^2$$

8. The set of all values of $k>-1$, for which the equation $(3x^2+4x+3)^2-(k+1)(3x^2+4x+3)(3x^2+4x+2)+k(3x^2+4x+2)^2=0$ has real roots, is

(1) $\left(\frac{1}{2},\frac{3}{1}\right]-\{1\}$

(2) $\left[-\frac{1}{2},1\right)$

(3) [2, 3)

(4) $\left(1,\frac{5}{2}\right]$

Answer (4)

Sol. $3x^2+4x+2>0\quad\forall x\in R\quad(\because D<0)$

$(3x^2+4x+3)^2-(k+1)(3x^2+4x+3)(3x^2+4x+2)+k(3x^2+4x+2)^2=0$

$$\Rightarrow\quad\left(\frac{3x^2+4x+3}{3x^2+4x+2}\right)^2-(k+1)\left(\frac{3x^2+4x+3}{3x^2+4x+2}\right)+k=0\quad\ldots\text{(i)}$$

Let $\frac{3x^2+4x+3}{3x^2+4x+2}=t$

$$t=\frac{3x^2+4x+2+1}{3x^2+4x+2}=1+\frac{1}{3x^2+4x+2}$$

$$3x^2+4x+2\in\left[\frac{2}{3},\infty\right)$$

$$\frac{1}{3x^2+4x+2}\in\left(0,\frac{3}{2}\right]$$

$$t=1+\frac{1}{3x^2+4x+2}\in\left(1,\frac{5}{2}\right]$$

$$\Rightarrow\quad t^2-(k+1)t+k=0\text{ where }t\in\left(1,\frac{5}{2}\right]\quad\ldots\text{(ii)}$$

(ii) should have at least one root in $\left(1,\frac{5}{2}\right]$

$(t-1)(t-k)=0$

$t=1,\ t=k$

$$\therefore\quad k\in\left(1,\frac{5}{2}\right]$$

9. Each of the person A and B independently tosses three fair coins. The probability that both of them get the same number of heads is :

(1) $\frac{1}{8}$

(2) 1

(3) $\frac{5}{8}$

(4) $\frac{5}{16}$

Answer (4)

Sol. Let $A_i\to$ A gets i heads

$B_i\to$ B gets i heads

$$P=P\left(\sum_{i=0}^{3}(A_i\cap B_i)\right)$$

$$=P\left(\sum_{i=0}^{3}P(A_i)P(B_i)\right)$$

$$=P(A_0)P(B_0)+P(A_1)P(B_1)+P(A_2)P(B_2)+P(A_3)P(B_3)$$

$$=\frac{1}{8}\times\frac{1}{8}+\frac{3}{8}\times\frac{3}{8}+\frac{3}{8}\times\frac{3}{8}+\frac{1}{8}\times\frac{1}{8}$$

$$=\frac{5}{16}$$

10. Let $A = \begin{pmatrix} [x+1] & [x+2] & [x+3] \\ [x] & [x+3] & [x+3] \\ [x] & [x+2] & [x+4] \end{pmatrix}$, where [t] denotes the greatest integer less than or equal to t. If det(A) = 192, then the set of values of x is the interval

(1) [60, 61) (2) [65, 66)

(3) [62, 63) (4) [68, 69)

Answer (3)

Sol. $|A| = \begin{vmatrix} [x+1] & [x+2] & [x+3] \\ [x] & [x+3] & [x+3] \\ [x] & [x+2] & [x+4] \end{vmatrix}$

$= \begin{vmatrix} [x]+1 & [x]+2 & [x]+3 \\ [x] & [x]+3 & [x]+3 \\ [x] & [x]+2 & [x]+4 \end{vmatrix}$

$C_3 \to C_3 - C_2$, $C_2 \to C_2 - C_1$

$= \begin{vmatrix} [x]+1 & 1 & 1 \\ [x] & 3 & 0 \\ [x] & 2 & 2 \end{vmatrix}$

(Expanding by C_3)

$= 1(2[x]-3[x]) + 2(3[x]+3-[x]) = -[x] + 2(2[x]+3)$

$= 3[x] + 6$

|A| = 192

3[x] + 6 = 192

[x] = 62

$x \in$ [62, 63)

11. The value of the integral $\int_0^1 \frac{\sqrt{x}dx}{(1+x)(1+3x)(3+x)}$ is

(1) $\frac{\pi}{4}\left(1-\frac{\sqrt{3}}{6}\right)$ (2) $\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{2}\right)$

(3) $\frac{\pi}{4}\left(1-\frac{\sqrt{3}}{2}\right)$ (4) $\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{6}\right)$

Answer (2)

Sol. Put $x = t^2$

$$\int_0^1 \frac{2t^2}{(1+t^2)(1+3t^2)(3+t^2)}dt$$

$$\frac{2t^2}{(1+t^2)(1+3t^2)(3+t^2)} = \frac{A}{1+t^2} + \frac{B}{1+3t^2} + \frac{C}{3+t^2}$$

$\Rightarrow A = \frac{1}{2}, B = -\frac{3}{8}, C = -\frac{3}{8}$

$\Rightarrow \frac{1}{2}\int_0^1 \frac{dt}{1+t^2} - \frac{3}{8}\int_0^1 \frac{dt}{1+3t^2} - \frac{3}{8}\int_0^1 \frac{dt}{3+t^2}$

$= \frac{\pi}{8} - \frac{3}{8}\left(\frac{1}{\sqrt{3}}\cdot\frac{\pi}{3}\right) - \frac{3}{8}\left(\frac{1}{\sqrt{3}}\frac{\pi}{6}\right)$

$= \frac{\pi}{8} - \frac{\sqrt{3}}{16}\pi = \frac{(2-\sqrt{3})\pi}{16}$

12. If $y(x) = \cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right), x \in \left(\frac{\pi}{2}, \pi\right)$, then $\frac{dy}{dx}$ at $x = \frac{5\pi}{6}$ is

(1) $-\frac{1}{2}$ (2) 0

(3) –1 (4) $\frac{1}{2}$

Answer (1)

Sol. $\because \sqrt{1+\sin x} = \sin\frac{x}{2} + \cos\frac{x}{2}$ and

$\sqrt{1-\sin x} = \sin\frac{x}{2} - \cos\frac{x}{2}$

$y(x) = \cot^{-1}\left(\frac{2\sin\frac{x}{2}}{2\cos\frac{x}{2}}\right) = \cot^{-1}\left(\tan\frac{x}{2}\right) = \frac{\pi}{2} - \frac{x}{2}$

$\frac{dy}{dx} = -\frac{1}{2}$

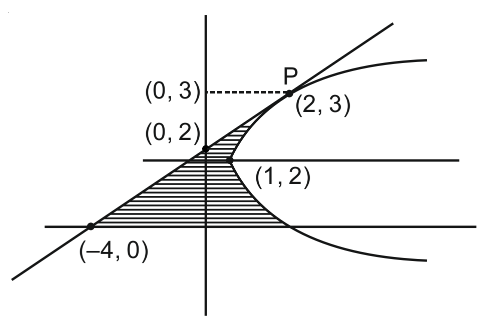
13. The area of the region bounded by the parabola $(y-2)^2 = (x-1)$, the tangent to it at the point whose ordinate is 3 and the x-axis is

(1) 4 (2) 6

(3) 10 (4) 9

Answer (4)

Sol.



Equation of tangent at $P(2, 3)$ is $2y - x = 4$

Required area

$$= \frac{1}{2}(2\times 4)+\int_0^3((y-2)^2+1)dy-\frac{1}{2}(1\times 2)$$

$$= 3+\left[\frac{(y-2)^3}{3}+y\right]_0^3 = 3+6=9$$

14. A differential equation representing the family of parabolas with axis parallel to y-axis and whose length of latus rectum is the distance of the point (2, –3) from the line $3x + 4y = 5$, is given by

(1) $11\frac{d^2y}{dx^2}=10$ (2) $11\frac{d^2x}{dy^2}=10$

(3) $10\frac{d^2x}{dy^2}=11$ (4) $10\frac{d^2y}{dx^2}=11$

Answer (1)

Sol. $\because$ Length of latus rectum = $\frac{11}{5}$

Let equation of parabola : $(x-a)^2=\frac{11}{5}(y-b)$

$\Rightarrow\ 2(x-a)=\frac{11}{5}\frac{dy}{dx}$

$\Rightarrow\ 2=\frac{11}{5}\frac{d^2y}{dx^2}$

$\Rightarrow\ 11\frac{d^2y}{dx^2}=10$

15. Let M and m respectively be the maximum and minimum values of the function $f(x) = \tan^{-1}(\sin x + \cos x)$ in $\left[0, \frac{\pi}{2}\right]$. Then the value of $\tan(M - m)$ is equal to

(1) $2-\sqrt{3}$ (2) $2+\sqrt{3}$

(3) $3+2\sqrt{2}$ (4) $3-2\sqrt{2}$

Answer (4)

Sol. Range of $\sin x + \cos x$ for $x\in\left[0,\frac{\pi}{2}\right]$ is $[1,\sqrt{2}]$

So, $M=\tan^{-1}\sqrt{2}$ and $m = \tan^{-1}1$

$\Rightarrow\ M-m=\tan^{-1}\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)$

$\Rightarrow\ \tan(M-m)=\frac{\sqrt{2}-1}{\sqrt{2}+1}=3-2\sqrt{2}$

16. The angle between the straight lines, whose direction cosines are given by the equations $2l + 2m - n = 0$ and $mn + nl + lm = 0$, is

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{2}$

(3) $\pi-\cos^{-1}\left(\frac{4}{9}\right)$ (4) $\cos^{-1}\left(\frac{8}{9}\right)$

Answer (2)

Sol. $\because$ $2l + 2m - n = 0$ …(i)

and $mn + nl + lm = 0$ …(ii)

From equation (i) and (ii)

$(m + l)(2l + 2m) + lm = 0$

$2l^2 + 5lm + 2m^2 = 0$

$2l^2 + 4lm + lm + 2m^2 = 0$

$2l(l + 2m) + m(l + 2m) = 0$

$\therefore$ $(2l + m)(l + 2m) = 0$

$\therefore$ D.R.^{s} of lines are < 1, –2, –2 > and < 2, –1, 2 >

Here $l_1l_2 + m_1m_2 + n_1n_2 = 0$

$\therefore$ Lines are perpendicular to each other.

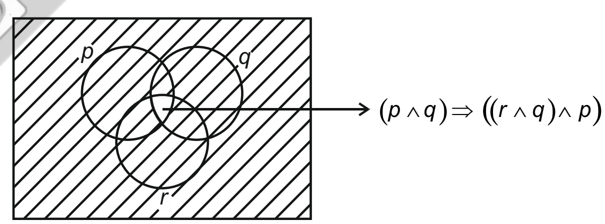
17. The Boolean expression $(p \wedge q) \Rightarrow ((r \wedge q) \wedge p)$ is equivalent to

(1) $(p \wedge q) \Rightarrow (r \wedge q)$ (2) $(q \wedge r) \Rightarrow (p \wedge q)$

(3) $(p \wedge r) \Rightarrow (p \wedge q)$ (4) $(p \wedge q) \Rightarrow (r \vee q)$

Answer (1)

Sol.



This is equivalent to $(p \wedge q) \Rightarrow (r \wedge q)$

18. If two tangents drawn from a point P to the parabola $y^2 = 16(x - 3)$ are at right angles, then the locus of point P is

(1) $x + 2 = 0$ (2) $x + 4 = 0$

(3) $x + 1 = 0$ (4) $x + 3 = 0$

Answer (3)

Sol. From directrix of parabola perpendicular tangent are drawn.

and directrix of parabola $y^2 = 16(x - 3)$ is $x = -1$

$\therefore$ Required locus is $x + 1 = 0$

19. Let $\mathbb{Z}$ be the set of all integers,

$$A = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : (x-2)^2 + y^2 \le 4\},$$

$$B = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x^2 + y^2 \le 4\} \text{ and}$$

$$C = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : (x-2)^2 + (y^2-2)^2 \le 4\}$$

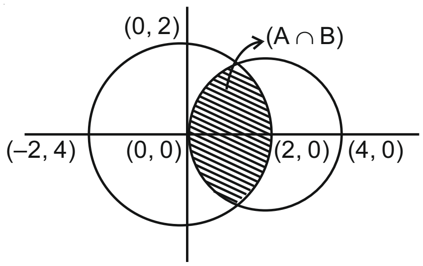
If the total number of relations from A $\cap$ B to A $\cap$ C is 2^p, then the value of p is

(1) 16 (2) 49

(3) 25 (4) 9

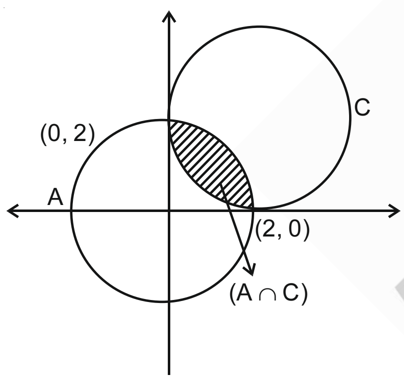
Answer (3)

Sol. The set A and set B are represented as :



$\therefore$ A $\cap$ B = {(0, 0), (1, 0), (2, 0), (1, 1), (1, –1)}

The set A and set C are represented as :



$\therefore$ A $\cap$ C = {(1, 1), (2, 0), (2, 1), (2, 2), (3, 2)}

$\therefore$ Total number relations from A $\cap$ B to A $\cap$ C = $2^{5\times5}$

$\therefore$ p = 25

20. The equation of the plane passing through the line of intersection of the planes $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k}) = 1$ and $\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4=0$ and parallel to the x-axis is

(1) $\vec{r}\cdot(\hat{i}+3\hat{k})+6=0$ (2) $\vec{r}\cdot(\hat{j}-3\hat{k})+6=0$

(3) $\vec{r}\cdot(\hat{j}-3\hat{k})-6=0$ (4) $\vec{r}\cdot(\hat{i}-3\hat{k})+6=0$

Answer (2)

Sol. Equation of plane passing through line of intersection of planes $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k}) = 1$ and $\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4=0$ is

$(x + y + z - 1) + \lambda(2x + 3y - z + 4) = 0$

$\Rightarrow$ $(1 + 2\lambda)x + (1 + 3\lambda)y + (1 - \lambda)z + (4\lambda - 1) = 0$ ...(i)

$\because$ This plane is parallel to x-axis

$\therefore$ $1\cdot(1 + 2\lambda) + 0\cdot(1 + 3\lambda) + 0\cdot(1 - \lambda) = 0$

$\therefore$ $\lambda = -\frac{1}{2}$

$\therefore$ Required equation of plane is

$$-\frac{1}{2}y + \frac{3}{z}z - 3 = 0$$

$\therefore$ $y - 3z + 6 = 0$

$\therefore$ $\vec{r}\cdot(\hat{j}-3\hat{k})+6=0$

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

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1. Let S be the mirror image of the point $Q(1, 3, 4)$ with respect to the plane $2x - y + z + 3 = 0$ and let $R(3, 5, \gamma)$ be a point of this plane. Then the square of the length of the line segment SR is ______.

Answer (72)

Sol. Let S be (x_1, y_1, z_1)

$$\frac{x_1-1}{2} = \frac{y_1-3}{-1} = \frac{z_1-4}{1} = -2\cdot\frac{2\times1-3+4}{2^2+(-1)^2+1^2}$$

$\Rightarrow$ $x_1 = -3,\ y_1 = 5,\ z_1 = 2$

Hence S is $(-3, 5, 2)$

as $R(3, 5, y)$ lies on $2x - y + z + 3 = 0$

$\Rightarrow$ $6 - 5 + \gamma + 3 = 0 \Rightarrow \gamma = -4$

Hence R is $(3, 5, -4)$

$(SR)^2 = 36 + 0 + 36 = 72$

2. The probability distribution of random variable X is given by :

X	1	2	3	4	5
$P(X)$	K	$2K$	$2K$	$3K$	K

Let $p = P(1 < X < 4 \mid X < 3)$. If $5p = \lambda K$, then λ is equal to ______.

Answer (30)

Sol. $p = P\left(\frac{1<X<4}{X<3}\right) = \frac{P((1<X<4)\cap(X<3))}{P(X<3)}$

$$= \frac{P(X=2)}{P(X<3)} = \frac{2K}{K+2K} = \frac{2}{3}$$

Also, $K + 2K + 2K + 3K + K = 1 \Rightarrow K = \frac{1}{9}$

Now, $5p = \lambda K \Rightarrow 5\times\frac{2}{3} = \frac{1}{9}\lambda \Rightarrow \lambda = 30$

3. If $\int \frac{2e^x+3e^{-x}}{4e^x+7e^{-x}}dx = \frac{1}{14}(ux + v\log_e(4e^x + 7e^{-x})) + C$, where C is a constant of integration, then $u + v$ is equal to ______.

Answer (07)

Sol. Write $2e^x + 3e^{-x} = A(4e^x + 7e^{-x}) + B(4e^x - 7e^{-x})$

Comparing both sides

$4A + 4B = 2$ ...(i)

$7A - 7B = 3$ ...(ii)

on solving $A = \frac{13}{28}$ and $B = \frac{1}{28}$

$$I = \int\frac{2e^x+3e^{-x}}{4e^x+7e^{-x}}dx = \int\left(\frac{\frac{13}{28}(4e^x+7e^{-x})+\frac{1}{28}(4e^x-7e^{-x})}{4e^x+7e^{-x}}\right)dx$$

$$= \frac{13}{28}x + \frac{1}{18}\ln(4e^x+7e^{-x}) + C$$

Comparing LHS and RHS gives $u = \frac{13}{2}$ and $v = \frac{1}{2}$

$\Rightarrow u + v = 7$

4. Let $A(\sec\theta, 2\tan\theta)$ and $B(\sec\phi, 2\tan\phi)$, where $\theta + \phi = \frac{\pi}{2}$, be two points on the hyperbola $2x^2 - y^2 = 2$. If (α, β) is the point of the intersection of the normals to the hyperbola at A and B, then $(2\beta)^2$ is equal to ______.

Answer (*)

Sol. Points A and B do not lie on the given curve so it is not possible to solve the question with given data.

5. $3 \times 7^{22} + 2 \times 10^{22} - 44$ when divided by 18 leaves the remainder ______.

Answer (15)

Sol. $3 \times 7^{22} + 2 \times 10^{22} - 44$

$= 3 \times (1 + 6)^{22} + 2(1 + 9)^{22} - 44$

$$= 3\left[{}^{22}C_0 + {}^{22}C_1(6) + {}^{22}C_2(6)^2 + ... + {}^{22}C_{22}(6)^{22}\right]$$

$$+2\left[{}^{22}C_0 + {}^{22}C_1(9) + ... + {}^{22}C_{22}(9)^{22}\right] - 44$$

$= 3.{}^{22}C_0 + 18k_1 + 2.{}^{22}C_0\cdot 18k_2 - 44$

Remainder when divided by 18 = 3 + 2 – 44 = –39

Remainder = (–39 + 54) – 54 ⇒ 15 – 54

= 15

6. Let z_1 and z_2 be two complex numbers such that $\arg(z_1 - z_2) = \frac{\pi}{4}$ and z_1, z_2 satisfy the equation $|z - 3| = \text{Re}(z)$. Then the imaginary part of $z_1 + z_2$ is equal to ______.

Answer (6)

Sol. $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ and $z_1 - z_2 = (x_1 - x_2) + 2(y_1 - y_2)$

$\arg(z_1 - z_2) = \frac{\pi}{4} \Rightarrow \tan^{-1}\left(\frac{y_1-y_2}{x_1-x_2}\right) = \frac{\pi}{4}$

$y_1 - y_2 = x_1 - x_2$ ...(i)

$|z - 3| = \text{Re}(z) \Rightarrow |[x - 3) + 2y| = x$

$(x - 3)^2 + (y)^2 = x^2$

$y^2 = 6\left(x - \frac{3}{2}\right)$

Let point on this parabola

$\left(\frac{3}{2}+at_1^2, 2at_1\right)$ and $\left(\frac{3}{2}+at_2^2, 2at_2\right)$, where $a = \frac{6}{4}$

$y_1 - y_2 = x_1 - x_2$

$2a(t_1 - t_2) = a(t_1^2 - t_2^2)$

$t_1 + t_2 = 2$

Now, $\text{img}(z_1 + z_2) = y_1 + y_2$

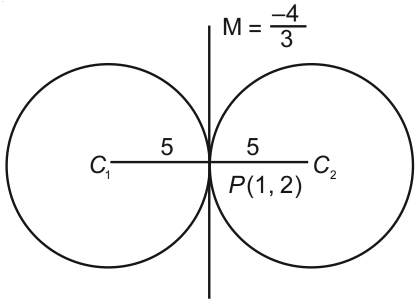
$= 2a(t_1 + t_2)$

$= 2\times\frac{6}{4}(2) = 6$

7. Two circles each of radius 5 units touch each other at the point (1, 2). If the equation of their common tangent is $4x + 3y = 10$, and $C_1(\alpha, \beta)$ and $C_2(\gamma, \delta)$, $C_1 \neq C_2$ are their centres, then $|(\alpha + \beta)(\gamma + \delta)|$ is equal to ______.

Answer (40)

Sol.



$$M_{C_1C_2} = \frac{3}{4} \Rightarrow \cos\theta = \frac{4}{5}, \sin\theta = \frac{3}{5}$$

Point $\dfrac{x-1}{\frac{4}{5}} = \dfrac{y-2}{\frac{3}{5}} = \pm 5$ (by parametric form of line)

$x - 1 = \pm 4$ or $y - 2 = \pm 3$

$x = 5, y = 5$ or $x = -3, y = -1$

$C_1(5, 5)$ and $C_2(-3, -1)$

$|(\alpha + \beta)(\gamma + \delta)| = |(5 + 5)(-3 - 1)| = 40$

8. An online exam is attempted by 50 candidates out of which 20 are boys. The average marks obtained by boys is 12 with a variance 2. The variance of marks obtained by 30 girls is also 2. The average marks of all 50 candidates is 15. If μ is the average marks of girls and σ^2 is the variance of marks of 50 candidates, then $\mu + \sigma^2$ is equal to ________.

Answer (25)

Sol. Sum of marks of boys $\sum X_B = 240$

Total marks $\Rightarrow \sum X = 750$

So, sum of marks of girls $= 510 = \sum X_G$

$$\Rightarrow \frac{\sum X_B^2}{20} - (12)^2 = 2 \text{ and } \frac{\sum X_G^2}{30} - (\bar{X}_G)^2 = 2$$

$$\sum X_B^2 = 2920 \text{ and } \frac{\sum X_B^2}{30} - (17)^2 = 2$$

$$\therefore \quad \sum X_G^2 = 8730$$

$$(\text{variance})_{\text{overall}} = \frac{\sum X_B^2 + X_G^2}{50} - (\bar{X})^2$$

$$= \frac{2920 + 8730}{50} - (15)^2 = 8$$

$\mu = 17, \sigma^2 = 8$

9. Let S = {1, 2, 3, 4, 5, 6, 9}. Then the number of elements in the set T = {A ⊆ S : A ≠ ϕ and the sum of all the elements of A is not a multiple of 3} is ________.

Answer (80)

Sol. There 2 numbers of the type $3\lambda + 1$, 2 numbers of the type $3\lambda - 1$ and 3 numbers of the type 3λ.

So number of subsets whose sum of divisible by 3

$$= 2^3 \cdot ({}^2C_0^2 + {}^2C_1^2 + {}^2C_2^2)$$

$= 48$

Required number of subsets $= 2^7 - 48 = 80$

10. Let S be the sum of all solutions (in radians) of the equation $\sin^4\theta + \cos^4\theta - \sin\theta\cos\theta = 0$ in $[0, 4\pi]$.

Then $\dfrac{8S}{\pi}$ is equal to ________.

Answer (56)

Sol. $(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta - \sin\theta\cos\theta = 0$

Let $\sin\theta \cdot \cos\theta = t$, $1 - 2t^2 - t = 0$

$2t^2 + t - 1 = 0 \quad \Rightarrow \quad t = \frac{1}{2}$ OR -1

$\sin\theta \cdot \cos\theta = \frac{1}{2}$	$\sin\theta\cos\theta = -1$
$\sin 2\theta = 1$	$\sin 2\theta = -2$
$\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4},$	(Not Possible)

$S = 7\pi$

$$\frac{8s}{\pi} = 56$$

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