

27/08/2021
Morning



Corporate Office: Aakash Tower, 8, Pusa Road, New Delhi-110005 Ph. 011-47623456

Time : 3 hrs.

Answers & Solutions

M.M. : 300

for

JEE (MAIN)-2021 (Online) Phase-4

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS :

- (1) The test is of **3 hours** duration.
- (2) The Test Booklet consists of 90 questions. The maximum marks are 300.
- (3) There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry and Mathematics** having 30 questions in each part of equal weightage. Each part has two sections.
 - (i) Section-I : This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **-1 mark** for wrong answer.
 - (ii) Section-II : This section contains 10 questions. In Section-II, attempt any **five questions out of 10**. There will be **no negative marking for Section-II**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and there is no negative marking for wrong answer.

PART-A : PHYSICS

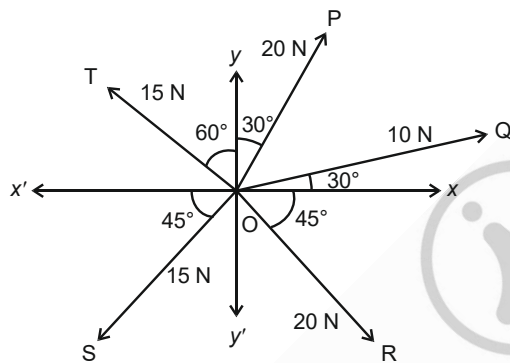
SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The resultant of these forces \vec{OP} , \vec{OQ} , \vec{OR} , \vec{OS} and \vec{OT} is approximately _____ N.

[Take $\sqrt{3} = 1.7$, $\sqrt{2} = 1.4$. Given \hat{i} and \hat{j} unit vectors along x, y axis]



- (1) $9.25\hat{i} + 5\hat{j}$ (2) $3\hat{i} + 15\hat{j}$
 (3) $-1.5\hat{i} - 15.5\hat{j}$ (4) $2.5\hat{i} - 14.5\hat{j}$

Answer (1)

Sol. $\sum F_x = 10 \times \frac{\sqrt{3}}{2} + 20 \times \frac{1}{2} - 15 \times \frac{\sqrt{3}}{2} - 15 \times \frac{1}{\sqrt{2}} + 20 \times \frac{1}{\sqrt{2}}$
 $= 8.5 + 10 - 12.75 - 10.71 + 14.28 = 9.32$

$\sum F_y = 10 \times \frac{1}{2} + 20 \times \frac{\sqrt{3}}{2} + 15 \times \frac{1}{2} - 15 \times \frac{1}{\sqrt{2}} - 20 \times \frac{1}{\sqrt{2}}$
 $= 5 + 17 + 7.5 - 10.71 - 14.28 = 4.54$

$\vec{F}_{net} = 9.32\hat{i} + 4.54\hat{j}$

2. An object is placed beyond the centre of curvature C of the given concave mirror. If the distance of the object is d_1 from C and the distance of the image formed is d_2 from C, the radius of curvature of this mirror is:

- (1) $\frac{d_1 d_2}{d_1 - d_2}$ (2) $\frac{d_1 d_2}{d_1 + d_2}$
 (3) $\frac{2d_1 d_2}{d_1 + d_2}$ (4) $\frac{2d_1 d_2}{d_1 - d_2}$

Answer (4)

Sol. Here distance from focus

object distance, $x = \left(\frac{R}{2} + d_1\right)$

image distance, $y = \left(\frac{R}{2} - d_2\right)$

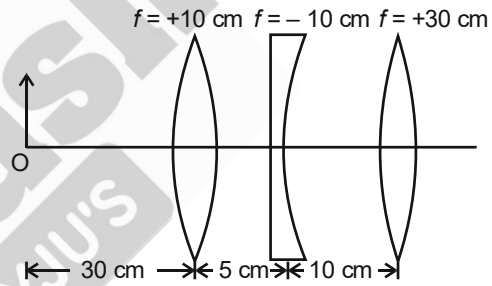
Now $xy = f^2$

$\Rightarrow \left(\frac{R}{2} + d_1\right)\left(\frac{R}{2} - d_2\right) = \frac{R^2}{4}$

$\Rightarrow \frac{R^2}{4} + \frac{R}{2}(d_1 - d_2) - d_1 d_2 = \frac{R^2}{4}$

$\Rightarrow R = \frac{2d_1 d_2}{d_1 - d_2}$

3. Find the distance of the image from object O, formed by the combination of lenses in the figure:



- (1) 75 cm (2) 10 cm
 (3) Infinity (4) 20 cm

Answer (1)

Sol. 1st refraction

$\frac{1}{v_1} - \frac{1}{-30} = \frac{1}{10} \Rightarrow v_1 = 15$

for 2nd refraction

$\frac{1}{v_2} - \frac{1}{10} = \frac{1}{-10} \Rightarrow v_2 = \infty$

for 3rd refraction

Ray will converge at focus of L_3 at 30 cm right of it.

4. A huge circular arc of length 4.4 ly subtends an angle '4s' at the centre of the circle. How long it would take for a body to complete 4 revolution if its speed is 8 AU per second?

Given: 1 ly = 9.46×10^{15} m
 1 AU = 1.5×10^{11} m

- (1) 3.5×10^6 s (2) 4.5×10^{10} s
 (3) 4.1×10^8 s (4) 7.2×10^8 s

Answer (2)

Sol. $l = r\theta$

and $T = \frac{2\pi r}{V} = \frac{2\pi}{V} \times \frac{l}{\theta}$

$\therefore t = 4T = \frac{8\pi \times l}{V\theta}$

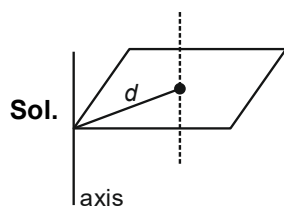
$= \frac{8\pi \times 4.4 \times 9.46 \times 10^{15}}{8 \times 1.5 \times 10^{11} \times 4 \times \frac{1}{3600} \times \frac{\pi}{180}}$

$= 4.5 \times 10^{10}$ s

5. Moment of inertia of a square plate of side l about the axis passing through one of the corner and perpendicular to the plane of square plate is given by:

- (1) $\frac{2}{3}Ml^2$ (2) $\frac{Ml^2}{6}$
 (3) Ml^2 (4) $\frac{Ml^2}{12}$

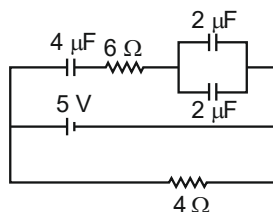
Answer (1)



Sol.

$I_{axis} = I_{cm} + Md^2$
 $= \frac{Ml^2}{6} + M \times \left(\frac{l}{\sqrt{2}}\right)^2$
 $= \frac{Ml^2}{6} + \frac{Ml^2}{2}$
 $= \frac{2}{3}Ml^2$

6. Calculate the amount of charge on capacitor of $4 \mu\text{F}$. The internal resistance of battery is 1Ω :



- (1) zero (2) $8 \mu\text{C}$
 (3) $4 \mu\text{C}$ (4) $16 \mu\text{C}$

Answer (2)

Sol. $C_{eq} = 2 \mu\text{F}$

$Q = C_{eq}V, V = 5 - \frac{5}{5} \times 1 = 4$ volt

$= 2 \times 4 = 8 \mu\text{C}$

7. Five identical cells each of internal resistance 1Ω and emf 5 V are connected in series and in parallel with an external resistance 'R'. For what value of 'R', current in series and parallel combination will remain the same?

- (1) 10Ω
 (2) 5Ω
 (3) 1Ω
 (4) 25Ω

Answer (3)

Sol. For parallel combination

$I_1 = \frac{E_{eq}}{r_{eq} + R} = \frac{5}{\left(\frac{1}{5} + R\right)}$

For series combination

$I_2 = \frac{5 \times 5}{5 + R}$

$I_1 = I_2 \Rightarrow \frac{25}{1 + 5R} = \frac{25}{5 + R}$

$\Rightarrow R = 1 \Omega$

8. A uniformly charged disc of radius R having surface charge density σ is placed in the xy plane with its center at the origin. Find the electric field intensity along the z -axis at a distance Z from origin:

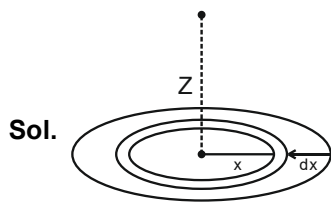
(1) $E = \frac{\sigma}{2\epsilon_0} \left(1 - \frac{Z}{(Z^2 + R^2)^{1/2}}\right)$

(2) $E = \frac{2\epsilon_0}{\sigma} \left(\frac{1}{(Z^2 + R^2)^{1/2}} + Z\right)$

(3) $E = \frac{\sigma}{2\epsilon_0} \left(\frac{1}{(Z^2 + R^2)} + \frac{1}{Z^2}\right)$

(4) $E = \frac{\sigma}{2\epsilon_0} \left(1 + \frac{Z}{(Z^2 + R^2)^{1/2}}\right)$

Answer (1)



$$dE = \frac{(k)(dq)x}{(x^2 + Z^2)^{3/2}}$$

$$E = \int dE$$

$$= \int_0^R \left(\frac{1}{4\pi\epsilon_0} \right) \frac{(\sigma)(2\pi x)(dx)x}{(x^2 + Z^2)^{3/2}}$$

$$= \frac{\sigma}{2\epsilon_0} \left[1 - \frac{Z}{\sqrt{R^2 + Z^2}} \right]$$

9. A balloon carries a total load of 185 kg at normal pressure and temperature of 27°C. What load will the balloon carry on rising to a height at which the barometric pressure is 45 cm of Hg and the temperature is -7°C. Assuming the volume constant?

- (1) 214.15 kg (2) 181.46 kg
(3) 219.07 kg (4) 123.54 kg

Answer (4)

Sol. $\frac{\rho_1 T_1}{P_1} = \frac{\rho_2 T_2}{P_2}$

$$M = V\rho_1 \Rightarrow \frac{M_1}{M_2} = \frac{\rho_1}{\rho_2} = \frac{P_1 T_2}{P_2 T_1}$$

$$\Rightarrow M_2 = \frac{P_2 T_1}{P_1 T_2} M_1$$

$$= \frac{(45) \times 300}{76 \times 266} \times 185 = 123.54 \text{ kg}$$

10. There are 10^{10} radioactive nuclei in a given radioactive element. Its half-life time is 1 minute. How many nuclei will remain after 30 seconds?

$$(\sqrt{2} = 1.414)$$

- (1) 2×10^{10} (2) 10^5
(3) 4×10^{10} (4) 7×10^9

Answer (4)

Sol. $N = N_0 e^{-\lambda t}$

$$\Rightarrow N = (10^{10}) e^{\frac{-\ln 2}{2}}$$

$$= (10^{10}) = \frac{1}{\sqrt{2}}$$

$$= 7 \times 10^9$$

11. An ideal gas is expanding such that $PT^3 = \text{constant}$. The coefficient of volume expansion of the gas is:

(1) $\frac{4}{T}$

(2) $\frac{3}{T}$

(3) $\frac{1}{T}$

(4) $\frac{2}{T}$

Answer (1)

Sol. $PT^3 = C$

$$PV = nRT$$

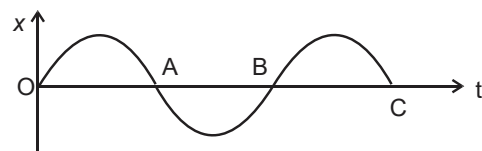
$$P = \frac{nRT}{V}$$

$$\frac{nRT}{V} T^3 = C$$

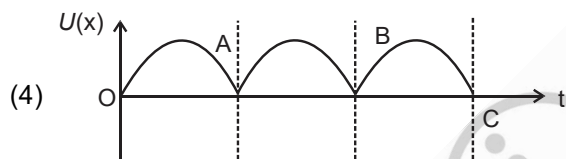
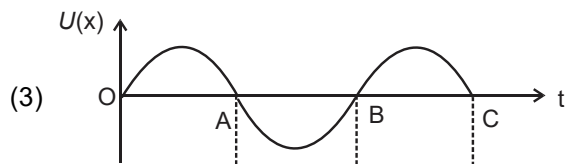
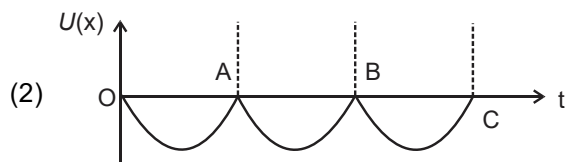
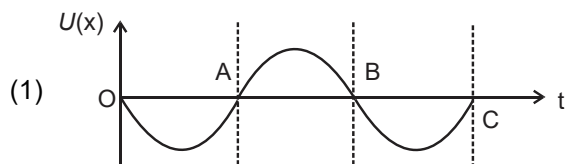
$$V = \frac{nR}{C} T^4 \quad \dots(i)$$

$$\frac{dV}{V} = \frac{4}{T} dT \quad \frac{1}{V} \frac{dV}{dT} = \frac{4}{T}$$

12. The variation of displacement with time of a particle executing free simple harmonic motion is shown in the figure.



The potential energy $U(x)$ versus time (t) plot of the particle is correctly shown in figure.



Answer (4)

Sol. $U = \frac{1}{2}kx^2$

So option (4)

13. For a transistor in CE mode to be used as an amplifier, it must be operated in:

- (1) Cut-off region only
- (2) Saturation region only
- (3) Both cut-off and saturation
- (4) The active region only

Answer (4)

Sol. In CE mode transistor is used as an amplifier in active region only

14. Two ions of masses 4 amu and 16 amu have charges + 2e and + 3e respectively. These ions pass through the region of constant perpendicular magnetic field. The kinetic energy of both ions is same. Then:

- (1) no ion will be deflected
- (2) both ions will be deflected equally
- (3) lighter ion will be deflected more than heavier ion
- (4) lighter ion will be deflected less than heavier ion

Answer (3)

Sol. Radius = $\frac{mV}{qB} = \frac{\sqrt{2Km}}{qB}$

$$\frac{R_1}{R_2} = \frac{\sqrt{m_1}}{q_1} \times \frac{q_2}{\sqrt{m_2}}$$

⇒ $R_1 < R_2$ so deviation of lighter ion is more

15. Electric field in a plane electromagnetic wave is given by $E = 50 \sin(500x - 10 \times 10^{10}t)$ V/m

The velocity of electromagnetic wave in this medium is:

(Given C = speed of light in vacuum)

- (1) $\frac{C}{2}$
- (2) $\frac{2}{3}C$
- (3) $\frac{3}{2}C$
- (4) C

Answer (2)

Sol. $E = 50 \sin(500 - 10 \times 10^{10} t)$ V/m

$\omega = 10 \times 10^{10}$

$K = 500$

⇒ $\frac{\omega}{v} = k$

$v = \frac{\omega}{k} = \frac{10 \times 10^{10}}{500} = \frac{1000}{500} \times 10^8$

$= 2 \times 10^8$

$= \frac{2}{3}(3 \times 10^8)$

$= \frac{2}{3}C$

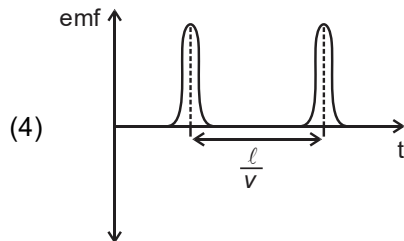
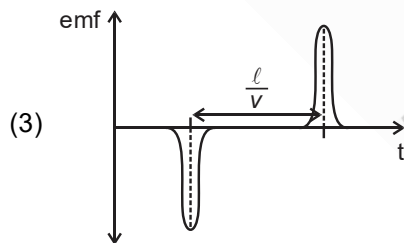
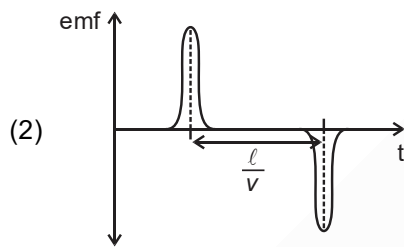
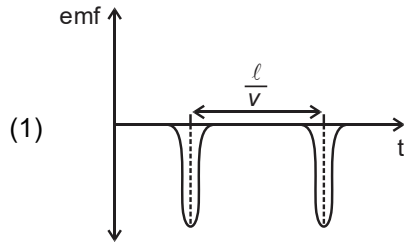
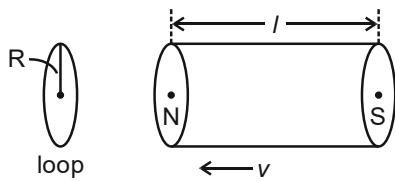
16. Which of the following is not a dimensionless quantity?

- (1) Quality factor
- (2) Power factor
- (3) Permeability of free space(μ_0)
- (4) Relative magnetic permeability(μ_r)

Answer (3)

Sol. Permeability of free space is not a dimensionless quantity.

17. A bar magnet is passing through a conducting loop of radius R with velocity v . The radius of the bar magnet is such that it just passes through the loop. The induced e.m.f. in the loop can be represented by the approximate curve :



Answer (3)

Sol. As the magnet goes into the loop, the flux increases and as it goes out, flux decreases.

\therefore Induced emf changes its polarity.

$$\varepsilon = -\frac{d\phi}{dt}$$

18. In Millikan's oil drop experiment, what is viscous force acting on an uncharged drop of radius $2.0 \times 10^{-5} \text{ m}$ and density $1.2 \times 10^3 \text{ kgm}^{-3}$? Take viscosity of liquid = $1.8 \times 10^{-5} \text{ Nsm}^{-2}$. (Neglect buoyancy due to air).

- (1) $5.8 \times 10^{-10} \text{ N}$
 (2) $1.8 \times 10^{-10} \text{ N}$
 (3) $3.8 \times 10^{-11} \text{ N}$
 (4) $3.9 \times 10^{-10} \text{ N}$

Answer (4)

Sol. At steady state,

Viscous force = Gravity force

$$= \frac{4}{3} \pi r^3 \times \rho g$$

$$= \frac{4}{3} \times 3.14 \times 8 \times 10^{-15} \times 1.2 \times 10^3 \times 9.8$$

$$= 3.9 \times 10^{-10} \text{ N}$$

19. In a photoelectric experiment, increasing the intensity of incident light :

- (1) Increases the frequency of photons incident and the K.E. of the ejected electrons remains unchanged.
 (2) Increases the frequency of photons incident and increases the K.E. of the ejected electrons.
 (3) Increases the number of photons incident and also increases the K.E. of the ejected electrons.
 (4) Increases the number of photons incident and the K.E. of the ejected electrons remains unchanged.

Answer (4)

Sol. Intensity change does not affect the maximum kinetic energy of emitted electron.

20. If E and H represents the intensity of electric field and magnetising field the unit of E/H will be respectively, then

- (1) joule (2) ohm
 (3) newton (4) mho

Answer (2)

Sol. Unit of $E = \frac{V}{m}$

Unit of $H = \frac{A}{m}$

$$\frac{E}{H} = \frac{V}{A} = \Omega$$

SECTION - II

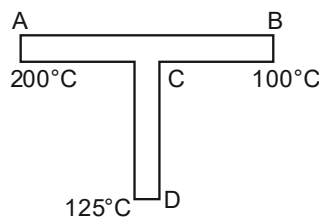
Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. A transmitting antenna has a height of 320 m and that of receiving antenna is 200 m. The maximum distance between them for satisfactory communication in line of sight mode is 'd'. The value of 'd' is _____ km.

Answer (224)

Sol. $d = \sqrt{2Rh_T} + \sqrt{2Rh_R}$
 $= \sqrt{R} (\sqrt{0.64} + \sqrt{4})$
 $= 224 \text{ km}$

2. A rod CD of thermal resistance 10.0 KW^{-1} is joined at the middle of an identical rod AB as shown in figure. The ends A, B and D are maintained at 200°C , 100°C and 125°C respectively. The heat current in CD is P watt. The value of P is _____.



Answer (2)

Sol. $\frac{200 - T}{R/2} + \frac{100 - T}{R/2} = \frac{T - 125}{R}$
 $\Rightarrow T = 145^\circ\text{C}$
 $H = \frac{145 - 125}{10} = 2 \text{ W}$

3. The cars X and Y are approaching each other with velocities 36 km/h and 72 km/h respectively. The frequency of a whistle sound as emitted by a passenger in car X, heard by the passenger in car Y is 1320 Hz. If the velocity of sound in air is 340 m/s, the actual frequency of the whistle sound produced is _____ Hz.

Answer (1210)

Sol. $f = f_0 \left(\frac{v + v_0}{v - v_s} \right)$
 $f_0 = f \left(\frac{v - v_s}{v + v_0} \right) = 1320 \left(\frac{340 - 10}{340 + 20} \right)$
 $= 1210 \text{ Hz}$

4. The alternating current is given by

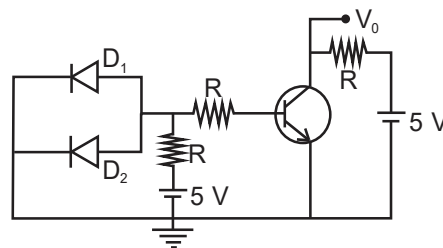
$$i = \left\{ \sqrt{42} \sin\left(\frac{2\pi}{T}t\right) + 10 \right\} \text{ A}$$

The r.m.s. value of this current is _____ A.

Answer (11)

Sol. $i = 10 + \sqrt{42} \sin\left(\frac{2\pi}{T}t\right) \text{ A}$
 $i_{\text{rms}} = \sqrt{10^2 + \frac{(\sqrt{42})^2}{2}} = 11 \text{ A}$

5. A circuit is arranged as shown in figure. The output voltage V_0 is equal to _____ V.



Answer (5)

Sol. The transistor is not sufficiently biased at
 Base - Collector junction
 $\Rightarrow I_c = 0$
 $\Rightarrow V_0 = 5 \text{ V}$

6. First, a set of n equal resistors of 10Ω each are connected in series to a battery of emf 20 V and internal resistance 10Ω . A current I is observed to flow. Then, the n resistors are connected in parallel to the same battery. It is observed that the current is increased 20 times, the value of n is _____.

Answer (20)

Sol. $I_1 = \frac{20}{10(n+1)} = \frac{2}{n+1}$

$$I_2 = \frac{20}{10 + \frac{10}{n}} = \frac{2n}{n+1}$$

$$I_2 = 20I_1 \Rightarrow n = 20$$

7. Two persons A and B perform same amount of work in moving a body through a certain distance d with application of forces acting at angles 45° and 60° with the direction of displacement respectively. The ratio of force applied by person A to the force applied by person B is $\frac{1}{\sqrt{x}}$. The value of x is _____.

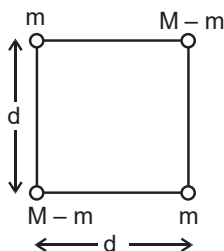
Answer (2)

Sol. $F_1 d \cos 45^\circ = F_2 d \cos 60^\circ$

$$\Rightarrow \frac{F_2}{F_1} = \sqrt{2}$$

$$\frac{F_1}{F_2} = \frac{1}{\sqrt{2}}$$

8. A body of mass $(2M)$ splits into four masses $\{m, M - m, m, M - m\}$, which are rearranged to form a square as shown in the figure. The ratio of $\frac{M}{m}$ for which, the gravitational potential energy of the system becomes maximum is $x : 1$. The value of x is _____.



Answer (2)

Sol. $|G.P.E|_{\text{system}} = \frac{Gm^2}{d\sqrt{2}} + \frac{G(M-m)^2}{d\sqrt{2}} + \frac{4Gm(M-m)}{d}$

Differentiation with respect to m should be equal to zero.

$$\Rightarrow 2m - 2(M-m) + 4\sqrt{2}(M-m) - 4\sqrt{2}(m) = 0$$

$$\Rightarrow m(2 - 4\sqrt{2}) + (M-m)(4\sqrt{2} - 2) = 0$$

$$\Rightarrow m = \frac{M}{2}$$

9. A uniform conducting wire of length is $24a$, and resistance R is wound up as a current carrying coil in the shape of an equilateral triangle of side 'a' and then in the form of a square of side 'a'. The coil is connected to a voltage source V_0 . The ratio of magnetic moment of the coils in case of equilateral triangle to that for square is $1:\sqrt{y}$, where y is _____.

Answer (3)

Sol. Magnetic moment of triangular coil

$$= \frac{V_0}{R} \times \left(\frac{\sqrt{3}}{4} a^2 \right) 8 = M_1$$

Magnetic moment of square coil = $\frac{V_0}{R} (a^2) 6 = M_2$

$$\frac{M_1}{M_2} = \frac{2\sqrt{3}}{6} = \frac{1}{\sqrt{3}}$$

10. If the velocity of a body related to displacement x is given by $v = \sqrt{5000 + 24x} \text{ m/s}$, then the acceleration of the body is _____ m/s^2

Answer (12)

Sol. $v = \sqrt{5000 + 24x}$

$$v^2 = 5000 + 24x$$

$$2v \frac{dv}{dx} = 24$$

$$v \frac{dv}{dx} = 12$$

PART-B : CHEMISTRY

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Acidic ferric chloride solution on treatment with excess of potassium ferrocyanide gives a Prussian blue coloured colloidal species. It is :

- (1) $KFe[Fe(CN)_6]$ (2) $K_5Fe[Fe(CN)_6]_2$
 (3) $Fe_4[Fe(CN)_6]_3$ (4) $HFe[Fe(CN)_6]$

Answer (1)

Sol. $FeCl_3 + K_4[Fe(CN)_6] \longrightarrow KFe[Fe(CN)_6] + 3KCl$
 (Prussian blue)

2. The unit of the van der Waals gas equation

parameter 'a' in $\left(P + \frac{an^2}{V^2}\right)(V - nb) = nRT$ is :

- (1) $\text{atm dm}^6 \text{ mol}^{-2}$ (2) kg m s^{-1}
 (3) kg m s^{-2} (4) $\text{dm}^3 \text{ mol}^{-1}$

Answer (1)

Sol. The unit of 'a' in van der Waals gas equation is $\text{atm dm}^6 \text{ mol}^{-2}$.

3. The number of water molecules in gypsum, dead burnt plaster and plaster of Paris, respectively

- (1) 2, 0 and 1
 (2) 0.5, 0 and 2
 (3) 5, 0 and 0.5
 (4) 2, 0 and 0.5

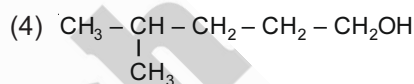
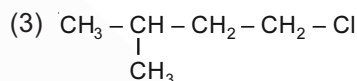
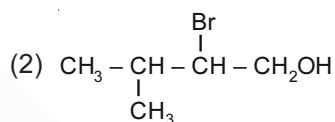
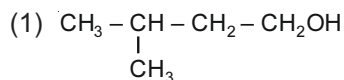
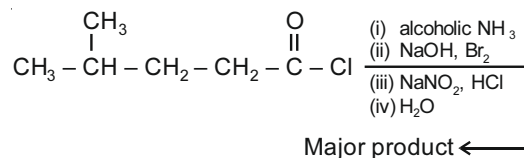
Answer (4)

Sol. The chemical formulae of the given compounds are

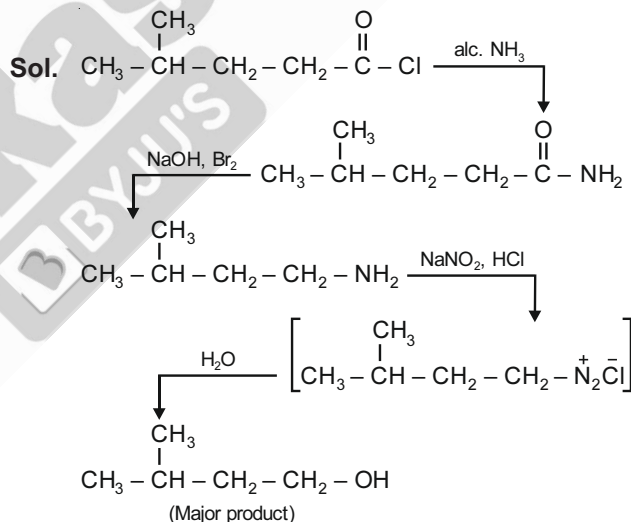
- (i) Gypsum $CaSO_4 \cdot 2H_2O$
 (ii) Dead burnt plaster $CaSO_4$
 (iii) Plaster of Paris $CaSO_4 \cdot \frac{1}{2}H_2O$

\therefore The number of water molecules in gypsum, dead burnt plaster and plaster of Paris are respectively, 2, 0 and 0.5.

4. The major product of the following reaction is :



Answer (1)



5. The nature of oxides V_2O_3 and CrO is indexed as 'X' and 'Y' type respectively. The correct set of X and Y is :

- (1) X = amphoteric Y = basic
 (2) X = basic Y = basic
 (3) X = basic Y = amphoteric
 (4) X = acidic Y = acidic

Answer (2)

Sol. V_2O_3 (X) is basic and CrO (Y) is also basic. The transition metal oxides in lower oxidation states are basic.

6. Match List - I with List - II :

List - I (Species)	List - II (No. of lone pairs of electrons on the central atom)
-----------------------	---

- | | |
|-------------------------------------|---------|
| (a) XeF ₂ | (i) 0 |
| (b) XeO ₂ F ₂ | (ii) 1 |
| (c) XeO ₃ F ₂ | (iii) 2 |
| (d) XeF ₄ | (iv) 3 |

Choose the **most appropriate** answer from the options given below :

- (1) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)
- (2) (a)-(iv), (b)-(i), (c)-(ii), (d)-(iii)
- (3) (a)-(iii), (b)-(ii), (c)-(iv), (d)-(i)
- (4) (a)-(iv), (b)-(ii), (c)-(i), (d)-(iii)

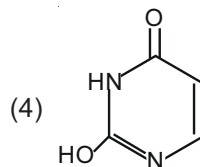
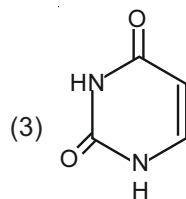
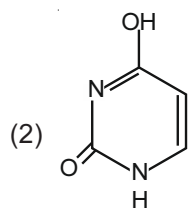
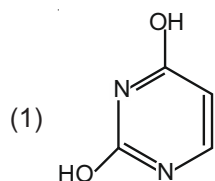
Answer (4)

Sol. No. of lone pairs of electrons on the central atom

$$\frac{(\text{No. of valence electrons on central atom}) - 2 \times (\text{No. of bivalent atoms}) - (\text{No. of monovalent atoms})}{2}$$

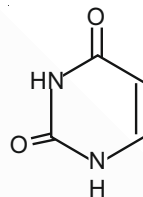
- | | |
|---------------------------------------|--|
| (a) XeF ₂ : | $n = \frac{8-2}{2} = 3$ |
| (b) XeO ₂ F ₂ : | $n = \frac{8 - (2 \times 2) - 2}{2} = 1$ |
| (c) XeO ₃ F ₂ : | $n = \frac{8 - (2 \times 3) - 2}{2} = 0$ |
| (d) XeF ₄ : | $n = \frac{8-4}{2} = 2$ |

7. Out of following isomeric forms of uracil, which one is present in RNA ?



Answer (3)

Sol. The isomeric form of uracil present in RNA is

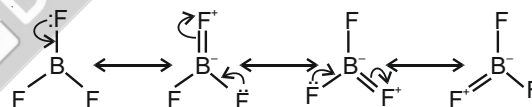


8. In which one of the following molecules strongest back donation of an electron pair from halide to boron is expected?

- | | |
|----------------------|----------------------|
| (1) BBr ₃ | (2) BCl ₃ |
| (3) BI ₃ | (4) BF ₃ |

Answer (4)

Sol. Among the given boron trihalides, the extent of back donation is maximum in BF₃ due to smaller size of F - atom



9. Match items of List - I with those of List - II :

List - I (Property)	List - II (Example)
(a) Diamagnetism	(i) MnO
(b) Ferrimagnetism	(ii) O ₂
(c) Paramagnetism	(iii) NaCl
(d) Antiferromagnetism	(iv) Fe ₃ O ₄

Choose the **most appropriate** answer from the options given below :

- (1) (a)-(iv), (b)-(ii), (c)-(i), (d)-(iii)
- (2) (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)
- (3) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)
- (4) (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)

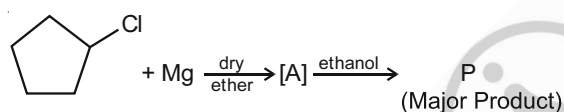
Answer (3)

- Sol.** (a) Diamagnetism — (iii) NaCl
 (b) Ferrimagnetism — (iv) Ne_3O_4
 (c) Paramagnetism — (ii) O_2
 (d) Antiferromagnetism — (i) MnO
10. Tyndall effect is more effectively shown by :
- (1) Lyophobic colloid
 - (2) Lyophilic colloid
 - (3) True solution
 - (4) Suspension

Answer (1)

Sol. Tyndall effect is more effectively shown by lyophobic colloids than Lyophilic colloids. True solution and suspension do not show Tyndall effect.

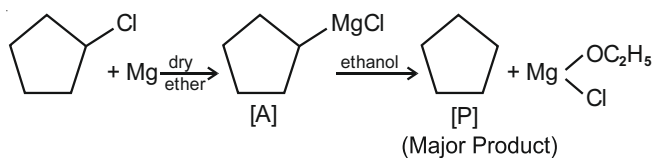
11. In the following sequence of reactions the P is :



- (1)
- (2)
- (3)
- (4)

Answer (3)

Sol.



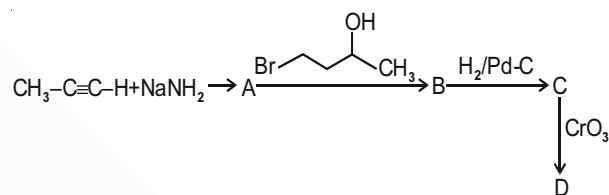
The alkyl part of Grignard's reagent [A] picks up proton from ethanol forming cyclopentane as major product.

12. Deuterium resembles hydrogen in properties but :
- (1) Reacts vigorously than hydrogen
 - (2) Reacts slower than hydrogen
 - (3) emits β^+ particles
 - (4) Reacts just as hydrogen

Answer (2)

Sol. Deuterium resembles hydrogen in properties but reacts slower than hydrogen due to its higher bond dissociation energy.

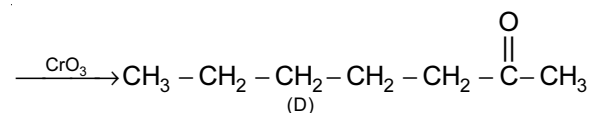
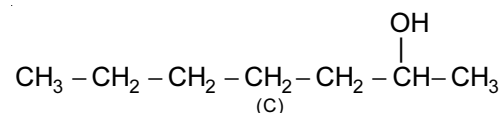
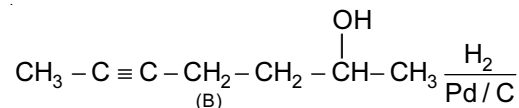
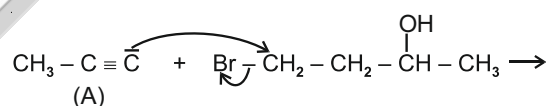
13. In the following sequence of reactions, the final product D is :



- (1) $\text{CH}_3\text{-CH}_2\text{-CH}_2\text{-CH}_2\text{-CH}_2\text{-C(=O)-CH}_3$
- (2) $\text{H}_3\text{C-CH=CH-CH(OH)-CH}_2\text{-CH}_2\text{-CH}_3$
- (3) $\text{CH}_3\text{-CH=CH-CH}_2\text{-CH}_2\text{-CH}_2\text{-COOH}$
- (4) $\text{H}_3\text{C-CH}_2\text{-CH}_2\text{-CH}_2\text{-CH}_2\text{-C(=O)-H}$

Answer (1)

Sol. $\text{CH}_3\text{-C}\equiv\text{C-H} + \text{NaNH}_2 \rightarrow \text{CH}_3\text{-C}\equiv\text{C}^-\text{Na}^+ + \text{NH}_3(\text{g})$



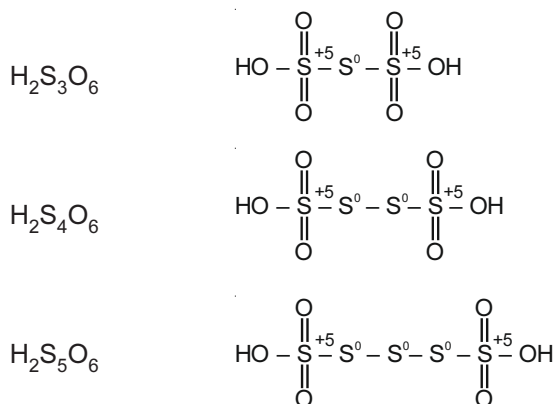
Note : The anion of (A) is a strong base and on its reaction with 4-bromobutan-2-ol, it is likely to pick up proton from alcohol. But looking at the options, nucleophilic substitution is considered to get (B).

14. In polythionic acid, $H_2S_xO_6$ ($x = 3$ to 5) the oxidation state(s) of sulphur is/are :

- (1) 0 and +5 only (2) +5 only
(3) +6 only (4) +3 and +5 only

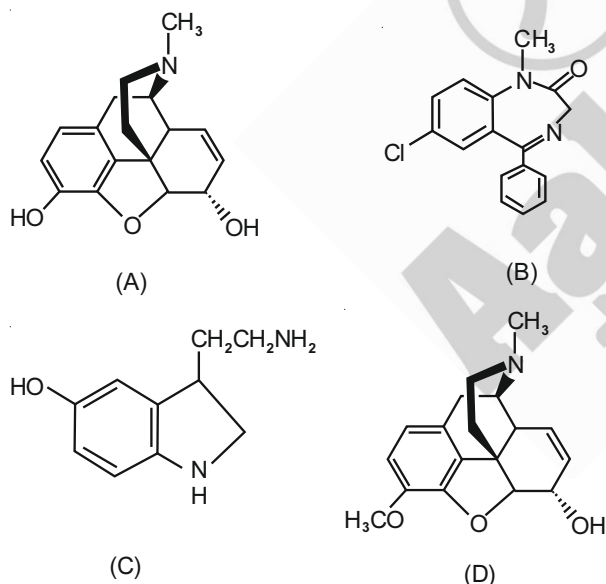
Answer (1)

Sol. $H_2S_nO_6$ ($x = 3$ to 5)



∴ Oxidation state(s) of sulphur in the above compounds are +5 and 0 only

15.



The correct statement about (A), (B), (C) and (D) is :

- (1) (B), (C) and (D) are tranquilizers
(2) (B) and (C) are tranquilizers
(3) (A) and (D) are tranquilizers
(4) (A), (B) and (C) are narcotic analgesics

Answer (2)

Sol. Compound (B) is Valium and compound (C) is serotonin. They are used as tranquilizers.

16. Given below are two statements: one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Synthesis of ethyl phenyl ether may be achieved by Williamson synthesis.

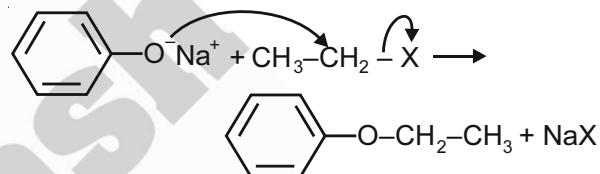
Reason (R) : Reaction of bromobenzene with sodium ethoxide yields ethyl phenyl ether.

In the light of the above statement, choose the **most appropriate** answer from the options given below:

- (1) (A) is not correct but (R) is correct
(2) (A) is correct but (R) is not correct
(3) Both (A) and (R) correct but (R) is NOT the correct explanation of (A)
(4) Both (A) and (R) are correct and (R) is the correct explanation of (A)

Answer (2)

Sol. Assertion is correct



But the reason is not correct because aryl halides do not undergo nucleophilic substitution reactions.

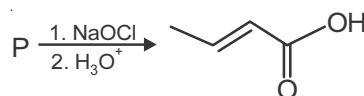
17. Which refining process is generally used in the purification of low melting metals?

- (1) Electrolysis
(2) Zone refining
(3) Liquefaction
(4) Chromatographic method

Answer (3)

Sol. Metals having low melting points like tin are refined by liquation.

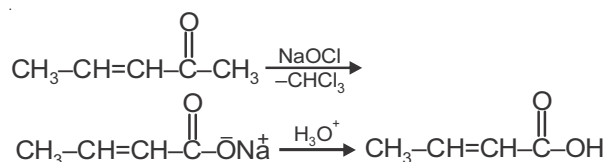
18. The structure of the starting compound P used in the reaction given below is:



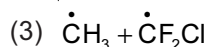
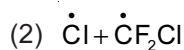
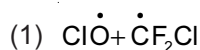
- (1)
- (2)
- (3)
- (4)

Answer (2)

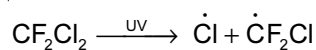
Sol. Aldehydes and ketones having 3 α -hydrogen atoms undergo haloform reaction with NaOCl. The carbonyl compound may or may not have a double bond.



19. The gas 'A' is having very low reactivity reaches to stratosphere. It is non-toxic and non-flammable but dissociated by UV-radiations in stratosphere. The intermediates formed initially from the gas 'A' are:

**Answer (2)**

Sol. The gas 'A' is likely to be freon (CF_2Cl_2) which is non-reactive, non-flammable and non-toxic. Once it reaches stratosphere, it is broken down by powerful UV-radiation to give Cl and CF_2Cl radicals as intermediates.



20. Which of the following is **not** a correct statement for primary aliphatic amines?

- (1) Primary amines on treating with nitrous acid solution form corresponding alcohols except methyl amine.
- (2) The intermolecular association in primary amines is less than the intermolecular association in secondary amines.
- (3) Primary amines can be prepared by the Gabriel phthalimide synthesis.
- (4) Primary amines are less basic than the secondary amines.

Answer (2)

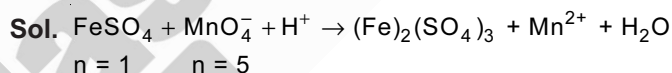
Sol. Intermolecular association due to H-bonding in primary amines is more than in secondary amines because primary amines have more number of H-atoms bonded to N-atom.

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. When 10 mL of an aqueous solution of KMnO_4 was titrated in acidic medium, equal volume of 0.1 M of an aqueous solution of ferrous sulphate was required for complete discharge of colour. The strength of KMnO_4 in grams per litre is _____ $\times 10^{-2}$. (Nearest integer)

[Atomic mass of K = 39, Mn = 55, O = 16]

Answer (316)

Equivalents of KMnO_4 = Equivalents of FeSO_4

$$= \frac{0.1 \times 1 \times 10}{1000} = 10^{-3}$$

$$\text{Moles of } \text{KMnO}_4 \text{ in 10 mL} = \frac{10^{-3}}{5} = 2 \times 10^{-4}$$

$$\text{Moles of } \text{KMnO}_4 \text{ in 1 L} = 2 \times 10^{-4} \times 100 = 0.02$$

$$\text{Mass of } \text{KMnO}_4 \text{ in 1 L} = 158 \times 0.02 = 316 \times 10^{-2} \text{ g/L}$$

2. The kinetic energy of an electron in the second

$$\text{Bohr orbit of a hydrogen atom is equal to } \frac{h^2}{x m a_0^2}.$$

The value of 10x is _____. (a_0 is radius of Bohr's orbit) (Nearest integer)

[Given : $\pi = 3.14$]

Answer (3155)

Sol. Kinetic energy of an electron in n^{th} orbit of Bohr

$$\text{atom} = \frac{1}{2}mv^2 = \frac{(mv)^2}{2m} = \frac{n^2h^2}{2(4\pi^2mr^2)}$$

For 2nd orbit of H-atom

$$n = 2 \text{ and } r = 4a_0$$

$$\therefore \text{KE} = \frac{h^2}{8\pi^2m \times 4a_0^2} = \frac{h^2}{315.5 \text{ ma}_0^2}$$

$$\therefore x = 315.5; 10x = 3155$$

3. 1 kg of 0.75 molal aqueous solution of sucrose can be cooled up to -4°C before freezing. The amount of ice (in g) that will be separated out is _____. (Nearest integer)

$$[\text{Given} : K_f(\text{H}_2\text{O}) = 1.86 \text{ K kg mol}^{-1}]$$

Answer (518)

Sol. Molality of sucrose solution = 0.75 m

$$\text{Mass of sucrose} = 0.75 \times 342 \text{ g} = 256.5 \text{ g}$$

$$\text{Mass of solutions} = 1256.5 \text{ g}$$

Mass of sucrose in 1 kg solution

$$= \frac{256.5 \times 1000}{1256.5} = 204.1 \text{ g}$$

$$\begin{aligned} \text{Mass of water in 1 kg solution} &= 1000 - 204.1 \\ &= 795.9 \text{ g} \end{aligned}$$

After colling the solution to -4°C

$$4 = \frac{1.86 \times 204.1 \times 1000}{342 \times w_B}; w_B = 277.5 \text{ g}$$

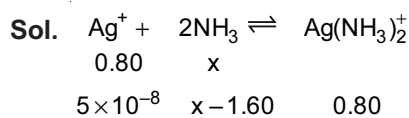
(w_B is the mass of water left)

$$\begin{aligned} \text{Mass of ice separated} &= 795.9 - 277.5 \\ &= 518.4 \approx 518 \text{ g} \end{aligned}$$

4. The number of moles of NH_3 , that must be added to 2 L of 0.80 M AgNO_3 in order to reduce the concentration of Ag^+ ions to 5.0×10^{-8} M ($K_{\text{formation}}$ for $[\text{Ag}(\text{NH}_3)_2]^+ = 1.0 \times 10^8$) is _____. (Nearest integer)

[Assume no volume change on adding NH_3]

Answer (4)



$$K_f = \frac{[\text{Ag}(\text{NH}_3)_2^+]}{[\text{Ag}^+][\text{NH}_3]^2} = \frac{0.80}{5 \times 10^{-8}(x - 1.60)^2} = 10^8$$

$$(x - 1.60)^2 = 6.25; x = 2.5 + 1.6 = 4.1 \text{ moles}$$

Number of moles of NH_3 required for 2L solution

$$= 2 \times 4.1 = 8$$

5. The number of f electrons in the ground state electronic configuration of Np ($Z = 93$) is _____. (Integer answer)

Answer (4)

Sol. The electronic configuration of neptunium in ground state is $[\text{Rn}] 5f^4 6d^1 7s^2$

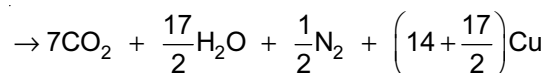
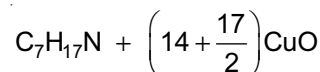
\therefore It has 4 electrons in the f subshell of the anti penultimate shell.

6. The number of moles of CuO , that will be utilized in Dumas method for estimating nitrogen in a sample of 57.5 g of N,N -dimethylaminopentane is _____ $\times 10^{-2}$. (Nearest integer)

Answer (1125)

Sol. The chemical formula of N,N -dimethylaminopentane is $\text{C}_7\text{H}_{17}\text{N}$ (Molar mass = 115)

$$\text{Number of moles of } \text{C}_7\text{H}_{17}\text{N} \text{ taken} = \frac{57.5}{115} = 0.5$$



For 0.5 moles of $\text{C}_7\text{H}_{17}\text{N}$, number of moles of CuO

$$\text{required} = \frac{1}{2} \left(14 + \frac{17}{2}\right) = \frac{45}{4} = 1125 \times 10^{-2}$$

7. In Carius method for estimation of halogens, 0.2 g of an organic compound gave 0.188 g of AgBr . The percentage of bromine in the compound is _____. (Nearest integer)

[Atomic mass : $\text{Ag} = 108, \text{Br} = 80$]

Answer (40)

Sol. Mass of organic compound = 0.2 gm

Mass of AgBr = 0.188

$$\text{Mass of Br} = \frac{0.188 \times 80}{188} = 0.08 \text{ gm}$$

$$\begin{aligned} \text{Percentage of Br in the compound} &= \frac{0.08 \times 100}{0.2} \\ &= 40\% \end{aligned}$$

8. 1 mol of an octahedral metal complex with formula $\text{MCl}_3 \cdot 2\text{L}$ on reaction with excess of AgNO_3 gives 1 mol of AgCl . The denticity of Ligand L is _____. (Integer answer)

Answer (2)

Sol. 1 mol of octahedral complex $\text{MCl}_3 \cdot 2\text{L}$ on reaction with AgNO_3 gives 1 mol of AgCl

\therefore Formula of complex is $[\text{MCl}_2 \cdot 2\text{L}]\text{Cl}$

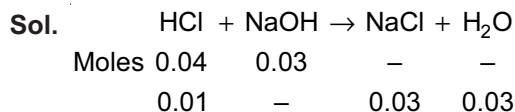
Since co-ordination number of M is 6, the denticity of L must be 2.

9. 200 mL of 0.2 M HCl is mixed with 300 mL of 0.1 M NaOH . The molar heat of neutralization of this reaction is -57.1 kJ . The increase in temperature in $^\circ\text{C}$ of the system on mixing is $x \times 10^{-2}$. The value of x is _____. (Nearest integer)

[Given : Specific heat of water = $4.18 \text{ J g}^{-1} \text{ K}^{-1}$

Density of water = 1.00 g cm^{-3}]

(Assume no volume change on mixing)

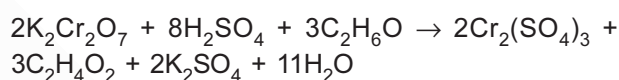
Answer (82)

$$Q, \text{ Heat released} = 0.03 \times 57.1 \text{ kJ} = 1.713 \text{ kJ}$$

$$Q = m \times s \times \Delta T$$

$$\Delta T = \frac{1.713 \times 1000}{500 \times 4.18} = 81.96 \times 10^{-2} \approx 82 \times 10^{-2}$$

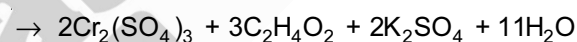
10. The reaction that occurs in a breath analyser, a device used to determine the alcohol level in a person's blood stream is



If the rate of appearance of $\text{Cr}_2(\text{SO}_4)_3$ is $2.67 \text{ mol min}^{-1}$ at a particular time, the rate of disappearance of $\text{C}_2\text{H}_6\text{O}$ at the same time is _____. (Nearest integer)

Answer (4)

Sol. $2\text{K}_2\text{Cr}_2\text{O}_7 + 8\text{H}_2\text{SO}_4 + 3\text{C}_2\text{H}_6\text{O}$



$$\text{Rate} = -\frac{1}{3} \frac{d[\text{C}_2\text{H}_6\text{O}]}{dt} = \frac{1}{2} \frac{d[\text{Cr}_2(\text{SO}_4)_3]}{dt}$$

$$\frac{d[\text{Cr}_2(\text{SO}_4)_3]}{dt} = 2.67 \text{ mol min}^{-1}$$

$$-\frac{d[\text{C}_2\text{H}_6\text{O}]}{dt} = \frac{3}{2} \frac{d[\text{Cr}_2(\text{SO}_4)_3]}{dt}$$

$$= \frac{3}{2} \times 2.67 = 4 \text{ mol min}^{-1}$$

PART-C : MATHEMATICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. If $(\sin^{-1} x)^2 - (\cos^{-1} x)^2 = a$; $0 < x < 1$, $a \neq 0$, then the value of $2x^2 - 1$ is

- (1) $\cos\left(\frac{2a}{\pi}\right)$ (2) $\sin\left(\frac{4a}{\pi}\right)$
 (3) $\cos\left(\frac{4a}{\pi}\right)$ (4) $\sin\left(\frac{2a}{\pi}\right)$

Answer (4)

Sol. $(\sin^{-1} x)^2 - (\cos^{-1} x)^2 = (\sin^{-1} x + \cos^{-1} x)(\sin^{-1} x - \cos^{-1} x) = a$

$$= \frac{\pi}{2} \left(\frac{\pi}{2} - 2\cos^{-1} x \right) = a$$

$$\Rightarrow \frac{\pi}{2} - 2\cos^{-1} x = \frac{2a}{\pi}$$

take sine both sides

$$\therefore \sin\left(\frac{\pi}{2} - 2\cos^{-1} x\right) = \sin\left(\frac{2a}{\pi}\right)$$

$$\Rightarrow \cos\left(2\cos^{-1} x\right) = \sin\left(\frac{2a}{\pi}\right)$$

$$\Rightarrow 2\cos^2\left(\cos^{-1} x\right) - 1 = \sin\left(\frac{2a}{\pi}\right)$$

$$\Rightarrow 2x^2 - 1 = \sin\left(\frac{2a}{\pi}\right)$$

2. Let us consider a curve, $y = f(x)$ passing through the point $(-2, 2)$ and slope of the tangent to the curve at any point $(x, f(x))$ is given by $f(x) + xf'(x) = x^2$. Then

- (1) $x^3 + xf(x) + 12 = 0$ (2) $x^2 + 2xf(x) + 4 = 0$
 (3) $x^2 + 2xf(x) - 12 = 0$ (4) $x^3 - 3xf(x) - 4 = 0$

Answer (4)

Sol. Let $f(x) = y$

$$\Rightarrow y + x \frac{dy}{dx} = x^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2 - y}{x}$$

$$\Rightarrow \frac{dy}{dx} + \frac{y}{x} = x \quad \text{(linear D.E)}$$

$$\text{I.F} = e^{\int \frac{1}{x} dx} = x$$

$$\Rightarrow \int d(xy) = \frac{x^3}{3} + C$$

$$\downarrow (-2, 2)$$

$$-4 = \frac{-8}{3} + C$$

$$\Rightarrow C = \frac{-4}{3}$$

$$\Rightarrow 3xy = x^3 - 4$$

$$\Rightarrow x^3 - 3xf(x) - 4 = 0$$

3. A wire of length 20 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a regular hexagon. Then the length of the side (in meters) of the hexagon, so that the combined area of the square and the hexagon is minimum, is

(1) $\frac{5}{2 + \sqrt{3}}$ (2) $\frac{5}{3 + \sqrt{3}}$

(3) $\frac{10}{3 + 2\sqrt{3}}$ (4) $\frac{10}{2 + 3\sqrt{3}}$

Answer (3)

Sol. Let side of square be a and that of hexagon be b

Hence, $4a + 6b = 20$... (i)

Area of square = a^2 and area of hexagon = $\frac{6\sqrt{3}}{4} b^2$

$$A = a^2 + \frac{6\sqrt{3}}{4} b^2 \quad \dots \text{(ii)}$$

$$\therefore A = \left(\frac{10 - 3b}{2}\right)^2 + \frac{6\sqrt{3}}{4} b^2$$

$$\frac{dA}{db} = 0$$

$$\Rightarrow 2\left(\frac{10-3b}{2}\right)\left(\frac{-3}{2}\right) + 2\left(\frac{6\sqrt{3}}{4}\right)b = 0$$

$$\therefore b = \frac{10}{(3+2\sqrt{3})}$$

(As $\frac{d^2A}{db} > 0$ area is minimum for $b = \frac{10}{3+2\sqrt{3}}$)

4. $\sum_{k=0}^{20} \binom{20}{k}^2$ is equal to

- (1) ${}^{41}C_{20}$ (2) ${}^{40}C_{19}$
 (3) ${}^{40}C_{21}$ (4) ${}^{40}C_{20}$

Answer (4)

Sol. $\sum_{k=0}^{20} \binom{20}{k}^2 = \binom{20}{0}^2 + \binom{20}{1}^2 + \binom{20}{2}^2 + \dots$
 $\dots + \binom{20}{20}^2 = {}^{40}C_{20}$

(Using binomial series)

$${}^nC_0^2 + {}^nC_1^2 + \dots + {}^nC_n^2 = 2n C_n$$

5. Let A be a fixed point (0, 6) and B be a moving point (2t, 0). Let M be the mid-point of AB and the perpendicular bisector of AB meets the y-axis at C. The locus of the mid-point P of MC is

- (1) $3x^2 + 2y - 6 = 0$ (2) $2x^2 + 3y - 9 = 0$
 (3) $3x^2 - 2y - 6 = 0$ (4) $2x^2 - 3y + 9 = 0$

Answer (2)

Sol. A(0, 6) and B(2t, 0)

Let mid point AB be $m = (t, 3)$

and $m_{AB} = \frac{-6}{2t} = \frac{-3}{t}$

\therefore Equation of perpendicular bisector is

$$y - 3 = \frac{t}{3}(x - t)$$

$$\Rightarrow 3y - 9 = tx - t^2$$

$$\therefore C \equiv \left(0, \frac{9-t^2}{3}\right)$$

Let mid point of MC be (h, k)

$$\therefore \frac{t+0}{2} = h \text{ and } \frac{3+9-t^2}{3} = 2k$$

$$\Rightarrow 9 + 9 - (2h)^2 = 6k$$

$$\Rightarrow 18 - 4x^2 = 6y$$

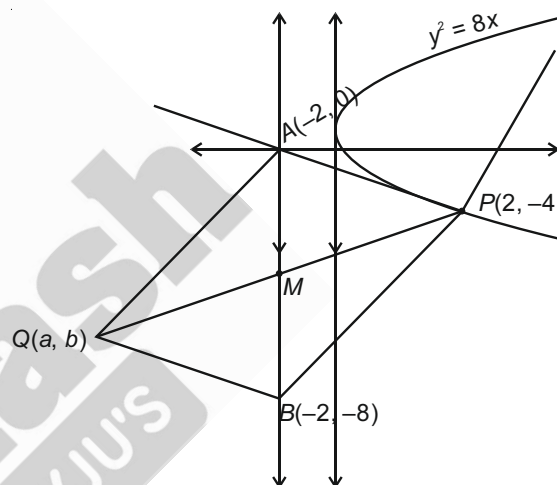
$$\Rightarrow 2x^2 + 3y - 9 = 0$$

6. A tangent and a normal are drawn at the point P(2, -4) on the parabola $y^2 = 8x$, which meet the directrix of the parabola at the points A and B respectively. If Q(a, b) is a point such that AQB P is a square, then $2a + b$ is equal to

- (1) -18 (2) -12
 (3) -16 (4) -20

Answer (3)

Sol.



Directrix: $x = -2$

Tangent at (2, -4)

$$-4y = 4(x + 2)$$

$$x + y + 2 = 0$$

$$\text{if } x = -2 \Rightarrow y = 0$$

$$A \equiv (-2, 0)$$

Normal at (2, -4)

$$x - y = 6$$

$$x = -2$$

$$\Rightarrow y = -8$$

$$B \equiv (-2, -8)$$

$$\frac{a+2}{2} = \frac{(-2)+(-2)}{2}$$

$$\Rightarrow a = -6$$

$$\frac{b+(-4)}{2} = \frac{0+(-8)}{2}$$

$$\Rightarrow b = -4$$

$$2a + b = -16$$

7. If for $x, y \in \mathbf{R}, x > 0, y = \log_{10}x + \log_{10}x^{1/3} + \log_{10}x^{1/9} + \dots$ upto ∞ terms and $\frac{2+4+6+\dots+2y}{3+6+9+\dots+3y} = \frac{4}{\log_{10}x}$, then the ordered pair (x, y) is equal to :
- (1) $(10^6, 9)$ (2) $(10^6, 6)$
(3) $(10^4, 6)$ (4) $(10^2, 3)$

Answer (1)

Sol. $y = \log_{10}x + \log_{10}x^{1/3} + \log_{10}x^{1/9} + \dots$
 $= \log_{10}(x \cdot x^{1/3} \cdot x^{1/9} \dots \infty)$
 $= \log_{10}\left(x^{1 + \frac{1}{3} + \frac{1}{9} + \dots}\right)$
 $y = \log_{10}\left(x^{\frac{1}{1 - \frac{1}{3}}}\right) = \log_{10}x^{3/2} = \frac{3}{2}\log_{10}x$

$$\frac{2+4+6+\dots+2y}{3+6+9+\dots+3y} = \frac{4}{\log_{10}x}$$

$$\Rightarrow \frac{2(1+2+3+\dots+y)}{3(1+2+3+\dots+y)} = \frac{4}{\log_{10}x}$$

$$\Rightarrow \frac{2}{3} = \frac{4}{\log_{10}x}$$

$$\Rightarrow \log_{10}x = 6$$

$$\Rightarrow x = 10^6$$

$$\Rightarrow y = \frac{3}{2} \times 6 = 9$$

8. If α, β are the distinct roots of $x^2 + bx + c = 0$, then

$$\lim_{x \rightarrow \beta} \frac{e^{2(x^2+bx+c)} - 1 - 2(x^2+bx+c)}{(x-\beta)^2}$$
 is equal to :

- (1) $b^2 - 4c$ (2) $b^2 + 4c$
(3) $2(b^2 + 4c)$ (4) $2(b^2 - 4c)$

Answer (4)

Sol. α, β are roots of $x^2 + bx + c = 0$
 $\therefore x^2 + bx + c = (x - \alpha)(x - \beta)$... (i)
 Also $\beta^2 + b\beta + c = 0$ (ii)

$$L = \lim_{x \rightarrow \beta} \frac{e^{2(x^2+bx+c)} - 1 - 2(x^2+bx+c)}{(x-\beta)^2 \times (x-\alpha)^2} \times (x-\alpha)^2$$

$$L = \lim_{x \rightarrow \beta} \frac{e^{2(x^2+bx+c)} - 1 - 2(x^2+bx+c)}{(x^2+bx+c)^2} \times \lim_{x \rightarrow \beta} (x-\alpha)^2$$

Let $x^2 + bx + c = t$
 $x \rightarrow \beta \Rightarrow t \rightarrow 0$

$$L = \lim_{t \rightarrow 0} \frac{e^{2t} - 1 - 2t}{t^2} \times (\beta - \alpha)^2$$

$$L = \lim_{t \rightarrow 0} \frac{\left(1 + 2t + \frac{(2t)^2}{2!} + \frac{(2t)^3}{3!} + \dots\right) - 1 - 2t}{t^2} \times (\alpha - \beta)^2$$

$$L = 2(\alpha - \beta)^2$$

$$= 2[(\alpha + \beta)^2 - 4\alpha\beta]$$

$$= 2[(-b)^2 - 4c]$$

$$L = 2(b^2 - 4c)$$

9. If $x^2 + 9y^2 - 4x + 3 = 0, x, y \in \mathbf{R}$, then x and y respectively lie in the intervals

- (1) $[1, 3]$ and $[1, 3]$ (2) $\left[-\frac{1}{3}, \frac{1}{3}\right]$ and $\left[-\frac{1}{3}, \frac{1}{3}\right]$
(3) $\left[-\frac{1}{3}, \frac{1}{3}\right]$ and $[1, 3]$ (4) $[1, 3]$ and $\left[-\frac{1}{3}, \frac{1}{3}\right]$

Answer (4)

Sol. $9y^2 = -x^2 + 4x - 3$... (i)

$$9y^2 \geq 0$$

$$\Rightarrow -x^2 + 4x - 3 \geq 0$$

$$x^2 - 4x + 3 \leq 0$$

$$(x - 1)(x - 3) \leq 0$$

$$x \in [1, 3]$$

Let $f(x) = -x^2 + 4x - 3$

$$(f(x))_{\max} = f(2) = 1$$

$$(f(x))_{\min} = f(1) \text{ or } f(3) = 0,$$

$$0 \leq -x^2 + 4x - 3 \leq 1$$

$$0 \leq 9y^2 \leq 1$$

$$0 \leq y^2 \leq \frac{1}{9}$$

$$0 \leq |y| \leq \frac{1}{3}$$

$$-\frac{1}{3} \leq y \leq \frac{1}{3}$$

Sol. $\frac{\sin(B+C)}{\sin(A+C)} = \frac{\sin(A-C)}{\sin(C-B)}$

$\Rightarrow \sin^2 C - \sin^2 B = \sin^2 A - \sin^2 C$
 $= a^2 + b^2 = 2c^2$

14. $\int_6^{16} \frac{\log_e x^2}{\log_e x^2 + \log_e(x^2 - 44x + 484)} dx$ is equal to

- (1) 6 (2) 10
(3) 5 (4) 8

Answer (3)

Sol. $I = \int_6^{16} \frac{2 \ln x}{2 \ln x + 2 \ln(22-x)} dx$

$I = \int_6^{16} \frac{\ln x}{\ln x + \ln(22-x)} dx \dots(1)$

$I = \int_6^{16} \frac{\ln(22-x)}{\ln(22-x) + \ln x} dx \dots(2)$

Adding (1) and (2) we get

$2I = \int_6^{16} dx = 10$

$\Rightarrow I = 5$

15. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = 2(y + 2 \sin x - 5)x - 2 \cos x$ such that $y(0) = 7$. Then $y(\pi)$ is equal to

- (1) $7e^{\pi^2} + 5$ (2) $2e^{\pi^2} + 5$
(3) $e^{\pi^2} + 5$ (4) $3e^{\pi^2} + 5$

Answer (2)

Sol. $\frac{dy}{dx} - 2xy = 4x \sin x - 2 \cos x - 10x$

I.F. = $e^{\int -2x dx} = e^{-x^2}$

$y \cdot e^{-x^2} = \int e^{-x^2} (4x \sin x - 2 \cos x - 10x) dx + C$

$\Rightarrow ye^{-x^2} = \int e^{-x^2} \cdot (-2x)(-2 \sin x) dx -$

$\int 2 \cos x \cdot e^{-x^2} dx + 5 \int (-2xe^{-x^2}) dx + C$

$\Rightarrow ye^{-x^2} = -2 \sin x \cdot e^{-x^2} + 5e^{-x^2} + C$

Put $x = 0, 7 = 5 + C \Rightarrow C = 2$

Put $x = \pi, y = 5 + 2e^{\pi^2}$

16. The statement $(p \wedge (p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow r$ is

- (1) a fallacy
(2) equivalent to $q \rightarrow \sim r$
(3) equivalent to $p \rightarrow \sim r$
(4) a tautology

Answer (4)

Sol.

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$p \wedge (p \rightarrow q)$	$p \wedge (p \rightarrow q) \wedge (q \rightarrow r)$	$p \wedge (p \rightarrow q) \wedge (q \rightarrow r) \rightarrow r$
T	T	T	T	T	T	T	T
T	T	F	T	F	T	F	T
T	F	T	F	T	F	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	F	F	T
F	T	F	T	F	F	F	T
F	F	T	T	T	F	F	T
F	F	F	T	T	F	F	T

Clearly it is a tautology

17. If $0 < x < 1$, then $\frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 + \dots$, is equal to

- (1) $x \left(\frac{1+x}{1-x} \right) + \log_e(1-x)$
(2) $x \left(\frac{1-x}{1+x} \right) + \log_e(1-x)$
(3) $\frac{1+x}{1-x} + \log_e(1-x)$
(4) $\frac{1-x}{1+x} + \log_e(1-x)$

Answer (1)

Sol. $\therefore \frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 + \dots, x \in (0, 1)$

$= \left(2 - \frac{1}{2}\right)x^2 + \left(2 - \frac{1}{3}\right)x^3 + \left(2 - \frac{1}{4}\right)x^4 + \dots$

$= 2x^2(1+x+x^2+\dots) - \left(\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots\right)$

$$= 2x^2 \cdot \frac{1}{1-x} + x - \left(\frac{x}{1} + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots \infty \right)$$

$$= x \left(\frac{1+x}{1-x} \right) + \ln(1-x)$$

18. The distance of the point (1, -2, 3) from the plane $x - y + z = 5$ measured parallel to a line, whose direction ratios are 2, 3, -6 is

- (1) 2 (2) 5
(3) 3 (4) 1

Answer (4)

Sol. Equation of line through point (1, -2, 3) and parallel to line with direction ratios 2, 3, -6 is

$$L: \frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = (\lambda \text{ say})$$

a point on line L is $P(2\lambda + 1, 3\lambda - 2, -6\lambda + 3)$

$\therefore P$ lies on plane $x - y + z = 5$ we get

$$2\lambda + 1 - 3\lambda + 2 - 6\lambda + 3 = 5$$

$$\therefore \lambda = \frac{1}{7}$$

\therefore Coordinate of $P = \left(\frac{9}{7}, -\frac{11}{7}, \frac{15}{7} \right)$

\therefore Required distance

$$= \sqrt{\left(\frac{9}{7} - 1 \right)^2 + \left(-\frac{11}{7} + 2 \right)^2 + \left(\frac{15}{7} - 3 \right)^2}$$

$$= 1$$

19. When a certain biased die is rolled, a particular face occurs with probability $\frac{1}{6} - x$ and its opposite face

occurs with probability $\frac{1}{6} + x$. All other faces occur

with probability $\frac{1}{6}$. Note that opposite faces sum to

7 in any die. If $0 < x < \frac{1}{6}$, and the probability of

obtaining total sum = 7, when such a die is rolled twice, is $\frac{13}{96}$, then the value of x is

- (1) $\frac{1}{12}$ (2) $\frac{1}{8}$
(3) $\frac{1}{16}$ (4) $\frac{1}{9}$

Answer (2)

Sol. The required probability

$$= 2 \left\{ \left(\frac{1-x}{6} \right) \left(\frac{1+x}{6} \right) + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} \right\}$$

$$\therefore 2 \left(\frac{1-x^2}{36} + \frac{1}{36} + \frac{1}{36} \right) = \frac{13}{96}$$

$$\frac{1}{6} - 2x^2 = \frac{13}{96}$$

$$2x^2 = \frac{1}{6} - \frac{13}{96} = \frac{16-13}{96}$$

$$\therefore x^2 = \frac{1}{64}$$

$$\therefore x = \frac{1}{8}$$

20. If $U_n = \left(1 + \frac{1}{n^2} \right) \left(1 + \frac{2^2}{n^2} \right) \dots \left(1 + \frac{n^2}{n^2} \right)$, then

$\lim_{n \rightarrow \infty} (U_n)^{\frac{-4}{n^2}}$ is equal to

- (1) $\frac{4}{e^2}$ (2) $\frac{4}{e}$
(3) $\frac{16}{e^2}$ (4) $\frac{e^2}{16}$

Answer (4)

Sol. Let $L = \lim_{n \rightarrow \infty} (U_n)^{\frac{-4}{n^2}}$

taking log of both sides we get:

$$\log L = \lim_{n \rightarrow \infty} -\frac{4}{n^2} \left\{ \ln \left(1 + \frac{1}{n^2} \right) + 2 \ln \left(1 + \frac{2^2}{n^2} \right) + \right.$$

$$\left. 3 \cdot \ln \left(1 + \frac{3^2}{n^2} \right) + \dots + n \cdot \ln \left(1 + \frac{n^2}{n^2} \right) \right\}$$

$$\log L = -4 \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n \frac{r}{n} \ln \left(1 + \left(\frac{r}{n} \right)^2 \right)$$

$$= -4 \int_0^1 x \ln(1+x^2) dx$$

$$= -2 \int_0^1 2x \ln(1+x^2) dx \quad \left\{ \begin{array}{l} \text{Let } 1+x^2 = t \\ \therefore 2x dx = dt \end{array} \right.$$

$$= -2 \int_1^2 \ln t dt$$

$$= -2(2 \ln 2 - 1) = \ln \left(\frac{e^2}{16} \right)$$

$$\therefore L = \frac{e^2}{16}$$

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. If the minimum area of the triangle formed by a tangent to the ellipse $\frac{x^2}{b^2} + \frac{y^2}{4a^2} = 1$ and the co-ordinate axis is kab , then k is equal to _____.

Answer (2)

Sol. Any point on ellipse is $P(b\cos\theta, 2a\sin\theta)$

tangent at $P: \frac{x}{b}\cos\theta + \frac{y}{2a}\sin\theta = 1$

$x = 0 \Rightarrow y = \frac{2a}{\sin\theta}$ and $y = 0 \Rightarrow x = \frac{b}{\cos\theta}$

Area of triangle = $\frac{1}{2} \left| \frac{2a}{\sin\theta} \times \frac{b}{\cos\theta} \right| = \left| \frac{2ab}{\sin 2\theta} \right|$

Minimum area = $2ab$ where $\sin 2\theta = \pm 1$
 $\Rightarrow k = 2$.

2. let n be an odd natural number such that the variance of 1, 2, 3, 4, ..., n is 14. Then n is equal to _____.

Answer (13)

Sol. $\frac{\sum x_1^2}{n} - (\bar{x})^2 = 14$

$$\frac{1^2 + 2^2 + \dots + n^2}{n} - \left(\frac{\frac{n(n+1)}{2}}{n} \right)^2 = 14$$

$$\frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4} = 14$$

$n^2 - 1 = 168 \Rightarrow n = 13$

3. If the system of linear equations

$2x + y - z = 3$

$x - y - z = \alpha$

$3x + 3y + \beta z = 3$

has infinitely many solution, then $\alpha + \beta - \alpha\beta$ is equal to _____.

Answer (5)

Sol. For infinite solutions

First requirement

$$\Delta = \begin{vmatrix} 2 & 1 & -1 \\ 1 & -1 & -1 \\ 3 & 3 & \beta \end{vmatrix} = 0, \Rightarrow \beta = -1$$

Now the equations are :

$2x + y - z = 3 \quad \dots(i)$

$x - y - z = \alpha \quad \dots(ii)$

$3x + 3y - z = 3 \quad \dots(iii)$

For infinite solutions, one equation should be obtainable as linear combination of other two equation.

Adding (ii) and (iii) and dividing by 2 given LHS of (ii)

$\Rightarrow \frac{3+\alpha}{2} = 3 \Rightarrow \alpha = 3$. Hence $\alpha + \beta - \alpha\beta = 5$

4. Let $\vec{a} = \hat{i} + 5\hat{j} + \alpha\hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} + \beta\hat{k}$ and $\vec{c} = -\hat{i} + 2\hat{j} - 3\hat{k}$ be three vectors such that, $|\vec{b} \times \vec{c}| = 5\sqrt{3}$ and \vec{a} is perpendicular to \vec{b} . Then the greatest amongst the values of $|\vec{a}|^2$ is _____.

Answer (90)

Sol. $\vec{b} \times \vec{c} = (-9 - 2\beta)\hat{i} + (3 - \beta)\hat{j} + 5\hat{k}$

$|\vec{b} \times \vec{c}| = 5\sqrt{3} \Rightarrow (9 + 2\beta)^2 + (3 - \beta)^2 + 25 = 75$

$\Rightarrow \beta^2 + 6\beta + 8 = 0$

$\Rightarrow \beta = -2$ or $\beta = -4$

\vec{a} is perpendicular to $\vec{b} \Rightarrow \vec{a} \cdot \vec{b} = 0$

$\Rightarrow (\hat{i} + 5\hat{j} + \alpha\hat{k}) \cdot (\hat{i} + 3\hat{j} + \beta\hat{k}) = 0$

$\Rightarrow 1 + 15 + \alpha\beta = 0 \quad \dots(i)$

$|\vec{a}|^2 = 1 + 25 + \alpha^2 = 26 + \left(\frac{-16}{\beta} \right)^2$

from greatest value of $|\vec{a}|^2$ take $\beta = 2$

\Rightarrow greatest value of $|\vec{a}|^2 = 90$

5. If $y^{1/4} + y^{-1/4} = 2x$, and

$$(x^2 - 1) \frac{d^2y}{dx^2} + \alpha x \frac{dy}{dx} + \beta y = 0,$$

then $|\alpha - \beta|$ is equal to _____.

Answer (17)

Sol. $y^{1/4} + y^{-1/4} = 2x \Rightarrow (y^{1/4} - y^{-1/4})^2$

$$= (y^{1/4} + y^{-1/4})^2 - 4 = 4(x^2 - 1)$$

$$\frac{1}{4y} \cdot (y^{1/4} - y^{-1/4}) \cdot \frac{dy}{dx} = 2 \Rightarrow \frac{1}{8} \cdot (y^{1/4} - y^{-1/4}) \cdot \frac{dy}{dx} = y \dots(i)$$

$$(y^{1/4} - y^{-1/4}) \frac{dy}{dx} = 8y$$

$$\frac{1}{4y} \cdot (y^{1/4} + y^{-1/4}) \cdot \left(\frac{dy}{dx}\right)^2 + (y^{1/4} - y^{-1/4}) \cdot \frac{d^2y}{dx^2} = 8 \cdot \frac{dy}{dx}$$

$$(y^{1/4} + y^{-1/4}) \cdot \left(\frac{dy}{dx}\right)^2 + (y^{1/4} - y^{-1/4}) \cdot \frac{d^2y}{dx^2} (4y) = 4 \cdot (8y) \frac{dy}{dx}$$

$$2x \cdot \left(\frac{dy}{dx}\right)^2 + (y^{1/4} - y^{-1/4}) \cdot \frac{d^2y}{dx^2} \cdot \left(\frac{y^{1/4} - y^{-1/4}}{2}\right) \frac{dy}{dx} = 4 \cdot (8y) \frac{dy}{dx}$$

$$2x \cdot \frac{dy}{dx} + \frac{4(x^2 - 1)}{2} \cdot \frac{d^2y}{dx^2} = 32y$$

$$(x^2 - 1) \cdot \frac{d^2y}{dx^2} + x \cdot \frac{dy}{dx} - 16y = 0 \Rightarrow \alpha = 1, \beta = -16$$

$$|\alpha - \beta| = 17$$

6. If $A = \{x \in \mathbf{R} : |x - 2| > 1\}$, $B = \{x \in \mathbf{R} : \sqrt{x^2 - 3} > 1\}$,

$$C = \{x \in \mathbf{R} : |x - 4| \geq 2\}$$

and Z is the set of all integers, then the number of subsets of the set $(A \cap B \cap C)^c \cap Z$ is _____.

Answer (256)

Sol. $A = \{x \in \mathbf{R} : |x - 2| > 1\}$

$$|x - 2| > 1 \Rightarrow x - 2 > 1 \text{ or } x - 2 < -1$$

$$x > 3 \text{ or } x < 1 \dots(i)$$

$$B = \{x \in \mathbf{R} : \sqrt{x^2 - 3} > 1\}$$

$$x^2 - 3 \geq 0 \text{ or } x^2 - 3 > 1$$

$$x^2 - 4 > 0 \Rightarrow x > 2 \text{ or } x < -2 \dots(ii)$$

$$C = \{x \in \mathbf{R} : |x - 4| \geq 2\}$$

$$x - 4 \geq 2 \text{ or } x - 4 \leq -2$$

$$x \geq 6 \text{ or } x \leq 2 \dots(iii)$$

$$(A \cap B \cap C) = x < -2 \text{ or } x \geq 6$$

$$(A \cap B \cap C)^c \cap Z = -2 \leq x < 6$$

$$n\{(A \cap B \cap C)^c \cap Z\} = 8$$

$$\text{No. of subset} = 2^8 = 256$$

7. A number is called a palindrome if it reads the same backward as well as forward. For example 285582 is a six digit palindrome. The number of six digit palindromes, which are divisible by 55, is _____.

Answer (100)

Sol. For divisible by 55 it shall be divisible by 11 and 5 both, for divisibility by 5 unit digit shall be 0 or 5 but as the number is six digit palindrome unit digit is 5.

\therefore Number is of form 5 _ _ _ _ 5

\therefore Now for divisibility by 11 remaining odd places have 10 options each & then even place will have same value as their difference of sum shall be multiple of +1.

\therefore No. of ways = $10 \times 10 = 100$

8. The number of distinct real roots of equation $3x^4 + 4x^3 - 12x^2 + 4 = 0$ is _____.

Answer (4)

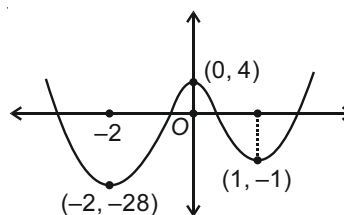
Sol. $f(x) = 12x^3 + 12x^2 - 24x = 12x(x + 2)(x - 1)$

points of extrema are at $x = 0, -2, 1$

$$f(0) = 4 \qquad f(-2) = -28$$

$$f(1) = -1$$

So, 4 Real Roots



9. If $\int \frac{dx}{(x^2+x+1)^2} = a \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + b\left(\frac{2x+1}{x^2+x+1}\right) + C$,
 $x > 0$ where C is the constant of integration, then the
 value of $9(\sqrt{3}a+b)$ is equal to _____.

Answer (15)

Sol. $\int \frac{dx}{\left(\left(x+\frac{1}{2}\right)^2 + \frac{3}{4}\right)^2}$ Let $x+\frac{1}{2} = \frac{\sqrt{3}}{2} \tan \theta$

$$\int \frac{\frac{\sqrt{3}}{2} \cdot \sec^2 \theta}{\frac{9}{16} \sec^4 \theta} d\theta = \frac{8}{3\sqrt{3}} \int \cos^2 \theta d\theta = \frac{4}{3\sqrt{3}} \int 2\cos^2 \theta d\theta$$

$$= \frac{4}{\sqrt{3}} \left(\theta + \frac{\sin 2\theta}{2} \right)$$

$$\tan \theta = \frac{2x+1}{\sqrt{3}} \Rightarrow \sin 2\theta = \frac{2x+1}{2(x^2+x+1)} \cdot \sqrt{3}$$

So $\int \frac{dx}{(x^2+x+1)^2}$

$$= \frac{4}{3\sqrt{3}} \left(\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + \frac{\sqrt{3}}{4} \frac{2x+1}{x^2+x+1} \right)$$

$$a = \frac{4}{3\sqrt{3}}, b = \frac{1}{3}$$

$$9(a+b) = 15$$

10. Let the equation $x^2 + y^2 + px + (1-p)y + 5 = 0$
 represent circles of varying radius $r \in (0, 5]$. Then
 the number of elements in the set $S = \{q : q = p^2 \text{ and } q \text{ is an integer}\}$ is _____.

Answer (61)

Sol. $r^2 = \frac{p^2}{4} + \frac{(1-p)^2}{4} - 5 \Rightarrow 0 < p^2 + (1-p)^2 - 20 \leq 100$
 $20 < p^2 + (1-p)^2 \leq 120$

$$p \in \left(\frac{1-\sqrt{239}}{2}, \frac{1-\sqrt{39}}{2} \right) \cup \left(\frac{1+\sqrt{39}}{2}, \frac{1+\sqrt{239}}{2} \right)$$

$$p^2 \in [7, 67]$$

Number of integral values = 61