31/08/2021
Evening

Aakash + BYJU'S

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Time : 3 hrs. M.M. : 300

Answers & Solutions

for

JEE (MAIN)-2021 (Online) Phase-4

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS :

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part has two sections.

(i) Section-I : This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) Section-II : This section contains 10 questions. In Section-II, attempt any **five questions out of 10.** There will be **no negative marking for Section-II**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and there is no negative marking for wrong answer.

PART–A : PHYSICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. If R_E be the radius of Earth, then the ratio between the acceleration due to gravity at a depth 'r' below and a height 'r' above the earth surface is:

(Given : $r < R_E$)

(1) $1-\frac{r}{R_E}-\frac{r^2}{R_E^2}-\frac{r^3}{R_E^3}$ (2) $1+\frac{r}{R_E}+\frac{r^2}{R_E^2}+\frac{r^3}{R_E^3}$

(3) $1+\frac{r}{R_E}-\frac{r^2}{R_E^2}-\frac{r^3}{R_E^3}$ (4) $1+\frac{r}{R_E}-\frac{r^2}{R_E^2}+\frac{r^3}{R_E^3}$

Answer (3)

Sol. at a depth r

$$a_1=\frac{Gm}{R_E^3}\times(R_E-r)=\frac{GM}{R_E^2}\left(1-\frac{r}{R_E}\right)$$

at a height r

$$a_2=\frac{Gm}{(R_E+r)^2}=\frac{GM}{R_E^2\left(1+\frac{r}{R_E}\right)^2}$$

$$\therefore \frac{a_1}{a_2}=\left(1-\frac{r}{R_E}\right)\left(1+\frac{r}{R_E}\right)^2$$

$$=1+\frac{r}{R_E}-\frac{r^2}{R_E^2}-\frac{r^3}{R_E^3}$$

2. Two thin metallic spherical shells of radii r_1 and r_2 ($r_1 < r_2$) are placed with their centres coinciding. A material of thermal conductivity K is filled in the space between the shells. The inner shell is maintained at temperature θ_1 and the outer shell at temperature $\theta_2(\theta_1 < \theta_2)$. The rate at which heat flows radially through the material is:

(1) $\frac{K(\theta_2-\theta_1)(r_2-r_1)}{4\pi r_1 r_2}$ (2) $\frac{4\pi K r_1 r_2(\theta_2-\theta_1)}{r_2-r_1}$

(3) $\frac{\pi r_1 r_2(\theta_2-\theta_1)}{r_2-r_1}$ (4) $\frac{K(\theta_2-\theta_1)}{r_2-r_1}$

Answer (2)

Sol. $\frac{dQ}{dt}=\frac{\theta_2-\theta_1}{R_{th}}$

Now, $R_{th}=\int_{r_1}^{r_2}\frac{dx}{K\times 4\pi x^2}$

$$=\frac{1}{4\pi K}\left(\frac{r_2-r_1}{r_1 r_2}\right)$$

$$\therefore \frac{dQ}{dt}=\frac{4\pi K r_1 r_2(\theta_2-\theta_1)}{(r_2-r_1)}$$

3. A bob of mass 'm' suspended by a thread of length l undergoes simple harmonic oscillations with time period T. If the bob is immersed in a liquid that has density $\frac{1}{4}$ times that of the bob and the length of the thread is increased by 1/3[rd] of the original length, then the time period of the simple harmonic oscillations will be:

(1) $\frac{3}{4}T$ (2) T

(3) $\frac{4}{3}T$ (4) $\frac{3}{2}T$

Answer (3)

Sol. $T=2\pi\sqrt{\frac{l}{g}}$

$$T'=2\pi\sqrt{\frac{\frac{4l}{3}}{g-\frac{1}{4}g}}=2\pi\sqrt{\frac{\frac{4}{3}l}{\frac{3}{4}g}}=\frac{4}{3}T$$

4. A mixture of hydrogen and oxygen has volume 500 cm^3, temperature 300 K, pressure 400 kPa and mass 0.76 g. The ratio of masses of oxygen to hydrogen will be:

(1) 3 : 16 (2) 16 : 3

(3) 3 : 8 (4) 8 : 3

Answer (2)

Sol. PV = nRT

$$\Rightarrow \quad n = \frac{400 \times 10^3 \times 500 \times 10^{-6}}{R \times 300}$$

$$n = \frac{8}{100} = \frac{m_{H_2}}{2} + \frac{m_{O_2}}{32} \quad \text{...(i)}$$

and $m_{H_2} + m_{O_2} = 0.76$ ...(ii)

$\therefore \; m_{H_2} = 0.12$ g and $m_{O_2} = 0.64$ g

So ratio $= \frac{16}{3}$

5. If velocity [V], time [T] and force [F] are chosen as the base quantities, the dimensions of the mass will be:

(1) $[FT^{-1}V^{-1}]$ (2) $[FVT^{-1}]$

(3) $[FT^2V]$ (4) $[FTV^{-1}]$

Answer (4)

Sol. Mass, $M = [V]^a [T]^b [F]^c$

$\Rightarrow M = [LT^{-1}]^a [T]^b [MLT^{-2}]^c$

$\Rightarrow c = 1$

$\Rightarrow a + c = 0 \Rightarrow a = -c = -1$

$\Rightarrow -a + b - 2c = 0$

$\Rightarrow b = a + 2c = -1 + 2 \times 1 = 1$

$M = [FTV^{-1}]$

6. For a body executing S.H.M.:

(a) Potential energy is always equal to its K.E.

(b) Average potential and kinetic energy over any given time interval are always equal.

(c) Sum of the kinetic and potential energy at any point of time is constant.

(d) Average K.E. in one time period is equal to average potential energy in one time period.

Choose the **most appropriate** option from the options given below:

(1) Only (b)

(2) (b) and (c)

(3) Only (c)

(4) (c) and (d)

Answer (4)

Sol. $v = \omega\sqrt{A^2 - x^2}$

$$KE = \frac{1}{2}m\omega^2(A^2 - x^2)$$

$$PE = \frac{1}{2}m\omega^2x^2$$

E_{Total} = KE + PE

$$= \frac{1}{2}m\omega^2A^2$$

Also $(KE)_{avg} = (PE)_{avg}$

7. A free electron of 2.6 eV energy collides with a H^+ ion. This results in the formation of a hydrogen atom in the first excited state and a photon is released. Find the frequency of the emitted photon. ($h = 6.6 \times 10^{-34}$ J s)

(1) 0.19×10^{15} MHz (2) 1.45×10^9 MHz

(3) 9.0×10^{27} MHz (4) 1.45×10^{16} MHz

Answer (2)

Sol. $hf = 2.6 + 13.6 \times \frac{1}{4}$

= 6 eV

$$\Rightarrow f = \frac{6 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}}$$

$= 1.45 \times 10^9$ MHz

8. Consider two separate ideal gases of electrons and protons having same number of particles. The temperature of both the gases are same. The ratio of the uncertainty in determining the position of an electron to that of a proton is proportional to:

(1) $\sqrt{\frac{m_p}{m_e}}$ (2) $\frac{m_p}{m_e}$

(3) $\left(\frac{m_p}{m_e}\right)^{3/2}$ (4) $\sqrt{\frac{m_e}{m_p}}$

Answer (1)

Sol. $(\Delta x)\Delta p \geq \frac{h}{4\pi}$

$$\Delta x \propto \frac{1}{\Delta p}$$

$$\Rightarrow \frac{\Delta x_e}{\Delta x_p} = \frac{\Delta p_p}{\Delta p_e} = \sqrt{\frac{m_p}{m_e}}$$

9. A block moving horizontally on a smooth surface with a speed of 40 m/s splits into two parts with masses in the ratio of 1 : 2. If the smaller part moves at 60 m/s in the same direction, then the fractional change in kinetic energy is:

(1) $\frac{1}{3}$ (2) $\frac{2}{3}$

(3) $\frac{1}{4}$ (4) $\frac{1}{8}$

Answer (4)

Sol. $(m)\,40 = \left(\frac{m}{3}\right)\times 60 + \left(\frac{2m}{3}\right)V$

$\Rightarrow \frac{2V}{3} = 20 \quad \Rightarrow \quad V = 30 \text{ m/s}$

$k_f = \frac{1}{2}\left(\frac{m}{3}\right)\times(60)^2 + \frac{1}{2}\left(\frac{2m}{3}\right)(30)^2$

$\frac{\Delta k}{k} = \frac{(k_f - k_i)}{k_i} = \frac{1}{8}$

10. **Statement I :**

To get a steady dc output from the pulsating voltage received from a full wave rectifier we can connect a capacitor across the output parallel to the load R_L.

Statement II :

To get a steady dc output from the pulsating voltage received from a full wave rectifier we can connect an inductor in series with R_L.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) Both statement I and statement II are true

(2) Statement I is false but statement II is true

(3) Both statement I and statement II are false

(4) Statement I is true but statement II is false

Answer (1)

Sol. To get steady dc output from the pulsating voltage received from a full wave rectifier we can use capacitor.

Both statements are true.

11. The magnetic field vector of an electromagnetic wave is given by $B = B_0 \frac{\hat{i}+\hat{j}}{\sqrt{2}}\cos(kz-\omega t)$; where $\hat{i}, \hat{j}$ represents unit vector along *x* and *y*-axis respectively. At t = 0 s, two electric charges q_1 of 4π coulomb and q_2 of 2π coulomb located at $\left(0, 0, \frac{\pi}{k}\right)$ and $\left(0, 0, \frac{3\pi}{k}\right)$, respectively, have the same velocity of 0.5 c $\hat{i}$, (where c is the velocity of light). The ratio of the force acting on charge q_1 to q_2 is:

(1) $\sqrt{2}:1$ (2) 2 : 1

(3) $2\sqrt{2}:1$ (4) $1:\sqrt{2}$

Answer (2)

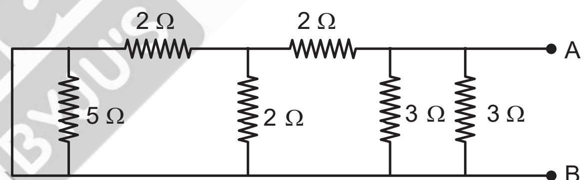
Sol. At given positions value of $\vec{E}$ and $\vec{B}$ is same

Force on 1 = $4\pi\left(\vec{E}+\vec{V}\times\vec{B}\right)$

Force on 2 = $2\pi\left(\vec{E}+\vec{V}\times\vec{B}\right)$

So $\frac{\text{Force on 1}}{\text{Force on 2}} = 2 : 1$

12. The equivalent resistance of the given circuit between the terminals A and B is:

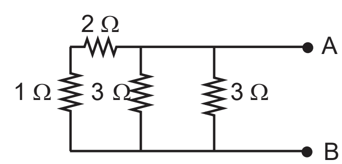


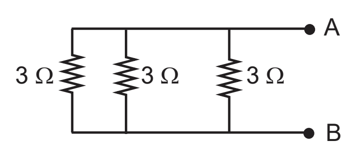
(1) 3 Ω (2) 1 Ω

(3) $\frac{9}{2}$ Ω (4) 0 Ω

Answer (2)

Sol. On simplification

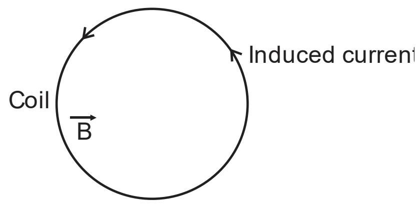




$\frac{1}{R_{AD}} = \frac{1}{3}+\frac{1}{3}+\frac{1}{3}$

R_{AB} = 1 Ω

13. A coil is placed in a magnetic field $\vec{B}$ as shown below:



A current is induced in the coil because $\vec{B}$ is:

(1) parallel to the plane of coil and increasing with time

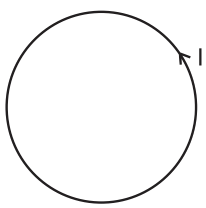
(2) outward and decreasing with time

(3) outward and increasing with time

(4) parallel to the plane of coil and decreasing with time

Answer (2)

Sol.



As current is induced

So, B is decreasing and outward

14. **Statement I:** If three forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ are represented by three sides of a triangle and $\vec{F}_1 + \vec{F}_2 = -\vec{F}_3$, then these three forces are concurrent forces and satisfy the condition for equilibrium.

Statement II: A triangle made up of three forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ as its sides taken in the same order, satisfy the condition for translatory equilibrium.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) Statement I is false but statement II is true

(2) Both statement I and statement II are false

(3) Both statement I and statement II are true

(4) Statement I is true but statement II is false

Answer (3)

Sol. As, $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$

So three forces are concurrent and object is in equilibrium.

Also if $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$

then $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ lie along sides of triangle taken in order

15. Choose the **incorrect** statement:

(a) The electric lines of force entering into a Gaussian surface provide negative flux.

(b) A charge 'q' is placed at the centre of a cube. The flux through all the faces will be the same.

(c) In a uniform electric field net flux through a closed Gaussian surface containing no net charge, is zero.

(d) When electric field is parallel to a Gaussian surface, it provides a finite non-zero flux.

Choose the **most appropriate** answer from the options given below:

(1) (d) Only

(2) (c) and (d) Only

(3) (a) and (c) Only

(4) (b) and (d) Only

Answer (1)

Sol. We know, total flux through close surface, $\phi = \frac{Q_{in}}{\varepsilon_0}$

if $\phi = 0$, $Q_{in} = 0$

If electric field is parallel to surface then electric flux = 0

16. A system consists of two identical spheres each of mass 1.5 kg and radius 50 cm at the ends of a light rod. The distance between the centres of the two spheres is 5 m. What will be the moment of inertia of the system about an axis perpendicular to the rod passing through its midpoint?

(1) 1.875×10^5 kgm^2

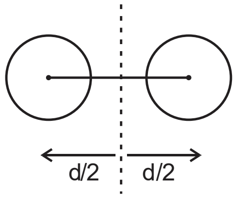
(2) 19.05 kgm^2

(3) 18.75 kgm^2

(4) 1.905×10^5 kgm^2

Answer (2)

Sol.



$$I = 2\times\left[\frac{2}{5}mR^2 + \frac{md^2}{4}\right]$$

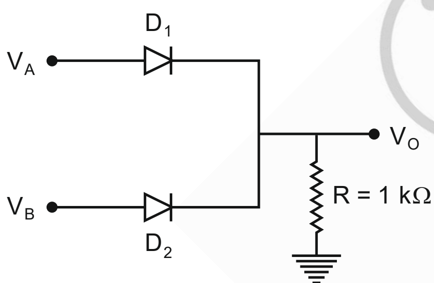
$$I = 2\left[\frac{2}{5}\times 1.5\times(0.5)^2 + 1.5\times(2.5)^2\right]$$

$= 1.2 \times (0.5)^2 + 3 \times [2.5]^2$

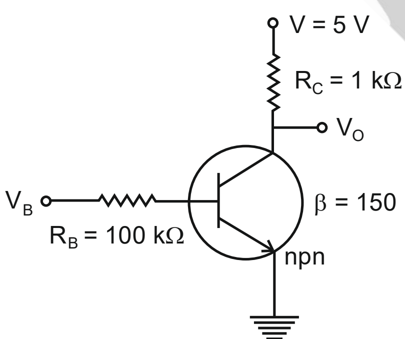
$I = 18.75 + 0.30$

$= 19.05$

17. If V_A and V_B are the input voltages (either 5 V or 0 V) and V_O is the output voltage then the two gates represented in the following circuits (A) and (B) are :



(A)



(B)

(1) NAND and NOR Gate

(2) AND and OR Gate

(3) OR and NOT Gate

(4) AND and NOT Gate

Answer (3)

Sol. In case (A), current flows through R if either V_A or V_B is high leading to non zero output.

$\Rightarrow$ A represent OR Gate.

In case (B), for high input voltage, $I_B \neq 0$

So $I_C \neq 0$.

$V_O = V - I_C R_C$

V_O will be low.

18. A current of 1.5 A is flowing through a triangle, of side 9 cm each. The magnetic field at the centroid of the triangle is :

(Assume that the current is flowing in the clockwise direction.)

(1) $2\sqrt{3}\times 10^{-7}$ T, outside the plane of triangle

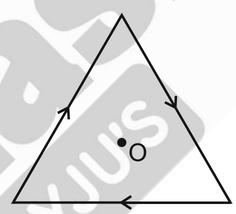
(2) 3×10^{-7} T, outside the plane of triangle

(3) $2\sqrt{3}\times 10^{-5}$ T, inside the plane of triangle

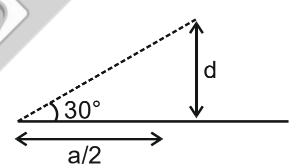
(4) 3×10^{-5} T, inside the plane of triangle

Answer (4)

Sol. Field due to each wire will add up.



$B_{net} = 3B_0$[B_0: Field due to single wire]



$$B_0 = \frac{\mu_0 I_0}{4\pi d}\times 2\sin 60^\circ$$

$$B_0 = \frac{\sqrt{3}\mu_0 I_0}{4\pi d}$$

$$d = \frac{a}{2\sqrt{3}}$$

$$B_0 = \frac{3\mu_0 I_0}{2\pi a}$$

$$B_{net} = \frac{9\mu_0 I_0}{2\pi a} = \frac{18\times 10^{-7}\times 1.5}{9\times 10^{-2}}$$

$= 3 \times 10^{-5}$ T

19. **Statement I :** Two forces $(\vec{P}+\vec{Q})$ and $(\vec{P}-\vec{Q})$ where $\vec{P} \perp \vec{Q}$, when act at an angle θ_1 to each other, the magnitude of their resultant is $\sqrt{3(P^2+Q^2)}$, when they act at an angle θ_2, the magnitude of their resultant becomes $\sqrt{2(P^2+Q^2)}$. This is possible only when $\theta_1 < \theta_2$.

Statement II : In the situation given above.

$\theta_1 = 60°$ and $\theta_2 = 90°$

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) Both Statement I and Statement II are true.

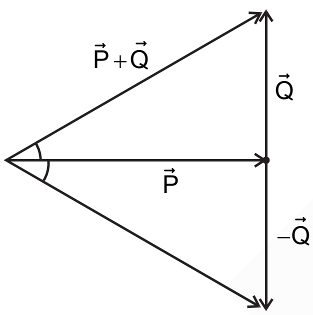
(2) Statement I is true but Statement II is false.

(3) Statement I is false but Statement II is true.

(4) Both Statement I and Statement II are false.

Answer (1)

Sol.



$$|\vec{P}+\vec{Q}+\vec{P}-\vec{Q}| = \sqrt{3}\sqrt{P^2+Q^2}$$

$4P^2 = 3(P^2 + Q^2)$

$P^2 = 3Q^2$

$$\theta_1 = 2\tan^{-1}\left(\frac{Q}{P}\right)$$

$= 60°$

For case-II

$4P^2 = 2(P^2 + Q^2)$

$P = Q$

$$\theta_2 = 2\tan^{-1}\left(\frac{Q}{P}\right) = 90°$$

20. Four identical hollow cylindrical columns of mild steel support a big structure of mass 50×10^3 kg. The inner and outer radii of each column are 50 cm and 100 cm respectively. Assuming uniform local distribution, calculate the compression strain of each column.

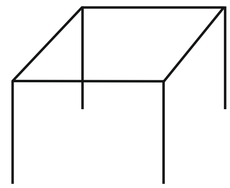
[use $Y = 2.0 \times 10^{11}$ Pa, $g = 9.8$ m/s^2]

(1) 3.60×10^{-8} (2) 1.87×10^{-3}

(3) 7.07×10^{-4} (4) 2.60×10^{-7}

Answer (4)

Sol.



Compressive force on each column $= \frac{mg}{4}$

$$\text{Stress} = \frac{F}{A}$$

$$\text{Strain} = \frac{F}{AY}$$

$$= \frac{mg}{4\pi Y[r_2^2 - r_1^2]}$$

$$= \frac{50 \times 10^3 \times 9.8}{4\pi \times 2 \times 10^{11}[1^2 - (0.50)^2]}$$

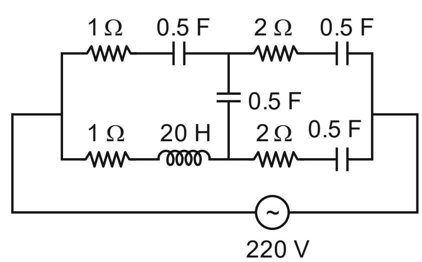
$$= \frac{50 \times 10^3 \times 9.8}{6\pi \times 10^{11}}$$

$$= \frac{5 \times 9.8 \times 10^{-7}}{6\pi} = 2.6 \times 10^{-7}$$

SECTION - II

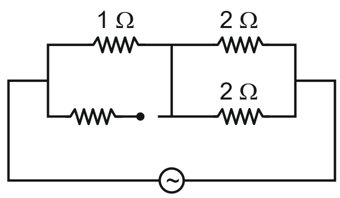
Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. At very high frequencies, the effective impedance of the given circuit will be ______ Ω.



Answer (2)

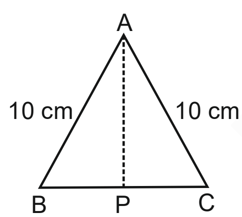
Sol. At high frequencies, $X_C \to 0$ & $X_L \to \infty$



$$Z = 1 + \frac{2 \times 2}{2+2}$$

$= 2\ \Omega$

2. Cross-section view of a prism is the equilateral triangle ABC shown in the figure. The minimum deviation is observed using this prism when the angle of incidence is equal to the prism angle. The time taken by light to travel from P (midpoint of BC) to A is ________ × 10^{-10} s. (Given, speed of light in vacuum = 3 × 10^8 m/s and cos30° = $\frac{\sqrt{3}}{2}$)



Answer (5)

Sol. $$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}}$$

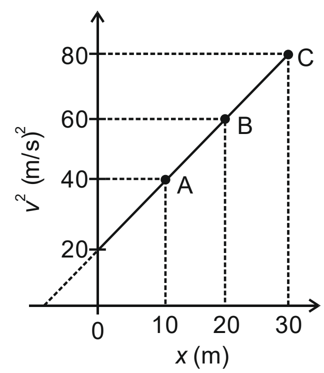
$$i = \frac{A+\delta_m}{2} = A \Rightarrow \delta_m = A$$

$$\mu = 2\cos\frac{A}{2} = \sqrt{3}$$

$$t = \frac{PA}{v} = \frac{0.10 \times \frac{\sqrt{3}}{2}}{3 \times \frac{10^8}{\sqrt{3}}}$$

= 5 × 10^{-10} sec

3. A particle is moving with constant acceleration 'a'. Following graph shows v^2 versus x(displacement) plot. The acceleration of the particle is _____ m/s^2.



Answer (1)

Sol. $v^2 = u^2 + 2ax$

$$a = \frac{1}{2}(\text{slope of } v^2 \text{ vs } x \text{ graph})$$

$$= \frac{1}{2}\left(\frac{40}{20}\right)$$

= 1 m/s^2

4. In a Young's double slit experiment, the slits are separated by 0.3 mm and the screen is 1.5 m away from the plane of slits. Distance between fourth bright fringes on both sides of central bright fringe is 2.4 cm. The frequency of light used is ___ × 10^{14} Hz.

Answer (5)

Sol. $$\Delta y = 2 \cdot \frac{4D\lambda}{d}$$

$$2.4 \times 10^{-2} = \frac{8 \times 1.5 \times \lambda}{0.3 \times 10^{-3}}$$

$\Rightarrow$ λ = 600 nm

$$f = \frac{c}{\lambda} = 5 \times 10^{14} \text{ Hz}$$

5. The diameter of a spherical bob is measured using a vernier callipers. 9 divisions of the main scale, in the vernier callipers, are equal to 10 divisions of vernier scale. One main scale division is 1 mm. The main scale reading is 10 mm and 8^{th} division of vernier scale was found to coincide exactly with one of the main scale division. If the given vernier callipers has positive zero error of 0.04 cm, then the radius of the bob is ___ × 10^{-2} cm.

Answer (52)

Sol. Reading = 10 + 8 × 0.1 = 10.8 mm

Diameter = Reading – Zero error

= 10.4 mm

Radius = 5.2 mm

6. A parallel plate capacitor of capacitance 200 μF is connected to a battery of 200 V. A dielectric slab of dielectric constant 2 is now inserted into the space between plates of capacitor while the battery remain connected. The change in the electrostatic energy in the capacitor will be ___ J.

Answer (4)

Sol. $U_{initial} = \frac{1}{2} \times (200 \times 10^{-6}) \times (200)^2$

$U_{final} = \frac{1}{2}(200 \times 10^{-6} \times 2) \times (200)^2$

$\Delta U = \frac{1}{2} \times 200 \times 10^{-6} \times 4 \times 10^4 = 4J$

7. A sample of gas with γ = 1.5 is taken through an adiabatic process in which the volume is compressed from 1200 cm^3 to 300 cm^3. If the initial pressure is 200 kPa. The absolute value of the workdone by the gas in the process = ____ J.

Answer (480)

Sol. $P_{initial}$ = 200 kPa = P_1(say)

P_{final} = P_2(Say) = 1600 kPa

$$W = \frac{P_1V_1 - P_2V_2}{\gamma - 1} = \frac{240 - 480}{1.5 - 1} = -480 \text{ J}$$

8. A resistor dissipates 192 J of energy in 1 s when a current of 4 A is passed through it. Now, when the current is doubled, the amount of thermal energy dissipated in 5 s is _____ J.

Answer (3840)

Sol. $I_1^2R_1(t_1) = 192$

$I_2^2R_1(t_2) = X$

$$\frac{192}{X} = \frac{1}{20} \Rightarrow X = 3840 \text{ J}$$

9. A long solenoid with 1000 turns/m has a core material with relative permeability 500 and volume 10^3 cm^3. If the core material is replaced by another material having relative permeability of 750 with same volume maintaining same current of 0.75 A in the solenoid, the fractional change in the magnetic moment of the core would be approximately $\left(\frac{x}{499}\right)$.

Find the value of *x*.

Answer (250)

Sol. $\frac{M_1}{M_2} = \frac{\chi_1}{\chi_2} = \frac{\mu_{r_1} - 1}{\mu_{r_2} - 1} = \frac{749}{499}$

$$\frac{\Delta M}{M_2} = \frac{M_1 - M_2}{M_2} = \frac{250}{499}$$

10. A bandwidth of 6 MHz is available for A.M. transmission. If the maximum audio signal frequency used for modulating the carrier wave is not to exceed 6 kHz. The number of stations that can be broadcasted within this band simultaneously without interfering with each other will be _____.

Answer (500)

Sol. Bandwidth = 6 × 10^6 Hz

Number of stations that can be broadcasted

$$= \frac{6 \times 10^6}{2 \times 6 \times 10^3} = 500$$

PART–B : CHEMISTRY

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The Eu^{2+} ion is a strong reducing agent in spite of its ground state electronic configuration (outermost) : [Atomic number of Eu = 63]

(1) $4f^6$

(2) $4f^7$

(3) $4f^6 6s^2$

(4) $4f^7 6s^2$

Answer (2)

Sol. Outermost electronic configuration of Eu

Eu (63) = $4f^7 6s^2$

Eu^{2+} = $4f^7$

2. Match List-I with List-II.

List-I (Parameter)	List-II (Unit)
(a) Cell constant	(i) S cm^2 mol^{-1}
(b) Molar conductivity	(ii) Dimensionless
(c) Conductivity	(iii) m^{-1}
(d) Degree of dissociation of electrolyte	(iv) Ω^{-1} m^{-1}

Choose the most appropriate answer from the options given below

(1) (a)-(iii), b(i), (c)-(ii), (d)-(iv)

(2) (a)-(i), b(iv), (c)-(iii), (d)-(ii)

(3) (a)-(ii), b(i), (c)-(iii), (d)-(iv)

(4) (a)-(iii), b(i), (c)-(iv), (d)-(ii)

Answer (4)

Sol.

Parameter	Unit
Cell constant	m^{-1}
Molar conductivity	S cm^2 mol^{-1}
Conductivity	Ω^{-1} m^{-1}
Degree of dissociation of electrolyte	Dimensionless

3. Which of the following is **NOT** an example of fibrous protein?

(1) Albumin

(2) Collagen

(3) Myosin

(4) Keratin

Answer (1)

Sol. When the polypeptide chains run parallel and are held together by hydrogen and disulphide bonds, then fibre-like structure is formed. Such proteins are generally insoluble in water. Some common examples are keratin (present in hair, wool, silk) and myosin (present in muscles), etc.

This structure results when the chains of polypeptides coil around to give a spherical shape. These are usually soluble in water. Insulin and albumins are the common examples of globular proteins.

4. Match List-I with List-II.

List-I (Metal Ion)	List-II (Group in Qualitative analysis)
(a) Mn^{2+}	(i) Group-III
(b) As^{3+}	(ii) Group-IIA
(c) Cu^{2+}	(iii) Group-IV
(d) Al^{3+}	(iv) Group-IIB

Choose the most appropriate answer from the options given below

(1) (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i)

(2) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

(3) (a)-(i), (b)-(iv), (c)-(ii), (d)-(iii)

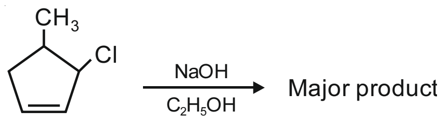
(4) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)

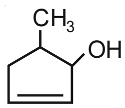
Answer (2)

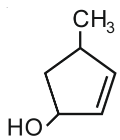
Sol.

Metal ion	Group in qualitative analysis
Mn^{2+}	Group-IV
As^{3+}	Group-IIB
Cu^{2+}	Group-IIA
Al^{3+}	Group-III

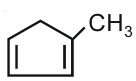
5. The major product of the following reaction is



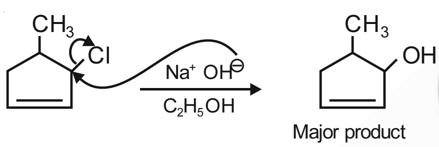
(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. 

6. Which one of the following statements is incorrect?

(1) Bond dissociation enthalpy of H_2 is highest among diatomic gaseous molecules which contain a single bond

(2) Atomic hydrogen is produced when H_2 molecules at a high temperature are irradiated with UV radiation

(3) At around 2000 K, the dissociation of dihydrogen into its atoms is nearly 8.1%

(4) Dihydrogen is produced on reacting zinc with HCl as well as $NaOH_{(aq)}$

Answer (3)

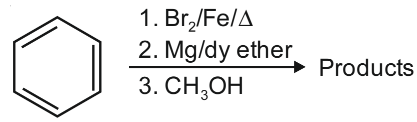
Sol. The H-H bond dissociation enthalpy is the highest for a single bond between two atoms of any element.

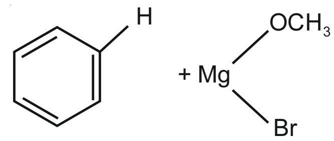
It is because of this factor that the dissociation of dihydrogen into its atoms is only ~0.081% around 2000 K. The atomic hydrogen is produced at a high temperature in an electric arc or under ultraviolet radiations.

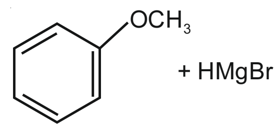
$$Zn + 2HCl \rightarrow ZnCl_2 + H_2$$

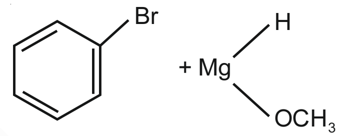
$$Zn + 2NaOH \rightarrow Na_2ZnO_2 + H_2$$

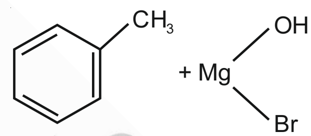
7. For the following sequence of reactions, the correct products are :



(1) 

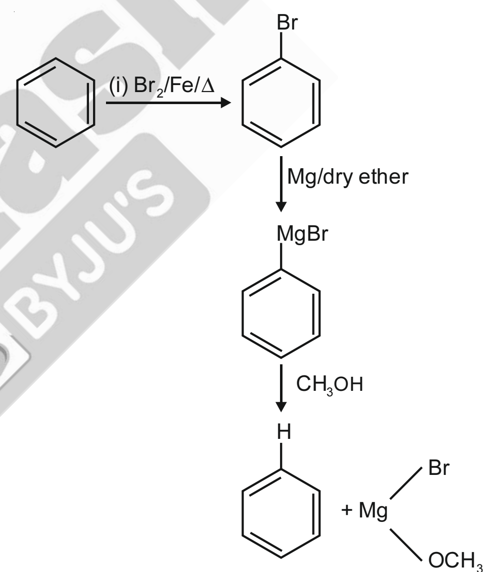
(2) 

(3) 

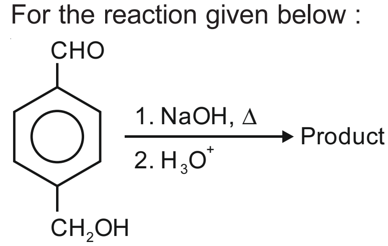
(4) 

Answer (1)

Sol.



8. For the reaction given below :



The compound which is not formed as a product in the reaction is a :

(1) Dicarboxylic acid

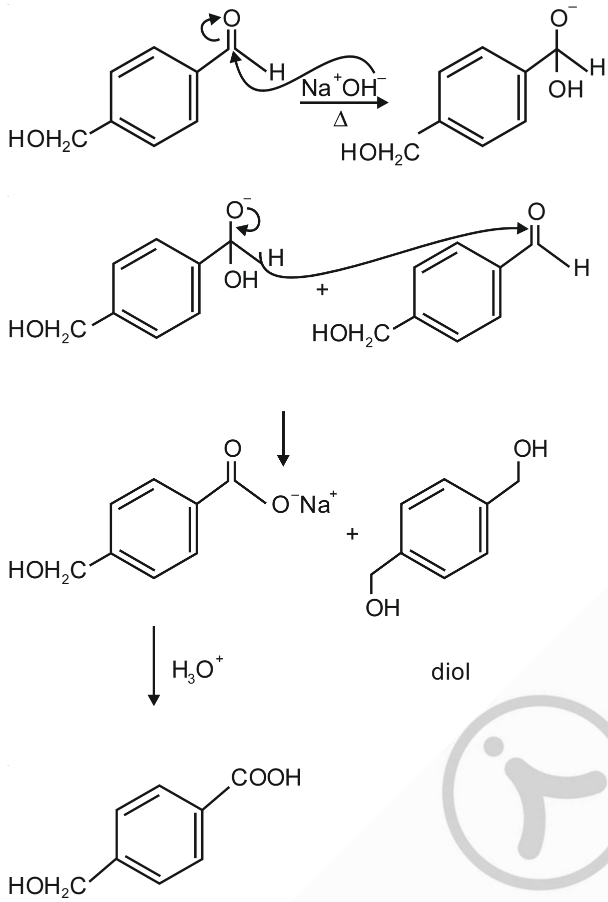
(2) Monocarboxylic acid

(3) Compound with both alcohol and acid functional groups

(4) Diol

Answer (1)

Sol.



9. The incorrect expression among the following is :

(1) For isothermal process $w_{reversible}$ $= -nRT \ln\frac{V_f}{V_i}$

(2) $\frac{\Delta G_{system}}{\Delta S_{Total}} = -T$ (at constant P)

(3) $\ln K = \frac{\Delta H^\circ - T\Delta S^\circ}{RT}$

(4) $K = e^{-\Delta G^\circ/RT}$

Answer (3)

Sol. Correct expressions are,

$$W_{rev} = -nRT \ln\left(\frac{V_f}{V_i}\right)$$

$$\Delta G_{sys} = -T\Delta S_{Total} \text{ (at constant-P)}$$

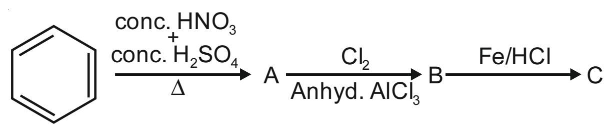
$$\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$$
$$-RT\ln K = \Delta H^\circ - T\Delta S^\circ$$
$$\ln K = \frac{\Delta H^\circ - T\Delta S^\circ}{-RT}$$
$$\ln K = \frac{T\Delta S^\circ - \Delta H^\circ}{RT}$$

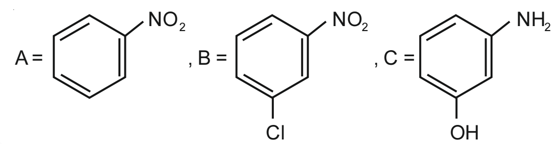
$$\Delta G^\circ = -RT \ln K$$

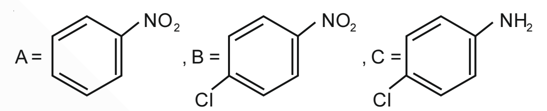
$$\frac{\Delta G^\circ}{-RT} = \ln K$$

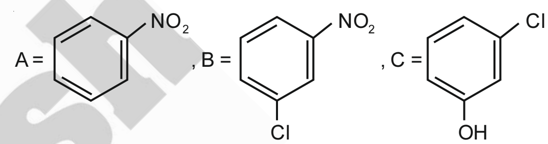
$$\Rightarrow K = e^{-\frac{\Delta G^\circ}{RT}}$$

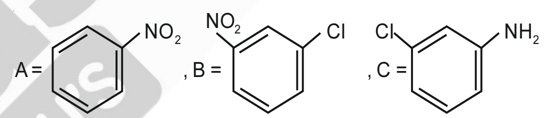
10. Identify correct A, B and C in the reaction sequence given below :



(1) 

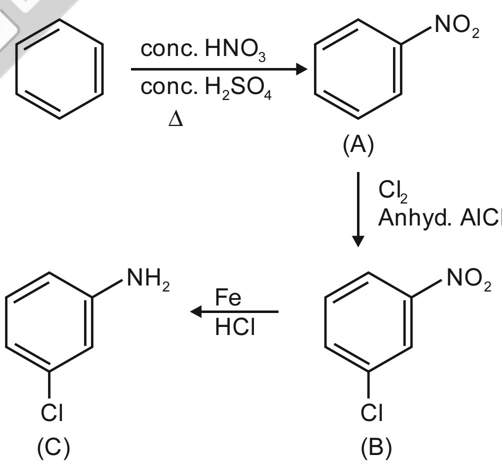
(2) 

(3) 

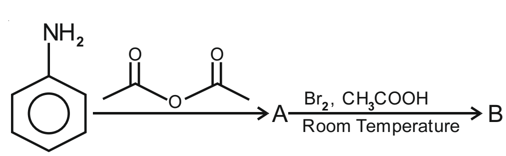
(4) 

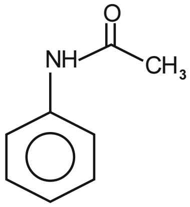
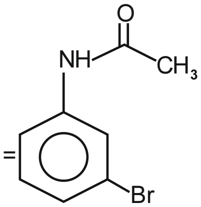
Answer (4)

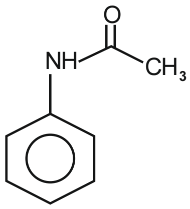
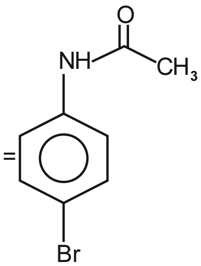
Sol.

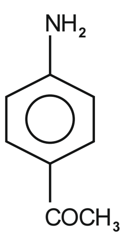
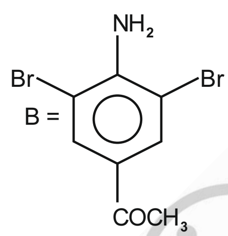


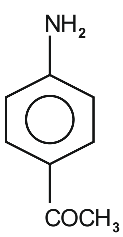
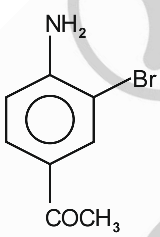
11. The major products A and B formed in the following reaction sequence are :



(1) A =  , B = 

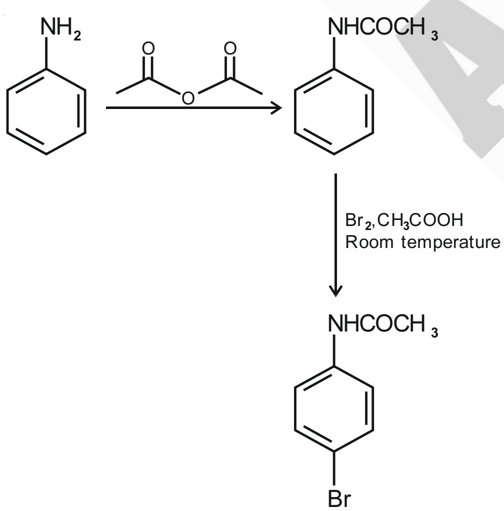
(2) A =  , B = 

(3) A =  , B = 

(4) A =  , B = 

Answer (2)

Sol.



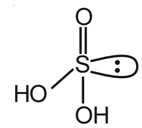
12. The number of S = O bonds present in sulphurous acid, peroxodisulphuric acid and pyrosulphuric acid, respectively are :

(1) 2, 3 and 4 (2) 2, 4 and 3

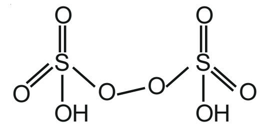
(3) 1, 4 and 4 (4) 1, 4 and 3

Answer (3)

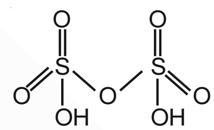
Sol. Sulphurous acid (H_2SO_3)



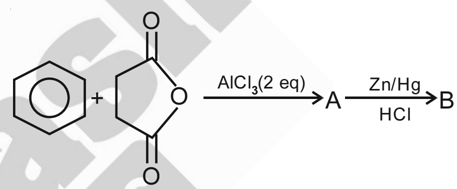
Peroxodisulphuric acid ($H_2S_2O_8$)

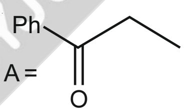
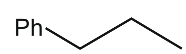


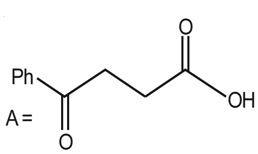
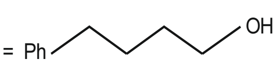
Pyrosulphuric acid ($H_2S_2O_7$)

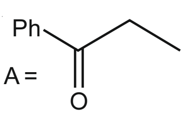
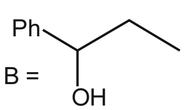


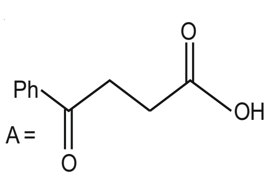
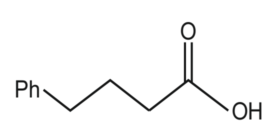
13. The structures of A and B formed in the following reaction are : [Ph = $-C_6H_5$]



(1) A =  , B = 

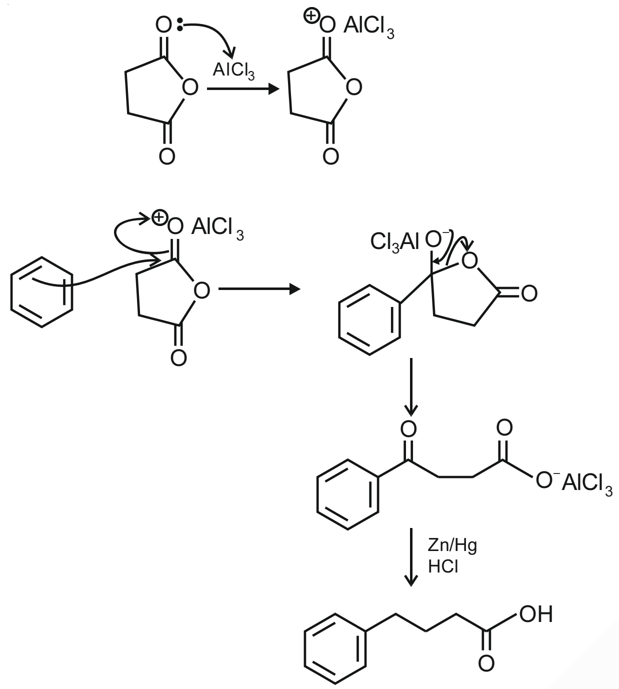
(2) A =  , B = 

(3) A =  , B = 

(4) A =  , B = 

Answer (4)

Sol.



(B)

14. Which one of the following correctly represents the order of stability of oxides, X_2O; (X = halogen)?

(1) I > Cl > Br (2) Cl > I > Br

(3) Br > Cl > I (4) Br > I > Cl

Answer (1)

Sol. A combination of kinetic and thermodynamic factors lead to the generally decreasing order of stability of oxides formed by halogens, I > Cl > Br. The higher oxide of halogen tend to be more stable than lower one.

15. Which among the following is not a polyester?

(1) Dacron (2) Glyptal

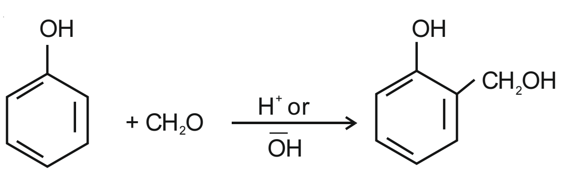
(3) PHBV (4) Novolac

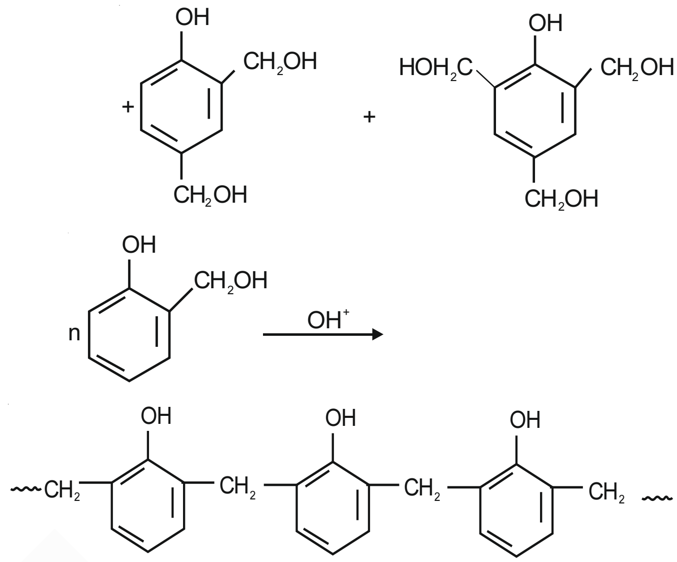
Answer (4)

Sol. Polyesters

These are the polycondensation products of dicarboxylic acids and diols.

Novolac is not a polyester.





Novolac

16. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**_____.

Assertion (A) : Lithium salts are hydrated.

Reason (R) : Lithium has higher polarising power than other alkali metal group members.

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(2) (A) is correct but (R) is not correct

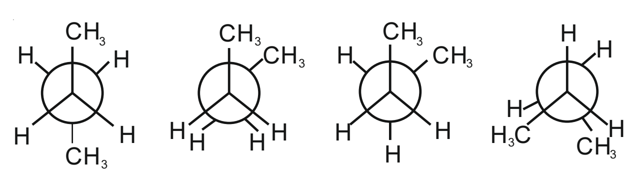
(3) (A) is not correct but (R) is correct

(4) Both (A) and (R) are correct but (R) is NOT the correct explanation of (A)

Answer (4)

Sol. Lithium has smallest size among the alkali metal. So charge to size ratio of lithium is very high. Due to this lithium salts are hydrated in nature. LiCl is deliquescent and crystallises as a hydrate, $LiCl.2H_2O$ whereas other alkali metal chlorides do not form hydrates. Polarising power is related to covlent character.

17. Arrange the following conformational isomers of n-butane in order of their increasing potential energy



I II III IV

(1) II < IV < III < I (2) I < III < IV < II

(3) II < III < IV < I (4) I < IV < III < II

Answer (2)

Sol. Energy in increasing order is,

Anti < Gauche < Partially Eclipsed < Eclipsed

I < III < IV < II

18. Spin only magnetic moment in BM of $[Fe(CO)_4(C_2O_4)]^+$ is :

(1) 5.92

(2) 1.73

(3) 1

(4) 0

Answer (2)

Sol. $\left[Fe(CO)_4(C_2O_4)\right]^+$

oxidation state of iron in the given compound is

X + 0 – 2 = +1

x = + 3

$Fe(26) = [Ar]\ 4s^2 3d^6$

$Fe^{3+} = [Ar]3d^5$

It is a low spin complex because of four strong CO ligand. So number of unpaired electron is 1.

$\mu = \sqrt{n(n+2)}$

$= \sqrt{1(1+2)}$

$= \sqrt{3} = 1.73$ B.M.

19. In which one of the following sets all species show disproportionation reaction?

(1) $ClO_4^-, MnO_4^-, ClO_2^-$ and F_2

(2) ClO_2^-, F_2, MnO_4^- and $Cr_2O_7^{2-}$

(3) MnO_4^-, ClO_2^-, Cl_2 and Mn^{3+}

(4) $Cr_2O_7^{2-}, MnO_4^-, ClO_2^-$ and Cl_2

Answer (Bonus)

Sol. $ClO_4^-, MnO_4^-, Cr_2O_7^{2-}$ – Cl, Mn, Cr in these anions are present in highest oxidation state. These will not undergo disproportionation.

20. The deposition of X and Y on ground surfaces is referred as wet and dry depositions, respectively. X and Y are

(1) X = SO_2, Y = Ammonium salts

(2) X = CO_2, Y = SO_2

(3) X = Ammonium salts, Y = SO_2

(4) X = Ammonium salts, Y = CO_2

Answer (3)

Sol. Ammonium salts are also formed and can be seen as an atmospheric haze (aerosol of fine particles). Aerosol particles of oxides or ammonium salts in rain drops result in wet-deposition. SO_2 is also absorbed directly on both solid and liquid ground surfaces and is thus deposited as dry-deposition.

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. 1.22 g of an organic acid is separately dissolved in 100 g of benzene (K_b = 2.6 K kg mol^{-1}) and 100 g of acetone (K_b = 1.7 K kg mol^{-1}). The acid is known to dimerize in benzene but remain as a monomer in acetone. The boiling point of the solution in acetone increases by 0.17°C. The increase in boiling point of solution in benzene in °C is x × 10^{-2}. The value of x is ________. (Nearest integer)

[Atomic mass : C = 12.0, H = 1.0, O = 16.0]

Answer (13)

Sol. K_b (benzene) = 2.6 K kg mol^{-1}

K_b (acetone) = 1.7 K kg mol^{-1}

In acetone

$$0.17 = \frac{1 \times 1.7 \times 1.22 \times 1000}{M \times 100}$$

M = 122 g/mol

In benzene

$$\Delta T_b = \frac{1}{2} \times \frac{1.22}{122} \times 2.6 \times 10$$

= 0.13 °C

= 13 × 10^{-2} °C

2. In the electrolytic refining of blister copper, the total number of main impurities, from the following, removed as anode mud is_____.

Pb, Sb, Se, Te, Ru, Ag, Au and Pt

Answer (6)

Sol. Impurities from the blister copper deposit as anode mud which contains antimony, selenium, tellurium, silver, gold and platinum.

3. For the reaction A → B, the rate constant k (in s^{-1}) is given by

$$\log_{10} k = 20.35 - \frac{(2.47 \times 10^3)}{T}$$

The energy of activation in kJ mol^{-1} is ________. (Nearest integer)

[Given : R = 8.314 JK^{-1} mol^{-1}]

Answer (47)

Sol. $$\log_{10} K = 20.35 - \frac{(2.47 \times 10^3)}{T} \quad \text{...(i)}$$

$$\log K = \log A - \frac{E_a}{2.303\ RT} \quad \text{...(ii)}$$

Comparing (i) and (ii),

$$\frac{E_a}{2.303\ RT} = \frac{2.47 \times 10^3}{T}$$

E_a = 2.47 × 10^3 × 2.303 × 8.314

= 47293.44 J mol^{-1}

= 47.2934 kJ mol^{-1}

4. The empirical formula for a compound with a cubic close packed arrangement of anions and with cations occupying all the octahedral sites in A_xB. The value of x is _______. (Integer answer)

Answer (1)

Sol. Cation$_{\text{(octahedral site)}}$ Anion$_{\text{(ccp)}}$

$A_{\left(1 + 12 \times \frac{1}{4}\right)}$ $B_{\left(6 \times \frac{1}{2} + 8 \times \frac{1}{8}\right)}$

A_4B_4

Empirical formula = AB

⇒ x = 1

5. The value of magnetic quantum number of the outermost electron of Zn^+ ion is ______. (Integer answer)

Answer (0)

Sol. Zn(30) = $[Ar]4s^23d^{10}$

Zn^+ = $[Ar]4s^13d^{10}$

Outermost electron is present in 4s

n = 4 l = 0 m_l = 0

6. CH_4 is adsorbed on 1 g charcoal at 0ºC following the Freundlich adsorption isotherm. 10.0 mL of CH_4 is adsorbed at 100 mm of Hg, whereas 15.0 mL is adsorbed at 200 mm of Hg. The volume of CH_4 adsorbed at 300 mm of Hg is 10^x mL. The value of x is _____ × 10^{-2}. (Nearest integer)

[Use $\log_{10}2$ = 0.3010, $\log_{10}3$ = 0.4771]

Answer (128)

Sol. $$\frac{x}{m} = kp^{\frac{1}{n}}$$

$$\frac{10}{1} = k(100)^{\frac{1}{n}}$$

$$\frac{15}{1} = k(200)^{\frac{1}{n}}$$

$$\frac{x}{1} = k(300)^{\frac{1}{n}}$$

$$\log 10 = \log k + \frac{1}{n}\log 100$$

$$\log 15 = \log k + \frac{1}{n}\log 200$$

$$\log\left(\frac{3}{2}\right) = \frac{1}{n}\log 2$$

$$n = \frac{0.3010}{0.1761} = 1.7$$

$$\log y = \log k + \frac{1}{n}\log 300$$

$$\log y - \log 10 = \frac{1}{n}\log 3$$

$$\log y - 1 = \frac{1}{1.7} \times 0.4771 = 0.28 + 1$$

$y = 10^{1.28}$

$10^{1.28} = 10^x$

$x = 1.28$

$= 128 \times 10^{-2}$

7. Sodium oxide reacts with water to produce sodium hydroxide. 20.0 g of sodium oxide is dissolved in 500 mL of water. Neglecting the change in volume, the concentration of the resulting NaOH solution is _____ × 10^{-1} M. (Nearest integer)

[Atomic mass : Na = 23.0, O = 16.0, H = 1.0]

Answer (13)

Sol. $Na_2O + H_2O \rightarrow 2NaOH$

Moles $\frac{20}{23\times2+16}$ $= \frac{2\times20}{(23\times2+16)}$

$$\text{Molarity} = \frac{0.645\times1000}{500}$$

$= 1.290$ M

$= 12.90 \times 10^{-1}$ M

$\simeq 13 \times 10^{-1}$ M

8. According to molecular orbital theory, the number of unpaired electron(s) in O_2^{2-} is _____.

Answer (0)

Sol. Electronic configuration of O_2^{2-} (according to MOT) is

$$\sigma 1s^2, \sigma^*1s^2, \sigma 2s^2, \sigma^*2s^2, \sigma 2p_x^2, \begin{Bmatrix}\pi 2p_y^2, \\ \pi 2p_z^2,\end{Bmatrix} \begin{Bmatrix}\pi^* 2p_y^2, \\ \pi^* 2p_z^2\end{Bmatrix} \sigma^* 2p_x$$

Total unpaired electron in O_2^{2-} is zero.

9. The pH of a solution obtained by mixing 50 mL of 1 M HCl and 30 mL of 1 M NaOH is x × 10^{-4}. The value of x is _____. (Nearest integer)

[log 2.5 = 0.3979]

Answer (6021)

Sol.

	HCl	+ NaOH	→ NaCl	+ H_2O
Initial (m mol)	50×1	30×1	–	–
Final (m mol)	20	–	30	30

pH = –log [H^+]

$$= -\log\left(\frac{20}{80}\right)$$

= – log (0.25)

= – log(2.5 × 10^{-1})

= – log(2.5) – log(10^{-1})

= – 0.3979 + 1

= 6021 × 10^{-4}

10. The transformation occurring in Duma's method is given below

$$C_2H_7N + \left(2x + \frac{y}{2}\right)CuO$$

$$\longrightarrow xCO_2 + \frac{y}{2}H_2O + \frac{z}{2}N_2 + \left(2x + \frac{y}{2}\right)Cu$$

The value of y is _____. (Integer answer)

Answer (7)

Sol. $C_2H_7N + \left(2x + \frac{y}{2}\right)CuO \longrightarrow$

$$xCO_2 + \frac{y}{2}H_2O + \frac{z}{2}N_2 + \left(2x + \frac{y}{2}\right)Cu \quad ...(i)$$

$$C_xH_yN_z + \left(2x + \frac{y}{2}\right)CuO \longrightarrow$$

$$xCO_2 + \frac{y}{2}H_2O + \frac{z}{2}N_2 + \left(2x + \frac{y}{2}\right)Cu \quad ...(ii)$$

Comparing (i) and (ii) we get,

x = 2

y = 7

z = 1

PART–C : MATHEMATICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. If $\frac{dy}{dx}=\frac{2^x y+2^y\cdot 2^x}{2^x+2^{x+y}\log_e 2}, y(0)=0$, then for y = 1, the value of x lies in the interval

(1) $\left(\frac{1}{2}, 1\right]$ (2) $\left(0, \frac{1}{2}\right]$

(3) (1, 2) (4) (2, 3)

Answer (3)

Sol. $\frac{dy}{dx}=\frac{2^x\left(y+2^y\right)}{2^x\left(1+2^y\ln 2\right)}$

$\Rightarrow \int\frac{\left(1+2^y\ln 2\right)dy}{\left(y+2^y\right)}=\int dx$

For LHS put $y + 2^y = t$

$\Rightarrow (1 + 2^y \ln 2)\, dy = dt$

$\Rightarrow \ln (y + 2^y) = x + c$

$\downarrow$ (0, 0)

c = 0

$\therefore \ln(y + 2^y) = x$

If y = 1

$\Rightarrow x = \ln 3$

$x \in (1, 2)$

2. If $\alpha + \beta + \gamma = 2\pi$, then the system of equations

$x + (\cos\gamma)y + (\cos\beta)z = 0$

$(\cos\gamma)x + y + (\cos\alpha)z = 0$

$(\cos\beta)x + (\cos\alpha)y + z = 0$

has :

(1) a unique solution

(2) no solution

(3) infinitely many solutions

(4) exactly two solutions

Answer (3)

Sol. $\Delta=\begin{vmatrix}1 & \cos\gamma & \cos\beta\\ \cos\gamma & 1 & \cos\alpha\\ \cos\beta & \cos\alpha & 1\end{vmatrix}$

$= (1 - \cos^2\alpha) - \cos\gamma(\cos\gamma - \cos\alpha\cos\beta)$
$+ \cos\beta(\cos\alpha\cos\gamma - \cos\beta)$

$= 1 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma) + 2\cos\alpha\cos\beta\cos\gamma$

(as A + B + C = 2π)

$= 1 - (1 - 2\cos\alpha\cos\beta\cos\gamma) + 2\cos\alpha\cos\beta\cos\gamma$

= 0

$\therefore$ System has infinite solution

3. The locus of mid-points of the line segments joining (–3, –5) and the points on the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ is

(1) $36x^2 + 16y^2 + 90x + 56y + 145 = 0$

(2) $9x^2 + 4y^2 + 18x + 8y + 145 = 0$

(3) $36x^2 + 16y^2 + 72x + 32y + 145 = 0$

(4) $36x^2 + 16y^2 + 108x + 80y + 145 = 0$

Answer (4)

Sol. Let general point on ellipse $(2\cos\theta, 3\sin\theta)$

Let mid-point of $(2\cos\theta, 3\sin\theta)$ & (–3, –5) be (h, k)

$\therefore 2\cos\theta - 3 = 2h$ & $3\sin\theta - 5 = 2k$

$\Rightarrow \cos\theta=\frac{2h+3}{2}$ & $\sin\theta=\frac{2k+5}{3}$

$\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \left(\frac{2y+5}{3}\right)^2+\left(\frac{2x+3}{2}\right)^2=1$

$\Rightarrow 4(2y + 5)^2 + 9(2x + 3)^2 = 36$

$\Rightarrow 36x^2 + 16y^2 + 108x + 80y + 145 = 0$

4. The number of solutions of the equation $32^{\tan^2 x}+32^{\sec^2 x}=81,\ 0\le x\le\frac{\pi}{4}$ is

(1) 3

(2) 1

(3) 2

(4) 0

Answer (2)

Sol. $32^{\tan^2 x} + 32^{1+\tan^2 x} = 81$

$32^{\tan^2 x}(33) = 81$

$32^{\tan^2 x} = \dfrac{81}{33}$

If $x \in \left[0, \dfrac{\pi}{4}\right]$ then $32^{\tan^2 x} \in [1, 32]$ & is always increasing.

Hence one solution only

5. Negation of the statement $(p \vee r) \Rightarrow (q \vee r)$ is

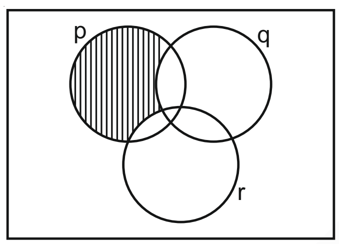
(1) $\sim p \wedge q \wedge \sim r$ (2) $p \wedge \sim q \wedge \sim r$

(3) $\sim p \wedge q \wedge r$ (4) $p \wedge q \wedge r$

Answer (2)

Sol. $\sim(p \vee r) \Rightarrow (q \vee r)$

$= (p \vee r) \wedge \sim(q \vee r)$



$\therefore \quad p \wedge \sim q \wedge \sim r$

6. An angle of intersection of the curves, $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ and $x^2 + y^2 = ab$, a > b is

(1) $\tan^{-1}\left(\dfrac{a+b}{\sqrt{ab}}\right)$ (2) $\tan^{-1}\left(\dfrac{a-b}{2\sqrt{ab}}\right)$

(3) $\tan^{-1}\left(\dfrac{a-b}{\sqrt{ab}}\right)$ (4) $\tan^{-1}\left(2\sqrt{ab}\right)$

Answer (3)

Sol. Let point of intersection be (x_1, y_1)

$x_1^2 + y_1^2 - ab = 0$

$b^2x_1^2 + a^2y_1^2 - a^2b^2 = 0$

$x_1^2 = \dfrac{a^2b}{a+b}, \; y_1^2 = \dfrac{ab^2}{a+b}$

$\Rightarrow \quad x_1 = a\sqrt{\dfrac{b}{a+b}}, \; y_1 = b\sqrt{\dfrac{a}{a+b}} \quad$ ...(i)

Tangent to ellipse $\dfrac{xx_1}{a^2} + \dfrac{yy_1}{b^2} = 1$

Slope $= m_1 = -\dfrac{b^2x_1}{a^2y_1}$

Tangent to circle $xx_1 + yy_1 = ab$

Slope $= m_2 = -\dfrac{x_1}{y_1}$

$$\tan\theta = \left|\dfrac{\dfrac{-b^2x_1}{a^2y_1} + \dfrac{x_1}{y_1}}{1 + \dfrac{b^2x_1}{a^2y_1}\cdot\dfrac{x_1}{y_1}}\right| = \dfrac{x_1y_1\left(a^2 - b^2\right)}{a^2b^2}$$

$$= \dfrac{ab\sqrt{ab}}{a+b}\dfrac{\left(a^2-b^2\right)}{a^2b^2} \quad \text{(Using (i))}$$

$\tan\theta = \dfrac{a-b}{\sqrt{ab}}$

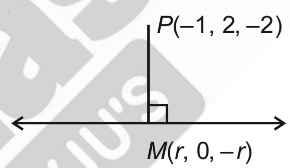
7. The distance of the point (–1, 2, –2) from the line of intersection of the planes $2x + 3y + 2z = 0$ and $x - 2y + z = 0$ is

(1) $\dfrac{5}{2}$ (2) $\dfrac{\sqrt{34}}{2}$

(3) $\dfrac{\sqrt{42}}{2}$ (4) $\dfrac{1}{\sqrt{2}}$

Answer (2)

Sol.



DRs of line of intersection (LOI).

$$\begin{vmatrix} i & j & k \\ 2 & 3 & 2 \\ 1 & -2 & 1 \end{vmatrix} = 7\vec{i} - 0\vec{j} - 7\vec{k}$$

DRs ≡ (1, 0, –1)

Equation of line of intersection

$\dfrac{x}{1} = \dfrac{y}{0} = \dfrac{z}{-1} = r$

$M(r, 0, -r)$

Direction ratios of PM $(r + 1, -2, -r + 2)$

Apply PM perpendicualr line $1(r + 1) + 0(-2) - 1(-r + 2) = 0$

$2r - 1 = 0$

$r = \dfrac{1}{2}$

$M\left(\dfrac{1}{2}, 0, \dfrac{-1}{2}\right)$

$PM = \sqrt{\dfrac{9}{4} + 4 + \dfrac{9}{4}} = \dfrac{\sqrt{34}}{2}$

8. Let A be the set of all points (α, β) such that the area of triangle formed by the points (5, 6), (3, 2) and (α, β) is 12 square units. Then the least possible length of a line segment joining the origin to a point in A, is

(1) $\frac{8}{\sqrt{5}}$ (2) $\frac{16}{\sqrt{5}}$

(3) $\frac{4}{\sqrt{5}}$ (4) $\frac{12}{\sqrt{5}}$

Answer (1)

Sol. $\frac{1}{2}\left\|\begin{matrix} \alpha & \beta & 1 \\ 5 & 6 & 1 \\ 3 & 2 & 1 \end{matrix}\right\| = 12$

$|4\alpha - 2\beta - 8| = 24$

$|2\alpha - \beta - 4| = 12$

Locus $= 2x - y - 4 = 12,\ \ 2x - y - 4 = -12$

$2x - y - 16 = 0,\quad 2x - y + 8 = 0$

Required length = minimum perpendicular distance from origin

$= \min\left\{\frac{16}{\sqrt{5}}, \frac{8}{\sqrt{5}}\right\} = \frac{8}{\sqrt{5}}$

9. If $y\frac{dy}{dx} = x\left[\frac{y^2}{x^2} + \frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)}\right]$, $x > 0$, $\phi > 0$, and $y(1) = -1$, then $\phi\left(\frac{y^2}{4}\right)$ is equal to

(1) $4\phi(2)$ (2) $4\phi(1)$

(3) $2\phi(1)$ (4) $\phi(1)$

Answer (2)

Sol. Let $\frac{y^2}{x^2} = u$

$\Rightarrow y^2 = ux^2$

$2yy' = u'x^2 + 2ux$ ...(i)

Given

$$yy' = x\left[\frac{y^2}{x^2} + \frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)}\right]$$

$\Rightarrow yy' = x\left[u + \frac{\phi(u)}{\phi'(u)}\right]$

$\Rightarrow \frac{1}{2}\left[u'x^2 + 2ux\right] = x\left[u + \frac{\phi(u)}{\phi'(u)}\right]$

$\frac{1}{2}u'x^2 = x\frac{\phi(u)}{\phi'(u)}$

$\Rightarrow xu' = \frac{2\phi(u)}{\phi'(u)}$

$x\frac{du}{dx} = \frac{2\phi(u)}{\phi'(u)}$

$\int \frac{\phi'(u)du}{\phi(u)} = \int \frac{2dx}{x}$

$\ln \phi(u) = 2 \ln x + \ln c$

$\phi(u) = cx^2$

$\phi\left(\frac{y^2}{x^2}\right) = cx^2$ ...(ii)

$x = 1 \Rightarrow y = -1$

$\therefore \phi(1) = c$

$\Rightarrow \phi\left(\frac{y^2}{x^2}\right) = \phi(1)x^2$

Put $x = 2$

$\phi\left(\frac{y^2}{4}\right) = 4\phi(1)$

10. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors mutually perpendicular to each other and have same magnitude. If a vector $\vec{r}$ satisfies

$\vec{a}\times\{(\vec{r}-\vec{b})\times\vec{a}\} + \vec{b}\times\{(\vec{r}-\vec{c})\times\vec{b}\} + \vec{c}\times\{(\vec{r}-\vec{a})\times\vec{c}\} = \vec{0}$,

then $\vec{r}$ is equal to

(1) $\frac{1}{3}(\vec{a}+\vec{b}+\vec{c})$ (2) $\frac{1}{2}(\vec{a}+\vec{b}+2\vec{c})$

(3) $\frac{1}{2}(\vec{a}+\vec{b}+\vec{c})$ (4) $\frac{1}{3}(2\vec{a}+\vec{b}-\vec{c})$

Answer (2)

Sol. $|\vec{a}| = |\vec{b}| = |\vec{c}| = d$ (say)

$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = 0$

$\vec{a}\times((\vec{r}-\vec{b})\times\vec{a}) + \vec{b}\times((\vec{r}-\vec{c})\times\vec{b}) + \vec{c}\times((\vec{r}-\vec{a})\times\vec{c}) = 0$

$= \sum (a\cdot a)(\vec{r}-\vec{b}) - (\vec{a}\cdot(\vec{r}-\vec{b}))\vec{a} = 0$

$= \sum d^2(\vec{r}-\vec{b}) - (\vec{a}-\vec{r})\vec{a} = 0 \quad [\because \ \vec{a}\cdot\vec{b} = \vec{0}]$

$= 3d^2\vec{r} - d^2(\vec{a}+\vec{b}+\vec{c}) - \{(\vec{r}\cdot\vec{a})\vec{a} + (\vec{r}\cdot\vec{b})b + (\vec{r}\cdot\vec{c})\vec{c}\} = 0$

$= 3d^2\vec{r} - d^2(\vec{a}+\vec{b}+\vec{c}) - d^2\vec{r} = 0$

[$\because$ Each term is component of $\vec{r}$]

$2\vec{r} - (\vec{a}+\vec{b}+\vec{c}) = 0$

$\vec{r} = \frac{1}{2}(\vec{a}+\vec{b}+\vec{c})$

11. If [x] is the greatest integer $\le x$, then $\pi^2 \int_0^2 \left(\sin\frac{\pi x}{2}\right)(x-[x])^{[x]}dx$ is equal to

(1) 4(π + 1) (2) 2(π – 1)

(3) 4(π – 1) (4) 2(π + 1)

Answer (3)

Sol. $\pi^2 \int_0^2 \left(\sin\frac{\pi x}{2}\right)(x-[x])^{[x]}dx$

$= \pi^2\left[\int_0^1 \sin\frac{\pi x}{2}dx + \int_1^2 (x-1)\sin\frac{\pi x}{2}dx\right]$

$= \pi^2\left[-\frac{2}{\pi}\cos\frac{\pi x}{2}\Big|_0^1 - (x-1)\frac{2}{\pi}\cos\frac{\pi x}{2}\Big|_1^2 + \int_1^2 \frac{2}{\pi}\cos\frac{\pi x}{2}dx\right]$

$= \pi^2\left[\frac{2}{\pi} + \frac{2}{\pi} + \frac{4}{\pi^2}\sin\frac{\pi x}{2}\Big|_1^2\right]$

$= \pi^2\left[\frac{4}{\pi} - \frac{4}{\pi^2}\right] = 4(\pi-1)$

12. Let S = {1, 2, 3, 4, 5, 6}. Then the probability that a randomly chosen onto function g from S to S satisfies g(3) = 2g(1) is

(1) $\frac{1}{30}$ (2) $\frac{1}{10}$

(3) $\frac{1}{15}$ (4) $\frac{1}{5}$

Answer (2)

Sol. Total number of onto functions = $\lfloor 6$

$\because g(3) = 2g(1)$ then $(g(1), g(3))$
= (1, 2) or (2, 4) or (3, 6)

In each case number of onto functions = $\lfloor 4$

Required probability = $\frac{3\lfloor 4}{\lfloor 6} = \frac{1}{10}$

13. Let f be any continuous function on [0, 2] and twice differentiable on (0, 2). if f(0) = 0, f(1) = 1 and f(2) = 2, then

(1) $f''(x) > 0$ for all $x \in (0, 2)$

(2) $f'(x) = 0$ for some $x \in [0, 2]$

(3) $f''(x) = 0$ for some $x \in (0, 2)$

(4) $f''(x) = 0$ for all $x \in (0, 2)$

Answer (3)

Sol. There exists a $C_1 \in (0,1)$

Such that $f'(C_1) = \frac{f(1)-f(0)}{1-0} = 1$

and there exists a $C_2 \in (1, 2)$

Such that $f'(C_2) = \frac{f(2)-f(1)}{2-1} = 1$

Hence there exists a $C \in (C_1, C_2)$ such that

$f''(C) = \frac{f'(C_1)-f'(C_2)}{C_1-C_2} = 0$

14. If $\alpha = \lim_{x\to\pi/4} \frac{\tan^3 x - \tan x}{\cos\left(x+\frac{\pi}{4}\right)}$ and $\beta = \lim_{x\to 0}(\cos x)^{\cot x}$ are the roots of the equation, $ax^2 + bx - 4 = 0$, then the ordered pair (a, b) is

(1) (–1, –3)

(2) (–1, 3)

(3) (1, 3)

(4) (1, –3)

Answer (3)

Sol. $\alpha = \lim_{x\to\frac{\pi}{4}} \frac{\tan x(-\cos 2x)}{\cos^2 x\cdot\cos\left(x+\frac{\pi}{4}\right)} = -2\lim_{x\to\frac{\pi}{4}} \frac{\cos 2x}{\cos\left(x+\frac{\pi}{4}\right)} = -4$

$\beta = \lim_{x\to 0}(\cos x)^{\cot x} = e^{\lim_{x\to 0}(\cos x - 1)\cot x} = e^0 = 1$

Quadratic equation having roots $\alpha_1\beta$ is

$x^2 + 3x - 4 = 0$

Clearly a = 1, b = 3

15. The domain of the function

$$f(x)=\sin^{-1}\left(\frac{3x^2+x-1}{(x-1)^2}\right)+\cos^{-1}\left(\frac{x-1}{x+1}\right) \text{ is}$$

(1) $\left[0, \frac{1}{4}\right]$ (2) $\left[0, \frac{1}{2}\right]$

(3) $\left[\frac{1}{4}, \frac{1}{2}\right]\cup\{0\}$ (4) $[-2, 0]\cup\left[\frac{1}{4}, \frac{1}{2}\right]$

Answer (3)

Sol. $\because \left(\frac{3x^2+x-1}{(x-1)^2}\right)\in[-1, 1]$ and $\frac{x-1}{x+1}\in[-1, 1]$

$\Rightarrow x\in\left[-2, \frac{1}{2}\right]$ and $x\in(-\infty, 0]\cup\left[\frac{1}{4}, \infty\right)-\{1\}$ and $x\in[0, \infty)$

finally $x\in\{0\}\cup\left[\frac{1}{4}, \frac{1}{2}\right]$

16. The mean and variance of 7 observations are 8 and 16 respectively. If two observations are 6 and 8, then the variance of the remaining 5 observations is

(1) $\frac{536}{25}$ (2) $\frac{134}{5}$

(3) $\frac{112}{5}$ (4) $\frac{92}{5}$

Answer (1)

Sol. $\bar{X}_{\text{old}}=8\Rightarrow\sum X_{\text{old}}=56$

$$\frac{\sum X_{\text{old}}^2}{7}-(\bar{X}_{\text{old}})^2=16\Rightarrow\sum X_{\text{old}}^2=560$$

Sum of remaining 5 observation

$=\sum X=56-14=42$

Sum of squares of 5 observation = $560-6^2-8^2$

$= 460$

Variance = $\frac{460}{5}-\left(\frac{42}{5}\right)^2=\frac{536}{25}$

17. The sum of the roots of the equation,

$x+1-2\log_2(3+2^x)+2\log_4(10-2^{-x})=0$, is

(1) $\log_2 14$ (2) $\log_2 12$

(3) $\log_2 11$ (4) $\log_2 13$

Answer (3)

Sol. $x+1-2\log_2(3+2^x)+2\log_4(10-2^{-x})=0$

$\Rightarrow x+1+\log_2(10-2^{-x})-\log_2(3+2^x)^2=0$

$$\Rightarrow x+1=\log_2\left(\frac{(3+2^x)^2}{(10-2^{-x})}\right)$$

$$\Rightarrow 2^{x+1}=\frac{9+6\cdot 2^x+2^{2x}}{10-2^{-x}}$$

$\Rightarrow 20\cdot 2^x-2=9+6\cdot 2^x+2^{2x}$

$\Rightarrow (2^x)^2-14(2^x)+11=0$

Let two roots are 2^{x_1} and 2^{x_2}

Then $2^{x_1}\cdot 2^{x_2}=11\Rightarrow x_1+x_2=\log_2 11$

$\therefore$ Sum of roots = $\log_2 11$

18. Let $f: N\to N$ be a function such that $f(m+n)=f(m)+f(n)$ for every $m, n\in N$. If $f(6)=18$, then $f(2).f(3)$ is equal to

(1) 54 (2) 18

(3) 6 (4) 36

Answer (1)

Sol. $\because f(m+n)=f(m)+f(n), f: N\to N$

then $f(x)=kx$

$\because f(6)=18\Rightarrow 18=k.6\Rightarrow k=3$

$\therefore f(x)=3x$

$\therefore f(2).f(3)=6\times 9$

$=54$

19. Let $a_1, a_2, a_3, \ldots$ be an A.P. If $\frac{a_1+a_2+\ldots+a_{10}}{a_1+a_2+\ldots+a_p}=\frac{100}{p^2}$, $p\neq 10$, then $\frac{a_{11}}{a_{10}}$ is equal to

(1) $\frac{19}{21}$ (2) $\frac{100}{121}$

(3) $\frac{21}{19}$ (4) $\frac{121}{100}$

Answer (3)

Sol. $\because a_1, a_2, a_3 \ldots$ are in A.P.

Let its common difference be d.

$$\therefore \frac{a_1+a_2+\ldots+a_{10}}{a_1+a_2+\ldots a_p}=\frac{100}{p^2}$$

$$\Rightarrow \frac{\frac{10}{2}\{2a_1+9d\}}{\frac{p}{2}\{2a_1+(p-1)d\}}=\frac{100}{p^2}$$

$$\Rightarrow \frac{2a_1+9d}{2a_1+(p-1)d}=\frac{10}{p}$$

$\Rightarrow$ $2pa_1 + 9pd = 20a_1 + 10(p-1)d$.

$= (2p-20)\,a_1 = (p-10)d$.

$\therefore$ $2a_1 = d$ $\quad\quad$ ($\because$ $p \neq 10$)

$$\therefore \frac{a_{11}}{a_{10}}=\frac{a_1+10d}{a_1 9d}=\frac{a_1 20a_1}{a_1+18a_1}=\frac{21}{19}$$

20. If z is a complex number such that $\frac{z-i}{z-1}$ is purely imaginary, then the minimum value of $|z-(3+3i)|$ is

(1) $6\sqrt{2}$ $\quad$ (2) $2\sqrt{2}$

(3) $3\sqrt{2}$ $\quad$ (4) $2\sqrt{2}-1$

Answer (2)

Sol. $\because$ $\frac{z-i}{z-1}$ is purely imaginary number

$$\therefore \frac{z-i}{z-1}+\frac{\bar{z}+i}{\bar{z}-1}=0$$

$$\Rightarrow (z-i)(\bar{z}-1)+(z-1)(\bar{z}+i)=0$$

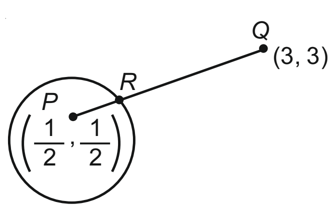
$$\Rightarrow 2z\bar{z}-(z+\bar{z})+i(z-\bar{z})=0$$

$\therefore$ $x^2+y^2-x-y=0$, let $z = x + iy$.

Which a circle of centre $\left(\frac{1}{2},\frac{1}{2}\right)$ and radius $\frac{1}{\sqrt{2}}$

$\therefore$ Minimum value of

$$|z-(3+3i)|=QR$$



$$=\sqrt{\left(3-\frac{1}{2}\right)^2+\left(3-\frac{1}{2}\right)^2}-\frac{1}{\sqrt{2}}$$

$$=\frac{5}{\sqrt{2}}-\frac{1}{\sqrt{2}}=2\sqrt{2}$$

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. The number of 4-digit numbers which are neither multiple of 7 nor multiple of 3 is _______.

Answer (5143)

Sol. A = set of all four digit integers divisible by 7.

B = set of all four digit integers divisible by 3.

$$n(A)=\left[\frac{9000}{7}\right]=1285$$

$$n(B)=\left[\frac{9000}{3}\right]=3000$$

$$n(A\cap B)=\left[\frac{9000}{21}\right]=428$$

$$n(A\cup B)=3857$$

$$n(\overline{A\cup B})=9000-3857$$

$= 5143$

2. If the coefficient of a^7b^8 in the expansion of $(a + 2b + 4ab)^{10}$ is $K{\cdot}2^{16}$, then K is equal to

Answer (315)

Sol. General term in expansion of $(a + 2b + 4ab)^{10}$

$$=\frac{10!a^p\cdot(2b)^q\cdot(4ab)^{10-p-q}}{p!q!(10-p-q)!}$$

$$=\frac{10!\cdot 2^{q+20-2p-2q}}{p!\cdot q!(10-p-q)!}\cdot a^{10-q}\cdot b^{10-p}$$

for a^7b^8, $p = 2$ & $q = 3$

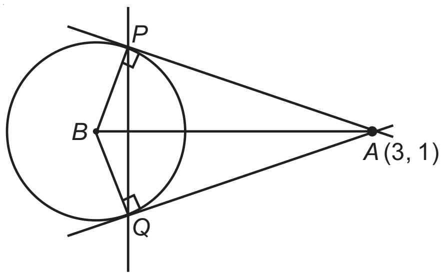
$$\Rightarrow \text{Coefficient of } a^7b^8=\frac{10!}{2!3!\cdot 5!}\cdot 2^{13}=K\cdot 2^{16}$$

$\Rightarrow$ $K = 315$

3. Let B be the centre of the circle $x^2 + y^2 - 2x + 4y + 1 = 0$. Let the tangents at two points P and Q on the circle intersect at the point $A(3, 1)$. Then $8\cdot\left(\frac{\text{area }\Delta APQ}{\text{area }\Delta BPQ}\right)$ is equal to _______.

Answer (18)

Sol.



Let L = length of tangent from A to the circle

& R = radius of circle

$\angle PAB = \angle BPQ = \theta$

$\Rightarrow$ area of $\Delta PAQ = 2.\frac{1}{2}\cdot L\sin\theta.\ L\cos\theta = L^2\cdot\sin\theta\cos\theta$

area of $\Delta PBQ = 2\cdot\frac{1}{2}\cdot R\sin\theta\cdot R\cos\theta = R^2\cdot\sin\theta.\cos\theta$

Hence $\frac{\text{area of }\Delta APQ}{\text{area of }\Delta BPQ} = \frac{L^2}{R^2}$

Now, $L = \sqrt{S_1}, = \sqrt{3^2+1^2-2\times3+4\times1+1} = 3$

& $R = 2$

$\Rightarrow\ 8\times\left(\frac{\text{area of }\Delta APQ}{\text{area of }\Delta BPQ}\right) = 8\times\left(\frac{3}{2}\right)^2 = 18$

4. If $S = \frac{7}{5}+\frac{9}{5^2}+\frac{13}{5^3}+\frac{19}{5^4}+...,$ then 160 S is equal to ________.

Answer (305)

Sol. $S = \frac{7}{5}+\frac{9}{5^2}+\frac{13}{5^3}+\frac{19}{5^4}+.....$

$\frac{1}{5}S = \frac{7}{5^2}+\frac{9}{5^3}+\frac{13}{5^4}+...$

Subtracting,

$\Rightarrow\ \frac{4S}{5} = \frac{7}{5}+\frac{2}{5^2}+\frac{4}{5^3}+\frac{6}{5^4}+...$

$\Rightarrow\ \frac{4S}{5} = \frac{7}{5}+M$ ...(i)

Where, $M = \frac{2}{5^2}+\frac{4}{5^3}+\frac{6}{5^4}+...$

$\frac{1}{5}M = \frac{2}{5^3}+\frac{4}{5^4}+\frac{6}{5^5}+...$

$\frac{4M}{5} = \frac{2}{5^2}+2\left(\frac{1}{5^3}+\frac{1}{5^4}+...\right)$

$\Rightarrow\ M = \frac{1}{8}$

Putting in (i) gives $S = \frac{61}{32}$

$\Rightarrow$ 160S = 305

5. Let $f(x)$ be a cubic polynomial with $f(1) = -10$, $f(-1) = 6$, and has a local minima at $x = 1$, and $f'(x)$ has a local minima at $x = -1$. Then $f(3)$ is equal to ________.

Answer (22)

Sol. $f(1) = -10,\ f(-1) = 6$

$f'(1) = 0$ and $f''(-1) = 0$ as given

$f(x)$ has minima at $x = 1$

and $f'(x)$ has minima at $x = -1$

So, $f''(x) = a(x + 1)$

Integrating both side

$f'(x) = \frac{a}{2}(x+1)^2 + c$

$f'(1) = 0 = 2a + c$

$c = -2a$

$f'(x) = \frac{a}{2}(x+1)^2 - 2a$

Integrating both side

$f(x) = \frac{a}{6}(x+1)^3 - 2xa + c'$

$f(1) = -10 = \frac{8a}{6} - 2a + c'$

$f(-1) = 6 = 2a + c'$

$2a + c' = 6$

$4a - 6c' = 60$

$a = 6,\ c' = -6$

$f(x) = (x + 1)^3 - 12x - 6$

$f(3) = (4)^3 - 36 - 6$

$f(3) = 22$

6. If $\int\frac{\sin x}{\sin^3 x+\cos^3 x}dx = \alpha\log_e|1+\tan x| + \beta\log_e |1-\tan x+\tan^2 x| + \gamma\tan^{-1}\left(\frac{2\tan x-1}{\sqrt{3}}\right)+\text{C},$ when C is constant of integration, then the value of $18(\alpha + \beta + \gamma^2)$ is ________.

Answer (3)

Sol. $\text{I} = \int\frac{\sin x}{\cos^3 x+\sin^3 x}dx = \int\frac{\frac{\sin x}{\cos^3 x}}{\frac{\cos^3 x}{\cos^3 x}+\frac{\sin^3 x}{\cos^3 x}}\cdot dx$

$\text{I} = \int\frac{\tan x\cdot\sec^2 x}{1+\tan^3 x}dx,$ Put tan$x$ = t

$\sec^2 x\cdot dx = dt$

$$I = \int \frac{t}{1+t^3} dt$$

$$\frac{t}{1+t^3} = \frac{A}{1+t} + \frac{Bt+C}{1+t^2-t}$$

$t = A(1 - t + t^2) + (1 + t)(Bt + C)$

By comparing coeff of x, x^2 and constant term,

$$A = -\frac{1}{3},\ B = \frac{1}{3},\ C = \frac{1}{3}$$

$$I = -\frac{1}{3}\int \frac{1}{1+t} dt + \frac{1}{3}\int \frac{t+1}{t^2-t+1} dt$$

$$I = -\frac{1}{3}\ln(1+t) + \frac{1}{6}\left[\int \frac{2t-1}{t^2-t+1} dt + 3\int \frac{1}{t^2-t+1} dt\right]$$

$$I = -\frac{1}{3}\ln(1+t) + \frac{1}{6}\left[\log\left(t^2-t+1\right) + 3\cdot\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2t-1}{\sqrt{3}}\right)\right] + C$$

$$I = -\frac{1}{3}\ln(1+\tan x) + \frac{1}{6}\cdot\log\left(\tan^2 x - \tan x + 1\right) + \frac{1}{\sqrt{3}}\cdot\tan^{-1}\left(\frac{2\tan x - 1}{\sqrt{3}}\right) + C$$

$$\alpha = -\frac{1}{3},\ \beta = \frac{1}{6},\ \gamma = \frac{1}{\sqrt{3}}$$

$18(\alpha + \beta + \gamma^2)$

$$18\left(-\frac{1}{3} + \frac{1}{6} + \frac{1}{3}\right) = 3$$

7. Suppose the line $\frac{x-2}{\alpha} = \frac{y-2}{-5} = \frac{z+2}{2}$ lies on the plane $x + 3y - 2z + \beta = 0$. Then $(\alpha + \beta)$ is equal to ________.

Answer (7)

Sol. Line $\frac{x-2}{\alpha} = \frac{y-2}{-5} = \frac{z+2}{2}$ lies in $x + 3y - 2z + \beta = 0$

Angle b/w line and plane = 0

$\Rightarrow \sin 0° = 0$

$(1)\cdot(\alpha) + (-5)(3) + (2)(-2) = 0$

$\alpha = 19$

Point (2, 2, –2) will be on plane

$(1)(2) + 3(2) - 2(-2) + \beta = 0$

$\beta = -12$

$\alpha + \beta = 19 - 12 = 7$

8. A tangent line L is drawn at the point (2, –4) on the parabola $y^2 = 8x$. If the line L is also tangent to the circle $x^2 + y^2 = a$, then 'a' is equal to ________.

Answer (2)

Sol. Equation of tangent $yy_1 = 4(x + x_1)$

$y(-4) = 4(x + 2)$

$-y = x + 2$

$\Rightarrow x + y + z = 0$

Length of perpendicular from centre (0, 0) should be equal to radius

Length of perpendicular $= \frac{2}{\sqrt{2}} = \sqrt{a} \Rightarrow a = 2$

9. The number of elements is the set

$$\left\{A = \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} : a, b, d \in \{-1, 0, 1\} \text{ and } (I - A)^3 = I - A^3\right\},$$

where I is 2 × 2 identity matrix, is ________.

Answer (8)

Sol. $(I - A)^3 = I - A^3$

$I - A^3 - 3A(I - A) = I - A^3$

$3A(I - A) = 0 \quad \Rightarrow A(I - A) = 0$

$$\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}\begin{bmatrix} 1-a & -b \\ 0 & 1-d \end{bmatrix} = 0$$

$$\begin{bmatrix} a(1-a) & -b(a-1+d) \\ 0 & d(1-d) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$a(1 - a) = 0$, $d(1 - d) = 0$, $b(a - 1 + d) = 0$

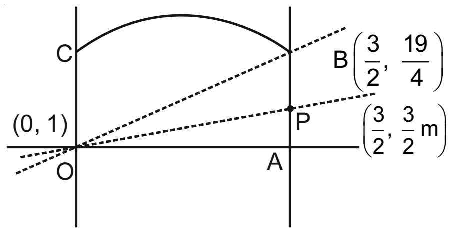
a = 0	a = 1	a = 0	a = 1
d = 0	d = 0	d = 1	d = 1
b = 0	b = –1, 0, 1	b = –1, 0, 1	b = 0

So Total 8 cases

10. If the line $y = mx$ bisects the area enclosed by the lines $x = 0$, $y = 0$, $x = \frac{3}{2}$ and the curve $y = 1 + 4x - x^2$, then 12 m is equal to _______.

Answer (26)

Sol.



Area OABC

$$\int_0^{3/2}\left(1+4x-x^2\right)dx = x + 2x^2 - \frac{x^3}{3}\Big|_0^{3/2} = \frac{39}{8}$$

If $y = mx$ passes through B then

Area of OAB $= \frac{1}{2}\cdot\frac{1}{3}\cdot\frac{19}{4} = \frac{57}{16}$

Which means for bisection it should pass through a point below B,

Let it be P, so for bisection ar(ΔOAP) $= \frac{39}{8}\cdot\frac{1}{2}$

So $\frac{1}{2}\cdot\frac{3}{2}\cdot\frac{3}{2}m = \frac{39}{8}\cdot\frac{1}{2}$

$$m = \frac{39}{8}\cdot\frac{4}{9} = \frac{26}{12}$$

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