31/08/2021
Morning

Corporate Office: Aakash Tower, 8, Pusa Road, New Delhi-110005 Ph. 011-47623456

Time : 3 hrs.

Answers & Solutions

M.M. : 300

for

JEE (MAIN)-2021 (Online) Phase-4

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS :

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part has two sections.

(i) Section-I : This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) Section-II : This section contains 10 questions. In Section-II, attempt any **five questions out of 10.** There will be **no negative marking for Section-II**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and there is no negative marking for wrong answer.

PART–A : PHYSICS

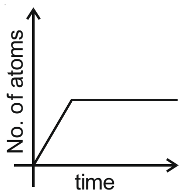
SECTION - I

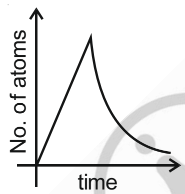
Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

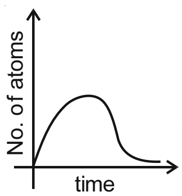
Choose the correct answer :

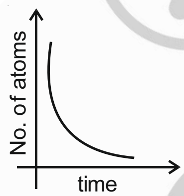
1. A sample of a radioactive nucleus A disintegrates to another radioactive nucleus B, which in turn disintegrates to some other stable nucleus C. Plot of a graph showing the variation of number of atoms of nucleus B versus time is:

   (Assume that at t = 0, there are no B atoms in the sample)

(1) 

(2) 

(3) 

(4) 

Answer (3)

Sol. No. of nuclei of B is initially zero and finally zero. It will reach a maximum value at some intermediate time.

2. A moving proton and electron have the same de-Broglie wavelength. If K and P denote the K.E. and momentum respectively. Then choose the correct option:

   (1) $K_P = K_e$ and $P_P = P_e$

   (2) $K_P < K_e$ and $P_P < P_e$

   (3) $K_P < K_e$ and $P_P = P_e$

   (4) $K_P > K_e$ and $P_P = P_e$

Answer (3)

Sol. $\lambda = \dfrac{h}{P}$

As $\lambda_P = \lambda_e \Rightarrow P_P = P_e$

$\Rightarrow P = \sqrt{2mK}$

$\Rightarrow K = \dfrac{P^2}{2M} \Rightarrow K \propto \dfrac{1}{M}$(for same P)

$\Rightarrow K_P < K_e$

3. A coil having N turns is wound tightly in the form of a spiral with inner and outer radii 'a' and 'b' respectively. Find the magnetic field at centre, when a current I passes through coil:

   (1) $\dfrac{\mu_0 I}{8}\left[\dfrac{a+b}{a-b}\right]$

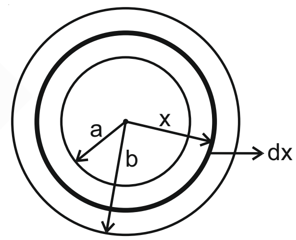
   (2) $\dfrac{\mu_0 I}{4(a-b)}\left[\dfrac{1}{a}-\dfrac{1}{b}\right]$

   (3) $\dfrac{\mu_0 I}{8}\left(\dfrac{a-b}{a+b}\right)$

   (4) $\dfrac{\mu_0 IN}{2(b-a)}\log_e\left(\dfrac{b}{a}\right)$

Answer (4)

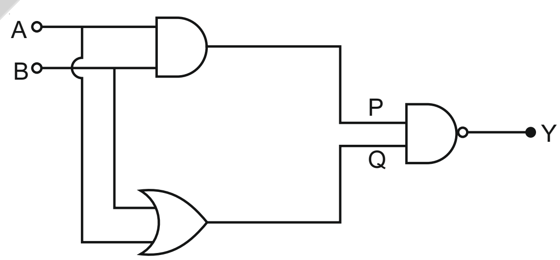
Sol. $dN = \dfrac{N}{(b-a)}dx$



$\therefore B = \int_a^b \dfrac{\mu_0 N \times I dx}{(b-a)\times 2x}$

$= \dfrac{\mu_0 NI}{2(b-a)}\ln\left(\dfrac{b}{a}\right)$

4. In the following logic circuit the sequence of the inputs A, B are (0, 0), (0, 1), (1, 0) and (1, 1). The output Y for this sequence will be:



   (1) 0, 1, 0, 1

   (2) 0, 0, 1, 1

   (3) 1, 0, 1, 0

   (4) 1, 1, 1, 0

Answer (4)

Sol.

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

5. For an ideal gas the instantaneous change in pressure 'p' with volume 'v' is given by the equation $\frac{dp}{dv} = -ap$. If p = p_0 at v = 0 is the given boundary condition, then the maximum temperature one mole of gas can attain is:

(Here R is the gas constant)

(1) Infinity (2) $\frac{ap_0}{eR}$

(3) 0°C (4) $\frac{p_0}{aeR}$

Answer (4)

Sol. $\frac{dp}{dv} = -ap \Rightarrow \int_{p_0}^{p} \frac{dp}{p} = -\int_0^v adv$

$\Rightarrow p = p_0 e^{-av}$

Now, $pv = nRT \Rightarrow T = \frac{pv}{R}$ [n = 1]

$\therefore \quad T = \frac{p_0 e^{-av} \times v}{R}$

So, T to be max, $\frac{dT}{dv} = 0 \Rightarrow v = \frac{1}{a}$

$\therefore \quad T_{max} = \frac{p_0}{eaR}$

6. An object is placed at the focus of concave lens having focal length *f*. What is the magnification and distance of the image from the optical centre of the lens?

(1) 1, ∞ (2) $\frac{1}{2}, \frac{f}{2}$

(3) $\frac{1}{4}, \frac{f}{4}$ (4) Very high, ∞

Answer (2)

Sol. $\frac{1}{v} - \frac{1}{-f} = \frac{1}{-f}$

$\Rightarrow \frac{1}{v} = -\frac{2}{f} \Rightarrow |v| = \frac{f}{2}$

$m = \frac{v}{u} \Rightarrow m = \frac{1}{2}$

7. A body of mass M moving at speed V_0 collides elastically with a mass 'm' at rest. After the collision, the two masses move at angles θ_1 and θ_2 with respect to the initial direction of motion of the body of mass M. The largest possible value of the ratio M/m, for which the angles θ_1 and θ_2 will be equal, is:

(1) 4 (2) 2

(3) 1 (4) 3

Answer (4)

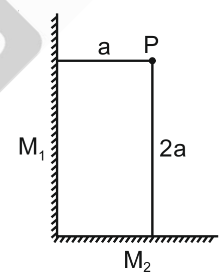
Sol. $2p\cos\theta = p_0$ ...(i)

Now, $\frac{p_0^2}{2M} = p^2\left(\frac{1}{2m} + \frac{1}{2M}\right)$ ...(ii)

$\Rightarrow (2\cos\theta)^2 = \frac{M}{m} + 1$

$\Rightarrow \left(\frac{M}{m}\right)_{max} = 4 - 1 = 3$

8. Two plane mirrors M_1 and M_2 are at right angle to each other shown. A point source 'P' is placed at 'a' and '2a' meter away from M_1 and M_2 respectively. The shortest distance between the images thus formed is: (Take $\sqrt{5} = 2.3$)

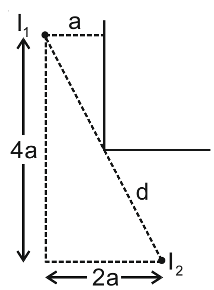


(1) $2\sqrt{10}a$ (2) 2.3a

(3) 4.6a (4) 3a

Answer (3)

Sol.

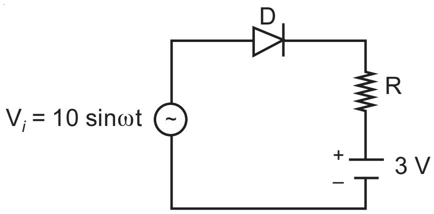


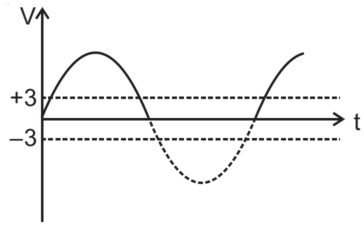
$\therefore \ d = \sqrt{(2a)^2 + (4a)^2}$

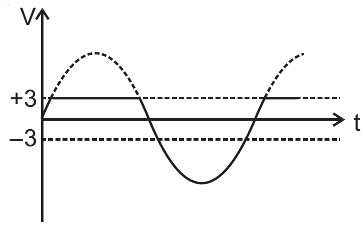
$= 2\sqrt{5}a$

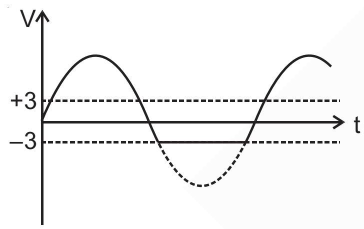
= 4.6a

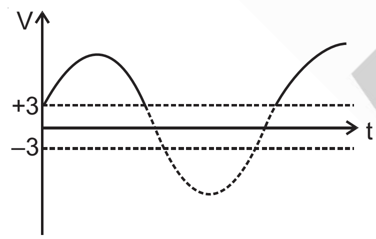
9. Choose the correct waveform that can represent the voltage across R of the following circuit, assuming the diode in ideal one:



(1) 

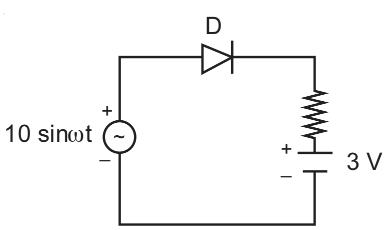
(2) 

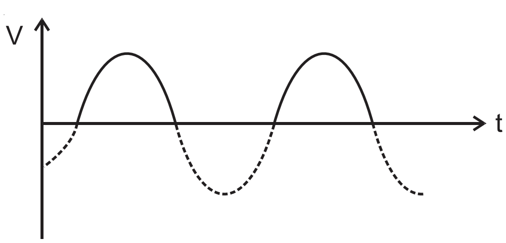
(3) 

(4) 

Answer (Bonus)

Sol. D will be forward biased if 10 sinωt > 3





10. An reversible engine has an efficiency of $\frac{1}{4}$. If the temperature of the sink is reduced by 58°C, its efficiency becomes double. Calculate the temperature of the sink:

(1) 382°C

(2) 280°C

(3) 174°C

(4) 180.4°C

Answer (Bonus)

Sol. $\frac{1}{4}=\left(1-\frac{T_2}{T_1}\right)$ ...(1)

$\frac{1}{2}=1-\frac{(T_2-58)}{T_1}$ ...(2)

From (1) and (2)

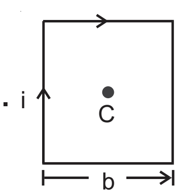
T_{sink} = 174 K [None of the option matches]

11. A small square loop of side 'a' and one turn is placed inside a larger square loop of side b and one turn (b > >a). The two loops are coplanar with their centres coinciding. If a current is passed in the square loop of side 'b', then the coefficient of mutual inductance between the two loops is:

(1) $\frac{\mu_0}{4\pi}\frac{8\sqrt{2}}{b}$ (2) $\frac{\mu_0}{4\pi}8\sqrt{2}\frac{a^2}{b}$

(3) $\frac{\mu_0}{4\pi}\frac{8\sqrt{2}}{a}$ (4) $\frac{\mu_0}{4\pi}8\sqrt{2}\frac{b^2}{a}$

Answer (2)

Sol. 

Magnetic field at C $(B)=\frac{\mu_0 i}{2\pi b}\sqrt{2}\times 4$

So flux through small loop = Ba^2

$=\frac{4\mu_0\sqrt{2}}{2\pi b}a^2 i$

$=\frac{\mu_0 8\sqrt{2}}{4\pi b}a^2 i$

$M=\frac{\mu_0}{4\pi}\frac{8\sqrt{2}a^2}{b}$

12. The masses and radii of the earth and moon are (M_1, R_1) and (M_2, R_2) respectively. Their centres are at a distance 'r' apart. Find the minimum escape velocity for a particle of mass 'm' to be projected from the middle to these two masses.

(1) $V = \frac{1}{2}\sqrt{\frac{4G(M_1+M_2)}{r}}$

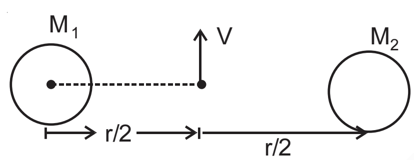
(2) $V = \frac{\sqrt{2G}\,(M_1+M_2)}{r}$

(3) $V = \sqrt{\frac{4G(M_1+M_2)}{r}}$

(4) $V = \frac{1}{2}\sqrt{\frac{2G(M_1+M_2)}{r}}$

Answer (3)

Sol.



$$-\frac{GM_1m}{\frac{r}{2}} + \frac{-GM_2m}{\frac{r}{2}} + \frac{1}{2}mV^2 = 0$$

$$\frac{V^2}{2} = \frac{2G}{r}(M_1+M_2)$$

$$V = \sqrt{\frac{4G}{r}(M_1+M_2)}$$

13. A helicopter is flying horizontally with a speed 'v' at an altitude 'h' has to drop a food packet for a man on the ground. What is the distance of helicopter from the man when the food packet is dropped?

(1) $\sqrt{\frac{2gh}{v^2}+h^2}$ (2) $\sqrt{2ghv^2+h^2}$

(3) $\sqrt{\frac{2v^2h}{g}+h^2}$ (4) $\sqrt{\frac{2ghv^2+1}{h^2}}$

Answer (3)

Sol. Time of flight $= \sqrt{\frac{2h}{g}}$

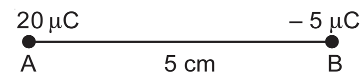
Now displacement in horizontal $= \sqrt{\frac{2h}{g}}V$

So, distance of man from helicopter

$$= \sqrt{h^2 + \left(\sqrt{\frac{2h}{g}}V\right)^2}$$

$$= \sqrt{h^2 + \frac{2V^2h}{g}}$$

14. Two particles A and B having charges 20 μC and –5 μC respectively are held fixed with a separation of 5 cm. At what position a third charged particle should be placed so that it does not experience a net electric force?



(1) At 5 cm from –5 μC on the right side

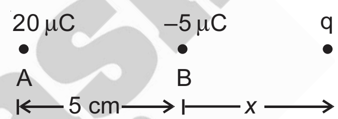
(2) At 1.25 cm from a –5 μC between two charges

(3) At 5 cm from 20 μC on the left side of system

(4) At midpoint between two charges

Answer (1)

Sol.



$$\frac{kq\times 20}{(5+x)^2} = \frac{kq5}{x^2}$$

$$\frac{2}{5+x} = \frac{1}{x}$$

2x = 5 + x

x = 5 cm

15. Match **List-I** with **List-II.**

List-I	**List-II**
(a) Torque	(i) MLT^{-1}
(b) Impulse	(ii) MT^{-2}
(c) Tension	(iii) ML^2T^{-2}
(d) Surface tension	(iv) MLT^{-2}

Choose the **most appropriat**e answer from the otpion given below:

(1) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

(2) (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)

(3) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

(4) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

Answer (3)

Sol. Torque = [ML^2T^{-2}]

Impulse = F × time

$= MLT^{-1}$

Tension $= MLT^{-2}$

Surface tension $= \frac{\text{force}}{\text{length}} = MT^{-2}$

So, (3) is correct option.

16. Which of the following equations is dimensionally incorrect?

Where t = time, h = height, s = surface tension, θ = angle, ρ = density, a, r = radius, g = acceleration due to gravity, v = volume, p = pressure, W = work done, τ = torque e = permittivity, ε = electric field, J = current density, L = length

(1) $h = \frac{2s\cos\theta}{prg}$ (2) $W = \tau\theta$

(3) $v = \frac{\pi pa^4}{8\eta L}$ (4) $J = \varepsilon\frac{\partial E}{\partial t}$

Answer (3)

Sol. Option (3) is dimensionally incorrect as dimension of $\frac{\pi Pa^4}{8\eta l}$ is L^3T^{-1} by Poiseuille formula.

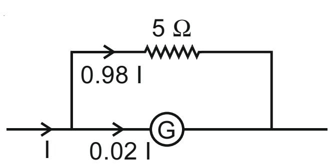
17. Consider a galvanometer shunted with 5 Ω resistance and 2% of current passes through it. What is the resistance of the given galvanometer?

(1) 344 Ω (2) 245 Ω

(3) 226 Ω (4) 300 Ω

Answer (2)

Sol. By KVL



$0.98\ I \times 5 = 0.02 \times R_G$

$R_G = 245\ \Omega$

18. Angular momentum of a single particle moving with constant speed along circular path :

(1) Is zero

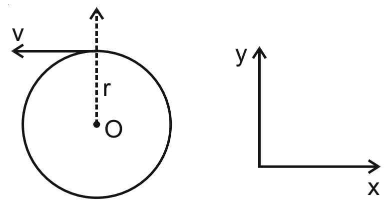
(2) Changes in magnitude but remains same in the direction

(3) Remains same in magnitude and direction

(4) Remains same in magnitude but changes in the direction

Answer (3)

Sol.



$\vec{L}_0 = \vec{r} \times \vec{p}$

$= mvr(\hat{k})$

It does not change in direction and magnitude.

19. In an ac circuit, an inductor, a capacitor and a resistor are connected in series with $X_L = R = X_C$. Impedance of this circuit is :

(1) $R\sqrt{2}$ (2) $2R^2$

(3) R (4) Zero

Answer (3)

Sol. $Z = \sqrt{(X_L - X_C)^2 + R^2}$

$X_L = X_C$

$\Rightarrow Z = R$

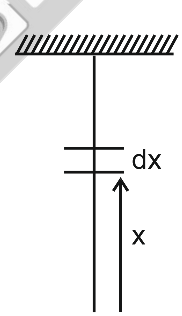
20. A uniform heavy rod of weight 10 kg ms^{-2}, cross-sectional area 100 cm^2 and length 20 cm is hanging from a fixed support. Young's modulus of the material of the rod is 2×10^{11} Nm^{-2}. Neglecting the lateral contraction, find the elongation of rod due to its own weight :

(1) 5×10^{-8} m (2) 5×10^{-10} m

(3) 2×10^{-9} m (4) 4×10^{-8} m

Answer (2)

Sol.



Tension in the rod at distance x is, $T = \frac{m}{l}xg$

$YA\frac{dy}{dx} = T$

$dy = \frac{T\,dx}{YA}$

$\int dy = \int \frac{mg}{AY} x dx$

$y = \frac{mgl}{2AY}$

$= \frac{10 \times 0.20}{2 \times 100 \times 10^{-4} \times 2 \times 10^{11}}$

$= 0.50 \times 10^{-9}$

$= 5 \times 10^{-10}$

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. A particle of mass 1 kg is hanging from a spring of force constant 100 Nm^{-1}. The mass is pulled slightly downward and released so that it executes free simple harmonic motion with time period T. The time when the kinetic energy and potential energy of the system will become equal, is $\frac{T}{x}$. The value of x is______.

Answer (8)

Sol. K.E = P.E.

$\Rightarrow \quad x = \frac{A}{\sqrt{2}}$

$x = A\cos\frac{2\pi}{T}\cdot t = \frac{A}{\sqrt{2}}$

$\Rightarrow \quad t = \frac{T}{8}$

2. A block moving horizontally on a smooth surface with a speed of 40 ms^{-1} splits into two equal parts. If one of the parts moves at 60 ms^{-1} in the same direction, then the fractional change in the kinetic energy will be x : 4 where x = _____.

Answer (1)

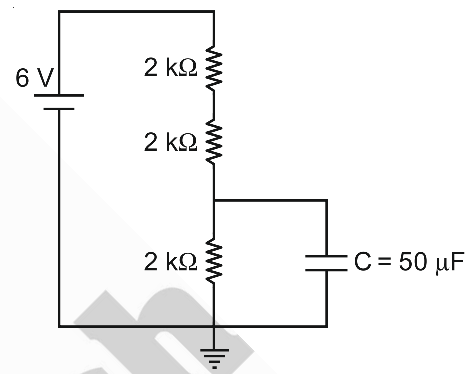
Sol. $m \cdot 40 = \frac{m}{2}\cdot 60 + \frac{m}{2} V$

$\Rightarrow$ V = 20 m/s

$f = \frac{\frac{1}{2}\cdot\frac{m}{2}(60^2 + 20^2) - \frac{1}{2}m\,40^2}{\frac{1}{2}m\,40^2}$

$= \frac{1}{4}$

3. A capacitor of 50 μF is connected in a circuit as shown in figure. The charge on the upper plate of the capacitor is _______ μC.



Answer (100)

Sol. In steady state

$i = \frac{V}{R_{eq}} = 10^{-3}$ ampere

$Q = CV_C = 50 \times 10^{-3} \times 2 \times 10^3$ μC

= 100 μC

4. If the sum of the heights of transmitting and receiving antennas in the line of sight of communication is fixed at 160 m, then the maximum range of LOS communication is ___ km. (Take radius of Earth = 6400 km)

Answer (64)

Sol. $d = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

$h_T + h_R = 160 \text{ m} = h_o$

$d_{max} = 2\sqrt{Rh_o}$ for $h_T = h_R = \frac{h_o}{2}$

= 64 km

5. A square shaped wire with resistance of each side 3 Ω is bent to form a complete circle. The resistance between two diametrically opposite points of the circle in unit of Ω will be ____

Answer (3)

Sol. $R_{eq} = \dfrac{\frac{R}{2}\frac{R}{2}}{\frac{R}{2}+\frac{R}{2}} = \dfrac{R}{4} = 3\ \Omega$

where R $\rightarrow$ resistance of complete wire.

6. The electric field in an electromagnetic wave is given by E = (50 NC^{-1}) sinω (t - x/c)

The energy contained in a cylinder of volume V is 5.5 × 10^{-12} J. The value of V is ___ cm^3.

(given ε_0 = 8.8 × 10^{-12} C^2N^{-1} m^{-2})

Answer (500)

Sol. Energy U = Energy density × V

$$5.5\times10^{-12} = \frac{1}{2}\varepsilon_0 E_0^2 \times V$$

$$V = \frac{5.5\times10^{-12}\times2}{8.8\times10^{-12}\times2500} m^3 = 500\ cm^3$$

7. A wire having a linear mass density 9.0 ×10^{-4} kg/m is stretched between two rigid supports with a tension of 900 N. The wire resonates at a frequeucy of 500 Hz. The next higher frequency at which the same wire resonates is 550 Hz. The length of the wire is ___ m.

Answer (10)

Sol. μ = 9 × 10^{-4} kg/m

T = 900 N

v = 10^3 m/s

f_n = 500 Hz, f_{n+1} = 550 Hz

Fundamental frequency = 50 Hz

$\Rightarrow$ λ = 20 m

ℓ = 10 m

8. When a rubber ball is taken to a depth of _____ m in deep sea, its volume decreases by 0.5%.

(The bulk modulus of rubber = 9.8 × 10^8 Nm^{-2},

Density of sea water = 10^3 kgm^{-3},

g = 9.8 m/s^2)

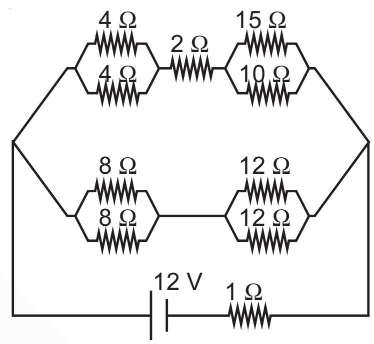
Answer (500)

Sol. $B = -\dfrac{VdP}{dV} = -\dfrac{V}{dV}(\rho gh)$

$$h = \frac{9.8\times10^8\times0.005}{10^3\times9.8}$$

= 500 m

9. The voltage drop across 15 Ω resistance in the given figure will be _____ V.



Answer (6)

Sol. Current through cell = 2A

$$V_{15\Omega} = \frac{10\times6}{2+2+6} = 6\ V$$

10. A car is moving on a plane inclined at 30° to the horizontal with an acceleration of 10 ms^{-2} parallel to the plane upward. A bob is suspended by a string from the roof of the car. The angle in degrees which the string makes with the vertical is ______.

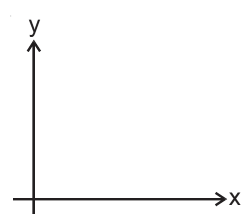
(Take g = 10 ms^{-2})

Answer (30)

Sol. Acceleration of point of suspension

$= 10\cos30^\circ\hat{i} + 10\sin30^\circ\hat{j}$

$= 5\sqrt{3}\hat{i} + 5\hat{j}$



$g_{eff} = -\left[\left(5\sqrt{3}\hat{i}+5\hat{j}\right)+10\hat{j}\right]$

$= -\left(5\sqrt{3}\hat{i}+15\hat{j}\right)$

$\theta = \tan^{-1}\left(\dfrac{5\sqrt{3}}{15}\right) = 30^\circ$

PART–B : CHEMISTRY

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

   Assertion (A) : Metallic character decreases and non-metallic character increases on moving from left to right in a period.

   Reason (R) : It is due to increase in ionisation enthalpy and decrease in electron gain enthalpy, when one moves from left to right in a period.

   In the light of the above statements, choose the **most appropriate** answer from the options given below :

   (1) Both (A) and (R) are correct and (R) is the correct explanation of (A)

   (2) (A) is true but (R) is false

   (3) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

   (4) (A) is false but (R) is true

Answer (2)

Sol. Metallic character decreases on moving left to right and non-metallic character increases, it is due to increase in ionisation enthalpy. But electron gain enthalpy also increases from left to right.

Reason is not correct.

2. BOD values (in ppm) for clean water (A) and polluted water (B) are expected respectively as :

   (1) A > 25, B < 17

   (2) A > 50, B < 27

   (3) A > 15, B > 47

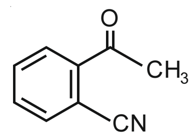
   (4) A < 5, B > 17

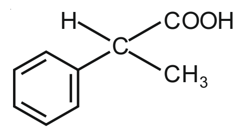
Answer (4)

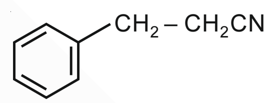
Sol. For clean water, BOD should be less than 5 ppm and for highly polluted water, it is more than 17 ppm.

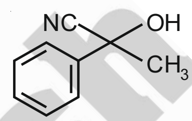
3. The structure of product C, formed by the following sequence of reactions is :

$$CH_3COOH + SOCl_2 \longrightarrow A \xrightarrow[AlCl_3]{\text{Benzene}} B \xrightarrow[^{-}OH]{KCN} C$$

(1) 

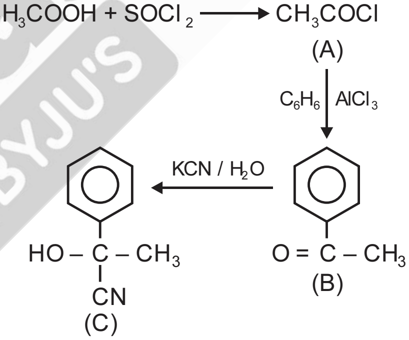
(2) 

(3) 

(4) 

Answer (4)

Sol. $CH_3COOH + SOCl_2 \longrightarrow CH_3COCl$



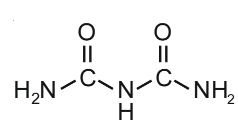
4. The denticity of an organic ligand, biuret is :

   (1) 3 (2) 2

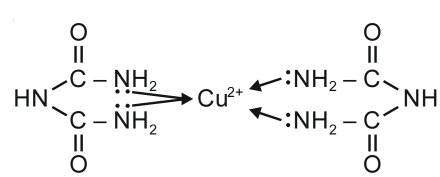
   (3) 4 (4) 6

Answer (2)

Sol. Biuret ligand is



It forms complexes like



5. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : A simple distillation can be used to separate a mixture of propanol and propanone.

Reason (R) : Two liquids with a difference of more than 20°C in their boiling points can be separated by simple distillations.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) (A) is false but (R) is true

(2) (A) is true but (R) is false

(3) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(4) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

Answer (3)

Sol.
- Simple distillation can be used to separate a mixture of propanol and propanone.
- Two liquid with difference in B.P. around 20°C can be separated by simple distillation.
- B.P. of propanol is 370 K.

  B.P. of acetone is 329 K.

6. The major component/ingredient of Portland Cement is :

(1) Tricalcium aluminate

(2) Dicalcium silicate

(3) Dicalcium aluminate

(4) Tricalcium silicate

Answer (4)

Sol. The important ingredients present in Portland Cement are dicalcium silicate (26%), tricalcium silicate (51%), tricalcium aluminate (11%)

7. Given below are two statements :

Statement I : The process of producing syn-gas is called gasification of coal.

Statement II : The composition of syn-gas is $CO + CO_2 + H_2$ (1 : 1 : 1).

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) **Statement I** is true but **Statement II** is false

(2) Both **Statement I** and **Statement II** are true

(3) Both **Statement I** and **Statement II** are false

(4) **Statement I** is false but **Statement II** is true

Answer (1)

Sol.
- The process of producing syn - gas is called 'coal gasification' — statement 1 is true
- Composition of syn - gas is $CO : H_2$ as 1 : 1

8. In the structure of the dichromate ion, there is a :

(1) Linear symmetrical Cr – O – Cr bond

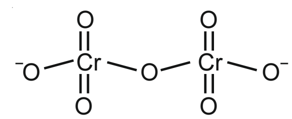
(2) Non-linear symmetrical Cr – O – Cr bond

(3) Linear unsymmetrical Cr – O – Cr bond

(4) Non-linear unsymmetrical Cr – O – Cr bond

Answer (2)

Sol. Structure of dichromate is



Structure is non — linear with symmetrical Cr – O – Cr bond

9. Monomer of Novolac is :

(1) o-Hydroxymethylphenol

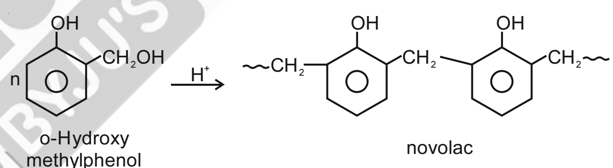
(2) 1,3-Butadiene and styrene

(3) Phenol and melamine

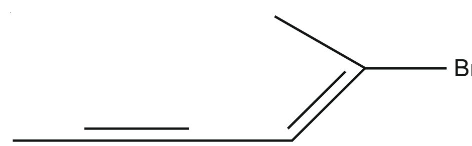
(4) 3-Hydroxybutanoic acid

Answer (1)

Sol.



10. Choose the **correct** name for compound given below:



(1) (4E)-5-Bromo-hex-2-en-4-yne

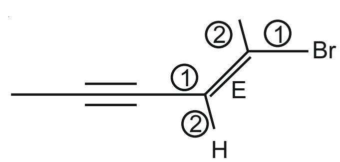
(2) (2E)-2-Bromo-hex-4-yn-2-ene

(3) (2E)-2-Bromo-hex-2-en-4-yne

(4) (4E)-5-Bromo-hex-4-en-2-yne

Answer (3)

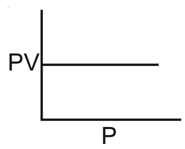
Sol.

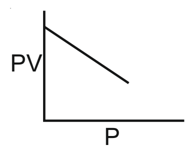


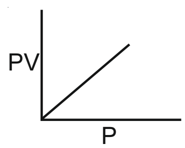
First prioritise the groups according to CIP rule

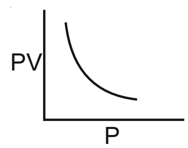
It is (2E)-2-Bromo-hex-2-en-4-yne

11. Which one of the following is the correct PV vs P plot at constant temperature for an ideal gas? (P and V stand for pressure and volume of the gas respectively)

(1) 

(2) 

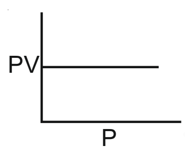
(3) 

(4) 

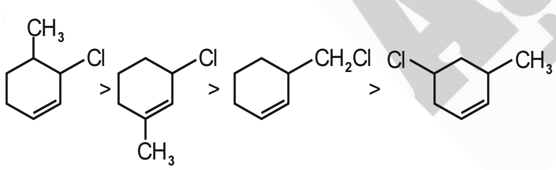
Answer (1)

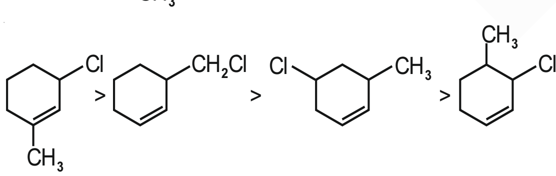
Sol. According to Boyle's law $P \propto \frac{1}{V}$ (at const. n, T)

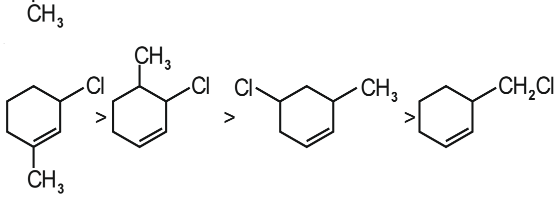
PV = constant

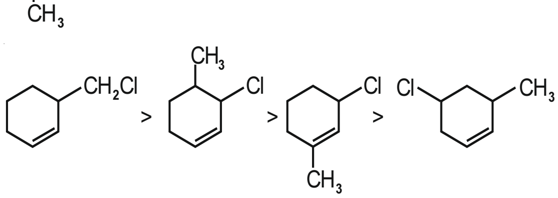
∴ 

12. The correct order of reactivity of the given chlorides with acetate in acetic acid is :

(1) 

(2) 

(3) 

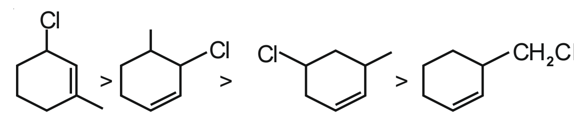
(4) 

Answer (3)

Sol. Acetate in acetic acid with R-X cause S_N1

- Allylic halides undergo faster S_N1

∴ Correct order is



13. Which one of the following lanthanides exhibits +2 oxidation state with diamagnetic nature? (Given Z for Nd = 60, Yb = 70, La = 57, Ce = 58)

(1) Nd (2) Yb

(3) La (4) Ce

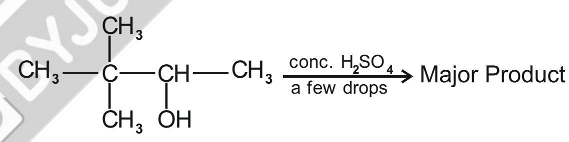
Answer (2)

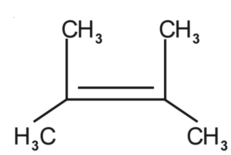
Sol. Yb (70) = $4f^{14}\ 6s^2$

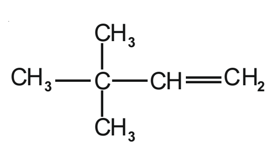
$Yb^{+2} = 4f^{14}\ 6s^0$

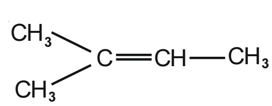
∵ All the electrons are paired hence Yb^{+2} is diamagnetic

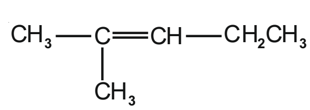
14. The major product formed in the following reaction is :



(1) 

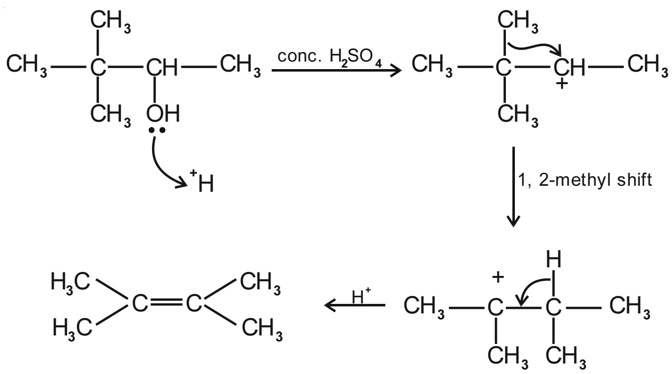
(2) 

(3) 

(4) 

Answer (1)

Sol.



15. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : Aluminium is extracted from bauxite by the electrolysis of molten mixture of Al_2O_3 with cryolite.

Reason (R) : The oxidation state of Al in cryolite is +3.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) (A) is true but (R) is false

(2) (A) is false but (R) is true

(3) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(4) Both (A) and (R) are correct and (R) is the correct explanation of (A)

Answer (3)

Sol. (A) is correct – Aluminium is extracted from bauxite by electrolysis of molten mixture of Al_2O_3 with cryolite

- Statement given in (R) is correct i.e, oxidation state of Al in cryolite (Na_3AlF_6) is +3 but is not the correct reason

16. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : Treatment of bromine water with propene yields 1-bromopropan-2-ol.

Reason (R) : Attack of water on bromonium ion follows Markovnikov rule and results in 1-bromopropan-2-ol.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) Both **(A)** and (R) are true and **(R)** is the correct explanation of (A)

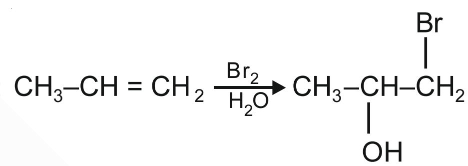
(2) **(A)** is true but **(R)** is false

(3) Both **(A)** and **(R)** are true but **(R)** is NOT the correct explanation of (A)

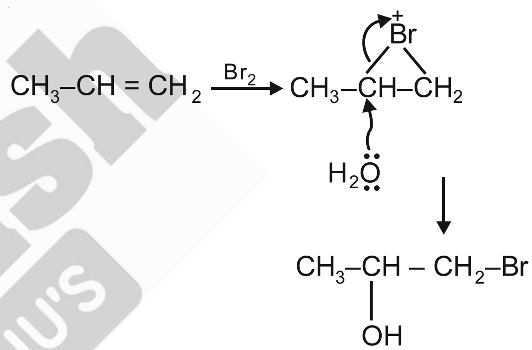
(4) **(A)** is false but **(R)** is true

Answer (1)

Sol. (A) :



(R) :



17. Which one of the following compounds contains β–C_1–C_4 glycosidic linkage?

(1) Maltose

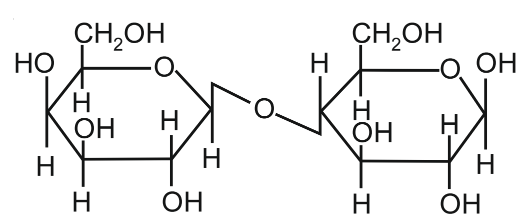
(2) Lactose

(3) Sucrose

(4) Amylose

Answer (2)

Sol. Lactose is



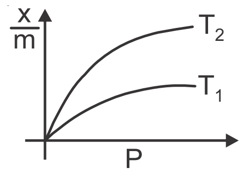
it has β–C_1–C_4 linkage

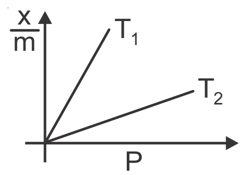
18. Select the graph that correctly describes the adsorption isotherms at two temperatures T_1 and T_2 ($T_1 > T_2$) for a gas :

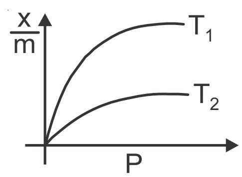
(x – mass of the gas adsorbed

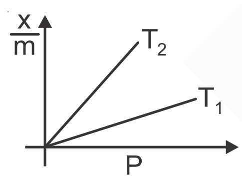
m – mass of adsorbent

P – pressure)

(1) 

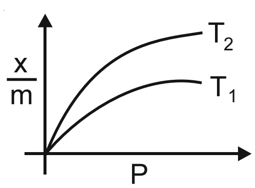
(2) 

(3) 

(4) 

Answer (1)

Sol. Extent of adsorption decreases with the increase in temperature



$[T_2 < T_1]$

19. Which one of the following 0.10 M aqueous solutions will exhibit the largest freezing point depression?

(1) glycine

(2) $KHSO_4$

(3) hydrazine

(4) glucose

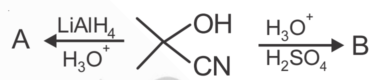
Answer (2)

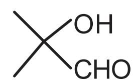
Sol. $\Delta T_f \propto i \times m$

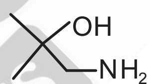
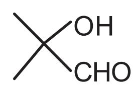
greater the value of i, greater will be the ΔT_f value.

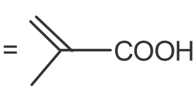
Solute	**i**
glycine	1
$KHSO_4$	3
hydrazine	1
glucose	1

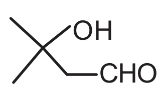
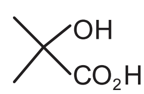
20. The major products A and B in the following set of reactions are



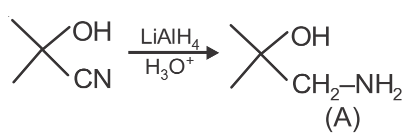
(1) A =  , B = 

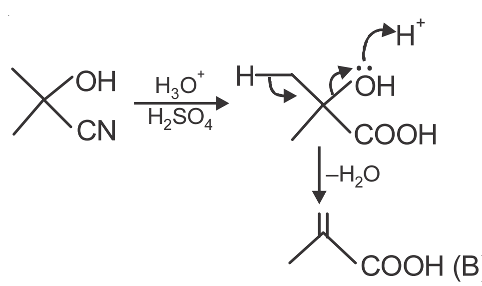
(2) A =  , B = 

(3) A =  , B = 

(4) A =  , B = 

Answer (3)

Sol. 



SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. The number of hydrogen bonded water molecule(s) associated with stoichiometry $CuSO_4 \cdot 5H_2O$ is ______.

Answer (1)

Sol. The number of hydrogen bonded water molecule associated with stoichiometry in $CuSO_4 \cdot 5H_2O$ is 1.

2. For a first order reaction, the ratio of the time for 75% completion of a reaction to the time for 50% completion is _______. (Integer answer)

Answer (2)

Sol. For first order reaction

$t_{75\%} = 2 \times t_{50\%}$

$\because \quad kt = \ln\dfrac{A_0}{A}$ (for first order reaction)

At $t_{50\%} \Rightarrow A = \dfrac{A_0}{2}$

$\therefore \quad kt_{50\%} = \ln 2$

At $t_{75\%} \Rightarrow A = \dfrac{A_0}{4}$

$\therefore \quad kt_{75\%} = \ln 4$

3. The molarity of the solution prepared by dissolving 6.3 g of oxalic acid ($H_2C_2O_4 \cdot 2H_2O$) in 250 mL of water in mol L^{-1} is $x \times 10^{-2}$. The value of x is _______. (Nearest integer)

[Atomic mass : H : 1.0, C : 12.0, O : 16.0]

Answer (20)

Sol. Molar mass of oxalic acid $H_2C_2O_4 \cdot 2H_2O$ = 126 g/mol

$$\text{Molarity} = \frac{\text{Number of moles of solute}}{\text{Vol. of solution (in L)}}$$

$$= \frac{6.3 \times 1000}{126 \times 250}$$

$$= 0.2 \text{ molar}$$

4. Consider the sulphides HgS, PbS, CuS, Sb_2S_3, As_2S_3 and CdS. Number of these sulphides soluble in 50% HNO_3 is_______.

Answer (4)

Sol. Except HgS and Sb_2S_3 rest of the compounds are soluble in 50% HNO_3

5. Consider the following cell reaction

$$Cd_{(s)} + Hg_2SO_{4(s)} + \frac{9}{5}H_2O_{(l)} \rightleftharpoons CdSO_4 \cdot \frac{9}{5}H_2O_{(s)} + 2Hg_{(l)}.$$

The value of E^0_{cell} is 4.315 V at 25°C. If $\Delta H^o = -825.2$ kJ mol^{-1}, the standard entropy change ΔS^o in J K^{-1} is _______. (Nearest integer)

[Given : Faraday constant = 96487 C mol^{-1}]

Answer (25)

Sol. $\Delta G = -nFE^o_{cell} = \Delta H - T\Delta S$

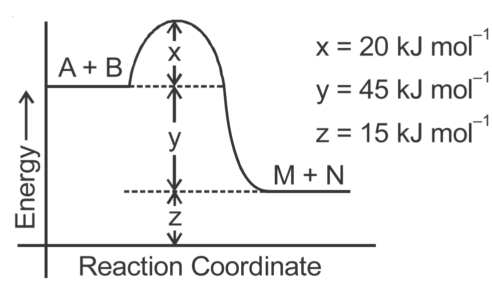
$$= \frac{-2 \times 96487 \times 4.315 + 825.2 \times 10^3}{298} = -\Delta S$$

$$= \Delta S \approx 25 \text{ JK}^{-1}$$

6. According to the following figure, the magnitude of the enthalpy change of the reaction

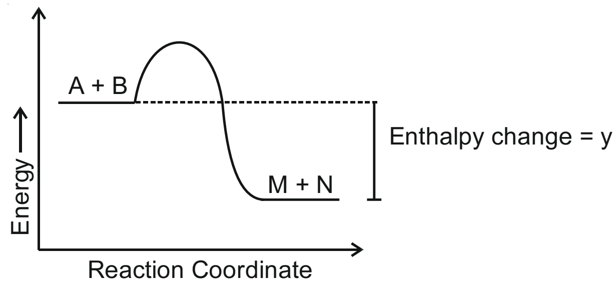
$A + B \rightarrow M + N$ in kJ mol^{-1}

is equal to _____. (Integer answer)



Answer (45)

Sol.



y = 45 kJ/mol

7. The total number of reagents from those given below, that can convert nitrobenzene into aniline is _____. (Integer answer)

I. Sn – HCl

II. Sn – NH_4OH

III. Fe – HCl

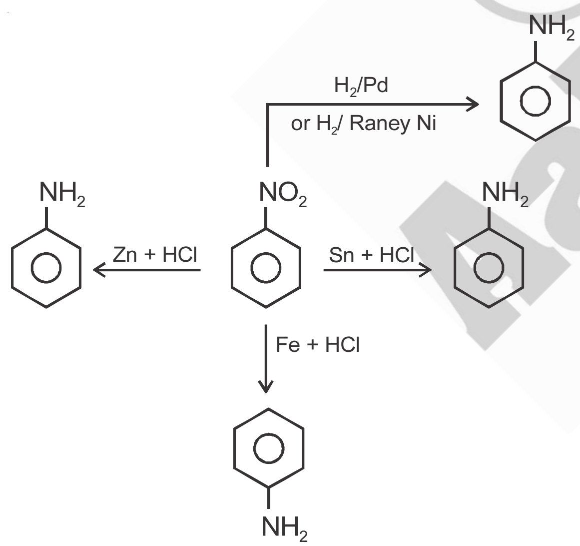
IV. Zn – HCl

V. H_2 – Pd

VI. H_2 – Raney Nickel

Answer (5)

Sol.



8. Ge (Z = 32) in its ground state electronic configuration has x completely filled orbitals with $m_l = 0$. The value of x is _____.

Answer (7)

Sol. Ge (32) = $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6$
$4s^2\ 3d^{10}\ 4p^2$

$m_l = 0$ $1s, 2s, 2p_z, 3s, 3p_z, 4s, 3d_{z^2}$

9. The number of halogen/(s) forming halic (V) acid is _____.

Answer (3)

Sol. Except F and At, all other halide can form Halic (V) acid.

- F cannot go in +5 oxidation state.
- At is radioactive.

10. A_3B_2 is a sparingly soluble salt of molar mass M (g mol^{-1}) and solubility x g L^{-1}. The solubility product satisfies $K_{sp} = a\left(\frac{x}{M}\right)^5$. The value of a is _____. (Integer answer)

Answer (108)

Sol. $A_3B_2 \rightleftharpoons 3A^{2+} + 2B^{3-}$
$\quad\quad\quad\quad \frac{3x}{M} \quad \frac{2x}{M}$

$$K_{sp} = \left(\frac{3x}{M}\right)^3 \times \left(\frac{2x}{M}\right)^2 = 108\left(\frac{x}{M}\right)^5$$

a = 108

PART–C : MATHEMATICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The number of real roots of the equation $e^{4x} + 2e^{3x} - e^x - 6 = 0$ is

   (1) 1 (2) 2

   (3) 4 (4) 0

Answer (1)

Sol. Let $e^x = t$

and $f(t) = t^4 + 2t^3 - t$ $(t > 0)$

$f'(t) = 4t^3 + 6t^2 - 1$

$f''(t) = 12t^2 + 12t > 0$ always as $t > 0$

$\therefore$ $f'(t)$ has only 1 root

Also $f'(0) = -1$ and $f'(-1) = 1$

$\Rightarrow$ $f'(t)$ vanishes between $(-1, 0)$

(but $t > 0$ $f'(t)$ has no root in given situation)

$\therefore$ $f(t)$ is always increasing if $t > 0$

$f(t) = 6$ will intersects only once

Hence 1 solution

2. cosec18° is a root of the equation

   (1) $x^2 - 2x + 4 = 0$

   (2) $x^2 + 2x - 4 = 0$

   (3) $x^2 - 2x - 4 = 0$

   (4) $4x^2 + 2x - 1 = 0$

Answer (3)

Sol. We know that $\text{cosec}18^\circ = \frac{4}{\sqrt{5}-1}$

As equation is with real coefficients other root will be $\frac{4(-\sqrt{5}+1)}{4} = -\sqrt{5}+1$

$\therefore$ Sum of root $\sqrt{5}+1-\sqrt{5}+1 = 2$

Product of roots = 1 – 5 = –4

$\therefore$ Equation is $x^2 - 2x - 4 = 0$

3. The line $12x\cos\theta + 5y\sin\theta = 60$ is tangent to which of the following curves?

   (1) $25x^2 + 12y^2 = 3600$

   (2) $144x^2 + 25y^2 = 3600$

   (3) $x^2 + y^2 = 169$

   (4) $x^2 + y^2 = 60$

Answer (2)

Sol. $x\left(\frac{\cos\theta}{5}\right) + y\left(\frac{\sin\theta}{12}\right) = 1$ ...(i)

Let tangent to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

i.e., $\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$ ...(ii)

Comparing (i) and (ii) we get $a = 5$ & $b = 12$

Curve is $\frac{x^2}{25} + \frac{y^2}{144} = 1$

$\Rightarrow$ $144x^2 + 25y^2 = 3600$

4. Three numbers are in an increasing geometric progression with common ratio r. If the middle number is doubled, then the new numbers are in an arithmetic progression with common difference d. If the fourth term of GP is $3r^2$, then $r^2 - d$ is equal to

   (1) $7-7\sqrt{3}$ (2) $7+\sqrt{3}$

   (3) $7-\sqrt{3}$ (4) $7+3\sqrt{3}$

Answer (2)

Sol. Let number be a, ar, ar^2

$\therefore$ $d = 2ar - a = ar^2 - 2ar$ ...(i)

and $ar^3 = 3r^2 \Rightarrow r = \frac{3}{a}$ ...(ii)

$\Rightarrow \frac{9}{a^2} + 1 = \frac{12}{a}$ (by (i) and (ii))

Put $\frac{1}{a} = t$

$\Rightarrow 9t^2 - 12t + 1 = 0$

$t = \frac{2\pm\sqrt{3}}{3} = \frac{1}{a} \Rightarrow \frac{3}{a} = r = 2\pm\sqrt{3}$

As G.P. is increasing $r = 2+\sqrt{3}, a = 3(2-\sqrt{3})$

$d = 6 - a = 6 - 6 + 3\sqrt{3}$

$\therefore$ $r^2 - d = 4 + 3 + 4\sqrt{3} - (3\sqrt{3})$

$= 7 + \sqrt{3}$

5. The sum of 10 terms of the series

$$\frac{3}{1^2\times 2^2}+\frac{5}{2^2\times 3^2}+\frac{7}{3^2\times 4^2}+.... \text{ is}$$

(1) $\frac{143}{144}$ (2) $\frac{99}{100}$

(3) $\frac{120}{121}$ (4) 1

Answer (3)

Sol. $T_n=\frac{2n+1}{n^2(n+1)^2}=\frac{(n+1)^2-n^2}{(n+1)^2n^2}$

$$S_n=\sum_{n=1}^{10}T_n=\sum_{n=1}^{10}\left(\frac{1}{n^2}-\frac{1}{(n+1)^2}\right)$$

$$=\left(1-\frac{1}{2^2}\right)+\left(\frac{1}{2^2}-\frac{1}{3^2}\right)+....+\left(\frac{1}{10^2}-\frac{1}{11^2}\right)$$

$$=1-\frac{1}{(11)^2}=\frac{120}{121}$$

6. If the following system of linear equations

$2x + y + z = 5$

$x - y + z = 3$

$x + y + az = b$

has no solution, then :

(1) $a=-\frac{1}{3}, b\neq\frac{7}{3}$

(2) $a\neq-\frac{1}{3}, b=\frac{7}{3}$

(3) $a\neq\frac{1}{3}, b=\frac{7}{3}$

(4) $a=\frac{1}{3}, b\neq\frac{7}{3}$

Answer (4)

Sol. $\Delta=\begin{vmatrix}2&1&1\\1&-1&1\\1&1&a\end{vmatrix}=1-3a$

$$\Delta_3=\begin{vmatrix}2&1&5\\1&-1&3\\1&1&b\end{vmatrix}=7-3b$$

For no solution, $\Delta = 0$, $\Delta_3 \neq 0$

$1 - 3a = 0$, $7 - 3b \neq 0$

$a=\frac{1}{3},\ b\neq\frac{7}{3}$

7. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|2\vec{a}+3\vec{b}|=|3\vec{a}+3\vec{b}|$ and the angle between $\vec{a}$ and $\vec{b}$ is 60°. If $\frac{1}{8}\vec{a}$ is a unit vector, then $|\vec{b}|$ is equal to :

(1) 8 (2) 4

(3) 5 (4) 6

Answer (3)

Sol. $\left|\frac{\vec{a}}{8}\right|=1\Rightarrow|\vec{a}|=8$

$$|2\vec{a}+3\vec{b}|^2=|3\vec{a}+3\vec{b}|^2$$

$$\therefore\quad 4|\vec{a}|^2+9|\vec{b}|^2+12\vec{a}.\vec{b}=9|\vec{a}|^2+|\vec{b}|^2+6\vec{a}\cdot\vec{b}$$

$$8|\vec{b}|^2+6|\vec{a}\cdot\vec{b}|-5|\vec{a}|^2=0$$

$$8|\vec{b}|^2+6|\vec{a}|\cdot|\vec{b}|\cos 60-5\times 64=0$$

$$8|\vec{b}|^2+24|\vec{b}|-320=0$$

$$|\vec{b}|^2+3|\vec{b}|-40=0$$

$$\left(|\vec{b}|+8\right)\left(|\vec{b}|-5\right)=0$$

$|\vec{b}|=5$, ($|\vec{b}|=-8$ rejected)

8. Which of the following is **not** correct for relation R on the set of real numbers?

(1) $(x, y) \in R \Leftrightarrow |x - y| \leq 1$ is reflexive and symmetric.

(2) $(x, y) \in R \Leftrightarrow 0\ |x| - |y| \leq 1$ is neither transitive nor symmetric

(3) $(x, y) \in R \Leftrightarrow 0 < |x - y| \leq 1$ is symmetric and transitive

(4) $(x, y) \in R \Leftrightarrow |x| - |y| \leq 1$ is reflexive but not symmetric

Answer (3)

Sol. $(x, y) \in R \Leftrightarrow 0 < |x - y| \leq 1$.

R is symmetric because $|x - y| = |y - x|$

But R is not transitive

For example

$x = 0.2$, $y = 0.9$, $z = 1.5$

$0 \leq |x - y| = 0.7 \leq 1$

$0 \leq |y - z| = 0.6 \leq 1$

But $|x - z| = 1.3 > 1$

9. $\lim\limits_{x\to 0}\dfrac{\sin^2\left(\pi\cos^4 x\right)}{x^4}$ is equal to :

(1) 4π (2) π^2

(3) $4\pi^2$ (4) $2\pi^2$

Answer (3)

Sol. $\lim\limits_{x\to 0}\dfrac{\sin^2\left(\pi\cos^4 x\right)}{x^4}$

$= \lim\limits_{x\to 0}\dfrac{\sin^2\left(\pi-\pi\cos^4 x\right)}{x^4}$

$= \lim\limits_{x\to 0}\left(\dfrac{\sin\left(\pi\left(1-\cos^4 x\right)\right)}{x^2}\right)^2$

$= \lim\limits_{x\to 0}\left(\dfrac{\sin\left(\pi\sin^2 x\left(1+\cos^2 x\right)\right)}{x^2}\right)^2$

$= \lim\limits_{x\to 0}\left(\dfrac{\sin\left(\pi\sin^2 x\left(1+\cos^2 x\right)\right)}{\pi\sin^2 x\left(1+\cos^2 x\right)}\times\dfrac{\pi\sin^2 x\left(1+\cos^2 x\right)}{x^2}\right)^2$

$= \lim\limits_{x\to 0}\left(\pi\dfrac{\sin^2 x}{x^2}\left(1+\cos^2 x\right)\right)^2$

$= (\pi \times 1 \times (1 + 1))^2$

$= 4\pi^2$

10. Let $*, \square \in \{\wedge, \vee\}$ be such that the Boolean expression $(p * \sim q) \Rightarrow (p \,\square\, q)$ is a tautology. Then:

(1) $* = \wedge$, $\square = \wedge$ (2) $* = \vee$, $\square = \wedge$

(3) $* = \vee$, $\square = \vee$ (4) $* = \wedge$, $\square = \vee$

Answer (4)

Sol. $(p * \sim q) \Rightarrow (p \,\square\, q)$

$= \sim (p * \sim q) \vee (p \,\square\, q)$

$= (\sim p \,\square\, q) \vee (p \,\square\, q)$

$= T$

$\Rightarrow \square = \vee$

and $* = \sim \square = \wedge$

11. The function $f(x) = |x^2 - 2x - 3| \cdot e^{|9x^2-12x+4|}$ is not differentiable at exactly :

(1) One point (2) Four points

(3) Two points (4) Three points

Answer (3)

Sol. $f(x) = |(x + 1)(x - 3)| \cdot e^{(3x-2)^2}$

Clearly $f(x)$ is not differentiable at $x = -1$ and 3.

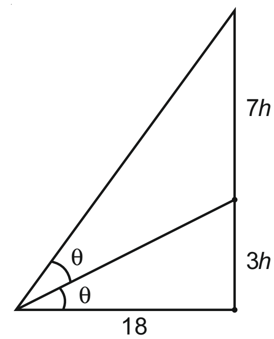
12. A vertical pole fixed to the horizontal ground is divided in the ratio 3 : 7 by a mark on it with lower part shorter than the upper part. If the two parts subtend equal angles at a point on the ground 18 m away from the base of the pole, then the height of the pole (in meters) is :

(1) $8\sqrt{10}$ (2) $12\sqrt{10}$

(3) $12\sqrt{15}$ (4) $6\sqrt{10}$

Answer (2)

Sol. $\tan\theta = \dfrac{h}{6}$



and $\tan 2\theta = \dfrac{5h}{9}$

$\because \dfrac{2\left(\frac{h}{6}\right)}{1-\frac{h^2}{36}} = \dfrac{5h}{9}$

$\Rightarrow h^2 = \dfrac{72}{5}$

Height of the pole $= 10h = 10\sqrt{\dfrac{72}{5}} = 12\sqrt{10}$

13. If $\dfrac{dy}{dx} = \dfrac{2^{x+y}-2^x}{2^y}, y(0) = 1,$ then $y(1)$ is equal to :

(1) $\log_2(2e)$ (2) $\log_2(1 + e^2)$

(3) $\log_2(1 + e)$ (4) $\log_2(2 + e)$

Answer (3)

Sol. $\dfrac{dy}{dx} = 2^x\left(\dfrac{2^y-1}{2^y}\right)$

$\Rightarrow \dfrac{2^y \ln 2\, dy}{2^y-1} = 2^x \ln 2\, dx$

$\Rightarrow \ln(2^y - 1) = 2^x + c$

put $x = 0$ $c = -1$

put $x = 1$ $\ln(2^y - 1) = 1$

$\Rightarrow 2^y - 1 = e$

$\Rightarrow y = \log_2(1 + e)$

14. If p and q are the lengths of the perpendiculars from the origin on the lines, $x\operatorname{cosec}\alpha - y\sec\alpha = k\cot 2\alpha$ and $x\sin\alpha + y\cos\alpha = k\sin 2\alpha$ respectively, then k^2 is equal to :

(1) $2p^2 + q^2$ (2) $p^2 + 4q^2$

(3) $4p^2 + q^2$ (4) $p^2 + 2q^2$

Answer (3)

Sol. $p = \left|\dfrac{k\cot 2\alpha}{\sqrt{\sec^2\alpha + \operatorname{cosec}^2\alpha}}\right| \Rightarrow p^2 = \dfrac{k^2}{4}\cos^2 2\alpha$

and $q = \left|\dfrac{k\sin 2\alpha}{\sqrt{\sin^2\alpha + \cos^2\alpha}}\right| \Rightarrow q^2 = k^2\sin^2 2\alpha$

So, $4p^2 + q^2 = k^2$

15. If the function $f(x) = \begin{cases} \dfrac{1}{x}\log_e\left(\dfrac{1+\frac{x}{a}}{1-\frac{x}{b}}\right) & , \quad x<0 \\ k & , \quad x = 0 \\ \dfrac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2+1}-1} & , \quad x>0 \end{cases}$

is continuous at x = 0, then $\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{4}{k}$ is equal to :

(1) 5

(2) 4

(3) –4

(4) –5

Answer (4)

Sol. $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^+} f(x) = f(0)$

$$\Rightarrow \lim_{x\to 0^-}\frac{\ln\left(1+\frac{x}{a}\right)-\ln\left(1-\frac{x}{b}\right)}{x} = \lim_{x\to 0^+}\frac{-2\sin^2 x}{\sqrt{x^2+1}-1} = k$$

$$\Rightarrow \frac{1}{a}+\frac{1}{b} = \lim_{x\to 0^+}\left(\frac{-2\sin^2 x}{x^2}\right)\left(\sqrt{x^2+1}+1\right) = k$$

$$\Rightarrow \frac{1}{a}+\frac{1}{b} = -4 = k$$

So, $\dfrac{1}{a}+\dfrac{1}{b}=\dfrac{4}{k} = -4-1 = -5$

16. The integral $\displaystyle\int\frac{1}{\sqrt[4]{(x-1)^3(x+2)^5}}dx$ is equal to :

(where C is a constant of integration)

(1) $\dfrac{3}{4}\left(\dfrac{x+2}{x-1}\right)^{\frac{5}{4}}+C$ (2) $\dfrac{4}{3}\left(\dfrac{x-1}{x+2}\right)^{\frac{5}{4}}+C$

(3) $\dfrac{3}{4}\left(\dfrac{x+2}{x-1}\right)^{\frac{1}{4}}+C$ (4) $\dfrac{4}{3}\left(\dfrac{x-1}{x+2}\right)^{\frac{1}{4}}+C$

Answer (4)

Sol. $I = \displaystyle\int\frac{1}{\sqrt[4]{(x-1)^3(x+2)^5}}dx$

$$= \int\frac{dx}{(x+2)^2\left(\frac{x-1}{x+2}\right)^{3/4}}$$

Let $\dfrac{x-1}{x+2} = t \Rightarrow \dfrac{3dx}{(x+2)^2} = dt$

$\therefore \quad I = \displaystyle\int\frac{1}{t^{3/4}}\cdot\frac{1}{3}dt$

$$= \frac{1}{3}\left(\frac{t^{-3/4+1}}{-\frac{3}{4}+1}\right)+C$$

$$= \frac{4}{3}\left(t^{1/4}\right)+C$$

$\therefore \quad I = \dfrac{4}{3}\left(\dfrac{x-1}{x+2}\right)^{\frac{1}{4}}+C$

17. Let the equation of the plane, that passes through the point (1, 4, –3) and contains the line of intersection of the planes $3x - 2y + 4z - 7 = 0$ and $x + 5y - 2z + 9 = 0$, be $\alpha x + \beta y + \gamma z + 3 = 0$, then $\alpha + \beta + \gamma$ is equal to :

(1) –23

(2) –15

(3) 23

(4) 15

Answer (1)

Sol. Let the equation of required plane is :

$(3x - 2y + 4z - 7) + \lambda(x + 5y - 2z + 9) = 0$

$\therefore \quad (3+\lambda)x + (-2+5\lambda)y + (4-2\lambda)z + (9\lambda - 7) = 0$

$\because \quad$ This plane passing through point (1, 4, –3), we get

$3 + \lambda - 8 + 20\lambda - 12 + 6\lambda + 9\lambda - 7 = 0$

$36\lambda - 24 = 0 \Rightarrow \lambda = \dfrac{2}{3}$

$\therefore \quad \dfrac{11}{3}x + \dfrac{4}{3}y + \dfrac{8}{3}z - 1 = 0$

$\therefore \quad -11x - 4y - 8z + 3 = 0$

On comparing with $\alpha x + \beta y + \gamma z + 3 = 0$, we get

$\alpha + \beta + \gamma = -23$

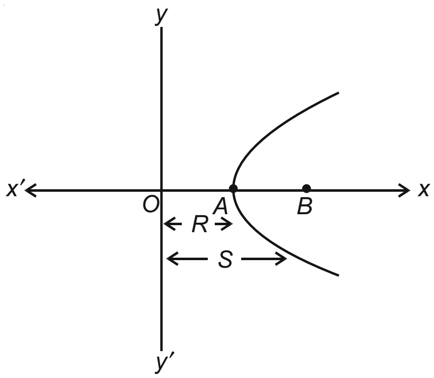
18. The length of the latus rectum of a parabola, whose vertex and focus are on the positive x-axis at a distance *R* and *S* (>*R*) respectively from origin, is

(1) $2(S - R)$ (2) $4(S - R)$

(3) $2(S + R)$ (4) $4(S + R)$

Answer (2)

Sol.



$\therefore$ $OA = R$ and $OB = S$

$\therefore$ $AB = a = S - R$

$\therefore$ Length of latus rectum = $4(S - R)$

19. If $a_r = \cos\frac{2r\pi}{9} + i\sin\frac{2r\pi}{9}, r = 1, 2, 3...., i = \sqrt{-1}$, then the determinant $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is equal to :

(1) $a_1a_9 - a_3a_7$

(2) $a_2a_6 - a_4a_8$

(3) a_9

(4) a_5

Answer (1)

Sol. $a_r = \cos\frac{2r\pi}{9} + i\sin\frac{2r\pi}{9} = e^{i\frac{2r\pi}{9}}$ where r = 1, 2, 3...

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} = \begin{vmatrix} e^{i\frac{2\pi}{9}} & e^{i\frac{4\pi}{9}} & e^{i\frac{6\pi}{9}} \\ e^{i\frac{8\pi}{9}} & e^{i\frac{10\pi}{9}} & e^{i\frac{12\pi}{9}} \\ e^{i\frac{14\pi}{9}} & e^{i\frac{16\pi}{9}} & e^{i\frac{18\pi}{9}} \end{vmatrix}$$

$$= e^{i\left(\frac{2\pi}{9}+\frac{8\pi}{9}+\frac{14\pi}{9}\right)} \begin{vmatrix} 1 & e^{i\frac{2\pi}{9}} & e^{i\frac{4\pi}{9}} \\ 1 & e^{i\frac{2\pi}{9}} & e^{i\frac{4\pi}{9}} \\ 1 & e^{i\frac{2\pi}{9}} & e^{i\frac{18\pi}{9}} \end{vmatrix} = 0$$

Now, $a_1 \cdot a_9 - a_3 \cdot a_7 = e^{i\frac{2\pi}{9}} \cdot e^{i\frac{18\pi}{9}} - e^{i\frac{6\pi}{9}} \cdot e^{i\frac{14\pi}{9}}$

$= e^{i\frac{20\pi}{9}} - e^{i\frac{20\pi}{9}}$

$= 0$

20. Let *f* be a non-negative function in [0, 1] and twice differentiable in (0, 1).

If $\int_0^x \sqrt{1-(f'(t))^2}\, dt = \int_0^x f(t)dt,\ 0 \le x \le 1$ and $f(0) = 0$,

then $\lim\limits_{x\to\infty} \frac{1}{x^2}\int_0^x f(t)dt$

(1) Equals 1

(2) Does not exist

(3) Equals 0

(4) Equals $\frac{1}{2}$

Answer (4)

Sol. $\int_0^x \sqrt{1-(f'(t))^2}\, dt = \int_0^x f(t)dt,\ x \in [0, 1]$

On differentiating both sides we get

$\sqrt{1-(f'(x))^2} = f(x)$

$1-(f(x))^2 = (f'(x))^2$

$\Rightarrow \frac{dy}{dx} = \sqrt{1-y^2}$

$\int \frac{dy}{\sqrt{1-y^2}} = \int dx \Rightarrow \sin^{-1} y = x + c$

$\because$ $f(0) = 0 \Rightarrow c = 0$

$\therefore$ $y = f(x) = \sin x$

$\therefore$ $\lim\limits_{x\to 0} \frac{\int_0^x f(t)dt}{x^2} = \lim\limits_{x\to 0} \frac{f(x)}{2x} = \lim\limits_{x\to 0} \frac{\sin x}{2x}$

$= \frac{1}{2}$

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. If the variable line $3x + 4y = \alpha$ lies between the two circles $(x-1)^2 + (y-1)^2 = 1$ and $(x-9)^2 + (y-1)^2 = 4$, without intercepting a chord on either circle, then the sum of all the integral values of α is_____.

Answer (165)

Sol. $C_1 \equiv (1, 1)$ and $r_1 = 1$

$C_1 \equiv (9, 1)$ and $r_2 = 2$

$L \equiv 3x + 4y - \alpha - 0$

Distance of line from C_1 should be greater than r_i $(i = 1, 2)$

$$\Rightarrow \left|\frac{7-\alpha}{5}\right| > 1$$

$$\Rightarrow |\alpha - 7| > 5$$

$$\Rightarrow \alpha \in (-\infty, 2) \cup (12, \infty) \quad \text{...(i)}$$

Also, $\left|\frac{27+4-\alpha}{5}\right| > 2 \Rightarrow |\alpha - 31| > 10$

$$\Rightarrow \alpha \in (-\infty, 21) \cup (41, \infty) \quad \text{...(ii)}$$

Further C_1 and C_2 should lie on opposite sides.

w.r.t. given lines

$$\Rightarrow (3+4-\alpha)\cdot(27+4-\alpha) < 0$$

$$\Rightarrow (\alpha - 7)(\alpha - 31) < 0$$

$$\Rightarrow \alpha \in (7, 31) \quad \text{...(iii)}$$

From (i), (ii) and (iii)

$\alpha \in (12, 21)$

Sum of all the integral values of α.

$= 12 + 13 + 14 \text{.......} + 21$

$$= \frac{21 \times 22}{2} - \frac{11 \times 12}{2}$$

$= 165$

2. The square of the distance of the point of intersection of the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+1}{6}$ and the plane $2x - y + z = 6$ from the point $(-1, -1, 2)$ is _____.

Answer (61)

Sol. Any point on line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+1}{6}$

is $P(2\lambda + 1, 3\lambda + 2, 6\lambda - 1)$

If P lies on the plane $2x - y + z = 6$

$\Rightarrow 2(2\lambda + 1) - (3\lambda + 2) + (6\lambda - 1) = 6$

$\Rightarrow \lambda = 1$

Hence point of intersection is $P(3, 5, 5)$

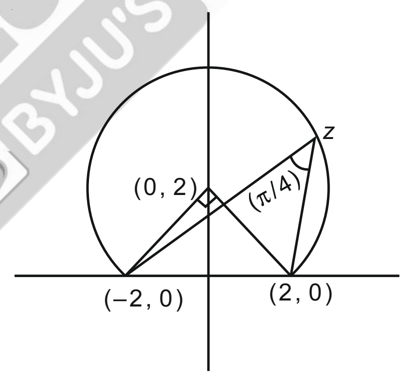
Distance of P from $(-1, -1, 2)$

$$= \left(\sqrt{16+36+9}\right)^2 = 61$$

3. A point z moves in the complex plane such that $\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{4}$, then the minimum value of $\left|z - 9\sqrt{2} - 2i\right|^2$ is equal to ______.

Answer (98)

Sol.



If $\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{4}$ then z lies on an arc of a circle as shown in figure.

Centre of this circle is $(0, 2)$ and radius $= 2\sqrt{2}$.

$\left|z - (9\sqrt{2} + 2i)\right| =$ Distance of z from $(9\sqrt{2}, 2)$.

Distance of $(9\sqrt{2}, 2)$ from centre $(0, 2)$

$= 9\sqrt{2}$

Minimum value of $\left|z - 9\sqrt{2} - 2i\right|^2$

is equal to $\left(9\sqrt{2} - 2\sqrt{2}\right)^2 = 98$

4. The mean of 10 numbers

7×8, 10×10, 13×12, 16×14, ... is _____.

Answer (398)

Sol. $S = 7\times 8 + 10 \times 10 + 13 \times 12 + \ldots\ldots$ 10 term

$$S = \sum_{r=1}^{10}(3r+4)(2r+6) = 2\cdot\sum_{r=1}^{10}\left(3r^2+13r+12\right)$$

$$= 2\cdot\left(3\times\frac{10\times11\times21}{6}+13\times\frac{10\times11}{2}+12\times10\right)$$

$= 3980$

$$\text{Mean} = \frac{3980}{10} = 398$$

5. If $\left(\frac{3^6}{4^4}\right)k$ is the term, independent of x, in the binomial expansion of $\left(\frac{x}{4}-\frac{12}{x^2}\right)^{12}$, then k is equal to _______.

Answer (55)

Sol. In expansion of $\left(\frac{x}{4}-\frac{12}{x^2}\right)^{12}$,

General term

$$\Rightarrow\ T_{r+1} = {}^{12}C_r\cdot\left(\frac{x}{4}\right)^{12-r}\left(-\frac{12}{x^2}\right)^r$$

$$T_{r+1} = {}^{12}C_r\cdot\left(\frac{1}{4}\right)^{12-r}(-12)^r\cdot x^{12-3r}$$

term independent of x

$\Rightarrow\ 12 - 3r = 0$

$\Rightarrow\ r = 4$

$$\left(\frac{3^6}{4^4}\right)k = {}^{12}C_4\left(\frac{1}{4}\right)^8(-12)^4$$

$$\left(\frac{3^6}{4^4}\right)k = \frac{12\times11\times10\times9}{24}\left(\frac{3}{4}\right)^4$$

$k = 55$

6. An electric instrument consists of two units. Each unit must function independently for the instrument to operate. The probability that the first unit functions is 0.9 and that of the second unit is 0.8. The instrument is switched on and it fails to operate. If the probability that only the first unit failed and second unit is functioning is p, then 98 p is equal to ______.

Answer (28)

Sol. $p(E_1) = 0.9$, $p(E_2) = 0.8$

$$p(E) = \frac{p\left(\bar{E}_1\right)\cdot p\left(E_2\right)}{p\left(E_1\right)\cdot p\left(\bar{E}_2\right)+p\left(\bar{E}_1\right)\cdot p\left(E_2\right)+p\left(\bar{E}_1\right)\cdot p\left(\bar{E}_2\right)}$$

$$= \frac{(.1)(.8)}{(.9)(.2)+(.1)(.8)+(.1)(.2)}$$

$$p = \frac{.08}{.28} = \frac{2}{7}$$

$$98p = 98\times\frac{2}{7} = 28$$

7. If $x\phi(x) = \int_5^x\left(3t^2-2\phi'(t)\right)dt$, $x > -2$, and $\phi(0) = 4$, then $\phi(2)$ is ______.

Answer (4)

Sol. $x\phi(x) = \int_5^x\left(3t^2-2\phi'(t)\right)dx$

Differentiating both side w.r.t. x.

$$\phi(x)+x\cdot\phi'(x) = 3x^2-2\phi'(x)$$

Let $\phi(x) = y$

$$(x+2)\cdot\frac{dy}{dx}+y = 3x^2$$

$$\frac{dy}{dx}+\frac{y}{x+2} = \frac{3x^2}{x+2}$$

$$I.F = e^{\int\frac{1}{x+2}dx} = (x+2)$$

Solution of differentiating equation

$$\Rightarrow\ y\cdot(x+2) = \int\left(\frac{3x^2}{x+2}\right)(x+2)dx+c$$

$y(x + 2) = x^3 + c$

at $x = 0$, $y = 4$

$c = 8$

$y(x + 2) = x^3 + 8$

at $x = 2$

$y = 4$

8. The number of six letter words (with or without meaning), formed using all the letters of the word 'VOWELS', so that all the consonants never come together, is _______.

Answer (576)

Sol. Total possible words = 6! = 720

When 4 consonants are together (V, W, L, S)

Total case ⇒ .

0, *E*, [V W L S]

Such cases = 3! · 4! = 144

Required cases = 720 – 144 = 576

9. If '*R*' is the least value of '*a*' such that the function $f(x) = x^2 + ax + 1$ is increasing on [1, 2] and '*S*' is the greatest value of '*a*' such that the function $f(x) = x^2 + ax + 1$ is decreasing on [1, 2], then the value of $|R - S|$ is _____.

Answer (2)

Sol. $f'(x) = 2x + a$

For increasing function $2x + a|_{x=1} \geq 0$

$$a \geq -2$$

For decreasing function $2x + a|_{x=2} \leq 0$

$$a \leq -4$$

$R = -2,\ S = -4$

$|R - S| = 2$

10. Let [*t*] denote the greatest integer ≤ *t*. Then the value of $8 \cdot \int_{-\frac{1}{2}}^{1} ([2x] + |x|)\,dx$ is _____.

Answer (5)

Sol. $\int_{-\frac{1}{2}}^{1} ([2x] + |x|)\,dx = \int_{-\frac{1}{2}}^{1} [2x]dx + \int_{-\frac{1}{2}}^{0} -xdx + \int_{0}^{1} xdx$

$$= \int_{-\frac{1}{2}}^{0} -1dx + \int_{0}^{\frac{1}{2}} 0\,dx + \int_{\frac{1}{2}}^{1} 1dx + \int_{-\frac{1}{2}}^{0} -x\,dx + \int_{0}^{1} x\,dx$$

$$= \left(-\frac{1}{2} + 0 + \frac{1}{2}\right) + \left.\frac{-x^2}{2}\right|_{-\frac{1}{2}}^{0} + \left.\frac{x^2}{2}\right|_{0}^{1}$$

$$= 0 - \left(\frac{-1}{8}\right) + \frac{1}{2} = \frac{5}{8}$$

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