26/06/2022
Evening

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Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2022 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The dimension of mutual inductance is :

(A) $[ML^2T^{-2}A^{-1}]$

(B) $[ML^2T^{-3}A^{-1}]$

(C) $[ML^2T^{-2}A^{-2}]$

(D) $[ML^2T^{-3}A^{-2}]$

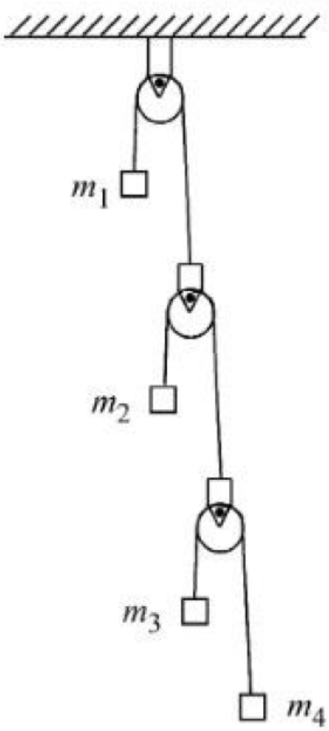
Answer (C)

Sol. $\because \quad U = \frac{1}{2}Mi^2$

$$\Rightarrow \quad [M] = \frac{[U]}{[i^2]} = \frac{ML^2T^{-2}}{A^2}$$

$$= [ML^2T^{-2}A^{-2}]$$

2. In the arrangement shown in figure a_1, a_2, a_3 and a_4 are the accelerations of masses m_1, m_2, m_3 and m_4 respectively. Which of the following relation is true for this arrangement?



(A) $4a_1 + 2a_2 + a_3 + a_4 = 0$

(B) $a_1 + 4a_2 + 3a_3 + a_4 = 0$

(C) $a_1 + 4a_2 + 3a_3 + 2a_4 = 0$

(D) $2a_1 + 2a_2 + 3a_3 + a_4 = 0$

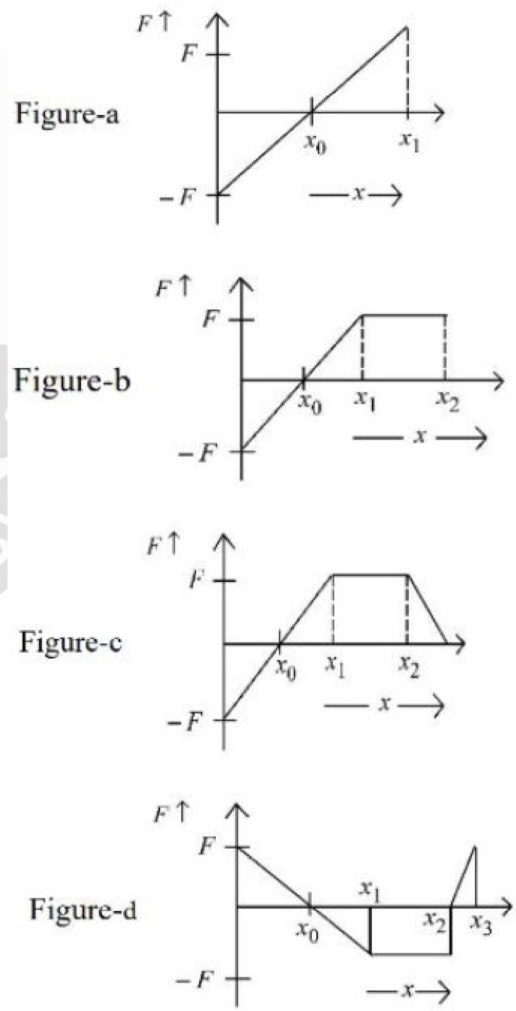
Answer (A)

Sol. From virtual work done method,

$4T \times a_1 + 2T \times a_2 + T \times a_3 + T \times a_4 = 0$

$\Rightarrow \quad 4a_1 + 2a_2 + a_3 + a_4 = 0$

3. Arrange the four graphs in descending order of total work done; where W_1, W_2, W_3 and W_4 are the work done corresponding to figure a, b, c and d respectively.



(A) $W_3 > W_2 > W_1 > W_4$ (B) $W_3 > W_2 > W_4 > W_1$

(C) $W_2 > W_3 > W_4 > W_1$ (D) $W_2 > W_3 > W_1 > W_4$

Answer (A)

Sol. $W_a = 0$, $W_b = +ve$, $W_c = +ve > W_b$, $W_d = -ve$

$\Rightarrow \quad W_c > W_b > W_a > W_d$

$\Rightarrow \quad W_3 > W_2 > W_1 > W_4$

4. A solid spherical ball is rolling on a frictionless horizontal plane surface about its axis of symmetry. The ratio of rotational kinetic energy of the ball to its total kinetic energy is -

(A) $\frac{2}{5}$ (B) $\frac{2}{7}$

(C) $\frac{1}{5}$ (D) $\frac{7}{10}$

Answer (B)

Sol. $KE_R = \frac{1}{2}I\omega^2$

$= \frac{1}{2}\times\frac{2}{5}\times\omega^2\times(mR^2)$

$KE_{total} = \frac{1}{2}\times\frac{7}{5}\times mR^2\times\omega^2$

$\therefore \quad \frac{KE_R}{KE_{total}} = \frac{2}{7}$

5. Given below are two statements: One is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A: If we move from poles to equator, the direction of acceleration due to gravity of earth always points towards the center of earth without any variation in its magnitude.

Reason R: At equator, the direction of acceleration due to the gravity is towards the center of earth.

In the light of above statements, choose the correct answer from the options given below

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Answer (D)

Sol. $g' = g_0 - \omega^2 R\cos^2\theta$

θ = latitude.

6. If ρ is the density and η is coefficient of viscosity of fluid which flows with a speed v in the pipe of diameter d, the correct formula for Reynolds number R_e is:

(A) $R_e = \frac{\eta d}{\rho v}$ (B) $R_e = \frac{\rho v}{\eta d}$

(C) $R_e = \frac{\rho v d}{\eta}$ (D) $R_e = \frac{\eta}{\rho v d}$

Answer (C)

Sol. $R_e = \frac{\rho v d}{\eta}$

Direct formula based.

7. A flask contains argon and oxygen in the ratio of 3 : 2 in mass and the mixture is kept at 27°C. The ratio of their average kinetic energy per molecule respectively will be

(A) 3 : 2 (B) 9 : 4

(C) 2 : 3 (D) 1 : 1

Answer (D)

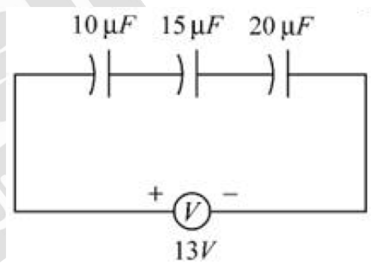
Sol. $KE_{avg} = \frac{3}{2}kT$ (At lower temperature)

As temperature is same for both the gases.

$\Rightarrow$ Both gases will have same average kinetic energy.

$\Rightarrow \quad \frac{(KE_{avg})_{argon}}{(KE_{avg})_{oxygen}} = \frac{1}{1}$

8. The charge on capacitor of capacitance 15 μF in the figure given below is



(A) 60 μC

(B) 130 μC

(C) 260 μC

(D) 585 μC

Answer (A)

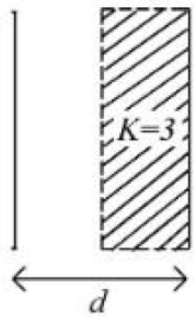
Sol. $C_{eq} = \frac{120}{26}\,\mu F$

$\Rightarrow \quad Q_{flown}$ or $Q = \frac{13\times120}{26}\,\mu C = 60\,\mu C$

$\Rightarrow$ Charge on 15 μF capacitor = 60 μC

As all the capacitors are in series.

9. A parallel plate capacitor with plate area A and plate separation d = 2 m has a capacitance of 4 μF. The new capacitance of the system if half of the space between them is filled with a dielectric material of dielectric constant K = 3 (as shown in figure) will be :



(A) 2 μF (B) 32 μF

(C) 6 μF (D) 8 μF

Answer (C)

Sol. Equivalent circuit is

$\frac{2\varepsilon_0 A}{d}$ $\quad$ $\frac{2\varepsilon_0 A}{d}\times 3$ $\quad\equiv\quad$ $\frac{12}{8}\frac{\varepsilon_0 A}{d}$

Now $\frac{\varepsilon_0 A}{d} = 4\ \mu\text{F}$

$\Rightarrow \frac{12}{8}\frac{\varepsilon_0 A}{d} = 6\ \mu\text{F}$

10. Sixty four conducting drops each of radius 0.02 m and each carrying a charge of 5 μC are combined to form a bigger drop. The ratio of surface density of bigger drop to the smaller drop will be:

(A) 1 : 4

(B) 4 : 1

(C) 1 : 8

(D) 8 : 1

Answer (B)

Sol. $q' = 64q$ … (i)

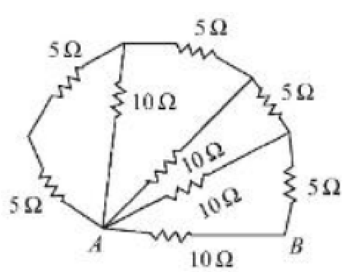
$A' = 16\ A$ … (ii)

Dividing (i) & (ii),

$\sigma' = 4\sigma$

$\Rightarrow \frac{\sigma'}{\sigma} = \frac{4}{1}$

11. The equivalent resistance between points A and B in the given network is:

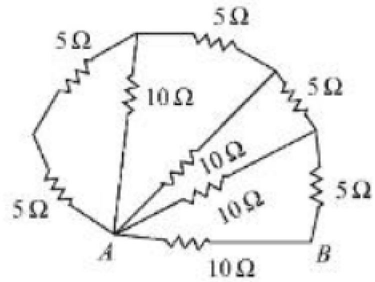


(A) 65 Ω (B) 20 Ω

(C) 5 Ω (D) 2 Ω

Answer (C)

Sol. Initially 5 Ω and 5 Ω are in series and then in parallel with 10 Ω this pattern continues thus



$R_{net} = 5\ \Omega$

12. A bar magnetic having a magnetic moment of 2.0×10^5 JT^{-1}, is placed along the direction of uniform magnetic field of magnitude $B = 14 \times 10^{-5}$ T. The work done in rotating the magnet slowly through 60° from the direction of field is:

(A) 14 J

(B) 8.4 J

(C) 4 J

(D) 1.4 J

Answer (A)

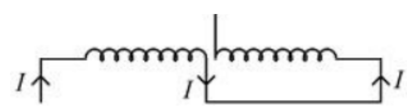
Sol. $U = -\vec{M}.\vec{B}$

So $U_f - U_i = -MB(1 - \cos\theta)$

$= -14$J

So $W = -\Delta U = 14$J

13. Two coils of self inductance L_1 and L_2 are connected in series combination having mutual inductance of the coils as M. The equivalent self inductance of the combination will be:



(A) $\frac{1}{L_1}+\frac{1}{L_2}+\frac{1}{M}$

(B) $L_1 + L_2 + M$

(C) $L_1 + L_2 + 2M$

(D) $L_1 + L_2 - 2M$

Answer (D)

Sol. Self inductances are in series but their mutual inductances are linked oppositely so equivalent self inductance

$L = L_1 + L_2 - M - M = L_1 + L_2 - 2M$

14. A metallic conductor of length 1 m rotates in a vertical plane parallel to east-west direction about one of its end with angular velocity 5 rad s^{-1}. If the horizontal component of earth's magnetic field is 0.2×10^{-4} T, then emf induced between the two ends of the conductor is:

(A) 5 μV (B) 50 μV

(C) 5 mV (D) 50 mV

Answer (B)

Sol. Emf = $\frac{1}{2}B\omega l^2$

$= \frac{1}{2} \times 0.2 \times 10^{-4} \times 5 \times 1^2$ V

$= 0.5 \times 10^{-4}$ V

$= 50\ \mu$V

15. Which is the correct ascending order of wavelengths?

(A) $\lambda_{visible} < \lambda_{X\text{-}ray} < \lambda_{gamma\text{-}ray} < \lambda_{microwave}$

(B) $\lambda_{gamma\text{-}ray} < \lambda_{X\text{-}ray} < \lambda_{visible} < \lambda_{microwave}$

(C) $\lambda_{X\text{-}ray} < \lambda_{gamma\text{-}ray} < \lambda_{visible} < \lambda_{microwave}$

(D) $\lambda_{microwave} < \lambda_{visible} < \lambda_{gamma\text{-}ray} < \lambda_{X\text{-}ray}$

Answer (B)

Sol. Wave length of microwave is maximum then visible light then X-rays and then gamma rays so the correct order will be

$\lambda_{gamma\text{-}ray} < \lambda_{X\text{-}ray} < \lambda_{visible} < \lambda_{microwave}$

16. For a specific wavelength 670 nm of light from a galaxy moving with velocity ν, the observed wavelength is 670.7 nm.

The value of ν is:

(A) 3×10^8 ms^{-1} (B) 3×10^{10} ms^{-1}

(C) 3.13×10^5 ms^{-1} (D) 4.48×10^5 ms^{-1}

Answer (C)

Sol. $\lambda_{obs} = \lambda_{source}\sqrt{\dfrac{1+\frac{\nu}{C}}{1-\frac{\nu}{C}}}$

For $\nu << C$,

$\frac{670.7}{670} = 1 + \frac{\nu}{C}$

$\Rightarrow \nu = \frac{0.7}{670} \times 3 \times 10^8$ m/s

$\Rightarrow \nu \simeq 3.13 \times 10^5$ m/s

17. A metal surface is illuminated by a radiation of wavelength 4500 Å. The rejected photo-electron enters a constant magnetic field of 2 mT making an angle of 90° with the magnetic field. If it starts revolving in a circular path of radius 2 mm, the work function of the metal is approximately:

(A) 1.36 eV (B) 1.69 eV

(C) 2.78 eV (D) 2.23 eV

Answer (A)

Sol. $\frac{hc}{\lambda} - \phi = KE$ …(i)

$R = \frac{mv}{Bq} = \frac{\sqrt{2m(KE)}}{Bq}$ …(ii)

Putting the values,

$\phi \simeq 1.36$ eV

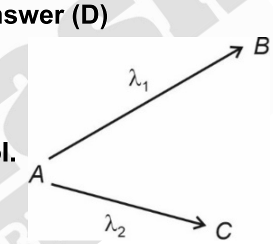
18. A radioactive nucleus can decay by two different processes. Half-life for the first process is 3.0 hours while it is 4.5 hours for the second process. The effective half-life of the nucleus will be:

(A) 3.75 hours (B) 0.56 hours

(C) 0.26 hours (D) 1.80 hours

Answer (D)

Sol.



$\frac{dA}{dt} - (-\lambda_1 A) + (-\lambda_2 A)$

$\Rightarrow \frac{dA}{dt} = -(\lambda_1 + \lambda_2)A$

$\Rightarrow \lambda_{eff} = \lambda_1 + \lambda_2$

$\Rightarrow \frac{\ln 2}{(t_{1/2})_{eff}} = \frac{\ln 2}{(t_{1/2})_1} + \frac{(\ln 2)}{(t_{1/2})_2}$

$\Rightarrow (t_{1/2})_{eff} = \frac{4.5 \times 3}{7.5}$ hours $= 1.8$ hours

19. The positive feedback is required by an amplifier to act an oscillator. The feedback here means:

(A) External input is necessary to sustain ac signal in output

(B) A portion of the output power is returned back to the input

(C) Feedback can be achieved by *LR* network

(D) The base-collector junction must be forward biased

Answer (B)

Sol. Feedback means a portion of the output power is fed to the inputs.

20. A sinusoidal wave $y(t) = 40\sin(10 \times 10^6\ \pi t)$ is amplitude modulated by another sinusoidal wave $x(t) = 20\sin(1000\pi t)$. The amplitude of minimum frequency component of modulated signal is:

(A) 0.5 (B) 0.25

(C) 20 (D) 10

Answer (D)

Sol. Modulate signal $s(t) \equiv [1 + 20\sin(1000\pi t)]\sin(10^7\pi t)$

$$\equiv \sin(10^7\pi t) + 10\cos(10^7\pi t - 10^3\pi t) + 10\cos(10^7\pi t + 10^3\pi t)$$

$\Rightarrow$ Required amplitude = 10

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. A ball is projected vertically upward with an initial velocity of 50 ms^{-1} at t = 0 s. At t = 2 s, another ball is projected vertically upward with same velocity. At t = ______s, second ball will meet the first ball (g = 10 ms^{-2})

Answer (6)

Sol At t = 2 s, v_1 = 50 – 2 × 10 = 30 m/s

$v_2 = v_2$

$\therefore$ $a_{rel} = g - g = 0$

$$S = \frac{u^2 - v^2}{2g} = \frac{50^2 - 30^2}{2 \times 10} = \frac{1600}{20} = 80 \text{ m}$$

$\therefore$ v_{rel} = 50 – 30 = 20 m/s

$$\therefore \quad \Delta t = \frac{80}{20} = 4 \text{ s}$$

$\therefore$ required time t = 2 + 4 = 6 s

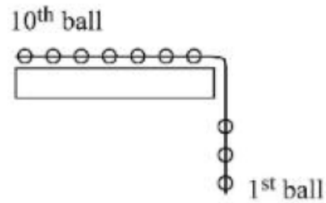
2. A batsman hits back a ball of mass 0.4 kg straight in the direction of the bowler without changing its initial speed of 15 ms^{-1}. The impulse imparted to the ball is _________ Ns.

Answer (12)

Sol. $I = m\Delta v$

= 0.4 × 2 × 15 = 12 Ns

3. A system to 10 balls each of mass 2 kg are connected via massless and unstretchable string. The system is allowed to slip over the edge of a smooth table as shown in figure. Tension on the string between the 7th and 8th ball is ______ N when 6th ball just leaves the table



Answer (36)

Sol. At given instant

$$a_{sys} = \frac{6\ m \times g}{10\ m} = \frac{6\ g}{10}$$

$\therefore$ $T_{78} = (3m) \times a_{sys}$

$$= (3m) \times \left(\frac{6g}{10}\right)$$

$$= \frac{3 \times 2 \times 6 \times 10}{10} = 36 \text{ N}$$

4. A geyser heats water flowing at a rate of 2.0 kg per minute from 30°C to 70°C. If geyser operates on a gas burner, the rate of combustion of fuel will be __________ g min^{-1}

[Heat of combustion = 8 × 10^3 Jg^{-1}, Specific heat of water = 4.2 Jg^{-1} °C^{-1}]

Answer (42)

Sol. $Q = ms\Delta T$

$$\frac{dQ}{dt} = \left(\frac{dm}{dt}\right)_{water} S\Delta T = \left(\frac{dm}{dt}\right)_{oil} C$$

$$\Rightarrow \quad 2 \times 4.2 \times 10^3 \times 40 = \left(\frac{dm}{dt}\right)_{oil} \times 8 \times 10^6$$

$$\Rightarrow \quad \left(\frac{dm}{dt}\right)_{oil} = \frac{8 \times 4.2 \times 10^4}{8 \times 10^6} \text{ kg / minute}$$

= 42 g/min

5. A heat engine operates with the cold reservoir at temperature 324 K. The minimum temperature of the hot reservoir, if the heat engine takes 300 J heat from the hot reservoir and delivers 180 J heat to the cold reservoir per cycle, is _____ K.

Answer (540)

Sol. $\left(1-\frac{324}{T_H}\right)=\frac{300-180}{300}$

$1-\frac{2}{5}=\frac{324}{T_H}$

$T_H=\frac{324\times5}{3}=540$

6. A set of 20 tuning forks is arranged in a series of increasing frequencies. If each fork gives 4 beats with respect to the preceding fork and the frequency of the last fork is twice the frequency of the first, then the frequency of last fork is ________Hz.

Answer (152)

Sol. Given $\nu_{20} = 2\nu_1$

Also $\nu_{20} = 4 \times 19 + \nu_1$

So $\nu_{20} = 152$ Hz

7. Two 10 cm long, straight wires, each carrying a current of 5 A are kept parallel to each other. If each wire experienced a force of 10^{-5} N, then separation between the wires is ______ cm.

Answer (5)

Sol. $\frac{dF}{dl}=\frac{\mu_o i_1 i_2}{2\pi d}$

So $\frac{2\times10^{-7}\times5\times5}{d}=\frac{10^{-5}}{10\times10^{-2}}$

$d=\frac{2\times10^{-7}\times5\times5}{10^{-4}}$

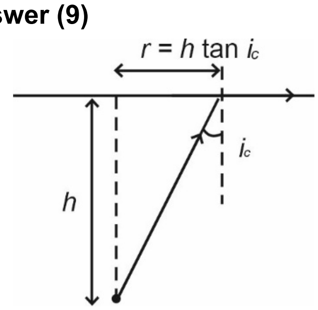
= 50 mm

= 5 cm

8. A small bulb is placed at the bottom of a tank containing water to a depth of $\sqrt{7}$ m. The refractive index of water is $\frac{4}{3}$. The area of the surface of water through which light from the bulb can emerge out is $x\pi$ m^2. The value of x is _________.

Answer (9)

Sol.



So $r=h\frac{\sin i_c}{\sqrt{1-\sin^2 i_c}}$

So $A = \pi r^2$

$=\frac{\pi h^2\sin^2 i_c}{1-\sin^2 i_c}$

$=\frac{\pi 7\times\frac{9}{16}}{1-\frac{9}{16}}=\frac{\pi\times7\times9}{7}=9\pi$

9. A travelling microscope is used to determine the refractive index of a glass slab. If 40 divisions are there in 1 cm on main scale and 50 Vernier scale divisions are equal to 49 main scale divisions, then least count of the travelling microscope is ________ × 10^{-6} m.

Answer (5)

Sol. 40 M = 1 cm

$\Rightarrow$ M = 0.025 cm ………..(1)

Also, 50 V = 49 M

$\Rightarrow$ Least count = M – V = $M-\frac{49}{50}M$

$=\frac{M}{50}$

$\Rightarrow LC=\frac{0.025}{50}$ cm

$=\frac{250}{50}\times10^{-6}$ m

$\Rightarrow$ LC = 5×10^{-6} m

10. The stopping potential for photoelectrons emitted from a surface illuminated by light of wavelength 6630 Å is 0.42 V. If the threshold frequency is $x \times 10^{13}$/s, where x is _____ (nearest integer).

(Given, speed of light = 3×10^8 m/s, Planck's constant = 6.63×10^{-34} Js)

Answer (35)

Sol. $\frac{hc}{\lambda}-\phi$ = KE = eV_0

$\Rightarrow \frac{6.63\times10^{-34}\times3\times10^{8}}{6630\times10^{-10}}-6.63\times10^{-34}f_{th}=1.6\times10^{-19}\times0.4$

$\Rightarrow f_{th} \simeq 35.11 \times 10^{13}$ H

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The number of radial and angular nodes in 4d orbital are, respectively

(A) 1 and 2 (B) 3 and 2

(C) 1 and 0 (D) 2 and 1

Answer (A)

Sol. (i) In 4d n = 4 l = 2

Radial nodes = n – l – 1

= 4 – 2 – 1

= 1

Angular nodes = l

= 2

2. Match List I with List II

List I Enzyme		List II Conversion of	
A	Invertase	I	Starch into maltose
B	Zymase	II	Maltose into glucose
C	Diastase	III	Glucose into ethanol
D	Maltase	IV	Cane sugar into glucose

Choose the most appropriate answer from the options given below

(A) A-III, B-IV, C-II, D-I (B) A-III, B-II, C-I, D-IV

(C) A-IV, B-III, C-I, D-II (D) A-IV, B-II, C-III, D-I

Answer (C)

Sol. (A) Invertase $\rightarrow$ Cane sugar into glucose

(B) Zymase $\rightarrow$ Glucose into ethanol

(C) Diastase $\rightarrow$ Starch into maltose

(D) Maltase $\rightarrow$ Maltose into glucose

3. Which of the following elements is considered as a metalloid?

(A) Sc (B) Pb

(C) Bi (D) Te

Answer (D)

Sol. Tellurium is metalloid

4. The role of depressants in 'Froth Floation method' is to

(A) Selectively prevent one component of the ore from coming to the froth

(B) Reduce the consumption of oil for froth formation

(C) Stabilize the froth

(D) Enhance non-wettability of the mineral particles.

Answer (A)

Sol. The role of depressants is to selectively prevent one component of the ore from coming to froth.

5. Boiling of hard water is helpful in removing the temporary hardness by converting calcium hydrogen carbonate and magnesium hydrogen carbonate to

(A) $CaCO_3$ and $Mg(OH)_2$

(B) $CaCO_3$ and $MgCO_3$

(C) $Ca(OH)_2$ and $MgCO_3$

(D) $Ca(OH)_2$ and $Mg(OH)_2$

Answer (A)

Sol. $Ca(HCO_3)_2 \xrightarrow{\Delta} CaCO_3\downarrow + H_2O + CO_2\uparrow$

$Mg(HCO_3)_2 \xrightarrow{\Delta} Mg(OH)_2 + 2CO_2\uparrow$

6. s-block element which **cannot** be qualitatively confirmed by the flame test is

(1) Li (2) Na

(3) Rb (4) Be

Answer (D)

Sol. Beryllium does not give flame test because of its small size and high ionization energy the energy of flame is not sufficient to excite the electrons to higher energy level

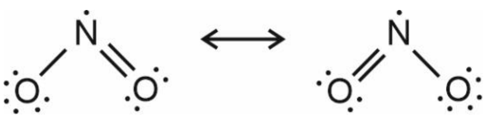
7. The oxide which contains an odd electron at the nitrogen atom is

(A) N_2O (B) NO_2

(C) N_2O_3 (D) N_2O_5

Answer (B)

Sol. The oxide of nitrogen which contains odd electron is NO_2



8. Which one of the following is an example of disproportionation reaction?

(A) $3MnO_4^{2-} + 4H^+ \rightarrow 2MnO_4^- + MnO_2 + 2H_2O$

(B) $MnO_4^- + 4H^+ + 4e^- \rightarrow MnO_2 + 2H_2O$

(C) $10I^- + 2MnO_4^- + 16H^+ \rightarrow 2Mn^{2+} + 8H_2O + 5I_2$

(D) $8MnO_4^- + 3S_2O_3^{2-} + H2O \rightarrow 8MnO_2 + 6SO_4^{2-} + 2OH^-$

Answer (A)

Sol. $\overset{+6}{Mn}O_4^{-2} \rightarrow \overset{+7}{Mn}O_4^-$

$\overset{+6}{Mn}O_4^{-2} \rightarrow \overset{+4}{Mn}O_2$

MnO_4^{-2} is an intermediate oxidation state and is converted into compounds having higher and lower oxidation states.

9. The most common oxidation state of Lanthanoid elements is +3. Which of the following is likely to deviate easily from +3 oxidation state?

(1) Ce(At. No. 58)

(2) La (At. No. 57)

(3) Lu (At. No. 71)

(4) Gd(At. No. 64)

Answer (A)

Sol. $Ce \rightarrow [Xe]4f^1\ 5d^1\ 6s^2$

$Ce^{+4} \rightarrow [xe]\ 4f^\circ\ 5d^\circ\ 6s^\circ$

Cerium in +4 oxidation state acquires inert gas configuration.

10. The measured BOD values for four different water samples (A-D) are as follows:

A = 3 ppm; B = 18 ppm; C = 21 ppm; D = 4 ppm;. The water samples which can be called as highly polluted with organic wastes, are

(A) A and B (B) A and D

(C) B and C (D) B and D

Answer (C)

Highly polluted water should have BOD value of 17 ppm or more

11. The correct order of nucleophilicity is

(A) $F^- > OH^-$ (B) $H_2\ddot{\underset{\cdot\cdot}{O}} > OH^-$

(C) $R\ddot{\underset{\cdot\cdot}{O}}H > RO^-$ (D) $NH_2^- > NH_3$

Answer (D)

Sol. $\underset{\text{Acid}}{NH_3} \longrightarrow \underset{\text{Conjugate base}}{NH_2^-} + H^+$

Conjugate base of acid is always a stronger nucleophile.

12. Oxidation of toluene to benzaldehyde can be easily carried out with which of the following reagents?

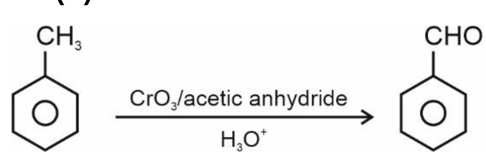
(A) CrO_3/acetic acid, H_3O^+

(B) CrO_3/acetic anhydride, H_3O^+

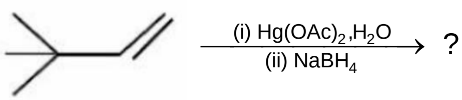
(C) $KMnO_4$/HCl, H_3O^+

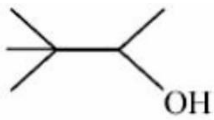
(D) CO/HCl, anhydrous $AlCl_3$

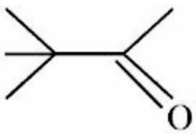
Answer (B)

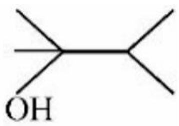
Sol. 

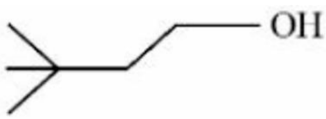
13. The major product in the following reaction



(A) 

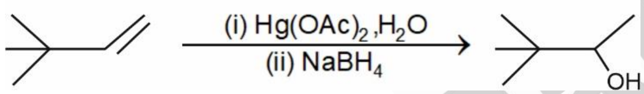
(B) 

(C) 

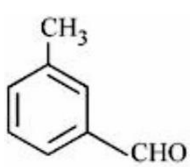
(D) 

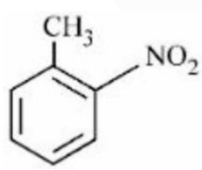
Answer (A)

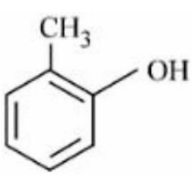
Sol. Oxymercuration-demercuration follows Markovnikov's addition of water without rearrangement.

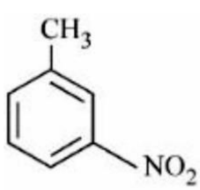


14. Halogenation of which one of the following will yield m-substituted product with respect to methyl group as a major product?

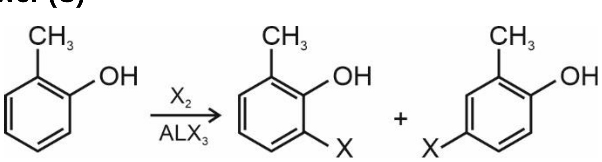
(A) 

(B) 

(C) 

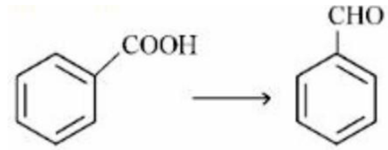
(D) 

Answer (C)

Sol. 

Both products are meta with respect to $-CH_3$.

15. The reagent, from the following, which converts benzoic acid to benzaldehyde in one step is



(A) $LiAlH_4$ (B) $KMnO_4$

(C) MnO (D) $NaBH_4$

Answer (C)

Sol. Benzoic acid can be converted to benzaldehyde in presence of MnO.

16. The final product 'A' in the following reaction sequence

$$CH_3CH_2-\underset{\underset{O}{\|}}{C}-CH_3 \xrightarrow{HCN} ? \xrightarrow[\text{Heat}]{95\%H_2SO_4} A$$

(A) $CH_3-CH=\overset{\overset{CH_3}{|}}{C}-COOH$

(B) $CH_3-CH=\underset{\underset{CH_3}{|}}{C}-CN$

(C) $CH_3-CH_2-\overset{\overset{OH}{|}}{\underset{\underset{CH_3}{|}}{C}}-COOH$

(D) $CH_3-CH=\underset{\underset{CH_3}{|}}{C}-CONH_2$

Answer (A)

Sol.

$$CH_3-CH_2-\overset{\overset{O}{\|}}{C}-CH_3 \xrightarrow{HCN}$$

$$CH_3-CH_2-\overset{\overset{OH}{|}}{\underset{\underset{CN}{|}}{C}}-CH_3$$

$$\Big\downarrow \frac{95\%H_2SO_4}{\Delta}$$

$$CH_3-CH=\overset{\overset{CH_3}{|}}{C}-COOH$$

[A]

17. Which statement is NOT correct for p-toluenesulphonyl chloride?

(A) It is known as Hinsberg's reagent

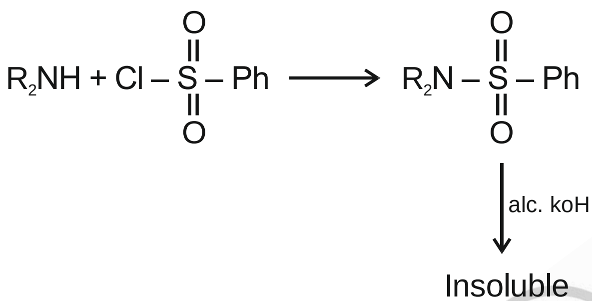
(B) It is used to distinguish primary and secondary amines

(C) On treatment with secondary amine, it leads to a product, that is soluble in alkali

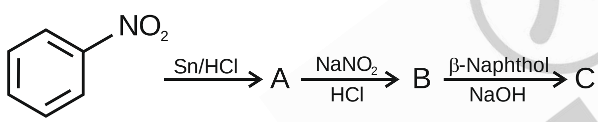
(D) It doesn't react with tertiary amines

Answer (C)

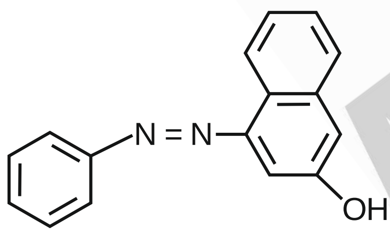
Sol.



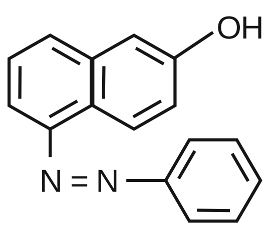
18. The final product 'C' in the following series of reactions



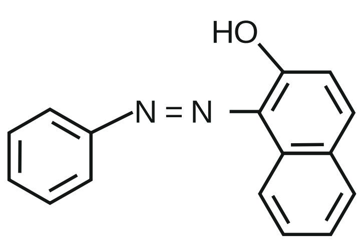
(A)



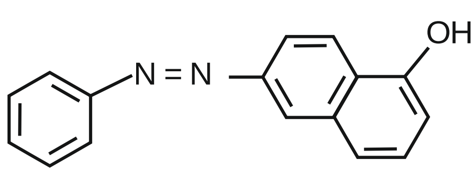
(B)



(C)

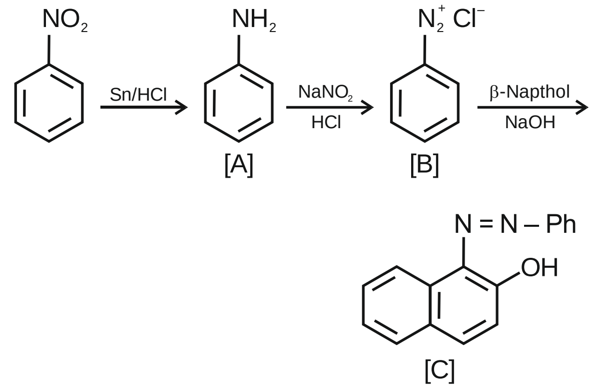


(D)



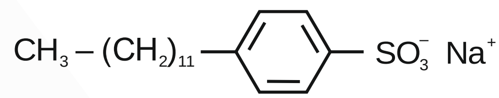
Answer (C)

Sol.



19. Which of the following is NOT an example of synthetic detergent?

(A)



(B) $CH_3-(CH_2)_{16}-COO^- Na^+$

(C) $\left[CH_3-(CH_2)_{15}-\overset{\overset{\large CH_3}{|}}{\underset{\underset{\large CH_3}{|}}{N}}-CH_3\right]^+ Br^-$

(D) $CH_3(CH_2)_{16}COO(CH_2CH_2O)nCH_2CH_2OH$

Answer (B)

Sol. $CH_3-(CH_2)_{16}-COO^- Na^+$

Sodium stearate is example of soap.

20. Which one of the following is a water soluble vitamin, that is not excreted easily?

(A) Vitamin B_2 (B) Vitamin B_1

(C) Vitamin B_6 (D) Vitamin B_{12}

Answer (D)

Sol. Vitamin B_{12} is water soluble and not excreted easily.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. CNG is an important transportation fuel. When 100 g CNG is mixed with 208 g oxygen in vehicles, it leads to the formation of CO_2 and H_2O and produced large quantity of heat during this combustion, then the amount of carbon dioxide, produced in grams is _____. [nearest integer]

[Assume CNG to be methane]

Answer (143)

Sol. $CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O$

wt. of $CH_4 = 100$ g
wt. of $O_2 = 208$ g $\qquad n_{O_2} = \frac{208}{32}$

In this reaction O_2 is limiting reagent

2 moles of $O_2 \longrightarrow$ 1 mole of CO_2

1 mole of $O_2 \longrightarrow \frac{1}{2}$ mole of CO_2

$\frac{208}{32}$ mole of $O_2 \longrightarrow \frac{208}{32}\times\frac{1}{2}$ mole of CO_2

$\longrightarrow \frac{208}{32}\times\frac{1}{2}\times 44$ gm of CO_2

$\longrightarrow$ 143 gm of CO_2

2. In a solid AB, A atoms are in ccp arrangement and B atoms occupy all the octahedral sites. If two atoms from the opposite faces are removed, then the resultant stoichiometry of the compound is A_xB_y. The value of x is _____. [nearest integer]

Answer (3)

Sol. A atoms are in CCP contribution of A is

A = 4

If atoms from opposite faces are removed

then $\quad A = 4 - x\times\frac{1}{x}$

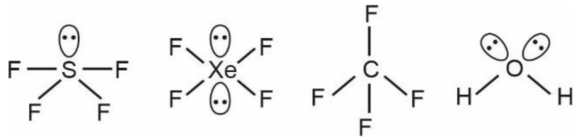
A = 3

Value of x = 3

3. Amongst SF_4, XeF_4, CF_4 and H_2O, the number of species with two lone pair of electrons is _____.

Answer (2)

Sol.



XeF_4 and H_2O have 2 lone pairs.

4. A fish swimming in water body when taken out from the water body is covered with a film of water of weight 36 g. When it is subjected to cooking at 100 °C, then the internal energy for vaporization in kJ mol^{-1} is ________. [nearest integer]

[Assume steam to be an ideal gas. Given $\Delta_{vap}H^\ominus$ for water at 373 K and 1 bar is 41.1 kJ mol^{-1}; R = 8.31 J K^{-1} mol^{-1}]

Answer (38)

Sol. $\underset{36\,g}{H_2O(\ell)} \longrightarrow \underset{36\,g}{H_2O(g)}$ (evaporation)

$n_{H_2O} = \frac{36}{18} = 2 \qquad \Delta n_g = 1 - 0 = 1$

$\Delta U_{vap} = \Delta H_{vap} - \Delta n_g RT$

$= 41.1 - (1) \times 8.31 \times 10^{-3} \times 373$

$= 41.1 - 3.099$

$= 38$ kJ/mol

5. The osmotic pressure exerted by a solution prepared by dissolving 2.0 g of protein of molar mass 60 kg mol^{-1} in 200 mL of water at 27°C is _________ Pa. [Integer value]

(use R = 0.083 L bar mol^{-1} K^{-1})

Answer (415)

Sol. $\pi = i\,CRT \qquad (i = 1)$

$\pi = \frac{2\times 1000}{60\times 10^3 \times 200}\times .083\times 300$

$\pi = .00415$ atm

$\pi = 415$ Pa

6. 40% of HI undergoes decomposition to H_2 and I_2 at 300 K. ΔG^o for this decomposition reaction at one atmosphere pressure is _____ J mol^{-1}. [nearest integer]

(Use R = 8.31 J K^{-1} mol^{-1}; log 2 = 0.3010, ln 10 = 2.3, log 3 = 0.477)

Answer (2735)

Sol. $HI \rightleftharpoons \frac{1}{2}H_2 + \frac{1}{2}I_2$

$1 \quad 0 \quad 0$

$1-\alpha \quad \alpha/2 \quad \alpha/2$

$\Delta G^\circ = -RT\ln K$

$= -RT\ln\frac{\left(\frac{\alpha}{2}\right)^{1/2}\left(\frac{\alpha}{2}\right)^{1/2}}{1-\alpha}$

$= -RT\ln\frac{\alpha}{2(1-\alpha)} \qquad (\alpha = 0.4)$

$= -8.314\times 300\ln\frac{0.4}{2\times 0.6}$

$= +8.314\times 300\ \ln 3$

$= 2735\ J/mol.$

7. $Cu(s) + Sn^{2+}(0.001M) \rightarrow Cu^{2+}(0.01\,M) + Sn(s)$

The Gibbs free energy change for the above reaction at 298 K is x × 10^{-1}kJ mol^{-1}. The value of x is _______.[nearest integer]

[Given

$E^{\ominus}_{Cu^{2+}/Cu} = 0.34V; E^{\ominus}_{Sn^{2+}/Sn} = -0.14\,V; F = 96500\,C\ mol^{-1}$]

Answer (983)

Sol. $Cu + Sn^{+2} \longrightarrow Cu^{+2} + Sn(s)$

$E^\circ_{cell} = E^\circ_{OX} + E^\circ_{Red}$

$= -0.34 - 0.14$

$= -0.48\ V$

$E = E^\circ - \frac{0.0591}{2}\log\frac{[Cu^{+2}]}{[Sn^{+2}]}$

$= -0.48 - 0.0295\ \log 10$

$= -0.5095\ V$

$\Delta G = -nFE$

$= -2\times 96500\times -0.5095\ J/mol$

$= 98333.5\times 10^{-3}\ kJ/mol$

$= 983.3\times 10^{-1}\ kJ/mol$

$= 983\times 10^{-1}\ kJ/mol$

8. Catalyst A reduces the activation energy for a reaction by 10 kJ mol^{-1} at 300 K. The ratio of rate constants, $\frac{K', \text{Catalysed}}{K, \text{Uncatalysed}}$ is e^x. The value of x is ____.[nearest integer]

[Assume that the pre-exponential factor is same in both the cases Given R = 8.31 J K^{-1} mol^{-1}]

Answer (4)

Sol. $\ln\frac{K'}{K} = \frac{Ea - Ea'}{RT}$

$= \frac{10\times 10^3}{8.314\times 300}$

$\ln\frac{K'}{K} = \frac{100}{8.314\times 3}$

$\frac{K'}{K} = e^4$

$x = 4$

9. Reaction of $[Co(H_2O)_6]^{2+}$ with excess ammonia and in the presence of oxygen results into a diamagnetic product. Number of electrons present in t_{2g}-orbitals of the product is _______.

Answer (6)

Sol. $[Co(H_2O)_6]^{+2} \xrightarrow[O_2]{NH_3} [Co(NH_3)_6]^{+3} + e^-$

$Co^{+3} \longrightarrow 3d^6$

NH_3 is a strong field ligand.

$3d^6 \longrightarrow t_{2g}^6\ eg^\circ$

10. The moles of methane required to produce 81 g of water after complete combustion is _____ × 10^{-2} mol. [nearest integer]

Answer (225)

Sol. $CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O$

$1\,mol\ CH_4 \longrightarrow 2\ mole\ H_2O$

$36\,gm\ H_2O \longrightarrow 1\,mole\ CH_4$

$81\,gm\ H_2O \longrightarrow \frac{1}{36}\times 81\,mole\ CH_4$

$\longrightarrow 2.25\ mole$

$\longrightarrow 225\times 10^{-2}$

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Let $f:\mathbb{R}\to\mathbb{R}$ be defined as $f(x) = x - 1$ and $g:\mathbb{R}-\{1,-1\}\to\mathbb{R}$ be defined as $g(x)=\dfrac{x^2}{x^2-1}$.

   Then the function *fog* is:

   (A) One-one but not onto

   (B) Onto but not one-one

   (C) Both one-one and onto

   (D) Neither one-one nor onto

Answer (D)

Sol. $f:\mathbb{R}\to\mathbb{R}$ defined as

$f(x) = x - 1$ and $g:\mathbb{R}\to\{1,-1\}\to\mathbb{R}$, $g(x)=\dfrac{x^2}{x^2-1}$

Now $fog(x)$ $\dfrac{x^2}{x^2-1}-1=\dfrac{1}{x^2-1}$

$\therefore$ Domain of $fog(x) = \mathbb{R} - \{-1, 1\}$

And range of $fog(x) = (-\infty, -1] \cup (0, \infty)$

Now, $\dfrac{d}{dx}(fog(x))=\dfrac{-1}{x^2-1}\cdot 2x=\dfrac{2x}{1-x^2}$

$\therefore\ \dfrac{d}{dx}(fog(x))>0$ for $\dfrac{2x}{(1-x)(1+x)}>0$

$\Rightarrow\ \dfrac{x}{(x-1)(x+1)}<0$

$\therefore\ x\in(-\infty,-1)\cup(0,1)$

and $\dfrac{d}{dx}(fog(x))<0$ for $x\in(-1,0)\cup(1,\infty)$

$\therefore$ $fog(x)$ is neither one-one nor onto.

2. If the system of equations

   $\alpha x + y + z = 5$, $x + 2y + 3z = 4$, $x + 3y + 5z = \beta$ has infinitely many solutions, then the ordered pair (α, β) is equal to:

   (A) (1, –3)

   (B) (–1, 3)

   (C) (1, 3)

   (D) (–1, –3)

Answer (C)

Sol. Given system of equations

$\alpha x + y + z = 5$

$x + 2y + 3z = 4$, has infinite solution

$x + 3y + 5z = \beta$

$$\therefore\quad \Delta=\begin{vmatrix}\alpha & 1 & 1\\ 1 & 2 & 3\\ 1 & 3 & 5\end{vmatrix}=0\Rightarrow \alpha(1)-1(2)+1(1)=0$$

$$\Rightarrow\ \boxed{\alpha=1}$$

and $$\Delta_1=\begin{vmatrix}5 & 1 & 1\\ 4 & 2 & 3\\ \beta & 3 & 5\end{vmatrix}=0$$

$\Rightarrow$ $5(1) - 1(20 - 3\beta) + 1(12 - 2\beta) = 0$

$\Rightarrow$ $\beta = 3$

And $$\Delta_2=\begin{vmatrix}1 & 5 & 1\\ 1 & 4 & 3\\ 1 & \beta & 5\end{vmatrix}=0\Rightarrow(20-3\beta)-5(2)+1(\beta-4)=0$$

$\Rightarrow$ $-2\beta + 6 = 0$

$\Rightarrow$ $\beta = 3$

Similarly,

$$\Rightarrow\ \Delta_3=\begin{vmatrix}1 & 1 & 5\\ 1 & 2 & 4\\ 1 & 3 & \beta\end{vmatrix}=0\Rightarrow\beta=3$$

$\therefore$ $(\alpha, \beta) = (1, 3)$

3. If $A=\sum_{n=1}^{\infty}\dfrac{1}{\left(3+(-1)^n\right)^n}$ and $B=\sum_{n=1}^{\infty}\dfrac{(-1)^n}{\left(3+(-1)^n\right)^n}$, then $\dfrac{A}{B}$ is equal to:

   (A) $\dfrac{11}{9}$

   (B) 1

   (C) $-\dfrac{11}{9}$

   (D) $-\dfrac{11}{3}$

Answer (C)

Sol. $A=\sum_{n=1}^{\infty}\dfrac{1}{\left(3+(-1)^n\right)^n}$ and $B=\sum_{n=1}^{\infty}\dfrac{(-1)^n}{\left(3+(-1)^n\right)^n}$

$$A=\frac{1}{2}+\frac{1}{4^2}+\frac{1}{2^3}+\frac{1}{4^4}+....$$

$$B=\frac{-1}{2}+\frac{1}{4^2}-\frac{1}{2^3}+\frac{1}{4^4}+....$$

$$A=\frac{\frac{1}{2}}{1-\frac{1}{4}}+\frac{\frac{1}{16}}{1-\frac{1}{16}},\ B=\frac{-\frac{1}{2}}{1-\frac{1}{4}}+\frac{\frac{1}{16}}{1-\frac{1}{16}}$$

$$A=\frac{11}{15},\ B=\frac{-9}{15}$$

$$\therefore\quad \frac{A}{B}=\frac{-11}{9}$$

4. $\lim\limits_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^4}$ is equal to:

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$

(C) $\frac{1}{6}$ (D) $\frac{1}{12}$

Answer (C)

Sol. $\lim\limits_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^4}=\lim\limits_{x\to 0}\frac{2\sin(x+\sin x)\cdot\sin\left(\frac{x-\sin x}{2}\right)}{x^4}$

$$=\lim_{x\to 0}2\cdot\left(\frac{\left(\frac{x+\sin x}{2}\right)\left(\frac{x-\sin x}{2}\right)}{x^4}\right)$$

$$=\lim_{x\to 0}\frac{1}{2}\cdot\left(\frac{\left(x+x-\frac{x^3}{3!}+\frac{x^5}{5!}...\right)\left(x-x+\frac{x^3}{3!}...\right)}{x^4}\right)$$

$$=\lim_{x\to 0}\frac{1}{2}\cdot\left(2-\frac{x^2}{3!}+\frac{x^4}{5!}...\right)\left(\frac{1}{3!}-\frac{x^2}{5!}-1\right)$$

$$=\frac{1}{6}$$

5. Let $f(x)$ = min $\{1, 1 + x\sin x\}$, $0 \le x \le 2\pi$. If m is the number of points, where f is not differentiable and n is the number of points, where f is not continuous, then the ordered pair (m, n) is equal to

(A) (2, 0) (B) (1, 0)

(C) (1, 1) (D) (2, 1)

Answer (B)

Sol. $f(x)$ = min{1, 1 + $x\sin x$}, $0 \le x \le 2\pi$

$$f(x)=\begin{cases}1, & 0\le x<\pi\\ 1+x\sin x, & \pi\le x\le 2\pi\end{cases}$$

Now at $x = \pi$, $\lim\limits_{x\to\pi^-}f(x)=1=\lim\limits_{x\to\pi^+}f(x)$

$\therefore$ $f(x)$ is continuous in $[0, 2\pi]$

Now, at $x = \pi$ L.H.D = $\lim\limits_{h\to 0}\frac{f(\pi-h)-f(\pi)}{-h}=0$

R.H.D. = $\lim\limits_{h\to 0}\frac{f(\pi+h)-f(\pi)}{h}=1-\frac{(\pi+h)\sin h-1}{h}$

$$=-\pi$$

$\therefore$ $f(x)$ is not differentiable at $x = \pi$

$\therefore$ $(m, n) = (1, 0)$

6. Consider a cuboid of sides $2x$, $4x$ and $5x$ and a closed hemisphere of radius r. If the sum of their surface areas is a constant k, then the ratio $x : r$, for which the sum of their volumes is maximum, is

(A) 2 : 5 (B) 19 : 45

(C) 3 : 8 (D) 19 : 15

Answer (B)

Sol. $\because\quad s_1+s_2=k$

$$76x^2+3\pi r^2=k$$

$$\therefore\quad 152x\frac{dx}{dr}+6\pi r=0$$

$$\therefore\quad \frac{dx}{dr}=\frac{-6\pi r}{152x}$$

Now $V=40x^3+\frac{2}{3}\pi r^3$

$$\therefore\quad \frac{dv}{dr}=120x^2\cdot\frac{dx}{dr}+2\pi r^2=0$$

$$\Rightarrow\quad 120x^2\cdot\left(\frac{-6\pi}{152}\frac{r}{x}\right)+2\pi r^2=0$$

$$\Rightarrow\quad 120\left(\frac{x}{r}\right)=2\pi\left(\frac{152}{6\pi}\right)$$

$$\Rightarrow\quad \left(\frac{x}{r}\right)=\frac{152}{3}\ \frac{1}{120}=\frac{19}{45}$$

7. The area of the region bounded by $y^2 = 8x$ and $y^2 = 16(3 - x)$ is equal to

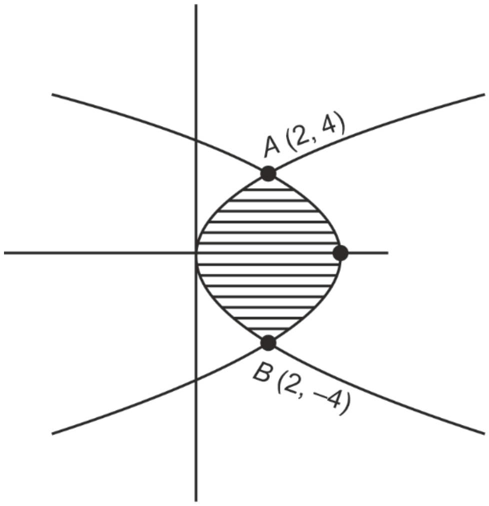
(A) $\frac{32}{3}$ (B) $\frac{40}{3}$

(C) 16 (D) 19

Answer (C)

Sol. $c_1 : y^2 = 8x$

$$c_2 : y^2 = 16(3-x)$$



Solving c_1 and c_2

$48 - 16x = 8x$

$\boxed{x = 2}$

$\therefore \quad y = \pm 4$

$\therefore$ Area of shaded region

$$= 2\int_0^4 \left\{ \left(\frac{48 - y^2}{16} \right) - \left(\frac{y^2}{8} \right) \right\} dy$$

$$= \frac{1}{8}\left[48y - y^3 \right]_0^4 = 16$$

8. If $\int \frac{1}{x}\sqrt{\frac{1-x}{1+x}}dx = g(x) + c,\ g(1) = 0,$ then $g\left(\frac{1}{2}\right)$ is equal to

(A) $\log_e\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right) + \frac{\pi}{3}$ (B) $\log_e\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right) + \frac{\pi}{3}$

(C) $\log_e\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right) - \frac{\pi}{3}$ (D) $\frac{1}{2}\log_e\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right) - \frac{\pi}{6}$

Answer (A)

Sol. $\because \quad \int \frac{1}{x}\sqrt{\frac{1-x}{1+x}}dx = g(x) + c$

$$\int_1^{\frac{1}{2}} \frac{1}{x}\sqrt{\frac{1-x}{1+x}}dx = g\left(\frac{1}{2}\right) - g(1)$$

$$\therefore \quad g\left(\frac{1}{2}\right) = \int_1^{\frac{1}{2}} \frac{1}{x}\sqrt{\frac{1-x}{1+x}}dx$$

$\cot x = \cos 2\theta$

$$= \int_0^{\frac{\pi}{6}} \frac{1}{\cos 2\theta} \cdot \frac{\sin\theta}{\cos\theta}(-2\sin 2\theta)d\theta$$

$$= -\int_0^{\frac{\pi}{6}} \frac{4\sin^2\theta}{\cos 2\theta}d\theta$$

$$= 2\int_0^{\frac{\pi}{6}} \frac{\left(1 - 2\sin^2\theta\right) - 1}{\cos 2\theta}d\theta$$

$$= 2\int_0^{\frac{\pi}{6}} (1 - \sec 2\theta)d\theta$$

$$= \frac{\pi}{3} - 2\cdot\frac{1}{2}\left[\ln\left|\sec 2\theta + \tan 2\theta\right|\right]_0^{\frac{\pi}{6}}$$

$$= \frac{\pi}{3} - \left[\ln\left|2 + \sqrt{3}\right| - \ln 1\right]$$

$$= \frac{\pi}{3} + \ln\left(\frac{1}{2 + \sqrt{3}}\right)$$

$$= \frac{\pi}{3} + \ln\left|\frac{\sqrt{3}-1}{\sqrt{3}+1}\right|$$

9. If $y = y(x)$ is the solution of the differential equation $x\frac{dy}{dx} + 2y = xe^x, y(1) = 0$ then the local maximum value of the function $z(x) = x^2y(x) - e^x, x \in R$ is

(A) $1 - e$ (B) 0

(C) $\frac{1}{2}$ (D) $\frac{4}{e} - e$

Answer (D)

Sol. $x\frac{dy}{dx} + 2y = xe^x,\ y(1) = 0$

$\frac{dy}{dx} + \frac{2}{x}y = e^x$, then $e^{\int \frac{2}{x}dx}dx = x^2$

$y \cdot x^2 = \int x^2e^x dx$

$yx^2 = x^2e^x - \int 2xe^x dx$

$= x^2e^x - 2\left(xe^x - e^x\right) + c$

$yx^2 = x^2e^x - 2xe^x + 2e^x + c$

$yx^2 = \left(x^2 - 2x + 2\right)e^x + c$

$0 = e + c \Rightarrow c = -e$

$y(x)\cdot x^2 - e^x = (x-1)^2 e^x - e$

$z(x) = (x-1)^2 e^x - e$

For local maximum $z'(x) = 0$

$\therefore \quad 2(x-1)e^x + (x-1)^2 e^x = 0$

$\therefore \quad x = -1$

And local maximum value $= z(-1)$

$$= \frac{4}{e} - e$$

10. If the solution of the differential equation

$\frac{dy}{dx} + e^x\left(x^2 - 2\right)y = \left(x^2 - 2x\right)\left(x^2 - 2\right)e^{2x}$ satisfies $y(0) = 0$, then the value of $y(2)$ is ______.

(A) –1 (B) 1

(C) 0 (D) e

Answer (C)

Sol. $\because \quad \frac{dy}{dx} + e^x\left(x^2 - 2\right)y = \left(x^2 - 2x\right)\left(x^2 - 2\right)e^{2x}$

Here, I.F. $= e^{\int e^x\left(x^2 - 2\right)dx}$

$= e^{\left(x^2 - 2x\right)e^x}$

$\therefore$ Solution of the differential equation is

$$y \cdot e^{\left(x^2-2x\right)e^x} = \int \left(x^2 - 2x\right)\left(x^2 - 2\right)e^{2x} \cdot e^{\left(x^2-2x\right)e^x} dx$$

$$= \int \left(x^2 - 2x\right)e^x \cdot \left(x^2 - 2\right)e^x \cdot e^{\left(x^2-2x\right)e^x} dx$$

Let $\left(x^2 - 2x\right)e^x = t$

$\therefore \quad \left(x^2 - 2\right)e^x dx = dt$

$y \cdot e^{\left(x^2-2x\right)e^x} = \int t \cdot e^t dt$

$y \cdot e^{\left(x^2-2x\right)e^x} = \left(x^2 - 2x - 1\right)e^{\left(x^2-2x\right)e^x} + c$

$\therefore \quad y(0) = 0$

$\therefore \quad c = 1$

$\therefore \quad y = \left(x^2 - 2x - 1\right) + e^{\left(2x - x^2\right)e^x}$

$\therefore \quad y(2) = -1 + 1$

$= 0$

11. If m is the slope of a common tangent to the curves $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and $x^2 + y^2 = 12$, then $12m^2$ is equal to:

(A) 6 (B) 9

(C) 10 (D) 12

Answer (B)

Sol. $C_1 : \frac{x^2}{16} + \frac{y^2}{9} = 1$ and $C_2 : x^2 + y^2 = 12$

Let $y = mx \pm \sqrt{16m^2 + 9}$ be any tangent to C_1 and if this is also tangent to C_2 then

$$\left|\frac{\sqrt{16m^2 + 9}}{\sqrt{m^2 + 1}}\right| = \sqrt{12}$$

$\Rightarrow \quad 16m^2 + 9 = 12m^2 + 12$

$\Rightarrow \quad 4m^2 = 3 \Rightarrow 12m^2 = 9$

12. The locus of the mid-point of the line segment joining the point (4, 3) and the points on the ellipse $x^2 + 2y^2 = 4$ is an ellipse with eccentricity:

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2\sqrt{2}}$

(C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{2}$

Answer (C)

Sol. Let $P(2\cos\theta, \sqrt{2}\sin\theta)$ be any point on ellipse $\frac{x^2}{4} + \frac{y^2}{2} = 1$ and $Q(4, 3)$ and let (h, k) be the mid point of PQ

then $h = \frac{2\cos\theta + 4}{2}$, $k = \frac{\sqrt{2}\sin\theta + 3}{2}$

$\therefore \quad \cos\theta = h - 2, \sin\theta = \frac{2k-3}{\sqrt{2}}$

$\therefore \quad (h-2)^2 + \left(\frac{2k-3}{\sqrt{2}}\right)^2 = 1$

$\Rightarrow \quad \frac{(x-2)^2}{1} + \frac{\left(y - \frac{3}{2}\right)^2}{\frac{1}{2}} = 1$

$\therefore \quad e = \sqrt{1 - \frac{1}{2}} = \frac{1}{\sqrt{2}}$

13. The normal to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{9}=1$ at the point $\left(8, 3\sqrt{3}\right)$ on it passes through the point:

(A) $\left(15, -2\sqrt{3}\right)$ (B) $\left(9, 2\sqrt{3}\right)$

(C) $\left(-1, 9\sqrt{3}\right)$ (D) $\left(-1, 6\sqrt{3}\right)$

Answer (C)

Sol. Given hyperbola : $\frac{x^2}{a^2}-\frac{y^2}{9}=1$

$\because$ It passes through $(8, 3\sqrt{3})$

$\because \frac{64}{a^2}-\frac{27}{9}=1 \Rightarrow a^2=16$

Now, equation of normal to hyperbola

$\frac{16x}{8}+\frac{9y}{3\sqrt{3}}=16+9$

$\Rightarrow 2x+\sqrt{3}y=25$ ...(i)

$\left(-1, 9\sqrt{3}\right)$ satisfies (i)

14. If the plane $2x + y - 5z = 0$ is rotated about its line of intersection with the plane $3x - y + 4z - 7 = 0$ by an angle of $\frac{\pi}{2}$, then the plane after the rotation passes through the point:

(A) (2, –2, 0) (B) (–2, 2, 0)

(C) (1, 0, 2) (D) (–1, 0, –2)

Answer (C)

Sol. $P_1 : 2x + y - 52 = 0$, $P_2 : 3x - y + 4z - 7 = 0$

Family of planes P_1 and P_2

$P : P_1 + \lambda P_2$

$\therefore P : (2+3\lambda)x+(1-\lambda)y+(-5+4\lambda)z-7\lambda=0$

$\because P \perp P_1 \therefore 4+6\lambda+1-\lambda+25-20\lambda=0$

$\boxed{\lambda=2}$

$\therefore P : 8x - y + 32 - 14 = 0$

It passes through the point (1, 0, 2)

15. If the lines $\vec{r}=\left(\hat{i}-\hat{j}+\hat{k}\right)+\lambda\left(3\hat{j}-\hat{k}\right)$ and $\vec{r}=\left(\alpha\hat{i}-\hat{j}\right)+\mu\left(2\hat{j}-3\hat{k}\right)$ are co-planar, then the distance of the plane containing these two lines from the point $(\alpha, 0, 0)$ is :

(A) $\frac{2}{9}$ (B) $\frac{2}{11}$

(C) $\frac{4}{11}$ (D) 2

Answer (B)

Sol. $\because$ Both lines are coplanar, so

$$\begin{vmatrix} \alpha-1 & 0 & -1 \\ 0 & 3 & -1 \\ 2 & 0 & -3 \end{vmatrix}=0$$

$\Rightarrow \alpha=\frac{5}{3}$

Equation of plane containing both lines

$$\begin{vmatrix} x-1 & y+1 & z-1 \\ 0 & 3 & -1 \\ 2 & 0 & -3 \end{vmatrix}=0$$

$\Rightarrow 9x + 2y + 6z = 13$

So, distance of $\left(\frac{5}{3}, 0, 0\right)$ from this plane

$=\frac{2}{\sqrt{81+4+36}}=\frac{2}{11}$

16. Let $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$, $\vec{b}=2\hat{i}-3\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}+\hat{k}$ be three given vectors. Let $\vec{v}$ be a vector in the plane of $\vec{a}$ and $\vec{b}$ whose projection on $\vec{c}$ is $\frac{2}{\sqrt{3}}$. If $\vec{v}\cdot\hat{j}=7$, then $\vec{v}\cdot\left(\hat{i}+\hat{k}\right)$ is equal to :

(A) 6 (B) 7

(C) 8 (D) 9

Answer (D)

Sol. Let $\vec{v}=\lambda_1\vec{a}+\lambda_2\vec{b}$, where $\lambda_1, \lambda_2 \in \mathbb{R}$.

$=(\lambda_1+2\lambda_2)\hat{i}+(\lambda_1-3\lambda_2)\hat{j}+(2\lambda_1+\lambda_2)\hat{k}$

$\because$ Projection of $\vec{v}$ on $\vec{c}$ is $\frac{2}{\sqrt{3}}$

$\therefore \frac{\lambda_1+2\lambda_2-\lambda_1+3\lambda_2+2\lambda_1+\lambda_2}{\sqrt{3}}=\frac{2}{\sqrt{3}}$

$\therefore \lambda_1 + 3\lambda_2 = 1$ ...(i)

and $\vec{v}\cdot\hat{j}=7 \Rightarrow \lambda_1-3\lambda_2=7$ ...(ii)

from equation (i) and (ii)

$\lambda_1 = 4$, $\lambda_2 = -1$

$\therefore \vec{v}=2\hat{i}+7\hat{j}+7\hat{k}$

$\therefore \vec{v}\cdot(\hat{i}+\hat{k})=2+7$

$= 9$

17. The mean and standard deviation of 50 observations are 15 and 2 respectively. It was found that one incorrect observation was taken such that the sum of correct and incorrect observations is 70. If the correct mean is 16, then the correct variance is equal to :

(A) 10 (B) 36

(C) 43 (D) 60

Answer (C)

Sol. Given $\bar{x} = 15, \sigma = 2 \Rightarrow \sigma^2 = 4$

$\therefore$ $x_2 + x_2 + \ldots\ldots + x_{50} = 15 \times 50 = 750$

$$4 = \frac{x_1^2 + x_2^2 + \ldots. + x_{50}^2}{50} - 225$$

$$\therefore \quad x_1^2 + x_2^2 + \ldots\ldots + x_{50}^2 = 50 \times 229$$

Let a be the correct observation and b is the incorrect observation

then $a + b = 70$

and $16 = \dfrac{750 - b + a}{50}$

$\therefore$ $a - b = 50 \Rightarrow a = 60, b = 10$

$\therefore$ Correct variance $= \dfrac{50 \times 229 + 60^2 - 10^2}{50} - 256$

$= 43$

18. 16 sin(20°) sin(40°) sin(80°) is equal to :

(A) $\sqrt{3}$ (B) $2\sqrt{3}$

(C) 3 (D) $4\sqrt{3}$

Answer (B)

Sol. 16sin20° · sin40° · sin80°

$= 4\sin 60°$ {$\because$ $4\sin\theta \cdot \sin(60° - \theta) \cdot \sin(60° + \theta) = \sin 3\theta$ }

$= 2\sqrt{3}$

19. If the inverse trigonometric functions take principal values, then $\cos^{-1}\left(\frac{3}{10}\cos\left(\tan^{-1}\left(\frac{4}{3}\right)\right) + \frac{2}{5}\sin\left(\tan^{-1}\left(\frac{4}{3}\right)\right)\right)$ is equal to :

(A) 0 (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

Answer (C)

Sol. $\cos^{-1}\left(\frac{3}{10}\cos\left(\tan^{-1}\left(\frac{4}{3}\right)\right) + \frac{2}{5}\sin\left(\tan^{-1}\left(\frac{4}{3}\right)\right)\right)$

$= \cos^{-1}\left(\frac{3}{10} \cdot \frac{3}{5} + \frac{2}{5} \cdot \frac{4}{5}\right)$

$= \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

20. Let $r \in \{p, q, \sim p, \sim q\}$ be such that the logical statement $r \vee (\sim p) \Rightarrow (p \wedge q) \vee r$ is a tautology. Then r is equal to :

(A) p (B) q

(C) $\sim p$ (D) $\sim q$

Answer (C)

Sol. Clearly r must be equal to $\sim p$

$\because$ $\sim p \vee \sim p = \sim p$

and $(p \wedge q) \vee \sim p = p$

$\therefore$ $\sim p \Rightarrow p$ = tautology.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse andw the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Let $f : \mathbb{R} \to \mathbb{R}$ satisfy $f(x + y) = 2^x f(y) + 4^y f(x)$, $\forall x, y \in \mathbb{R}$. If $f(2) = 3$, then $14 \cdot \dfrac{f'(4)}{f'(2)}$ is equal to ____.

Answer (248)

Sol. $\because$ $f(x + y) = 2^x f(y) + 4^y f(x)$ …(1)

Now, $f(y + x)$ $2^y f(x) + 4^x f(y)$ …(2)

$\therefore$ $2^x f(y) + 4^y f(x) = 2^y f(x) + 4^x f(y)$

$(4^y - 2^y) f(x) = (4^x - 2^x) f(y)$

$\dfrac{f(x)}{4^x - 2^x} = \dfrac{f(y)}{4^y - 2^y} = k$ (Say)

$\therefore$ $f(x) = k(4^x - 2^x)$

$\because$ $f(2) = 3$ then $k = \dfrac{1}{4}$

$\therefore \quad f(x) = \dfrac{4^x - 2^x}{4}$

$\therefore \quad f'(x) = \dfrac{4^x \ln 4 - 2^x \ln 2}{4}$

$f'(x) = \dfrac{(2.4^x - 2^x)\, ln\, 2}{4}$

$\therefore \quad \dfrac{f'(4)}{f'(2)} = \dfrac{2.256 - 16}{2.16 - 4}$

$\therefore \quad 14\dfrac{f'(4)}{f'(2)} = 248$

2. Let p and q be two real numbers such that $p + q = 3$ and $p^4 + q^4 = 369$. Then $\left(\dfrac{1}{p} + \dfrac{1}{q}\right)^{-2}$ is equal to _______.

Answer (4)

Sol. $\because \quad p + q = 3 \qquad \ldots(i)$

and $p^4 + q^4 = 369 \qquad \ldots(ii)$

$\{(p + q)^2 - 2pq\}^2 - 2p^2q^2 = 369$

or $(9 - 2pq)^2 - 2(pq)^2 = 369$

or $(pq)^2 - 18pq - 144 = 0$

$\therefore \quad pq = -6$ or 24

But $pq = 24$ is not possible

$\therefore \quad pq = -6$

Hence, $\left(\dfrac{1}{p} + \dfrac{1}{q}\right)^{-2} = \left(\dfrac{pq}{p+q}\right)^2 = (-2)^2 = 4$

3. If $z^2 + z + 1 = 0$, $z \in \mathbb{C}$, then $\left|\sum_{n=1}^{15}\left(z^n + (-1)^n \dfrac{1}{z^n}\right)^2\right|$ is equal to ________.

Answer (2)

Sol. $\because \quad z^2 + z + 1 = 0 \qquad \Rightarrow \quad \omega$ or ω^2

$\because \quad \left|\sum_{n=1}^{15}\left(z^n + (-1)^n \dfrac{1}{z^n}\right)^2\right|$

$= \left|\sum_{n=1}^{15} z^{2n} + \sum_{n=1}^{15} z^{-2n} + 2\cdot\sum_{n=1}^{15}(-1)^n\right|$

$= |0 + 0 - 2|$

$= 2$

4. Let $X = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, $Y = \alpha I + \beta X + \gamma X^2$ and

$Z = \alpha^2 I - \alpha\beta X + \left(\beta^2 - \alpha\gamma\right) X^2, \alpha, \beta, \gamma \in \mathbb{R}$.

If $Y^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{-2}{5} & \frac{1}{5} \\ 0 & \frac{1}{5} & \frac{-2}{5} \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$, then $(\alpha - \beta + \gamma)^2$ is equal to ___________.

Answer (100)

Sol. $\because \quad X = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$\therefore \quad X^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\therefore \quad Y = \alpha I + \beta X + \gamma X^2 \begin{bmatrix} \alpha & \beta & \gamma \\ 0 & \alpha & \beta \\ 0 & 0 & \alpha \end{bmatrix}$

$\because \quad Y \cdot Y^{-1} = I$

$\therefore \quad \begin{bmatrix} \alpha & \beta & \gamma \\ 0 & \alpha & \beta \\ 0 & 0 & \alpha \end{bmatrix} \begin{bmatrix} \frac{1}{5} & \frac{-2}{5} & \frac{1}{5} \\ 0 & \frac{1}{5} & \frac{-2}{5} \\ 0 & 0 & \frac{1}{5} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore \quad \begin{bmatrix} \frac{\alpha}{5} & \frac{\beta - 2\alpha}{5} & \frac{\alpha - 2\beta + \gamma}{5} \\ 0 & \frac{\alpha}{5} & \frac{\beta - 2\alpha}{5} \\ 0 & 0 & \frac{\alpha}{5} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore \quad \alpha = 5, \beta = 10, \gamma = 15$

$\therefore \quad (\alpha - \beta + \gamma)^2 = 100$

5. The total number of 3-digit numbers, whose greatest common divisor with 36 is 2, is _______-.

Answer (150)

Sol. $\because$ $x \in [100, 999]$, $x \in N$

Then $\frac{x}{2} \in [50, 499], \frac{x}{2} \in N$

Number whose G.C.D with 18 is 1 in this range have the required condition. There are 6 such number from 18 × 3 to 18 × 4. Similarly from 18 × 4 to 18 × 5….., 26 × 18 to 27 × 18

$\therefore$ Total numbers = 24 × 6 + 6 = 150

The extra numbers are 53, 487, 491, 493, 497 and 499.

6. If $\left({}^{40}C_0\right)+\left({}^{41}C_1\right)+\left({}^{42}C_2\right)+...+\left({}^{60}C_{20}\right)=\frac{m}{n}\,{}^{60}C_{20}$ m and n are coprime, then $m + n$ is equal to _____.

Answer (102)

Sol. ${}^{40}C_0 + {}^{41}C_1 + {}^{42}C_2 + \ ......+\ {}^{60}C_{20}$

$= {}^{40}C_{40} + {}^{41}C_{40} + {}^{42}C_{40} + \ ......+\ {}^{60}C_{40}$

$= {}^{61}C_{41}$

$\frac{61}{41} \cdot {}^{60}C_{40}$

$\therefore$ m = 61, n = 41

$\therefore$ $m + n$ = 102

7. If a_1 (> 0), a_2, a_3, a_4, a_5 are in a G.P., $a_2 + a_4 = 2a_3 + 1$ and $3a_2 + a_3 = 2a_4$, then $a_2 + a_4 + 2a_5$ is equal to _______.

Answer (40)

Sol. Let G.P. be $a_1 = a$, $a_2 = ar$, $a_3 = ar^2$, ……

$\because$ $3a_2 + a_3 = 2a_4$

$\Rightarrow$ $3ar + ar^2 = 2ar^3$

$\Rightarrow$ $2ar^2 - r - 3 = 0$

$\therefore$ $r = -1$ or $\frac{3}{2}$

$\because$ $a_1 = a > 0$ then $r \neq -1$

Now, $a_2 + a_4 = 2a_3 + 1$

$ar + ar^3 = 2ar^2 + 1$

$a\left(\frac{3}{2}+\frac{27}{8}-\frac{9}{2}\right)=1$

$\therefore$ $a = \frac{8}{3}$

$\therefore$ $a_2 + a_4 + 2a_5 = a\left(r + r^3 + 2r^4\right)$

$= \frac{8}{3}\left(\frac{3}{2}+\frac{27}{8}+\frac{81}{8}\right)$

$= 40$

8. The integral $\frac{24}{\pi}\int_0^{\sqrt{2}} \frac{\left(2-x^2\right)dx}{\left(2+x^2\right)\sqrt{4+x^4}}$ is equal to _______.

Answer (3)

Sol. $I = \frac{24}{\pi}\int_0^{\sqrt{2}} \frac{2-x^2}{\left(2+x^2\right)\sqrt{4+x^4}}dx$

Let $x = \sqrt{2}t \Rightarrow dx = \sqrt{2}dt$

$I = \frac{24}{\pi}\int_0^{1} \frac{\left(2-2t^2\right)\cdot\sqrt{2}dt}{\left(2+2t^2\right)\sqrt{4+4t^4}}$

$= \frac{12\sqrt{2}}{\pi}\int_0^{1} \frac{\left(\frac{1}{t^2}-1\right)dt}{\left(t+\frac{1}{t}\right)\sqrt{\left(t+\frac{1}{t}\right)^2-2}}$

Let $t + \frac{1}{t} = u$

$\Rightarrow$ $\left(1-\frac{1}{t^2}\right)dt = du$

$= \frac{12\sqrt{2}}{\pi}\int_{\infty}^{2} \frac{-du}{u\sqrt{4^2-2}}$

$= \frac{12\sqrt{2}}{\pi}\int_{2}^{\infty} \frac{du}{u^2\sqrt{-\left(\frac{\sqrt{2}}{u}\right)^2}}$

$= \frac{12\sqrt{2}}{\pi}\int_{\frac{1}{\sqrt{2}}}^{0} \frac{-\frac{1}{\sqrt{2}}dp}{\sqrt{1-p^2}}$

$= \frac{12}{\pi}\left[\sin^{-1}p\right]_0^{\frac{1}{\sqrt{2}}}$

$= \frac{12}{\pi}\cdot\frac{\pi}{4}$

= 3

9. Let a line L_1 be tangent to the hyperbola $\frac{x^2}{16}-\frac{y^2}{4}=1$ and let L_2 be the line passing through the origin and perpendicular to L_1. If the locus of the point of intersection of L_1 and L_2 is $(x^2+y^2)^2=\alpha x^2+\beta y^2$, then $\alpha+\beta$ is equal to____.

Answer (12)

Sol. Equation of L_1 is

$$\frac{x\sec\theta}{4}-\frac{y\tan\theta}{2}=1 \qquad \ldots(i)$$

Equation of line L_2 is

$$\frac{x\tan\theta}{2}+\frac{y\sec\theta}{4}=0 \qquad \ldots(ii)$$

$\because$ Required point of intersection of L_1 and L_2 is (x_1, y_1) then

$$\frac{x_1\sec\theta}{4}-\frac{y_1\tan\theta}{2}-1=0 \quad \ldots(iii)$$

and $\frac{y_1\sec\theta}{4}+\frac{x_1\tan\theta}{2}=0 \qquad \ldots(iv)$

From equations (iii) and (iv)

$$\sec\theta=\frac{4x_1}{x_1^2+y_1^2} \text{ and } \tan\theta=\frac{-2y_1}{x_1^2+y_1^2}$$

$\therefore$ Required locus of (x_1, y_1) is

$$(x^2+y^2)^2=16x^2-4y^2$$

$\therefore \quad \alpha=16, \beta=-4$

$\therefore \quad \alpha+\beta=12$

10. If the probability that a randomly chosen 6-digit number formed by using digits 1 and 8 only is a multiple of 21 is p, then 96 p is equal to _________ .

Answer (33)

Sol. Total number of numbers from given

Condition = $n(s) = 2^6$.

Every required number is of the form

$$A = 7\cdot(10^{a_1}+10^{a_2}+10^{a_3}+......)+111111$$

Here 111111 is always divisible by 21.

$\therefore$ If A is divisible by 21 then

$10^{a_1}+10^{a_2}+10^{a_3}+......$ must be divisible by 3.

For this we have ${}^6C_0+{}^6C_3+{}^6C_6$ cases are there

$\therefore \quad n(E)={}^6C_0+{}^6C_3+{}^6C_6=22$

$\therefore$ Required probability = $\frac{22}{2^6}=p$

$\therefore \quad \frac{11}{32}=p$

$\therefore \quad 96p = 33$

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