

26/06/2022

Morning



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Answers & Solutions

Time : 3 hrs.

for

M.M. : 300

JEE (Main)-2022 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

- (1) The test is of **3 hours** duration.
- (2) The Test Booklet consists of 90 questions. The maximum marks are 300.
- (3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.
 - (i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **-1 mark** for wrong answer.
 - (ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **-1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. An expression for a dimensionless quantity P is given by $P = \frac{\alpha}{\beta} \log_e \left(\frac{kt}{\beta x} \right)$; where α and β are constants, x is distance; k is Boltzmann constant and t is the temperature. Then the dimensions of α will be
- (A) $[M^0L^{-1}T^0]$ (B) $[ML^0T^{-2}]$
 (C) $[MLT^{-2}]$ (D) $[ML^2T^{-2}]$

Answer (C)

Sol. $[\alpha] = [\beta] = \left[\frac{kt}{x} \right]$
 $= \left[\frac{ML^2T^{-2}}{L} \right]$
 $= [MLT^{-2}]$

2. A person is standing in an elevator. In which situation, he experiences weight loss?
- (A) When the elevator moves upward with constant acceleration
 (B) When the elevator moves downward with constant acceleration
 (C) When the elevator moves upward with uniform velocity
 (D) When the elevator moves downward with uniform velocity

Answer (B)

Sol. Apparent weight = $m(g - a)$

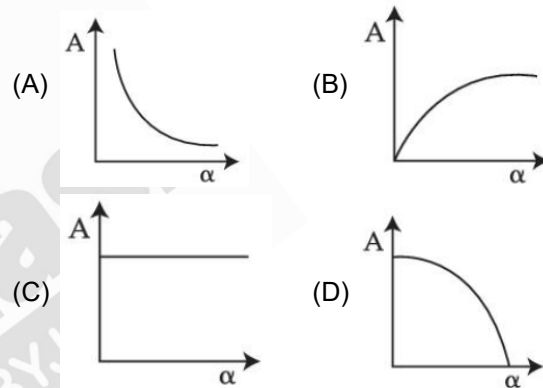
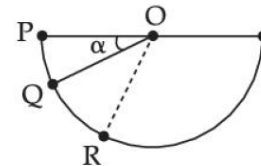
\Rightarrow Weight loss in downward accelerated elevator

3. An object is thrown vertically upwards. At its maximum height, which of the following quantity becomes zero?
- (A) Momentum
 (B) Potential Energy
 (C) Acceleration
 (D) Force

Answer (A)

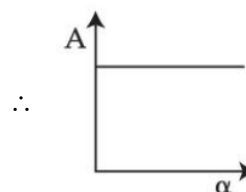
Sol. At topmost position,
 $v = 0$
 \Rightarrow momentum = 0

4. A ball is released from rest from point P of a smooth semi-spherical vessel as shown in figure. The ratio of the centripetal force and normal reaction on the ball at point Q is A while angular position of point Q is α with respect to point P . Which of the following graphs represent the correct relation between A and α when ball goes from Q to R ?



Answer (C)

Sol. $N = mg \sin \alpha + \frac{mv^2}{R}$
 and, $v^2 = 2g \times R \sin \alpha$
 $\therefore N = mg \sin \alpha + m \times (2g \sin \alpha)$
 $= 3mg \sin \alpha$
 \therefore ratio, $A = \frac{mv^2}{N}$
 $= \frac{2mg \sin \alpha}{3mg \sin \alpha}$
 $= \frac{2}{3}$



5. A thin circular ring of mass M and radius R is rotating with a constant angular velocity 2 rad s^{-1} in a horizontal plane about an axis vertical to its plane and passing through the center of the ring. If two objects each of mass m be attached gently to the opposite ends of a diameter of ring, the ring will then rotate with an angular velocity (in rad s^{-1}).

- (A) $\frac{M}{(M+m)}$ (B) $\frac{(M+2m)}{2M}$
 (C) $\frac{2M}{(M+2m)}$ (D) $\frac{2(M+2m)}{M}$

Answer (C)

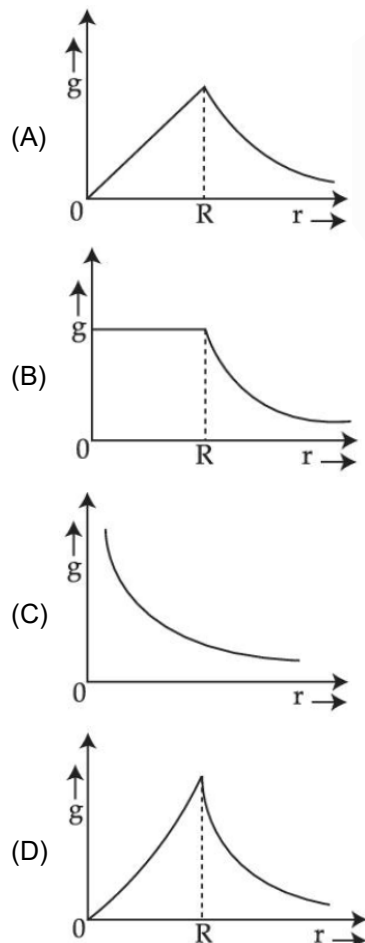
Sol. $I_1\omega_1 = I_2\omega_2$

$$\Rightarrow MR^2 \times 2 = (MR^2 + 2mR^2)\omega_2$$

$$\Rightarrow \omega_2 = \frac{2M}{M+2m}$$

6. The variation of acceleration due to gravity (g) with distance (r) from the center of the earth is correctly represented by

(Given $R =$ radius of earth)



Answer (A)

Sol. For $r < R$ $g = \frac{Gmr}{R^3} = Cr$ ($C = \text{Constant}$)

For $r > R$ $g = \frac{Gm}{r^2} = \frac{C'}{r^2}$ ($C' = \text{Constant}$)

For the above equations the best suited graph is as given in option (A)

7. The efficiency of a Carnot's engine, working between steam point and ice point, will be

- (A) 26.81%
 (B) 37.81%
 (C) 47.81%
 (D) 57.81%

Answer (A)

Sol. $\eta = 1 - \frac{T_C}{T_H} = \frac{T_H - T_C}{T_H}$

$$= \frac{100}{373} \times 100\%$$

$$= 26.81\%$$

\Rightarrow option (A)

8. Time period of a simple pendulum in a stationary lift is ' T '. If the lift accelerates with $\frac{g}{6}$ vertically upwards

then the time period will be

(Where $g =$ acceleration due to gravity)

(A) $\sqrt{\frac{6}{5}}T$

(B) $\sqrt{\frac{5}{6}}T$

(C) $\sqrt{\frac{6}{7}}T$

(D) $\sqrt{\frac{7}{6}}T$

Answer (C)

Sol. $T' = 2\pi\sqrt{\frac{l}{g_{\text{eff}}}}$

$$T' = 2\pi\sqrt{\frac{l}{g + \frac{g}{6}}} = 2\pi\sqrt{\frac{6l}{7g}}$$

$$\Rightarrow T' = \sqrt{\frac{6}{7}}T$$

\Rightarrow Option (C)

9. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats 1.4. Vessel is moving with speed v and is suddenly brought to rest. Assuming no heat is lost to the surrounding and vessel temperature of the gas increases by

(R = universal gas constant)

- (A) $\frac{Mv^2}{7R}$
 (B) $\frac{Mv^2}{5R}$
 (C) $2\frac{Mv^2}{7R}$
 (D) $7\frac{Mv^2}{5R}$

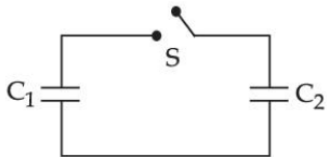
Answer (B)

Sol. $\frac{1}{2}mv^2 = n\frac{5}{2}R\Delta T$

$$\Rightarrow \Delta T = \frac{mv^2}{5nR} = \frac{Mv^2}{5R}$$

Option (B)

10. Two capacitors having capacitance C_1 and C_2 respectively are connected as shown in figure. Initially, capacitor C_1 is charged to a potential difference V volt by a battery. The battery is then removed and the charged capacitor C_1 is now connected to uncharged capacitor C_2 by closing the switch S . The amount of charge on the capacitor C_2 , after equilibrium, is



- (A) $\frac{C_1C_2}{(C_1+C_2)}V$ (B) $\frac{(C_1+C_2)}{C_1C_2}V$
 (C) $(C_1+C_2)V$ (D) $(C_1-C_2)V$

Answer (A)

Sol. $V_{\text{common}} = \frac{C_1V}{C_1+C_2}$

\Rightarrow Charge on capacitor C_2

$$= C_2 V_{\text{common}} = \frac{C_1C_2V}{C_1+C_2}$$

\Rightarrow Option (A)

11. Given below two statements: One is labelled as Assertion (A) and other is labelled as Reason (R).

Assertion (A) : Non-polar materials do not have any permanent dipole moment.

Reason (R) : When a non-polar material is placed in an electric field, the centre of the positive charge distribution of its individual atom or molecule coincides with the centre of the negative charge distribution.

In the light of above statements, choose the most appropriate answer from the options given below.

- (A) Both (A) and (R) are correct and (R) is the correct explanation of (A).
 (B) Both (A) and (R) are correct and (R) is not the correct explanation of (A).
 (C) (A) is correct but (R) is not correct.
 (D) (A) is not correct but (R) is correct.

Answer (C)

Sol. Non polar material does not have any permanent dipole moment and when placed in an electric field the positive and negative charges displace in opposite directions and result into an induced dipole moment as long as the field is applied.

12. The magnetic flux through a coil perpendicular to its plane is varying according to the relation $\phi = (5t^3 + 4t^2 + 2t - 5)$ Weber. If the resistance of the coil is 5 ohm, then the induced current through the coil at $t = 2$ s will be,

- (A) 15.6 A
 (B) 16.6 A
 (C) 17.6 A
 (D) 18.6 A

Answer (A)

Sol. $\text{Emf} = -\frac{d\phi}{dt} = -(15t^2 + 8t + 2)$

So, $i = \frac{|\text{Emf}|}{R} = \frac{(15t^2 + 8t + 2)}{5}$

at $t = 2$

$i = 15.6 \text{ A}$

13. An aluminium wire is stretched to make its length, 0.4% larger. The percentage change in resistance is :

- (A) 0.4% (B) 0.2%
(C) 0.8% (D) 0.6%

Answer (C)

Sol. When the wire is stretched, volume remains constant. If length is increased by 0.4% area will decrease by 0.4% so

$$\text{From } R = \rho \frac{l}{A}$$

$$\frac{dR}{R} \times 100 = \frac{dl}{l} \times 100 + \frac{dA}{A} \times 100$$

$$\%R = 0.4 + 0.4 = 0.8\%$$

14. A proton and an alpha particle of the same velocity enter in a uniform magnetic field which is acting perpendicular to their direction of motion. The ratio of the radii of the circular paths described by the alpha particle and proton is :

- (A) 1:4 (B) 4:1
(C) 2:1 (D) 1:2

Answer (C)

Sol. $R = \frac{mv}{qB}$

$$\frac{R_\alpha}{R_p} = \frac{m_\alpha / q_\alpha}{m_p / q_p} = 2$$

15. If electric field intensity of a uniform plane electromagnetic wave is given as

$$E = -301.6 \sin(kz - \omega t) \hat{a}_x + 452.4 \sin(kz - \omega t) \hat{a}_y \frac{V}{m}$$

Then, magnetic intensity 'H' of this wave in Am⁻¹ will be :

[Given : Speed of light in vacuum $c = 3 \times 10^8$ ms⁻¹, Permeability of vacuum $\mu_0 = 4\pi \times 10^{-7}$ NA⁻²]

- (A) $+0.8 \sin(kz - \omega t) \hat{a}_y + 0.8 \sin(kz - \omega t) \hat{a}_x$
(B) $+1.0 \times 10^{-6} \sin(kz - \omega t) \hat{a}_y + 1.5 \times 10^{-6} \sin(kz - \omega t) \hat{a}_x$
(C) $-0.8 \sin(kz - \omega t) \hat{a}_y - 1.2 \sin(kz - \omega t) \hat{a}_x$
(D) $-1.0 \times 10^{-6} \sin(kz - \omega t) \hat{a}_y - 1.5 \times 10^{-6} \sin(kz - \omega t) \hat{a}_x$

Answer (C)

Sol. We know

$$\vec{B} \times \vec{C} = \vec{E}$$

Taking cross product of \vec{C} both the sides

$$\vec{C} \times (\vec{B} \times \vec{C}) = \vec{C} \times \vec{E}$$

$$\text{So } \vec{B} = \frac{\vec{C} \times \vec{E}}{C^2}$$

$$\vec{C} = C \hat{k}$$

$$\vec{E} = -301.6 \sin(kz - \omega t) \hat{a}_x + 452.4 \sin(kz - \omega t) \hat{a}_y$$

$$\text{and } \vec{H} = \frac{\vec{B}}{\mu_0}$$

On solving

$$\vec{H} = -0.8 \sin(kz - \omega t) \hat{a}_y - 1.2 \sin(kz - \omega t) \hat{a}_x$$

16. In free space, an electromagnetic wave of 3 GHz frequency strikes over the edge of an object of size $\frac{\lambda}{100}$, where λ is the wavelength of the wave in free space. The phenomenon, which happens there will be:

- (A) Reflection
(B) Refraction
(C) Diffraction
(D) Scattering

Answer (D)

Sol. Since size is of the order of $\frac{\lambda}{100}$, hence scattering will take place.

17. An electron with speed v and a photon with speed c have the same de-Broglie wavelength. If the kinetic energy and momentum of electron are E_e and p_e and that of photon are E_{ph} and p_{ph} respectively. Which of the following is correct?

- (A) $\frac{E_e}{E_{ph}} = \frac{2c}{v}$
(B) $\frac{E_e}{E_{ph}} = \frac{v}{2c}$
(C) $\frac{p_e}{p_{ph}} = \frac{2c}{v}$
(D) $\frac{p_e}{p_{ph}} = \frac{v}{2c}$

Answer (B)

Sol. $\lambda_e = \lambda_{ph} \Rightarrow \frac{h}{p_e} = \frac{hc}{E_{ph}}$

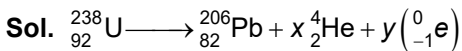
$\Rightarrow E_{ph} = p_e \times c = 2E_e \frac{c}{v}$

$\Rightarrow \frac{E_e}{E_{ph}} = \frac{v}{2c}$

18. How many alpha and beta particles are emitted when Uranium ${}_{92}\text{U}^{238}$ decays to lead ${}_{82}\text{Pb}^{206}$?

- (A) 3 alpha particles and 5 beta particles
- (B) 6 alpha particles and 4 beta particles
- (C) 4 alpha particles and 5 beta particles
- (D) 8 alpha particles and 6 beta particles

Answer (D)

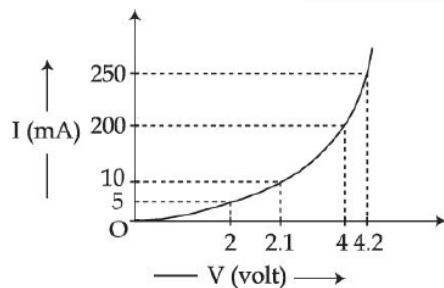


$\Rightarrow 206 + 4x = 238 \quad \dots(1)$

and $82 + 2x - y = 92 \quad \dots(2)$

$\Rightarrow x = 8$ and $y = 6$

19. The I - V characteristics of a p-n junction diode in forward bias is shown in the figure. The ratio of dynamic resistance, corresponding to forward bias voltage of 2 V and 4 V respectively, is :



- (A) 1 : 2
- (B) 5 : 1
- (C) 1 : 40
- (D) 20 : 1

Answer (B)

Sol. Dynamic resistance $= \frac{dV}{dI}$

$\Rightarrow r_1 = \frac{2.1 - 2}{10 - 5} \text{ k}\Omega$

& $r_2 = \frac{4.2 - 4}{250 - 200} \text{ k}\Omega$

$\Rightarrow r_1 : r_2 = 5 : 1.$

20. Choose the correct statement for amplitude modulation :

- (A) Amplitude of modulating signal is varied in accordance with the information signal.
- (B) Amplitude of modulated signal is varied in accordance with the information signal.
- (C) Amplitude of carrier signal is varied in accordance with the information signal.
- (D) Amplitude of modulated signal is varied in accordance with the modulating signal.

Answer (C)

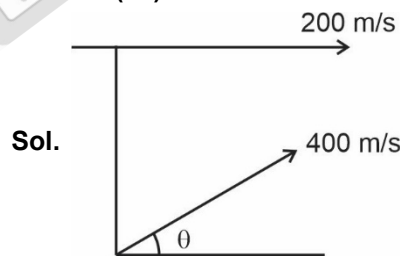
Sol. In amplitude modulation, amplitude of carrier signal is varied according to the message signal.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. A fighter jet is flying horizontally at a certain altitude with a speed of 200 ms^{-1} . When it passes directly overhead an anti-aircraft gun, a bullet is fired from the gun, at an angle θ with the horizontal, to hit the jet. If the bullet speed is 400 m/s , the value of θ will be ____°.

Answer (60)



Sol. To hit the jet
 $400 \cos\theta = 200$

$\Rightarrow \cos\theta = \frac{1}{2}$

$\Rightarrow \theta = 60^\circ$

2. A ball of mass 0.5 kg is dropped from the height of 10 m . The height, at which the magnitude of velocity becomes equal to the magnitude of acceleration due to gravity, is ____ m.

[Use $g = 10 \text{ m/s}^2$]

Answer (5)

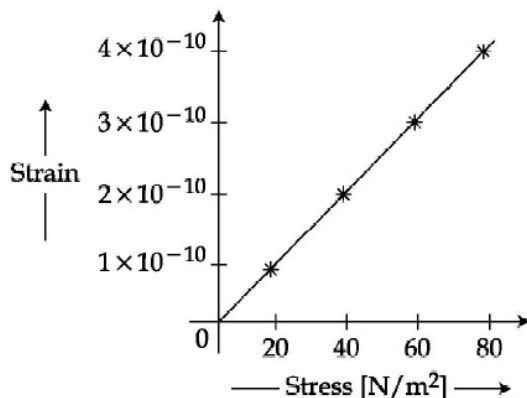
Sol. $gt = g$

$$\Rightarrow t = 1 \text{ sec}$$

$$\Delta h = \frac{1}{2}gt^2 = \frac{1}{2} \times 5 \times 1^2 = 5 \text{ m}$$

$$\begin{aligned} \therefore h &= H - \Delta h \\ &= 10 - 5 \\ &= 5 \text{ m} \end{aligned}$$

3. The elastic behaviour of material for linear stress and linear strain, is shown in the figure. The energy density for a linear strain of 5×10^{-4} is ____ kJ/m^3 . Assume that material is elastic upto the linear strain of 5×10^{-4} .



Answer (25)

Sol. $u_d = \frac{1}{2} \times Y \times (\text{strain})^2$

$$\begin{aligned} &= \frac{1}{2} \times \left(\frac{20}{10^{-10}} \right) \times (5 \times 10^{-4})^2 \\ &= 10^{11} \times 25 \times 10^{-8} \\ &= 25 \times 10^3 \text{ J/m}^3 \\ &= 25 \text{ kJ/m}^3 \end{aligned}$$

4. The elongation of a wire on the surface of the earth is 10^{-4} m. The same wire of same dimensions is elongated by 6×10^{-5} m on another planet. The acceleration due to gravity on the planet will be ____ ms^{-2} . (Take acceleration due to gravity on the surface of earth = 10 ms^{-2})

Answer (6)

Sol. $\Delta l = \frac{M'gl}{2A y}$

$$\Rightarrow \Delta l \propto g$$

$$\Rightarrow \frac{g_p}{g_e} = \frac{\Delta l_p}{\Delta l_e} = \frac{6 \times 10^{-5}}{10 \times 10^{-5}}$$

$$\Rightarrow g_p = 6 \text{ m/s}^2 \text{ as } g_e = 10 \text{ m/s}^2$$

5. A 10Ω , 20 mH coil carrying constant current is connected to a battery of 20 V through a switch. Now after switch is opened current becomes zero in $100 \mu\text{s}$. The average e.m.f. induced in the coil is ____ V .

Answer (400)

Sol. Initial flux through inductor = LI

$$\Rightarrow \phi_i = 20 \times 10^{-3} \times \frac{20}{10}$$

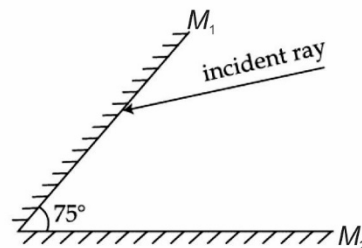
$$= 4 \times 10^{-2} \text{ weber}$$

Final flux = 0

$$\Rightarrow \text{average emf}$$

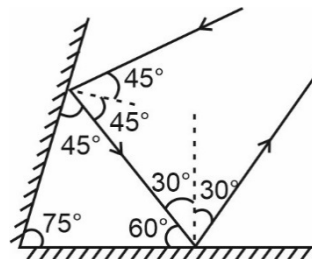
$$\begin{aligned} &= \frac{|\phi_i - \phi_f|}{100 \mu\text{s}} \\ &= \frac{4 \times 10^{-2}}{10^{-4}} = 400 \text{ V} \end{aligned}$$

6. A light ray is incident, at an incident angle θ_1 , on the system of two plane mirrors M_1 and M_2 having an inclination angle 75° between them (as shown in figure). After reflecting from mirror M_1 it gets reflected back by the mirror M_2 with an angle of reflection 30° . The total deviation of the ray will be ____ degree.



Answer (210)

Sol.



On first reflection angle of deviation is 90° and on second reflection angle of deviation is 120°

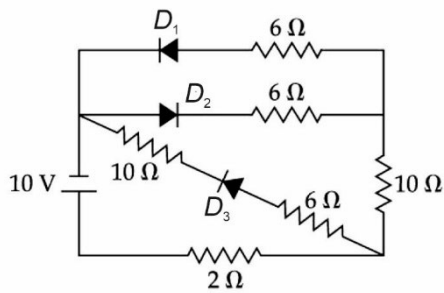
so total deviation is $\delta = 90^\circ + 120^\circ = 210^\circ$

7. In a vernier callipers, each cm on the main scale is divided into 20 equal parts. If tenth vernier scale division coincides with ninth main scale division. Then the value of vernier constant will be $\text{---} \times 10^{-2}$ mm.

Answer (5)

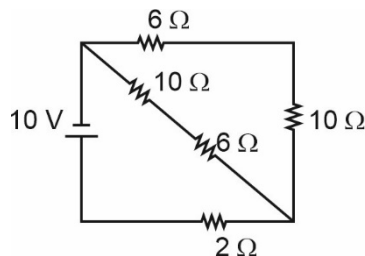
$$\begin{aligned} \text{Sol. LC} &= \frac{1 \text{ MSD}}{\text{VSD}} = \frac{\frac{1}{20} \text{ cm}}{10} \\ &= \frac{1}{200} \text{ cm} \\ &= 5 \times 10^{-2} \text{ mm} \end{aligned}$$

8. As per the given circuit, the value of current through the battery will be --- A.



Answer (1)

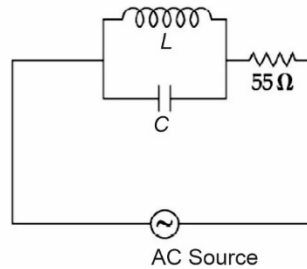
Sol. Because of diode D_2 current will not flow through it so new circuit diagram is



so $R_{\text{net}} = 10 \Omega$

$$\text{and } i = \frac{V}{R_{\text{net}}} = 1 \text{ A}$$

9. A 110 V, 50 Hz, AC source is connected in the circuit (as shown in figure). The current through the resistance 55Ω , at resonance in the circuit, will be --- A.

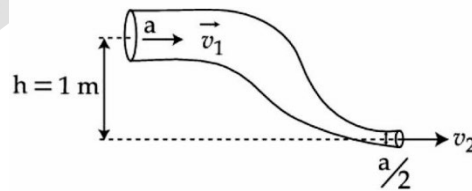


Answer (0)

Sol. At resonance $\left(\omega = \frac{1}{\sqrt{LC}} \right)$, impedance of the circuit is infinite.

$$\Rightarrow \text{Current through resistance} = 0.$$

10. An ideal fluid of density 800 kg m^{-3} , flows smoothly through a bent pipe (as shown in figure) that tapers in cross-sectional area from a to $\frac{a}{2}$. The pressure difference between the wide and narrow sections of pipe is 4100 Pa . At wider section, the velocity of fluid is $\frac{\sqrt{x}}{6} \text{ ms}^{-1}$ for $x = \text{---}$. (Given $g = 10 \text{ ms}^{-2}$)



Answer (363)

Sol. Applying Bernoulli's theorem:

$$P_1 + \rho gh + \frac{1}{2} \rho v^2 = P_2 + 0 + \frac{1}{2} \rho (2v)^2$$

Putting the values,

$$4100 = 800 \left\{ \frac{3}{2} v^2 - 10 \right\}$$

$$\Rightarrow v = \frac{\sqrt{363}}{6} \text{ m/s}$$

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. A commercially sold conc. HCl is 35% HCl by mass. If the density of this commercial acid is 1.46 g/mL, the molarity of this solution is :

(Atomic mass : Cl = 35.5 amu, H = 1 amu)

- (A) 10.2 M (B) 12.5 M
(C) 14.0 M (D) 18.2 M

Answer (C)

Sol. Molarity = $\frac{35}{(36.5) \times \frac{100}{1.46}} \times 1000 = 14.0 \text{ M}$

2. An evacuated glass vessel weighs 40.0 g when empty, 135.0 g when filled with a liquid of density 0.95 g mL⁻¹ and 40.5 g when filled with an ideal gas at 0.82 atm at 250 K. The molar mass of the gas in g mol⁻¹ is:

(Given : R = 0.082 L atm K⁻¹ mol⁻¹)

- (A) 35
(B) 50
(C) 75
(D) 175

Answer (Bonus)

Sol. Weight of liquid = 135.0 – 40.0 = 95.0 g

Volume of liquid = $\frac{95}{0.95} = 100 \text{ ml} = 0.1 \text{ L}$

Weight of gas = 0.5 g

Moles of gas = $\frac{0.5}{M}$

PV = nRT

$0.82 \times 0.1 = \frac{0.5}{M} \times 0.082 \times 250$

M = 0.5 × 250 = 125 g mol⁻¹

3. If the radius of the 3rd Bohr's orbit of hydrogen atom is r₃ and the radius of 4th Bohr's orbit is r₄. Then :

- (A) $r_4 = \frac{9}{16} r_3$ (B) $r_4 = \frac{16}{9} r_3$
(C) $r_4 = \frac{3}{4} r_3$ (D) $r_4 = \frac{4}{3} r_3$

Answer (B)

Sol. $r_n = r_0 \frac{n^2}{Z}$

$r_4 = r_0 \times \frac{4^2}{1}$

and $r_3 = r_0 \times \frac{3^2}{1}$

Therefore $\frac{r_4}{r_3} = \frac{4^2}{3^2} \Rightarrow r_4 = \frac{16}{9} r_3$

4. Consider the ions/molecules



For increasing bond order the correct option is:

- (A) O₂²⁻ < O₂⁻ < O₂ < O₂⁺
(B) O₂⁻ < O₂²⁻ < O₂ < O₂⁺
(C) O₂⁻ < O₂²⁻ < O₂⁺ < O₂
(D) O₂⁻ < O₂⁺ < O₂²⁻ < O₂

Answer (A)

Sol. Species	Bond order
O ₂ ⁺	2.5
O ₂	2
O ₂ ⁻	1.5
O ₂ ²⁻	1

5. The $\left(\frac{\partial E}{\partial T}\right)_p$ of different types of half cells are as follows:

- A B C D
1 × 10⁻⁴ 2 × 10⁻⁴ 0.1 × 10⁻⁴ 0.2 × 10⁻⁴

(Where E is the electromotive force)

Which of the above half cells would be preferred to be used as reference electrode?

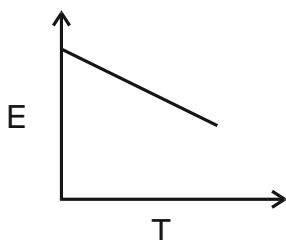
- (A) A (B) B
(C) C (D) D

Answer (C)

Sol. We know that

$$E = E^\circ - \frac{2.303RT}{nF} \log \left[\frac{1}{(\text{oxidized form})} \right]$$

$$\text{Slope} = \frac{-2.303 R}{nF} \log \left[\frac{1}{(\text{oxidized form})} \right]$$



Higher the value of slope $\left(\frac{\partial E}{\partial T} \right)_p$, more the deviation between E and E° . Thus, half cell with least value of $\left(\frac{\partial E}{\partial T} \right)_p$ can be used as reference electrode.

6. Choose the correct stability order of group 13 elements in their +1 oxidation state.

- (A) Al < Ga < In < Tl (B) Tl < In < Ga < Al
(C) Al < Ga < Tl < In (D) Al < Tl < Ga < In

Answer (A)

Sol. Due to inert pair effect, stability of +3 oxidation state decreases and that of +1 oxidation state increases for (down the group) group 13 elements.

So, the correct order of stability of group 13 elements in their +1 oxidation state is Al < Ga < In < Tl.

7. Given below are two statements:

Statement I : According to the Ellingham diagram, any metal oxide with higher ΔG° is more stable than the one with lower ΔG° .

Statement II : The metal involved in the formation of oxide placed lower in the Ellingham diagram can reduce the oxide of a metal placed higher in the diagram.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

- (A) Both **Statement I** and **Statement II** are correct.
(B) Both **Statement I** and **Statement II** are incorrect.
(C) **Statement I** is correct but **Statement II** is incorrect.
(D) **Statement I** is incorrect but **Statement II** is correct.

Answer (D)

Sol. Ellingham diagram is plot of ΔG vs T.

The criterion for the feasibility of a thermal reduction is that at a given temperature Gibbs energy change of a reaction must be negative. The change in Gibbs energy, ΔG for any process at any specified temperature, is given by the equation

$$\Delta G = \Delta H - T\Delta S$$

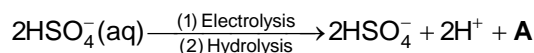
where ΔH = enthalpy change and

ΔS = entropy change

According to the Ellingham diagram, any metal oxide with higher ΔG° has a tendency of getting reduced by the metal whose metal oxide has lower value of ΔG° .

Therefore, Statement I is incorrect but Statement II is correct.

8. Consider the following reaction:



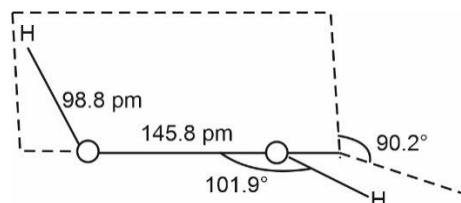
The dihedral angle in product **A** in its solid phase at 110 K is :

- (A) 104° (B) 111.5°
(C) 90.2° (D) 111.0°

Answer (C)

Sol. A should be H_2O_2

Structure of H_2O_2 is solid phase



Dihedral angle = 90.2°

9. The correct order of melting point is :

- (A) Be > Mg > Ca > Sr
 (B) Sr > Ca > Mg > Be
 (C) Be > Ca > Mg > Sr
 (D) Be > Ca > Sr > Mg

Answer (D)

Sol. Element	M.P.
Be	1560 K
Mg	924 K
Ca	1124 K
Sr	1062

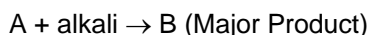
10. The correct order of melting points of hydrides of group 16 elements is:

- (A) H₂S < H₂Se < H₂Te < H₂O
 (B) H₂O < H₂S < H₂Se < H₂Te
 (C) H₂S < H₂Te < H₂Se < H₂O
 (D) H₂Se < H₂S < H₂Te < H₂O

Answer (A)

Sol. Hydride	M.P.
H ₂ O	273 K
H ₂ S	188 K
H ₂ Se	208 K
H ₂ Te	222 K

11. Consider the following reaction:

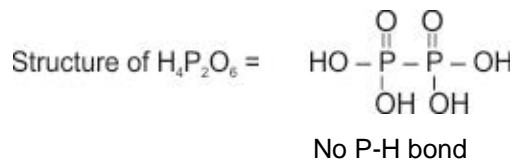


If B is an oxoacid of phosphorus with no P-H bond, then A is:

- (A) White P₄
 (B) Red P₄
 (C) P₂O₃
 (D) H₃PO₃

Answer (B)

Sol. White P₄ + alkali → H₃PO₂

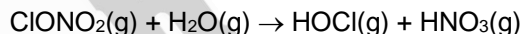
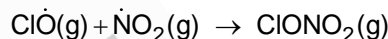


12. Polar stratospheric clouds facilitate the formation of:

- (A) ClONO₂
 (B) HOCl
 (C) ClO
 (D) CH₄

Answer (B)

Sol. In summer season nitrogen dioxide and methane react with chlorine monoxide and chlorine atoms forming, Chlorine sinks, preventing much ozone depletion, whereas in winter, special types of clouds called polar stratospheric clouds are formed over Antarctica. These polar stratospheric clouds provide surface on which chlorine nitrate formed gets hydrolysed to form hypochlorous acid.



13. Given below are two statements:

Statement I: In 'Lassaigne's Test', when both nitrogen and sulphur are present in an organic compound, sodium thiocyanate is formed.

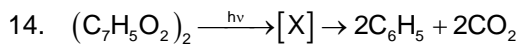
Statement II: If both nitrogen and sulphur are present in an organic compound, then the excess of sodium used in sodium fusion will decompose the sodium thiocyanate formed to give NaCN and Na₂S.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

- (A) Both **Statement I** and **Statement II** are correct.
 (B) Both **Statement I** and **Statement II** are incorrect.
 (C) **Statement I** is correct but **Statement II** is incorrect.
 (D) **Statement I** is incorrect but **Statement II** is correct.

Answer (A)

Sol. NaSCN + 2Na → NaCN + Na₂S

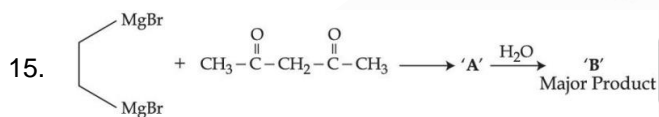
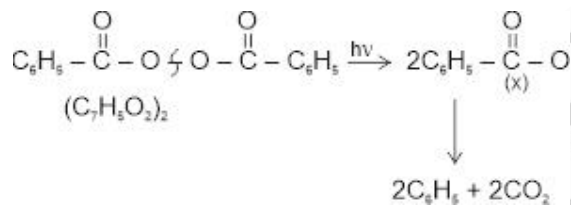


Consider the above reaction and identify the intermediate 'X'

- (A) $C_6H_5-C^{\oplus}$
- (B) $C_6H_5-C-O^{\ominus}$
- (C) $C_6H_5-C-\ddot{O}$
- (D) $C_6H_5-C-\ddot{O}$

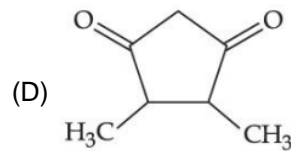
Answer (D)

Sol.



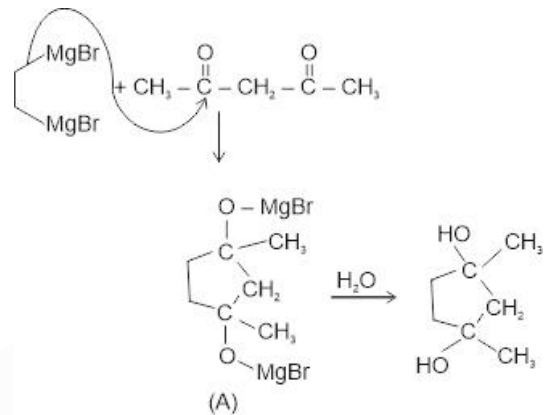
Consider the above reaction sequence and identify the product B.

- (A)
- (B)
- (C)



Answer (A)

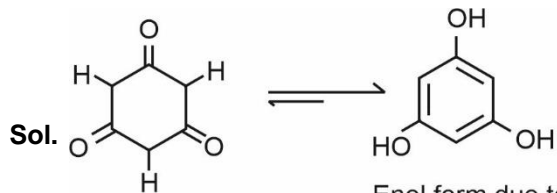
Sol.



16. Which will have the highest enol content?

- (A)
- (B)
- (C)
- (D)

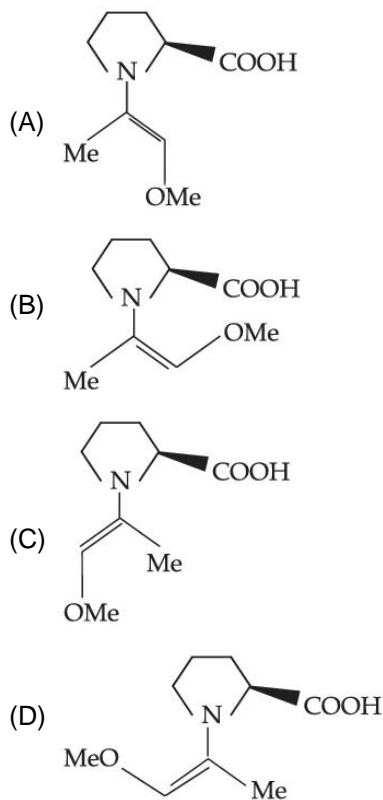
Answer (C)



Enol form due to aromaticity is more stable

17. Among the following structures, which will show the most stable enamine formation?

(Where Me is $-\text{CH}_3$)



Answer (C)

Sol. The most appropriate option is (C) as one group is far enough from $-\text{COOH}$ group.

18. Which of the following sets are correct regarding polymer?

- (A) Copolymer : Buna-S
 (B) Condensation polymer : Nylon-6,6
 (C) Fibres : Nylon-6,6
 (D) Thermosetting polymer : Terylene
 (E) Homopolymer : Buna-N

Choose the correct answer from given options below:

- (A) (A), (B) and (C) are correct
 (B) (B), (C) and (D) are correct
 (C) (A), (C) and (E) are correct
 (D) (A), (B) and (D) are correct

Answer (A)

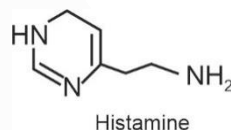
- Sol.** (A) Buna-S – Copolymer
 (B) Nylon-6,6 – Condensation polymer
 (C) Nylon-6,6 – Fibre
 (D) Terylene – Thermoplastic
 (E) Buna-N – Copolymer
 (A) A, B and C are correct.

19. A chemical which stimulates the secretion of pepsin is:

- (A) Anti-histamine (B) Cimetidine
 (C) Histamine (D) Zantac

Answer (C)

Sol. Histamine stimulates the secretion of pepsin and hydrochloric acid in the stomach.

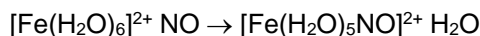
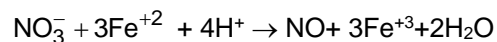


20. Which statement is **not** true with respect to nitrate ion test?

- (A) A dark brown ring is formed at the junction of two solutions.
 (B) Ring is formed due to nitroferrous sulphate complex.
 (C) The brown complex is $[\text{Fe}(\text{H}_2\text{O})_5(\text{NO})]\text{SO}_4$.
 (D) Heating the nitrate salt with conc. H_2SO_4 , light brown fumes are evolved.

Answer (B)

Sol. Brown ring test

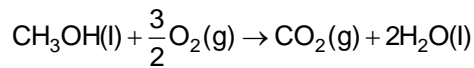


Brown ring

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

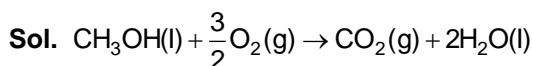
1. For complete combustion of methanol



the amount of heat produced as measured by bomb calorimeter is 726 kJ mol⁻¹ at 27°C. The enthalpy of combustion for the reaction is -x kJ mol⁻¹, where x is _____. (Nearest integer)

(Given : R = 8.3 JK⁻¹ mol⁻¹)

Answer (727)



$$\Delta H = \Delta U + \Delta n_g RT$$

$$= -726 \text{ kJ} + \left(\frac{-1}{2}\right) \times 8.3 \times 300$$

$$\approx -727 \text{ kJ mol}^{-1}$$

2. A 0.5 percent solution of potassium chloride was found to freeze at -0.24°C. The percentage dissociation of potassium chloride is _____. (Nearest integer)

(Molal depression constant for water is 1.80 K kg mol⁻¹ and molar mass of KCl is 74.6 g mol⁻¹)

Answer (98)

Sol. $\Delta T_f = iK_b m$

$$i = \frac{0.24 \times 99.5 \times 74.6}{1.80 \times 0.5 \times 1000}$$

$$= 1.98$$

$$\alpha = \frac{i-1}{n-1} = \frac{0.98}{1} = 0.98$$

3. 50 mL of 0.1 M CH₃COOH is being titrated against 0.1 M NaOH. When 25 mL of NaOH has been added, the pH of the solution will be ____ × 10⁻². (Nearest integer)

(Given : pK_a (CH₃COOH) = 4.76)

$$\log 2 = 0.30$$

$$\log 3 = 0.48$$

$$\log 5 = 0.69$$

$$\log 7 = 0.84$$

$$\log 11 = 1.04$$

Answer (476)



at initially $\frac{50 \times 0.1}{1000}$ m moles $\frac{25 \times 0.1}{1000}$ m mole

at time t 2.5 m moles 0 2.5 m mol

$$\text{pH} = \text{pK}_a + \log \left| \frac{(\text{salt})}{(\text{acid})} \right|$$

$$\text{pH} = 4.76 + \log \left| \frac{2.5}{2.5} \right|$$

$$\text{pH} = 4.76$$

4. A flask is filled with equal moles of A and B. The half lives of A and B are 100 s and 50 s respectively and are independent of the initial concentration. The time required for the concentration of A to be four times that of B is _____ s.

(Given : ln 2 = 0.693)

Answer (200)



at initially 1 1

 mole mole

after 100 s $\frac{1}{2}$ $\frac{1}{4}$

after 200 s $\frac{1}{4}$ $\frac{1}{16}$

Ans. 200 seconds

5. 2.0 g of H₂ gas is adsorbed on 2.5 g of platinum powder at 300 K and 1 bar pressure. The volume of the gas adsorbed per gram of the adsorbent is ____ mL.

(Given : R = 0.083 L bar K⁻¹ mol⁻¹)

Answer (9960)

Sol. $PV = nRT$

$$V = \frac{2 \times 0.083 \times 300}{2 \times 1} = 24.9 \text{ litre}$$

\therefore Volume of the gas adsorbed per gram of the

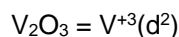
$$\text{adsorbent} = \frac{24.9}{2.5} = 9.96 \text{ L}$$

$$= 9960 \text{ ml}$$

6. The spin-only magnetic moment value of the most basic oxide of vanadium among V_2O_3 , V_2O_4 and V_2O_5 is _____ B.M. (Nearest integer)

Answer (3)

Sol. The most basic oxide among V_2O_3 , V_2O_4 and V_2O_5 is V_2O_3

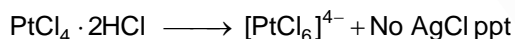
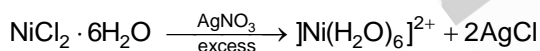
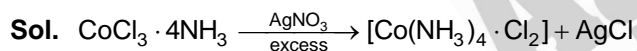


$$\text{Magnetic moment} = \sqrt{2(2+2)} = \sqrt{8}$$

$$= 2.83 \approx 3$$

7. The spin-only magnetic moment value of an octahedral complex among $CoCl_3 \cdot 4NH_3$, $NiCl_2 \cdot 6H_2O$ and $PtCl_4 \cdot 2HCl$, which upon reaction with excess of $AgNO_3$ gives 2 moles of $AgCl$ is _____ B.M. (Nearest Integer)

Answer (3)



$$[Ni^{+2}(H_2O)_6] = d^8 = t_{2g}^6 e_g^2 = 2 \text{ unpaired electrons}$$

$$\text{Magnetic moment} = \sqrt{2(2+2)}$$

$$2\sqrt{8} \approx 3$$

8. On complete combustion 0.30 g of an organic compound gave 0.20 g of carbon dioxide and 0.10 g of water. The percentage of carbon in the given organic compound is _____.

(Nearest Integer)

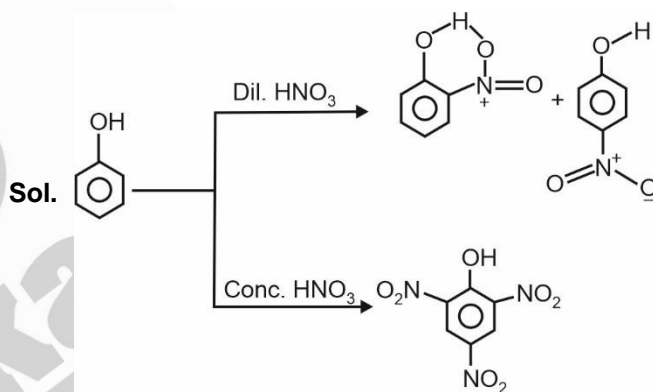
Answer (18)

$$\text{Sol. } C\% = \frac{12}{44} \times \frac{0.20}{0.30} \times 100$$

$$= \frac{200}{11} = 18.18 \approx 18$$

9. Compound 'P' on nitration with dil. HNO_3 yields two isomers (A) and (B) show the intramolecular and intermolecular hydrogen bonding respectively. Compound (P) on reaction with conc. HNO_3 yields a yellow compound 'C', a strong acid. The number of oxygen atoms is present in compound 'C' _____.

Answer (7)

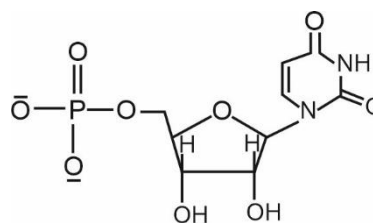


The number of oxygen atoms = 7

10. The number of oxygens present in a nucleotide formed from a base, that is present only in RNA is _____.

Answer (9)

Sol. Nucleotide formed by Uracil, the base present in RNA, is



The number of oxygen = 9

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Let $f(x) = \frac{x-1}{x+1}$, $x \in \mathbb{R} - \{-1, 1\}$. If $f^{n+1}(x) = f(f^n(x))$ for all $n \in \mathbb{N}$, then $f^6(6) + f^7(7)$ is equal to :
- (A) $\frac{7}{6}$ (B) $-\frac{3}{2}$
(C) $\frac{7}{12}$ (D) $-\frac{11}{12}$

Answer (B)

Sol. $f(x) = \frac{x-1}{x+1} \Rightarrow f(f(x)) = \frac{\frac{x-1}{x+1} - 1}{\frac{x-1}{x+1} + 1} = -\frac{1}{x}$

$\Rightarrow f^3(x) = -\frac{x+1}{x-1} \Rightarrow f^4(x) = -\frac{\frac{x-1}{x+1} + 1}{\frac{x-1}{x+1} - 1} = x$

So, $f^6(6) + f^7(7) = f^2(6) + f^3(7)$

$= -\frac{1}{6} - \frac{7+1}{7-1} = -\frac{9}{6} = -\frac{3}{2}$

2. Let $A = \left\{ z \in \mathbb{C} : \left| \frac{z+1}{z-1} \right| < 1 \right\}$
and $B = \left\{ z \in \mathbb{C} : \arg\left(\frac{z-1}{z+1}\right) = \frac{2\pi}{3} \right\}$.

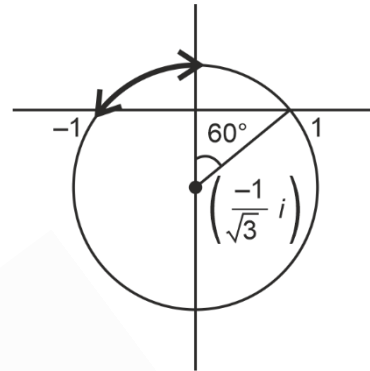
Then $A \cap B$ is :

- (A) A portion of a circle centred at $\left(0, -\frac{1}{\sqrt{3}}\right)$ that lies in the second and third quadrants only
(B) A portion of a circle centred at $\left(0, -\frac{1}{\sqrt{3}}\right)$ that lies in the second quadrant only
(C) An empty set
(D) A portion of a circle of radius $\frac{2}{\sqrt{3}}$ that lies in the third quadrant only

Answer (B)

Sol. $\left| \frac{z+1}{z-1} \right| < 1 \Rightarrow |z+1| < |z-1| \Rightarrow \text{Re}(z) < 0$

and $\arg\left(\frac{z-1}{z+1}\right) = \frac{2\pi}{3}$ is a part of circle as shown.



3. Let A be a 3×3 invertible matrix. If $|\text{adj}(24A)| = |\text{adj}(3 \text{adj}(2A))|$, then $|A|^2$ is equal to :
- (A) 6^6 (B) 2^{12}
(C) 2^6 (D) 1

Answer (C)

Sol. $|\text{adj}(24A)| = |\text{adj}(3 \text{adj}(2A))|$
 $\Rightarrow |24A|^2 = |3 \text{adj}(2A)|^2$
 $\Rightarrow (24^3)^2 \cdot |A|^2 = (3^3)^2 |\text{adj}(2A)|^2$
 $\Rightarrow 24^6 \cdot |A|^2 = 3^6 |2A|^4$
 $\Rightarrow 24^6 |A|^2 = 3^6 \cdot (2^3)^4 |A|^4$
 $\Rightarrow |A|^2 = \frac{24^6}{3^6 \cdot 2^{12}} = \frac{2^{18} \cdot 3^6}{3^6 \cdot 2^{12}} = 2^6$

4. The ordered pair (a, b) , for which the system of linear equations
 $3x - 2y + z = b$
 $5x - 8y + 9z = 3$
 $2x + y + az = -1$
has no solution, is :

- (A) $\left(3, \frac{1}{3}\right)$ (B) $\left(-3, \frac{1}{3}\right)$
(C) $\left(-3, -\frac{1}{3}\right)$ (D) $\left(3, -\frac{1}{3}\right)$

Answer (C)

Sol. $\begin{vmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{vmatrix} = 0 \Rightarrow -14a - 42 = 0 \Rightarrow a = -3$

Now 3(equation (1)) – (equation (2)) – 2(equation (3)) is

$$3(3x - 2y + z - b) - (5x - 8y + 9z - 3) - 2(2x + y + az + 1) = 0$$

$$\Rightarrow -3b + 3 - 2 = 0 \Rightarrow b = \frac{1}{3}$$

So for no solution $a = -3$ and $b \neq \frac{1}{3}$

5. The remainder when $(2021)^{2023}$ is divided by 7 is :

- (A) 1 (B) 2
(C) 5 (D) 6

Answer (C)

Sol. $2021 \equiv -2 \pmod{7}$

$$\begin{aligned} \Rightarrow (2021)^{2023} &\equiv (-2)^{2023} \pmod{7} \\ &\equiv -2(8)^{674} \pmod{7} \\ &\equiv -2(1)^{674} \pmod{7} \\ &\equiv -2 \pmod{7} \\ &\equiv 5 \pmod{7} \end{aligned}$$

So when $(2021)^{2023}$ is divided by 7, remainder is 5.

6. $\lim_{x \rightarrow \frac{1}{\sqrt{2}}} \frac{\sin(\cos^{-1} x) - x}{1 - \tan(\cos^{-1} x)}$ is equal to :

- (A) $\sqrt{2}$ (B) $-\sqrt{2}$
(C) $\frac{1}{\sqrt{2}}$ (D) $-\frac{1}{\sqrt{2}}$

Answer (D)

Sol. $\lim_{x \rightarrow \frac{1}{\sqrt{2}}} \frac{\sin(\cos^{-1} x) - x}{1 - \tan(\cos^{-1} x)}$ let $\cos^{-1} x = \frac{\pi}{4} + \theta$

$$= \lim_{\theta \rightarrow 0} \frac{\sin\left(\frac{\pi}{4} + \theta\right) - \cos\left(\frac{\pi}{4} + \theta\right)}{1 - \tan\left(\frac{\pi}{4} + \theta\right)}$$

$$= \lim_{\theta \rightarrow 0} \frac{\sqrt{2} \sin\left(\frac{\pi}{4} + \theta - \frac{\pi}{4}\right)}{1 - \frac{1 + \tan \theta}{1 - \tan \theta}}$$

$$= \lim_{\theta \rightarrow 0} \frac{\sqrt{2} \sin \theta}{-2 \tan \theta} (1 - \tan \theta) = -\frac{1}{\sqrt{2}}$$

7. $f, g : \mathbf{R} \rightarrow \mathbf{R}$ be two real valued functions defined

as $f(x) = \begin{cases} -|x+3|, & x < 0 \\ e^x, & x \geq 0 \end{cases}$ and

$$g(x) = \begin{cases} x^2 + k_1 x, & x < 0 \\ 4x + k_2, & x \geq 0 \end{cases}$$
 where k_1 and k_2 are real constants. If $(g \circ f)$ is differentiable at $x = 0$, then $(g \circ f)(-4) + (g \circ f)(4)$ is equal to :

(A) $4(e^4 + 1)$ (B) $2(2e^4 + 1)$
(C) $4e^4$ (D) $2(2e^4 - 1)$

Answer (D)

Sol. $\because g \circ f$ is differentiable at $x = 0$

So R.H.D = L.H.D

$$\frac{d}{dx}(4e^x + k_2) = \frac{d}{dx}((-|x+3|)^2 - k_1|x+3|)$$

$$\Rightarrow 4 = 6 - k_1 \Rightarrow k_1 = 2$$

Also $g(f(0^+)) = g(f(0^-))$

$$\Rightarrow 4 + k_2 = 9 - 3k_1 \Rightarrow k_2 = -1$$

Now $g(f(-4)) + g(f(4))$

$$\begin{aligned} &= g(-1) + g(e^4) = (1 - k_1) + (4e^4 + k_2) \\ &= 4e^4 - 2 \\ &= 2(2e^4 - 1) \end{aligned}$$

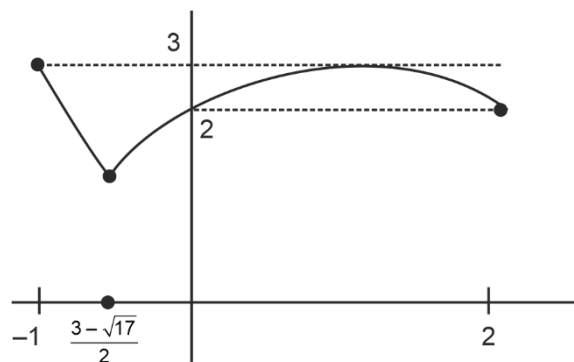
8. The sum of the absolute minimum and the absolute maximum values of the function $f(x) = |3x - x^2 + 2| - x$ in the interval $[-1, 2]$ is :

- (A) $\frac{\sqrt{17} + 3}{2}$ (B) $\frac{\sqrt{17} + 5}{2}$
(C) 5 (D) $\frac{9 - \sqrt{17}}{2}$

Answer (A)

Sol. $f(x) = |x^2 - 3x - 2| - x \forall x \in [-1, 2]$

$$\Rightarrow f(x) = \begin{cases} x^2 - 4x - 2 & \text{if } -1 \leq x < \frac{3 - \sqrt{17}}{2} \\ -x^2 + 2x + 2 & \text{if } \frac{3 - \sqrt{17}}{2} \leq x \leq 2 \end{cases}$$



$$f(x)_{\max} = 3$$

$$f(x)_{\min} = f\left(\frac{3-\sqrt{17}}{2}\right) = \frac{\sqrt{17}-3}{2}$$

9. Let S be the set of all the natural numbers, for which the line $\frac{x}{a} + \frac{y}{b} = 2$ is a tangent to the curve

$$\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2 \text{ at the point } (a, b), ab \neq 0. \text{ Then :}$$

- (A) $S = \phi$ (B) $n(S) = 1$
(C) $S = \{2k : k \in \mathbf{N}\}$ (D) $S = \mathbf{N}$

Answer (D)

Sol. $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$

$$\Rightarrow \frac{n}{a}\left(\frac{x}{a}\right)^{n-1} + \frac{n}{b}\left(\frac{y}{b}\right)^{n-1} \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{b}{a}\left(\frac{bx}{ay}\right)^{n-1}$$

$$\Rightarrow \frac{dy}{dx}_{(a,b)} = -\frac{b}{a}$$

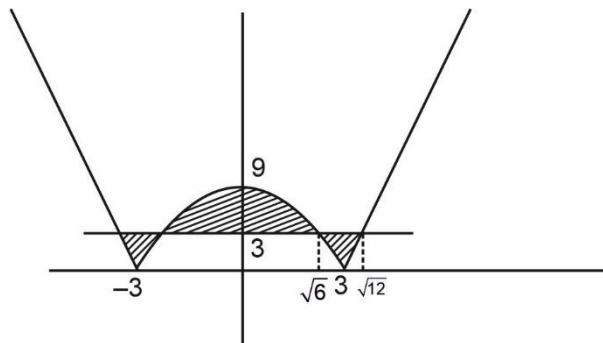
So line always touches the given curve.

10. The area bounded by the curve $y = |x^2 - 9|$ and the line $y = 3$ is
(A) $4(2\sqrt{3} + \sqrt{6} - 4)$ (B) $4(4\sqrt{3} + \sqrt{6} - 4)$
(C) $8(4\sqrt{3} + 3\sqrt{6} - 9)$ (D) $8(4\sqrt{3} + \sqrt{6} - 9)$

Answer (*)

Sol. $y = 3$ and $y = |x^2 - 9|$

Intersect in first quadrant at $x = \sqrt{6}$ and $x = \sqrt{12}$



Required area

$$= 2 \left[\frac{2}{3}(6 \times \sqrt{6}) + \int_{\sqrt{6}}^3 (3 - (9 - x^2)) dx + \int_3^{\sqrt{12}} (3 - (x^2 - 9)) dx \right]$$

$$= 2 \left[4\sqrt{6} + \left(\frac{x^3}{3} - 6x \right) \Big|_{\sqrt{6}}^3 + \left(12x - \frac{x^3}{3} \right) \Big|_3^{\sqrt{12}} \right]$$

$$= 2 \left[4\sqrt{6} + (4\sqrt{6} - 9) + (8\sqrt{12} - 27) \right]$$

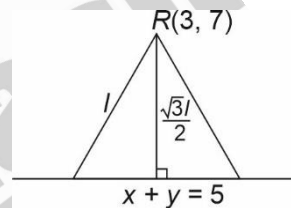
$$= 2 \left[8\sqrt{6} + 16\sqrt{3} - 36 \right] = 8 \left[2\sqrt{6} + 4\sqrt{3} - 9 \right]$$

11. Let R be the point $(3, 7)$ and let P and Q be two points on the line $x + y = 5$ such that PQR is an equilateral triangle, Then the area of ΔPQR is :

- (A) $\frac{25}{4\sqrt{3}}$ (B) $\frac{25\sqrt{3}}{2}$
(C) $\frac{25}{\sqrt{3}}$ (D) $\frac{25}{2\sqrt{3}}$

Answer (D)

Sol.



Altitude of equilateral triangle,

$$\frac{\sqrt{3}l}{2} = \frac{5}{\sqrt{2}}$$

$$l = \frac{5\sqrt{2}}{\sqrt{3}}$$

$$\text{Area of triangle} = \frac{\sqrt{3}}{4} l^2 = \frac{\sqrt{3}}{4} \cdot \frac{50}{3} = \frac{25}{2\sqrt{3}}$$

12. Let C be a circle passing through the points $A(2, -1)$ and $B(3, 4)$. The line segment AB is not a diameter of C . If r is the radius of C and its centre lies on the circle $(x-5)^2 + (y-1)^2 = \frac{13}{2}$, then r^2 is equal to :

- (A) 32 (B) $\frac{65}{2}$
(C) $\frac{61}{2}$ (D) 30

Answer (B)

If image of $\left(2, -\frac{1}{2}, 2\right)$ in plane P is (a, b, c) then

$$\frac{a-2}{1} = \frac{b+\frac{1}{2}}{-2} = \frac{c-2}{4}$$

$$\text{and } \left(\frac{a+2}{2}\right) - 2\left(\frac{b-\frac{1}{2}}{2}\right) + 4\left(\frac{c+2}{2}\right) = 4$$

Clearly $a = \frac{4}{3}, b = \frac{5}{6}$ and $c = -\frac{2}{3}$

So, $a : b : c = 8 : 5 : -4$

16. If $\vec{a} \cdot \vec{b} = 1, \vec{b} \cdot \vec{c} = 2$ and $\vec{c} \cdot \vec{a} = 3$, then the value of

$$\left[\vec{a} \times (\vec{b} \times \vec{c}), \vec{b} \times (\vec{c} \times \vec{a}), \vec{c} \times (\vec{b} \times \vec{a})\right] \text{ is :}$$

(A) 0 (B) $-6\vec{a} \cdot (\vec{b} \times \vec{c})$

(C) $12\vec{c} \cdot (\vec{a} \times \vec{b})$ (D) $-12\vec{b} \cdot (\vec{c} \times \vec{a})$

Answer (A)

Sol. $\therefore \vec{a} \times (\vec{b} \times \vec{c}) = 3\vec{b} - \vec{c} = \vec{u}$

$$\vec{b} \times (\vec{c} \times \vec{a}) = \vec{c} - 2\vec{a} = \vec{v}$$

$$\vec{c} \times (\vec{b} \times \vec{a}) = 3\vec{b} - 2\vec{a} = \vec{w}$$

$$\therefore \vec{u} + \vec{v} = \vec{w}$$

So vectors \vec{u}, \vec{v} and \vec{w} are coplanar, hence their Scalar triple product will be zero.

17. Let a biased coin be tossed 5 times. If the probability of getting 4 heads is equal to the probability of getting 5 heads, then the probability of getting atmost two heads is:

(A) $\frac{275}{6^5}$ (B) $\frac{36}{5^4}$

(C) $\frac{181}{5^5}$ (D) $\frac{46}{6^4}$

Answer (D)

Sol. Let probability of getting head = p

$$\text{So, } {}^5C_4 p^4 (1-p) = {}^5C_5 p^5$$

$$\Rightarrow p = 5(1-p) \Rightarrow p = \frac{5}{6}$$

Probability of getting atmost two heads =

$${}^5C_0 (1-p)^5 + {}^5C_1 p(1-p)^4 + {}^5C_2 p^2(1-p)^3$$

$$= \frac{1+25+250}{6^5}$$

$$= \frac{276}{6^5} = \frac{46}{6^4}$$

18. The mean of the numbers $a, b, 8, 5, 10$ is 6 and their variance is 6.8. If M is the mean deviation of the numbers about the mean, then $25M$ is equal to:

(A) 60 (B) 55

(C) 50 (D) 45

Answer (A)

Sol. $\therefore \bar{x} = 6 = \frac{a+b+8+5+10}{5} \Rightarrow a+b=7 \dots(i)$

$$\text{And } \sigma^2 = \frac{a^2+b^2+8^2+5^2+10^2}{5} - 6^2 = 6.8$$

$$\Rightarrow a^2 + b^2 = 25 \dots(ii)$$

From (i) and (ii) $(a, b) = (3, 4)$ or $(4, 3)$

Now mean deviation about mean

$$M = \frac{1}{5}(3+2+2+1+4) = \frac{12}{5}$$

$$\Rightarrow 25M = 60$$

19. Let $f(x) = 2\cos^{-1}x + 4\cot^{-1}x - 3x^2 - 2x + 10, x \in [-1, 1]$,

If $[a, b]$ is the range of the function, f then $4a - b$ is equal to :

(A) 11

(B) $11 - \pi$

(C) $11 + \pi$

(D) $15 - \pi$

Answer (B)

Sol. $f(x) = 2\cos^{-1}x + 4\cot^{-1}x - 3x^2 - 2x + 10 \forall x \in [-1, 1]$

$$\Rightarrow f'(x) = -\frac{2}{\sqrt{1-x^2}} - \frac{4}{1+x^2} - 6x - 2 < 0 \forall x \in [-1, 1]$$

So $f(x)$ is decreasing function and range of $f(x)$ is $[f(1), f(-1)]$, which is $[\pi + 5, 5\pi + 9]$

$$\text{Now } 4a - b = 4(\pi + 5) - (5\pi + 9)$$

$$= 11 - \pi$$

20. Let $\Delta, \nabla \in \{\wedge, \vee\}$ be such that $p \nabla q \Rightarrow ((p \Delta q) \nabla r)$ is a tautology. Then $(p \nabla q) \Delta r$ is logically equivalent to :

- (A) $(p \Delta r) \vee q$ (B) $(p \Delta r) \wedge q$
 (C) $(p \wedge r) \Delta q$ (D) $(p \nabla r) \wedge q$

Answer (A)

Sol. Case-I

If ∇ is same as \wedge

Then $(p \wedge q) \Rightarrow ((p \Delta q) \wedge r)$ is equivalent to $\sim (p \wedge q) \vee ((p \Delta q) \wedge r)$ is equivalent to $(\sim (p \wedge q) \vee (p \Delta q)) \wedge (\sim (p \wedge q) \vee r)$

Which cannot be a tautology

For both Δ (i.e. \vee or \wedge)

Case-II

If ∇ is same as \vee

Then $(p \vee q) \Rightarrow ((p \Delta q) \vee r)$ is equivalent to

$\sim (p \vee q) \vee (p \Delta q) \vee r$ which can be a tautology if Δ is also same as \vee .

Hence both Δ and ∇ are same as \vee .

Now $(p \nabla q) \Delta r$ is equivalent to $(p \vee q \vee r)$.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. The sum of the cubes of all the roots of the equation $x^4 - 3x^3 - 2x^2 + 3x + 1 = 0$ is _____.

Answer (36)

Sol. $x^4 - 3x^3 - x^2 - x^2 + 3x + 1 = 0$

$$(x^2 - 1)(x^2 - 3x - 1) = 0$$

Let the root of $x^2 - 3x - 1 = 0$ be α and β and other two roots of given equation are 1 and -1

$$\begin{aligned} \text{So sum of cubes of roots} &= 1^3 + (-1)^3 + \alpha^3 + \beta^3 \\ &= (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) \\ &= (3)^3 - 3(-1)(3) \\ &= 36 \end{aligned}$$

2. There are ten boys B_1, B_2, \dots, B_{10} and five girls G_1, G_2, \dots, G_5 in a class. Then the number of ways of forming a group consisting of three boys and three girls, if both B_1 and B_2 together should not be the members of a group, is _____.

Answer (1120)

Sol. Required number of ways = Total ways of selection - ways in which B_1 and B_2 are present together.

$$\begin{aligned} &= {}^{10}C_3 \cdot {}^5C_3 - {}^8C_1 \cdot {}^5C_3 = 10(120 - 8) \\ &= 1120 \end{aligned}$$

3. Let the common tangents to the curves $4(x^2 + y^2) = 9$ and $y^2 = 4x$ intersect at the point Q. Let an ellipse, centered at the origin O, has lengths of semi-minor and semi-major axes equal to OQ and 6, respectively. If e and l respectively denote the eccentricity and the length of the latus rectum

of this ellipse, then $\frac{l}{e^2}$ is equal to _____.

Answer (4)

Sol. Let $y = mx + c$ is the common tangent

$$\text{So } c = \frac{1}{m} = \pm \frac{3}{2} \sqrt{1+m^2} \Rightarrow m^2 = \frac{1}{3}$$

So equation of common tangents will be

$$y = \pm \frac{1}{\sqrt{3}} x \pm \sqrt{3}, \text{ which intersects at } Q(-3, 0)$$

Major axis and minor axis of ellipse are 12 and 6. So eccentricity

$$\begin{aligned} e^2 &= 1 - \frac{1}{4} = \frac{3}{4} \text{ and length of latus rectum} \\ &= \frac{2b^2}{a} = 3 \end{aligned}$$

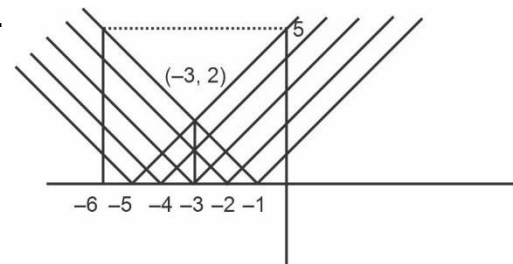
$$\text{Hence } \frac{l}{e^2} = \frac{3}{3/4} = 4$$

4. Let $f(x) = \max\{|x+1|, |x+2|, \dots, |x+5|\}$. Then

$$\int_{-6}^0 f(x) dx \text{ is equal to } \underline{\hspace{2cm}}.$$

Answer (21)

Sol.



$$\int_{-6}^0 f(x) dx = 2 \left[\frac{1}{2} (2+5) 3 \right] = 21$$

5. Let the solution curve $y = y(x)$ of the differential equation $(4 + x^2)dy - 2x(x^2 + 3y + 4)dx = 0$ pass through the origin. Then $y(2)$ is equal to _____.

Answer (12)

Sol. $(4 + x^2) dy - 2x(x^2 + 3y + 4)dx = 0$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{6x}{x^2 + 4} \right) y + 2x$$

$$\Rightarrow \frac{dy}{dx} - \left(\frac{6x}{x^2 + 4} \right) y = 2x$$

$$\text{I.F.} = e^{-3\ln(x^2+4)} = \frac{1}{(x^2 + 4)^3}$$

$$\text{So } \frac{y}{(x^2 + 4)^3} = \int \frac{2x}{(x^2 + 4)^3} dx + c$$

$$\Rightarrow y = -\frac{1}{2}(x^2 + 4) + c(x^2 + 4)^3$$

When $x = 0, y = 0$ gives $c = \frac{1}{32}$,

So, for $x = 2, y = 12$

6. If $\sin^2(10^\circ)\sin(20^\circ)\sin(40^\circ)\sin(50^\circ)\sin(70^\circ)$
 $= \alpha - \frac{1}{16}\sin(10^\circ)$, then $16 + \alpha^{-1}$ is equal to _____.

Answer (80)

Sol: $(\sin 10^\circ \cdot \sin 50^\circ \cdot \sin 70^\circ) \cdot (\sin 10^\circ \cdot \sin 20^\circ \cdot \sin 40^\circ)$

$$= \left(\frac{1}{4} \sin 30^\circ \right) \cdot \left[\frac{1}{2} \sin 10^\circ (\cos 20^\circ - \cos 60^\circ) \right]$$

$$= \frac{1}{16} \left[\sin 10^\circ \left(\cos 20^\circ - \frac{1}{2} \right) \right]$$

$$= \frac{1}{32} [2\sin 10^\circ \cdot \cos 20^\circ - \sin 10^\circ]$$

$$= \frac{1}{32} [\sin 30^\circ - \sin 10^\circ - \sin 10^\circ]$$

$$= \frac{1}{64} - \frac{1}{16} \sin 10^\circ$$

Clearly $\alpha = \frac{1}{64}$

Hence $16 + \alpha^{-1} = 80$

7. Let $A = \{n \in \mathbf{N} : \text{H.C.F.}(n, 45) = 1\}$ and
 Let $B = \{2k : k \in \{1, 2, \dots, 100\}\}$. Then the sum of all the elements of $A \cap B$ is _____.

Answer (5264)

Sol: Sum of all elements of $A \cap B = 2$ [Sum of natural numbers upto 100 which are neither divisible by 3 nor by 5]

$$= 2 \left[\frac{100 \times 101}{2} - 3 \left(\frac{33 \times 34}{2} \right) - 5 \left(\frac{20 \times 21}{2} \right) + 15 \left(\frac{6 \times 7}{2} \right) \right]$$

$$= 10100 - 3366 - 2100 + 630$$

$$= 5264$$

8. The value of the integral

$$\frac{48}{\pi^4} \int_0^\pi \left(\frac{3\pi x^2}{2} - x^3 \right) \frac{\sin x}{1 + \cos^2 x} dx$$
 is equal to _____.

Answer (6)

Sol: $I = \frac{48}{\pi^4} \int_0^\pi \left[\left(\frac{\pi}{2} - x \right)^3 - \frac{3\pi^2}{4} \left(\frac{\pi}{2} - x \right) + \frac{\pi^3}{4} \right] \frac{\sin x}{1 + \cos^2 x} dx$

Using $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$ we get

$$I = \frac{48}{\pi^4} \int_0^\pi \left[-\left(\frac{\pi}{2} - x \right)^3 + \frac{3\pi^2}{4} \left(\frac{\pi}{2} - x \right) + \frac{\pi^3}{4} \right] \frac{\sin x}{1 + \cos^2 x} dx$$

Adding these two equations, we get

$$2I = \frac{48}{\pi^4} \int_0^\pi \frac{\pi^3}{2} \cdot \frac{\sin x}{1 + \cos^2 x} dx$$

$$\Rightarrow I = \frac{12}{\pi} \left[-\tan^{-1}(\cos x) \right]_0^\pi = \frac{12}{\pi} \cdot \frac{\pi}{2} = 6$$

9. Let $A = \sum_{i=1}^{10} \sum_{j=1}^{10} \min\{i, j\}$ and $B = \sum_{i=1}^{10} \sum_{j=1}^{10} \max\{i, j\}$.

Then $A + B$ is equal to _____.

Answer (1100)

Sol: Each element of ordered pair $\{i, j\}$ is either present in A or in B .

So, $A + B =$ Sum of all elements of all ordered pairs $\{i, j\}$ for $1 \leq i \leq 10$ and $1 \leq j \leq 10$

$$= 20(1 + 2 + 3 + \dots + 10)$$

$$= 1100$$

10. Let $S = (0, 2\pi) - \left\{ \frac{\pi}{2}, \frac{3\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4} \right\}$. Let $y = y(x)$, $x \in S$, be the solution curve of the differential equation $\frac{dy}{dx} = \frac{1}{1 + \sin 2x}$, $y\left(\frac{\pi}{4}\right) = \frac{1}{2}$. If the sum of abscissas of all the points of intersection of the curve $y = y(x)$ with the curve $y = \sqrt{2} \sin x$ is $\frac{k\pi}{12}$, then k is equal to _____.

Answer (42)

Sol: $\frac{dy}{dx} = \frac{1}{1 + \sin 2x}$

$$\Rightarrow dy = \frac{\sec^2 x \, dx}{(1 + \tan x)^2}$$

$$\Rightarrow y = -\frac{1}{1 + \tan x} + c$$

When $x = \frac{\pi}{4}$, $y = \frac{1}{2}$ gives $c = 1$

So $y = \frac{\tan x}{1 + \tan x} \Rightarrow y = \frac{\sin x}{\sin x + \cos x}$

Now, $y = \sqrt{2} \sin x \Rightarrow \sin x = 0$

or $\sin x + \cos x = \frac{1}{\sqrt{2}}$

$\sin x = 0$ gives $x = \pi$ only.

and $\sin x + \cos x = \frac{1}{\sqrt{2}} \Rightarrow \sin\left(x + \frac{\pi}{4}\right) = \frac{1}{2}$

So $x + \frac{\pi}{4} = \frac{5\pi}{6}$ or $\frac{13\pi}{6} \Rightarrow x = \frac{7\pi}{12}$ or $\frac{23\pi}{12}$

Sum of all solutions = $\pi + \frac{7\pi}{12} + \frac{23\pi}{12} = \frac{42\pi}{12}$

Hence $k = 42$.

