27/06/2022
Evening

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Answers & Solutions

for

Time : 3 hrs. M.M. : 300

JEE (Main)-2022 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The SI unit of a physical quantity is pascal-second. The dimensional formula of this quantity will be :

(A) $[ML^{-1}T^{-1}]$ (B) $[ML^{-1}T^{-2}]$

(C) $[ML^{2}T^{-1}]$ (D) $[M^{-1}L^{3}T^{0}]$

Answer (A)

Sol. [pascal-second] = $\frac{MLT^{-2}}{L^2} \times T$

$= ML^{-1}T^{-1}$

2. The distance of the Sun from earth is 1.5×10^{11} m and its angular diameter is (2000) s when observed from the earth. The diameter of the Sun will be :

(A) 2.45×10^{10} m (B) 1.45×10^{10} m

(C) 1.45×10^{9} m (D) 0.14×10^{9} m

Answer (C)

Sol. Diameter = $r \times \delta$

$$= 1.5 \times 10^{11} \times (2000) \times \left(\frac{1}{3600}\right) \times \left(\frac{\pi}{180}\right)$$

$= 1.45 \times 10^{9}$ m

3. When a ball is dropped into a lake from a height 4.9 m above the water level, it hits the water with a velocity v and then sinks to the bottom with the constant velocity v. It reaches the bottom of the lake 4.0 s after it is dropped. The approximate depth of the lake is :

(A) 19.6 m (B) 29.4 m

(C) 39.2 m (D) 73.5 m

Answer (B)

Sol. $t_1 = \sqrt{\frac{2h}{g}}$

$$= \sqrt{\frac{2 \times 4.9}{9.8}} = 1\,\text{s}$$

$\Delta t = 4 - 1 = 3$ s,

$v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 4.9} = 9.8$ m/s

$\therefore$ depth = $9.8 \times 3 = 29.4$ m

4. One end of a massless spring of spring constant k and natural length l_0 is fixed while the other end is connected to a small object of mass m lying on a frictionless table. The spring remains horizontal on the table. If the object is made to rotate at an angular velocity ω about an axis passing through fixed end, then the elongation of the spring will be :

(A) $\frac{k - m\omega^2 l_0}{m\omega^2}$ (B) $\frac{m\omega^2 l_0}{k + m\omega^2}$

(C) $\frac{m\omega^2 l_0}{k - m\omega^2}$ (D) $\frac{k + m\omega^2 l_0}{m\omega^2}$

Answer (C)

Sol. $m\omega^2(l_0 + x) = kx$

$\Rightarrow m\omega^2 l_0 = (k - m\omega^2) \times x$

$\Rightarrow x = \frac{m\omega^2 l_0}{(k - m\omega^2)}$

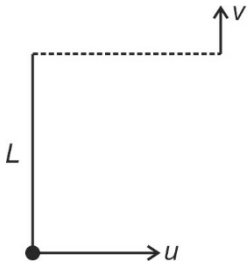
5. A stone tied to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time, the stone is at its lowest position and has a speed u. The magnitude of change in its velocity, as it reaches a position where the string is horizontal, is $\sqrt{x(u^2 - gL)}$. The value of x is

(A) 3 (B) 2

(C) 1 (D) 5

Answer (B)

Sol. $\vec{v} = \sqrt{u^2 - 2gL}\,\hat{j}$



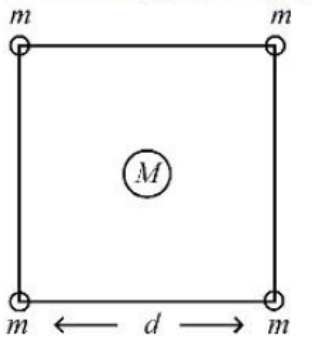
$\vec{u} = u\hat{i}$

$\therefore \quad |\vec{v} - \vec{u}| = \sqrt{(u^2 - 2gL) + u^2}$

$= \sqrt{2u^2 - 2gL}$

$\therefore \quad x = 2$

6. Four spheres each of mass m form a square of side d (as shown in figure). A fifth sphere of mass M is situated at the centre of square. The total gravitational potential energy of the system is:



(A) $-\frac{Gm}{d}\left[\left(4+\sqrt{2}\right)m+4\sqrt{2}M\right]$

(B) $-\frac{Gm}{d}\left[\left(4+\sqrt{2}\right)M+4\sqrt{2}m\right]$

(C) $-\frac{Gm}{d}\left[3m^2+4\sqrt{2}M\right]$

(D) $-\frac{Gm}{d}\left[6m^2+4\sqrt{2}M\right]$

Answer (A)

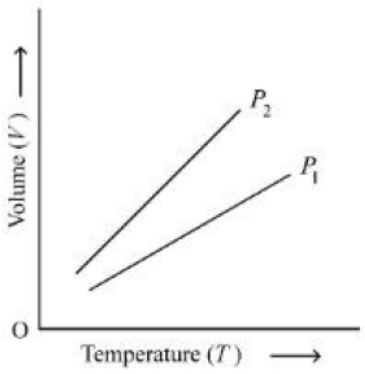
Sol. Total gravitational potential energy

$$=-\left\{\frac{4GMm}{d/\sqrt{2}}+\frac{4Gm^2}{d}+\frac{2Gm^2}{\sqrt{2}d}\right\}$$

$$=-\frac{Gm}{d}\left\{M4\sqrt{2}+(4+\sqrt{2})m\right\}$$

$$=-\frac{Gm}{d}\left\{4\sqrt{2}M+(4+\sqrt{2})m\right\}$$

7. For a perfect gas, two pressures P_1 and P_2 are shown in figure. The graph shows:



(A) $P_1>P_2$

(B) $P_1<P_2$

(C) $P_1 = P_2$

(D) Insufficient data to draw any conclusion

Answer (A)

Sol. As per ideal gas equation, $V=\frac{nR}{P}T$

$\Rightarrow$ Slope of V-T graph is inversely proportional to P.

As $m_2 > m_1 \Rightarrow P_1 > P_2$

8. According to kinetic theory of gases,

A. The motion of the gas molecules freezes at 0°C

B. The mean free path of gas molecules decreases if the density of molecules is increased.

C. The mean free path of gas molecules increases if temperature is increased keeping pressure constant.

D. Average kinetic energy per molecule per degree of freedom is $\frac{3}{2}k_BT$ (for monoatomic gases).

Choose the most appropriate answer from the options given below:

(A) A and C only (B) B and C only

(C) A and B only (D) C and D only

Answer (B)

Sol. According to kinetic theory of gases,

A. The motion of the gas molecules freezes at 0 K.

B. The mean free path decreases on increasing the number density of the molecules as $\mu=\frac{1}{\sqrt{2}\pi nd^2}\Rightarrow \mu\propto\frac{1}{n}$.

C. The mean free path increases on increasing the volume. Now if temperature is increased by keeping the pressure constant the volume should increase that is mean free path increases.

D. K.E.$_{avg}$ per molecule per degree of freedom is $\frac{1}{2}k_BT$.

$\Rightarrow$ Option (B) and (C) only are correct.

9. A lead bullet penetrates into a solid object and melts. Assuming that 40% of its kinetic energy is used to heat it, the initial speed of bullet is:

(Given initial temperature of the bullet = 127°C),

Melting point of the bullet = 327°C,

Latent heat of fusion of lead = 2.5×10^4 J kg^{-1}

Specific heat capacity of lead = 125 J/kg K)

(A) 125 ms^{-1} (B) 500ms^{-1}

(C) 250ms^{-1} (D) 600ms^{-1}

Answer (B)

Sol. $\frac{2}{5}\times\frac{1}{2}mv^2 = mL + ms\Delta T$

$\Rightarrow \frac{v^2}{5} = 2.5\times10^4 + 125 + 200$

$\Rightarrow \frac{v^2}{5} = 5\times10^4$

$\Rightarrow v = 500$ m/s

10. The equation of a particle executing simple harmonic motion is given by $x = \sin\pi\left(t+\frac{1}{3}\right)m$. At $t = 1$ s, the speed of particle will be

(A) 0 cm s^{-1} (B) 157cm s^{-1}

(C) 272cm s^{-1} (D) 314cm s^{-1}

Answer (B)

Sol. $x = \sin\left(\pi t + \frac{\pi}{3}\right)$m

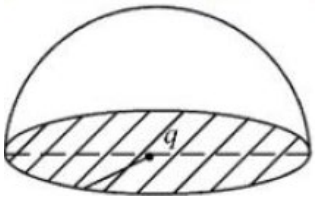
$\Rightarrow \frac{dx}{dt} = \pi\cos\left(\pi t + \frac{\pi}{3}\right)$

$= \pi\cos\left(\pi + \frac{\pi}{3}\right)$ at $t = 1$ s

$= -\frac{\pi}{2}$ m/s

or $\left|\frac{dx}{dt}\right| = 157$ cm/s

11. If a charge q is placed at the centre of a closed hemispherical non-conducting surface, the total flux passing through the flat surface would be:



(A) $\frac{q}{\varepsilon_0}$ (B) $\frac{q}{2\varepsilon_0}$

(C) $\frac{q}{4\varepsilon_0}$ (D) $\frac{q}{2\pi\varepsilon_0}$

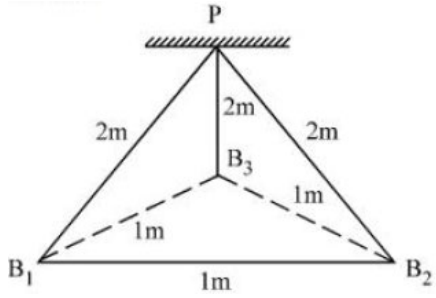
Answer (B)

Sol. Flux passing through flat surface = Flux passing through curved surface.

So $\phi = \frac{q}{2\varepsilon_0}$

12. Three identical charged balls each of charge 2 C are suspended from a common point P by silk threads of 2 m each (as shown in figure). They form an equilateral triangle of side 1 m.

The ratio of net force on a charged ball to the force between any two charged balls will be:

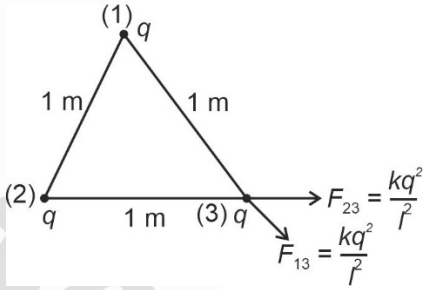


(A) 1 : 1 (B) 1 : 4

(C) $\sqrt{3}:2$ (D) $\sqrt{3}:1$

Answer (D)

Sol.



F_{net} on charge 3, $F_1 = \frac{\sqrt{3}kq^2}{1^2}$

Force between any 2 charges

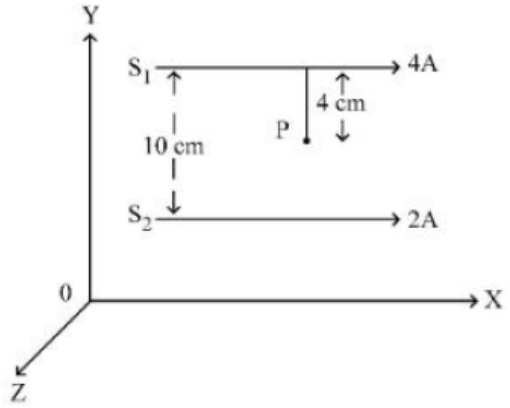
$F_2 = \frac{kq^2}{1^2}$

So, $\frac{F_1}{F_2} = \sqrt{3}$

13. Two long parallel conductors S_1 and S_2 are separated by a distance 10 cm and carrying currents of 4 A and 2 A respectively. The conductors are placed along x-axis in X-Y plane. There is a point P located between the conductors (as shown in figure).

A charge particle of 3π coulomb is passing through the point P with velocity $\vec{v} = \left(2\hat{i} + 3\hat{j}\right)$ m/s; where $\hat{i}$ and $\hat{j}$ represents unit vector along x & y axis respectively.

The force acting on the charge particle is $4\pi\times10^{-5}\left(-x\hat{i} + 2\hat{j}\right)N$. The value of x is:



(A) 2 (B) 1

(C) 3 (D) –3

Answer (C)

Sol. Field at P is $= \left(\frac{\mu_0 \times i_1}{2\pi r_1} - \frac{\mu_0 i_2}{2\pi r_2}\right)(-\hat{k})$

$$= -\left(\frac{\mu_0 4}{2\pi \times 0.04} - \frac{\mu_0 \times 2}{2\pi \times 0.06}\right)\hat{k} = -\frac{\mu_0 \times 200}{6\pi}\hat{k}$$

So, force $\vec{F} = q\vec{v} \times \vec{B}$

$$= 3\pi(2\hat{i} + 3\hat{j}) \times \left(-\left(\frac{\mu_0 \times 200}{6\pi}\hat{k}\right)\right)$$

$$= 3\pi\left(\frac{200\mu_0}{3\pi}\hat{j} - \frac{100\mu_0}{\pi}\hat{i}\right)$$

$$= 200\mu_0\hat{j} - 300\mu_0\hat{i}$$

$$= 4\pi \times 10^{-5}(2\hat{j} - 3\hat{i})$$

So, $x = 3$

14. If L, C and R are the self-inductance, capacitance and resistance respectively, which of the following does not have the dimension of time?

(A) RC (B) $\frac{L}{R}$

(C) $\sqrt{LC}$ (D) $\frac{L}{C}$

Answer (D)

Sol. $U = \frac{1}{2}Li^2 = \frac{1}{2}CV^2$

So, $\left[\frac{L}{C}\right] = \frac{V^2}{i^2} = R^2$ is not the dimension of time.

15. Given below are two statements:

Statement I: A time varying electric field is a source of changing magnetic field and vice-versa. Thus a disturbance in electric or magnetic field creates *EM* waves.

Statement II: In a material medium, the *EM* wave travels with speed $v = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$.

In the light of the above statements, choose the correct answer from the options given below.

(A) Both statement I and statement II are true

(B) Both statement I and statement II are false

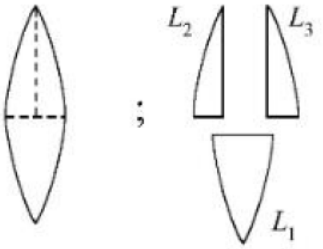
(C) Statement I is correct but statement II is false

(D) Statement I is incorrect but statement II is true

Answer (C)

Sol. In a material medium speed of light is given by $v = \frac{1}{\sqrt{\varepsilon_0\varepsilon_r\mu_0\mu_r}}$. So statement 2 is false.

16. A convex lens has power P. It is cut into two halves along its principal axis. Further one piece (out of the two halves) is cut into two halves perpendicular to the principal axis (as shown in figures). Choose the incorrect option for the reported pieces.



(A) Power of $L_1 = \frac{P}{2}$

(B) Power of $L_2 = \frac{P}{2}$

(C) Power of $L_3 = \frac{P}{2}$

(D) Power of $L_1 = P$

Answer (A)

Sol. We know $P = \frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$L_1 : \frac{1}{f_1} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = P_1 = (\mu - 1)\left(\frac{2}{R}\right) = P$$

$$L_2 : \frac{1}{f_2} = (\mu - 1)\left(\frac{1}{R_1}\right) = P_2 = \frac{\mu - 1}{R}$$

$$L_3 : \frac{1}{f_3} = (\mu - 1)\left(-\frac{1}{R_2}\right) = P_3 = \frac{\mu - 1}{R}$$

17. If a wave gets refracted into a denser medium, then which of the following is true?

(A) Wavelength, speed and frequency decreases

(B) Wavelength increases, speed decreases and frequency remains constant

(C) Wavelength and speed decreases but frequency remains constant

(D) Wavelength, speed and frequency increases

Answer (C)

Sol. Frequency is independent of medium. For denser medium, wavelength and speed both would decrease.

18. Given below are two statements:

Statement I: In hydrogen atom, the frequency of radiation emitted when an electron jumps from lower energy orbit (E_1) to higher energy orbit (E_2), is given as $hf = E_1 - E_2$.

Statement II: The jumping of electron from higher energy orbit (E_2) to lower energy orbit (E_1) is associated with frequency of radiation given as $f = \frac{(E_2 - E_1)}{h}$.

This condition is Bohr's frequency condition.

In the light of the above statements, choose the correct answer from the options given below:

(A) Both statement I and statement II are true

(B) Both statement I and statement II are false

(C) Statement I is correct but statement II is false

(D) Statement I is incorrect but statement II is true

Answer (D)

Sol. Radiation is not emitted but absorbed when an electron jumps from low energy to high energy.

Also, $E_2 - E_1$ is the energy of photon

$\Rightarrow E_2 - E_1 = hf$

$\Rightarrow f = \frac{E_2 - E_1}{h}$

19. For a transistor to act as a switch, it must be operated in

(A) Active region

(B) Saturation state only

(C) Cut-off state only

(D) Saturation and cut-off state

Answer (D)

Sol. A transistor acts as a switch when it is operated in saturation and cut-off state.

20. We do not transmit low frequency signal to long distance because-

(a) The size of the antenna should be comparable to signal wavelength which is unreal solution for a signal of longer wavelength

(b) Effective power radiated by a long wavelength baseband signal would be high

(c) We want to avoid mixing up signals transmitted by different transmitter simultaneously

(d) Low frequency signal can be sent to long distances by superimposing with a high frequency wave as well

Therefore, the most suitable option will be:

(A) All statements are true

(B) (a), (b) and (c) are true only

(C) (a), (c) and (d) are true only

(D) (b), (c) and (d) are true only

Answer (C)

Sol. For longer wavelength, size of antenna would increase. Also, mixing of signals needs to be avoided.

Also, we can use modulation to send low frequency signal by superimposing them with high frequency signals.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

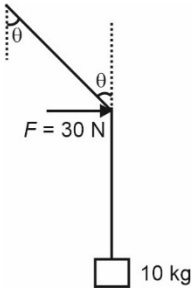
1. A mass of 10 kg is suspended vertically by a rope of length 5 m from the roof. A force of 30 N is applied at the middle point of rope in horizontal direction. The angle made by upper half of the rope with vertical is $\theta = \tan^{-1}(x \times 10^{-1})$. The value of x is

_____

(Given, g = 10 m/s^2)

Answer (3)

Sol.



$T\cos\theta = mg$

$T\cos\theta = 100$ N …(i)

$T\sin\theta = 30$ …(ii)

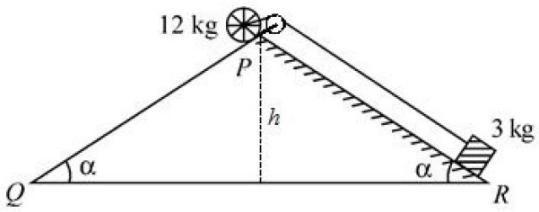
$\Rightarrow \dfrac{T\sin\theta}{T\cos\theta} = \dfrac{30}{100}$

$\Rightarrow \tan\theta = \dfrac{3}{10}$

$\therefore \quad x = 3$

2. A rolling wheel of 12 kg is on an inclined plane at position P and connected to a mass of 3 kg through a string of fixed length and pulley as shown in figure. Consider PR as friction free surface.

The velocity of centre of mass of the wheel when it reaches at the bottom Q of the inclined plane PQ will be $\dfrac{1}{2}\sqrt{xgh}$ m/s. The value of x is _____.



Answer (3)

Sol. For rolling wheel

$[12g\sin\alpha - 3g\sin\alpha] \times R = (2 \times 12R^2 + 3R^2) \times \dfrac{a}{R}$

$\Rightarrow \dfrac{9g\sin\alpha}{27} = a$

$\Rightarrow a = \dfrac{g\sin\alpha}{3}$

$\therefore v = \sqrt{2 \times \dfrac{g\sin\alpha}{3} \times \dfrac{h}{\sin\alpha}} = \sqrt{\dfrac{2}{3}gh}$

$= \dfrac{1}{2} \times \sqrt{\dfrac{8}{3}gh}$

$\therefore \quad x = \dfrac{8}{3} = 2.67$

3. A diatomic gas ($\gamma = 1.4$) does 400 J of work when it is expanded isobarically. The heat given to the gas in the process is _____ J.

Answer (1400)

Sol. $W = nR\Delta T = 400$ J

$\therefore \quad \Delta Q = nC_P\Delta T$

$= n \times \dfrac{7}{2}R \times \Delta T = \dfrac{7}{2} \times (400) = 1400$

4. A particle executes simple harmonic motion. Its amplitude is 8 cm and time period is 6 s. The time it will take to travel from its position of maximum displacement to the point corresponding to half of its amplitude, is _____ s.

Answer (1)

Sol. $A = 8$ cm

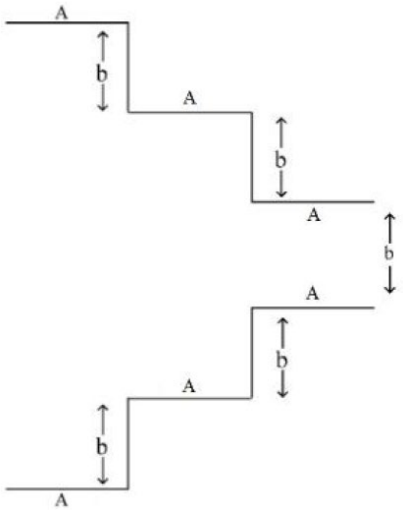
$T = 6$ s

$A\cos\left(\dfrac{2\pi t}{T}\right) = \dfrac{A}{2}$

$\Rightarrow \dfrac{2\pi t}{T} = \dfrac{\pi}{3}$

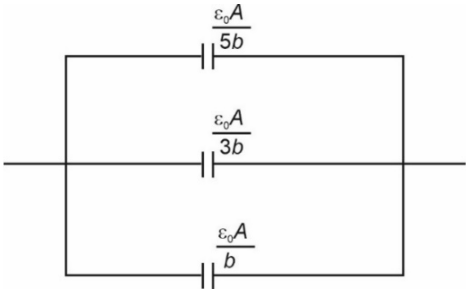
or $t = \dfrac{T}{6} = 1$ s

5. A parallel plate capacitor is made up of stair like structure with a plate area A of each stair and that is connected with a wire of length b, as shown in the figure. The capacitance of the arrangement is $\dfrac{x}{15}\dfrac{\varepsilon_0 A}{b}$, the value of x is _____?



Answer (23)

Sol. The circuit is equivalent to 3 capacitors in parallel as shown



$$C_{eq} = \frac{\varepsilon_0 A}{b}\left(1+\frac{1}{3}+\frac{1}{5}\right) = \frac{23}{15}\frac{\varepsilon_0 A}{b}$$

$\Rightarrow x = 23$

6. The current density in a cylindrical wire of radius r = 4.0 mm is 1.0 x 10^6 A/m^2. The current through the outer portion of the wire between radial distances $\frac{r}{2}$ and r is $x\pi$ A; where x is _____

Answer (12)

Sol. $i = A \times j$

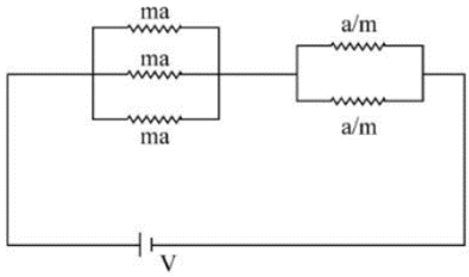
$$= \pi\left(R^2 - \frac{R^2}{4}\right)j$$

$$= \frac{3\pi R^2}{4} \times j$$

$$= \frac{3\pi \times \left(4\times 10^{-3}\right)^2}{4} \times 1.0 \times 10^6$$

$= 12\ \pi$

7. In the given circuit 'a' is an arbitrary constant. The value of m for which the equivalent circuit resistance is minimum, will be $\sqrt{\frac{x}{2}}$. The value of x is_____.



Answer (3)

Sol. $R_{net} = \frac{ma}{3} + \frac{a}{2m}$

$$= a\left[\frac{m}{3} + \frac{1}{2m} - \frac{2}{\sqrt{6}} + \frac{2}{\sqrt{6}}\right]$$

$$= a\left[\left(\sqrt{\frac{m}{3}} - \frac{1}{\sqrt{2m}}\right)^2 + \sqrt{\frac{2}{3}}\right]$$

This will be minimum when

$$\sqrt{\frac{m}{3}} = \frac{1}{\sqrt{2m}}$$

or $m = \sqrt{\frac{3}{2}}$ so $x = 3$

8. A deuteron and a proton moving with equal kinetic energy enter into a uniform magnetic field at right angle to the field. If r_d and r_p are the radii of their circular paths respectively, then the ratio $\frac{r_d}{r_p}$ will be $\sqrt{x}:1$ where x is _____.

Answer (2)

Sol. $R = \frac{\sqrt{2mK}}{qB}$

So $\frac{r_d}{r_p} = \frac{\sqrt{m_d}/q_d}{\sqrt{m_p}/q_p}$

$= \sqrt{2}$

So $x = 2$

9. A metallic rod of length 20 cm is placed in North-South direction and is moved at a constant speed of 20 m/s towards East. The horizontal component of the Earth's magnetic field at that place is 4 × 10^{-3} T and the angle of dip is 45º. The emf induced in the rod is _____mV.

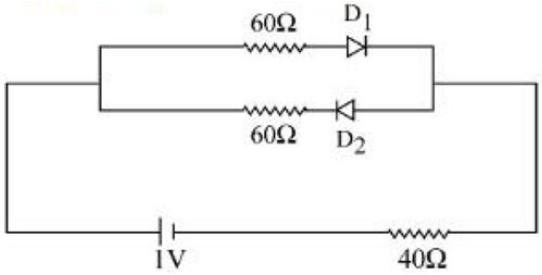
Answer (16)

Sol. $E = Blv$

$$= 4 \times 10^{-3} \times \frac{20}{100} \times 20 \text{ Volts}$$

$= 16$ mV

10. The cut-off voltage of the diodes (shown in figure) in forward bias is 0.6 V. The current through the resistor of 40 Ω is ____ mA.



Answer (4)

Sol. D_1 : conducting

D_2 : open circuit

$$\Rightarrow i = \frac{1-0.6}{60+40}\text{A}$$

$$= \frac{0.4}{100}\text{A}$$

$\Rightarrow i = 4$ mA

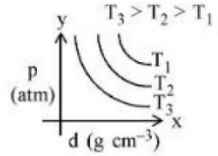
CHEMISTRY

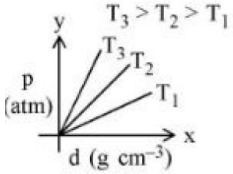
SECTION - A

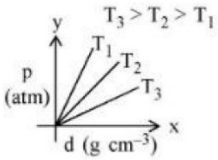
Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

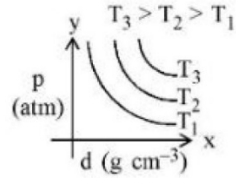
Choose the correct answer :

1. Which amongst the given plots is the correct plot for pressure (p) vs density (d) for an ideal gas?

(A) 

(B) 

(C) 

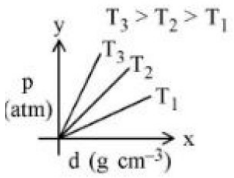
(D) 

Answer (B)

Sol. $\because d = \frac{pM}{RT}$

Hence, $dRT = pM$

$p \propto T$



2. Identify the **incorrect** statement for PCl_5 from the following.

(A) In this molecule, orbitals of phosphorous are assumed to undergo sp^3d hybridization.

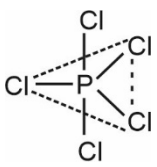
(B) The geometry of PCl_5 is trigonal bipyramidal.

(C) PCl_5 has two axial bonds stronger than three equatorial bonds.

(D) The three equatorial bonds of PCl_5 lie in a plane

Answer (C)

Sol. PCl_5



- All three equatorial bonds in a plane
- sp^3d hybridization
- Trigonal bipyramidal
- Axial bonds are weaker than equatorial bonds.

3. Statement-I : Leaching of gold with cyanide ion in absence of air/O_2 leads to cyano complex of Au(III).

Statement-II : Zinc is oxidized during the displacement reaction carried out for gold extraction.

In the light of the above statements, choose the **correct** answer from the options given below.

(A) Both statement-I and statement-II are correct

(B) Both statement-I and statement-II are incorrect

(C) Statement-I is correct but statement-II is incorrect

(D) Statement-I is incorrect but statement-II is correct

Answer (D)

Sol. Leaching of gold with cyanide ion is done in presence of air/O_2 leading to cyano complex $[Au(CN)_2]^-$ where Au is in +1 oxidation state.

$$4Au(s) + 8CN^-(aq) + 2H_2O(aq) + O_2(g) \longrightarrow 4\left[Au(CN)_2\right]^-_{aq} + 4OH^-$$

$$2\left[Au(CN)_2\right]^-_{(aq)} + Zn(s) \longrightarrow \left[Zn(CN)_4\right]^{2-}_{(aq)} + 2Au(s)$$

Zinc is oxidised from (0) to +2 oxidation state during displacement reaction carried out for gold extraction.

4. The correct order of increasing intermolecular hydrogen bond strength is

(A) $HCN < H_2O < NH_3$

(B) $HCN < CH_4 < NH_3$

(C) $CH_4 < HCN < NH_3$

(D) $CH_4 < NH_3 < HCN$

Answer (C)

Sol. Due to high difference in electronegativity of H and N the H-bond strength of NH_3 is highest. There is no H-bond in CH_4.

$CH_4 < HCN < NH_3$

Hence, correct option is (C)

5. The correct order of increasing ionic radii is

(A) $Mg^{2+} < Na^+ < F^- < O^{2-} < N^{3-}$

(B) $N^{3-} < O^{2-} < F^- < Na^+ < Mg^{2+}$

(C) $F^- < Na^+ < O^{2-} < Mg^{2+} < N^{3-}$

(D) $Na^+ < F^- < Mg^{2+} < O^{2-} < N^{3-}$

Answer (A)

Sol. For isoelectronic species

$$\text{Ionic radii} \propto \frac{(-)\text{ve charge}}{(+)\text{ve charge}}$$

Hence, correct order of ionic radii is

$Mg^{2+} < Na^+ < F^- < O^{2-} < N^{3-}$

6. The gas produced by treating an aqueous solution of ammonium chloride with sodium nitrite is

(A) NH_3 (B) N_2

(C) N_2O (D) Cl_2

Answer (B)

Sol. N_2 gas is produced by treating an aqueous solution of ammonium chloride with sodium nitrite.

$$NH_4Cl(aq) + NaNO_2(aq) \longrightarrow N_2(g) + 2H_2O(l) + NaCl(aq)$$

Ammonium chloride	Sodium nitrite	Nitrogen gas	Water	Sodium chloride

7. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A: Fluorine forms one oxoacid.

Reason R: Fluorine has smallest size amongst all halogens and is highly electronegative.

In the light of the above statements, choose the *most appropriate* answer from the option given below.

(1) Both **A** and **R** are correct and **R** is the correct explanation of **A**.

(2) Both **A** and **R** are correct but **R** is NOT the correct explanation of **A**.

(3) **A** is correct but **R** is not correct.

(4) **A** is not correct but **R** is correct.

Answer (A)

Sol. Due to smaller size, fluorine forms only one oxoacid.

Both the Assertion and Reason are correct and Reason is the correct explanation.

8. In 3d series, the metal having the highest M^{2+}/M standard electrode potential is

(A) Cr (B) Fe

(C) Cu (D) Zn

Answer (C)

Sol.

Metal	E° M^{2+}/M
Cr	–0.90 V
Fe	–0.44 V
Cu	+ 0.34 V
Zn	(–0.76 V)

The metal having highest $E°(M^{2+}/M)$ standard reduction potential is Cu.

9. The 'f' orbitals are half and completely filled, respectively in lanthanide ions

[Given: Atomic no. Eu, 63; Sm, 62; Tm, 69; Tb, 65; Yb, 70; Dy, 66]

(A) Eu^{2+} and Tm^{2+} (B) Sm^{2+} and Tm^{3+}

(C) Tb^{4+} and Yb^{2+} (D) Dy^{3+} and Yb^{3+}

Answer (C)

Sol.

	+2	+3	+4
Eu	$4f^7$	$4f^6$	
Tm	$4f^{13}$	$4f^{12}$	
Sm	$4f^6$	$4f^5$	
Tb	$4f^9$	$4f^8$	$4f^7$
Yb	$4f^{14}$	$4f^{13}$	
Dy	$4f^{10}$	$4f^9$	

Hence, the pair Tb^{+4} Yb^{+2} have half filled and completely filled f subshells respectively.

10. Arrange the following coordination compounds in the increasing order of magnetic moments. (Atomic numbers: Mn = 25; Fe = 26)

(1) $[FeF_6]^{3-}$ (2) $[Fe(CN)_6]^{3-}$

(3) $[MnCl_6]^{3-}$ (high spin) (4) $[Mn(CN)_6]^{3-}$

Choose the correct answer from the options given below:

(A) 1 < 2 < 4 < 3 (B) 2 < 4 < 3 < 1

(C) 1 < 3 < 4 < 2 (D) 2 < 4 < 1 < 3

Answer (B)

Sol.

Coordination Compound	Number of unpaired e^- (n)	Magnetic moment (μ) (B.M)
A $[FeF_6]^{3-}$ – d^5	5	5.91
B $[Fe(CN)_6]^{3-}$ – d^5	1	1.73
C $[MnCl_6]^{3-}$ – d^4	4	4.89
D $[Mn(CN)_6]^{3-}$ – d^4	2	2.82

Hence, correct order of magnetic moment is 2 < 4 < 3 < 1

11. On the surface of polar stratospheric clouds, hydrolysis of chlorine nitrate gives A and B while its reaction with HCl produces B and C. A, B and C are, respectively

(A) HOCl, HNO_3, Cl_2

(B) Cl_2, HNO_3, HOCl

(C) $HClO_2$, HNO_2, HOCl

(D) HOCl, HNO_2, Cl_2O

Answer (A)

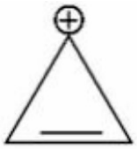
Sol. On the surface of polar stratospheric clouds, hydrolysis of chlorine nitrate as

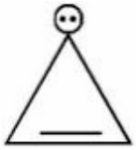
$$ClONO_2 + H_2O \longrightarrow \underset{A}{HOCl} + \underset{B}{HNO_3}$$

$$ClONO_2 + HCl \longrightarrow \underset{C}{Cl_2} + \underset{B}{HNO_3}$$

Hence A, B and C are HOCl, HNO_3 and Cl_2 respectively.

12. Which of the following is most stable?

(A) 

(B) 

(C) 

(D) 

Answer (D)

Sol.  — Aromatic compound ($2\pi e^-$)

 — Carbene (electron deficient)

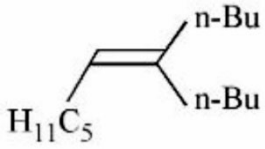
 — Anti Aromatic compound ($4\pi e^-$)

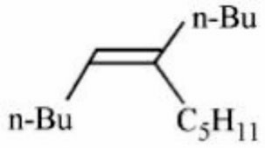
 — Non-aromatic conjugated diene

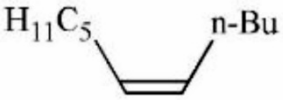
1,3-cyclohexadiene is most stable because it is a neutral molecule. All others are intermediates and hence less stable.

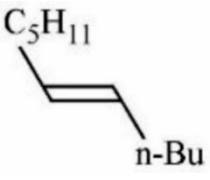
13. What will be the major product of following sequence of reactions?

$$n-Bu-\equiv \xrightarrow[(ii)\ Lindlar\ cat,\ H_2]{(i)\ n-BuLi,\ n-C_5H_{11}Cl}$$

(A) 

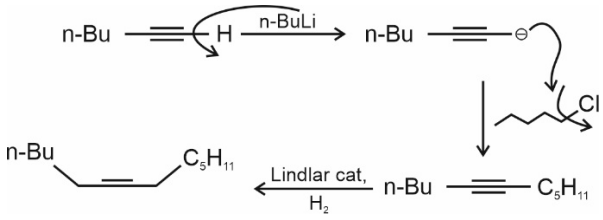
(B) 

(C) 

(D) 

Answer (C)

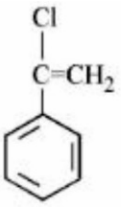
Sol.

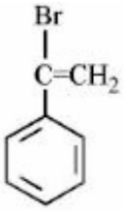


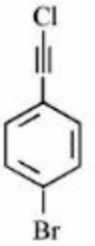
Hence correct option is (C).

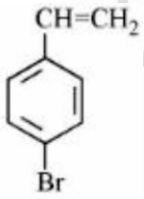
14. Product 'A' of following sequence of reactions is

Ethylbenzene $\xrightarrow[\text{(b) } Cl_2, \Delta \\ \text{(c) alc. KOH}]{\text{(a) } Br_2, Fe}$ 'A' Major Product

(A) 

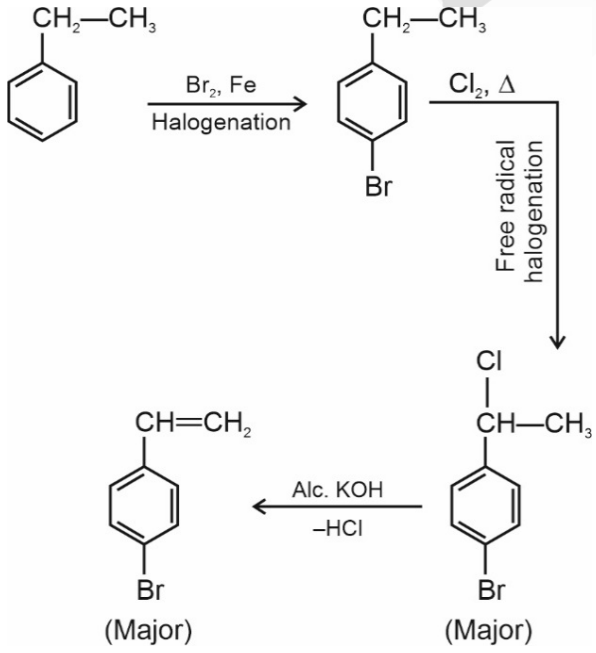
(B) 

(C) 

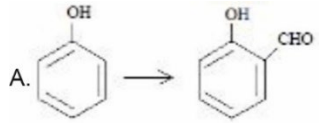
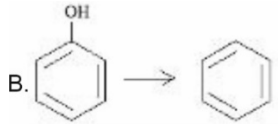
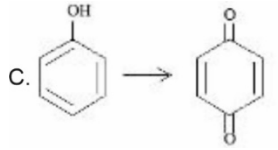
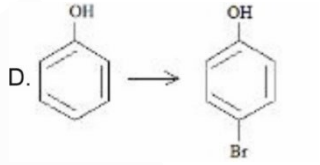
(D) 

Answer (D)

Sol.



15. Match List I with List II.

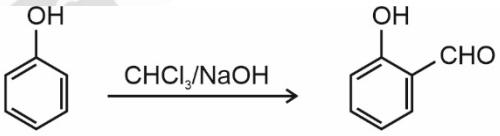
List I	List II
A. 	I. Br_2 in CS_2
B. 	II. $Na_2Cr_2O_7$/ H_2SO_4
C. 	III. Zn
D. 	IV. $CHCl_3$/NaOH

Choose the correct answer from the options given below:

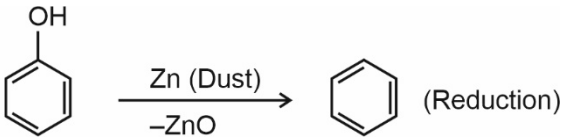
(A) A-IV, B-III, C-II, D-I (B) A-IV, B-III, C-I, D-II

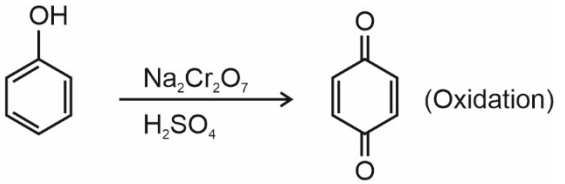
(C) A-II, B-III, C-I, D-IV (D) A-IV, B-II, C-III, D-I

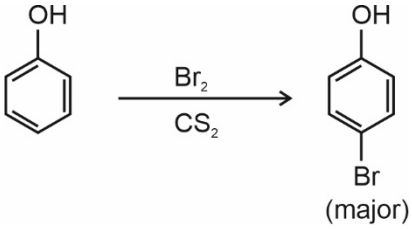
Answer (A)

Sol. A. 

Reimer Tiemann reaction

B. 

C. 

D. 

∴Correct match is

(A) - IV, (B) - III, (C) - II, (D) - I

16. Decarboxylation of all six possible forms of diaminobenzoic acid $C_6H_3(NH_2)_2COOH$ yields three products A, B and C. Three acids give a product 'A', two acids give a product 'B' and one acid gives a product 'C'. The melting point of product 'C' is

(A) 63ºC

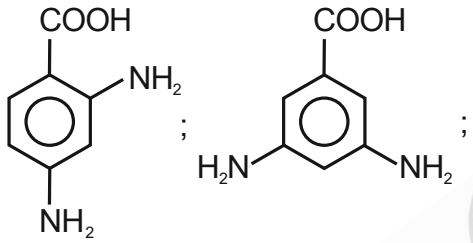
(B) 90ºC

(C) 104ºC

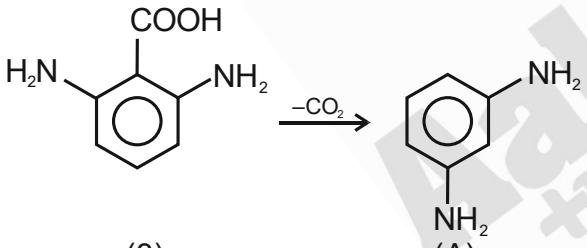
(D) 142ºC

Answer (D)

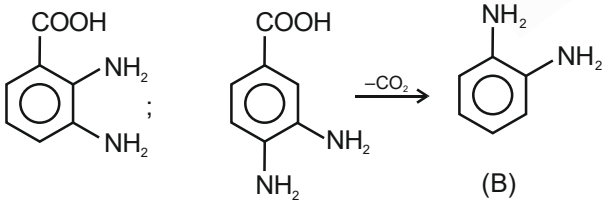
Sol. The six possible forms of diaminobenzoic acid are



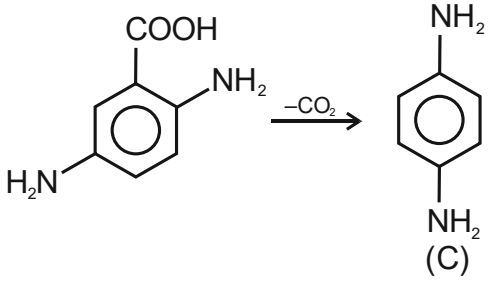
(1) (2)



(3) (A)



(B)



(C)

Melting point of product (C) = 142ºC

17. Which is true about Buna-N?

(A) It is a linear polymer of 1, 3-butadiene

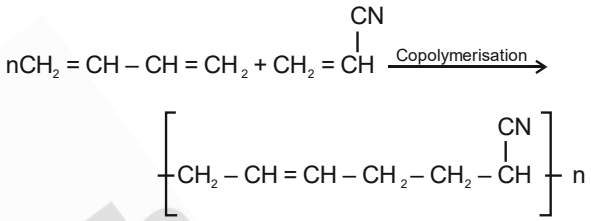
(B) It is obtained by copolymerization of 1, 3-butadiene and styrene

(C) It is obtained by copolymerization of 1, 3-butadiene and acrylonitrile

(D) The suffix N in Buna-N stands for its natural occurrence.

Answer (C)

Sol. Buna-N is formed by copolymerisation of 1-3-butadiene and acrylonitrile



18. Given below are two statements

Statement I: Maltose has two α-D-glucose units linked at C_1 and C_4 and is a reducing sugar.

Statement II: Maltose has two monosaccharides: α-D-glucose and β-D-glucose linked at C_1 and C_6 and it is a non-reducing sugar.

In the light of the above statements, choose the **correct** answer from the options given below.

(A) Both Statement I and Statement II are true

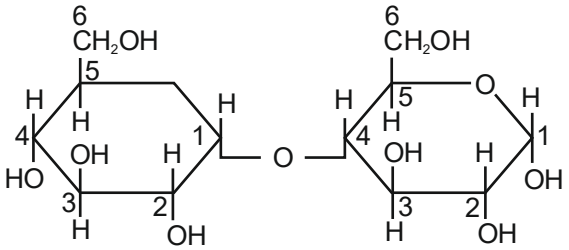
(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Answer (C)

Sol. Maltose is composed of two α-D-glucose units in which C_1 of one glucose unit and C_4 of second glucose unit are linked.



19. Match **List I** with **List II**.

List I		List II	
A.	Antipyretic	I.	Reduces pain
B.	Analgesic	II.	Reduces stress
C.	Tranquilizer	III.	Reduces fever
D.	Antacid	IV.	Reduces acidity (stomach)

Choose the correct answer from the options given below:

(A) A-III, B-I, C-II, D-IV (B) A-III, B-I, C-IV, D-II

(C) A-I, B-IV, C-II, D-III (D) A-I, B-III, C-II, D-IV

Answer (A)

Sol. Antipyretic – Reduces fever

Analgesic – Reduces pain

Tranquilizer – Reduces stress

Antacid – Reduces Acidity (stomach)

20. Match **List I** with **List II**.

List I (Anion)		List II (gas evolved on reaction with dil H_2SO_4)	
A.	CO_3^{2-}	I.	Colourless gas which turns lead acetate paper black.
B.	S^{2-}	II.	Colourless gas which turns acidified potassium dichromate solution green
C.	SO_3^{2-}	III.	Brown fumes which turns acidified KI solution containing starch blue.
D.	NO_2^-	IV.	Colourless gas evolved with brisk effervescence, which turns lime water milky.

Choose the correct answer from the options given below:

(A) A-III, B-I, C-II, D-IV (B) A-II, B-I, C-IV, D-III

(C) A-IV, B-I, C-III, D-II (D) A-IV, B-I, C-II, D-III

Answer (D)

Sol. CO_3^{2-} : On action of dil sulphuric acid, CO_2 gas is released which turns lime water milky.

S^{2-} : On action of dil sulphuric acid, H_2S gas is released which turns lead acetate paper black.

SO_3^{2-} : On action of dil H_2SO_4, SO_2 gas is evolved which turns acidified potassium dichromate solution green.

NO_2^- : On action of dil H_2SO_4, NO_2 gas is evolved which turns KI solution containg starch blue.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. 116 g of a substance upon dissociation reaction, yields 7.5 g of hydrogen, 60 g of oxygen and 48.5 g of carbon. Given that the atomic masses of H, O and C are 1, 16 and 12, respectively. The data agrees with how many formulae of the following? _____.

A. CH_3COOH B. HCHO

C. CH_3OOCH_3 D. CH_3CHO

Answer (2)

Sol.

Element	Mass%	Moles%	Relative moles
H	6.46	6.46	2
O	51.72	3.23	1
C	41.81	3.48	1

∴ Empirical formula = COH_2

The empirical formula goes with acetic acid CH_3COOH and formaldehyde HCHO.

Thus data agrees with 2 formulae.

2. Consider the following set of quantum numbers.

	n	l	m_l
A.	3	3	–3
B.	3	2	–2
C.	2	1	+1
D.	2	2	+2

The number of correct sets of quantum numbers is _____.

Answer (2)

Sol. The correct sets of Quantum numbers are, (02)

n = 3 l = 2 m_l = –2

and n = 2 l = 1 m_l = +1

I can have values from 0 to (n – 1) and m can have values from $-l$..... 0 .....+l $(2l + 1)$

3. BeO reacts with HF in presence of ammonia to give [A] which on thermal decomposition produces [B] and ammonium fluoride. Oxidation state of Be in [A] is _____

Answer (2)

Sol. $BeO + 2NH_3 + 4HF \longrightarrow \underset{(A)}{(NH_4)_2[BeF_4]}$

$(NH_4)_2(BeF_4) \xrightarrow{\text{Heating}} \underset{(B)}{BeF_2} + 2NH_4F$

Oxidation State of Be in (A) is (+2)

4. When 5 moles of He gas expand isothermally and reversibly at 300 K from 10 litre to 20 litre, the magnitude of the maximum work obtained is _____ J. [nearest integer] (Given : R = 8.3 J K^{-1} mol^{-1} and log 2 = 0.3010)

Answer (8630)

Sol. $W_{rev} = -2.303\ nRT \log_{10}\left(\frac{V_2}{V_1}\right)$

$= -2.303 \times 5 \times 8.3 \times 300 \times \log_{10}\left(\frac{20}{10}\right)$

$\simeq -8630$ J

5. A solution containing 2.5×10^{-3} kg of a solute dissolved in 75×10^{-3} kg of water boils at 373.535 K. The molar mass of the solute is _____ g mol^{-1}. [nearest integer] (Given : $K_b(H_2O)$ = 0.52 K kg mol^{-1} and boiling point of water = 373.15 K)

Answer (45)

Sol. $W_{solute} = 2.5 \times 10^{-3}$ kg

$W_{solvent} = 75 \times 10^{-3}$ kg

$\Delta T_b = 373.535 - 373.15$

$= 0.385$ K

$K_b(H_2O)$ = 0.52 K kg mol^{-1}

$$\Delta T_b = \frac{K_b \times 10^3 \times W_{solute}}{M_{solute} \times W_{solvent}}$$

$$M_{solute} = \frac{0.52 \times 10^3 \times 2.5 \times 10^{-3}}{75 \times 10^{-3} \times 0.385}$$

$= 45.02$

≈ 45

6. pH value of 0.001 M NaOH solution is_____.

Answer (11)

Sol. $[OH^-] = 0.001 = 10^{-3}$ M

$[H^+][OH^-] = 10^{-14}$

$[H^+] = 10^{-11}$

$pH = -\log[H^+]$

$= -\log(10^{-11})$

pH = 11

7. For the reaction taking place in the cell:

$Pt(s)\,|\,H_2(g)\,|\,H^+(aq)\,||\,Ag^+(aq)\,|\,Ag(s)$

$E^\circ_{cell} = +0.5332$ V.

The value of $\Delta_f G^\ominus$ is _____kJ mol^{-1} [in nearest integer]

Answer (51)

Sol. $Pt(s)\left|H_2(g)\right|H^+(aq)\left\|Ag^+(aq)\right|Ag(s)$

$$\frac{1}{2}H_2 + Ag^+ \longrightarrow Ag + H^+$$

n = 1

$E^\circ_{cell} = 0.5332$

$\Delta G^\circ = -nFE^\circ$

$= -1 \times 96500 \times 0.5332$

$= -51.453$ kJ/mole

$\simeq -51$ kJ/mole

8. It has been found that for a chemical reaction with rise in temperature by 9 K the rate constant gets doubled. Assuming a reaction to be occurring at 300 K, the value of activation energy is found to be _____kJ mol^{-1}. [nearest integer]

(Given ln10 = 2.3, R = 8.3 J K^{-1} mol^{-1}, log 2 = 0.30)

Answer (59)

Sol. $T_1 = 300$ K

(Rate constant)

$K_2 = 2K_1$, on increase temperature by 9K

$T_2 = 309$ K

Ea = ?

$$\log\frac{K_2}{K_1} = \frac{Ea}{2.3R}\left[\frac{T_2 - T_1}{T_2 \cdot T_1}\right]$$

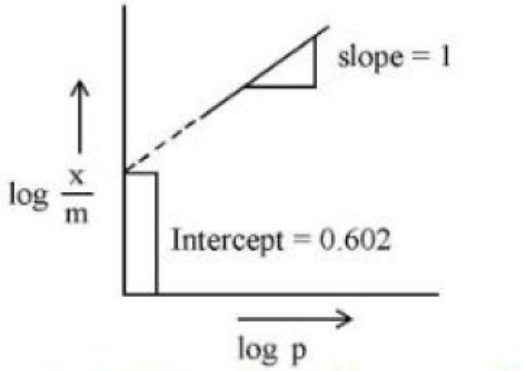
$$\log 2 = \frac{Ea}{2.3 \times 8.3}\left[\frac{9}{309 \times 300}\right]$$

$$Ea = \frac{0.3 \times 309 \times 300 \times 2.3 \times 8.3}{9}$$

$= 58988.1$ J / mole

$\simeq 59$ kJ/mole

9.



If the initial pressure of a gas 0.03 atm, the mass of the gas absorbed per gram of the adsorbent is _____× 10^{-2}g.

Answer (12)

Sol. Given that log K = intercept = 0.602 = log 4

$\therefore$ K = 4

$$\text{Slope} = \frac{1}{n} = 1$$

and initial pressure = 0.03 atm

$\frac{x}{m} = K(p)^{1/n} = 4 \times 0.03 = 0.12 = 12 \times 10^{-2}$

$\therefore$ mass of gas absorbed per gm of adsorbent

$= 12 \times 10^{-2}$ g

10. 0.25 g of an organic compound containing chlorine gave 0.40 g of silver chloride in Carius estimation. The percentage of chlorine present in the compound is _____. [in nearest integer]

(Given : Molar mass of Ag is 108 g mol^{-1} and that of Cl is 35.5 g mol^{-1})

Answer (40)

Sol. Mass of organic compound = 0.25 g

Mass of AgCl = 0.40 g

$$\%\ Cl = \frac{35.5 \times (\text{mass of AgCl})}{143.5 \times (\text{mass of organic compound})} \times 100$$

$$= \frac{35.5 \times 0.40 \times 100}{143.5 \times 0.25}$$

$= 39.581$

$\simeq 40$

% Cl = 40 %

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The number of points of intersection of $|z-(4+3i)|=2$ and $|z|+|z-4|=6$, $z \in C$, is

(A) 0 (B) 1

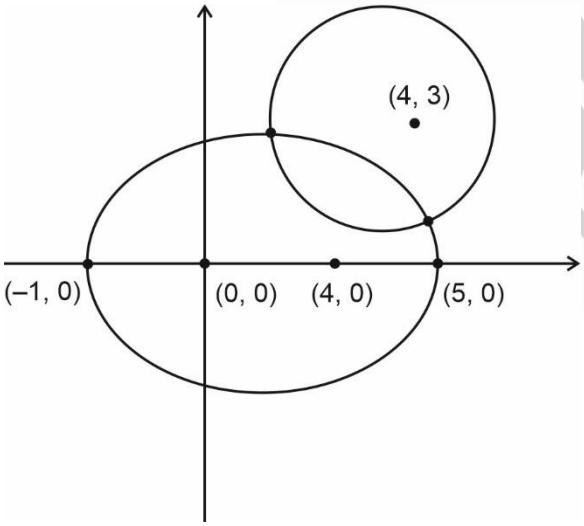
(C) 2 (D) 3

Answer (C)

Sol. $C_1 : |z-(4+3i)|=2$ and $C_2 : |z|+|z-4|=6$, $z \in C$

C_1: represents a circle with centre (4, 3) and radius 2 and C_2 represents a ellipse with focii at (0, 0) and (4, 0) and length of major axis = 6, and length of semi-major axis $2\sqrt{5}$ and (4, 2) lies inside the both C_1 and C_2 and (4, 3) lies outside the C_2

$\therefore$



So, number of intersection points = 2

2. Let $f(x)=\begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, $a \in R$. Then the sum of the square of all the values of a, for which $2f'(10)-f'(5)+100=0$, is

(A) 117 (B) 106

(C) 125 (D) 136

Answer (C)

Sol. $f(x)=\begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, $a \in R$

$f(x) = a(a^2+ax)+1(a^2x+ax^2)$

$= a(x+a)^2$

$f'(x) = 2a(x+a)$

Now, $2f'(10)-f'(5)+100=0$

$\Rightarrow 2\cdot 2a(10+a)-2a(5+a)+100=0$

$\Rightarrow 2a(a+15)+100=0$

$\Rightarrow a^2+15a+50=0$

$\Rightarrow a=-10,-5$

$\therefore$ Sum of squares of values of a = 125.

3. Let for some real numbers α and β, $a=\alpha-i\beta$. If the system of equations $4ix+(1+i)y=0$ and $8\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)x+\bar{a}y=0$ has more than one solution, then $\frac{\alpha}{\beta}$ is equal to

(A) $-2+\sqrt{3}$ (B) $2-\sqrt{3}$

(C) $2+\sqrt{3}$ (D) $-2-\sqrt{3}$

Answer (B)

Sol. Given $a=\alpha-i\beta$ and

$4ix+(1+i)y=0$ ...(i)

$8\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)x+\bar{a}y=0$ ..(ii)

By (i)

$$\frac{x}{y}=\frac{-(1+i)}{4i} \quad \text{...(iii)}$$

By (ii)

$$\frac{x}{y}=\frac{-\bar{a}}{8\left(\frac{-1}{2}+\frac{\sqrt{3}i}{2}\right)} \quad \text{...(iv)}$$

Now by (iii) and (iv)

$$\frac{1+i}{4i}=\frac{\bar{a}}{4\left(-1+\sqrt{3}i\right)}$$

$$\Rightarrow \bar{a}=\left(\sqrt{3}-1\right)+\left(\sqrt{3}+1\right)i$$

$$\Rightarrow \alpha+i\beta=\left(\sqrt{3}-1\right)+\left(\sqrt{3}+1\right)i$$

$$\therefore \frac{\alpha}{\beta}=\frac{\sqrt{3}-1}{\sqrt{3}+1}=2-\sqrt{3}$$

4. Let A and B be two 3×3 matrices such that $AB = I$ and $|A| = \frac{1}{8}$. Then $|adj\,(B\,adj(2A))|$ is equal to

(A) 16 (B) 32

(B) 64 (D) 128

Answer (C)

Sol. A and B are two matrices of order 3×3.

and $AB = I$, $\quad |A| = \frac{1}{8}$

Now, $|A|\,|B| = 1$

$|B| = 8$

$\therefore \quad |\text{adj}(B(\text{adj}(2A))| = |B(\text{adj}(2A))|^2$

$= |B|^2 |\text{adj}(2A)|^2$

$= 2^6 |2A|^{2\times 2}$

$= 2^6 \cdot 2^{12} \cdot \frac{1}{2^{12}} = 64$

5. Let $S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + \ldots$ Then $4S$ is equal to

(A) $\left(\frac{7}{3}\right)^2$ (B) $\frac{7^3}{3^2}$

(C) $\left(\frac{7}{3}\right)^3$ (D) $\frac{7^2}{3^3}$

Answer (C)

Sol. $S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + \ldots.$ …(i)

$\frac{1}{7}S = \frac{2}{7} + \frac{6}{7^2} + \frac{12}{7^3} + \frac{20}{7^4} + \ldots.$ …(ii)

(i) – (ii)

$\frac{6}{7}S = 2 + \frac{4}{7} + \frac{6}{7^2} + \frac{8}{7^3} + \ldots.$ …(iii)

$\frac{6}{7^2}S = \frac{2}{7} + \frac{4}{7^2} + \frac{6}{7^3} + \ldots.$ …(iv)

(iii) – (iv)

$\left(\frac{6}{7}\right)^2 S = 2 + \frac{2}{7} + \frac{2}{7^2} + \frac{2}{7^3} + \ldots..$

$= 2\left[\frac{1}{1-\frac{1}{7}}\right] = 2\left(\frac{7}{6}\right)$

$\therefore \quad 4S = 8\left(\frac{7}{6}\right)^3 = \left(\frac{7}{3}\right)^3$

6. If a_1, a_2, a_3 ….. and b_1, b_2, b_3 ….. are A.P., and $a_1 = 2$, $a_{10} = 3$, $a_1 b_1 = 1 = a_{10} b_{10}$, then $a_4 b_4$ is equal to

(A) $\frac{35}{27}$ (B) 1

(C) $\frac{27}{28}$ (D) $\frac{28}{27}$

Answer (D)

Sol. a_1, a_2, a_3 ... are in A.P. (Let common difference is d_1)

b_1, b_2, b_3 ... are in A.P. (Let common difference is d_2)

and $a_1 = 2$, $a_{10} = 3$, $a_1 b_1 = 1 = a_{10} b_{10}$

$\because \quad a_1 b_1 = 1 \qquad \therefore \quad b_1 = \frac{1}{2}$

$a_{10} b_{10} = 1 \qquad \therefore \quad b_{10} = \frac{1}{3}$

Now, $a_{10} = a_1 + 9d_1 \Rightarrow d_1 = \frac{1}{9}$

$b_{10} = b_1 + 9d_2 \Rightarrow d_2 = \frac{1}{9}\left[\frac{1}{3} - \frac{1}{2}\right] = -\frac{1}{54}$

Now, $a_4 = 2 + \frac{3}{9} = \frac{7}{3}$

$b_4 = \frac{1}{2} - \frac{3}{54} = \frac{4}{9}$

$\therefore \quad a_4 b_4 = \frac{28}{27}$

7. If m and n respectively are the number of local maximum and local minimum points of the function $f(x) = \int_0^{x^2} \frac{t^2 - 5t + 4}{2 + e^t} dt$, then the ordered pair (m, n) is equal to

(A) (3, 2) (B) (2, 3)

(C) (2, 2) (D) (3, 4)

Answer (B)

Sol. $f(x) = \int_0^{x^2} \frac{t^2 - 5t + 4}{2 + e^t} dt$

$f'(x) = 2x\left(\frac{x^4 - 5x^2 + 4}{2 + e^{x^2}}\right) = 0$

$x = 0$, or $\left(x^2-4\right)\left(x^2-1\right)=0$

$x = 0,\ x = \pm 2,\ \pm 1$

Now, $f'(x) = \dfrac{2x(x+1)(x-1)(x+2)(x-2)}{\left(e^{x^2}+2\right)}$

$f'(x)$ changes sign from positive to negative at

$x = -1, 1$ So, number of local maximum points = 2

$f'(x)$ changes sign from negative to positive at

$x = -2, 0, 2$ So, number of local minimum points = 3

$\therefore\quad m = 2,\ n = 3$

8. Let f be a differentiable function in $\left(0,\dfrac{\pi}{2}\right)$ If $\int\limits_{\cos x}^{1} t^2 f(t)\,dt = \sin^3 x + \cos x$, then $\dfrac{1}{\sqrt{3}} f'\left(\dfrac{1}{\sqrt{3}}\right)$ is equal to

(A) $6-9\sqrt{2}$ (B) $6-\dfrac{9}{\sqrt{2}}$

(D) $\dfrac{9}{2}-6\sqrt{2}$ (D) $\dfrac{9}{\sqrt{2}}-6$

Answer (B)

Sol. $\int\limits_{\cos x}^{1} t^2 f(t)\,dt = \sin^3 x + \cos x$

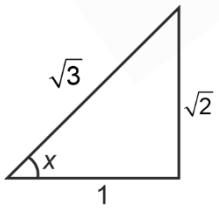
$\Rightarrow\quad \sin x\cos^2 x\, f(\cos x) = 3\sin^2 x\cos x - \sin x$

$\Rightarrow\quad f(\cos x) = 3\tan x - \sec^2 x$

$\Rightarrow\quad f'(\cos x)\cdot(-\sin x) = 3\sec^2 x - 2\sec^2 x\tan x$

Put $\cos x = \dfrac{1}{\sqrt{3}},$

$\therefore\quad f'\left(\dfrac{1}{\sqrt{3}}\right)\left(-\dfrac{\sqrt{2}}{\sqrt{3}}\right) = 9-6\sqrt{2}$



$\dfrac{1}{\sqrt{3}} f'\left(\dfrac{1}{\sqrt{3}}\right) = 6-\dfrac{9}{\sqrt{2}}$

9. The integral $\int\limits_{0}^{1} \dfrac{1}{7^{\left[\frac{1}{x}\right]}}dx$, where [·] denotes the greatest integer function, is equal to

(A) $1+6\log_e\left(\dfrac{6}{7}\right)$ (B) $1-6\log_e\left(\dfrac{6}{7}\right)$

(C) $\log_e\left(\dfrac{7}{6}\right)$ (D) $1-7\log_e\left(\dfrac{6}{7}\right)$

Answer (A)

Sol. $\int_0^1 \dfrac{1}{7^{\left[\frac{1}{x}\right]}}dx$, let $\dfrac{1}{x} = t$

$$\frac{-1}{x^2}dx = dt$$

$$= \int_{\infty}^{1} \frac{1}{-t^2 7^{[t]}}dt = \int_1^{\infty} \frac{1}{t^2 7^{[t]}}dt$$

$$= \int_1^2 \frac{1}{7t^2}dt + \int_2^3 \frac{1}{7^2 t^2}dt + \ldots.$$

$$= \frac{1}{7}\left[-\frac{1}{t}\right]_1^2 + \frac{1}{7^2}\left[\frac{-1}{t}\right]_2^3 + \frac{1}{7^3}\left[-\frac{1}{t}\right]_2^3 + \ldots.$$

$$= \sum_{n=1}^{\infty} \frac{1}{7^n}\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

$$= \sum_{n=1}^{\infty} \frac{\left(\frac{1}{7}\right)^n}{n} - 7\sum_{n=1}^{\infty} \frac{\left(\frac{1}{7}\right)^{n+1}}{n+1}$$

$$= -\log\left(1-\frac{1}{7}\right) + 7\log\left(1-\frac{1}{7}\right) + 1$$

$$= 1 + 6\log\frac{6}{7}$$

10. If the solution curve of the differential equation $\left(\left(\tan^{-1}y\right)-x\right)dy = \left(1+y^2\right)dx$ passes through the point (1, 0), then the abscissa of the point on the curve whose ordinate is tan(1), is

(A) $2e$ (B) $\dfrac{2}{e}$

(C) 2 (D) $\dfrac{1}{e}$

Answer (B)

Sol. $\left(\left(\tan^{-1}y\right)-x\right)dy = \left(1+y^2\right)dx$

$$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$$

I.F. $= e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$

$\therefore$ Solution

$$x\cdot e^{\tan^{-1}y} = \int \frac{e^{\tan^{-1}y}\tan^{-1}y}{1+y^2}dy$$

Let $e^{\tan^{-1}y} = t$

$$\frac{e^{\tan^{-1}y}}{1+y^2} = dt$$

$$= xe^{\tan^{-1}y} = \int \ln t\,dt = t\ln t - t + c$$

$$\therefore \quad = xe^{\tan^{-1}y} = e^{\tan^{-1}y}\tan^{-1}y - e^{\tan^{-1}y} + c \ldots(i)$$

$\because$ It passes through (1, 0) $\Rightarrow c = 2$

Now put $y = \tan 1$, then

$ex = e - e + 2$

$$\Rightarrow \quad x = \frac{2}{e}$$

11. If the equation of the parabola, whose vertex is at (5, 4) and the directrix is $3x + y - 29 = 0$, is $x^2 + ay^2 + bxy + cx + dy + k = 0$, then $a + b + c + d + k$ is equal to

(A) 575 (B) –575

(C) 576 (D) –576

Answer (D)

Sol. Given vertex is (5, 4) and directrix $3x + y - 29 = 0$

Let foot of perpendicular of (5, 4) on directrix is (x_1, y_1)

$$\frac{x_1-5}{3} = \frac{y_1-4}{1} = \frac{-(-10)}{10}$$

$$\therefore \quad (x_1,\ y_1) \equiv (8, 5)$$

So, focus of parabola will be $S = (2, 3)$

Let $P(x, y)$ be any point on parabola, then

$$(x-2)^2 + (y-3)^2 = \frac{(3x+y-29)^2}{10}$$

$$\Rightarrow \quad 10(x^2+y^2-4x-6y+13) = 9x^2+y^2+841+6xy-58y-174x$$

$$\Rightarrow \quad x^2 + 9y^2 - 6xy + 134x - 2y - 711 = 0$$

and given parabola

$$x^2 + ay^2 + bxy + cx + dy + k = 0$$

$\therefore$ $a = 9$, $b = -6$, $c = 134$, $d = -2$, $k = -711$

$\therefore$ $a + b + c + d + k = -576$

12. The set of values of k, for which the circle $C : 4x^2 + 4y^2 - 12x + 8y + k = 0$ lies inside the fourth quadrant and the point $\left(1, -\frac{1}{3}\right)$ lies on or inside the circle C, is

(A) An empty set

(B) $\left(6, \frac{65}{9}\right]$

(C) $\left[\frac{80}{9}, 10\right)$

(D) $\left(9, \frac{92}{9}\right]$

Answer (D)

Sol. $C : 4x^2 + 4y^2 - 12x + 8y + k = 0$

$\because$ $\left(1, -\frac{1}{3}\right)$ lies on or inside the C

then $4 + \frac{4}{9} - 12 - \frac{8}{3} + k \le 0$

$$\Rightarrow \quad k \le \frac{92}{9}$$

Now, circle lies in 4th quadrant centre $\equiv \left(\frac{3}{2}, -1\right)$

$$\therefore \quad r < 1 \Rightarrow \sqrt{\frac{9}{4} + 1 - \frac{k}{4}} < 1$$

$$\Rightarrow \quad \frac{13}{4} - \frac{k}{4} < 1$$

$$\Rightarrow \quad \frac{k}{4} > \frac{9}{4}$$

$$\Rightarrow \quad k > 9$$

$$\therefore \quad k \in \left(9, \frac{92}{9}\right)$$

13. Let the foot of the perpendicular from the point (1, 2, 4) on the line $\frac{x+2}{4} = \frac{y-1}{2} = \frac{z+1}{3}$ be P, Then the distance of P from the plane $3x + 4y + 12z + 23 = 0$ is

(A) 5 (B) $\frac{50}{13}$

(C) 4 (D) $\frac{63}{13}$

Answer (A)

Sol. $L: \frac{x+2}{4}=\frac{y-1}{2}=\frac{z+1}{3}=t$

Let $P=(4t-2, 2t+1, 3t-1)$

$\because$ P is the foot of perpendicular of (1, 2, 4)

$\therefore$ $4(4t-3)+2(2t-1)+3(3t-5)=0$

$\Rightarrow$ $29t=29 \Rightarrow \boxed{t=1}$

$\therefore$ $P=(2, 3, 2)$

Now, distance of P from the plane

$3x+4y+12z+23=0$, is

$$\left|\frac{6+12+24+23}{\sqrt{9+16+144}}\right|=\frac{65}{13}=5$$

14. The shortest distance between the lines $\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-1}{-1}$ and $\frac{x+3}{2}=\frac{y-6}{1}=\frac{z-5}{3}$, is

(A) $\frac{18}{\sqrt{5}}$ (B) $\frac{22}{3\sqrt{5}}$

(C) $\frac{46}{3\sqrt{5}}$ (D) $6\sqrt{3}$

Answer (A)

Sol. $L_1: \frac{x-3}{2}=\frac{y-2}{3}=\frac{z-1}{-1}$

$L_2: \frac{x+3}{2}=\frac{y-6}{1}=\frac{z-5}{3}$

Now, $\vec{p}\times\vec{q}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & -1\\ 2 & 1 & 3\end{vmatrix}=10\hat{i}-8\hat{j}-4\hat{k}$

and $\vec{a}_2-\vec{a}_1=6\hat{i}-4\hat{j}-4\hat{k}$

$\therefore$ S.D $=\left|\frac{60+32+16}{\sqrt{100+64+16}}\right|=\frac{108}{\sqrt{180}}=\frac{18}{\sqrt{5}}$

15. Let $\vec{a}$ and $\vec{b}$ be the vectors along the diagonals of a parallelogram having area $2\sqrt{2}$. Let the angle between $\vec{a}$ and $\vec{b}$ be acute, $|\vec{a}|=1$, and $|\vec{a}\cdot\vec{b}|=|\vec{a}\times\vec{b}|$. If $\vec{c}=2\sqrt{2}(\vec{a}\times\vec{b})-2\vec{b}$, then an angle between $\vec{b}$ and $\vec{c}$ is

(A) $\frac{\pi}{4}$ (B) $-\frac{\pi}{4}$

(C) $\frac{5\pi}{6}$ (D) $\frac{3\pi}{4}$

Answer (D)

Sol. $\because$ $\vec{a}$ and $\vec{b}$ be the vectors along the diagonals of a parallelogram having area $2\sqrt{2}$.

$\therefore$ $\frac{1}{2}|\vec{a}\times\vec{b}|=2\sqrt{2}$

$|\vec{a}||\vec{b}|\sin\theta=4\sqrt{2}$

$\Rightarrow$ $|\vec{b}|\sin\theta=4\sqrt{2}$ ...(i)

and $|\vec{a}\cdot\vec{b}|=|\vec{a}\times\vec{b}|$

$|\vec{a}||\vec{b}|\cos\theta=|\vec{a}||\vec{b}|\sin\theta$

$\Rightarrow$ $\boxed{\tan\theta=1}$ $\therefore \theta=\frac{\pi}{4}$

By (i) $|\vec{b}|=8$

Now $\vec{c}=2\sqrt{2}(\vec{a}\times\vec{b})-2\vec{b}$

$\Rightarrow$ $\vec{c}\cdot\vec{b}=-2|\vec{b}|^2=-128$ ...(ii)

and $\vec{c}\cdot\vec{c}=8|\vec{a}\times\vec{b}|^2+4|\vec{b}|^2$

$\Rightarrow$ $|\vec{c}|^2=8.32+4.64$

$\Rightarrow$ $|\vec{c}|=16\sqrt{2}$ ...(iii)

From (ii) and (iii)

$|\vec{c}||\vec{b}|\cos\alpha=-128$

$\Rightarrow$ $\cos\alpha=\frac{-1}{\sqrt{2}}$

$\alpha=\frac{3\pi}{4}$

16. The mean and variance of the data 4, 5, 6, 6, 7, 8, x, y, where $x < y$, are 6 and $\frac{9}{4}$ respectively. Then x^4+y^2 is equal to

(A) 162 (B) 320

(C) 674 (D) 420

Answer (B)

Sol. Mean $=\frac{4+5+6+6+7+8+x+y}{8}=6$

$\therefore$ $x+y=12$ ...(i)

And variance

$$=\frac{2^2+1^2+0^2+0^2+1^2+2^2+(x-6)^2+(y-6)^2}{8}$$

$$=\frac{9}{4}$$

$\therefore$ $(x-6)^2+(y-6)^2=8$ ...(ii)

From (i) and (ii)

$x=4$ and $y=8$

$\therefore$ $x^4+y^2=320$

17. If a point $A(x, y)$ lies in the region bounded by the y-axis, straight lines $2y + x = 6$ and $5x - 6y = 30$, then the probability that $y < 1$ is

(A) $\frac{1}{6}$

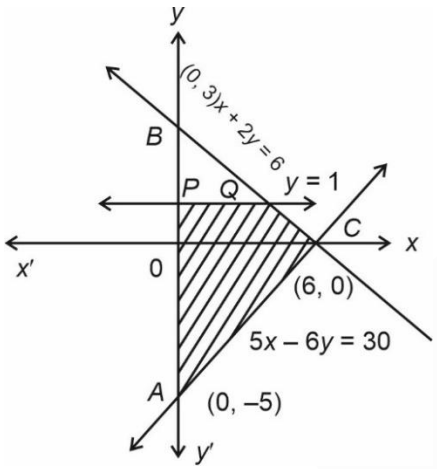
(B) $\frac{5}{6}$

(C) $\frac{2}{3}$

(D) $\frac{6}{7}$

Answer (B)

Sol. The required probability



$$= \frac{\text{Area of Region } PQCAP}{\text{Area of Region } ABCA}$$

$$= \frac{\frac{1}{2}\times 8\times 6 - \frac{1}{2}\times 2\times 4}{\frac{1}{2}\times 8\times 6}$$

$$= \frac{5}{6}$$

18. The value of $\cot\left(\sum_{n=1}^{50}\tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right)$ is

(A) $\frac{26}{25}$

(B) $\frac{25}{26}$

(C) $\frac{50}{51}$

(D) $\frac{52}{51}$

Answer (A)

Sol. $\cot\left(\sum_{n=1}^{50}\tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right)$

$$= \cot\left(\sum_{n=1}^{50}\tan^{-1}\left(\frac{(n+1)-n}{1+(n+1)n}\right)\right)$$

$$= \cot\left(\sum_{n=1}^{50}\left(\tan^{-1}(n+1)-\tan^{-1}n\right)\right)$$

$$= \cot\left(\tan^{-1}51-\tan^{-1}1\right)$$

$$= \cot\left(\tan^{-1}\left(\frac{51-1}{1+51}\right)\right)$$

$$= \cot\left(\cot^{-1}\left(\frac{52}{50}\right)\right)$$

$$= \frac{26}{25}$$

19. $\alpha = \sin 36^\circ$ is a root of which of the following equation?

(A) $16x^4 - 10x^2 - 5 = 0$

(B) $16x^4 + 20x^2 - 5 = 0$

(C) $16x^4 - 20x^2 + 5 = 0$

(D) $16x^4 - 10x^2 + 5 = 0$

Answer (C)

Sol. $\alpha = \sin 36^\circ = x$(say)

$\therefore \quad x = \frac{\sqrt{10-2\sqrt{5}}}{4}$

$\Rightarrow \quad 16x^2 = 10 - 2\sqrt{5}$

$\Rightarrow \quad (8x^2 - 5)^2 = 5$

$\Rightarrow \quad 16x^4 - 80x^2 + 20 = 0$

$\therefore \quad 4x^4 - 20x^2 + 5 = 0$

20. Which of the following statement is a tautology?

(A) $((\sim q) \wedge p) \wedge q$

(B) $((\sim q) \wedge p) \wedge (p \wedge (\sim p))$

(C) $((\sim q) \wedge p) \vee (p \vee (\sim p))$

(D) $(p \wedge q) \wedge (\sim (p \wedge q))$

Answer (C)

Sol. $\because \quad ((\sim q) \wedge p) \vee (p \vee (\sim p))$

$= (\sim q \wedge p) \vee t \qquad (t \text{ is tautology})$

$\equiv t$

$\therefore$ option (C) is correct.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Define $f : S \to S$ as

$$f(n) = \begin{cases} 2n & , \text{if } n = 1, 2, 3, 4, 5 \\ 2n - 11, & \text{if } n = 6, 7, 8, 9, 10 \end{cases}.$$

Let $g : S \to S$ be a function such that

$$fog(n) = \begin{cases} n+1 & , \text{ if } n \text{ is odd} \\ n-1 & , \text{ if } n \text{ is even} \end{cases},$$

Then $g(10)\,(g(1) + g(2) + g(3) + g(4) + g(5))$ is equal to _____.

Answer (190)

Sol. $\because$ $f(n) = \begin{cases} 2n & , \ n = 1, 2, 3, 4, 5 \\ 2n - 11, & \ n = 6, 7, 8, 9, 10 \end{cases}$

$\therefore$ $f(1) = 2, f(2) = 4, \ldots, f(5) = 10$

and $f(6) = 1, f(7) = 3, f(8) = 5, \ldots, f(10) = 9$

Now, $f(g(n)) = \begin{cases} n+1, & \text{if } n \text{ is odd} \\ n-1, & \text{if } n \text{ is even} \end{cases}$

$\therefore$ $f(g(10)) = 9 \quad \Rightarrow g(10) = 10$

$f(g(1)) = 2 \quad \Rightarrow g(1) = 1$

$f(g(2)) = 1 \quad \Rightarrow g(2) = 6$

$f(g(3)) = 4 \quad \Rightarrow g(3) = 2$

$f(g(4)) = 3 \quad \Rightarrow g(4) = 7$

$f(g(5)) = 6 \quad \Rightarrow g(5) = 3$

$\therefore$ $g(10)\,(g(1) + g(2) + g(3) + g(4) + g(5)) = 190$

2. Let α, β be the roots of the equation $x^2 - 4\lambda x + 5 = 0$ and α, γ be the roots of the equation $x^2 - \left(3\sqrt{2} + 2\sqrt{3}\right)x + 7 + 3\lambda\sqrt{3} = 0, \quad \lambda > 0$. If $\beta + \gamma = 3\sqrt{2}$, then $(\alpha + 2\beta + \gamma)^2$ is equal to _____.

Answer (98)

Sol.

$\because$ α, β are roots of $x^2 - 4\lambda x + 5 = 0$

$\therefore$ $\alpha + \beta = 4\lambda$ and $\alpha\beta = 5$

Also, α, γ are roots of

$x^2 - \left(3\sqrt{2} + 2\sqrt{3}\right)x + 7 + 3\sqrt{3}\lambda = 0, \lambda > 0$

$\therefore$ $\alpha + \gamma = 3\sqrt{2} + 2\sqrt{3}, \quad \alpha\gamma = 7 + 3\sqrt{3}\lambda$

$\because$ α is common root

$\therefore$ $\alpha^2 - 4\lambda\alpha + 5 = 0$ …(i)

and $\alpha^2 - (3\sqrt{2} + 2\sqrt{3})\alpha + 7 + 3\sqrt{3}\lambda = 0$ …(ii)

From (i) – (ii): we get $\alpha = \dfrac{2 + 3\sqrt{3}\lambda}{3\sqrt{2} + 2\sqrt{3} - 4\lambda}$

$\because$ $\beta + \gamma = 3\sqrt{2}$

$\therefore$ $4\lambda + 3\sqrt{2} + 2\sqrt{3} - 2\alpha = 3\sqrt{2}$

$\Rightarrow 3\sqrt{2} = 4\lambda + 3\sqrt{2} + 2\sqrt{3} - \dfrac{4 + 6\sqrt{3}\lambda}{3\sqrt{2} + 2\sqrt{3} - 4\lambda}$

$\Rightarrow 8\lambda^2 + 3(\sqrt{3} - 2\sqrt{2})\lambda - 4 - 3\sqrt{6} = 0$

$\therefore$ $\lambda = \dfrac{6\sqrt{2} - 3\sqrt{3} \pm \sqrt{9(11 - 4\sqrt{6}) + 32(4 + 3\sqrt{6})}}{16}$

$\therefore$ $\lambda = \sqrt{2}$

$\therefore$ $(\alpha + 2\beta + \gamma)^2 = (\alpha + \beta + \beta + \gamma)^2$

$= \left(4\sqrt{2} + 3\sqrt{2}\right)^2$

$= \left(7\sqrt{2}\right)^2$

$= 98$

3. Let A be a matrix of order 2×2, whose entries are from the set $\{0, 1, 3, 4, 5\}$. If the sum of all the entries of A is a prime number p, $2 < p < 8$, then the number of such matrices A is _____.

Answer (180)

Sol. $\because$ Sum of all entries of matrix A must be prime p such that $2 < p < 8$ then sum of entries may be 3, 5 or 7.

If sum is 3 then possible entries are (0, 0, 0, 3), (0, 0, 1, 2) or (0, 1, 1, 1).

$\therefore$ Total number of matrices = 4 + 4 + 12 = 20

If sum of 5 then possible entries are

(0, 0, 0, 5), (0, 0, 1, 4), (0, 0, 2, 3), (0, 1, 1, 3), (0, 1, 2, 2) and (1, 1, 1, 2).

$\therefore$ Total number of matrices = 4 + 12 + 12 + 12 + 12 + 4 = 56

If sum is 7 then possible entries are

(0, 0, 2, 5), (0, 0, 3, 4), (0, 1, 1, 5), (0, 3, 3, 1), (0, 2, 2, 3), (1, 1, 1, 4), (1, 2, 2, 2), (1, 1, 2, 3) and (0, 1, 2, 4)

Total number of matrices with sum 7 = 104

$\therefore$ Total number of required matrices

= 20 + 56 + 104

= 180

4. If the sum of the coefficients of all the positive powers of x, in the Binomial expansion of $\left(x^n + \frac{2}{x^5}\right)^7$ is 939, then the sum of all the possible integral values of n is _____.

Answer (57)

Sol.

$$\left(x^n + \frac{2}{x^5}\right)^7 = \sum_{r=0}^{7} {}^7C_r (x^n)^{7-r} \cdot \left(\frac{2}{x^5}\right)^r$$

$$= \sum_{r=0}^{7} {}^7C_r \cdot 2^r \cdot x^{7n-nr-5r}$$

$\therefore$ ${}^7C_0 \cdot 2^0 + {}^7C_1 \cdot 2^1 + {}^7C_2 \cdot 2^2 + {}^7C_3 \cdot 2^3 + {}^7C_4 \cdot 2^4$

= 939

$\therefore$ $r = 4$

$\because$ $7n - nr - 5r = 0$

and $r = 4$ then $n > \frac{20}{3}$

and r should not be 5

$\therefore$ $n < \frac{25}{2}$

$\therefore$ Possible values of n are 7, 8, 9, 10, 11, 12

$\therefore$ Sum of integral value of n = 57

5. Let $[t]$ denote the greatest integer $\le t$ and $\{t\}$ denote the fractional part of t. The integral value of α for which the left hand limit of the function

$f(x) = [1 + x] + \frac{\alpha^{2[x]+\{x\}} + [x] - 1}{2[x] + \{x\}}$ at $x = 0$ is equal to $\alpha - \frac{4}{3}$, is _____.

Answer (3)

$$f(x) = [1+x] + \frac{a^{2[x]+\{x\}} + [x] - 1}{2[x] + \{x\}}$$

$$\lim_{x \to 0^-} f(x) = \alpha - \frac{4}{3}$$

$$\Rightarrow \lim_{x \to 0^-} 1 + [x] + \frac{\alpha^{x+[x]} + [x] - 1}{x + [x]} = \alpha - \frac{4}{3}$$

$$\Rightarrow \lim_{h \to 0^-} 1 - 1 + \frac{\alpha^{-h-1} - 1 - 1}{-h - 1} = \alpha - \frac{4}{3}$$

$$\therefore \quad \frac{\alpha^{-1} - 2}{-1} = \alpha - \frac{4}{3}$$

$\Rightarrow 3\alpha^2 - 10\alpha + 3 = 0$

$\therefore$ $\alpha = 3$ or $\frac{1}{3}$

$\because$ α in integer, hence $\alpha = 3$

6. If $y(x) = (x^x)^x$, $x > 0$, then $\frac{d^2x}{dy^2} + 20$ at $x = 1$ is equal to _____.

Answer (16)

Sol. $\because$ $y(x) = \left(x^x\right)^x$

$\therefore$ $y = x^{x^2}$

$\therefore$ $\frac{dy}{dx} = x^2 \cdot x^{x^2-1} + x^{x^2} \ln x \cdot 2x$

$\therefore$ $\frac{dx}{dy} = \frac{1}{x^{x^2+1}(1 + 2\ln x)}$ ..(i)

Now, $\frac{d^2x}{dx} = \frac{d}{dx}\left(\left(x^{x^2+1}(1+2\ln x)\right)^{-1}\right) \cdot \frac{dx}{dy}$

$$= \frac{-x\left(x^{x^2+1}(1+2\ln x)\right)^{-2} \cdot x^{x^2}(1+2\ln x)\left(x^2 + 2x^2 \ln x + 3\right)}{x^{x^2}(1+2\ln x)}$$

$$= \frac{-x^{x^2}(1+2\ln x)\left(x^2+3+2x^2\ln x\right)}{\left(x^{x^2}(1+2\ln x)\right)^3}$$

$$\frac{d^2x}{dy^2\,(\text{at } x=1)} = -4$$

$$\therefore \quad \frac{d^2x}{dy^2\,(\text{at } x=1)} + 20 = 16$$

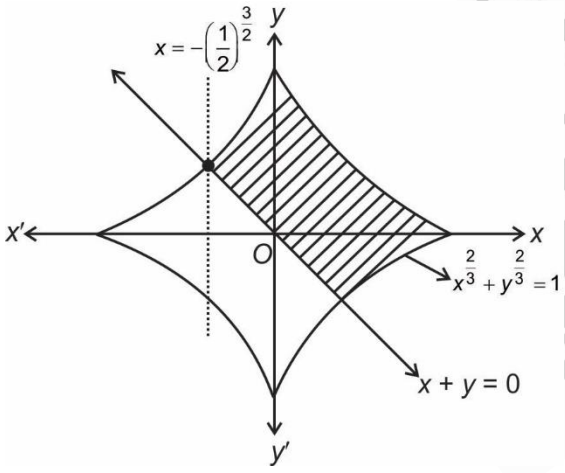
7. If the area of the region

$$\left\{(x,y): x^{\frac{2}{3}} + y^{\frac{2}{3}} \le 1,\ x+y \ge 0, y \ge 0\right\}$$ is A, then

$\frac{256A}{\pi}$ is equal to _____.

Answer (36)

Sol.



$\therefore$ Area of shaded region

$$= \int_{-\left(\frac{1}{2}\right)^{\frac{3}{2}}}^{0} \left(\left(1-x^{\frac{2}{3}}\right)^{\frac{3}{2}} + x\right)dx + \int_{0}^{1}\left(1-x^{\frac{2}{3}}\right)^{\frac{3}{2}}dx$$

$$= \int_{-\left(\frac{1}{2}\right)^{\frac{3}{2}}}^{0} \left(1-x^{\frac{2}{3}}\right)^{\frac{3}{2}}dx + \int_{-\left(\frac{1}{2}\right)^{\frac{3}{2}}}^{0} x\,dx$$

Let $x = \sin^3\theta$

$\therefore \quad dx = 3\sin^2\theta\cos\theta\,d\theta$

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{2}} 3\sin^2\theta\cos^4\theta\,d\theta + \left(0 - \frac{1}{16}\right)$$

$$= \frac{9\pi}{64} + \frac{1}{16} - \frac{1}{16} = \frac{36\pi}{256} = A$$

$\therefore \quad \frac{256A}{\pi} = 36$

8. Let $y = y(x)$ be the solution of the differential equation $\left(1-x^2\right)dy = \left(xy + \left(x^3+2\right)\sqrt{1-x^2}\right)dx$, $-1 < x < 1$, and $y(0) = 0$. If $\int_{-\frac{1}{2}}^{\frac{1}{2}} \sqrt{1-x^2}\,y(x)\,dx = k$, then k^{-1} is equal to _____.

Answer (320)

Sol. $\left(1-x^2\right)dy = \left(xy + \left(x^3+2\right)\sqrt{1-x^2}\right)dx$

$$\therefore \quad \frac{dy}{dx} - \frac{x}{1-x^2}y = \frac{x^3+3}{\sqrt{1-x^2}}$$

$$\therefore \quad \text{I.F.} = e^{\int -\frac{x}{1-x^2}dx} = \sqrt{1-x^2}$$

Solution is

$$y\cdot\sqrt{1-x^2} = \int\left(x^3+3\right)dx$$

$$y\cdot\sqrt{1-x^2} = \frac{x^4}{4} + 3x + c$$

$$\because \quad y(0) = 0 \Rightarrow c = 0$$

$$\therefore \quad y(x) = \frac{x^4+12x}{4\sqrt{1-x^2}}$$

$$\therefore \quad \int_{-\frac{1}{2}}^{\frac{1}{2}} \sqrt{1-x^2}\,y(x)\,dx = \int_{-\frac{1}{2}}^{\frac{1}{2}} \left(\frac{x^4+12x}{4}\right)dx$$

$$= \int_{0}^{\frac{1}{2}} \frac{x^4}{2}dx$$

$$\therefore \quad k = \frac{1}{320}$$

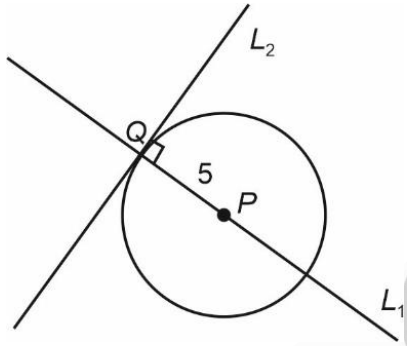
$$\therefore \quad k^{-1} = 320$$

9. Let a circle C of radius 5 lie below the x-axis. The line $L_1 : 4x + 3y + 2 = 0$ passes through the centre P of the circle C and intersects the line $L_2 : 3x - 4y - 11 = 0$ at Q. The line L_2 touches C at the point Q. Then the distance of P from the line $5x - 12y + 51 = 0$ is _____.

Answer (11)

Sol. $L_1 : 4x + 3y + 2 = 0$

$L_2 : 3x - 4y - 11 = 0$



Since circle C touches the line L_2 at Q intersection point Q of L_1 and L_2, is (1, –2)

$\because$ P lies of L_1

$\therefore \quad P\left(x, -\frac{1}{3}(2+4x)\right)$

Now, $PQ = 5 \Rightarrow (x-1)^2 + \left(\frac{4x+2}{3} - 2\right)^2 = 25$

$\Rightarrow \quad (x-1)^2\left[1+\frac{16}{9}\right] = 25$

$\Rightarrow \quad (x-1)^2 = 9$

$\Rightarrow \quad x = 4, -2$

$\because$ Circle lies below the x-axis

$\therefore \quad y = -6$

$P(4, -6)$

Now distance of P from $5x - 12y + 51 = 0$

$= \left|\frac{20+72+51}{13}\right| = \frac{143}{13} = 11$

10. Let $S = \{E_1, E_2, \ldots\ldots\ldots\ldots, E_8\}$ be a sample space of a random experiment such that $P(E_n) = \frac{n}{36}$ for every n = 1, 2, ……, 8. Then the number of elements in the set $\left\{A \subseteq S : P(A) \geq \frac{4}{5}\right\}$ is _____ .

Answer (19)

Sol. Here $P(E_n) = \frac{n}{36}$ for n = 1, 2, 3, ……, 8

Here $P(A)$

$= \frac{\text{Any possible sum of } (1, 2, 3, ..., 8) (= a \text{ say})}{36}$

$\because \quad \frac{a}{36} \geq \frac{4}{5}$

$\therefore \quad a \geq 29$

If one of the number from {1, 2, …., 8} is left then total $a \geq 29$ by 3 ways.

Similarly by leaving terms more 2 or 3 we get 16 more combinations.

$\therefore$ Total number of different set A possible is 16 + 3 = 19

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