27/06/2022
Morning

Corporate Office : Aakash Tower, 8, Pusa Road, New Delhi-110005 | Ph.: 011-47623456

Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2022 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. A projectile is launched at an angle 'α' with the horizontal with a velocity 20 ms^{-1}. After 10 s, its inclination with horizontal is 'β'. The value of tanβ will be (g = 10 ms^{-2}).

(A) tanα + 5secα (B) tanα – 5secα

(C) 2tanα – 5secα (D) 2tanα + 5secα

Answer (B)

Sol. $v_y = 20\times\sin\alpha - 10\times10$

$v_x = 20\cos\alpha$

$\therefore \quad \tan\beta = \frac{v_y}{v_x} = \frac{20\sin\alpha - 100}{20\cos\alpha}$

$= \tan\alpha - 5\sec\alpha$

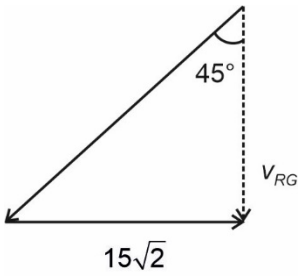
2. A girl standing on road holds her umbrella at 45° with the vertical to keep the rain away. If she starts running without umbrella with a speed of $15\sqrt{2}$ kmh^{-1}, the rain drops hit her head vertically. The speed of rain drops with respect to the moving girl is

(A) 30 kmh^{-1} (B) $\frac{25}{\sqrt{2}}$ kmh^{-1}

(C) $\frac{30}{\sqrt{2}}$ kmh^{-1} (D) 25 kmh^{-1}

Answer (C)

Sol.



From graph,

$v_{RG} = 15\sqrt{2}\tan45°$

$= 15\sqrt{2}$

$= \frac{30}{\sqrt{2}}$

3. A silver wire has a mass (0.6 ± 0.006) g, radius (0.5 ± 0.005) mm and length (4 ± 0.04) cm. The maximum percentage error in the measurement of its density will be

(A) 4%

(B) 3%

(C) 6%

(D) 7%

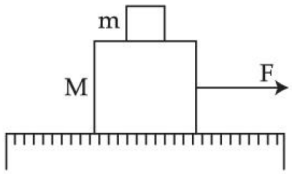
Answer (A)

Sol. $\rho = \frac{m}{V} = \frac{m}{\pi r^2 \times l}$

$\therefore \quad$ % error in $\rho = \left(\frac{0.006}{0.6} + 2\times\frac{0.005}{0.5} + \frac{0.04}{4}\right)\times100$

= 4%

4. A system of two blocks of masses m = 2 kg and M = 8 kg is placed on a smooth table as shown in figure. The coefficient of static friction between two blocks is 0.5. The maximum horizontal force F that can be applied to the block of mass M so that the blocks move together will be



(A) 9.8 N

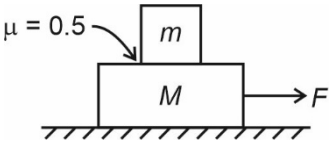
(B) 39.2 N

(C) 49 N

(D) 78.4 N

Answer (C)

Sol.



$\therefore \quad a_{max} = \mu g$

$= 0.5 \times 9.8 = 4.9$ m/s^2

$\therefore \quad F_{max} = (8 + 2) \times 4.9 = 49$ N

5. Two blocks of masses 10 kg and 30 kg are placed on the same straight line with coordinates (0, 0) cm and (x, 0) cm respectively. The block of 10 kg is moved on the same line through a distance of 6 cm towards the other block. The distance through which the block of 30 kg must be moved to keep the position of centre of mass of the system unchanged is

(A) 4 cm towards the 10 kg block

(B) 2 cm away from the 10 kg block

(C) 2 cm towards the 10 kg block

(D) 4 cm away from the 10 kg block

Answer (C)

Sol. For COM to remain unchanged,

$m_1x_1 = m_2x_2$

$\Rightarrow$ $10 \times 6 = 30 \times x_2$

$\Rightarrow$ $x_2 = 2$ cm towards 10 kg block.

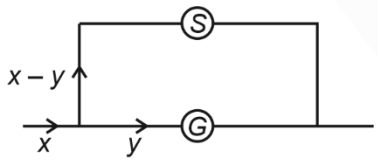
6. A 72 Ω galvanometer is shunted by a resistance of 8 Ω. The percentage of the total current which passes through the galvanometer is

(A) 0.1% (B) 10%

(C) 25% (D) 0.25%

Answer (B)

Sol.



From the given setup

$y \times R_G = (x - y)(R_S)$

$\Rightarrow$ $y \times 72 = (x - y) \times 8$

$\Rightarrow$ $9y = x - y$

$\Rightarrow$ $y = \frac{x}{10}$ or 10% of x

Option (B)

7. Given below are two statements

Statement-I: The law of gravitation holds good for any pair of bodies in the universe.

Statement-II: The weight of any person becomes zero when the person is at the centre of the earth.

In the light of the above statements, choose the **correct** answer from the options given below

(A) Both **Statement I** and **Statement II** are true

(B) Both **Statement I** and **Statement II** are false

(C) **Statement I** is true but **Statement II** is false

(D) **Statement I** is false but **Statement II** is true

Answer (A)

Sol. **Statement-I** is true as law of gravitation is a universal law

Statement-II is also true as gravitational field at centre of earth is zero.

8. What percentage of kinetic energy of a moving particle is transferred to a stationary particle when it strikes the stationary particle of 5 times its mass?

(Assume the collision to be head-on elastic collision)

(A) 50.0% (B) 66.6%

(C) 55.6% (D) 33.3%

Answer (C)

Sol. For a head on elastic collision

$$v_2 = \frac{mu_1}{m+5m} + \frac{mu_1}{m+5m}$$

$$= \frac{2u_1}{6} \text{ or } \frac{u_1}{3}$$

Initial kinetic energy of first mass $= \frac{1}{2}mu_1^2$

Final kinetic energy of second mass

$$= \frac{1}{2} \times 5m\left(\frac{u_1}{3}\right)^2$$

$$= \frac{5}{9}\left(\frac{1}{2}mu_1^2\right)$$

$\Rightarrow$ kinetic energy transferred = 55% of initial kinetic energy of first colliding mass

9. The velocity of a small ball of mass 'm' and density d_1, when dropped in a container filled with glycerine, becomes constant after some time. If the density of glycerine is d_2, then the viscous force acting on the ball, will be

(A) $mg\left(1 - \frac{d_1}{d_2}\right)$ (B) $mg\left(1 - \frac{d_2}{d_1}\right)$

(C) $mg\left(\frac{d_1}{d_2} - 1\right)$ (D) $mg\left(\frac{d_2}{d_1} - 1\right)$

Answer (B)

Sol. Viscous force acting on the ball will be equal and opposite to net of weight and buoyant force

$$\Rightarrow F_0 = \frac{4}{3}\pi r^3 d_1 g - \frac{4}{3}\pi r^3 d_2 g$$

$$= \frac{4}{3}\pi r^3 d_1 g\left(1-\frac{d_2}{d_1}\right)$$

$$= mg\left(1-\frac{d_2}{d_1}\right)$$

$\Rightarrow$ Option (B) is correct

10. The susceptibility of a paramagnetic material is 99. The permeability of the material in Wb/A-m, is

[Permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ Wb/A-m]

(A) $4\pi \times 10^{-7}$

(B) $4\pi \times 10^{-4}$

(C) $4\pi \times 10^{-5}$

(D) $4\pi \times 10^{-6}$

Answer (C)

Sol. $\mu_r = x + 1$

$= 99 + 1 = 100$

$\Rightarrow \mu = \mu_r\mu_0 = 100 \times 4\pi \times 10^{-7}$ Wb/Am

$= 4\pi \times 10^{-5}$ Wb/Am

$\Rightarrow$ Option (C) is correct

11. The current flowing through an ac circuit is given by

$I = 5\sin(120\pi t)$A

How long will the current take to reach the peak value starting from zero?

(A) $\frac{1}{60}$s (B) 60s

(C) $\frac{1}{120}$s (D) $\frac{1}{240}$s

Answer (D)

Sol. $\omega = 120\pi$

$$\Rightarrow T = \frac{1}{60}\text{sec}$$

The current will take its peak value in $\frac{T}{4}$ time

So $t = \frac{T}{4}$

$$= \frac{1}{240}\text{s}$$

12. Match List-I with List-II

	List-I		List-II
(a)	Ultraviolet rays	(i)	Study crystal structure
(b)	Microwaves	(ii)	Greenhouse effect
(c)	Infrared waves	(iii)	Sterilizing surgical instrument
(d)	X-rays	(iv)	Radar system

Choose the **correct** answer from the options given below :

(A) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

(B) (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)

(C) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)

(D) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

Answer (A)

Sol. UV rays are used to sterilize surgical material. Microwaves are used in radar system, infrared are used for green house effect and X-rays are used to study crystal structure.

13. An α particle and a carbon 12 atom has same kinetic energy K. The ratio of their de-Broglie wavelengths ($\lambda_\alpha : \lambda_{C12}$) is :

(A) $1:\sqrt{3}$

(B) $\sqrt{3}:1$

(C) 3:1

(D) $2:\sqrt{3}$

Answer (B)

Sol. $K_\alpha = K_C$

$$\frac{p_\alpha^2}{2m_\alpha} = \frac{p_C^2}{2m_C}$$

$$\frac{p_\alpha}{p_C} = \sqrt{\frac{m_\alpha}{m_C}}$$

$$So\ \frac{\lambda_\alpha}{\lambda_C} = \frac{h/p_\alpha}{h/p_C} = \sqrt{\frac{m_C}{m_\alpha}}$$

$$So\ \frac{\lambda_\alpha}{\lambda_C} = \sqrt{3}$$

14. A force of 10 N acts on a charged particle placed between two plates of a charged capacitor. If one plate of capacitor is removed, then the force acting on that particle will be

(A) 5 N (B) 10 N

(C) 20 N (D) Zero

Answer (A)

Sol. E between two plates is $\frac{\sigma}{\varepsilon_0}$ and due to one plate is $\frac{\sigma}{2\varepsilon_0}$ so the force will be halved

So new force $F = 5$ N

15. The displacement of simple harmonic oscillator after 3 seconds starting from its mean position is equal to half of its amplitude. The time period of harmonic motion is :

(A) 6 s (B) 8 s

(C) 12 s (D) 36 s

Answer (D)

Sol. Time taken by the harmonic oscillator to move from mean position to half of amplitude is $\frac{T}{12}$

So, $\frac{T}{12} = 3$

$T = 36$ sec

16. An observer moves towards a stationary source of sound with a velocity equal to one-fifth of the velocity of sound. The percentage change in the frequency will be:

(A) 20%

(B) 10%

(C) 5%

(D) 0%

Answer (A)

Sol. $f' = f_0\left[\frac{v - v_0}{v - v_s}\right]$

$\Rightarrow f' = f_0\left[\frac{v + \frac{v}{5}}{v}\right]$

$\Rightarrow f' = \frac{6f_0}{5}$

$\Rightarrow$ % change = 20

17. Consider a light ray travelling in air is incident into a medium of refractive index $\sqrt{2n}$. The incident angle is twice that of refracting angle. Then, the angle of incidence will be:

(A) $\sin^{-1}\left(\sqrt{n}\right)$

(B) $\cos^{-1}\left(\sqrt{\frac{n}{2}}\right)$

(C) $\sin^{-1}\left(\sqrt{2n}\right)$

(D) $2\cos^{-1}\left(\sqrt{\frac{n}{2}}\right)$

Answer (D)

Sol. According to the law,

$1 \times \sin\theta = \sqrt{2n} \times \sin\left(\frac{\theta}{2}\right)$

$\Rightarrow \cos\frac{\theta}{2} = \sqrt{\frac{n}{2}}$

$\Rightarrow \theta = 2\cos^{-1}\left(\sqrt{\frac{n}{2}}\right)$

18. A hydrogen atom in its ground state absorbs 10.2 eV of energy. The angular momentum of electron of the hydrogen atom will increase by the value of:

(Given, Planck's constant = 6.6×10^{-34} Js).

(A) 2.10×10^{-34} Js

(B) 1.05×10^{-34} Js

(C) 3.15×10^{-34} Js

(D) 4.2×10^{-34} Js

Answer (B)

Sol. $-13.6 + 10.2 = \frac{-13.6}{n^2}$

$\Rightarrow \frac{13.6}{n^2} = 3.4$

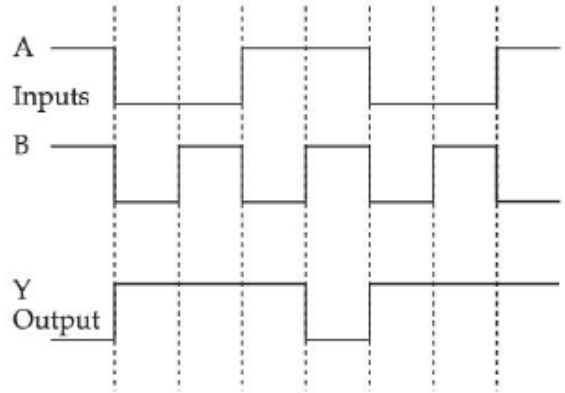
$\Rightarrow n = 2$

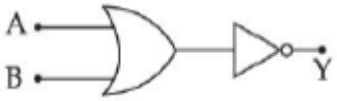
$\Rightarrow \Delta L = 2 \times \frac{h}{2\lambda} - 1 \times \frac{h}{2\lambda}$

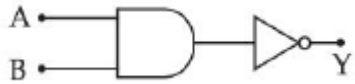
$= \frac{h}{2\lambda}$

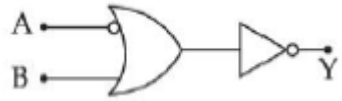
$\Rightarrow \Delta L \simeq 1.05 \times 10^{-34}$ Js

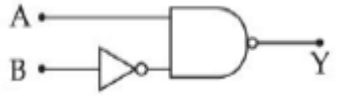
19. Identify the correct Logic Gate for the following output (Y) of two inputs A and B.



(A) 

(B) 

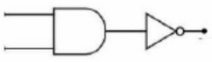
(C) 

(D) 

Answer (B)

Sol.

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

$\Rightarrow Y = (AB)' \Rightarrow$ 

20. A mixture of hydrogen and oxygen has volume 2000 cm^3, temperature 300 K, pressure 100 kPa and mass 0.76 g. The ratio of number of moles of hydrogen to number of moles of oxygen in the mixture will be:

[Take gas constant R = 8.3 JK^{-1} mol^{-1}]

(A) $\frac{1}{3}$

(B) $\frac{3}{1}$

(C) $\frac{1}{16}$

(D) $\frac{16}{1}$

Answer (B)

Sol. $P_1V = n_1RT$

$P_2V = n_2RT$

$\Rightarrow$ (100 kPa) $V = (n_1 + n_2)RT$

$$\Rightarrow n_1 + n_2 = \frac{(100\text{ kPa})(2000\text{ cm}^3)}{8.3 \times 300} \quad \ldots(1)$$

Also, $n_1 \times 2 + n_2 \times 32 = 0.76$ …(2)

Solving (1) and (2),

$n_1 = 0.06$

$n_2 = 0.02$

$$\Rightarrow \frac{n_1}{n_2} = 3$$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. In a carnot engine, the temperature of reservoir is 527°C and that of sink is 200 K. If the work done by the engine when it transfers heat from reservoir to sink is 12000 kJ, the quantity of heat absorbed by the engine from reservoir is ___ × 10^6J.

Answer (16)

Sol. $\eta = 1 - \frac{T_2}{T_1}$

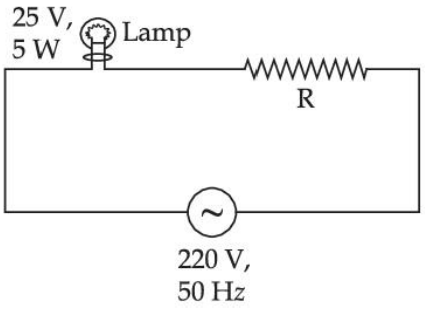
$$= 1 - \frac{200}{800} = \frac{3}{4}$$

$$\therefore \quad \eta = \frac{W}{Q_1}$$

$$\Rightarrow \frac{3}{4} = \frac{12000 \times 10^3}{Q_1}$$

$\Rightarrow Q_1 = 16 \times 10^6$ J

2. A 220 V, 50 Hz AC source is connected to a 25 V, 5 W lamp and an additional resistance R in series (as shown in figure) to run the lamp at its peak brightness, then the value of R (in ohm) will be ___.



Answer (975)

Sol. $R_b = \frac{(25)^2}{5} = 125\ \Omega$

$I_{rms} = \sqrt{\frac{5}{125}} = \frac{1}{5}A$

$\Rightarrow \frac{220}{R+125} = \frac{1}{5}$

$\Rightarrow$ R = 1100 – 125

= 975 Ω

3. In young's double slit experiment the two slits are 0.6 mm distance apart. Interference pattern is observed on a screen at a distance 80 cm from the slits. The first dark fringe is observed on the screen directly opposite to one of the slits. The wavelength of light will be ___ nm.

Answer (450)

Sol. $y = \frac{d}{2}$,

$\therefore \quad \Delta x = y\frac{d}{D}$

$\Rightarrow \frac{d^2}{2D} = \frac{\lambda}{2}$

$\Rightarrow \lambda = \frac{\left(0.6\times10^{-3}\right)^2}{0.8}$

= 450 nm

4. A beam of monochromatic light is used to excite the electron in Li^{++} from the first orbit to the third orbit. The wavelength of monochromatic light is found to be $x \times 10^{-10}$m. The value of x is ___.

[Given hc = 1242 eV nm]

Answer (114)

Sol. $E(\text{in eV}) = 13.6\times9\left(1-\frac{1}{9}\right)$

= 13.6 × 8 eV

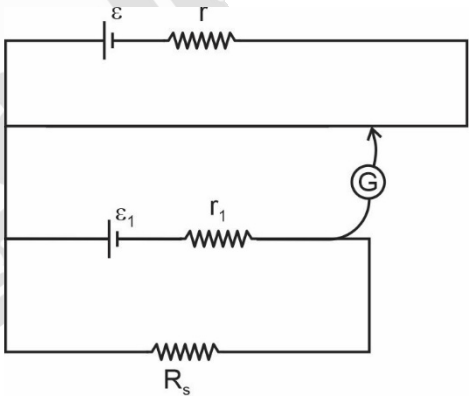
$\Rightarrow \lambda = \frac{12420}{13.6\times8}$Å

= 114.15 Å

5. A cell, shunted by a 8 Ω resistance, is balanced across a potentiometer wire of length 3 m. The balancing length is 2 m when the cell is shunted by 4 Ω resistance. The value of internal resistance of the cell will be ___ Ω.

Answer (8)

Sol.



$\frac{\varepsilon_1 8}{r_1+8}3c$

$\frac{\varepsilon_1 4}{r_1+4}2c$

$\Rightarrow \frac{2(r_1+4)}{r_1+8} = \frac{3}{2}$

$\Rightarrow r_1 = 8\ \Omega$

6. The current density in a cylindrical wire of radius 4 mm is 4×10^6 Am^{-2}. The current through the outer portion of the wire between radial distances $\frac{R}{2}$ and R is _____ π A.

Answer (48)

Sol. $i = A \times j$

$$= \pi\left(R^2 - \frac{R^2}{4}\right)j$$

$$= \frac{3\pi R^2}{4} \times j$$

$$= \frac{3\pi \times \left(4 \times 10^{-3}\right)^2}{4} \times 4 \times 10^6$$

$$= 48\pi$$

7. A capacitor of capacitance 50 pF is charged by 100 V source. It is then connected to another uncharged identical capacitor. Electrostatic energy loss in the process is _____ nJ.

Answer (125)

Sol. Electrical energy lost = $\frac{1}{2}\left(\frac{1}{2}CV^2\right)$

$$= \frac{1}{2} \times \frac{1}{2} \times 50 \times 10^{-12} \times (100)^2$$

$$= \frac{500}{4} \text{ nJ}$$

$$= 125 \text{ nJ}$$

8. The height of a transmitting antenna at the top of a tower is 25 m and that of receiving antenna is, 49 m. The maximum distance between them, for satisfactory communication in LOS (Line-Of-Sight) is $K\sqrt{5} \times 10^2$ m. The value of K is _____.

(Assume radius of Earth is 64 × 10^{+5} m) [Calculate upto nearest integer value]

Answer (192)

Sol. $d = \sqrt{2h_t R_e} + \sqrt{2 \times h_R R_e}$

$$= \sqrt{2 \times 25 \times 64 \times 10^5} + \sqrt{2 \times 49 \times 64 \times 10^5}$$

$$= 8000\sqrt{5} + 11200\sqrt{5} \text{ m}$$

$$= 19200\sqrt{5} \text{ m}$$

$$= 192\sqrt{5} \times 10^2 \text{ m}$$

9. The area of cross-section of a large tank is 0.5 m^2. It has a narrow opening near the bottom having area of cross-section 1 cm^2. A load of 25 kg is applied on the water at the top in the tank. Neglecting the speed of water in the tank, the velocity of the water, coming out of the opening at the time when the height of water level in the tank is 40 cm above the bottom, will be _____ cms^{-1}.

[Take g = 10 ms^{-2}]

Answer (300)

Sol. By Bernoulli's theorem:

$$\frac{250}{0.5} + \rho gh = \frac{1}{2}\rho v^2$$

$\Rightarrow v$ = 3 m/s

$\Rightarrow v$ = 300 cm/s

10. A pendulum of length 2 m consists of a wooden bob of mass 50 g. A bullet of mass 75 g is fired towards the stationary bob with a speed v. The bullet emerges out of the bob with a speed $\frac{v}{3}$ and the bob just completes the vertical circle. The value of v is _____ ms^{-1}. (if g =10 m/s^2).

Answer (10)

Sol. $v_{bob} = \sqrt{5gl} = \sqrt{5 \times 10 \times 2}$ = 10 m/s

Conserving momentum:

$$75 \times v = 75 \times \frac{v}{3} + 50 \times 10$$

$\Rightarrow 50v = 50 \times 10$

$\Rightarrow v$ = 10 m/s

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

   Assertion (A) : At 10°C, the density of a 5 M solution of KCl [atomic masses of K & Cl are 39 & 35.5 g mol^{-1} respectively], is 'x' g ml^{-1}. The solution is cooled to –21°C. The molality of the solution will remain unchanged.

   Reason (R) : The molality of a solution does not change with temperature as mass remains unaffected with temperature.

   In the light of the above statements, choose the **correct** answer from the options given below.

   (A) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**.

   (B) Both **(A)** and **(R)** are true but **(R)** is not the correct explanation of **(A)**.

   (C) **(A)** is true but **(R)** is false.

   (D) **(A)** is false but **(R)** is true.

Answer (A)

Sol. Density = 'x' gm ml^{-1}

$$\therefore \text{ molality, } m = \frac{5 \times 1000}{[x(1000) - 372.5]} = 7.96$$

$\simeq 8$ m (Assuming x = 1)

$\therefore \Delta T_f = iK_f m$

Assuming complete dissociation of salt (100%)

(i = 2)

$\Delta T_f = 2 \times 1.86 \times 8 \simeq 29.76$

Hence, the solution does not freeze at –21°C. This means that molality of the solution won't change as $x \geq 1$.

Statement (II) is also correct as molality is mass dependent and hence, does not change with temperature. However, as solvents are not mentioned, statement (I) can also be incorrect.

2. Based upon VSEPR theory, match the shape (geometry) of the molecules in **List-I** with the molecules in **List-II** and select the **most appropriate** option.

List-I (Shape)	**List-II (Molecules)**
(A) T-shaped	(I) XeF_4
(B) Trigonal planar	(II) SF_4
(C) Square planar	(III) ClF_3
(D) See-saw	(IV) BF_3

(A) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

(B) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

(C) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

(D) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

Answer (B)

Sol.

(Shape)	**(Molecules)**
(A) T-shaped	(III) ClF_3
(B) Trigonal planar	(IV) BF_3
(C) Square planar	(I) XeF_4
(D) See-saw	(II) SF_4

Hence, (B) is the correct option.

3. Match **List-I** with **List-II**

List-I

(A) Spontaneous process

(B) Process with $\Delta P = 0$, $\Delta T = 0$

(C) $\Delta H_{reaction}$

(D) Exothermic Process

List-II

(I) $\Delta H < 0$

(II) $\Delta G_{T,P} < 0$

(III) Isothermal and isobaric process

(IV) [Bond energies of molecules in reactants] – [Bond energies of product molecules]

Choose the **correct** answer from the options given below :

(A) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

(B) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

(C) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

(D) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)

Answer (B)

Sol. Correct match is

(A) Spontaneous process	(II) $\Delta G_{T,P} < 0$ (Constant temperature and pressure condition)
(B) Process with $\Delta P = 0, \Delta T = 0$	(III) Isothermal and isobaric process
(C) $\Delta H_{reaction}$	(IV) [Bond energies of molecules in reactants – bond energies of product molecules]
(D) Exothermic process	(I) $\Delta H < 0$

Hence, the correct option is (B).

4. Match **List-I** with **List-II**

List-I	**List-II**
(A) Lyophilic colloid	(I) Liquid-liquid colloid
(B) Emulsion	(II) Protective colloid
(C) Positively charged colloid	(III) $FeCl_3$ + NaOH
(D) Negatively charged colloid	(IV) $FeCl_3$ + hot water

Choose the **correct** answer from the options given below :

(A) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

(B) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(C) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)

(D) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

Answer (A)

Sol. Correct match of List-I and List-II is:

(A) Lyophilic colloid	(II) Protective colloid
(B) Emulsion	(I) Liquid-liquid colloid
(C) Positively charged colloid	(IV) $FeCl_3$ + hot water (It forms a positively charged sol of $Fe(OH)_3$)
(D) Negatively charged colloid	(III) $FeCl_3$ + NaOH (Negatively charged colloid is formed due to adsorption of OH^- ions on $Fe(OH)_3$)

Hence, the correct option is (A).

5. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : The ionic radii of O^{2-} and Mg^{2+} are same.

Reason (R) : Both O^{2-} and Mg^{2+} are isoelectronic species.

In the light of the above statements, choose the **correct** answer from the options given below.

(A) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**.

(B) Both **(A)** and **(R)** are true but **(R)** is not the correct explanation of **(A)**.

(C) **(A)** is true but **(R)** is false.

(D) **(A)** is false but **(R)** is true.

Answer (D)

Sol. Correct order of ionic radii:

$$O^{-2} > Mg^{+2}$$

This is because among isoelectronic species, the size of anions are greater than the size of cations.
Statement (II) is correct as both O^{-2} and Mg^{+2} are isoelectronic.

6. Match **List-I** with **List-II.**

List-I	**List-II**
(A) Concentration of Gold ore	(I) Aniline
(B) Leaching of alumina	(II) NaOH
(C) Froth stabiliser	(III) SO_2
(D) Blister copper	(IV) NaCN

Choose the **correct** answer from the options given below.

(A) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(B) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

(C) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

(D) (A)-(II), (B)-(IV), (C)-(III), (D)-(I)

Answer (B)

Sol.

	List-I		List-II
(A)	Concentration of Gold ore	(IV)	NaCN
(B)	Leaching of alumina	(II)	NaOH
(C)	Froth stabiliser	(I)	Aniline (Aniline and cresols are used as froth stabilisers in froth floatation process)
(D)	Blister copper	(III)	SO_2 (During self reduction process used in the formation of blister copper SO_2 gas is evolved)

Hence (B) is most appropriate option.

7. Addition of H_2SO_4 to BaO_2 produces:

(A) BaO, SO_2 and H_2O

(B) $BaHSO_4$ and O_2

(C) $BaSO_4$, H_2 and O_2

(D) $BaSO_4$ and H_2O_2

Answer (D)

Sol. $BaO_2 \cdot 8H_2O + H_2SO_4 \rightarrow BaSO_4 + H_2O_2 + H_2O$

Hence, the correct option is (D)

8. $BeCl_2$ reacts with $LiAlH_4$ to give:

(A) Be + $Li[AlCl_4]$ + H_2

(B) Be + AlH_3 + LiCl + HCl

(C) BeH_2 + LiCl + $AlCl_3$

(D) BeH_2 + $Li[AlCl_4]$

Answer (C)

Sol. $BeCl_2 + LiAlH_4 \rightarrow BeH_2 + LiCl + AlCl_3$

The above reaction using $LiAlH_4$ is an important preparation method for production of hydrides.

9. Match **List-I** with **List-II.**

	List-I (Si-Compounds)		**List-II (Si-Polymeric/other Products)**
(A)	$(CH_3)_4Si$	(I)	Chain Silicone
(B)	$(CH_3)Si(OH)_3$	(II)	Dimeric Silicone
(C)	$(CH_3)_2Si(OH)_2$	(III)	Silane
(D)	$(CH_3)_3Si(OH)$	(IV)	2D-Silicone

Choose the **correct** answer from the options given below:

(A) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

(B) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

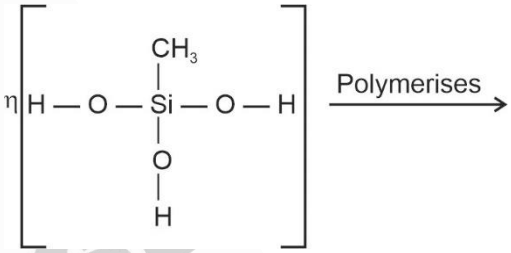
(C) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

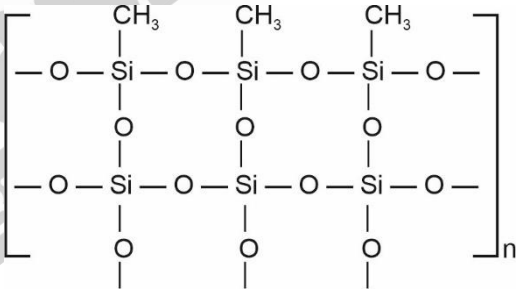
(D) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

Answer (D)

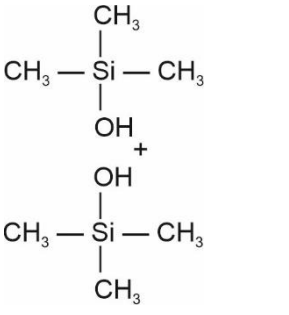
Sol.

List-I (Si-Compounds)		**List-II (Si-Polymeric/other Products)**
(A)	$(CH_3)_4Si$	(III) Silane
(B)	$(CH_3)Si(OH)_3$	(IV) 2D-Silicone
(C)	$(CH_3)_2Si(OH)_2$	(I) Chain Silicone
(D)	$(CH_3)_3Si(OH)$	(II) Dimeric Silicone

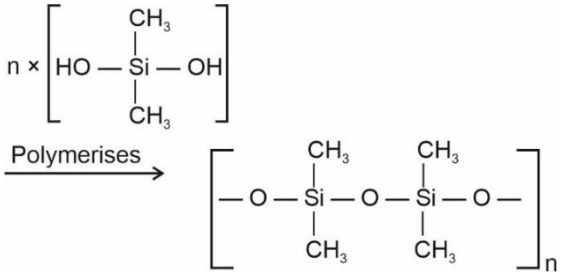




2D-Silicone



Dimeric Silicone



10. Heating white phosphorus with conc. NaOH solution gives mainly:

(A) Na_3P and H_2O

(B) H_3PO and NaH

(C) $P(OH)_3$ and NaH_2PO_4

(D) PH_3 and NaH_2PO_2

Answer (D)

Sol. P_4(white) + NaOH

$\rightarrow PH_3 + NaH_2PO_2 + H_2O$

11. Which of the following will have maximum stabilization due to crystal field?

(A) $[Ti(H_2O)_6]^{3+}$ (B) $[Co(H_2O)_6]^{2+}$

(C) $[Co(CN)_6]^{-3}$ (D) $[Cu(NH_3)_4]^{2+}$

Answer (C)

Sol. The given complexes are:

$[Ti(H_2O)_6]^{3+}$, $[Co(H_2O)_6]^{2+}$, $[Co(CN)_6]^{-3}$, $[Cu(NH_3)_4]^{2+}$

CN^- is the strongest ligand among the given complexes CFSE value for the $[Co(CN)_6]^{-3}$ complex will be highest as it has d^6 configuration with a CFSE value of $-2.40\ \Delta_0 + 2P$, where P represents pairing energy and Δ_0 represents splitting energy in octahedral field.

The value of Δ_0 is high for cyanide complexes.

12. Given below are two Statements:

Statement I: Classical smog occurs in cool humid climate. It is a reducing mixture of smoke, fog and sulphur dioxide.

Statement II: Photochemical smog has components, ozone, nitric oxide, acrolein, formaldehyde, PAN etc.

In the light of the above statements, choose the most appropriate answer from the options given below.

(A) Both **Statement I** and **Statement II** are correct.

(B) Both **Statement I** and **Statement II** are incorrect.

(C) **Statement I** is correct but **Statement II** is incorrect.

(D) **Statement I** is incorrect but **Statement II** is correct

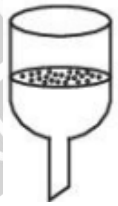
Answer (A)

Sol. (I) Classical smog occurs in cool humid climate. It is a reducing mixture of smoke, fog and sulphur dioxide. This is a correct statement.

(II) This statement is also based on fact and is a correct statement.

13. Which of the following is structure of a separating funnel?

(A)  (B) 

(C)  (D) 

Answer (A)

Sol. The diagram is option (A) clearly represents separating funnel which is used to separate two immiscible liquids.

14. 'A' and 'B' respectively are:

$A \xrightarrow[(2)\ Zn-H_2O]{(1)\ O_3}$ Ethane-1, 2-dicarbaldehyde + Glyoxal/Oxaldehyde

$B \xrightarrow[(2)\ Zn-H_2O]{(1)\ O_3} 5-$oxohexanal

(A) 1-methylcyclohex-1, 3-diene & cyclopentene

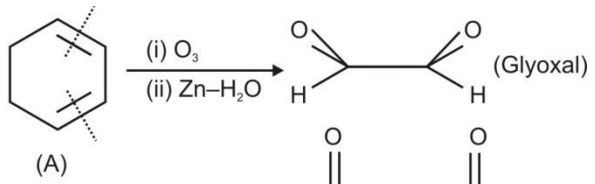
(B) Cyclohex-1, 3-diene & cyclopentene

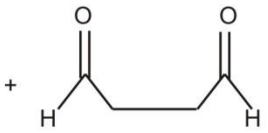
(C) 1-methylcyclohex-1, 4-diene & 1-methylcyclo-pent-ene

(D) Cyclohex-1, 3-diene & 1-methylcyclopent-1-ene

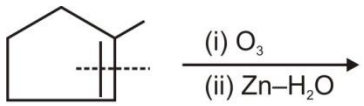
Answer (D)

Sol.

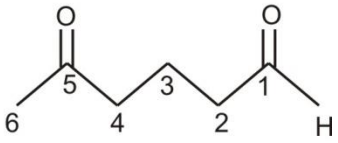




(Ethane-1,2-dicarbaldeyde)



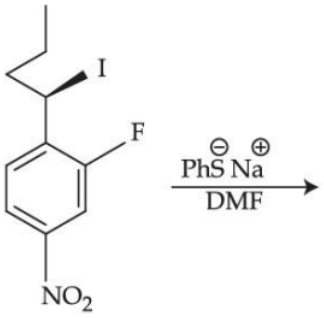
(B)

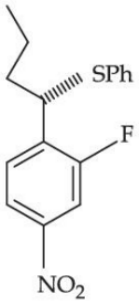


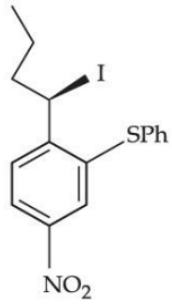
(S-oxohexanal)

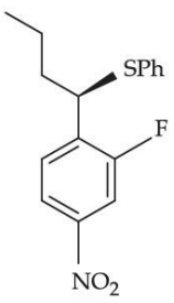
(B) should be 1-methylcyclopent-1-ene.

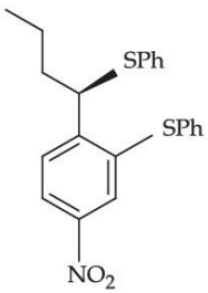
15. The major product of the following reaction is:



(A) 

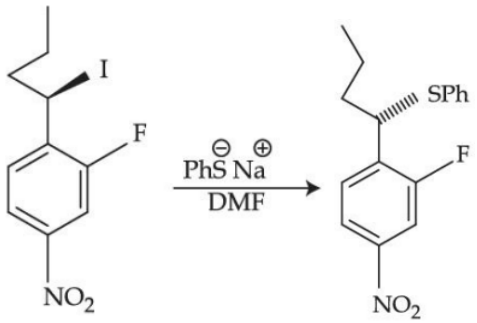
(B) 

(C) 

(D) 

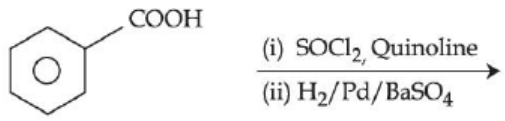
Answer (A)

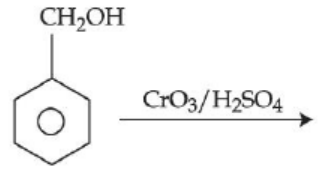
Sol.

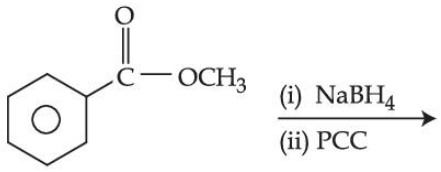


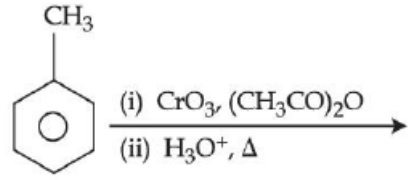
Rate of S_N2 >S_N2 (AR)

16. Which of the following reactions will yield benzaldehyde as a product?

(A) 

(B) 

(C) 

(D) 

(A) (B) and (C)

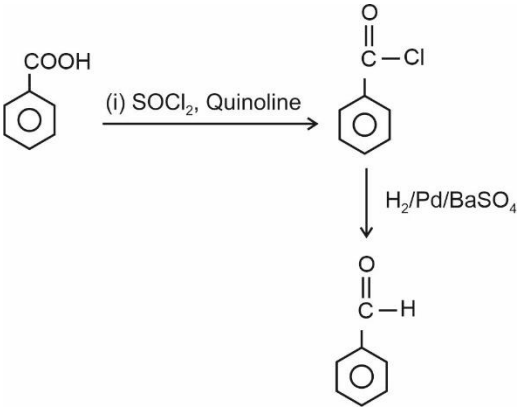
(B) (C) and (D)

(C) (A) and (D)

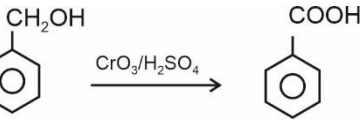
(D) (A) and (C)

Answer (C)

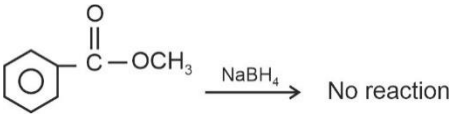
Sol. (A)



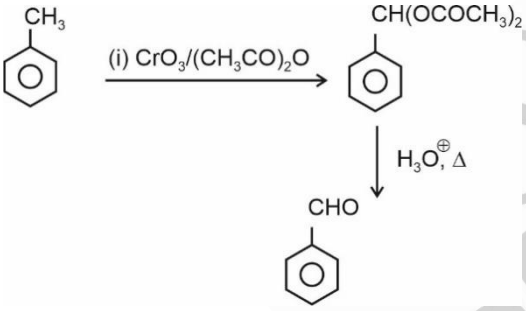
(B)



(C)



(D)



17. Given below are two statements:

Statement-I : In Hofmann degradation reaction, the migration of only an alkyl group takes place from carbonyl carbon of the amide to the nitrogen atom.

Statement-II : The group is migrated in Hofmann degradation reaction to electron deficient atom.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(A) Both **Statement I** and **Statement II** are correct

(B) Both **Statement I** and **Statement II** are incorrect

(C) **Statement I** is correct but **Statement II** is incorrect

(D) **Statement I** is incorrect but **Statement II** is correct

Answer (D)

Sol. Hofmann bromamide degradation

In this degradation, the migration of the alkyl/aryl group occurs to the electron deficient nitrogen (nitrene).

Statement (I) is not absolutely correct as it mentions only the alkyl group, whereas migration of aryl groups may also occur depending on migratory aptitude.

Statement (II) is correct as migration occurs to electron deficient atom.

18. Match List-I with List-II

List-I (Polymer)	**List-II (Used in)**
(A) Bakelite	(I) Radio and television cabinets
(B) Glyptal	(II) Electrical switches
(C) PVC	(III) Paints and Lacqures
(D) Polystyrene	(IV) Water pipes

Choose the **correct** answer from the options given below:

(A) (A) –(II) (B) – (III), (C) – (IV), (D) – (I)

(B) (A) –(I) (B) – (II), (C) – (III), (D) – (IV)

(C) (A) –(IV) (B) – (III), (C) – (II), (D) – (I)

(D) (A) –(II) (B) – (III), (C) – (I), (D) – (IV)

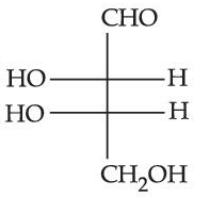
Answer (A)

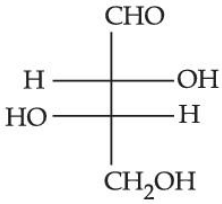
Sol.

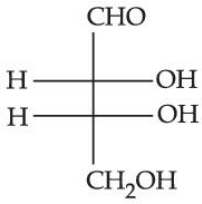
List-I (Polymer)	**List-II (Used in)**
(A) Bakelite	(II) Electrical switches
(B) Glyptal	(III) Paints and Lacqures
(C) PVC	(IV) Water pipes
(D) Polystyrene	(I) Radio and television Cabinets

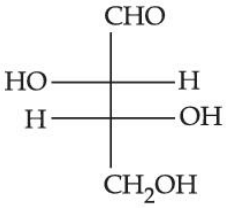
Therefore, the correct option is (A).

19. L-isomer of a compound 'A' ($C_4H_8O_4$) gives a positive test with $[Ag(NH_3)_2]^+$. Treatment of 'A' with acetic anhydride yields triacetate derivative. Compound 'A' produces an optically active compound (B) and an optically inactive compound (C) on treatment with bromine water and HNO_3 respectively. Compound (A) is:

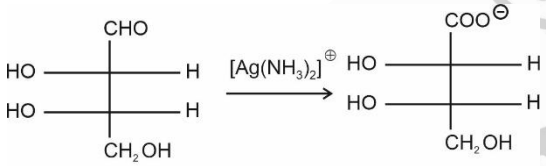
(A) 

(B) 

(C) 

(D) 

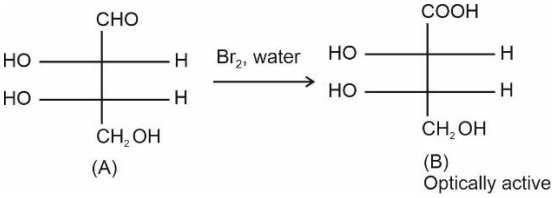
Answer (A)

Sol. 

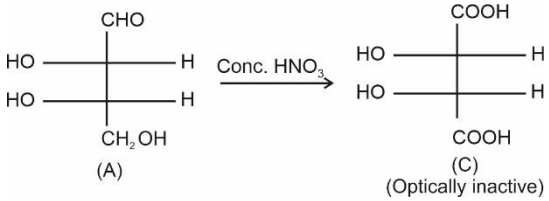
L-erythreose

(A)

When (A) is heated with acetic anhydride, acetylation occurs and –OH group is replaced by $-O-\overset{\overset{O}{\|}}{C}-CH_3$ and hence, triacetate is formed.



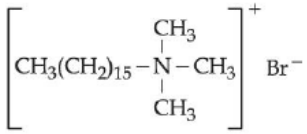
(A) (B) Optically active

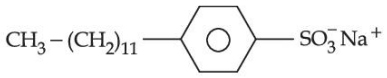


(A) (C) (Optically inactive)

20. Match List-I with List-II

List-I

(A) 

(B) 

(C) $C_{17}H_{35}COO^-Na^+ + Na_2CO_3$ + Rosinate

(D) $CH_3(CH_2)_{16}COO(CH_2CH_2O)CH_2CH_2OH$

List-II

(I) Dishwashing power

(II) Toothpaste

(III) Laundry soap

(IV) Hair conditional

Choose the correct answer from the options given below:

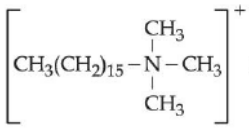
(A) (A) –(III) (B) – (II), (C) – (IV), (D) – (I)

(B) (A) –(IV) (B) – (II), (C) – (III), (D) – (I)

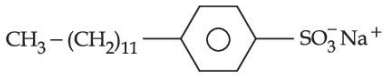
(C) (A) –(IV) (B) – (III), (C) – (II), (D) – (I)

(D) (A) –(III) (B) – (IV), (C) – (I), (D) – (II)

Answer (B)

Sol.  → Cationic detergent

(Hair conditioner)

(B) 

→ Toothpaste

(Anionic detergent)

(C) $C_{17}H_{35}COO^-Na^+ + Na_2CO_3$ + Rosinate

→ Laundry soap

(D) $CH_3(CH_2)_{16}COO(CH_2CH_2O)CH_2CH_2OH$

→ Dishwashing powder

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Metal deficiency defect is shown by $Fe_{0.93}O$. In the crystal, some Fe^{2+} cations are missing and loss of positive charge is compensated by the presence of Fe^{3+} ions. The percentage of Fe^{2+} ions in the $Fe_{0.93}O$ crystals is _____. (Nearest integer)

Answer (85)

Sol. $Fe_{0.93}O$

Let the number of O^{-2} ions be 100

and the number of Fe^{+2} ions be X

The number of Fe^{+3} ions be (93 – X)

$\therefore$ X(2) + (93 – X)3 = 200

279 – X = 200

X = 79

$\therefore$ % of Fe^{+2} ions = $\frac{79}{93} \times 100$

$\simeq$ 85%

2. If the uncertainty in velocity and position of a minute particle in space are, 2.4×10^{-26} (m s^{-1}) and 10^{-7} (m) respectively. The mass of the particle in g is _____. (Nearest integer)

(Given : h = 6.626×10^{-34} Js)

Answer (22)

Sol. $\Delta v = 2.4 \times 10^{-26} m\, s^{-1}$

$\Delta x = 10^{-7} m$

$\therefore \quad m \geq \frac{h}{4\pi(\Delta x)(\Delta v)}$

$\geq \frac{6.626 \times 10^{-34}}{4 \times 3.14 \times (10^{-7})(2.4) \times 10^{-26}}$

$\geq \frac{6.626 \times 10^{-1}}{4 \times 2.4 \times 3.14}$

$\geq$ 0.02198 kg

$\therefore$ Mass of the particle $\simeq$ 22 g

3. 2 g of a non-volatile non-electrolyte solute is dissolved in 200 g of two different solvents A and B whose ebullioscopic constants are in the ratio of 1 : 8. The elevation in boiling points of A and B are in the ratio $\frac{x}{y}$ (x : y). The value of y is _____.

(Nearest Integer)

Answer (8)

Sol. $\Delta Tb = k_b m$

$\frac{(\Delta T_b)_A}{(\Delta T_b)_B} = \frac{(k_b)_A}{(k_b)_B}$

$= \frac{1}{8} = \frac{x}{y}$

$\therefore \quad y = 8$

4. $2NOCl(g) \rightleftharpoons 2NO(g) + Cl_2(g)$

In an experiment, 2.0 moles of NOCl was placed in a one-litre flask and the concentration of NO after equilibrium established, was found to be 0.4 mol/ L. The equilibrium constant at 30°C is _____ $\times 10^{-4}$.

Answer (125)

Sol. $2NOCl(g) \rightleftharpoons 2NO(g) + Cl_2(g)$

t = 0 2

$t = t_{eq}$ 2 – 0.4 0.4 0.2

$k_C = \frac{(0.2) \times (0.4)^2}{(1.6)^2}$

$= \frac{0.2}{16} = \frac{1}{8} \times 10^{-1}$

$= 0.125 \times 10^{-1}$

$= 125 \times 10^{-4}$

5. The limiting molar conductivities of NaI, $NaNO_3$ and $AgNO_3$ are 12.7, 12.0 and 13.3 mS m^2 mol–1, respectively (all at 25°C). The limiting molar conductivity of AgI at this temperature is ____ mS m^2 mol–1.

Answer (14)

Sol. $\Lambda_m^0(AgI) = \Lambda_m^0(NaI) + \Lambda_m^0(AgNO_3) - \Lambda_m^0(NaNO_3)$

= 12.7 + 13.3 – 12.0

= 26 – 12

= 14 mS m^2 mol^{-1}

6. The rate constant for a first order reaction is given by the following equation :

$$\ln k = 33.24 - \frac{2.0\times10^4\,K}{T}$$

The activation energy for the reaction is given by _____ kJ mol^{-1}. (In nearest integer)

(Given : R = 8.3 J K^{-1} mol^{-1})

Answer (166)

Sol. $\ln k = 33.24 - \frac{2\times10^4}{T}$

$\therefore \quad \frac{E_a}{R} = 2\times10^4$

$E_a = 2 \times 10^4 \times 8.3$

= 166 kJ/mol

7. The number of statement(s) **correct** from the following for Copper (at. no. 29) is/are _____.

(A) Cu(II) complexes are always paramagnetic

(B) Cu(I) complexes are generally colourless

(C) Cu(I) is easily oxidized

(D) In Fehling solution, the active reagent has Cu(I)

Answer (3)

Sol. (A) Cu(II) complexes are always paramagnetic as they have one unpaired electron due to d^9 configuration of Cu(II)

(B) Cu(I) complexes are generally colourless due to d^{10} configuration.

(C) Cu(I) is easily oxidised to Cu^{+2} in aqueous solution

$2Cu^+ \rightarrow Cu^{+2} + Cu$

Cu^{+1} disproportionates to Cu^{+2} and Cu

($E^o_{cell} > 0$ for this cell reaction in aqueous solution)

In Fehling's solution, active reagent has Cu(II) which is reduced to Cu(I) on reaction with aldehydes.

Hence (D) statement is incorrect

8. Acidified potassium permanganate solution oxidises oxalic acid. The spin-only magnetic moment of the manganese product formed from the above reaction is _____ B.M.

(Nearest Integer)

Answer (6)

Sol. $KMnO_4(\text{acidic medium}) + H_2C_2O_4 \rightarrow CO_2 + Mn^{+2}$

Mn^{+2} has 5 unpaired electrons

$\therefore$ Spin only magnetic moment = $\sqrt{5(5+2)}$

= $\sqrt{5\times7}$

= $\sqrt{35}$

$\simeq$ 5.92 B.M.

$\simeq$ 6 B.M.

9. Two elements A and B which form 0.15 moles of A_2B and AB_3 type compounds. If both A_2B and AB_3 weigh equally, then the atomic weight of A is _____ times of atomic weight of B.

Answer (2)

Sol. Mole of A_2B = moles of AB_3

$$\frac{W}{2A+B} = \frac{W}{A+3B}$$

A + 3B = 2A + B

2B = A

Atomic weight of A is 2 times that of B.

10. Total number of possible stereoisomers of dimethyl cyclopentane is _____.

Answer (Bonus)

Sol. Position of methyl groups not mentioned.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The area of the polygon, whose vertices are the non-real roots of the equation $\bar{z} = iz^2$ is :

(A) $\frac{3\sqrt{3}}{4}$ (B) $\frac{3\sqrt{3}}{2}$

(C) $\frac{3}{2}$ (D) $\frac{3}{4}$

Answer (A)

Sol. $\bar{z} = iz^2$

Let $z = x + iy$

$x - iy = i(x^2 - y^2 + 2xiy)$

$x - iy = i(x^2 - y^2) - 2xy$

$\therefore \quad x = -2yx \quad \text{or} \quad x^2 - y^2 = -y$

$x = 0 \quad \text{or} \quad y = -\frac{1}{2}$

Case-I

$x = 0$

$-y^2 = -y$

$y = 0, 1$

Case-II

$y = -\frac{1}{2}$

$\Rightarrow \quad x^2 - \frac{1}{4} = \frac{1}{2} \quad \Rightarrow x = \pm\frac{\sqrt{3}}{2}$

$z = \left\{0, i, \frac{\sqrt{3}}{2} - \frac{i}{2}, \frac{-\sqrt{3}}{2} - \frac{i}{2}\right\}$

$$\text{Area of polygon} = \frac{1}{2}\begin{vmatrix} 0 & 1 & 1 \\ \frac{\sqrt{3}}{2} & \frac{-1}{2} & 1 \\ \frac{-\sqrt{3}}{2} & \frac{-1}{2} & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left|-\sqrt{3} \quad -\frac{\sqrt{3}}{2}\right| = \frac{3\sqrt{3}}{4}$$

2. Let the system of linear equations $x + 2y + z = 2$, $\alpha x + 3y - z = \alpha$, $-\alpha x + y + 2z = -\alpha$ be inconsistent. Then α is equal to :

(A) $\frac{5}{2}$ (B) $-\frac{5}{2}$

(C) $\frac{7}{2}$ (D) $-\frac{7}{2}$

Answer (D)

Sol. $x + 2y + z = 2$

$\alpha x + 3y - z = \alpha$

$-\alpha x + y + 2z = -\alpha$

$$\Delta = \begin{vmatrix} 1 & 2 & 1 \\ \alpha & 3 & -1 \\ -\alpha & 1 & 2 \end{vmatrix} = 1(6+1) - 2(2\alpha - \alpha) + 1(\alpha + 3\alpha)$$

$= 7 + 2\alpha$

$\Delta = 0 \Rightarrow \alpha = -\frac{7}{2}$

$$\Delta_1 = \begin{vmatrix} 2 & 2 & 1 \\ \alpha & 3 & -1 \\ -\alpha & 1 & 2 \end{vmatrix} = 14 + 2\alpha \neq 0 \text{ for } \alpha = -\frac{7}{2}$$

$\therefore$ For no solution $\alpha = -\frac{7}{2}$

3. If $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$, where a, b, c are in A.P. and $|a| < 1$, $|b| < 1$, $|c| < 1$, $abc \neq 0$, then :

(A) x, y, z are in A.P.

(B) x, y, z are in G.P.

(C) $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P.

(D) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1 - (a + b + c)$

Answer (C)

Sol. $x = \sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$; $y = \sum_{n=0}^{\infty} b^n = \frac{1}{1-b}$; $z = \sum_{n=0}^{\infty} c^n = \frac{1}{1-c}$

Now,

$a, b, c \rightarrow$ AP

$1 - a, 1 - b, 1 - c \rightarrow$ AP

$\frac{1}{1-a}, \frac{1}{1-b}, \frac{1}{1-c} \rightarrow$ HP

$x, y, z \rightarrow$ HP

$\therefore \quad \frac{1}{x}, \frac{1}{y}, \frac{1}{z} \rightarrow$ AP

4. Let $\frac{dy}{dx} = \frac{ax - by + a}{bx + cy + a}$, where a, b, c are constants, represent a circle passing through the point (2, 5). Then the shortest distance of the point (11, 6) from this circle is

(A) 10 (B) 8

(C) 7 (D) 5

Answer (B)

Sol. $\frac{dy}{dx} = \frac{ax - by + a}{bx + cy + a}$

$= bx\,dy + cy\,dy + a\,dy = ax\,dx - by\,dx + a\,dx$

$= cy\,dy + a\,dy - ax\,dx - a\,dx + b(x\,dy + y\,dx) = 0$

$= c\int y\,dy + a\int x\,dx - a\int dx + b\int d(xy) = 0$

$= \frac{cy^2}{2} + ay - \frac{ax^2}{2} - ax + bxy = k$

$= ax^2 - cy^2 + 2ax - 2ay - 2bxy = k$

Above equation is circle

$\Rightarrow \quad a = -c$ and $b = 0$

$ax^2 + ay^2 + 2ax - 2ay = k$

$\Rightarrow \quad x^2 + y^2 + 2x - 2y = \lambda \qquad \left[\lambda = \frac{k}{a}\right]$

Passes through (2, 5)

$4 + 25 + 4 - 10 = \lambda \Rightarrow \lambda = 23$

Circle $\equiv x^2 + y^2 + 2x - 2y - 23 = 0$

Centre (–1, 1) $r = \sqrt{(-1)^2 + 1^2 + 23} = 5$

Shortest distance of (11, 6) $= \sqrt{12^2 + 5^2} - 5$

$= 13 - 5$

$= 8$

5. Let a be an integer such that $\lim\limits_{x \to 7} \frac{18 - [1 - x]}{[x - 3a]}$ exists, where $[t]$ is greatest integer $\le t$. Then a is equal to :

(A) –6 (B) –2

(C) 2 (D) 6

Answer (A)

Sol. $\lim\limits_{x \to 7} \frac{18 - [1 - x]}{[x - 3a]}$ exist & $a \in I$.

$= \lim\limits_{x \to 7} \frac{17 - [-x]}{[x] - 3a}$ exist

RHL $= \lim\limits_{x \to 7^+} \frac{17 - [-x]}{[x] - 3a} = \frac{25}{7 - 3a} \qquad \left[a \ne \frac{7}{3}\right]$

LHL $= \lim\limits_{x \to 7^-} \frac{17 - [-x]}{[x] - 3a} = \frac{24}{6 - 3a} \qquad [a \ne 2]$

For limit to exist

LHL = RHL

$\frac{25}{7 - 3a} = \frac{24}{6 - 3a}$

$\Rightarrow \quad \frac{25}{7 - 3a} = \frac{8}{2 - a}$

$\therefore \quad \boxed{a = -6}$

6. The number of distinct real roots of $x^4 - 4x + 1 = 0$ is :

(A) 4 (B) 2

(C) 1 (D) 0

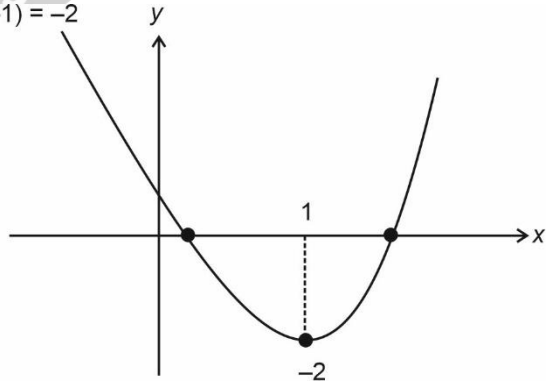
Answer (B)

Sol. $f(x) = x^4 - 4x + 1 = 0$

$f'(x) = 4x^3 - 4$

$= 4(x - 1)(x^2 + 1 + x)$

$f(-1) = -2$



$\Rightarrow$ Two solution

7. The lengths of the sides of a triangle are $10 + x^2$, $10 + x^2$ and $20 - 2x^2$. If for $x = k$, the area of the triangle is maximum, then $3k^2$ is equal to :

(A) 5

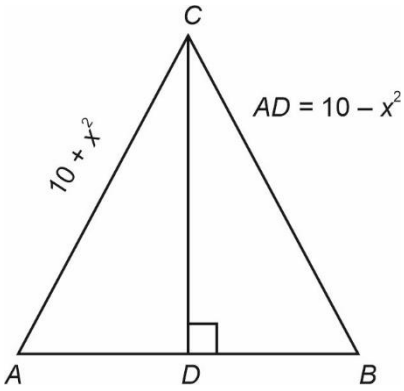
(B) 8

(C) 10

(D) 12

Answer (C)

Sol.



$$CD = \sqrt{\left(10+x^2\right)^2 - \left(10-x^2\right)^2} = 2\sqrt{10}|x|$$

$$\text{Area} = \frac{1}{2}\times CD \times AB = \frac{1}{2}\times 2\sqrt{10}\,|x|\left(20-2x^2\right)$$

$$A = \sqrt{10}\,|x|\left(10-x^2\right)$$

$$\frac{dA}{dx} = \sqrt{10}\frac{|x|}{x}\left(10-x^2\right) + \sqrt{10}\,|x|\,(-2x) = 0$$

$\Rightarrow$ $10 - x^2 = 2x^2$

$3x^2 = 10$

$x = k$

$$\boxed{3k^2 = 10}$$

8. If $\cos^{-1}\left(\frac{y}{2}\right) = \log_e\left(\frac{x}{5}\right)^5, |y| < 2,$ then :

(A) $x^2y'' + xy' - 25y = 0$

(B) $x^2y'' - xy' - 25y = 0$

(C) $x^2y'' - xy' + 25y = 0$

(D) $x^2y'' + xy' + 25y = 0$

Answer (D)

Sol. $\cos^{-1}\left(\frac{y}{2}\right) = \log_e\left(\frac{x}{5}\right)^5 \quad |y| < 2$

Differentating on both side

$$-\frac{1}{\sqrt{1-\left(\frac{y}{2}\right)^2}}\times\frac{y'}{2} = \frac{5}{x}\times\frac{1}{5}$$

$$\frac{-xy'}{2} = 5\sqrt{1-\left(\frac{y}{2}\right)^2}$$

Square on both side

$$\frac{x^2y'^2}{4} = 25\left(\frac{4-y^2}{4}\right)$$

Diff on both side

$$2xy'^2 + 2y'y''x^2 = -25\times 2yy'$$

$$xy' + y''x^2 + 25y = 0$$

9. If $\int\frac{\left(x^2+1\right)e^x}{\left(x+1\right)^2}dx = f(x)e^x + C,$ where C is a constant, then $\frac{d^3f}{dx^3}$ at $x = 1$ is equal to :

(A) $-\frac{3}{4}$

(B) $\frac{3}{4}$

(C) $-\frac{3}{2}$

(D) $\frac{3}{2}$

Answer (B)

Sol. $I = \int\frac{e^x\left(x^2+1\right)}{\left(x+1\right)^2}dx = f(x)e^x + c$

$$= \int\frac{e^x\left(x^2-1+1+1\right)}{\left(x+1\right)^2}dx$$

$$= \int e^x\left[\frac{x-1}{x+1} + \frac{2}{\left(x+1\right)^2}\right]dx$$

$$= e^x\left(\frac{x-1}{x+1}\right) + c$$

$\therefore$ $f(x) = \frac{x-1}{x+1}$

$$f(x) = 1 - \frac{2}{x+1}$$

$$f'(x) = 2\left(\frac{1}{x+1}\right)^2$$

$$f''(x) = -4\left(\frac{1}{x+1}\right)^3$$

$$f'''(x) = \frac{12}{\left(x+1\right)^4}$$

for $x = 1$

$$f'''(1) = \frac{12}{2^4} = \frac{12}{16} = \frac{3}{4}$$

10. The value of the integral $\int_{-2}^{2} \frac{|x^3+x|}{\left(e^{x|x|}+1\right)}dx$ is equal to:

(A) $5e^2$

(B) $3e^{-2}$

(C) 4

(D) 6

Answer (D)

Sol. $I = \int_{-2}^{2} \frac{|x^3+x|}{e^{x|x|}+1}dx$ ...(i)

$I = \int_{-2}^{2} \frac{|x^3+x|}{e^{-x|x|}+1}dx$ ...(ii)

$2I = \int_{-2}^{2} |x^3+x|dx$

$2I = 2\int_{0}^{2}(x^3+x)dx$

$I = \int_{0}^{2}(x^3+x)dx$

$= \frac{x^4}{4}+\frac{x^2}{2}\Big]_0^2$

$= \left(\frac{16}{4}+\frac{4}{2}\right)-0$

$= 4 + 2 = 6$

11. If $\frac{dy}{dx}+\frac{2^{x-y}(2^y-1)}{2^x-1}=0, x, y>0, y(1)=1$, then $y(2)$ is equal to :

(A) $2 + \log_2 3$

(B) $2 + \log_3 2$

(C) $2 - \log_3 2$

(D) $2 - \log_2 3$

Answer (D)

Sol. $\frac{dy}{dx}+\frac{2^{x-y}(2^y-1)}{2^x-1}=0$, $x, y > 0$, $y(1) = 1$

$\frac{dy}{dx}=-\frac{2^x(2^y-1)}{2^y(2^x-1)}$

$\int\frac{2^y}{2^y-1}dy=-\int\frac{2^x}{2^x-1}dx$

$= \frac{\log_e(2^y-1)}{\log_e 2}=-\frac{\log_e(2^x-1)}{\log_e 2}+\frac{\log_e c}{\log_e 2}$

$= |(2^y - 1)(2^x - 1)| = c$

$\because \quad y(1) = 1$

$\therefore \quad c = 1$

$= |(2^y - 1)(2^x - 1)| = 1$

For $x = 2$

$|(2^y - 1)3| = 1$

$2^y-1=\frac{1}{3}\Rightarrow 2y=\frac{4}{3}$

Taking log to base 2.

$\therefore \quad \boxed{y = 2-\log_2 3}$

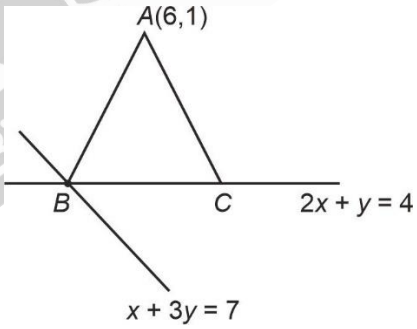
12. In an isosceles triangle ABC, the vertex A is (6, 1) and the equation of the base BC is $2x + y = 4$. Let the point B lie on the line $x + 3y = 7$. If (α, β) is the centroid of ΔABC, then $15(\alpha + \beta)$ is equal to :

(A) 39 (B) 41

(C) 51 (D) 63

Answer (C)

Sol.



$\left.\begin{aligned}2x+y&=4\\2x+6y&=14\end{aligned}\right\} y = 2, x = 3$

$B(1, 2)$

Let $C(k, 4 - 2k)$

Now $AB^2 = AC^2$

$5^2 + (-1)^2 = (6 - k)^2 + (-3 + 2k)^2$

$\Rightarrow \quad 5k^2 - 24k + 19 = 0$

$(5k - 19)(k - 1) = 0 \Rightarrow k=\frac{19}{5}$

$C\left(\frac{19}{5}, -\frac{18}{5}\right)$

Centroid (α, β)

$$\alpha = \frac{6+1+\frac{19}{5}}{3} = \frac{18}{5}$$

$$\beta = \frac{1+2-\frac{18}{5}}{3} = -\frac{1}{5}$$

Now $15(\alpha + \beta)$

$$15\left(\frac{17}{5}\right) = 51$$

13. Let the eccentricity of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$, be $\frac{1}{4}$. If this ellipse passes through the point $\left(-4\sqrt{\frac{2}{5}}, 3\right)$, then $a^2 + b^2$ is equal to :

(A) 29 (B) 31

(C) 32 (D) 34

Answer (B)

Sol. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$$\Rightarrow \frac{\left(-4\sqrt{\frac{2}{5}}\right)^2}{a^2}+\frac{32}{b^2}=1$$

$$\Rightarrow \frac{32}{5a^2}+\frac{9}{b^2}=1 \quad \ldots(i)$$

$a^2(1 - e^2) = b^2$

$$a^2\left(1-\frac{1}{16}\right)=b^2$$

$15a^2 = 16b^2 \Rightarrow a^2 = \frac{16b^2}{15}$

From (i)

$$\frac{6}{b^2}+\frac{9}{b^2}=1 \Rightarrow b^2=15 \ \& \ a^2 = 16$$

$a^2 + b^2 = 15 + 16 = 31$

14. If two straight lines whose direction cosines are given by the relations $l + m - n = 0$, $3l^2 + m^2 + cnl = 0$ are parallel, then the positive value of c is :

(A) 6 (B) 4

(C) 3 (D) 2

Answer (A)

Sol. $l + m - n = 0 \Rightarrow n = l + m$

$3l^2 + m^2 + cnl = 0$

$3l^2 + m^2 + cl(l + m) = 0$

$= (3 + c)l^2 + clm + m^2 = 0$

$$= (3+c)\left(\frac{l}{m}\right)^2 + c\left(\frac{l}{m}\right) + 1 = 0$$

$\because$ Lines are parallel

$D = 0$

$c^2 - 4(3 + c) = 0$

$c^2 - 4c - 12 = 0$

$(c - 4)(c + 3) = 0$

$c = 4$ (as $c > 0$)

15. Let $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} - 3\hat{j} + 2\hat{k}$. Then the number of vectors $\vec{b}$ such that $\vec{b} \times \vec{c} = \vec{a}$ and $|\vec{b}| \in \{1, 2, \ldots, 10\}$ is :

(A) 0 (B) 1

(C) 2 (D) 3

Answer (A)

Sol. $\vec{a} = \hat{i} + \hat{j} - \hat{k}$

$\vec{c} = 2\hat{i} - 3\hat{j} + 2\hat{k}$

Now, $\vec{b} \times \vec{c} = \vec{a}$

$\vec{c} \cdot (\vec{b} \times \vec{c}) = \vec{c} \cdot \vec{a}$

$\vec{c} \cdot \vec{a} = 0$

$\Rightarrow (\hat{i} + \hat{j} - \hat{k})(2\hat{i} - 3\hat{j} + 2\hat{k}) = 0$

$= 2 - 3 - 2 = 0$

$\Rightarrow -3 = 0$ (Not possible)

$\Rightarrow$ No possible value of $\vec{b}$ is possible.

16. Five numbers, x_1, x_2, x_3, x_4, x_5 are randomly selected from the numbers 1, 2, 3,....., 18 and are arranged in the increasing order ($x_1 < x_2 < x_3 < x_4 < x_5$). The probability that $x_2 = 7$ and $x_4 = 11$ is:

(A) $\frac{1}{136}$ (B) $\frac{1}{72}$

(C) $\frac{1}{68}$ (D) $\frac{1}{34}$

Answer (C)

Sol. Total cases $= {}^{18}C_5$

Favourable cases

$${}^{6}C_1 \quad {}^{3}C_1 \quad {}^{7}C_1$$
(Select x_1) (Select x_3) (Select x_5)

$$P = \frac{6 \cdot 3 \cdot 7}{{}^{18}C_5} = \frac{1}{68}$$

17. Let X be a random variable having binomial distribution $B(7, p)$. If $P(X = 3) = 5P(X = 4)$, then the sum of the mean and the variance of X is:

(A) $\frac{105}{16}$

(B) $\frac{7}{16}$

(C) $\frac{77}{36}$

(D) $\frac{49}{16}$

Answer (C)

Sol. Given $P(X = 3) = 5P(X = 4)$ and $n = 7$

$$\Rightarrow \quad {}^{7}C_3 p^3 q^4 = 5 \cdot {}^{7}C_4 p^4 q^3$$

$\Rightarrow$ $q = 5p$ and also $p + q = 1$

$$\Rightarrow \quad p = \frac{1}{6} \text{ and } q = \frac{5}{6}$$

$$\text{Mean} = \frac{7}{6} \text{ and variance} = \frac{35}{36}$$

$$\text{Mean + Variance} = \frac{7}{6} + \frac{35}{36} = \frac{77}{36}$$

18. The value of $\cos\left(\frac{2\pi}{7}\right) + \cos\left(\frac{4\pi}{7}\right) + \cos\left(\frac{6\pi}{7}\right)$ is equal to:

(A) –1

(B) $-\frac{1}{2}$

(C) $-\frac{1}{3}$

(D) $-\frac{1}{4}$

Answer (B)

Sol. $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7} = \frac{\sin 3\left(\frac{\pi}{7}\right)}{\sin\frac{\pi}{7}} \cos\frac{\left(\frac{2\pi}{7} + \frac{6\pi}{7}\right)}{2}$

$$= \frac{\sin\left(\frac{3\pi}{7}\right) \cdot \cos\left(\frac{4\pi}{7}\right)}{\sin\left(\frac{\pi}{7}\right)}$$

$$= \frac{2\sin\frac{4\pi}{7}\cos\frac{4\pi}{7}}{2\sin\frac{\pi}{7}}$$

$$= \frac{\sin\left(\frac{8\pi}{7}\right)}{2\sin\frac{\pi}{7}} = \frac{-\sin\frac{\pi}{7}}{2\sin\frac{\pi}{7}} = \frac{-1}{2}$$

19. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\left(\cos\frac{7\pi}{6}\right) + \tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ is equal to:

(A) $\frac{11\pi}{12}$ (B) $\frac{17\pi}{12}$

(C) $\frac{31\pi}{12}$ (D) $-\frac{3\pi}{4}$

Answer (A)

Sol. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \tan^{-1}(-1)$

$$= \frac{\pi}{3} + \frac{5\pi}{6} - \frac{\pi}{4}$$

$$= \frac{4\pi + 10\pi - 3\pi}{12} = \frac{11\pi}{12}$$

20. The boolean expression $(\sim(p \wedge q)) \vee q$ is equivalent to:

(A) $q \rightarrow (p \wedge q)$ (B) $p \rightarrow q$

(C) $p \rightarrow (p \rightarrow q)$ (D) $p \rightarrow (p \vee q)$

Answer (D)

Sol. Making truth table

p	q	$p \wedge q$	$\sim p \wedge q$	$(\sim(p \wedge q)) \vee q$	$p \vee q$	$p \rightarrow q$	$p \rightarrow (p \vee q)$
T	T	T	F	T	T	T	T
T	F	F	T	T	T	F	T
F	T	F	T	T	T	T	T
F	F	F	T	T	F	T	T
				Tautology			Tautology

$\therefore \quad (\sim(p \wedge q)) \vee q \equiv p \rightarrow (p \vee q)$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Let $f : \mathbf{R} \to \mathbf{R}$ be a function defined by $f(x) = \frac{2e^{2x}}{e^{2x}+e}$.

Then $f\left(\frac{1}{100}\right)+f\left(\frac{2}{100}\right)+f\left(\frac{3}{100}\right)+...+f\left(\frac{99}{100}\right)$ is equal to _____.

Answer (99)

Sol. $f(x) = \frac{2e^{2x}}{e^{2x}+e^{x}}$ and $f(1-x) = \frac{2e^{2-2x}}{e^{2-2x}+e^{1-x}}$

$\therefore \quad \frac{f(x)+f(1-x)}{2} = 1$

i.e. $f(x) + f(1 – x) = 2$

$\therefore \quad f\left(\frac{1}{100}\right)+f\left(\frac{2}{100}\right)+....+f\left(\frac{99}{100}\right)$

$= \sum_{x=1}^{49} f\left(\frac{x}{100}\right)+f\left(1-\frac{x}{100}\right)+f\left(\frac{1}{2}\right)$

$= 49 \times 2 + 1 = 99$

2. If the sum of all the roots of the equation $e^{2x} – 11e^{x} – 45e^{–x} + \frac{81}{2} = 0$ is $\log_e p$, then p is equal to _____.

Answer (45)

Sol. Let $e^x = t$ then equation reduces to

$t^2 - 11t - \frac{45}{t} + \frac{81}{2} = 0$

$\Rightarrow \quad 2t^3 – 22t^2 + 81t – 45 = 0 \qquad ...(i)$

if roots of $e^{2x} - 11e^{x} - 45e^{-x} + \frac{81}{2} = 0$ are α, β, γ then roots of (i) will be $e^{\alpha_1}e^{\alpha_2}e^{\alpha_3}$ using product of roots

$e^{\alpha_1+\alpha_2+\alpha_3} = 45$

$\Rightarrow \quad \alpha_1 + \alpha_2 + \alpha_3 = \ln 45 \quad \Rightarrow \quad p = 45$

3. The positive value of the determinant of the matrix A, whose

$\text{Adj}(\text{Adj}(A)) = \begin{pmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{pmatrix}$, is _____.

Answer (14)

Sol. $\left|adj\left(adj\left(A\right)\right)\right| = |A|^{2^2} = |A|^4$

$\therefore \quad |A|^4 = \begin{vmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{vmatrix}$

$= (14)^3 \begin{vmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix}$

$= (14)^3 \left(3 - 2(-5) - 1(-1)\right)$

$|A|^4 = (14)^4 \Rightarrow |A| = 14$

4. The number of ways, 16 identical cubes, of which 11 are blue and rest are red, can be placed in a row so that between any two red cubes there should be at least 2 blue cubes, is _____.

Answer (56)

Sol. First we arrange 5 red cubes in a row and assume x_1, x_2, x_3, x_4, x_5 and x_6 number of blue cubes between them

$\square_{x_1} \quad \square_{x_2} \quad \square_{x_3} \quad \square_{x_4} \quad \square_{x_5} \quad _{x_6}$

Here, $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 11$

and $x_2, x_3, x_4, x_5 \geq 2$

So $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 3$

No. of solutions = ${}^8C_5 = 56$

5. If the coefficient of x^{10} in the binomial expansion of $\left(\frac{\sqrt{x}}{5^{\frac{1}{4}}}+\frac{\sqrt{5}}{x^{\frac{1}{3}}}\right)^{60}$ is $5^k \cdot l$, where $l, k \in \mathbf{N}$ and l is co-prime to 5, then k is equal to _____.

Answer (5)

Sol. $T_{r+1} = {}^{60}C_r \left(x^{\frac{1}{2}}\right)^{60-r} \left(x^{-\frac{1}{3}}\right)^{r} \left(5^{\frac{-1}{4}}\right)^{60-r} \left(5^{\frac{1}{2}}\right)^{r}$

for x^{10} $\frac{60-r}{2} - \frac{r}{3} = 10$

$\Rightarrow \quad 180 – 3r – 2r = 60$

$\Rightarrow$ $r = 24$

$\therefore$ Coeff. of $x^{10} = \dfrac{^{60}C_{24}}{5^9}5^{12} = 5^k\, l$

as l and 5 are coprime

k = 3 + exponent of 5 in $^{60}C_{24}$

$$= 3 + \left(\left[\frac{60}{5}\right] + \left[\frac{60}{5^2}\right] - \left[\frac{24}{5}\right] - \left[\frac{24}{5^2}\right] - \left[\frac{36}{5}\right] - \left[\frac{36}{5^2}\right]\right)$$

= 3 + (12 + 2 – 4 – 0 – 7 – 1)

= 3 + 2 = 5

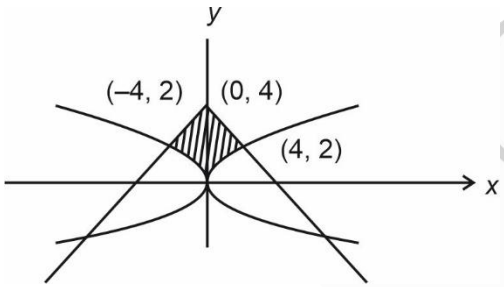
6. Let

$A_1 = \{(x, y) : |x| \le y^2, |x| + 2y \le 8\}$ and

$A_2 = \{(x, y) : |x| + |y| \le k\}$. If 27(Area A_1) = 5(Area A_2), then k is equal to :

Answer (6*)

Sol.

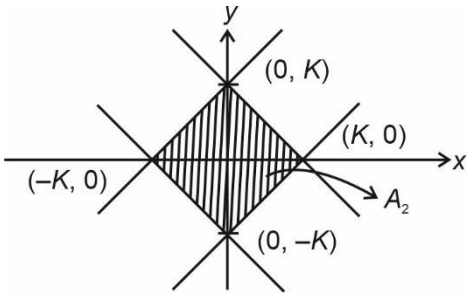


Required area (above x-axis)

$$A_1 = 2\int_0^4 \left(\frac{8-x}{2} - \sqrt{x}\right) dx$$

$$= 2\left(16 - \frac{16}{4} - \frac{8}{3/2}\right) = \frac{40}{3}$$

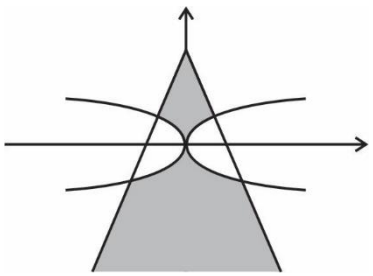
and $A_2 = 4\left(\frac{1}{2}\cdot k^2\right) = 2k^2$



$$\therefore \quad 27\cdot\frac{40}{3} = 5\cdot\left(2k^2\right)$$

$\Rightarrow$ $k = 6$

* A_1 is



Which tends to infinity if not mentioned above x-axis

7. If the sum of the first ten terms of the series $\frac{1}{5} + \frac{2}{65} + \frac{3}{325} + \frac{4}{1025} + \frac{5}{2501} + \ldots$ is $\frac{m}{n}$, where m and n are co-prime numbers, then $m + n$ is equal to _____.

Answer (276)

Sol. $T_r = \dfrac{r}{\left(2r^2\right)^2 + 1}$

$$= \frac{r}{\left(2r^2+1\right)^2 - (2r)^2}$$

$$= \frac{1}{4}\frac{4r}{\left(2r^2+2r+1\right)\left(2r^2-2r+1\right)}$$

$$S_{10} = \frac{1}{4}\sum_{r=1}^{10}\left(\frac{1}{\left(2r^2-2r+1\right)} - \frac{1}{\left(2r^2+2r+1\right)}\right)$$

$$= \frac{1}{4}\left[1 - \frac{1}{5} + \frac{1}{5} - \frac{1}{13} + \ldots.. + \frac{1}{181} - \frac{1}{221}\right]$$

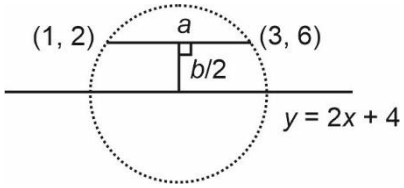
$\Rightarrow$ $S_{10} = \frac{1}{4}\cdot\frac{220}{221} = \frac{55}{221} = \frac{m}{n}$

$\therefore$ $m + n = 276$

8. A rectangle R with end points of one of its sides as (1, 2) and (3, 6) is inscribed in a circle. If the equation of a diameter of the circle is $2x – y + 4 = 0$, then the area of R is _____.

Answer (16)

Sol.



As slope of line joining (1, 2) and (3, 6) is 2 given diameter is parallel to side

$$\therefore \quad a = \sqrt{(3-1)^2 + (6-2)^2} = \sqrt{20}$$

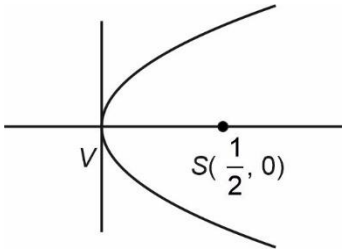
and $b/2 = \frac{4}{\sqrt{5}} \Rightarrow b = \frac{8}{\sqrt{5}}$

Area = $ab = 2\sqrt{5}\cdot\frac{8}{\sqrt{5}} = 16$.

9. A circle of radius 2 unit passes through the vertex and the focus of the parabola $y^2 = 2x$ and touches the parabola $y = \left(x - \frac{1}{4}\right)^2 + \alpha$, where $\alpha > 0$. Then $(4\alpha - 8)^2$ is equal to _____.

Answer (63)

Sol.



Let the equation of circle be

$$x\left(x - \frac{1}{2}\right) + y^2 + \lambda y = 0$$

$$\Rightarrow \quad x^2 + y^2 - \frac{1}{2}x + \lambda y = 0$$

Radius = $\sqrt{\frac{1}{16} + \frac{\lambda^2}{4}} = 2$

$$\Rightarrow \quad \lambda^2 = \frac{63}{4}$$

$$\Rightarrow \quad \left(x - \frac{1}{4}\right)^2 + \left(y + \frac{\lambda}{2}\right)^2 = 4$$

$\because$ This circle and parabola $y - \alpha = \left(x - \frac{1}{4}\right)^2$ touch each other, so

$$\alpha = -\frac{\lambda}{2} + 2$$

$$\Rightarrow \quad \alpha - 2 = -\frac{\lambda}{2}$$

$$\Rightarrow \quad (\alpha - 2)^2 = \frac{\lambda^2}{4} = \frac{63}{16}$$

$$\Rightarrow \quad (4\alpha - 8)^2 = 63$$

10. Let the mirror image of the point (a, b, c) with respect to the plane $3x - 4y + 12z + 19 = 0$ be $(a - 6, \beta, \gamma)$. If $a + b + c = 5$, then $7\beta - 9\gamma$ is equal to _____.

Answer (137)

Sol. $\frac{x-a}{3} = \frac{y-b}{-4} = \frac{z-c}{12} = \frac{-2(3a - 4b + 12c + 19)}{3^2 + (-4)^2 + 12^2}$

$$\frac{x-a}{3} = \frac{y-b}{-4} = \frac{z-c}{12} = \frac{-6a + 8b - 24c - 38}{169}$$

$$(x, y, z) \equiv (a - 6, \beta, \gamma)$$

$$\frac{(a-6)-a}{3} = \frac{\beta - b}{-4} = \frac{\gamma - c}{12} = \frac{-6a + 8b - 24c - 38}{169}$$

$$\frac{\beta - b}{-4} = -2 \Rightarrow \boxed{\beta = 8 + b}$$

$$\frac{\gamma - c}{12} = -2 \Rightarrow \boxed{\gamma = -24 + c}$$

$$\frac{-6a + 8b - 24c - 38}{169} = -2$$

$$\Rightarrow \quad 3a - 4b + 12c = 150 \quad ...(1)$$

$$a + b + c = 5$$

$$3a + 3b + 3c = 15 \quad ....(2)$$

Applying (1) – (2)

$$-7b + 9c = 135$$

$$7b - 9c = -135$$

$$7\beta - 9\gamma = 7(8 + b) - 9(-24 + c)$$

$$= 56 + 216 + 7b - 9c.$$

$$= 56 + 216 - 135 = 137.$$

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