

28/06/2022

Evening



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Answers & Solutions

Time : 3 hrs.

for

M.M. : 300

JEE (Main)-2022 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

- (1) The test is of **3 hours** duration.
- (2) The Test Booklet consists of 90 questions. The maximum marks are 300.
- (3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.
 - (i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **-1 mark** for wrong answer.
 - (ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **-1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Velocity (v) and acceleration (a) in two systems of units 1 and 2 are related as $v_2 = \frac{n}{m^2}v_1$ and $a_2 = \frac{a_1}{mn}$ respectively. Here m and n are constants.

The relations for distance and time in two systems respectively are :

- (A) $\frac{n^3}{m^3}L_1 = L_2$ and $\frac{n^2}{m}T_1 = T_2$
- (B) $L_1 = \frac{n^4}{m^2}L_2$ and $T_1 = \frac{n^2}{m}T_2$
- (C) $L_1 = \frac{n^2}{m}L_2$ and $T_1 = \frac{n^4}{m^2}T_2$
- (D) $\frac{n^2}{m}L_1 = L_2$ and $\frac{n^4}{m^2}T_1 = T_2$

Answer (A)

Sol. $[L] = \frac{[v^2]}{[a]}$

$$\text{so } \frac{[v_2]^2}{[a_2]} = \frac{\left[\frac{n}{m^2}v_1\right]^2}{\left[\frac{a_1}{mn}\right]}$$

$$\frac{[v_2]^2}{[a_2]} = \frac{n^3}{m^3} \frac{[v_1]^2}{[a_1]}$$

$$\text{or } [L_2] = \frac{n^3}{m^3}[L_1]$$

Similarly

$$[T] = \frac{[v]}{[a]}$$

$$\text{So, } [T_2] = \frac{n^2}{m}[T_1]$$

2. A ball is spun with angular acceleration $\alpha = 6t^2 - 2t$, where t is in second and α is in rads^{-2} . At $t = 0$, the ball has angular velocity of 10 rads^{-1} and angular position of 4 rad. The most appropriate expression for the angular position of the ball is :

- (A) $\frac{3}{4}t^4 - t^2 + 10t$
- (B) $\frac{t^4}{2} - \frac{t^3}{3} + 10t + 4$
- (C) $\frac{2t^4}{3} - \frac{t^3}{6} + 10t + 12$
- (D) $2t^4 - \frac{t^3}{2} + 5t + 4$

Answer (B)

Sol. $\alpha = \frac{d\omega}{dt} = 6t^2 - 2t$

$$\int_0^\omega d\omega = \int_0^t (6t^2 - 2t)dt$$

$$\text{so } \omega = 2t^3 - t^2 + 10$$

$$\text{and } \frac{d\theta}{dt} = 2t^3 - t^2 + 10$$

$$\text{so } \int_4^\theta d\theta = \int_0^t (2t^3 - t^2 + 10)dt$$

$$\theta = \frac{t^4}{2} - \frac{t^3}{3} + 10t + 4$$

3. A block of mass 2 kg moving on a horizontal surface with speed of 4 ms^{-1} enters a rough surface ranging from $x = 0.5 \text{ m}$ to $x = 1.5 \text{ m}$. The retarding force in this range of rough surface is related to distance by $F = -kx$ where $k = 12 \text{ Nm}^{-1}$. The speed of the block as it just crosses the rough surface will be :

- (A) Zero
- (B) 1.5 ms^{-1}
- (C) 2.0 ms^{-1}
- (D) 2.5 ms^{-1}

Answer (C)

Sol. $F = -12x$

$$mv \frac{dv}{dx} = -12x$$

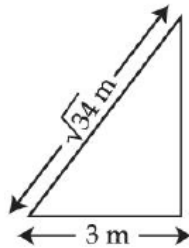
$$\int_4^v vdv = -6 \int_{0.5}^{1.5} xdx \quad (m = 2 \text{ kg})$$

$$\frac{v^2 - 16}{2} = -6 \left[\frac{1.5^2 - 0.5^2}{2} \right]$$

$$\frac{v^2 - 16}{2} = -6$$

$$v = 2 \text{ m/sec}$$

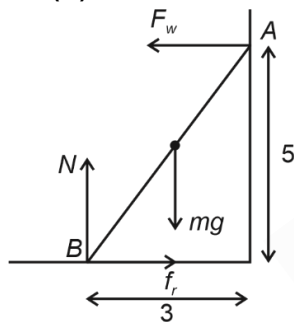
4. A $\sqrt{34}$ m long ladder weighing 10 kg leans on a frictionless wall. Its feet rest on the floor 3 m away from the wall as shown in the figure. If F_f and F_w are the reaction forces of the floor and the wall, then ratio of F_w/F_f will be:
(Use $g = 10 \text{ m/s}^2$)



- (A) $\frac{6}{\sqrt{110}}$ (B) $\frac{3}{\sqrt{113}}$
(C) $\frac{3}{\sqrt{109}}$ (D) $\frac{2}{\sqrt{109}}$

Answer (C)

Sol.



Taking torque from B

$$F_w \times 5 = \frac{3}{2} mg$$

$$\Rightarrow F_w = \frac{3}{10} \times 10 \times 10$$

$$= 30 \text{ N}$$

$$N = mg = 100 \text{ N}$$

$$\text{and } f_r = F_w = 30 \text{ N}$$

$$\text{so } F_f = \sqrt{N^2 + f_r^2} = \sqrt{10000 + 900} = 10\sqrt{109} \text{ N}$$

$$\text{so } \frac{F_w}{F_f} = \frac{3}{\sqrt{109}}$$

5. Water falls from a 40 m high dam at the rate of 9×10^4 kg per hour. Fifty percentage of gravitational potential energy can be converted into electrical energy. Using this hydro electric energy number of 100 W lamps, that can be lit, is :
(Take $g = 10 \text{ ms}^{-2}$)

- (A) 25 (B) 50
(C) 100 (D) 18

Answer (B)

Sol. Total gravitational PE of water per second = $\frac{mgh}{T}$

$$= \frac{9 \times 10^4 \times 10 \times 40}{3600} = 10^4 \text{ J/sec}$$

50% of this energy can be converted into electrical energy so total electrical energy = $\frac{10^4}{2} = 5000 \text{ W}$

$$\text{So total bulbs lit can be } = \frac{5000 \text{ W}}{100 \text{ W}} = 50 \text{ bulbs}$$

6. Two objects of equal masses placed at certain distance from each other attracts each other with a force of F . If one-third mass of one object is transferred to the other object, then the new force will be

- (A) $\frac{2}{9} F$ (B) $\frac{16}{9} F$
(C) $\frac{8}{9} F$ (D) F

Answer (C)



Let the masses are m and distance between them is l , then $F = \frac{Gm^2}{l^2}$.

When $1/3^{\text{rd}}$ mass is transferred to the other then masses will be $\frac{4m}{3}$ and $\frac{2m}{3}$. So new force will be

$$F' = \frac{G \frac{4m}{3} \times \frac{2m}{3}}{l^2} = \frac{8}{9} \frac{Gm^2}{l^2} = \frac{8}{9} F$$

7. A water drop of radius $1 \mu\text{m}$ falls in a situation where the effect of buoyant force is negligible. Co-efficient of viscosity of air is $1.8 \times 10^{-5} \text{ Nsm}^{-2}$ and its density is negligible as compared to that of water (10^6 gm^{-3}). Terminal velocity of the water drop is

(Take acceleration due to gravity = 10 ms^{-2})

- (A) $145.4 \times 10^{-6} \text{ ms}^{-1}$ (B) $118.0 \times 10^{-6} \text{ ms}^{-1}$
(C) $132.6 \times 10^{-6} \text{ ms}^{-1}$ (D) $123.4 \times 10^{-6} \text{ ms}^{-1}$

Answer (D)

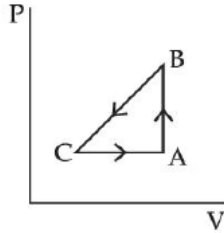
Sol. $6\pi\eta r v = mg$

$$6\pi\eta r v = \frac{4}{3}\pi r^3 \rho g$$

$$\text{or } v = \frac{2}{9} \frac{r^2 g}{\eta} = \frac{2}{9} \times \frac{10^3 \times (10^{-6})^2 \times 10}{1.8 \times 10^{-5}}$$

$$= 123.4 \times 10^{-6} \text{ m/s}$$

8. A sample of an ideal gas is taken through the cyclic process $ABCA$ as shown in figure. It absorbs, 40 J of heat during the part AB , no heat during BC and rejects 60 J of heat during CA . A work of 50 J is done on the gas during the part BC . The internal energy of the gas at A is 1560 J. The work done by the gas during the part CA is:



- (A) 20 J (B) 30 J
(C) -30 J (D) -60 J

Answer (B)

Sol. $\Delta U_{AB} = 40$ J as process is isochoric.

$$\Delta U_{BC} + W_{BC} = 0$$

$$\Delta U_{BC} = +50 \quad (W_{BC} = -50 \text{ J})$$

$$U_C = U_A + \Delta U_{AB} + \Delta U_{BC} = 1650$$

For CA process,

$$Q_{CA} = -60 \text{ J}$$

$$\Delta U_{CA} + W_{CA} = -60$$

$$-90 + W_{CA} = -60$$

$$\Rightarrow W_{CA} = +30 \text{ J}$$

The graph given is inconsistent with the statement BC may be adiabatic and CA cannot be like isobaric as shown, as increasing volume while rejecting heat at same time.

9. What will be the effect on the root mean square velocity of oxygen molecules if the temperature is doubled and oxygen molecule dissociates into atomic oxygen?
- (A) The velocity of atomic oxygen remains same
(B) The velocity of atomic oxygen doubles
(C) The velocity of atomic oxygen becomes half
(D) The velocity of atomic oxygen becomes four times

Answer (B)

Sol. As $v_{rms} = \sqrt{\frac{3RT}{M_0}}$

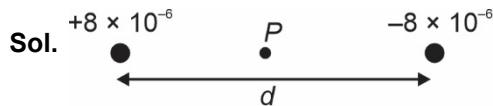
T is doubled and oxygen molecule is dissociated into atomic oxygen molar mass is halved.

$$\text{So, } v'_{rms} = \sqrt{\frac{3R \times 2T_0}{M_0/2}} = 2v_{rms}$$

So velocity of atomic oxygen is doubled.

10. Two point charges A and B of magnitude $+8 \times 10^{-6}$ C and -8×10^{-6} C respectively are placed at a distance d apart. The electric field at the middle point O between the charges is $6.4 \times 10^4 \text{ NC}^{-1}$. The distance ' d ' between the point charges A and B is:
- (A) 2.0 m (B) 3.0 m
(C) 1.0 m (D) 4.0 m

Answer (B)



Electric field at P will be

$$E = \frac{kq}{(d/2)^2} \times 2 = \frac{8kq}{d^2}$$

$$\text{So, } \frac{8 \times 9 \times 10^9 \times 8 \times 10^{-6}}{d^2} = 6.4 \times 10^4$$

$$\text{So, } d = 3 \text{ m}$$

11. Resistance of the wire is measured as 2Ω and 3Ω at 10°C and 30°C respectively. Temperature co-efficient of resistance of the material of the wire is:
- (A) 0.033°C^{-1} (B) $-0.033^\circ\text{C}^{-1}$
(C) 0.011°C^{-1} (D) 0.055°C^{-1}

Answer (A)

Sol. $R_{10} = 2 = R_0(1 + \alpha \times 10)$

$$R_{30} = 3 = R_0(1 + \alpha \times 30)$$

On solving

$$\alpha = 0.033^\circ\text{C}$$

12. The space inside a straight current carrying solenoid is filled with a magnetic material having magnetic susceptibility equal to 1.2×10^{-5} . What is fractional increase in the magnetic field inside solenoid with respect to air as medium inside the solenoid?
- (A) 1.2×10^{-5} (B) 1.2×10^{-3}
(C) 1.8×10^{-3} (D) 2.4×10^{-5}

Answer (A)

Sol. $\vec{B}' = \mu_0(1 + X)ni$ in the material

$$\vec{B} = \mu_0 ni$$
 without material

So fractional increase is

$$\frac{B' - B}{B} = X = 1.2 \times 10^{-5}$$

13. Two parallel, long wires are kept 0.20 m apart in vacuum, each carrying current of x A in the same direction. If the force of attraction per meter of each wire is 2×10^{-6} N, then the value of x is approximately:

- (A) 1 (B) 2.4
(C) 1.4 (D) 2

Answer (C)

Sol. $\frac{dF}{dl} = 2 \times 10^{-6} \text{ N/m} = \frac{\mu_0 i_1 i_2}{2\pi d}$

$$2 \times 10^{-6} = \frac{2 \times 10^{-7} \times x^2}{0.2}$$

$$x = \sqrt{2} = 1.4$$

14. A coil is placed in a time varying magnetic field. If the number of turns in the coil were to be halved and the radius of wire doubled, the electrical power dissipated due to the current induced in the coil would be:

- (Assume the coil to be short circuited.)
(A) Halved (B) Quadrupled
(C) The same (D) Doubled

Answer (D)

Sol. As number of turns are halved so length of wire is halved, and radius is doubled, then area will be 4 times the previous one if previous resistance is R then new resistance is $\frac{R}{8}$ and if previous emf is E

then new emf will be $\frac{E}{2}$ so

$$P_i = \frac{E^2}{R}$$

$$P_f = \frac{(E/2)^2}{R/8} = \frac{2E^2}{R} = 2P_i$$

As the answer key is changing students can challenge this question.

15. An EM wave propagating in x -direction has a wavelength of 8 mm. The electric field vibrating y -direction has maximum magnitude of 60 Vm^{-1} . Choose the correct equations for electric and magnetic field if the EM wave is propagating in vacuum:

(A) $E_y = 60 \sin \left[\frac{\pi}{4} \times 10^3 (x - 3 \times 10^8 t) \right] \hat{j} \text{ Vm}^{-1}$

$$B_z = 2 \sin \left[\frac{\pi}{4} \times 10^3 (x - 3 \times 10^8 t) \right] \hat{k} \text{ T}$$

(B) $E_y = 60 \sin \left[\frac{\pi}{4} \times 10^3 (x - 3 \times 10^8 t) \right] \hat{j} \text{ Vm}^{-1}$

$$B_z = 2 \times 10^{-7} \sin \left[\frac{\pi}{4} \times 10^3 (x - 3 \times 10^8 t) \right] \hat{k} \text{ T}$$

(C) $E_y = 2 \times 10^{-7} \sin \left[\frac{\pi}{4} \times 10^3 (x - 3 \times 10^8 t) \right] \hat{j} \text{ Vm}^{-1}$

$$B_z = 60 \sin \left[\frac{\pi}{4} \times 10^3 (x - 3 \times 10^8 t) \right] \hat{k} \text{ T}$$

(D) $E_y = 2 \times 10^{-7} \sin \left[\frac{\pi}{4} \times 10^4 (x - 4 \times 10^8 t) \right] \hat{j} \text{ Vm}^{-1}$

$$B_z = 60 \sin \left[\frac{\pi}{4} \times 10^4 (x - 4 \times 10^8 t) \right] \hat{k} \text{ T}$$

Answer (B)

Sol. In first 3 options speed of light is 3×10^8 m/sec and in the fourth option it is 4×10^8 m/sec.

Using

$$E = cB$$

We can check the option is B.

16. In Young's double slit experiment performed using a monochromatic light of wavelength λ , when a glass plate ($\mu = 1.5$) of thickness $x\lambda$ is introduced in the path of the one of the interfering beams, the intensity at the position where the central maximum occurred previously remains unchanged. The value of x will be:

- (A) 3 (B) 2
(C) 1.5 (D) 0.5

Answer (B)

Sol. For the intensity to remain same the position must be of a maxima so path difference must be $n\lambda$ so

$$(1.5 - 1)x\lambda = n\lambda$$

$$x = 2n \quad (n = 0, 1, 2 \dots)$$

So, value of x will be

$$x = 0, 2, 4, 6 \dots$$

17. Let K_1 and K_2 be the maximum kinetic energies of photo-electrons emitted when two monochromatic beams of wavelength λ_1 and λ_2 , respectively are incident on a metallic surface. If $\lambda_1 = 3\lambda_2$ then:

(A) $K_1 > \frac{K_2}{3}$ (B) $K_1 < \frac{K_2}{3}$

(C) $K_1 = \frac{K_2}{3}$ (D) $K_2 = \frac{K_1}{3}$

Answer (B)

Sol. $K_1 = \frac{hc}{\lambda_1} - \phi = \frac{hc}{3\lambda_2} - \phi \dots (i)$

and $K_2 = \frac{hc}{\lambda_2} - \phi \dots (ii)$

from (i) and (ii) we can say

$$3K_1 = K_2 - 2\phi$$

$$K_1 < \frac{K_2}{3}$$

18. Following statements related to radioactivity are given below:

- (A) Radioactivity is a random and spontaneous process and is dependent on physical and chemical conditions.
- (B) The number of un-decayed nuclei in the radioactive sample decays exponentially with time.
- (C) Slope of the graph of \log_e (no. of undecayed nuclei) Vs. time represents the reciprocal of mean life time (τ).
- (D) Product of decay constant (λ) and half-life time ($T_{1/2}$) is not constant.

Choose the most appropriate answer from the options given below:

- (A) (A) and (B) only (B) (B) and (D) only
- (C) (B) and (C) only (D) (C) and (D) only

Answer (C)

Sol. Radioactive decay is a random and spontaneous process it depends on unbalancing of nucleus.

$$N = N_0 e^{-\lambda t} \quad \dots(B)$$

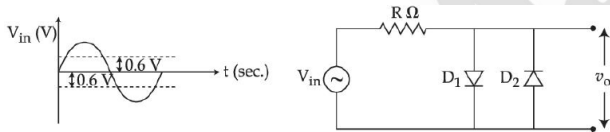
$$\ln N = -\lambda t + \ln N_0$$

$$\text{So, slope} = -\lambda \quad \dots(C)$$

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

$$\text{So } t_{1/2} \times \lambda = \ln 2 = \text{Constant}$$

19. In the given circuit the input voltage V_{in} is shown in figure. The cut-in voltage of $p-n$ junction diode (D_1 or D_2) is 0.6 V. Which of the following output voltage (V_o) waveform across the diode is correct?



- (A)
- (B)
- (C)
- (D)

Answer (D)

$$\text{Sol. Till } |V| \leq 0.6V \quad |V_o| = |V|$$

So correct graph will be D.

20. Amplitude modulated wave is represented by

$V_{AM} = 10[1 + 0.4 \cos(2\pi \times 10^4 t)] \cos(2\pi \times 10^7 t)$. The total bandwidth of the amplitude modulated wave is:

- (A) 10 kHz (B) 20 MHz
- (C) 20 kHz (D) 10 MHz

Answer (C)

$$\begin{aligned} \text{Sol. Bandwidth} &= 2 \times f_m \\ &= 2 \times 10^4 \text{ Hz} = 20 \text{ kHz} \end{aligned}$$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

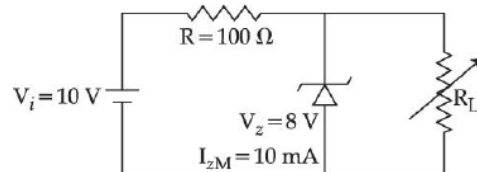
1. A student in the laboratory measures thickness of a wire using screw gauge. The readings are 1.22 mm, 1.23 mm, 1.19 mm and 1.20 mm. The percentage error is $\frac{x}{121}\%$. The value of x is _____.

Answer (150)

$$\begin{aligned} \text{Sol. } I_{\text{mean}} &= \frac{1.22 + 1.23 + 1.19 + 1.20}{4} = 1.21 \\ \Delta I_{\text{mean}} &= \frac{0.01 + 0.02 + 0.02 + 0.01}{4} = 0.015 \\ \text{So } \% I &= \frac{\Delta I_{\text{mean}}}{I_{\text{mean}}} \times 100 = \frac{0.015}{1.21} \times 100 \\ &= \frac{150}{121}\% \end{aligned}$$

$$x = 150$$

2. A zener of breakdown voltage $V_z = 8 \text{ V}$ and maximum Zener current, $I_{zM} = 10 \text{ mA}$ is subjected to an input voltage $V_i = 10 \text{ V}$ with series resistance $R = 100 \Omega$. In the given circuit R_L represents the variable load resistance. The ratio of maximum and minimum value of R_L is _____.



Answer (2)

Sol. Minimum value of R_L for which the diode is shorted

$$\text{is } \frac{R_L}{R_L + 100} \times 10 = 8 \Rightarrow R_L = 400 \Omega$$

For maximum value of R_L , current through diode is 10 mA

$$\text{So } i_R = i_{R_L} + I_{ZM}$$

$$\frac{2}{100} = \frac{8}{R_L} + 10 \times 10^{-3}$$

$$10 \times 10^{-3} = \frac{8}{R_L}$$

$$R_L = 800 \Omega$$

$$\text{So } \frac{R_{L\max}}{R_{L\min}} = 2$$

3. In a Young's double slit experiment, an angular width of the fringe is 0.35° on a screen placed at 2 m away for particular wavelength of 450 nm. The angular width of the fringe, when whole system is immersed in a medium of refractive index $7/5$, is $\frac{1}{\alpha}$. The value of α is _____.

Answer (4)

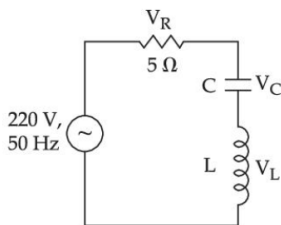
Sol. Angular fringe width $\theta = \frac{\lambda}{D}$

$$\text{So } \frac{\theta_1}{\lambda_1} = \frac{\theta_2}{\lambda_2}$$

$$\theta_2 = \frac{0.35^\circ}{450 \text{ nm}} \times \frac{450 \text{ nm}}{715} = 0.25^\circ = \frac{1}{4}$$

$$\text{So } \alpha = 4$$

4. In the given circuit, the magnitude of V_L and V_C are twice that of V_R . Given that $f = 50$ Hz, the inductance of the coil is $\frac{1}{K\pi}$ mH. The value of K is _____.



Answer (0)

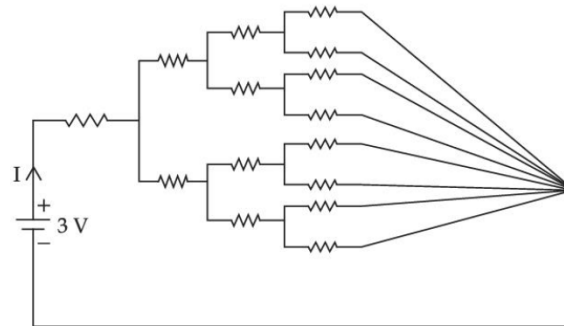
Sol. $V_L = 2V_R$

$$\text{So } \omega L i = 2 R i$$

$$\Rightarrow L = \frac{2R}{\omega} = \frac{2 \times 5}{2\pi \times 50} = \frac{1}{10\pi} \text{ H} = \frac{100}{\pi}$$

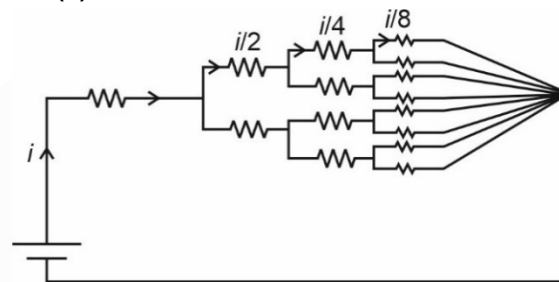
$$\text{So } k = \frac{1}{100} \approx 0$$

5. All resistances in figure are 1Ω each. The value of current ' I ' is $\frac{a}{5}$ A. The value of a is _____.



Answer (8)

Sol.



Let the current is i

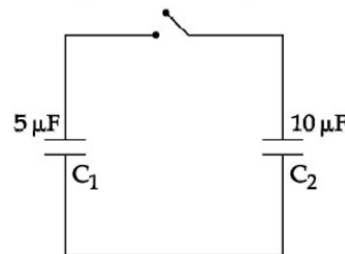
Using kirchhoff's law

$$iR + \frac{i}{2}R + \frac{i}{4}R + \frac{i}{8}R = 3$$

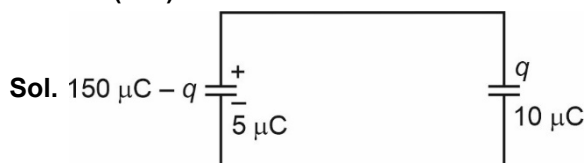
$$i = \frac{3 \times 8}{15} = \frac{8}{5} \text{ A}$$

$$\text{So } a = 8$$

6. A capacitor C_1 of capacitance $5 \mu\text{F}$ is charged to a potential of 30 V using a battery. The battery is then removed and the charged capacitor is connected to an uncharged capacitor C_2 of capacitance $10 \mu\text{F}$ as shown in figure. When the switch is closed charge flows between the capacitors. At equilibrium, the charge on the capacitor C_2 is _____ μC .



Answer (100)



Let the charge q is flown in the circuit.

So using Kirchoff's law

$$\frac{q}{10} = \frac{150 - q}{5}$$

$$q = 100 \mu\text{C}$$

7. A tuning fork of frequency 340 Hz resonates in the fundamental mode with an air column of length 125 cm in a cylindrical tube closed at one end. When water is slowly poured in it, the minimum height of water required for observing resonance once again is ____ cm.

(Velocity of sound in air is 340 ms^{-1})

Answer (50)

Sol. Given $340 = \frac{n}{4 \times 125} v$

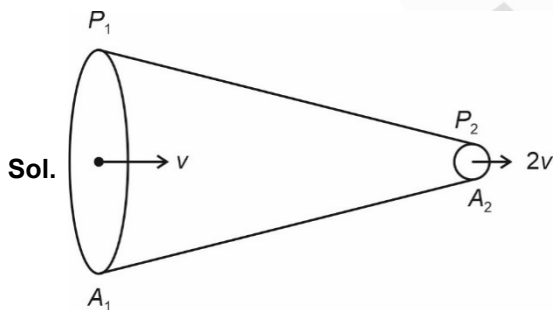
$$\Rightarrow n = 5$$

$$\text{So } \lambda = 100 \text{ cm}$$

$$\text{So minimum height is } \frac{\lambda}{2} = 50 \text{ cm}$$

8. A liquid of density 750 kgm^{-3} flows smoothly through a horizontal pipe that tapers in cross-sectional area from $A_1 = 1.2 \times 10^{-2} \text{ m}^2$ to $A_2 = \frac{A_1}{2}$. The pressure difference between the wide and narrow sections of the pipe is 4500 Pa. The rate of flow of liquid is ____ $\times 10^{-3} \text{ m}^3\text{s}^{-1}$.

Answer (24)



Using Bernoulli's equation

$$P_1 + \frac{1}{2} \rho v^2 = P_2 + \frac{1}{2} \rho 4v^2$$

$$\frac{3}{2} \rho v^2 = P_1 - P_2$$

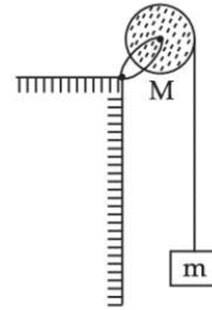
$$\Rightarrow v = \sqrt{\frac{2(P_1 - P_2)}{3\rho}}$$

$$= \sqrt{\frac{2 \times 4500}{3 \times 750}} = 2 \text{ m/sec}$$

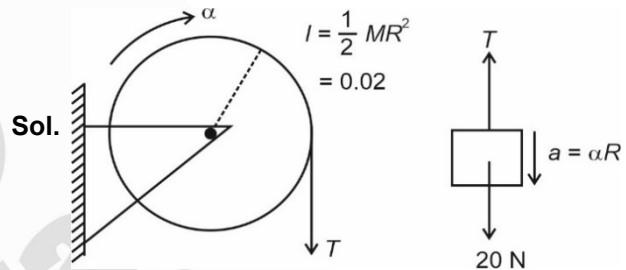
$$\text{So } Q = A_1 v = 24 \times 10^{-3} \text{ m}^3/\text{sec}$$

9. A uniform disc with mass $M = 4 \text{ kg}$ and radius $R = 10 \text{ cm}$ is mounted on a fixed horizontal axle as shown in figure. A block with mass $m = 2 \text{ kg}$ hangs from a massless cord that is wrapped around the rim of the disc. During the fall of the block, the cord does not slip and there is no friction at the axle. The tension in the cord is ____ N.

(Take $g = 10 \text{ ms}^{-2}$)



Answer (10)



Sol.

$$I = \frac{1}{2} MR^2 = 0.02$$

$$20 - T = 2a$$

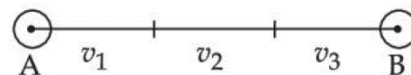
$$\text{and } 0.1 \times T = 0.02 \alpha = \frac{0.02 a}{0.1}$$

$$T = 2a$$

$$\Rightarrow a = 5 \text{ m/sec}^2$$

$$\text{So } T = 10 \text{ N}$$

10. A car covers AB distance with first one-third at velocity $v_1 \text{ ms}^{-1}$, second one-third at $v_2 \text{ ms}^{-1}$ and last one-third at $v_3 \text{ ms}^{-1}$. If $v_3 = 3v_1$, $v_2 = 2v_1$ and $v_1 = 11 \text{ ms}^{-1}$ then the average velocity of the car is ____ ms^{-1} .



Answer (18)

Sol.
$$v_{\text{mean}} = \frac{3v_1 v_2 v_3}{v_1 v_2 + v_2 v_3 + v_3 v_1}$$

$$= \frac{3 \times 11 \times 22 \times 33}{11 \times 22 + 22 \times 33 + 33 \times 11}$$

$$= 18 \text{ m/sec}$$

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Compound A contains 8.7% Hydrogen, 74% Carbon and 17.3% Nitrogen. The molecular formula of the compound is,

Given : Atomic masses of C, H and N are 12, 1 and 14 amu respectively.

The molar mass of the compound A is 162 g mol^{-1} .

- (A) $\text{C}_4\text{H}_6\text{N}_2$ (B) $\text{C}_2\text{H}_3\text{N}$
 (C) $\text{C}_5\text{H}_7\text{N}$ (D) $\text{C}_{10}\text{H}_{14}\text{N}_2$

Answer (D)

Sol.

Element	%mass	Moles	Whole number ratio
C	74	6.17	5
H	8.7	8.7	7
N	17.3	1.236	1

Empirical Formula = $\text{C}_5\text{H}_7\text{N}$

Empirical formula mass = 81 g

$$n \times 81 = 162$$

$$n = 2$$

Hence molecular formula is $\text{C}_{10}\text{H}_{14}\text{N}_2$

2. Consider the following statements :

- (A) The principal quantum number 'n' is a positive integer with values of 'n' = 1, 2, 3,
 (B) The azimuthal quantum number 'l' for a given 'n' (principal quantum number) can have values as 'l' = 0, 1, 2, ...n

(C) Magnetic orbital quantum number 'm_l' for a particular 'l' (azimuthal quantum number) has $(2l + 1)$ values.

(D) $\pm 1/2$ are the two possible orientations of electron spin.

(E) For $l = 5$, there will be a total of 9 orbital

Which of the above statements are correct?

- (A) (A), (B) and (C)
 (B) (A), (C), (D) and (E)
 (C) (A), (C) and (D)
 (D) (A), (B), (C) and (D)

Answer (C)

Sol. Possible values of l for a given 'n' = 0, 1, 2 ... (n - 1)

For $l = 5$, total orbitals = $2l + 1$

$$= 2(5) + 1 = 11 \text{ orbital}$$

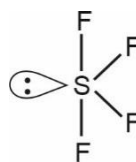
Hence A, C and D are correct statements

3. In the structure of SF_4 , the lone pair of electrons on S is in.

- (A) Equatorial position and there are two lone pair - bond pair repulsions at 90°
 (B) Equatorial position and there are three lone pair - bond pair repulsions at 90°
 (C) Axial position and there are three lone pair - bond pair repulsion at 90°
 (D) Axial position and there are two lone pair - bond pair repulsion at 90°

Answer (A)

Sol. $\text{SF}_4 \rightarrow sp^3d$ hybridisation.



The lone pair of electrons on S is in equatorial position and there are two lone pair-bond pair repulsions at 90° .

4. A student needs to prepare a buffer solution of propanoic acid and its sodium salt with pH 4.

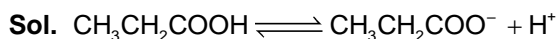
The ratio of $\frac{[\text{CH}_3\text{CH}_2\text{COO}^-]}{[\text{CH}_3\text{CH}_2\text{COOH}]}$ required to make

buffer is _____.

Given : $K_a(\text{CH}_3\text{CH}_2\text{COOH}) = 1.3 \times 10^{-5}$

- (A) 0.03 (B) 0.13
(C) 0.23 (D) 0.33

Answer (B)



From Henderson equation

$$\text{pH} = \text{p}K_a + \log \frac{[\text{CH}_3\text{CH}_2\text{COO}^-]}{[\text{CH}_3\text{CH}_2\text{COOH}]}$$

$$4 = -\log 1.3 \times 10^{-5} + \log \frac{[\text{CH}_3\text{CH}_2\text{COO}^-]}{[\text{CH}_3\text{CH}_2\text{COOH}]}$$

$$-\log 10^{-4} = -\log 1.3 \times 10^{-5} + \log \frac{[\text{CH}_3\text{CH}_2\text{COO}^-]}{[\text{CH}_3\text{CH}_2\text{COOH}]}$$

$$-\log 10^{-4} = -\log 1.3 \times 10^{-5} \frac{[\text{CH}_3\text{CH}_2\text{COOH}]}{[\text{CH}_3\text{CH}_2\text{COO}^-]}$$

$$10^{-4} = 1.3 \times 10^{-5} \frac{[\text{CH}_3\text{CH}_2\text{COOH}]}{[\text{CH}_3\text{CH}_2\text{COO}^-]}$$

$$\frac{[\text{CH}_3\text{CH}_2\text{COO}^-]}{[\text{CH}_3\text{CH}_2\text{COOH}]} = 0.13$$

5. Match List-I with List-II :

List-I	List-II
(A) Negatively charged sol	(I) $\text{Fe}_2\text{O}_3 \cdot x\text{H}_2\text{O}$
(B) Macromolecular colloid	(II) CdS sol
(C) Positively charged sol	(III) Starch
(D) Cheese	(IV) a gel

Choose the **correct** answer from the options given below:

- (A) (A) – (II), (B) – (III), (C) – (IV), (D) – (I)
(B) (A) – (II), (B) – (I), (C) – (III), (D) – (IV)
(C) (A) – (II), (B) – (III), (C) – (I), (D) – (IV)
(D) (A) – (I), (B) – (III), (C) – (II), (D) – (IV)

Answer (C)

- Sol.** (A) Negatively charged sol CdS sol
(B) Macromolecular colloid Starch
(C) Positively charged sol $\text{Fe}_2\text{O}_3 \cdot x\text{H}_2\text{O}$
(D) Cheese A gel

6. Match List-I with List-II:

List-I(Oxide)	List-II (Nature)
(A) Cl_2O_7	(I) Amphoteric
(B) Na_2O	(II) Basic
(C) Al_2O_3	(III) Neutral
(D) N_2O	(IV) Acidic

Choose the correct answer from the options given below:

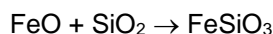
- (1) A-IV, B-III, C-I, D-II (2) A-IV, B-II, C-I, D-III
(3) A-II, B-IV, C-III, D-I (4) A-I, B-II, C-III, D-IV

Answer (B)

- Sol.** (A) $\text{Cl}_2\text{O}_7 \rightarrow$ Acidic
(B) $\text{Na}_2\text{O} \rightarrow$ Basic
(C) $\text{Al}_2\text{O}_3 \rightarrow$ Amphoteric
(D) $\text{N}_2\text{O} \rightarrow$ Neutral

Oxides of metals are basic in nature whereas oxides of non metals are acidic in nature. N_2O is a neutral oxide.

7. In the metallurgical extraction of copper, following reaction is used :



FeO and FeSiO_3 respectively are.

- (1) Gangue and flux (2) Flux and slag
(3) Slag and flux (4) Gangue and slag

Answer (D)



Gangue Slag

8. Hydrogen has three isotopes: protium (^1H), deuterium (^2H or D) and tritium (^3H or T). They have nearly same chemical properties but different physical properties. They differ in

- (1) Number of protons
(2) Atomic number
(3) Electronic configuration
(4) Atomic mass

Answer (D)

Sol. ^1H , ^2D and ^3H have same atomic number but their atomic masses are different.

Isotopes have same atomic number i.e. same number of protons

9. Among the following, basic oxide is:

- (1) SO_3 (2) SiO_2
 (3) CaO (4) Al_2O_3

Answer (C)

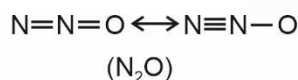
Sol. Since, oxides of metals are basic in nature. Hence CaO is a basic oxide

SO_3 and SiO_2 are acidic oxides and Al_2O_3 is a amphoteric oxide

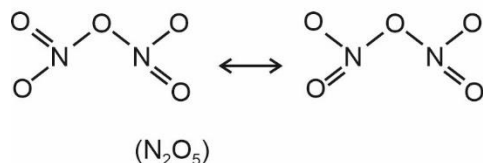
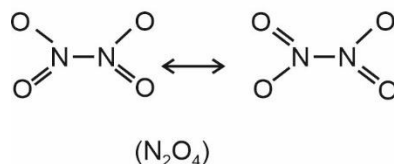
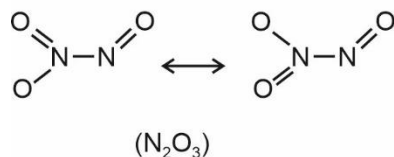
10. Among the given oxides of nitrogen ; N_2O , N_2O_3 , N_2O_4 and N_2O_5 , the number of compound/(s) having N – N bond is:

- (1) 1 (2) 2
 (3) 3 (4) 4

Answer (C)



Sol.



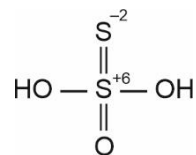
N_2O , N_2O_3 and N_2O_4 contain N – N bond

11. Which of the following oxoacids of sulphur contains "S" in two different oxidation states?

- (A) $\text{H}_2\text{S}_2\text{O}_3$ (B) $\text{H}_2\text{S}_2\text{O}_6$
 (C) $\text{H}_2\text{S}_2\text{O}_7$ (D) $\text{H}_2\text{S}_2\text{O}_8$

Answer (A)

Sol. In $\text{H}_2\text{S}_2\text{O}_3$, sulphur exhibits two different oxidation states +6 and –2.



12. Correct statement about photo-chemical smog is:

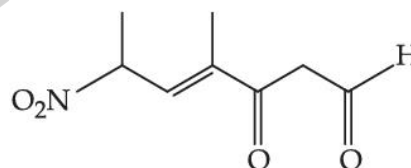
- (A) It occurs in humid climate.
 (B) It is a mixture of smoke, fog and SO_2 .
 (C) It is reducing smog.
 (D) It results from reaction of unsaturated hydrocarbons.

Answer (D)

Sol. Photochemical smog occurs in warm, dry and sunny climate. The main components of photochemical smog result from the action of unsaturated hydrocarbons and nitrogen oxides.

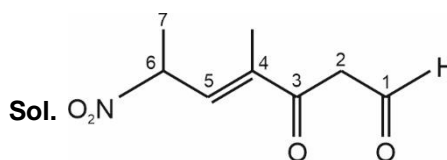
This is an oxidising smog.

13. The correct IUPAC name of the following compound is:



- (A) 4-methyl-2-nitro-5-oxohept-3-enal
 (B) 4-methyl-5-oxo-2-nitrohept-3-enal
 (C) 4-methyl-6-nitro-3-oxohept-4-enal
 (D) 6-formyl-4-methyl-2-nitrohex-3-enal

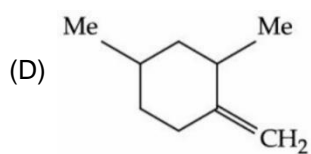
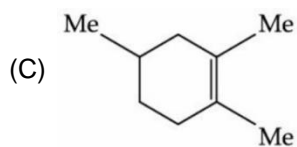
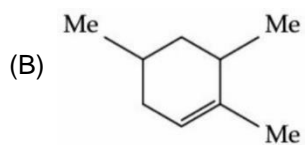
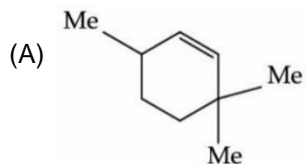
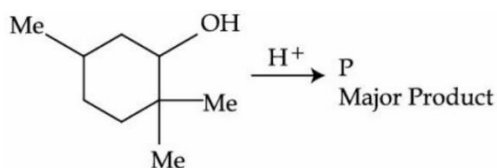
Answer (C)



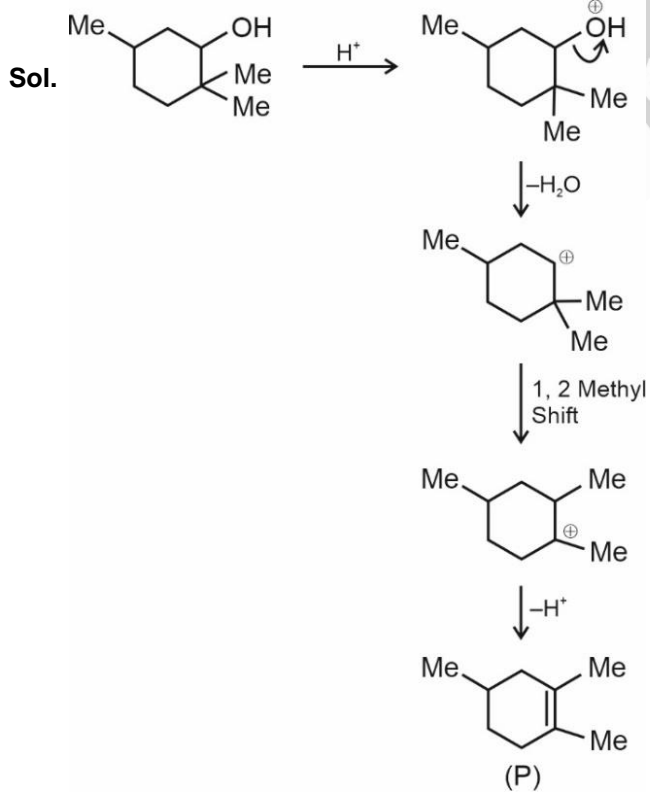
Sol.

4-methyl-6-nitro-3-oxohept-4-enal

14. The major product (P) of the given reaction is
(where, Me is $-\text{CH}_3$)

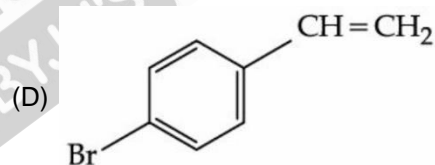
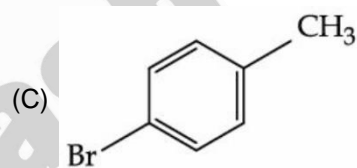
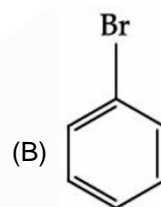
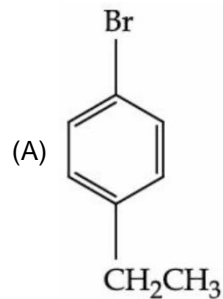


Answer (C)



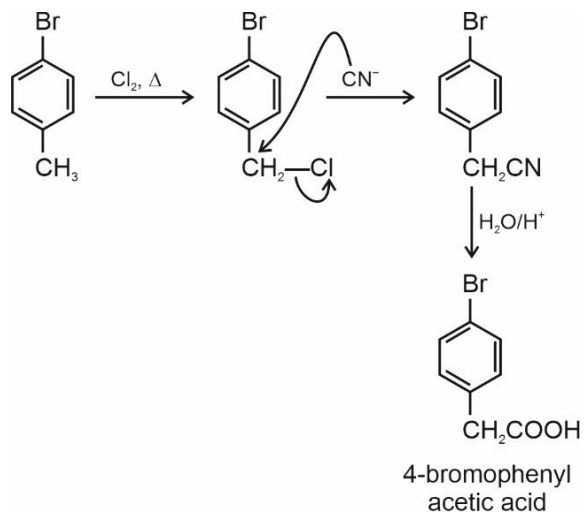
15. A $\xrightarrow[\text{(ii) } \text{CN}^-]{\text{(i) } \text{Cl}_2, \Delta}$ 4-Bromophenyl acetic acid.
(iii) $\text{H}_2\text{O}/\text{H}^+$

In the above reaction 'A' is

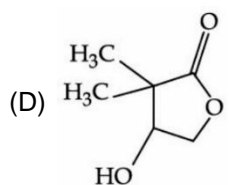
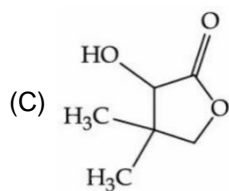
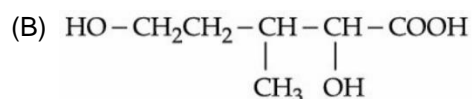
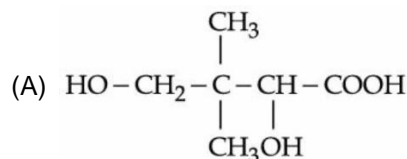


Answer (C)

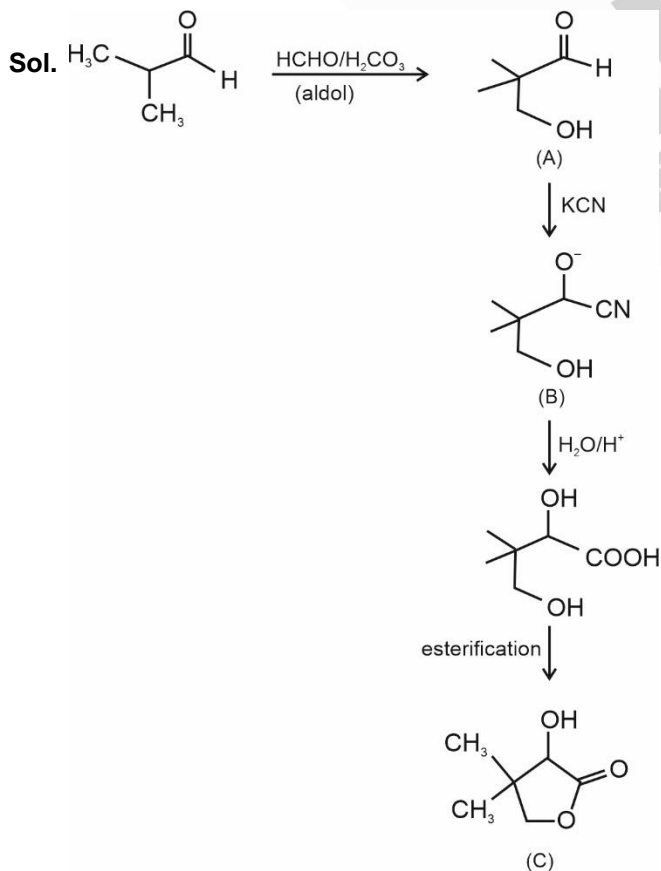
Sol.



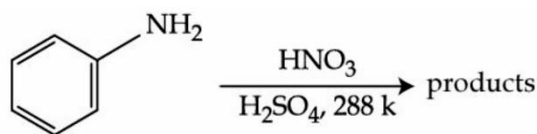
16. Isobutyraldehyde on reaction with formaldehyde and K_2CO_3 gives compound 'A'. Compound 'A' reacts with KCN and yields compound 'B', which on hydrolysis gives a stable compound 'C'. The compound 'C' is



Answer (C)



17. With respect to the following reaction, consider the given statements:

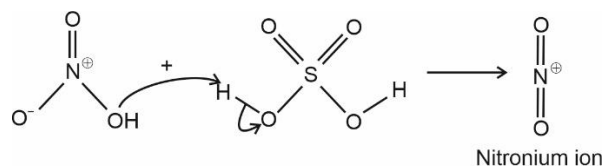
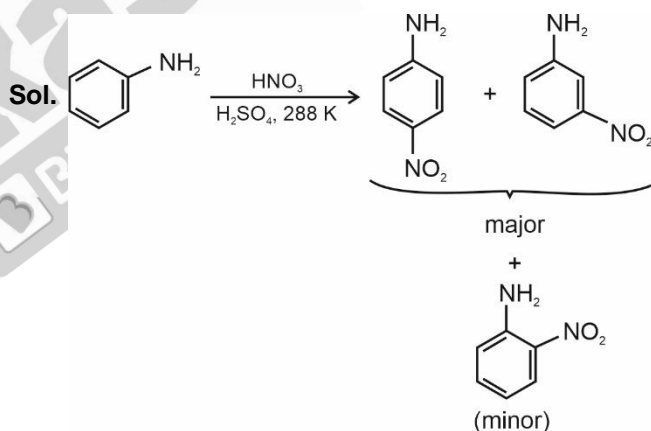


- (A) o-Nitroaniline and p-nitroaniline are the predominant products.
 (B) p-Nitroaniline and m-nitroaniline are the predominant products.
 (C) HNO_3 acts as an acid.
 (D) H_2SO_4 acts as an acid.

Choose the **correct** option.

- (A) (A) and (C) are correct statements.
 (B) (A) and (D) are correct statements.
 (C) (B) and (D) are correct statements.
 (D) (B) and (C) are correct statements.

Answer (C)



Hence H_2SO_4 acts as an acid

18. Given below are two statements, one is **Assertion (A)** and other is **Reason (R)**.

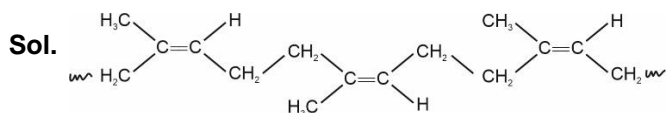
Assertion (A): Natural rubber is a linear polymer of isoprene called *cis*-polyisoprene with elastic properties.

Reason (R): The *cis*-polyisoprene molecules consist of various chains held together by strong polar interactions with coiled structure.

In the light of the above statements, choose the **correct** one from the options given below:

- (A) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**.
 (B) Both **(A)** and **(R)** are true but **(R)** is not the correct explanation of **(A)**.
 (C) **(A)** is true but **(R)** is false.
 (D) **(A)** is false but **(R)** is true.

Answer (C)



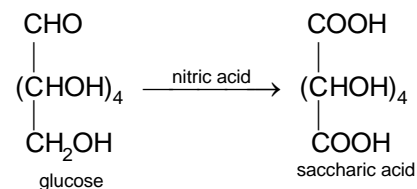
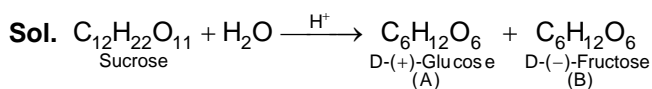
The *cis*-polyisoprene molecule consists of various chains held together by weak van der Waals interactions and has a coiled structure.

Hence assertion is true but reason is false.

19. When sugar 'X' is boiled with dilute H_2SO_4 in alcoholic solution, two isomers 'A' and 'B' are formed. 'A' on oxidation with HNO_3 yields saccharic acid whereas 'B' is laevorotatory. The compound 'X' is:

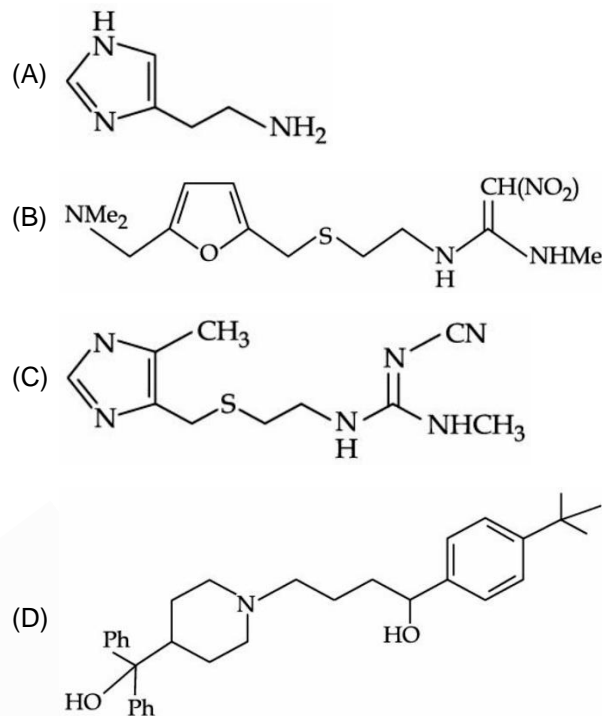
- (A) Maltose
 (B) Sucrose
 (C) Lactose
 (D) Starch

Answer (B)



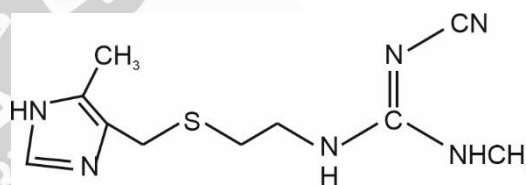
D-(-)-Fructose is a laevorotatory compound.

20. The drug tegamet is:



Answer (C)

Sol. Tegamet (cimetidine) is



SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. 100 g of an ideal gas is kept in a cylinder of 416 L volume at $27^\circ C$ under 1.5 bar pressure. The molar mass of the gas is _____ $g\ mol^{-1}$. (Nearest integer).
 (Given : $R = 0.083\ L\ bar\ K^{-1}\ mol^{-1}$)

Answer (4)

Sol. From combined gas law,

$$PV = nRT$$

$$PV = \frac{W}{M}RT$$

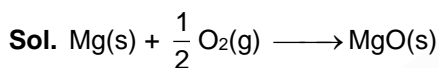
$$1.5 \times 416 = \frac{100}{M} \times 0.083 \times 300$$

$$M = 4 \text{ g/mol}$$

2. For combustion of one mole of magnesium in an open container at 300 K and 1 bar pressure, $\Delta_c H^\ominus = -601.70 \text{ kJ mol}^{-1}$, the magnitude of change in internal energy for the reaction is _____ kJ. (Nearest integer)

(Given : $R = 8.3 \text{ J K}^{-1} \text{ mol}^{-1}$)

Answer (600)



$$\Delta H = \Delta U + \Delta n_g RT$$

$$\Delta n_g = -\frac{1}{2}$$

$$-601.70 = \Delta U - \frac{1}{2} (8.3) (300) \times 10^{-3}$$

$$\Delta U = -601.70 + 1.245$$

$$\Delta U \approx -600 \text{ kJ}$$

Magnitude of change in internal energy is 600 kJ.

3. 2.5 g of protein containing only glycine ($\text{C}_2\text{H}_5\text{NO}_2$) is dissolved in water to make 500 mL of solution. The osmotic pressure of this solution at 300 K is found to be 5.03×10^{-3} bar. The total number of glycine units present in the protein is _____.

(Given : $R = 0.083 \text{ L bar K}^{-1} \text{ mol}^{-1}$)

Answer (330)

Sol. Since,

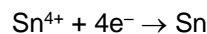
$$\pi = icRT$$

$$5.03 \times 10^{-3} = \frac{2.5}{M} \times \frac{1000}{500} \times 0.083 \times 300$$

$$\text{Molar mass of protein} = 24751.5 \text{ g/mol}$$

$$\begin{aligned} \text{Number of glycine units in protein} &= \frac{24751.5}{75} \\ &= 330 \end{aligned}$$

4. For the given reactions

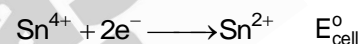
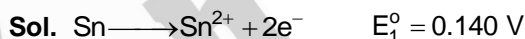


the electrode potentials are; $E_{\text{Sn}^{2+}/\text{Sn}}^\ominus = -0.140 \text{ V}$

and $E_{\text{Sn}^{4+}/\text{Sn}}^\ominus = 0.010 \text{ V}$. The magnitude of standard

electrode potential for $\text{Sn}^{4+} / \text{Sn}^{2+}$ i.e. $E_{\text{Sn}^{4+}/\text{Sn}^{2+}}^\ominus$ is _____ $\times 10^{-2} \text{ V}$. (Nearest integer)

Answer (16)



$$E_{\text{cell}}^\ominus = \frac{n_2 E_2^\ominus + n_1 E_1^\ominus}{n} = \frac{4(0.010) + 2(0.140)}{2}$$

$$E_{\text{cell}}^\ominus = 0.16 \text{ V} = 16 \times 10^{-2} \text{ V}$$

5. A radioactive element has a half life of 200 days. The percentage of original activity remaining after 83 days is _____ (Nearest integer)

(Given : $\text{antilog } 0.125 = 1.333$,

$\text{antilog } 0.693 = 4.93$)

Answer (75)

Sol. $\lambda = \frac{2.303}{t} \log \frac{A_0}{A}$

$$\frac{0.693}{200} = \frac{2.303}{83} \log \frac{A_0}{A}$$

$$\frac{A}{A_0} = 0.75$$

Hence, percentage of original activity remaining after 83 days is 75%

6. $[\text{Fe}(\text{CN})_6]^{4-}$
 $[\text{Fe}(\text{CN})_6]^{3-}$
 $[\text{Ti}(\text{CN})_6]^{3-}$
 $[\text{Ni}(\text{CN})_4]^{2-}$
 $[\text{Co}(\text{CN})_6]^{3-}$

Among the given complexes, number of paramagnetic complexes is _____.

Answer (2)

Sol.	Valence shell configuration	Magnetic nature
$[\text{Fe}(\text{CN})_6]^{4-}$	$3d^6$ (Pairing)	Diamagnetic
$[\text{Fe}(\text{CN})_6]^{3-}$	$3d^5$ (Pairing)	Paramagnetic
$[\text{Ti}(\text{CN})_6]^{3-}$	$3d^1$	Paramagnetic
$[\text{Ni}(\text{CN})_4]^{2-}$	$3d^8$ (Pairing)	Diamagnetic
$[\text{Co}(\text{CN})_6]^{3-}$	$3d^6$ (Pairing)	Diamagnetic

7. (a) $\text{CoCl}_3 \cdot 4 \text{NH}_3$, (b) $\text{CoCl}_3 \cdot 5 \text{NH}_3$, (c) $\text{CoCl}_3 \cdot 6 \text{NH}_3$ and (d) $\text{CoCl}(\text{NO}_3)_2 \cdot 5 \text{NH}_3$. Number of complex(es) which will exist in cis-trans form is/are _____.

Answer (1)

Sol. $\text{CoCl}_3 \cdot 4 \text{NH}_3 \Rightarrow [\text{Co}(\text{NH}_3)_4\text{Cl}_2]\text{Cl}$
 $\text{CoCl}_3 \cdot 5 \text{NH}_3 \Rightarrow [\text{Co}(\text{NH}_3)_5\text{Cl}]\text{Cl}_2$
 $\text{CoCl}_3 \cdot 6 \text{NH}_3 \Rightarrow [\text{Co}(\text{NH}_3)_6]\text{Cl}_3$
 Only $[\text{Co}(\text{NH}_3)_4\text{Cl}_2]$ can show geometrical isomerism. Hence can exist in cis-trans form.

8. The complete combustion of 0.492 g of an organic compound containing 'C', 'H' and 'O' gives 0.793 g of CO_2 and 0.442 g of H_2O . The percentage of oxygen composition in the organic compound is _____. [nearest integer]

Answer (46)

Sol. $\text{C}_x\text{H}_y\text{O}_z + \text{O}_2 \longrightarrow \text{CO}_2 + \text{H}_2\text{O}$

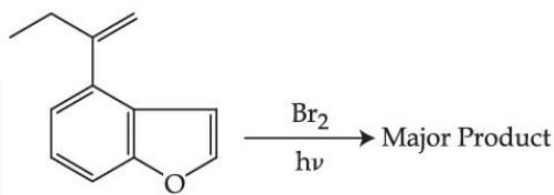
$$\text{weight of carbon} = \frac{0.793}{44} \times 12 = 0.216 \text{ g}$$

$$\text{weight of hydrogen} = \frac{0.442}{18} \times 2 = 0.05 \text{ g}$$

$$\begin{aligned} \text{weight of oxygen} &= 0.492 - (0.216 + 0.05) \\ &= 0.226 \text{ g} \end{aligned}$$

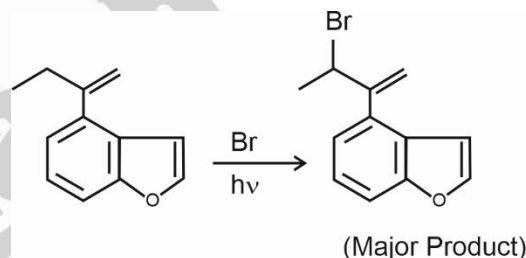
$$\begin{aligned} \% \text{ by mass of oxygen in compound} &= \frac{0.226}{0.492} \times 100 \\ &\approx 46\% \end{aligned}$$

9. The major product of the following reaction contains _____ bromine atom(s).



Answer (1)

Sol.



10. 0.01 M KMnO_4 solution was added to 20.0 mL of 0.05 M Mohr's salt solution through a burette. The initial reading of 50 mL burette is zero. The volume of KMnO_4 solution left in burette after the end point is _____ mL. [nearest integer]

Answer (30)

Sol. Meq of oxidising agent = Meq of reducing agent

$$(M \times V \times n_F)_{\text{KMnO}_4} = (M \times V \times n_F)_{\text{Mohr's salt}}$$

$$0.01 \times 20 \times 5 = 0.05 \times V \times 1$$

$$\text{Volume required} = 20 \text{ ml}$$

Since initial volume of KMnO_4 in burette is 50 mL. Hence volume of KMnO_4 left in the burette after end point is 30 mL.

$$= -{}^{11}C_7 \left(\frac{5}{2}\right)^4 \left(\frac{-1}{5}\right)^7 = \frac{11 \times 10 \times 9 \times 8}{24} \times \frac{1}{16 \times 125}$$

$$= \frac{33}{200}$$

5. If n arithmetic means are inserted between a and 100 such that the ratio of the first mean to the last mean is $1 : 7$ and $a + n = 33$, then the value of n is:

- (A) 21 (B) 22
(C) 23 (D) 24

Answer (C)

Sol. $a, A_1, A_2, \dots, A_n, 100$

Let d be the common difference of above A.P. then

$$\frac{a+d}{100-d} = \frac{1}{7}$$

$$\Rightarrow 7a + 8d = 100 \quad \dots(i)$$

$$\text{and } a + n = 33 \quad \dots(ii)$$

$$\text{and } 100 = a + (n+1)d$$

$$\Rightarrow 100 = a + (34-a) \frac{(100-7a)}{8}$$

$$\Rightarrow 800 = 8a + 7a^2 - 338a + 3400$$

$$\Rightarrow 7a^2 - 330a + 2600 = 0$$

$$\Rightarrow a = 10, \frac{260}{7}, \text{ but } a \neq \frac{260}{7}$$

$$\therefore n = 23$$

6. Let $f, g : R \rightarrow R$ be functions defined by

$$f(x) = \begin{cases} [x], & x < 0 \\ |1-x|, & x \geq 0 \end{cases} \text{ and } g(x) = \begin{cases} e^x - x, & x < 0 \\ (x-1)^2 - 1, & x \geq 0 \end{cases}$$

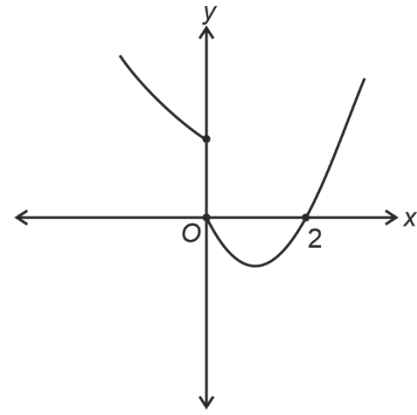
Where $[x]$ denote the greatest integer less than or equal to x . Then, the function fog is discontinuous at exactly :

- (A) one point (B) two points
(C) three points (D) four points

Answer (B)

Sol. $f(x) = \begin{cases} [x], & x < 0 \\ |1-x|, & x \geq 0 \end{cases}$ and $g(x) = \begin{cases} e^x - x, & x < 0 \\ (x-1)^2 - 1, & x \geq 0 \end{cases}$

$$fog(x) = \begin{cases} [g(x)], & g(x) < 0 \\ |1-g(x)|, & g(x) \geq 0 \end{cases}$$



(graph of $y = g(x)$)

$$= \begin{cases} |1+x-e^x|, & x < 0 \\ 1, & x = 0 \\ [(x-1)^2 - 1], & 0 < x < 2 \\ |2-(x-1)^2|, & x \geq 2 \end{cases}$$

So, $x = 0, 2$ are the two points where fog is discontinuous.

7. Let $f : R \rightarrow R$ be a differentiable function such that

$$f\left(\frac{\pi}{4}\right) = \sqrt{2}, f\left(\frac{\pi}{2}\right) = 0 \text{ and } f'\left(\frac{\pi}{2}\right) = 1 \text{ and let}$$

$$g(x) = \int_x^{\frac{\pi}{4}} (f'(t) \sec t + \tan t \sec f(t)) dt \text{ for}$$

$$x \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right]. \text{ Then } \lim_{x \rightarrow \left(\frac{\pi}{2}\right)^-} g(x) \text{ is equal to}$$

- (A) 2 (B) 3
(C) 4 (D) -3

Answer (B)

Sol. Given : $f\left(\frac{\pi}{4}\right) = \sqrt{2}, f\left(\frac{\pi}{2}\right) = 0$ and $f'\left(\frac{\pi}{2}\right) = 1$

$$g(x) = \int_x^{\frac{\pi}{4}} (f'(t) \sec t + \tan t \sec f(t)) dt$$

$$= [\sec t + f(t)]_x^{\frac{\pi}{4}} = 2 - \sec x f(x)$$

Now, $\lim_{x \rightarrow \frac{\pi}{2}^-} g(x) = \lim_{h \rightarrow 0} g\left(\frac{\pi}{2} - h\right)$

$$= \lim_{h \rightarrow 0} 2 - (\operatorname{cosec} h) f\left(\frac{\pi}{2} - h\right)$$

$$= \lim_{h \rightarrow 0} \left[2 - \frac{f\left(\frac{\pi}{2} - h\right)}{\sin h} \right]$$

$$= \lim_{h \rightarrow 0} \left[2 + \frac{f'\left(\frac{\pi}{2} - h\right)}{\cos h} \right]$$

$$= 3$$

8. Let $f: R \rightarrow R$ be a continuous function satisfying $f(x) + f(x+k) = n$, for all $x \in R$ where $k > 0$ and n is a positive integer. If $I_1 = \int_0^{4nk} f(x) dx$ and

$$I_2 = \int_{-k}^{3k} f(x) dx, \text{ then}$$

- (A) $I_1 + 2I_2 = 4nk$ (B) $I_1 + 2I_2 = 2nk$
 (C) $I_1 + nI_2 = 4n^2k$ (D) $I_1 + nI_2 = 6n^2k$

Answer (C)

Sol. $f: R \rightarrow R$ and $f(x) + f(x+k) = n \quad \forall x \in R$

$$x \rightarrow x+k$$

$$f(x+k) + f(x+2k) = n$$

$$\therefore f(x+2k) = f(x)$$

So, period of $f(x)$ is $2k$

$$\text{Now, } I_1 = \int_0^{4nk} f(x) dx = 2n \int_0^{2k} f(x) dx$$

$$= 2n \left[\int_0^k f(x) dx + \int_k^{2k} f(x) dx \right]$$

$$x = t+k \Rightarrow dx = dt \quad (\text{in second integral})$$

$$= 2n \left[\int_0^k f(x) dx + \int_0^k f(t+k) dt \right]$$

$$= 2n^2k$$

$$\text{Now, } I_2 = \int_{-k}^{3k} f(x) dx = 2 \int_0^{2k} f(x) dx$$

$$I_2 = 2(nk)$$

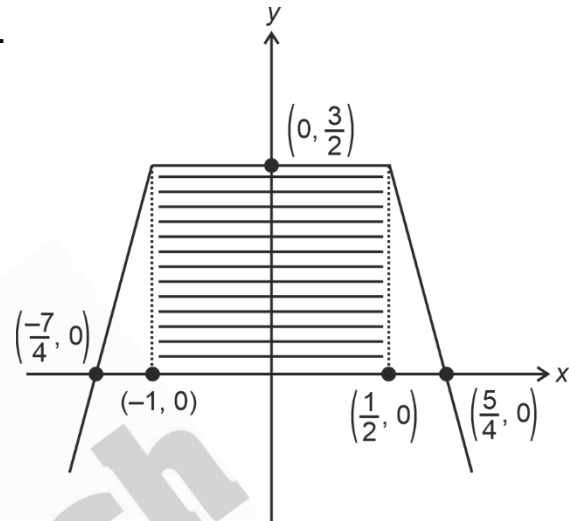
$$\therefore I_1 + nI_2 = 4n^2k$$

9. The area of the bounded region enclosed by the curve $y = 3 - \left| x - \frac{1}{2} \right| - |x+1|$ and the x-axis is

- (A) $\frac{9}{4}$ (B) $\frac{45}{16}$
 (C) $\frac{27}{8}$ (D) $\frac{63}{16}$

Answer (C)

Sol.



$$y = \begin{cases} 2x - \frac{7}{2} & x < -1 \\ \frac{3}{2} & -1 \leq x \leq \frac{1}{2} \\ \frac{5}{2} - 2x & x > \frac{1}{2} \end{cases}$$

$$y = 3 - \left| x - \frac{1}{2} \right| - |x+1|$$

Area of shaded region (required area)

$$= \frac{1}{2} \left(3 + \frac{3}{2} \right) \cdot \frac{3}{2} = \frac{27}{8}$$

10. Let $x = x(y)$ be the solution of the differential equation $2ye^{\frac{x}{y^2}} dx + \left(y^2 - 4xe^{\frac{x}{y^2}} \right) dy = 0$ such

that $x(1) = 0$. Then, $x(e)$ is equal to

- (A) $e \log_e(2)$ (B) $-e \log_e(2)$
 (C) $e^2 \log_e(2)$ (D) $-e^2 \log_e(2)$

Answer (D)

Sol. Given differential equation

$$2ye^{\frac{x}{y^2}} dx + \left(y^2 - 4xe^{\frac{x}{y^2}} \right) dy = 0, \quad x(1) = 0$$

$$\Rightarrow e^{\frac{x}{y^2}} [2y dx - 4xy dy] = -y^2 dy$$

$$\Rightarrow e^{\frac{x}{y^2}} \left[\frac{2y^2 dx - 4xy dy}{y^4} \right] = \frac{-1}{y} dy$$

$$\Rightarrow 2e^{\frac{x}{y^2}} d\left(\frac{x}{y^2}\right) = -\frac{1}{y} dy$$

$$\Rightarrow 2e^{\frac{x}{y^2}} = -\ln y + c \quad \dots(i)$$

Now, using $x(1) = 0$, $c = 2$

So, for $x(e)$, Put $y = e$ in (i)

$$2e^{\frac{x}{e^2}} = -1 + 2$$

$$\Rightarrow \frac{x}{e^2} = \ln\left(\frac{1}{2}\right) \Rightarrow x(e) = -e^2 \ln 2$$

11. Let the slope of the tangent to a curve $y = f(x)$ at (x, y) be given by $2 \tan x(\cos x - y)$. If the curve passes through the point $\left(\frac{\pi}{4}, 0\right)$, then the value of

$\int_0^{\pi/2} y dx$ is equal to :

- (A) $(2 - \sqrt{2}) + \frac{\pi}{\sqrt{2}}$ (B) $2 - \frac{\pi}{\sqrt{2}}$
 (C) $(2 + \sqrt{2}) + \frac{\pi}{\sqrt{2}}$ (D) $2 + \frac{\pi}{\sqrt{2}}$

Answer (B)

Sol. $\frac{dy}{dx} = 2 \tan x(\cos x - y)$

$$\Rightarrow \frac{dy}{dx} + 2 \tan xy = 2 \sin x$$

$$\text{I.F.} = e^{\int 2 \tan x dx} = \sec^2 x$$

\therefore Solution of D.E. will be

$$y(x) \sec^2 x = \int 2 \sin x \sec^2 x dx$$

$$y \sec^2 x = 2 \sec x + c$$

\therefore Curve passes through $\left(\frac{\pi}{4}, 0\right)$

$$\therefore c = -2\sqrt{2}$$

$$\therefore y = 2 \cos x - 2\sqrt{2} \cos^2 x$$

$$\therefore \int_0^{\pi/2} y dx = \int_0^{\pi/2} (2 \cos x - 2\sqrt{2} \cos^2 x) dx$$

$$= 2 - 2\sqrt{2} \cdot \frac{\pi}{4} = 2 - \frac{\pi}{\sqrt{2}}$$

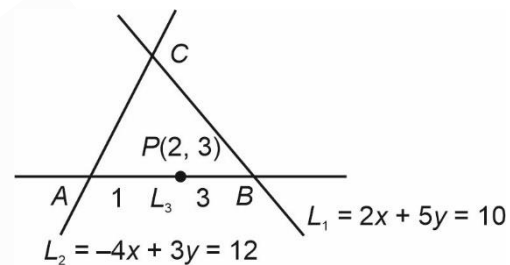
12. Let a triangle be bounded by the lines $L_1 : 2x + 5y = 10$; $L_2 : -4x + 3y = 12$ and the line L_3 , which passes through the point $P(2, 3)$, intersects L_2 at A and L_1 at B . If the point P divides the line-segment AB , internally in the ratio $1 : 3$, then the area of the triangle is equal to

- (A) $\frac{110}{13}$ (B) $\frac{132}{13}$
 (C) $\frac{142}{13}$ (D) $\frac{151}{13}$

Answer (B)

Sol. $L_1 : 2x + 5y = 10$

$$L_2 : -4x + 3y = 12$$



Solving L_1 and L_2 we get

$$C \equiv \left(\frac{-15}{13}, \frac{32}{13}\right)$$

Now, Let $A\left(x_1, \frac{1}{3}(12 - 4x_1)\right)$ and

$$B\left(x_2, \frac{1}{5}(10 - 2x_2)\right)$$

$$\therefore \frac{3x_1 + x_2}{4} = 2$$

$$\text{and } \frac{(12 + 4x_1) + \frac{10 - 2x_2}{5}}{4} = 3$$

$$\text{So, } 3x_1 + x_2 = 8 \text{ and } 10x_1 - x_2 = -5$$

$$\text{So, } (x_1, x_2) = \left(\frac{3}{13}, \frac{95}{13}\right)$$

$$A = \left(\frac{3}{13}, \frac{56}{13}\right) \text{ and } B = \left(\frac{95}{13}, \frac{-12}{13}\right)$$

$$= \left| \frac{1}{2} \left(3 \left(\frac{-44}{13} \right) - \frac{56}{13} \left(\frac{110}{13} \right) + 1 \left(\frac{2860}{169} \right) \right) \right|$$

$$= \frac{132}{13} \text{ sq. units}$$

13. Let $a > 0, b > 0$. Let e and l respectively be the eccentricity and length of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Let e' and l' respectively be the eccentricity and length of the latus rectum of its conjugate hyperbola. If $e^2 = \frac{11}{14}l$ and $(e')^2 = \frac{11}{8}l'$, then the value of $77a + 44b$ is equal to :
- (A) 100 (B) 110
(C) 120 (D) 130

Answer (D)

Sol. H : $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then

$$e^2 = \frac{11}{14}l \quad (l \text{ be the length of LR})$$

$$\Rightarrow a^2 + b^2 = \frac{11}{7}b^2a \quad \dots(i)$$

$$\text{and } e'^2 = \frac{11}{8}l'$$

(l' be the length of LR of conjugate hyperbola)

$$\Rightarrow a^2 + b^2 = \frac{11}{4}a^2b \quad \dots(ii)$$

By (i) and (ii)

$$\boxed{7a = 4b}$$

then by (i)

$$\frac{16}{49}b^2 + b^2 = \frac{11}{7}b^2 \cdot \frac{4b}{7}$$

$$\Rightarrow 44b = 65 \text{ and } 77a = 65$$

$$\therefore 77a + 44b = 130$$

14. Let, $\vec{a} = \alpha\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = -2\hat{i} + \alpha\hat{j} + \hat{k}$, where $\alpha \in \mathbf{R}$. If the area of the parallelogram whose adjacent sides are represented by the vectors \vec{a} and \vec{b} is $\sqrt{15(\alpha^2 + 4)}$, then the value of $2|\vec{a}|^2 + (\vec{a} \cdot \vec{b})|\vec{b}|^2$ is equal to :
- (A) 10 (B) 7
(C) 9 (D) 14

Answer (D)

Sol. $\vec{a} = \alpha\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = -2\hat{i} + \alpha\hat{j} + \hat{k}$

$$\therefore \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 2 & -1 \\ -2 & \alpha & 1 \end{vmatrix} = (2 + \alpha)\hat{i} - (\alpha - 2)\hat{j} + (\alpha^2 + 4)\hat{k}$$

$$\text{Now } |\vec{a} \times \vec{b}| = \sqrt{15(\alpha^2 + 4)}$$

$$\Rightarrow (2 + \alpha)^2 + (\alpha - 2)^2 + (\alpha^2 + 4)^2 = 15(\alpha^2 + 4)$$

$$\Rightarrow \alpha^4 - 5\alpha^2 - 36 = 0$$

$$\therefore \alpha = \pm 3$$

$$\text{Now, } 2|\vec{a}|^2 + (\vec{a} \cdot \vec{b})|\vec{b}|^2 = 2 \cdot 14 - 14 = 14$$

15. If vertex of a parabola is $(2, -1)$ and the equation of its directrix is $4x - 3y = 21$, then the length of its latus rectum is :
- (A) 2 (B) 8
(C) 12 (D) 16

Answer (B)

Sol. Vertex of Parabola : $(2, -1)$

and directrix : $4x - 3y = 21$

Distance of vertex from the directrix

$$a = \left| \frac{8 + 3 - 21}{\sqrt{25}} \right| = 2$$

$$\therefore \text{length of latus rectum} = 4a = 8$$

16. Let the plane $ax + by + cz = d$ pass through $(2, 3, -5)$ and is perpendicular to the planes $2x + y - 5z = 10$ and $3x + 5y - 7z = 12$.

If a, b, c, d are integers $d > 0$ and $\gcd(|a|, |b|, |c|, d) = 1$, then the value of $a + 7b + c + 20d$ is equal to :

- (A) 18 (B) 20
(C) 24 (D) 22

Answer (D)

Sol. Equation of plane through point $(2, 3, -5)$ and perpendicular to planes $2x + y - 5z = 10$ and $3x + 5y - 7z = 12$ is

$$\begin{vmatrix} x-2 & y-3 & z+5 \\ 2 & 1 & -5 \\ 3 & 5 & -7 \end{vmatrix} = 0$$

$$\therefore \text{Equation of plane is } (x-2)(-7+25) - (y-3)(-14+15) + (z+5) \cdot 7 = 0$$

$$\therefore 18x - y + 7z + 2 = 0$$

$$\Rightarrow 18x - y + 7z = -2$$

$$\therefore -18x + y - 7z = 2$$

On comparing with $ax + by + cz = d$ where $d > 0$ is

$$a = -18, b = 1, c = -7, d = 2$$

$$\therefore a + 7b + c + 20d = 22$$

17. The probability that a randomly chosen one-one function from the set $\{a, b, c, d\}$ to the set $\{1, 2, 3, 4, 5\}$ satisfies $f(a) + 2f(b) - f(c) = f(d)$ is :

- (A) $\frac{1}{24}$ (B) $\frac{1}{40}$
(C) $\frac{1}{30}$ (D) $\frac{1}{20}$

Answer (D)

Sol. Number of one-one function from $\{a, b, c, d\}$ to set $\{1, 2, 3, 4, 5\}$ is ${}^5P_4 = 120$ n(s).

The required possible set of value

$(f(a), f(b), f(c), f(d))$ such that $f(a) + 2f(b) - f(c) = f(d)$ are $(5, 3, 2, 1), (5, 1, 2, 3), (4, 1, 3, 5), (3, 1, 4, 5), (5, 4, 3, 2)$ and $(3, 4, 5, 2)$

$$\therefore n(E) = 6$$

$$\therefore \text{Required probability} = \frac{n(E)}{n(S)} = \frac{6}{120} = \frac{1}{20}$$

18. The value of $\lim_{n \rightarrow \infty} 6 \tan \left\{ \sum_{r=1}^n \tan^{-1} \left(\frac{1}{r^2 + 3r + 3} \right) \right\}$ is

equal to :

- (A) 1
(B) 2
(C) 3
(D) 6

Answer (C)

Sol. $\lim_{n \rightarrow \infty} 6 \tan \left\{ \sum_{r=1}^n \tan^{-1} \left(\frac{1}{r^2 + 3r + 3} \right) \right\}$

$$= \lim_{n \rightarrow \infty} 6 \tan \left\{ \sum_{r=1}^n \tan^{-1} \left(\frac{(r+2) - (r+1)}{1 + (r+2)(r+1)} \right) \right\}$$

$$= \lim_{n \rightarrow \infty} 6 \tan \left\{ \sum_{r=1}^n (\tan^{-1}(r+2) - \tan^{-1}(r+1)) \right\}$$

$$= \lim_{n \rightarrow \infty} 6 \tan \left\{ \tan^{-1}(n+2) - \tan^{-1} 2 \right\}$$

$$= 6 \tan \left\{ \frac{\pi}{2} - \cot^{-1} \left(\frac{1}{2} \right) \right\}$$

$$= 6 \tan \left(\tan^{-1} \left(\frac{1}{2} \right) \right)$$

$$= 3$$

19. Let \vec{a} be a vector which is perpendicular to the vector $3\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k}$. If $\vec{a} \times (2\hat{i} + \hat{k}) = 2\hat{i} - 13\hat{j} - 4\hat{k}$, then the projection of the vector \vec{a} on the vector $2\hat{i} + 2\hat{j} + \hat{k}$ is :

- (A) $\frac{1}{3}$ (B) 1
(C) $\frac{5}{3}$ (D) $\frac{7}{3}$

Answer (C)

Sol. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$$\text{and } \vec{a} \cdot \left(3\hat{i} - \frac{1}{2}\hat{j} + 2\hat{k} \right) = 0 \Rightarrow 3a_1 + \frac{a_2}{2} + 2a_3 = 0 \dots(i)$$

$$\text{and } \vec{a} \times (2\hat{i} + \hat{k}) = 2\hat{i} - 13\hat{j} - 4\hat{k}$$

$$\Rightarrow a_2\hat{i} + (2a_3 - a_1)\hat{j} - 2a_2\hat{k} = 2\hat{i} - 13\hat{j} - 4\hat{k}$$

$$\therefore a_2 = 2 \dots(ii)$$

$$\text{and } a_1 - 2a_3 = 13 \dots(iii)$$

From eq. (i) and (iii) : $a_1 = 3$ and $a_3 = -5$

$$\therefore \vec{a} = 3\hat{i} + 2\hat{j} - 5\hat{k}$$

$$\therefore \text{projection of } \vec{a} \text{ on } 2\hat{i} + 2\hat{j} + \hat{k} = \frac{6 + 4 - 5}{3} = \frac{5}{3}$$

20. If $\cot \alpha = 1$ and $\sec \beta = -\frac{5}{3}$, where

$\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$, then the value of $\tan(\alpha + \beta)$ and the quadrant in which $\alpha + \beta$ lies, respectively are :

- (A) $-\frac{1}{7}$ and IVth quadrant
(B) 7 and Ist quadrant
(C) -7 and IVth quadrant
(D) $\frac{1}{7}$ and Ist quadrant

Answer (A)

Sol. $\because \cot \alpha = 1, \quad \alpha \in \left(\pi, \frac{3\pi}{2} \right)$

$$\text{then } \tan \alpha = 1$$

$$\text{and } \sec \beta = -\frac{5}{3}, \quad \beta \in \left(\frac{\pi}{2}, \pi \right)$$

$$\text{then } \tan \beta = -\frac{4}{3}$$

$$\begin{aligned} \therefore \tan(\alpha + \beta) &= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta} \\ &= \frac{1 - \frac{4}{3}}{1 + \frac{4}{3}} \\ &= -\frac{1}{7} \end{aligned}$$

$$\alpha + \beta \in \left(\frac{3\pi}{2}, 2\pi \right) \text{ i.e. fourth quadrant}$$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Let the image of the point $P(1, 2, 3)$ in the line $L : \frac{x-6}{3} = \frac{y-1}{2} = \frac{z-2}{3}$ be Q . Let $R(\alpha, \beta, \gamma)$ be a point that divides internally the line segment PQ in the ratio $1 : 3$. Then the value of $22(\alpha + \beta + \gamma)$ is equal to _____.

Answer (125)

Sol. The point dividing PQ in the ratio $1 : 3$ will be mid-point of P & foot of perpendicular from P on the line.

\therefore Let a point on line be λ

$$\Rightarrow \frac{x-6}{3} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$$

$$\Rightarrow P'(3\lambda + 6, 2\lambda + 1, 3\lambda + 2)$$

as P' is foot of perpendicular

$$(3\lambda + 5)3 + (2\lambda - 1)2 + (3\lambda - 1)3 = 0$$

$$\Rightarrow 22\lambda + 15 - 2 - 3 = 0$$

$$\Rightarrow \lambda = \frac{-5}{11}$$

$$\therefore P' \left(\frac{51}{11}, \frac{1}{11}, \frac{7}{11} \right)$$

$$\text{Mid-point of } PP' \equiv \left(\frac{\frac{51}{11} + 1}{2}, \frac{\frac{1}{11} + 2}{2}, \frac{\frac{7}{11} + 3}{2} \right)$$

$$\equiv \left(\frac{62}{22}, \frac{23}{22}, \frac{40}{22} \right) \equiv (\alpha, \beta, \gamma)$$

$$\Rightarrow 22(\alpha + \beta + \gamma) = 62 + 23 + 40 = 125$$

2. Suppose a class has 7 students. The average marks of these students in the mathematics examination is 62, and their variance is 20. A student fails in the examination if he/she gets less than 50 marks, then in worst case, the number of students can fail is _____.

Answer (0)

Sol. According to given data

$$\frac{\sum_{i=1}^7 (x_i - 62)^2}{7} = 20$$

$$\Rightarrow \sum_{i=1}^7 (x_i - 62)^2 = 140$$

So for any x_i , $(x_i - 62)^2 \leq 140$

$$\Rightarrow x_i > 50 \quad \forall i = 1, 2, 3, \dots, 7$$

So no student is going to score less than 50.

3. If one of the diameters of the circle $x^2 + y^2 - 2\sqrt{2}x - 6\sqrt{2}y + 14 = 0$ is a chord of the circle $(x - 2\sqrt{2})^2 + (y - 2\sqrt{2})^2 = r^2$, then the value of r^2 is equal to _____.

Answer (10)

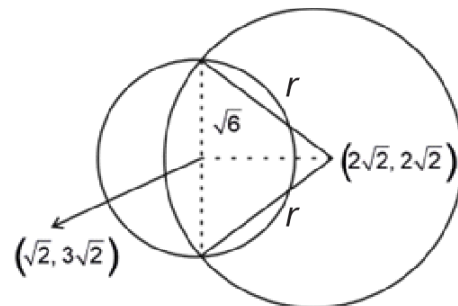
Sol. For $x^2 + y^2 - 2\sqrt{2}x - 6\sqrt{2}y + 14 = 0$

$$\text{Radius} = \sqrt{(\sqrt{2})^2 + (3\sqrt{2})^2 - 14} = \sqrt{6}$$

$$\Rightarrow \text{Diameter} = 2\sqrt{6}$$

If this diameter is chord to

$$(x - 2\sqrt{2})^2 + (y - 2\sqrt{2})^2 = r^2 \text{ then}$$



$$\Rightarrow r^2 = 6 + \left(\sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} \right)^2$$

$$\Rightarrow r^2 = 6 + 4 = 10$$

$$\Rightarrow r^2 = 10$$

4. If $\lim_{x \rightarrow 1} \frac{\sin(3x^2 - 4x + 1) - x^2 + 1}{2x^3 - 7x^2 + ax + b} = -2$, then the value of $(a - b)$ is equal to _____.

Answer (11)

$$\text{Sol. } \lim_{x \rightarrow 1} \frac{\left(\frac{\sin(3x^2 - 4x + 1)}{3x^2 - 4x + 1} \right) (3x^2 - 4x + 1) - x^2 + 1}{2x^3 - 7x^2 + ax + b} = -2$$

$$\Rightarrow \lim_{x \rightarrow 1} \frac{3x^2 - 4x + 1 - x^2 + 1}{2x^3 - 7x^2 + ax + b} = -2$$

$$\Rightarrow \lim_{x \rightarrow 1} \frac{2(x-1)^2}{2x^3 - 7x^2 + ax + b} = -2$$

So $f(x) = 2x^3 - 7x^2 + ax + b = 0$ has $x = 1$ as repeated root, therefore $f(1) = 0$ and $f'(1) = 0$ gives

$$a + b + 5 \text{ and } a = 8$$

So, $a - b = 11$

5. Let for $n = 1, 2, \dots, 50$, S_n be the sum of the infinite geometric progression whose first term is n^2 and whose common ratio is $\frac{1}{(n+1)^2}$. Then the value of

$$\frac{1}{26} + \sum_{n=1}^{50} \left(S_n + \frac{2}{n+1} - n - 1 \right) \text{ is equal to } \underline{\hspace{2cm}}.$$

Answer (41651)

$$\text{Sol. } S_n = \frac{n^2}{1 - \frac{1}{(n+1)^2}} = \frac{n(n+1)^2}{n+2} = (n^2 + 1) - \frac{2}{n+2}$$

$$\text{Now } \frac{1}{26} + \sum_{n=1}^{50} \left(S_n + \frac{2}{n+1} - n - 1 \right)$$

$$= \frac{1}{26} + \sum_{n=1}^{50} \left\{ (n^2 - n) + 2 \left(\frac{1}{n+1} - \frac{1}{n+2} \right) \right\}$$

$$= \frac{1}{26} + \frac{50 \times 51 \times 101}{6} - \frac{50 \times 51}{2} + 2 \left(\frac{1}{2} - \frac{1}{52} \right)$$

$$= 1 + 25 \times 17(101 - 3)$$

$$= 41651$$

6. If the system of linear equations

$$2x - 3y = \gamma + 5,$$

$\alpha x + 5y = \beta + 1$, where $\alpha, \beta, \gamma \in R$ has infinitely many solutions, then the value of $|9\alpha + 3\beta + 5\gamma|$ is equal to _____.

Answer (58)

Sol. If $2x - 3y = \gamma + 5$ and $\alpha x + 5y = \beta + 1$ have infinitely many solutions then

$$\frac{2}{\alpha} = \frac{-3}{5} = \frac{\gamma + 5}{\beta + 1}$$

$$\Rightarrow \alpha = -\frac{10}{3} \text{ and } 3\beta + 5\gamma = -28$$

$$\text{So } |9\alpha + 3\beta + 5\gamma| = |-30 - 28| = 58$$

7. Let $A = \begin{pmatrix} 1+i & 1 \\ -i & 0 \end{pmatrix}$ where $i = \sqrt{-1}$. Then, the number of elements in the set $\{n \in \{1, 2, \dots, 100\} : A^n = A\}$ is _____.

Answer (25)

$$\text{Sol. } \therefore A^2 = \begin{bmatrix} 1+i & 1 \\ -i & 0 \end{bmatrix} \begin{bmatrix} 1+i & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} i & 1+i \\ 1-i & -i \end{bmatrix}$$

$$A^4 = \begin{bmatrix} i & 1+i \\ 1-i & -i \end{bmatrix} \begin{bmatrix} i & 1+i \\ 1-i & -i \end{bmatrix} = I$$

So $A^5 = A$, $A^9 = A$ and so on.

Clearly $n = 1, 5, 9, \dots, 97$

Number of values of $n = 25$

8. Sum of squares of modulus of all the complex numbers z satisfying $\bar{z} = iz^2 + z^2 - z$ is equal to _____.

Answer (2)

Sol. Let $z = x + iy$

$$\text{So } 2x = (1 + i)(x^2 - y^2 + 2xyi)$$

$$\Rightarrow 2x = x^2 - y^2 - 2xy \quad \dots(i) \text{ and}$$

$$x^2 - y^2 + 2xy = 0 \quad \dots(ii)$$

From (i) and (ii) we get

$$x = 0 \text{ or } y = -\frac{1}{2}$$

When $x = 0$ we get $y = 0$

$$\text{When } y = -\frac{1}{2} \text{ we get } x^2 - x - \frac{1}{4} = 0$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{2}}{2}$$

So there will be total 3 possible values of z , which

$$\text{are } 0, \left(\frac{-1 + \sqrt{2}}{2} \right) - \frac{1}{2}i \text{ and } \left(\frac{-1 - \sqrt{2}}{2} \right) - \frac{1}{2}i$$

Sum of squares of modulus

$$= 0 + \left(\frac{\sqrt{2}-1}{2} \right)^2 + \frac{1}{4} + \left(\frac{\sqrt{2}+1}{2} \right)^2 = +\frac{1}{4}$$

$$= 2$$

9. Let $S = \{1, 2, 3, 4\}$. Then the number of elements in the set $\{f: S \times S \rightarrow S : f \text{ is onto and } f(a, b) = f(b, a) \geq a \forall (a, b) \in S \times S\}$ is _____.

Answer (37)

Sol. There are 16 ordered pairs in $S \times S$. We write all these ordered pairs in 4 sets as follows.

$$A = \{(1, 1)\}$$

$$B = \{(1, 4), (2, 4), (3, 4), (4, 4), (4, 3), (4, 2), (4, 1)\}$$

$$C = \{(1, 3), (2, 3), (3, 3), (3, 2), (3, 1)\}$$

$$D = \{(1, 2), (2, 2), (2, 1)\}$$

All elements of set B have image 4 and only element of A has image 1.

All elements of set C have image 3 or 4 and all elements of set D have image 2 or 3 or 4.

We will solve this question in two cases.

Case I : When no element of set C has image 3.

Number of onto functions = 2 (when elements of set D have images 2 or 3)

Case II : When atleast one element of set C has image 3.

$$\begin{aligned} \text{Number of onto functions} &= (2^3 - 1)(1 + 2 + 2) \\ &= 35 \end{aligned}$$

$$\text{Total number of functions} = 37$$

10. The maximum number of compound propositions, out of $p \vee r \vee s, p \vee r \vee \sim s, p \vee \sim q \vee s, \sim p \vee \sim r \vee s, \sim p \vee \sim r \vee \sim s, \sim p \vee q \vee \sim s, q \vee r \vee \sim s, q \vee \sim r \vee \sim s, \sim p \vee \sim q \vee \sim s$ that can be made simultaneously true by an assignment of the truth values to p, q, r and s , is equal to _____.

Answer (9)

Sol. There are total 9 compound propositions, out of which 6 contain $\sim s$. So if we assign s as false, these 6 propositions will be true.

In remaining 3 compound propositions, two contain p and the third contains $\sim r$. So if we assign p and r as true and false respectively, these 3 propositions will also be true.

Hence maximum number of propositions that can be true are 9.

