28/06/2022
Morning

Corporate Office : **Aakash Tower, 8, Pusa Road, New Delhi-110005 | Ph.: 011-47623456**

Answers & Solutions

Time : 3 hrs. *for* **M.M. : 300**

JEE (Main)-2022 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Given below are two statements : One is labelled as **Assertion A** and the other is labelled as **Reason R**.

   Assertion A : Product of Pressure (P) and time (t) has the same dimension as that of coefficient of viscosity.

   Reason R : Coefficient of viscosity

   $$= \frac{\text{Force}}{\text{Velocity gradient}}$$

   Choose the correct answer from the options given below.

   (A) Both **A** and **R** true, and **R** is correct explanation of **A**.

   (B) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**.

   (C) **A** is true but **R** is false.

   (D) **A** is false but **R** is true.

Answer (C)

Sol. $[\text{Pressure}][\text{Time}] = \left[\frac{\text{Force}}{\text{Area}}\right]\left[\frac{\text{Distance}}{\text{Velocity}}\right]$

$[\text{Coefficient of viscosity}] = \left[\frac{\text{Force}}{\text{Area}}\right]\left[\frac{\text{Distance}}{\text{Velocity}}\right]$

Statement 'A' is true

But statement R is false are coefficient of viscosity

$$= \frac{\text{Force}}{\text{Area} \times \text{Velocity gradient}}$$

2. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration (a) is varying with time t as $a = k^2rt^2$, where k is a constant. The power delivered to the particle by the force acting on it is given as

   (A) Zero (B) $mk^2r^2t^2$

   (C) mk^2r^2t (D) mk^2rt

Answer (C)

Sol. $a_r = k^2rt^2 = \frac{v^2}{r}$

$\Rightarrow v^2 = k^2r^2t^2$ or $v = krt$

and $\frac{d|v|}{dt} = kr$

$\Rightarrow a_t = kr$

$\Rightarrow |\bar{F}\cdot\bar{v}| = (mkr)(krt)$

$= mk^2r^2t$ = power delivered

3. Motion of a particle in x-y plane is described by a set of following equations $x = 4\sin\left(\frac{\pi}{2} - \omega t\right)$ m and $y = 4\sin(\omega t)$ m. The path of the particle will be

   (A) Circular (B) Helical

   (C) Parabolic (D) Elliptical

Answer (A)

Sol. $x = 4\sin\left(\frac{\pi}{2} - \omega t\right)$

$= 4\cos(\omega t)$

$y = 4\sin(\omega t)$

$\Rightarrow x^2 + y^2 = 4^2$

$\Rightarrow$ The particle is moving in a circular motion with radius of 4 m.

4. Match List-I with List-II

	List-I		List-II
A.	Moment of inertia of solid sphere of radius R about any tangent	I.	$\frac{5}{3}MR^2$
B.	Moment of inertia of hollow sphere of radius (R) about any tangent.	II.	$\frac{7}{5}MR^2$
C.	Moment of inertia of circular ring of radius (R) about its diameter.	III.	$\frac{1}{4}MR^2$
D.	Moment of inertia of circular disc of radius (R) about any diameter.	IV.	$\frac{1}{2}MR^2$

Choose the correct answer from the options given below.

(A) A-II, B-I, C-IV, D-III (B) A-I, B-II, C-IV, D-III

(C) A-II, B-I, C-III, D-IV (D) A-I, B-II, C-III, D-IV

Answer (A)

Sol. (A) Moment of inertia of solid sphere of radius R about a tangent $=\frac{2}{5}MR^2+MR^2=\frac{7}{5}MR^2$

$\Rightarrow$ A – (II)

(B) Moment of inertia of hollow sphere of radius R about a tangent $=\frac{2}{3}MR^2+MR^2=\frac{5}{3}MR^2$

$\Rightarrow$ B – (I)

(C) Moment of inertia of circular ring of radius (R) about its diameter $=\frac{\left(MR^2\right)}{2}$

$\Rightarrow$ C – (IV)

(D) Moment of inertia of circular ring of radius (R) about any diameter

$$=\frac{MR^2/2}{2}=\frac{MR^2}{4}$$

$\Rightarrow$ D – (III)

5. Two planets A and B of equal mass are having their period of revolutions T_A and T_B such that $T_A = 2T_B$. These planets are revolving in the circular orbits of radii r_A and r_B respectively. Which out of the following would be the correct relationship of their orbits?

(A) $2r_A^2=r_B^3$

(B) $r_A^3=2r_B^3$

(C) $r_A^3=4r_B^3$

(D) $T_A^2-T_B^2=\frac{\pi^2}{GM}\left(r_B^3-4r_A^3\right)$

Answer (C)

Sol. $T_A = 2T_B$

Now $T_A^2\propto r_A^3$

$$\Rightarrow \left(\frac{r_A}{r_B}\right)^3=\left(\frac{T_A}{T_B}\right)^2$$

$$\Rightarrow r_A^3=4r_B^3$$

6. A water drop of diameter 2 cm is broken into 64 equal droplets. The surface tension of water is 0.075 N/m. In this process the gain in surface energy will be

(A) 2.8×10^{-4} J

(B) 1.5×10^{-3} J

(C) 1.9×10^{-4} J

(D) 9.4×10^{-5} J

Answer (A)

Sol. $r'=\frac{r}{4}$

$\Rightarrow$ $\Delta E = T(\Delta S)$

$= T\times 4\pi(nr'^2-r^2)$, $n = 64$

$= T\times 4\pi\times(4-1)r^2$

$\Rightarrow$ $\Delta E = 0.075\times 4\times 3.142\,(3)\times 10^{-4}$ J

$= 2.8\times10^{-4}$ J

7. Given below are two statements

Statement-I: When μ amount of an ideal gas undergoes adiabatic change from state (P_1, V_1, T_1) to state (P_2, V_2, T_2), then work done is $W=\frac{\mu R(T_2-T_1)}{1-\gamma}$, where $\gamma=\frac{C_p}{C_v}$ and R = universal gas constant.

Statement-II: In the above case, when work is done on the gas, the temperature of the gas would rise.

Choose the correct answer from the options given below

(A) Both statement-I and statement-II are true

(B) Both statement-I and statement-II are false

(C) Statement-I is true but statement-II is false

(D) Statement-I is false but statement-II is true

Answer (A)

Sol. $W=\frac{\mu R(T_2-T_1)}{1-r}$ for a polytropic process for adiabatic process $r=\gamma$

$\Rightarrow$ Statement I is true

In an adiabatic process

$\Delta U = -\Delta W$

$\Rightarrow$ If work is done on the gas

$\Rightarrow$ ΔW is negative

$\Rightarrow$ ΔU is positive or temperature increases

$\Rightarrow$ Statement II is true

8. Given below are two statements

Statement-I: A point charge is brought in an electric field. The value of electric field at a point near to the charge may increase if the charge is positive.

Statement-II: An electric dipole is placed in a non-uniform electric field. The net electric force on the dipole will not be zero.

Choose the correct answer from the options given below

(A) Both statement-I and statement-II are true

(B) Both statement-I and statement-II are false

(C) Statement-I is true but statement-II is false

(D) Statement-I is false but statement-II is true

Answer (A)

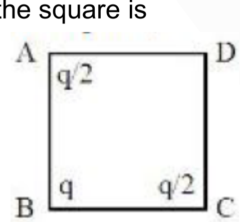
Sol. As one moves closer to a positive charge (isolated) the density of electric field line increases and so does the electric field intensity

$\Rightarrow$ Statement I is true

As opposite poles of an electric dipole would experience equal and opposite forces so net force on a dipole in a uniform electric field will be zero

$\Rightarrow$ Statement II is true

9. The three charges $\frac{q}{2}$, q and $\frac{q}{2}$ are placed at the corners A, B and C of a square of side 'a' as shown in figure. The magnitude of electric field (E) at the corner D of the square is



(A) $\frac{q}{4\pi \epsilon_0 a^2}\left(\frac{1}{\sqrt{2}}+\frac{1}{2}\right)$ (B) $\frac{q}{4\pi \epsilon_0 a^2}\left(1+\frac{1}{\sqrt{2}}\right)$

(C) $\frac{q}{4\pi \epsilon_0 a^2}\left(1-\frac{1}{\sqrt{2}}\right)$ (D) $\frac{q}{4\pi \epsilon_0 a^2}\left(\frac{1}{\sqrt{2}}-\frac{1}{2}\right)$

Answer (A)

Sol. $|E_0| = \frac{kq/2}{a^2}\sqrt{2} + \frac{kq}{(a\sqrt{2})^2}$

$= \frac{kq}{\sqrt{2}a^2} + \frac{kq}{2a^2}$

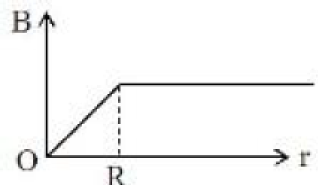
$= \frac{kq}{a^2}\left(\frac{1}{\sqrt{2}}+\frac{1}{2}\right),\ k = \frac{1}{4\pi\varepsilon_0}$

$\Rightarrow$ Option A is correct

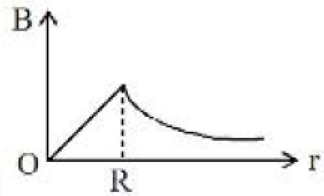
10. An infinitely long hollow conducting cylinder with radius R carries a uniform current along its surface.

Choose the correct representation of magnetic field (B) as a function of radial distance (r) from the axis of cylinder.

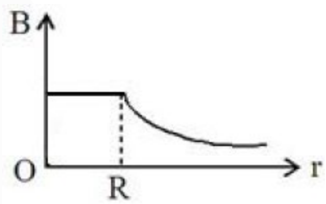
(A)



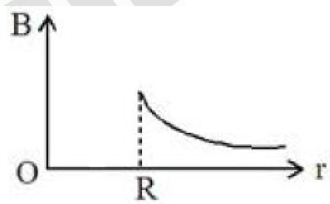
(B)



(C)



(D)



Answer (D)

Sol. Inside a hollow cylindrical conductor with uniform current distribution net magnetic field is zero in hollow space.

But outside the cylindrical conductor $B \propto \frac{1}{r}$

$\Rightarrow$ Graph in option D would be a correct one

11. A radar sends an electromagnetic signal of electric field (E_0) = 2.25 V/m and magnetic field (B_0) = 1.5×10^{-8} T which strikes a target on line of sight at a distance of 3 km in a medium. After that, a part of signal (echo) reflects back towards the radar with same velocity and by same path. If the signal was transmitted at time $t = 0$ from radar, then after how much time echo will reach to the radar?

(A) 2.0×10^{-5} s (B) 4.0×10^{-5} s

(C) 1.0×10^{-5} s (D) 8.0×10^{-5} s

Answer (B)

Sol. $E_0 = 2.25$ V/m

$B_0 = 1.5 \times 10^{-8}$ T

$\Rightarrow \frac{E_0}{B_0} = 1.5 \times 10^8$ m/s

$\Rightarrow$ Refractive index = 2

Distance to be travelled = 6 km

Time taken = $\frac{6 \times 10^3}{1.5 \times 10^8} = 4 \times 10^{-5}$ s

$\Rightarrow$ Option (B) is correct

12. The refracting angle of a prism is A and refractive index of the material of the prism is cot (A/2). Then the angle of minimum deviation will be :

(A) 180 – 2A (B) 90 – A

(C) 180 + 2A (D) 180 – 3A

Answer (A)

Sol. $\mu = \frac{\sin\left(\frac{\delta_m + A}{2}\right)}{\sin(A/2)} = \cot A/2$

$\Rightarrow \cos A/2 = \sin\left(\frac{\delta_m + A}{2}\right)$

$\Rightarrow \frac{\pi}{2} - \frac{A}{2} = \frac{\delta_m + A}{2}$

$\Rightarrow \pi - 2A = \delta_m$

Option (A) is correct

13. The aperture of the objective is 24.4 cm. The resolving power of this telescope, if a light of wavelength 2440 Å is used to see the object will be:

(A) 8.1×10^6

(B) 10.0×10^7

(C) 8.2×10^5

(D) 1.0×10^{-8}

Answer (C)

Sol. R.P. = $\frac{1}{1.22\,\lambda/a}$

$= \frac{24.4 \times 10^{-2}}{1.22 \times 2440 \times 10^{-10}}$

$= 8.2 \times 10^5$

Option (C) is correct

14. The de Broglie wavelengths for an electron and a photon are λ_e and λ_p respectively. For the same kinetic energy of electron and photon, which of the following presents the correct relation between the de Broglie wavelengths of two?

(A) $\lambda_p \propto \lambda_e^2$

(B) $\lambda_p \propto \lambda_e$

(C) $\lambda_p \propto \sqrt{\lambda_e}$

(D) $\lambda_p \propto \sqrt{\frac{1}{\lambda_e}}$

Answer (A)

Sol. $\lambda_p = \frac{h}{p} = \frac{hc}{E}$ ...(i)

$\lambda_e = \frac{h}{\sqrt{2mE}}$ ...(ii)

From (i) and (ii)

$\lambda_p \propto \lambda_e^2$

$\Rightarrow$ Option A is correct

15. The Q-value of a nuclear reaction and kinetic energy of the projectile particle, K_p are related as :

(A) $Q = K_p$ (B) $(K_p + Q) < 0$

(C) $Q < K_p$ (D) $(K_p + Q) > 0$

Answer (D)

Sol. $K_p > 0$

If Q is released $\Rightarrow Q > 0$

$\Rightarrow K_p + Q > 0$

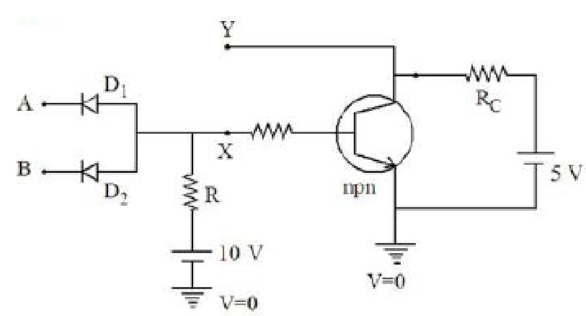
If Q is absorbed $\Rightarrow Q < 0$

Even then particle has to be given kinetic energy greater than magnitude of Q to maintain momentum conservation.

$\Rightarrow K + Q > 0$

$\Rightarrow$ Option D is correct

16. In the following circuit, the correct relation between output (Y) and inputs A and B will be:

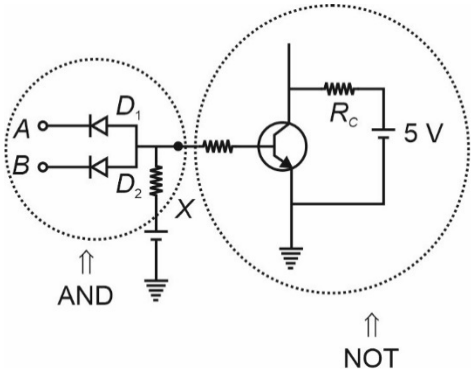


(A) $Y = AB$ (B) $Y = A + B$

(C) $Y = \overline{AB}$ (D) $Y = \overline{A+B}$

Answer (C)

Sol. The shown circuit is a combination of AND gate and a NOT gate



$\Rightarrow Y = \overline{AB}$

Option (C) is a correct option.

17. For using a multimeter to identify diode from electrical components, choose the correct statement out of the following about the diode:

(A) It is two terminal device which conducts current in both directions.

(B) It is two terminal device which conducts current in one direction only

(C) It does not conduct current gives an initial deflection which decays to zero.

(D) It is three terminal device which conducts current in one direction only between central terminal and either of the remaining two terminals.

Answer (B)

Sol. A diode is a two terminal device which conducts current in forward bias only

$\Rightarrow$ Option (B) is correct.

18. Given below are two statements : One is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : n-p-n transistor permits more current than a p-n-p transistor.

Reason R: Electrons have greater mobility as a charge carrier.

Choose the correct answer from the options given below:

(A) Both **A** and **R** are true, and **R** is correct explanation of **A**.

(B) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**.

(C) **A** is true but **R** is false.

(D) **A** is false but **R** is true.

Answer (A)

Sol. (A) is true as n-p-n transistor permits more current than p-n-p transistor as electrons which are majority charge carriers in n-p-n have higher mobility than holes which are majority carriers in p-n-p transistor

$\Rightarrow$ Statement R is correct explanation of statement A

19. Match List-I with List-II

	List-I		**List-II**
(A)	Television signal	I.	03 KHz
(B)	Radio signal	II.	20 KHz
(C)	High Quality Music	III.	02 MHz
(D)	Human speech	IV.	06 MHz

Choose the correct answer from the options given below:

(A) A-I, B-II, C-III, D-IV

(B) A-IV, B-III, C-I, D-II

(C) A-IV, B-III, C-II, D-I

(D) A-I, B-II, C-IV, D-III

Answer (C)

Sol. Television signal $\Rightarrow$ 6 MHz

Radio signal $\Rightarrow$ 2 MHz

High Quality music $\Rightarrow$ 20 kHz

Human speech $\Rightarrow$ 3 kHz

$\Rightarrow$ Option (C) is correct.

20. The velocity of sound in a gas, in which two wavelengths 4.08 m and 4.16 m produce 40 beats in 12 s, will be:

(A) 282.8 ms^{-1}

(B) 175.5 ms^{-1}

(C) 353.6 ms^{-1}

(D) 707.2 ms^{-1}

Answer (D)

Sol. $\frac{v}{4.08} - \frac{v}{4.16} = \frac{40}{12}$

$v = \frac{40}{12} \times \frac{4.08 \times 4.16}{0.08}$

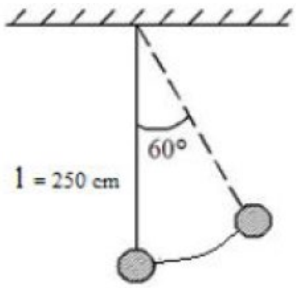
= 707.2 m/s

$\Rightarrow$ Option (D) is correct.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. A pendulum is suspended by a string of length 250 cm. The mass of the bob of the pendulum is 200 g. The bob is pulled aside until the string is at 60° with vertical as shown in the figure. After releasing the bob, the maximum velocity attained by the bob will be ___ ms^{-1}. (if g = 10 m/s²)



Answer (5)

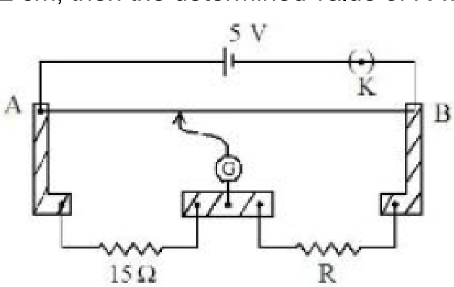
Sol. $\frac{1}{2}mv^2 = mgl(1-\cos\theta)$

$\Rightarrow \quad v = \sqrt{2gl(1-\cos\theta)}$

$= \sqrt{2\times10\times2.5\times\frac{1}{2}}$

= 5 m/s

2. A meter bridge setup is shown in the figure. It is used to determine an unknown resistance R using a given resistor of 15 Ω. The galvanometer (G) shows null deflection when tapping key is at 43 cm mark from end A. If the end correction for end A is 2 cm, then the determined value of R will be ___ Ω.

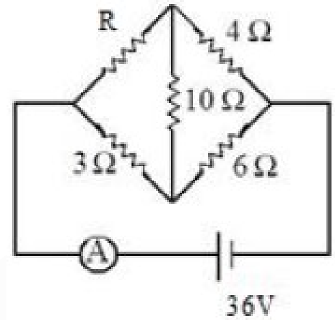


Answer (19)

Sol. $\frac{43+2}{15} = \frac{57}{R}$

$R = \frac{57\times15}{45} = 19\ \Omega$

3. Current measured by the ammeter Ⓐ in the reported circuit when no current flows through 10 Ω resistance, will be ___ A.



Answer (10)

Sol. For $I_{10} = 0$

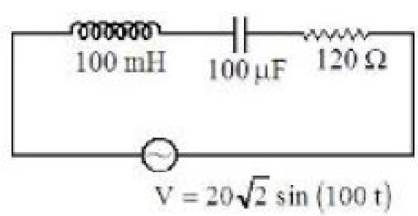
$\frac{R}{3} = \frac{4}{6}$

$\Rightarrow \quad R = 2\ \Omega$

$\Rightarrow \quad I_A = \frac{36\times(6+9)}{6\times9}$

$= \frac{36\times15}{6\times9} = 10$ A

4. An AC source is connected to an inductance of 100 mH, a capacitance of 100 μF and a resistance of 120 Ω as shown in figure. The time in which the resistance having a thermal capacity 2 J/°C will get heated by 16°C is ___ s.



Answer (15)

Sol. $L = 100 \times 10^{-3}$ H

$C = 100 \times 10^{-6}$ F

$R = 120\ \Omega$

$\omega L = 10\ \Omega$

$\frac{1}{\omega C} = \frac{1}{10^4\times10^{-6}} = 100\ \Omega$

$\Rightarrow X_C - X_L = 90\ \Omega$

$\Rightarrow Z = \sqrt{90^2 + 120^2} = 150\ \Omega$

$\Rightarrow I_{rms} = \frac{20}{150} = \frac{2}{15}$ A

For heat resistance by 16°C heat required = 32 J

$\Rightarrow \left(\frac{2}{15}\right)^2 \times (120) \times t = 32$

$t = \frac{32 \times 15 \times 15}{4 \times 120}$

= 15

5. The position vector of 1 kg object is $\vec{r} = (3\hat{i} - \hat{j})$ m and its velocity $\vec{v} = (3\hat{j} + \hat{k})$ ms^{-1}. The magnitude of its angular momentum is $\sqrt{x}$ Nm where x is

Answer (91)

Sol. $|\vec{l}| = |\vec{r} \times (m\vec{v})|$

$= |(3\hat{i} - \hat{j}) \times (3\hat{j} + \hat{k})|$

$= |-\hat{i} - 3\hat{j} + 9\hat{k}|$

$= \sqrt{91}$

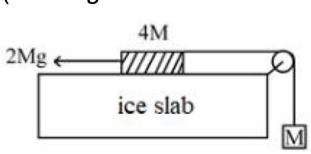
6. A man of 60 kg is running on the road and suddenly jumps into a stationary trolly car of mass 120 kg. Then, the trolly car starts moving with velocity 2 ms^{-1}. The velocity of the running man was _____ ms^{-1}, when he jumps into the car.

Answer (6)

Sol. $v_m = \frac{(120 + 60)v_T}{60}$

$= \frac{180 \times 2}{60}$ = 6 m/s

7. A hanging mass M is connected to a four times bigger mass by using a string-pulley arrangement, as shown in the figure. The bigger mass is placed on a horizontal ice-slab and being pulled by 2 Mg force. In this situation, tension in the string is $\frac{x}{5}$ Mg for x = __________. Neglect mass of the string and friction of the block (bigger mass) with ice slab. (Given g = acceleration due to gravity)



Answer (6)

Sol. $a = \frac{Mg}{4M + M} = \frac{g}{5}$ (in upward direction)

$T = M\left(g + \frac{g}{5}\right) = \frac{6Mg}{5}$

$\Rightarrow x = 6$

8. The total internal energy of two mole monoatomic ideal gas at temperature T = 300 K will be _____ J. (Given R = 8.31 J/mol.K)

Answer (7479)

Sol. $U = 2\left(\frac{3}{2}R\right)300$

= 3 × 8.31 × 300

= 7479 J

9. A singly ionized magnesium atom (A = 24) ion is accelerated to kinetic energy 5 keV, and is projected perpendicularly into a magnetic field B of the magnitude 0.5 T. The radius of path formed will be ________ cm.

Answer (10)

Sol. $R = \frac{mv}{qB}$

$R = \frac{\sqrt{2mKE}}{qB}$

$= \frac{\sqrt{2 \times 24 \times 1.67 \times 10^{-27} \times 5 \times 1.6 \times 10^{-16}}}{1.6 \times 10^{-19} \times 0.5}$

= 10.009 cm = 10 cm

10. A telegraph line of length 100 km has a capacity of 0.01 μF/km and it carries an alternating current at 0.5 kilo cycle per second. If minimum impedance is required, then the value of the inductance that needs to be introduced in series is_______ mH. (if $\pi = \sqrt{10}$)

Answer (100)

Sol. Total capacitance = 0.01 × 100 = 1 μF

ω = 500 × 2π = 1000π rad/s

$\omega L = \frac{1}{\omega C}$

$\Rightarrow L = \frac{1}{\omega^2 C} = \frac{1}{10^6 \pi^2 \times 10^{-6}} = \frac{1}{10}$H = 100 mH

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The **incorrect** statement about the imperfections in solids is:

   (A) Schottky defect decreases the density of the substance.

   (B) Interstitial defect increases the density of the substance.

   (C) Frenkel defect does not alter the density of the substance.

   (D) Vacancy defect increases the density of the substance.

Answer (D)

Sol. Vacancy defect causes decrease in density

2. The Zeta potential is related to which property of colloids?

   (A) Colour

   (B) Tyndall effect

   (C) Charge on the surface of colloidal particles

   (D) Brownian movement

Answer (C)

Sol. The potential difference between the fixed layer and the diffused layer of opposite charges is called zeta potential.

It is related with the charge on the surface of colloidal particles.

3. Element "E" belongs to the period 4 and group 16 of the periodic table. The valence shell electron configuration of the element, which is just above "E" in the group is

   (A) $3s^2, 3p^4$ (B) $3d^{10}, 4s^2, 4p^4$

   (C) $4d^{10}, 5s^2, 5p^4$ (D) $2s^2, 2p^4$

Answer (A)

Sol. Element E is Selenium

The element which is just above 'E' in periodic table is sulphur, its electronic configuration is $1s^2, 2s^2, 2p^6, 3s^2, 3p^4$

4. Given are two statements one is labelled as **Assertion A** and other is labelled as **Reason R**.

   Assertion A : Magnesium can reduce Al_2O_3 at a temperature below 1350°C, while above 1350°C aluminium can reduce MgO.

   Reason R : The melting and boiling points of magnesium are lower than those of aluminium.

   In light of the above statements, choose most appropriate answer from the options given below :

   (A) Both **A** and **R** are correct, and **R** is correct explanation of **A**.

   (B) Both **A** and **R** are correct, but **R** is NOT the correct explanation of **A**.

   (C) **A** is correct **R** is not correct.

   (D) **A** is not correct, **R** is correct.

Answer (B)

Sol. Magnesium can reduce Al_2O_3 at a temperature below 1350°C while above 1350°C aluminium can reduce MgO because below 1350°C ΔG of MgO (formation) is more negative and above 1350°C ΔG of Al_2O_3 (formation) is more negative.

The melting and boiling point of magnesium are lower than those of aluminium but it is not the correct reason.

5. Dihydrogen reacts with CuO to give

   (A) CuH_2 (B) Cu

   (C) Cu_2O (D) $Cu(OH)_2$

Answer (B)

Sol. $CuO + H_2 \rightarrow Cu + H_2O$

6. Nitrogen gas is obtained by thermal decomposition of

   (A) $Ba(NO_3)_2$

   (B) $Ba(N_3)_2$

   (C) $NaNO_2$

   (D) $NaNO_3$

Answer (B)

Sol. $Ba(N_3)_2 \xrightarrow{\Delta} Ba + 3N_2$

7. Given below are two statements :

Statement I : The pentavalent oxide of group-15 element, E_2O_5, is less acidic than trivalent oxide, E_2O_3, of the same element.

Statement II : The acidic character of trivalent oxide of group 15 elements, E_2O_3, decreases down the group.

In light of the above statements, choose **most appropriate** answer from the options given below:

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I true, but Statement II is false

(D) Statement I false, but Statement II is true

Answer (D)

Sol. Statement I is false, as E_2O_5 is more acidic than E_2O_3

Statement II is correct.

8. Which one of the lanthanoids given below is the most stable in divalent form?

(A) Ce (Atomic Number 58)

(B) Sm (Atomic Number 62)

(C) Eu (Atomic Number 63)

(D) Yb (Atomic Number 70)

Answer (C)

Sol. Eu^{+2} is $4f^7$

Yb^{+2} is $4f^{14}$

but Eu^{+2} is more stable than Yb^{+2} because

$E^{\circ}_{Eu|Eu^{+2}} > E^{\circ}_{Yb|Yb^{+2}}$

9. Given below are two statements:

Statement I: $[Ni(CN)_4]^{2-}$ is square planar and diamagnetic complex, with dsp^2 hybridization for Ni but $[Ni(CO)_4]$ is tetrahedral, paramagnetic and with sp^3-hybridization for Ni.

Statement II: $[NiCl_4]^{2-}$ and $[Ni(CO)_4]$ both have same *d*-electron configuration, have same geometry and are paramagnetic.

In light of the above statements, choose the **correct** answer from the options given below :

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is correct but Statement II is false

(D) Statement I is incorrect but Statement II is true

Answer (B)

Sol. $[Ni(CN)_4]^{2-}$ is square planar and diamagnetic with $\mu = 0$

its hybridisation is dsp^2

$Ni(CO)_4$ is tetrahedral but diamagnetic.

10. Which amongst the following is **not** a pesticide?

(A) DDT (B) Organophosphates

(C) Dieldrin (D) Sodium arsenite

Answer (D)

Sol. Sodium arsenite is a herbicide.

11. Which one of the following techniques is not used to spot components of a mixture separated on thin layer chromatographic plate?

(A) I_2 (Solid)

(B) U.V. Light

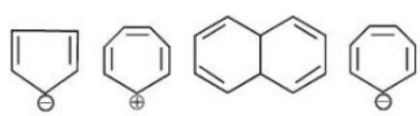
(C) Visualisation agent as a component of mobile phase

(D) Spraying of an appropriate reagent

Answer (C)

Sol. The function of mobile phase is to carry the components present on TLC.

12. Which of the following structures are aromatic in nature?



A B C D

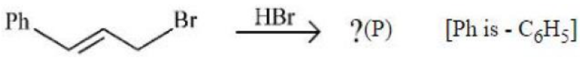
(A) A, B, C and D (B) Only A and B

(C) Only A and C (D) Only B, C and D

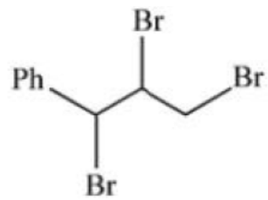
Answer (B)

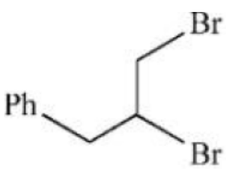
Sol.  and  are aromatic as they are cyclic, planar and has $4n + 2\ \pi\ e^-$ (n = 1)

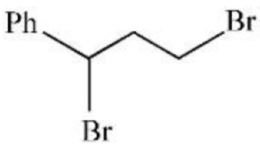
13. The major product (P) in the reaction

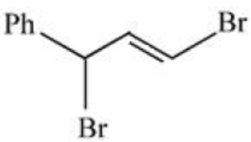


is

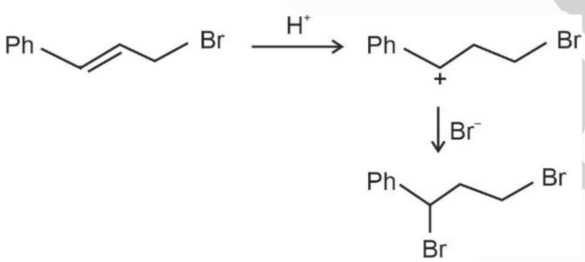
(A) 

(B) 

(C) 

(D) 

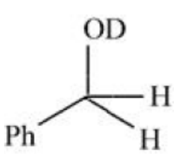
Answer (C)

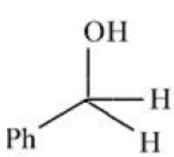
Sol. 

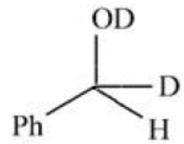
14. The correct structure of product 'A' formed in the following reaction,

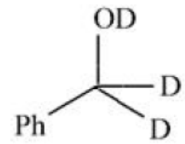
$$\text{PhCHO} + \text{Ph}\cdot\text{CHO} \xrightarrow[\text{in } D_2O]{\text{NaOD}} A + \text{Ph}-\overset{O}{\overset{\|}{C}}-O^-$$

(Ph is - C_6H_5) is

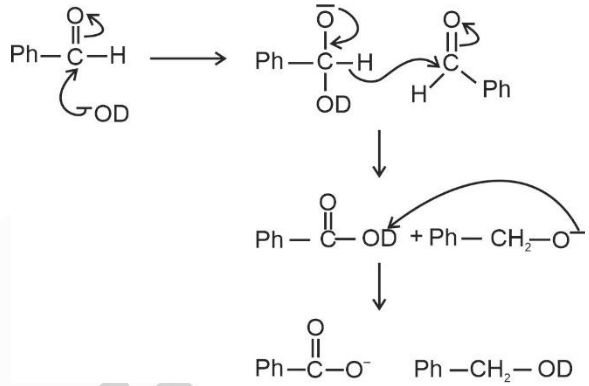
(A) 

(B) 

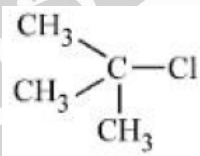
(C) 

(D) 

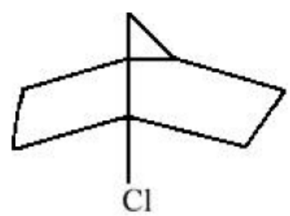
Answer (A)

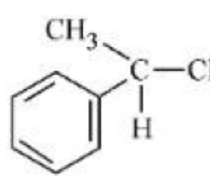
Sol. 

15. Which one of the following compounds is inactive towards S_N1 reaction?

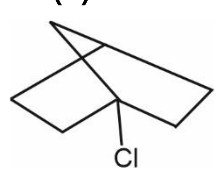
(A) 

(B) 

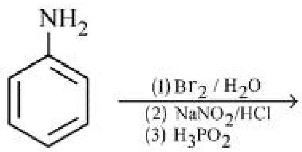
(C) 

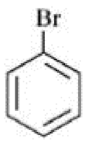
(D) 

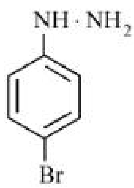
Answer (C)

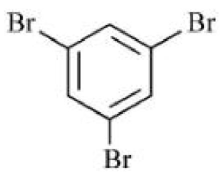
Sol.  is inactive towards S_N1 as halogen is attached to bridge head carbon atom, where formation of carbocation is not possible.

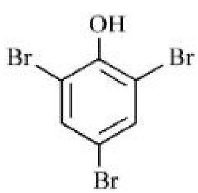
16. Identify the major product formed in the following sequence of reactions:



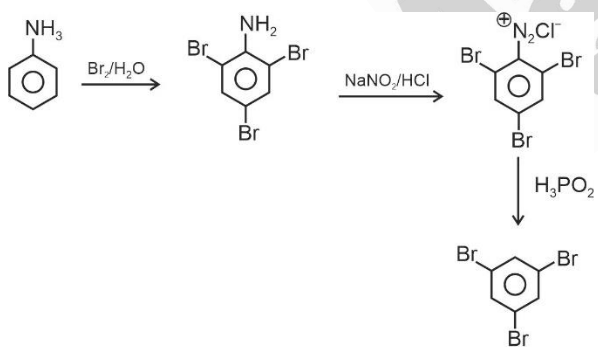
(A) 

(B) 

(C) 

(D) 

Answer (C)

Sol. 

17. A primary aliphatic amine on reaction with nitrous acid in cold (273 K) and there after raising temperature of reaction mixture to room temperature (298 K), gives

(A) nitrile (B) alcohol

(C) diazonium salt (D) secondary amine

Answer (B)

Sol. $R-NH_2 \xrightarrow[0°C]{HNO_2} [R-\overset{\oplus}{N_2}] \longrightarrow R-OH$

18. Which one of the following is **NOT** a copolymer?

(A) Buna-S

(B) Neoprene

(C) PHBV

(D) Butadiene-styrene

Answer (B)

Sol. Monomer of neoprene is 2-chloro-1,3-butadiene, and it is not a copolymer.

19. Stability of α-Helix structure of proteins depends upon

(A) dipolar interaction

(B) H-bonding interaction

(C) van der Walls forces

(D) π-stacking interaction

Answer (B)

Sol. Mostly H-bonding is responsible for the stability of α-helix form.

20. The formula of the purple colour formed in Laissaigne's test for sulphur using sodium nitroprusside is

(A) $NaFe[Fe(CN)_6]$

(B) $Na[Cr(NH_3)_2(NCS)_4]$

(C) $Na_2[Fe(CN)_5(NO)]$

(D) $Na_4[Fe(CN)_5(NOS)]$

Answer (D)

Sol. $S^{2-} + [Fe(CN)_5NO)]^{2-} \rightarrow [Fe(CN)_5(NOS)]^{4-}$

Purple

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. A 2.0 g sample containing MnO_2 is treated with HCl liberating Cl_2. The Cl_2 gas is passed into a solution of KI and 60.0 mL of 0.1 M $Na_2S_2O_3$ is required to titrate the liberated iodine. The percentage of MnO_2 in the sample is ______. (Nearest integer)

   [Atomic masses (in u) Mn = 55; Cl = 35.5; O = 16, I = 127, Na = 23, K = 39, S = 32]

Answer (13)

Sol. $MnO_2 + 4HCl \longrightarrow MnCl_2 + 2H_2O + Cl_2$

$$Cl_2 + KI \longrightarrow Cl^- + I_2$$

$$\downarrow 2Na_2S_2O_3$$

$$Na_2S_4O_6 + 2NaI$$

Equivalent of MnO_2 = HCl = Cl_2 = I_2 = $Na_2S_2O_3$

2 × number of moles of MnO_2 = 1 × number of moles of $Na_2S_2O_3$

Moles of MnO_2 = $\dfrac{60 \times 0.1 \times 10^{-3}}{2}$

$= 3 \times 10^{-3}$ mole

Mass of MnO_2 = 0.261 g

% of MnO_2 = $\dfrac{0.261}{2} \times 100 \simeq 13\%$

2. If the work function of a metal is 6.63×10^{-19} J, the maximum wavelength of the photon required to remove a photoelectron from the metal is ______ nm. (Nearest integer)

   [Given : h = 6.63×10^{-34} J s, and c = 3×10^8 m s^{-1}]

Answer (300)

Sol. $\because \quad E = \dfrac{hc}{\lambda}$

$$6.63 \times 10^{-19} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{\lambda}$$

$\lambda = 3 \times 10^{-7}$ m

$= 300 \times 10^{-9}$ m

$= 300$ nm

3. The hybridization of P exhibited in PF_5 is sp_xd_y. The value of y is _______

Answer (1)

Sol. PF_5 is sp^3d hybridised

y = 1

4. 4.0 L of an ideal gas is allowed to expand isothermally into vacuum until the total volume is 20 L. The amount of heat absorbed in this expansion is ____ L atm.

Answer (0)

Sol. Work done = $-P_{ext}\,\Delta v$

$\because \; P_{ext} = 0$ (vacuum)

$\therefore$ w = 0, $\Delta U = 0$ (as the process is isothermal)

So, q = 0

5. The vapour pressures of two volatile liquids A and B at 25°C are 50 Torr and 100 Torr, respectively. If the liquid mixture, contains 0.3 mole fraction of A, then the mole fraction of liquid B in the vapour phase is $\dfrac{x}{17}$. The value of x is _______.

Answer (14)

Sol. $P_T = P_A^0.x_A + P_B^0.x_B$

$= 50 \times 0.3 + 100 \times 0.7$

= 85 mm Hg

$$y_B = \frac{70}{85} = \frac{x}{17}$$

$$\frac{x}{17} = 14$$

6. The solubility product of a sparingly soluble salt A_2X_3 is 1.1×10^{-23}. If specific conductance of the solution is 3×10^{-5} S m^{-1}, the limiting molar conductivity of the solution is $x \times 10^{-3}$ S m^2 mol^{-1}. The value of x is _______.

Answer (3)

Sol. $A_2X_3 \rightleftharpoons 2A + 3X$

$\qquad\qquad 2S \quad 3S$

$K_{sp} = (2s)^2(3s)^3 = 1.1 \times 10^{-23}$

$S \approx 10^{-5}$

For sparingly soluble salts

$$\wedge_m = \wedge_m^o$$

$$\wedge_m = \frac{k}{S \times 10^3}$$

$$= \frac{3 \times 10^{-5}}{10^{-5}} \times 10^{-3}$$

$$= 3 \times 10^{-3} \text{ Sm}^2 \text{ mol}^{-1}$$

7. The quantity of electricity of Faraday needed to reduce 1 mol of $Cr_2O_7^{2-}$ to Cr^{3+} is

Answer (6)

Sol. $\overset{+6}{Cr_2}O_7^{2-} \longrightarrow 2Cr^{3+}$

$\because$ Each Cr is converting from +6 to +3

$\therefore$ 6 faradays of charge is required

8. For a first order reaction A → B, the rate constant, k = 5.5×10^{-14} s^{-1}. The time required for 67% completion of reaction is $x \times 10^{-1}$ times the half life of reaction. The value of x is _____ (Nearest integer)

Answer (16)

Sol. $\because$ $kt = \ln\frac{A_0}{A}$

$$\frac{\ln 2}{t_{\frac{1}{2}}} t_{67\%} = \ln\frac{A_0}{0.33A_0}$$

$$\frac{\log 2}{t_{\frac{1}{2}}} t_{67\%} = \log\frac{1}{0.33}$$

$t_{67\%} = 1.566\ t_{1/2}$

x = 15.66

Nearest integer = 16

9. Number of complexes which will exhibit synergic bonding amongst, $[Cr(CO)_6]$, $[Mn(CO)_5]$ and $[Mn_2(CO)_{10}]$ is ________.

Answer (3)

Sol. CO ligand shows synergic bonding, so all complexes can show synergic bonding.

10. In the estimation of bromine, 0.5 g of an organic compound gave 0.40 g of silver bromide. The percentage of bromine in the given compound is _____% (nearest integer)

(Relative atomic masses of Ag and Br are 108 u and 80 u, respectively).

Answer (34)

Sol. 188 g AgBr has 80 g of Br

$$\therefore \quad 0.4 \text{ g AgBr} = \frac{80}{188} \times 0.4$$

% of Br in given organic compound

$$= \frac{80 \times 0.4}{188 \times 0.5} \times 100$$

$\approx 34\%$

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. If $\sum_{k=1}^{31}\left({}^{31}C_k\right)\left({}^{31}C_{k-1}\right)-\sum_{k=1}^{30}\left({}^{30}C_k\right)\left({}^{30}C_{k-1}\right)=\frac{\alpha(60!)}{(30!)(31!)}$,

where $\alpha \in \mathbf{R}$, then the value of 16α is equal to

(A) 1411

(B) 1320

(C) 1615

(D) 1855

Answer (A)

Sol. $\sum_{k=1}^{31}{}^{31}C_k\,{}^{31}C_{k-1}-\sum_{k=1}^{30}{}^{30}C_k\,{}^{30}C_{k-1}$

$=\sum_{k=1}^{31}{}^{31}C_k\cdot{}^{31}C_{32-k}-\sum_{k=1}^{30}{}^{30}C_k\cdot{}^{30}C_{31-k}$

$= {}^{62}C_{32} - {}^{60}C_{31}$

$=\frac{60!}{31!29!}\left(\frac{62\cdot 61}{32\cdot 30}-1\right)=\frac{60!}{31!29!}\frac{2822}{32\cdot 30}$

$\alpha=\frac{2822}{32} \Rightarrow 16\alpha = 1411$

2. Let a function $f : \mathrm{N} \to \mathrm{N}$ be defined by

$$f(n)=\begin{cases}2n, & n=2,4,6,8,.....\\ n-1, & n=3,7,11,15,.....\\ \frac{n+1}{2}, & n=1,5,9,13,....\end{cases}$$

then, f is

(A) One-one but not onto

(B) Onto but not one-one

(C) Neither one-one nor onto

(D) One-one and onto

Answer (D)

Sol. When n = 1, 5, 9, 13 then $\frac{n+1}{2}$ will give all odd numbers.

When n = 3, 7, 11, 15 …

$n - 1$ will be even but not divisible by 4

When n = 2, 4, 6, 8, …

Then $2n$ will give all multiples of 4

So range will be N.

And no two values of n give same y, so function is one-one and onto.

3. If the system of linear equations

$2x + 3y - z = -2$

$x + y + z = 4$

$x - y + |\lambda|z = 4\lambda - 4$

where $\lambda \in$ R, has no solution, then

(A) $\lambda = 7$

(B) $\lambda = -7$

(C) $\lambda = 8$

(D) $\lambda^2 = 1$

Answer (B)

Sol. $\Delta=\begin{vmatrix}2 & 3 & -1\\ 1 & 1 & 1\\ 1 & -1 & |\lambda|\end{vmatrix}=0 \Rightarrow |\lambda| = 7$

But at $\lambda = 7$, $D_x = D_y = D_z = 0$

$P_1 : 2x + 3y - z = -2$

$P_2 : x + y + z = 4$

$P_3 : x - y + |\lambda|z = 4\lambda - 4$

So clearly $5P_2 - 2P_1 = P_3$, so at $\lambda = 7$, system of equation is having infinite solutions.

So $\lambda = -7$ is correct answer.

4. Let A be a matrix of order 3 × 3 and det (A) = 2. Then det (det (A) adj (5 adj (A^3))) is equal to ______.

(A) 512×10^6

(B) 256×10^6

(C) 1024×10^6

(D) 256×10^{11}

Answer (A)

Sol. $|A| = 2$

$||A|\ \text{adj}(5\ \text{adj}\ A^3)|$

$= |25|A|\ \text{adj}(\text{adj}\ A^3)|$

$= 25^3\ |A|^3 \cdot |\text{adj}\ A^3|^2$

$= 25^3 \cdot 2^3 \cdot |A^3|^4$

$= 25^3 \cdot 2^3 \cdot 2^{12} = 10^6 \cdot 512$

5. The total number of 5-digit numbers, formed by using the digits 1, 2, 3, 5, 6, 7 without repetition, which are multiple of 6, is

(A) 36 (B) 48

(C) 60 (D) 72

Answer (D)

Sol. Number should be divisible by 6 and it should be even.

Total sum = 1 + 2 + 3 + 5 + 6 + 7 = 24

So number removed should be of type 3.

C-1 : excluding 3 _ _ _ _ 2 ways = 4! × 2 = 48

C-2 : excluding 6 _ _ _ _ 1 way = 4! = 24

Total cases = 48 + 24 = 72

6. Let A_1, A_2, A_3, … be an increasing geometric progression of positive real numbers. If $A_1A_3A_5A_7 = \frac{1}{1296}$ and $A_2 + A_4 = \frac{7}{36}$, then, the value of $A_6 + A_8 + A_{10}$ is equal to

(A) 33 (B) 37

(C) 43 (D) 47

Answer (C)

Sol. $\frac{A_4}{r^3}.\frac{A_4}{r}.A_4r.A_4r^3 = \frac{1}{1296}$

$A_4 = \frac{1}{6}$

$A_2 = \frac{7}{36} - \frac{1}{6} = \frac{1}{36}$

So $A_6 + A_8 + A_{10} = 1 + 6 + 36$

$= 43$

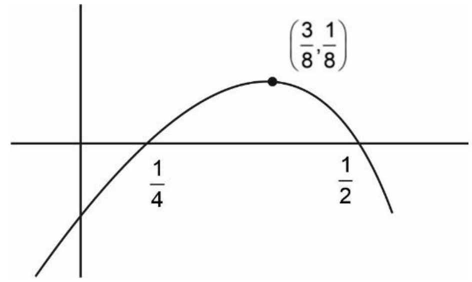
7. Let $[t]$ denote the greatest integer less than or equal to t. Then, the value of the integral $\int_0^1 \left[-8x^2 + 6x - 1\right]dx$ is equal to

(A) –1 (B) $\frac{-5}{4}$

(C) $\frac{\sqrt{17}-13}{8}$ (D) $\frac{\sqrt{17}-16}{8}$

Answer (C)

Sol. $\int_0^1 \left[-8x^2 + 6x - 1\right]dx$



$= \int_0^{\frac{1}{4}} (-1)dx + \int_{\frac{1}{4}}^{\frac{3}{4}} 0dx + \int_{\frac{1}{2}}^{\frac{3}{4}} -1dx + \int_{\frac{3}{4}}^{\frac{3+\sqrt{17}}{8}} -2dx + \int_{\frac{3+\sqrt{17}}{8}}^{1} -3dx$

$= -\frac{1}{4} - \frac{1}{4} - 2\left(\frac{3+\sqrt{17}}{8} - \frac{3}{4}\right) - 3\left(1 - \frac{3+\sqrt{17}}{8}\right)$

$= \frac{\sqrt{17}-13}{8}$

8. Let $f : \mathbb{R} \to \mathbb{R}$ be defined as

$$f(x) = \begin{cases} \left[e^x\right], & x < 0 \\ ae^x + [x-1], & 0 \le x < 1 \\ b + \left[\sin(\pi x)\right], & 1 \le x < 2 \\ \left[e^{-x}\right] - c, & x \ge 2 \end{cases}$$

Where $a, b, c \in \mathbb{R}$ and $[t]$ denotes greatest integer less than or equal to t. Then, which of the following statements is true?

(A) There exists $a, b, c \in \mathbb{R}$ such that f is continuous on $\mathbb{R}$.

(B) If f is discontinuous at exactly one point, then $a + b + c = 1$

(C) If f is discontinuous at exactly one point, then $a + b + c \neq 1$

(D) f is discontinuous at atleast two points, for any values of a, b and c

Answer (C)

Sol. $$f(x) = \begin{cases} 0 & x < 0 \\ ae^x - 1 & 0 \le x < 1 \\ b & x = 1 \\ b - 1 & 1 < x < 2 \\ -c & x \ge 2 \end{cases}$$

To be continuous at $x = 0$

$a - 1 = 0$

to be continuous at $x = 1$

$ae - 1 = b = b - 1 \Rightarrow$ not possible

to be continuous at $x = 2$

$b - 1 = -c \Rightarrow b + c = 1$

If $a = 1$ and $b + c = 1$ then $f(x)$ is discontinuous at exactly one point

9. The area of the region $S = \{(x, y) : y^2 \le 8x,\ y \ge \sqrt{2}x, x \ge 1\}$ is

(A) $\frac{13\sqrt{2}}{6}$

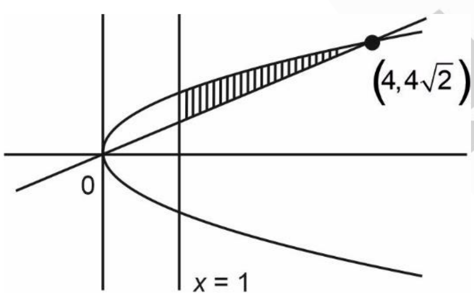
(B) $\frac{11\sqrt{2}}{6}$

(C) $\frac{5\sqrt{2}}{6}$

(D) $\frac{19\sqrt{2}}{6}$

Answer (B)

Sol.



Required area

$$= \int_1^4 \left(\sqrt{8x} - \sqrt{2}x\right)dx$$

$$= \frac{2\sqrt{8}}{3}x^{\frac{3}{2}} - \frac{x^2}{\sqrt{2}}\Bigg|_1^4$$

$$= \frac{16\sqrt{3}}{3} - \frac{16}{\sqrt{2}} - \frac{2\sqrt{8}}{3} + \frac{1}{\sqrt{2}}$$

$$= \frac{11\sqrt{2}}{6} \text{ sq. units}$$

10. Let the solution curve $y = y(x)$ of the differential equation

$$\left[\frac{x}{\sqrt{x^2 - y^2}} + e^{\frac{y}{x}}\right]x\frac{dy}{dx} = x + \left[\frac{x}{\sqrt{x^2 - y^2}} + e^{\frac{y}{x}}\right]y$$

pass through the points (1, 0) and $(2\alpha, \alpha)$, $\alpha > 0$. Then α is equal to

(A) $\frac{1}{2}\exp\left(\frac{\pi}{6} + \sqrt{e} - 1\right)$ (B) $\frac{1}{2}\exp\left(\frac{\pi}{3} + e - 1\right)$

(C) $\exp\left(\frac{\pi}{6} + \sqrt{e} + 1\right)$ (D) $2\exp\left(\frac{\pi}{3} + \sqrt{e} - 1\right)$

Answer (A)

Sol. $$\left(\frac{1}{\sqrt{1 - \frac{y^2}{x^2}}} + e^{\frac{y}{x}}\right)\frac{dy}{dx} = 1 + \left(\frac{1}{\sqrt{1 - \frac{y^2}{x^2}}} + e^{\frac{y}{x}}\right)\frac{y}{x}$$

Putting $y = tx$

$$\left(\frac{1}{\sqrt{1 - t^2}} + e^t\right)\left(t + x\frac{dt}{dx}\right) = 1 + \left(\frac{1}{\sqrt{1 - t^2}} + e^t\right)t$$

$$\Rightarrow x\left(\frac{1}{\sqrt{1 - t^2}} + e^t\right)\frac{dt}{dx} = 1$$

$\Rightarrow \sin^{-1} t + e^t = \ln x + C$

$$\Rightarrow \sin^{-1}\left(\frac{y}{x}\right) + e^{y/x} = \ln x + C$$

at $x = 1$, $y = 0$

So, $0 + e^0 = 0 + C \Rightarrow C = 1$

at $(2\alpha, \alpha)$

$$\sin^{-1}\left(\frac{y}{x}\right) + e^{y/x} = \ln x + 1$$

$$\Rightarrow \frac{\pi}{6} + e^{\frac{1}{2}} - 1 = \ln(2\alpha)$$

$$\Rightarrow \alpha = \frac{1}{2}e^{\left(\frac{\pi}{6} + e^{\frac{1}{2}} - 1\right)}$$

11. Let $y = y(x)$ be the solution of the differential equation $x\left(1 - x^2\right)\frac{dy}{dx} + \left(3x^2y - y - 4x^3\right) = 0$, $x > 1$, with $y(2) = -2$. Then $y(3)$ is equal to

(A) –18 (B) –12

(C) –6 (D) –3

Answer (A)

Sol. $\frac{dy}{dx}+\frac{y(3x^2-1)}{x(1-x^2)}=\frac{4x^3}{x(1-x^2)}$

$$IF=e^{\int\frac{3x^2-1}{x-x^3}dx}=e^{-\ln|x^3-x|}=e^{-\ln(x^3-x)}$$

$$=\frac{1}{x^3-x}$$

Solution of D.E. can be given by

$$y.\frac{1}{x^3-x}=\int\frac{4x^3}{x(1-x^2)}.\frac{1}{x(x^2-1)}dx$$

$$\Rightarrow\ \frac{y}{x^3-x}=\int\frac{-4x}{(x^2-1)^2}dx$$

$$\Rightarrow\ \frac{y}{x^3-x}=\frac{2}{(x^2-1)}+c$$

at $x = 2, y = -2$

$$\frac{-2}{6}=\frac{2}{3}+c\ \Rightarrow\ c=-1$$

$$\text{at } x=3\Rightarrow\frac{y}{24}=\frac{2}{8}-1\Rightarrow y=-18$$

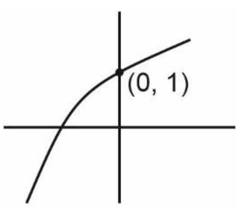
12. The number of real solutions of $x^7 + 5x^3 + 3x + 1 = 0$ is equal to ______.

(A) 0 (B) 1

(C) 3 (D) 5

Answer (B)

Sol.



$$f'(x)=7x^6+15x^2+3>0\ \forall x\in R$$

$f(x)$ is always increasing

So clearly it intersects

x-axis at only one point

13. Let the eccentricity of the hyperbola $H:\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be $\sqrt{\frac{5}{2}}$ and length of its latus rectum be $6\sqrt{2}$, If $y = 2x + c$ is a tangent to the hyperbola H. then the value of c^2 is equal to

(A) 18 (B) 20

(C) 24 (D) 32

Answer (B)

Sol. $1+\frac{b^2}{a^2}=\frac{5}{2}\Rightarrow\frac{b^2}{a^2}=\frac{3}{2}$

$$\frac{2b^2}{a}=6\sqrt{2}\Rightarrow 2.\frac{3}{2}.a=6\sqrt{2}$$

$$\Rightarrow\ a=2\sqrt{2}, b^2=12$$

$$c^2=a^2m^2-b^2=8.4-12=20$$

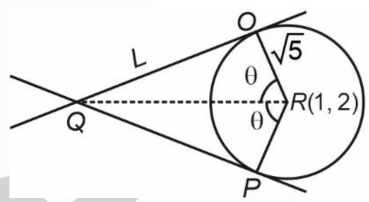
14. If the tangents drawn at the points $O(0, 0)$ and $P\left(1+\sqrt{5}, 2\right)$ on the circle $x^2 + y^2 - 2x - 4y = 0$ intersect at the point Q, then the area of the triangle OPQ is equal to

(A) $\frac{3+\sqrt{5}}{2}$ (B) $\frac{4+2\sqrt{5}}{2}$

(C) $\frac{5+3\sqrt{5}}{2}$ (D) $\frac{7+3\sqrt{5}}{2}$

Answer (C)

Sol.



$\frac{\sqrt{5}}{L}=\tan\theta$

$$\tan 2\theta=2\Rightarrow\frac{2\tan\theta}{1-\tan^2\theta}=2$$

$$\tan\theta=\frac{\sqrt{5}-1}{2}\qquad\text{(as }\theta\text{ is acute)}$$

$$\text{Area}=\frac{1}{2}L^2\sin 2\theta=\frac{1}{2}.\frac{5}{\tan^2\theta}.2\sin\theta\cos\theta$$

$$=\frac{5\sin\theta\cos\theta}{\sin^2\theta}.\cos^2\theta$$

$$=5\cot\theta.\cos^2\theta$$

$$=5.\frac{2}{\sqrt{5}-1}.\frac{1}{1+\left(\frac{\sqrt{5}-1}{2}\right)^2}$$

$$=\frac{10}{\sqrt{5}-1}.\frac{4}{4+6-2\sqrt{5}}$$

$$=\frac{40}{2\sqrt{5}(\sqrt{5}-1)^2}=\frac{4\sqrt{5}}{6-2\sqrt{5}}$$

$$=\frac{4\sqrt{5}(6+2\sqrt{5})}{16}$$

$$=\frac{\sqrt{5}(3+\sqrt{5})}{2}$$

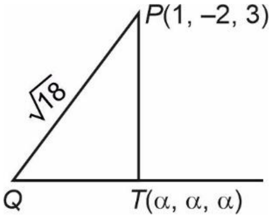
15. If two distinct points Q, R lie on the line of intersection of the planes $-x + 2y - z = 0$ and $3x - 5y + 2z = 0$ and $PQ = PR = \sqrt{18}$ where the point P is (1, –2, 3), then the area of the triangle PQR is equal to

(A) $\frac{2}{3}\sqrt{38}$ (B) $\frac{4}{3}\sqrt{38}$

(C) $\frac{8}{3}\sqrt{38}$ (D) $\sqrt{\frac{152}{3}}$

Answer (B)

Sol.



Line L is $x = y = z$

$\overrightarrow{PQ}.(\hat{i}+\hat{j}+\hat{k}) = 0$

$\Rightarrow (\alpha - 3) + \alpha + 2 + \alpha - 1 = 0$

$\Rightarrow \alpha = \frac{2}{3}$ so, $T = \left(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right)$

$PT = \sqrt{\frac{38}{3}}$

$\Rightarrow QT = \frac{4}{\sqrt{3}}$

So, Area $= \left(\frac{1}{2} \times \frac{4}{\sqrt{3}} \times \frac{\sqrt{38}}{\sqrt{3}}\right).2$

$= \frac{4\sqrt{38}}{3}$ sq units

16. The acute angle between the planes P_1 and P_2, when P_1 and P_2 are the planes passing through the intersection of the planes $5x + 8y + 13z - 29 = 0$ and $8x - 7y + z - 20 = 0$ and the points (2, 1, 3) and (0, 1, 2), respectively, is

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{12}$

Answer (A)

Sol. Family of Plane's equation can be given by

$(5 + 8\lambda)x + (8 - 7\lambda)y + (13 + \lambda)z - (29 + 20\lambda) = 0$

P_1 passes through (2, 1, 3)

$\Rightarrow (10 + 16\lambda) + (8 - 7\lambda) + (39 + 3\lambda) - (29 + 20\lambda) = 0$

$\Rightarrow -8\lambda + 28 = 0 \Rightarrow \lambda = \frac{7}{2}$

d.r, s of normal to P_1

$\left\langle 33, \frac{-33}{2}, \frac{33}{2}\right\rangle$ or $\left\langle 1, -\frac{1}{2}, \frac{1}{2}\right\rangle$

P_2 passes through (0, 1, 2)

$\Rightarrow 8 - 7\lambda + 26 + 2\lambda - (29 + 20\lambda) = 0$

$\Rightarrow 5 - 25\lambda = 0$

$\Rightarrow \lambda = \frac{1}{5}$

d.r, s of normal to P_2

$\left\langle \frac{33}{5}, \frac{33}{5}, \frac{66}{5}\right\rangle$ or $\langle 1, 1, 2\rangle$

Angle between normals

$$= \frac{\left(\hat{i} - \frac{1}{2}\hat{j} + \frac{1}{2}\hat{k}\right)\cdot\left(\hat{i}+\hat{j}+2\hat{k}\right)}{\frac{\sqrt{3}}{2} \qquad \sqrt{6}}$$

$$\cos\theta = \frac{1 - \frac{1}{2} + 1}{3} = \frac{1}{2}$$

$\theta = \frac{\pi}{3}$

17. Let the plane $P : \vec{r}\cdot\vec{a} = d$ contain the line of intersection of two planes $\vec{r}\cdot\left(\hat{i} + 3\hat{j} - \hat{k}\right) = 6$ and $\vec{r}\cdot\left(-6\hat{i} + 5\hat{j} - \hat{k}\right) = 7$. If the plane P passes through the point $\left(2, 3, \frac{1}{2}\right)$, then the value of $\frac{|13\vec{a}|^2}{d^2}$ is equal to

(A) 90

(B) 93

(C) 95

(D) 97

Answer (B)

Sol. $P_1: x + 3y - z = 6$

$P_2: -6x + 5y - z = 7$

Family of planes passing through line of intersection of P_1 and P_2 is given by $x(1 - 6\lambda) + y(3 + 5\lambda) + z(-1 - \lambda) - (6 + 7\lambda) = 0$

It passes through $\left(2, 3, \frac{1}{2}\right)$

So, $2(1-6\lambda)+3(3+5\lambda)+\frac{1}{2}(-1-\lambda)-(6+7\lambda)=0$

$\Rightarrow \quad 2-12\lambda+9+15\lambda-\frac{1}{2}-\frac{\lambda}{2}-6-7\lambda=0$

$\Rightarrow \quad \frac{9}{2}-\frac{9\lambda}{2}=0 \Rightarrow \lambda=1$

Required plane is

$-5x + 8y - 2z - 13 = 0$

Or $\vec{r}\cdot(-5\hat{i}+8\hat{j}-2\hat{k})=13$

$$\frac{|13\vec{a}|^2}{|d|^2}=\frac{13^2}{(13)^2}\cdot|\vec{a}|^2=93$$

18. The probability, that in a randomly selected 3-digit number at least two digits are odd, is

(A) $\frac{19}{36}$ (B) $\frac{15}{36}$

(C) $\frac{13}{36}$ (D) $\frac{23}{36}$

Answer (A)

Sol. Required cases = Total – all digits even – exactly one digit even

Total = 900 ways

All even $\Rightarrow$ 4 5 5 = 100 ways

One digit odd $\Rightarrow$ odd: 5 5 5 = 125 ways

4 odd: 5 5 = 100 ways

4 5 odd: 5 = 100 ways

Required probability $=\frac{900-425}{900}=\frac{19}{36}$

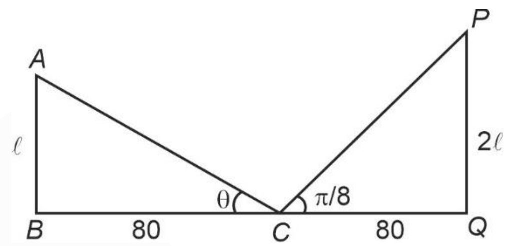
19. Let AB and PQ be two vertical poles, 160 m apart from each other. Let C be the middle point of B and Q, which are feet of these two poles. Let $\frac{\pi}{8}$ and θ be the angles of elevation from C to P and A, respectively. If the height of pole PQ is twice the height of pole AB, then $\tan^2\theta$ is equal to

(A) $\frac{3-2\sqrt{2}}{2}$ (B) $\frac{3+\sqrt{2}}{2}$

(C) $\frac{3-2\sqrt{2}}{4}$ (D) $\frac{3-\sqrt{2}}{4}$

Answer (C)

Sol.



$\frac{\ell}{80}=\tan\theta$ ...(i)

$\frac{2\ell}{80}=\tan\frac{\pi}{8}$ ...(ii)

From (i) and (ii)

$\frac{1}{2}=\frac{\tan\theta}{\tan\frac{\pi}{8}} \Rightarrow \tan^2\theta=\frac{1}{4}\tan^2\frac{\pi}{8}$

$\Rightarrow \quad \tan^2\theta=\frac{\sqrt{2}-1}{4\left(\sqrt{2}+1\right)}=\frac{3-2\sqrt{2}}{4}$

20. Let p, q, r be three logical statements. Consider the compound statements

S_1 : $((\sim p) \vee q) \vee ((\sim p) \vee r)$ and

S_2 : $p \rightarrow (q \vee r)$

Then, which of the following is **NOT** true?

(A) If S_2 is True, then S_1 is True

(B) If S_2 is False, then S_1 is False

(C) If S_2 is False, then S_1 is True

(D) If S_1 is False, then S_2 is False

Answer (C)

Sol. S_1: $(\sim p \vee q) \vee (\sim p \vee r)$

$\cong (\sim p \vee q \vee r)$

S_2: $\sim p \vee (q \vee r)$

Both are same

So, option (C) is incorrect.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Let R_1 and R_2 be relations on the set {1, 2, ..., 50} such that

   $R_1 = \{(p, p^n) : p$ is a prime and $n \geq 0$ is an integer} and

   $R_2 = \{(p, p^n) : p$ is a prime and $n = 0$ or 1}.

   Then, the number of elements in $R_1 - R_2$ is ______.

Answer (8)

Sol. $R_1 - R_2 = \{(2, 2^2), (2, 2^3), (2, 2^4), (2, 2^5), (3, 3^2)$ $(3, 3^3), (5, 5^2), (7, 7^2)\}$

So number of elements = 8

2. The number of real solutions of the equation $e^{4x} + 4e^{3x} - 58e^{2x} + 4e^x + 1 = 0$ is _____.

Answer (2)

Sol. Dividing by e^{2x}

$e^{2x} + 4e^x - 58 + 4e^{-x} + e^{-2x} = 0$

$\Rightarrow (e^x + e^{-x})^2 + 4(e^x + e^{-x}) - 60 = 0$

Let $e^x + e^{-x} = t \in [2, \infty)$

$\Rightarrow t^2 + 4t - 60 = 0$

$\Rightarrow t = 6$ is only possible solution

$e^x + e^{-x} = 6 \Rightarrow e^{2x} - 6e^x + 1 = 0$

Let $e^x = p$,

$p^2 - 6p + 1 = 0$

$$\Rightarrow p = \frac{3+\sqrt{5}}{2} \text{ OR } \frac{3-\sqrt{5}}{2}$$

$$\text{So } x = \ln\left(\frac{3+\sqrt{5}}{2}\right) \text{ OR } \ln\left(\frac{3-\sqrt{5}}{2}\right)$$

3. The mean and standard deviation of 15 observations are found to be 8 and 3 respectively. On rechecking it was found that, in the observations, 20 was misread as 5. Then, the correct variance is equal to _______.

Answer (17)

Sol. $\frac{\Sigma x_i^2}{15} - 8^2 = 9 \Rightarrow \Sigma x_i^2 = 15 \times 73 = 1095$

Let $\bar{x}_c$ be corrected mean $\bar{x}_c = 9$

$\Sigma x_c^2 = 1095 - 25 + 400 = 1470$

$$\text{Correct variance } = \frac{1470}{15} - (9)^2 = 98 - 81 = 17$$

4. If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ are coplanar vectors and $\vec{a} \cdot \vec{c} = 5$, $\vec{b} \perp \vec{c}$, then $122(c_1 + c_2 + c_3)$ is equal to _________.

Answer (150)

Sol. $2C_1 + C_2 + 3C_3 = 5$ …(i)

$3C_1 + 3C_2 + C_3 = 0$ …(ii)

$$[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} 2 & 1 & 3 \\ 3 & 3 & 1 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

$= 2(3C_3 - C_2) - 1(3C_3 - C_1) + 3(3C_2 - 3C_1)$

$= 3C_3 + 7C_2 - 8C_1$

$\Rightarrow 8C_1 - 7C_2 - 3C_3 = 0$ …(iii)

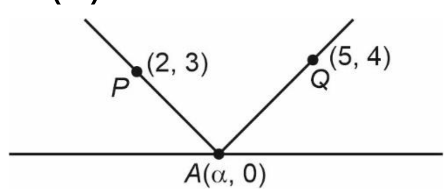
$$C_1 = \frac{10}{122}, C_2 = \frac{-85}{122}, C_3 = \frac{225}{122}$$

So $122(C_1 + C_2 + C_3) = 150$

5. A ray of light passing through the point $P(2, 3)$ reflects on the x-axis at point A and the reflected ray passes through the point $Q(5, 4)$. Let R be the point that divides the line segment AQ internally into the ratio 2 : 1. Let the co-ordinates of the foot of the perpendicular M from R on the bisector of the angle PAQ be (α, β). Then, the value of $7\alpha + 3\beta$ is equal to ___________.

Answer (31)

Sol.



$$\frac{4}{5-\alpha} = \frac{3}{\alpha-2} \Rightarrow 4\alpha - 8 = 15 - 3\alpha$$

$$\alpha = \frac{23}{7}$$

$$A = \left(\frac{23}{7}, 0\right) \quad Q = (5, 4)$$

$$R = \left(\frac{10+\frac{23}{7}}{3}, \frac{8}{3}\right)$$

$$= \left(\frac{31}{7}, \frac{8}{3}\right)$$

Bisector of angle PAQ is $X = \frac{23}{7}$

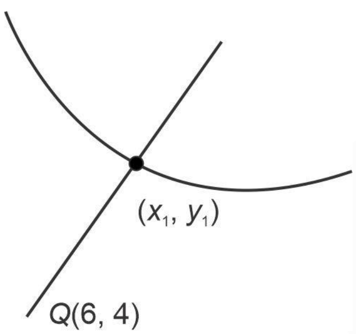
$$\Rightarrow \quad M = \left(\frac{23}{7}, \frac{8}{3}\right)$$

So, $7\alpha + 3\beta = 31$

6. Let l be a line which is normal to the curve $y = 2x^2 + x + 2$ at a point P on the curve. If the point $Q(6, 4)$ lies on the line l and O is origin, then the area of the triangle OPQ is equal to ___________.

Answer (13)

Sol.



$$\frac{y_1-4}{x_1-6} = -\frac{1}{4x_1+1}$$

$$\Rightarrow \quad \frac{2x_1^2+x_1-2}{x_1-6} = -\frac{1}{4x_1+1}$$

$$\Rightarrow \quad 6-x_1 = 8x_1^3+6x_1^2-7x_1-2$$

$$\Rightarrow \quad 8x_1^3+6x_1^2-6x_1-8 = 0$$

So $x_1 = 1 \Rightarrow y_1 = 5$

$$\text{Area} = \left|\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 6 & 4 & 1 \\ 1 & 5 & 1 \end{vmatrix}\right| = 13.$$

7. Let $A = \{1, a_1, a_2 \ldots a_{18}, 77\}$ be a set of integers with $1 < a_1 < a_2 < \ldots < a_{18} < 77$. Let the set $A + A = \{x + y : x, y \in A\}$ contain exactly 39 elements. Then, the value of $a_1 + a_2 + \ldots + a_{18}$ is equal to _____.

Answer (702)

Sol. If we write the elements of $A + A$, we can certainly find 39 distinct elements as $1 + 1, 1 + a_1, 1 + a_2, \ldots\ldots 1 + a_{18}, 1 + 77, a_1 + 77, a_2 + 77, \ldots\ldots a_{18} + 77, 77 + 77$.

It means all other sums are already present in these 39 values, which is only possible in case when all numbers are in A.P.

Let the common difference be 'd'.

$77 = 1 + 19d \Rightarrow d = 4$

$$\text{So, } \sum_{i=1}^{18} a_1 = \frac{18}{2}[2a_1 + 17d] = 9[10+68] = 702$$

8. The number of positive integers k such that the constant term in the binomial expansion of $\left(2x^3 + \frac{3}{x^k}\right)^{12}, x \neq 0$ is $2^8 \cdot \ell$, where ℓ is an odd integer, is______.

Answer (2)

Sol. $T_{r+1} = {}^{12}C_r \left(2x^3\right)^{12-r}\left(\frac{3}{x^k}\right)^r$

$$= {}^{12}C_r\, 2^{12-r} 3^r\, x^{36-3r-kr}$$

For constant term $36 - 3r - kr = 0$

$$r = \frac{36}{3+k}$$

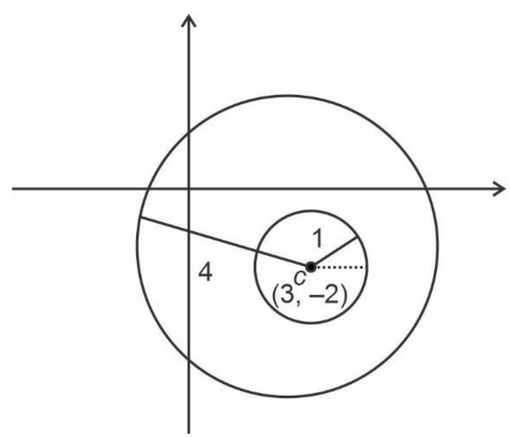
So, k can be 1, 3, 6, 9, 15, 33

In order to get 2^8, check by putting values of k and corresponding in general term. By checking, it is possible only where $k = 3$ or 6

9. The number of elements in the set $\{z = a + ib \in \mathbb{C} : a, b \in \mathbb{Z} \text{ and } 1 < |z - 3 + 2i| < 4\}$ is __________.

Answer (40)

Sol.



at line $y = -2$, we have $(5, -2)$ $(6, -2)$ $(1, -2)$ $(0, -2)$

$\Rightarrow$ 4 points

at line $y = -1$, we have (4, –1) (5, –1) (6, –1) (2, –1) (1, –1) (0, –1) $\Rightarrow$ 6 points

at line $y = 0$, we have (0, 0) (1, 0) (2, 0) (3, 0) (4, 0) (5, 0) (6, 0) $\Rightarrow$ 7 points

at line $y = 1$, we have (1, 1), (2, 1), (3, 1), (4, 1), (5, 1) i.e. 5 points

symmetrically

at line $y = -5$, we have 5 points

at line $y = -4$, we have 7 points

at line $y = -3$, we have 6 points

So Total integral points = 2(5 + 7 + 6) + 4

= 40

10. Let the lines $y+2x=\sqrt{11}+7\sqrt{7}$ and $2y+x=2\sqrt{11}+6\sqrt{7}$ be normal to a circle $C : (x - h)^2 + (y - k)^2 = r^2$. If the line $\sqrt{11}y-3x=\frac{5\sqrt{77}}{3}+11$ is tangent to the circle C, then the value of $(5h - 8k)^2 + 5r^2$ is equal to ________.

Answer (816)

Sol. $L_1 :\ y+2x=\sqrt{11}+7\sqrt{7}$

$L_2 :\ 2y+x=2\sqrt{11}+6\sqrt{7}$

Point of intersection of these two lines is centre of circle i.e. $\left(\frac{8}{3}\sqrt{7},\sqrt{11}+\frac{5}{3}\sqrt{7}\right)$

$\perp^r$ from centre to line $3x-\sqrt{11}y+\left(\frac{5\sqrt{77}}{3}+11\right)=0$ is radius of circle

$$\Rightarrow\quad r=\left|\frac{8\sqrt{7}-11-\frac{5}{3}\sqrt{77}+\frac{5\sqrt{77}}{3}+11}{\sqrt{20}}\right|$$

$$=\left|\sqrt[4]{\frac{7}{5}}\right|=\sqrt[4]{\frac{7}{5}}\text{ units}$$

So $(5h - 8K)^2 + 5r^2$

$$=\left(\frac{40}{3}\sqrt{7}-8\sqrt{11}-\frac{40}{3}\sqrt{7}\right)^2+5.16.\frac{7}{5}$$

= 64 × 11 + 112 = 816.

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