29/06/2022
Evening

Corporate Office : Aakash Tower, 8, Pusa Road, New Delhi-110005 | Ph.: 011-47623456

Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2022 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. A small toy starts moving from the position of rest under a constant acceleration. If it travels a distance of 10 m in *t* s, the distance travelled by the toy in the next *t* s will be :

(A) 10 m (B) 20 m

(C) 30 m (D) 40 m

Answer (C)

Sol. $\frac{1}{2}at^2 = 10$ m

$\frac{1}{2}a(2t)^2 = 40$ m

$\Rightarrow$ Distance travelled in next *t* s

= 40 – 10 = 30 m

2. At what temperature a gold ring of diameter 6.230 cm be heated so that it can be fitted on a wooden bangle of diameter 6.241 cm? Both the diameters have been measured at room temperature (27°C).

(Given: coefficient of linear thermal expansion of gold $\alpha_L = 1.4 \times 10^{-5}$ K^{-1})

(A) 125.7°C (B) 91.7°C

(C) 425.7°C (D) 152.7°C

Answer (D)

Sol. $\Delta D = D\alpha\Delta T$

$$\Delta T = \frac{0.011}{6.230 \times 1.4 \times 10^{-5}}$$

= 126.11°C

$\Rightarrow$ $T_f = T + \Delta T$

= (27 + 126.11)°C

= 153.11°C

$\Rightarrow$ Option (D) is correct

3. Two point charges *Q* each are placed at a distance *d* apart. A third point charge *q* is placed at a distance *x* from mid-point on the perpendicular bisector. The value of *x* at which charge *q* will experience the maximum Coulombs force is :

(A) $x = d$ (B) $x = \frac{d}{2}$

(C) $x = \frac{d}{\sqrt{2}}$ (D) $x = \frac{d}{2\sqrt{2}}$

Answer (D)

Sol. Force experienced by the charge *q*

$$F = \frac{kQqx}{\left[\left(\frac{d}{2}\right)^2 + x^2\right]^{\frac{3}{2}}}$$

For maximum Coulomb's force for *x*

$$\frac{dF}{dx} = 0$$

On solving $x = \frac{d}{2\sqrt{2}}$

4. The speed of light in media '*A*' and '*B*' are 2.0×10^{10} cm/s and 1.5×10^{10} cm/s respectively. A ray of light enters from the medium *B* to *A* at an incident angle 'θ'. If the ray suffers total internal reflection, then

(A) $\theta = \sin^{-1}\left(\frac{3}{4}\right)$ (B) $\theta > \sin^{-1}\left(\frac{2}{3}\right)$

(C) $\theta < \sin^{-1}\left(\frac{3}{4}\right)$ (D) $\theta > \sin^{-1}\left(\frac{3}{4}\right)$

Answer (D)

Sol. $\mu_A = \frac{3 \times 10^8}{2 \times 10^8} = 1.5$

$$\mu_B = \frac{3 \times 10^8}{1.5 \times 10^8} = 2$$

For *TIR*

$\theta > i_c$

$$\theta > \sin^{-1}\left(\frac{1.5}{2}\right)$$

$$\theta > \sin^{-1}\left(\frac{3}{4}\right)$$

5. In the following nuclear reaction,

$$D \xrightarrow{\alpha} D_1 \xrightarrow{\beta^-} D_2 \xrightarrow{\alpha} D_3 \xrightarrow{\gamma} D_4$$

Mass number of D is 182 and atomic number is 74. Mass number and atomic number of D_4 respectively will be ______

(A) 174 and 71

(B) 174 and 69

(C) 172 and 69

(D) 172 and 71

Answer (A)

Sol. Equivalent reaction can be written as

$D \longrightarrow D_4 + 2\alpha + \beta^- + \gamma$

$\Rightarrow$ Mass number of D_4 = Mass number of $D - 2 \times 4$

$= 182 - 8 = 174$

$\Rightarrow$ Atomic number of D_4

$=$ Atomic number of $D - 2 \times 2 + 1$

$= 74 - 4 + 1 = 71$

6. The electric field at a point associated with a light wave is given by

$E = 200[\sin(6 \times 10^{15})t + \sin(9 \times 10^{15})t]$ Vm^{-1}

Given : $h = 4.14 \times 10^{-15}$ eVs

If this light falls on a metal surface having a work function of 2.50 eV, the maximum kinetic energy of the photoelectrons will be

(A) 1.90 eV

(B) 3.27 eV

(C) 3.60 eV

(D) 3.42 eV

Answer (D)

Sol. Frequency of EM waves = $\frac{6}{2\pi}\times 10^{15}$ and $\frac{9}{2\pi}\times 10^{15}$

Energy of one photon of these waves

$= \left(4.14\times 10^{-15}\times\frac{6}{2\pi}\times 10^{15}\right)$ eV

and $\left(4.14\times 10^{-15}\times\frac{9}{2\pi}\times 10^{15}\right)$ eV

= 3.95 eV and 5.93 eV

$\Rightarrow$ Energy of maximum energetic electrons

= 5.93 – 2.50 = 3.43 eV

7. A capacitor is discharging through a resistor R. Consider in time t_1, the energy stored in the capacitor reduces to half of its initial value and in time t_2, the charge stored reduces to one eighth of its initial value. The ratio $\frac{t_1}{t_2}$ will be

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) $\frac{1}{6}$

Answer (D)

Sol. For a discharging capacitor when energy reduces to half the charge would become $\frac{1}{\sqrt{2}}$ times the initial value.

$$\Rightarrow \left(\frac{1}{2}\right)^{1/2} = e^{-t_1/\tau}$$

Similarly, $\left(\frac{1}{2}\right)^{3} = e^{-t_2/\tau}$

$$\Rightarrow \frac{t_1}{t_2} = \frac{1}{6}$$

8. Starting with the same initial conditions, an ideal gas expands from volume V_1 to V_2 in three different ways. The work done by the gas is W_1 if the process is purely isothermal, W_2, if the process is purely adiabatic and W_3 if the process is purely isobaric. Then, choose the correct option.

(A) $W_1 < W_2 < W_3$

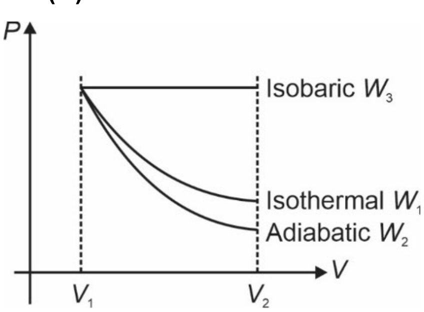
(B) $W_2 < W_3 < W_1$

(C) $W_3 < W_1 < W_2$

(D) $W_2 < W_1 < W_3$

Answer (D)

Sol.



Comparing the area under the PV graph

$A_3 > A_1 > A_2$

$\Rightarrow W_3 > W_1 > W_2$

9. Two long current carrying conductors are placed parallel to each other at a distance of 8 cm between them. The magnitude of magnetic field produced at mid-point between the two conductors due to current flowing in them is 30 μT. The equal current flowing in the two conductors is:

(A) 30 A in the same direction

(B) 30 A in the opposite direction

(C) 60 A in the opposite direction

(D) 300 A in the opposite direction

Answer (B)

Sol. As $B_{net} \neq 0$ that is the wires are carrying current in opposite direction.

$$\frac{\mu_0 I \times 2}{2\pi(4\times10^{-2})} = 30\times10^{-6} \text{ T}$$

$$\Rightarrow \quad I = \frac{30\times10^{-6}}{10^{-6}} \text{A} \text{ = 30 A in opposite direction.}$$

10. The time period of a satellite revolving around earth in a given orbit is 7 hours. If the radius of orbit is increased to three times its previous value, then approximate new time period of the satellite will be

(A) 40 hours (B) 36 hours

(C) 30 hours (D) 25 hours

Answer (B)

Sol. $T_2^2 = \left(\frac{R_2}{R_1}\right)^3 T_1^2$

$$\Rightarrow \quad T_2 = (3)^{3/2}\times7 \approx 5.2\times7$$

$$T_2 \cong 36 \text{ hrs}$$

11. The TV transmission tower at a particular station has a height of 125 m. For doubling the coverage of its range, the height of the tower should be increased by

(A) 125 m (B) 250 m

(C) 375 m (D) 500 m

Answer (C)

Sol. Range $R = \sqrt{2h\text{Re}}$

Let the height be h' to double the range so

$2R = \sqrt{2h'\text{Re}}$

On solving $h' = 4h$

$h' = 500$ m

So $\Delta h = 375$ m

12. The motion of a simple pendulum executing S.H.M. is represented by the following equation.

$y = A\sin(\pi t + \phi)$, where time is measured in second.

The length of pendulum is

(A) 97.23 cm (B) 25.3 cm

(C) 99.4 cm (D) 406.1 cm

Answer (C)

Sol. $\omega = \pi = \sqrt{\frac{g}{\ell}}$

So $\ell = \frac{g}{\pi^2}$

$\simeq$ 99.4 cm (Nearest value)

13. A vessel contains 16 g of hydrogen and 128 g of oxygen at standard temperature and pressure. The volume of the vessel in cm^3 is:

(A) 72×10^5 (B) 32×10^5

(C) 27×10^4 (D) 54×10^4

Answer (C)

Sol. Total number of moles are

$$n = n_{H_2} + n_{O_2}$$

$$= \frac{16}{2} + \frac{128}{32}$$

= 12 moles

Using $PV = nRT$

$$V = \frac{nRT}{P}$$

$$= \frac{12\times8.31\times273.15}{10^5} \text{ m}^3$$

$= 0.27 \text{ m}^3 = 27 \times 10^4 \text{ cm}^3$

14. Given below are two statements:

Statement I: The electric force changes the speed of the charged particle and hence changes its kinetic energy; whereas the magnetic force does not change the kinetic energy of the charged particle.

Statement II: The electric force accelerates the positively charged particle perpendicular to the direction of electric field. The magnetic force accelerates the moving charged particle along the direction of magnetic field.

In the light of the above statements, choose the most appropriate answer from the options given below:

(A) Both statement I and statement II are correct

(B) Both statement I and statement II are incorrect

(C) Statement I is correct but statement II is incorrect

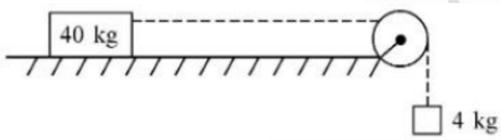
(D) Statement I is incorrect but statement II is correct

Answer (C)

Sol. Electric field accelerates the particle in the direction of field $\left(\vec{F} = q\vec{E} = m\vec{a}\right)$ and magnetic field accelerates the particle perpendicular to the field $\left(\vec{F} = q\vec{v} \times \vec{B} = m\vec{a}\right)$

15. A block of mass 40 kg slides over a surface, when a mass of 4 kg is suspended through an inextensible massless string passing over frictionless pulley as shown below.

The coefficient of kinetic friction between the surface and block is 0.02. The acceleration of block is (Given g = 10 ms^{-2}.)



(A) 1 ms^{-2}

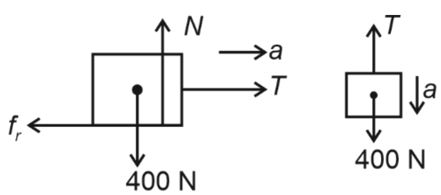
(B) 1/5 ms^{-2}

(C) 4/5 ms^{-2}

(D) 8/11 ms^{-2}

Answer (D)

Sol.



$f_{r_{max}} = \mu N$

$= 0.02 \times 400$

$= 8$ N

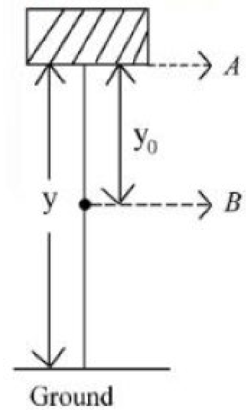
Let the acceleration is a as shown then.

$40 - T = 4a$

$T - 8 = 40a$

$\Rightarrow \quad a = \frac{32}{44} = \frac{8}{11}$ m/s^2

16. In the given figure, the block of mass m is dropped from the point 'A'. The expression for kinetic energy of block when it reaches point 'B' is



(A) $\frac{1}{2}mgy_0^2$ (B) $\frac{1}{2}mgy^2$

(C) $mg(y - y_0)$ (D) mgy_0

Answer (D)

Sol. Loss is potential energy = gain in kinetic energy

$-(mg(y - y_0) - mgy) = KE - 0$

$\Rightarrow KE = mgy_0$

17. A block of mass M placed inside a box descends vertically with acceleration 'a'. The block exerts a force equal to one-fourth of its weight on the floor of the box.

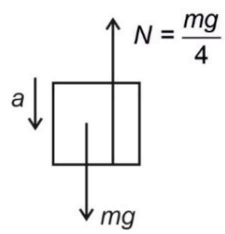
The value of 'a' will be

(A) $\frac{g}{4}$ (B) $\frac{g}{2}$

(C) $\frac{3g}{4}$ (D) g

Answer (C)

Sol.



Using Newton's second law

$$mg - \frac{mg}{4} = ma$$

$$\Rightarrow a = \frac{3g}{4}$$

18. If the electric potential at any point (x, y, z)m in space is given by $V = 3x^2$ volt.

The electric field at the point (1, 0, 3)m will be

(A) 3 Vm^{-1}, directed along positive x-axis

(B) 3 Vm^{-1}, directed along negative x-axis

(C) 6 Vm^{-1}, directed along positive x-axis

(D) 6 Vm^{-1}, directed along negative x-axis

Answer (D)

Sol. $\vec{E} = -\frac{dV}{dx}\hat{i}$

$\vec{E} = -6x\hat{i}$

So, $\vec{E}$ at (1, 0, 3) is

$\vec{E} = -6\hat{i}$ V/m

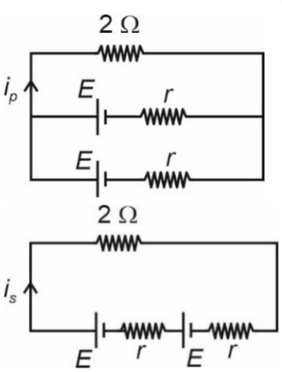
19. The combination of two identical cells, whether connected in series or parallel combination provides the same current through an external resistance of 2 Ω. The value of internal resistance of each cell is

(A) 2 Ω (B) 4 Ω

(C) 6 Ω (D) 8 Ω

Answer (A)

Sol.



From diagram

$i_p = \frac{E}{2+\frac{r}{2}}$ and $i_s = \frac{2E}{2+2r}$

given $i_p = i_s$

$\frac{1}{2+\frac{r}{2}} = \frac{1}{1+r}$

$1 + r = 2 + \frac{r}{2}$

$r = 2\ \Omega$

20. A person can throw a ball upto a maximum range of 100 m. How high above the ground he can throw the same ball?

(A) 25 m (B) 50 m

(C) 100 m (D) 200 m

Answer (B)

Sol. $R_{max} = \frac{u^2}{g} = 100$ m

So, $H_{max} = \frac{u^2}{2g} = 50$ m

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. The vernier constant of Vernier callipers is 0.1 mm and it has zero error of (–0.05) cm. While measuring diameter of a sphere, the main scale reading is 1.7 cm and coinciding vernier division is 5. The corrected diameter will be ________ × 10^{-2} cm.

Answer (180)

Sol. Since zero error is negative, we will add 0.05 cm.

$\Rightarrow$ Corrected reading = 1.7 cm + 5 × 0.1 mm + 0.05 cm

= 180 × 10^{-2} cm

2. A small spherical ball of radius 0.1 mm and density 10^4 kg m^{-3} falls freely under gravity through a distance h before entering a tank of water. If, after entering the water the velocity of ball does not change and it continue to fall with same constant velocity inside water, then the value of h will be ______ m.

(Given g = 10 ms^{-2}, viscosity of water = 1.0 × 10^{-5} N-sm^{-2}).

Answer (20)

Sol. $\sqrt{2gh}$ = terminal speed

$$\Rightarrow \sqrt{2gh} = \frac{2}{9}\frac{r^2 g(\rho-\rho')}{\eta}$$

$$= \frac{2}{9}\times\frac{10^{-8}\times 10\times 9000}{10^{-5}}$$

$$\Rightarrow h = \frac{400}{2g}$$

$\Rightarrow h$ = 20 m

3. In an experiment to determine the velocity of sound in air at room temperature using a resonance tube, the first resonance is observed when the air column has a length of 20.0 cm for a tuning fork of frequency 400 Hz is used. The velocity of the sound at room temperature is 336 ms^{-1}. The third resonance is observed when the air column has a length of _______ cm

Answer (104)

Sol. $400 = \dfrac{v}{4(L_1+e)}$ ...(i)

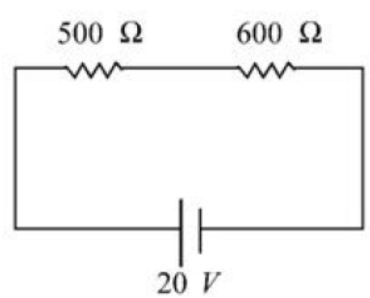
$400 = \dfrac{5v}{4(L_2+e)}$ ...(ii)

$$\Rightarrow L_1 + e = \frac{\lambda}{4} = 21\,\text{cm}$$

$$L_2 + e = \frac{5\lambda}{4} = 105\text{ cm}$$

$\Rightarrow$ e = 1 cm & L_2 = 104 cm

4. Two resistors are connected in series across a battery as shown in figure. If a voltmeter of resistance 2000 Ω is used to measure the potential difference across 500 Ω resistor, the reading of the voltmeter will be _____ V



Answer (8)

Sol. New $R_{eff} = \dfrac{2000\times 500}{2500} + 600\ \Omega = 1000\ \Omega$

$\Rightarrow$ Reading of voltmeter = $\dfrac{400}{1000}\times 20 = 8$ volts

5. A potential barrier of 0.4 V exists across a p-n junction. An electron enters the junction from the n-side with a speed of 6.0 × 10^{5} ms^{-1}. The speed with which electrons enters the p side will be $\dfrac{x}{3}\times 10^5$ ms^{-1} the value of x is __________.

(Give mass of electron = 9 × 10^{-31} kg, charge on electron = 1.6 × 10^{-19} C)

Answer (14)

Sol. Conserving energy,

$$\frac{1}{2}mv^2 = \frac{1}{2}m(6\times 10^5)^2 - 0.4\,\text{eV}$$

$$\Rightarrow v = \sqrt{(6\times 10^5)^2 - \frac{2\times 1.6\times 10^{-19}\times 0.4}{9\times 10^{-31}}}$$

$$= \sqrt{36\times 10^{10} - \frac{1.28}{9}\times 10^{12}}$$

$$\Rightarrow v = \frac{14}{3}\times 10^5\text{ m/s}$$

$\Rightarrow$ x = 14

6. The displacement current of 4.425 μA is developed in the space between the plates of parallel plate capacitor when voltage is changing at a rate of 10^{6} Vs^{-1}. The area of each plate of the capacitor is 40 cm^{2}. The distance between each plate of the capacitor x × 10^{-3} m. The value of x is,

(Permittivity of free space, E_0 = 8.85 × 10^{-12} C^{2} N^{-1} m^{-2})

Answer (8)

Sol. 4.425 μA = $\dfrac{E_0 A}{d}\times\dfrac{dV}{dt}$

$$\Rightarrow d = \frac{8.85\times 10^{-12}\times 40\times 10^{-4}}{4.425\times 10^{-6}}\times 10^6$$

$\Rightarrow$ d = 8 × 10^{-3} m

$\Rightarrow$ x = 8

7. The moment of inertia of a uniform thin rod about a perpendicular axis passing through one end is I_1. The same rod is bent into a ring and its moment of inertia about a diameter is I_2. If $\frac{I_1}{I_2}$ is $\frac{x\pi^2}{3}$, then the value of x will be ______.

Answer (8)

Sol. $I_1 = \frac{ML^2}{3}$ ....(1)

For ring : $I_2 = \frac{MR^2}{2}$

and $2\pi R = L$

$\Rightarrow I_2 = \frac{M}{2}\left(\frac{L^2}{4\pi^2}\right)$ ....(2)

$\Rightarrow \frac{I_1}{I_2} = \frac{8\pi^2}{3}$

$\Rightarrow x = 8$

8. The half life of a radioactive substance is 5 years. After x years, a given sample of the radioactive substance gets reduced to 6.25% of its initial value. The value of x is ______

Answer (20)

Sol. $N = N_0 e^{-\lambda t}$

$\Rightarrow \frac{6.25}{100} = e^{-\lambda t}$

$\Rightarrow e^{-\lambda t} = \frac{1}{16} = \left(\frac{1}{2}\right)^4$

$\Rightarrow t = 4t_{1/2}$

$\Rightarrow t = 20$ years

9. In a double slit experiment with monochromatic light, fringes are obtained on a screen placed at some distance from the plane of slits. If the screen is moved by 5×10^{-2} m towards the slits, the change in fringe width is 3×10^{-3} cm. If the distance between the slits is 1 mm, then the wavelength of the light will be _______ nm.

Answer (600)

Sol. Fringe width $\beta = \frac{\lambda D}{d}$

$\Rightarrow |d\beta| = \frac{\lambda}{d}|d(D)|$

$\Rightarrow 3 \times 10^{-3}\,\text{cm} = \frac{\lambda}{1\,\text{mm}}(5 \times 10^{-2}\,\text{m})$

$\Rightarrow \lambda = \frac{3 \times 10^{-8}}{5 \times 10^{-2}}$ m

$\Rightarrow \lambda = 600$ nm

10. An inductor of 0.5 mH, a capacitor of 200 μF and a resistor of 2 Ω are connected in series with a 220 V ac source. If the current is in phase with the emf, the frequency of ac source will be ______ × 10^2 Hz.

Answer (5)

Sol. Current will be in phase with emf when

$\omega L = \frac{1}{\omega C}$

$\Rightarrow \omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5 \times 10^{-4} \times 2 \times 10^{-4}}}$

$\Rightarrow \omega = \frac{10^4}{\sqrt{10}}$ rad/s

$\Rightarrow f = \frac{1}{2\pi} \times \frac{10^4}{\sqrt{10}}$ Hz

$\Rightarrow f \simeq 500$ Hz

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Using the rules for significant figures, the correct answer for the expression $\frac{0.02858 \times 0.112}{0.5702}$ will be

   (A) 0.005613

   (B) 0.00561

   (C) 0.0056

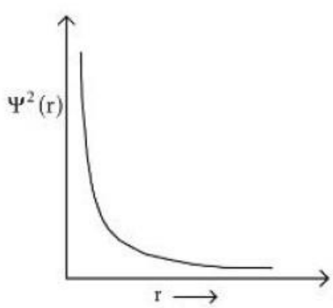
   (D) 0.006

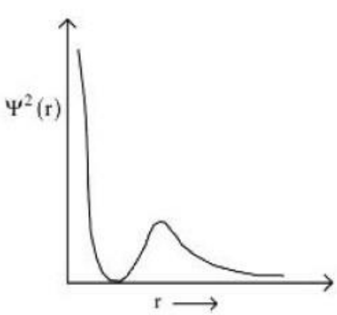
Answer (B)

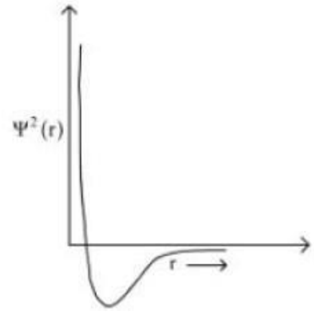
Sol. $\frac{0.02858 \times 0.112}{0.5702} = .00561$

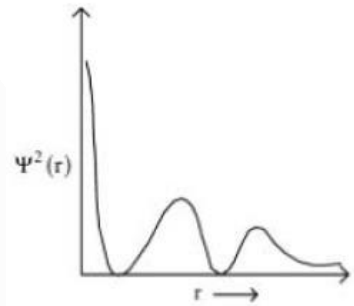
Answer expressed in 3 significant figures.

2. Which of the following is the correct plot for the probability density $\psi^2(r)$ as a function of distance 'r' of the electron from the nucleus for 2s orbital?

   (A)


   (B)


   (C)


   (D)


Answer (B)

Sol. 2s

radial node = n – l – 1

= 2 – 0 – 1

= 1

It will have one radial node

3. Consider the species CH_4, NH_4^+ and BH_4^-. Choose the correct option with respect to these species.

   (A) They are isoelectronic and only two have tetrahedral structures

   (B) They are isoelectronic and all have tetrahedral structures.

   (C) Only two are isoelectronic and all have tetrahedral structures.

   (D) Only two are isoelectronic and only two have tetrahedral structures.

Answer (B)

Sol. CH_4, NH_4^+ and BH_4^- are isoelectronic as well as tetrahedral.

4. 4.0 moles of argon and 5.0 moles of PCl_5 are introduced into an evacuated flask of 100 litre capacity at 610 K. The system is allowed to equilibrate. At equilibrium, the total pressure of mixture was found to be 6.0 atm. The K_p for the reaction is [Given : R = 0.082 L atm K^{-1} mol^{-1}]

(A) 2.25 (B) 6.24

(C) 12.13 (D) 15.24

Answer (A)

Sol. $PCl_5 \rightleftharpoons PCl_3 + Cl_2$

t = 0	5	0	0
t = t	5 – n	n	n

Total moles = 5 – n + n + n

= 5 + n.

For Argon

$n_{Ar} = 4$

Total moles = $n_{Ar} + nPCl_5 + nPCl_3 + nPCl_2$

= 4 + 5 + n

= 9 + n

$K_p = \frac{P_{PCl_3}.PCl_2}{P_{PCl_5}}$ PV = nRT

$6 \times 100 = (9 + n) \times 0.082 \times 610$

n = 3

$= \frac{\left(\frac{3}{12}\times 6\right)\times\left(\frac{3}{12}\times 6\right)}{\frac{2}{12}\times 6}$

$= \frac{27}{12} = \frac{9}{4} = 2.25$ atm

5. A 42.12% (w, v) solution of NaCl causes precipitation of a certain sol in 10 hours. The coagulating value of NaCl for the sol is

[Given : Molar mass : Na = 23.0 g mol^{-1}; Cl = 35.5 g mol^{-1}]

(A) 36 mmol L^{-1} (B) 36 mol L^{-1}

(C) 1440 mol L^{-1} (D) 1440 mmol L^{-1}

Answer (Bonus)

Sol. Data is insufficient.

6. Given below are two statements. One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: The first ionization enthalpy for oxygen is lower than that of nitrogen.

Reason R: The four electrons in 2p orbitals of oxygen experience more electron-electron repulsion.

In the light of the above statements, choose the *correct* answer from the options given below.

(A) Both A and R are correct and Rj is the correct explanation of A

(B) Both A and R are correct but R is NOT the correct explanation of A

(C) A is correct but R is not correct

(D) A is not correct but R is correct

Answer (B)

Sol. Nitrogen has half filled p-orbitals which is stable. Due to this it's 1st ionization energy is more than oxygen.

7. Match **List-I** with **List-II**

List-I Ore	**List-II Composition**
A. Siderite	I. $FeCO_3$
B. Malachite	II. $CuCO_3.\ Cu(OH)_2$
C. Sphalerite	III. ZnS
D. Calamine	IV. $ZnCO_3$

Choose the correct answer from the options given below:

(A) A-I, B-II, C-III, D-IV (B) A-III, B-IV, C-II, D-I

(C) A-IV, B-III, C-I, D-II (D) A-I, B-II, C-IV, D-III

Answer (A)

Sol. Siderite → $FeCO_3$

Malachite → $CuCO_3.\ Cu(OH)_2$

Sphalerite → ZnS

Calamine → $ZnCO_3$

8. Given below are two statements.

Statement-I: In $CuSO_4.5H_2O$, Cu-O bonds are present.

Statement-II: In $CuSO_4.5H_2O$, ligands coordinating with Cu(II) ion are O-and S-based ligands.

In the light of the above statements, choose the correct answer from the options given below:

(A) Both Statement-I and Statement-II are correct

(B) Both Statement-I and Statement-II are incorrect

(C) Statement-I is correct but Statement-II is incorrect

(D) Statement-I is incorrect but Statement-II is correct.

Answer (C)

Sol. Statement I is true but statement II is false. Only oxygen atom forms Co-ordinate bond with Cu^{+2} in $CuSO_4.5H_2O$

9. Amongst baking soda, caustic soda and washing soda, carbonate anion is present in

(A) Washing soda only

(B) Washing soda and caustic soda only

(C) Washing soda and baking soda only

(D) Baking soda, caustic soda and washing soda

Answer (A)

Sol. CO_3^{-2} ion is present only in washing soda.

10. Number of lone pair(s) of electrons on central atom and the shape of BrF_3 molecule respectively, are

(A) 0, triangular planar

(B) 1, pyramidal

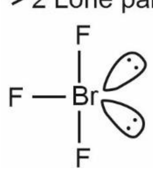
(C) 2, bent T-shape

(D) 1, bent T-shape

Answer (C)

Sol. $:\ddot{B}rF_3$ Shape is bent T-shape

↳ 2 Lone pairs



11. Aqueous solution of which of the following boron compounds will be strongly basic in nature?

(A) $NaBH_4$

(B) $LiBH_4$

(C) B_2H_6

(D) $Na_2B_4O_7$

Answer (D)

Sol. $Na_2B_4O_7 + 7H_2O \longrightarrow 2H_3BO_3 + 2Na[B(OH)_4]$

Aqueous solution of borax is buffer whose $pH \simeq 9$

Other compounds are less basic than this.

12. Sulphur dioxide is one of the components of polluted air. SO_2 is also a major contributor to acid rain. The correct and complete reaction to represent acid rain caused by SO_2 is

(A) $2SO_2 + O_2 \rightarrow 2SO_3$

(B) $SO_2 + O_3 \rightarrow SO_3 + O_2$

(C) $SO_2 + H_2O_2 \rightarrow H_2SO_4$

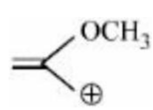
(D) $2SO_2 + O_2 + 2H_2O \rightarrow 2H_2SO_4$

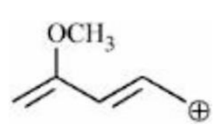
Answer (D)

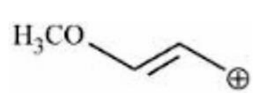
Sol. $2SO_2 + O_2 + 2H_2O \longrightarrow 2H_2SO_4$

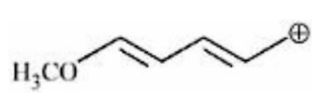
Acid rain occurs due to increased concentration of oxides of sulphur and Nitrogen.

13. Which of the following carbocations is most stable?

(A) 

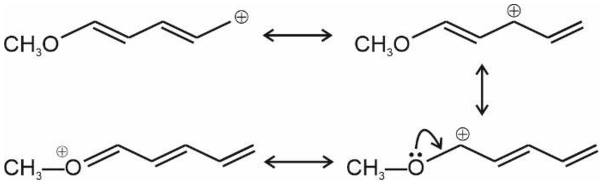
(B) 

(C) 

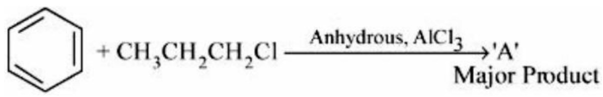
(D) 

Answer (D)

Sol.



14.



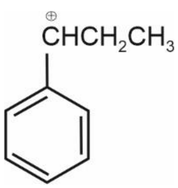
The stable carbocation formed in the above reaction is

(A) $CH_3CH_2\overset{\oplus}{C}H_2$

(B) $CH_3\overset{\oplus}{C}H_2$

(C) $CH_3-\overset{\oplus}{C}H-CH_3$

(D)

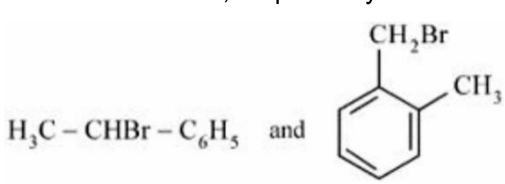


Answer (C)

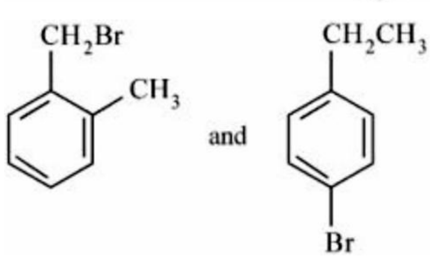
Sol. Initially $CH_3-CH_2-\overset{+}{C}H_2$ is formed. On rearrangement $CH_3-\overset{+}{C}H-CH_3$ stable carbocation is formed.

15. Two isomers (A) and (B) with Molar mass 184 g/mol and elemental composition C, 52.2%; H, 4.9 % and Br 42.9% gave benzoic acid and p-bromobenzoic acid, respectively on oxidation with $KMnO_4$. Isomer 'A' is optically active and gives a pale yellow precipitate when warmed with alcoholic $AgNO_3$. Isomers 'A' and 'B' are, respectively

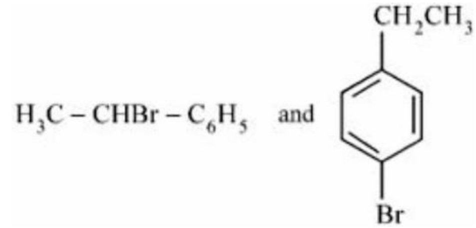
(A)



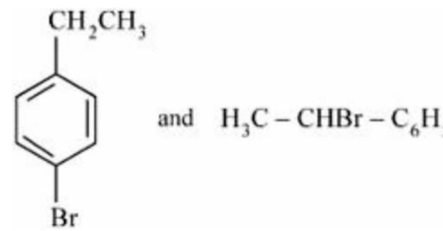
(B)



(C)



(D)



Answer (C)

Sol.

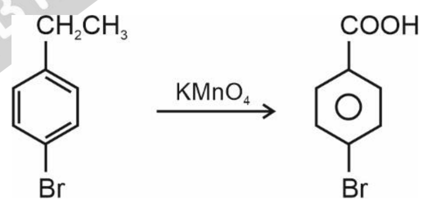
	moles	relative ratio		simplest ratio
C	52.2 = 52.2 / 12 =	4.35	→	8.7
H	4.9 = 4.9 / 1 =	4.9	→	9.8
Br	42.9 = 42.9 / 80 =	0.5	→	1

C_8H_9Br

A is optically active

$CH_3-\overset{Br}{\underset{*}{C}H}-C_6H_5$

B forms para bromo benzoic acid on reaction with $KMnO_4$.



16. In Friedel-Crafts alkylation of aniline, one gets

(A) Alkylated product with ortho and para substitution.

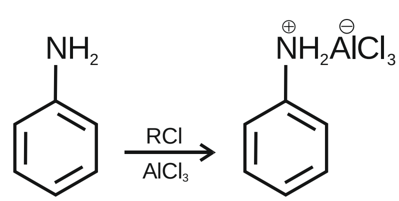
(B) Secondary amine after acidic treatment.

(C) An amide product.

(D) Positively charged nitrogen at benzene ring.

Answer (D)

Sol.



17. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : Dacron is an example of polyester polymer.

Reason R : Dacron is made up of ethylene glycol and terephthalic acid monomers.

In the light of the above statements, choose the **most appropriate** answer from the options given below.

(A) Both **A** and **R** are correct and **R** is the correct explanation of **A**.

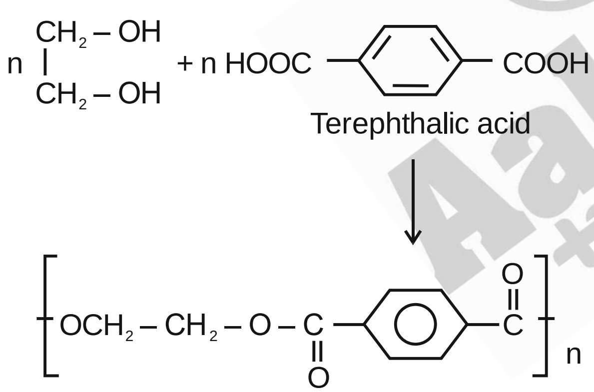
(B) Both **A** and **R** are correct but **R** is NOT the correct explanation of **A**.

(C) **A** is correct but **R** is not correct.

(D) **A** is not correct but **R** is correct.

Answer (A)

Sol.



Dacron

18. The structure of protein that is unaffected by heating is

(A) Secondary Structure

(B) Tertiary Structure

(C) Primary Structure

(D) Quaternary Structure

Answer (C)

Sol. Primary structure is unaffected by heating

19. The mixture of chloroxylenol and terpineol is an example of

(A) Antiseptic

(B) Pesticide

(C) Disinfectant

(D) Narcotic analgesic

Answer (A)

Sol. Mixture of chloroxylenol and terpineol is known as Dettol. It acts as Antiseptic

20. A white precipitate was formed when $BaCl_2$ was added to water extract of an inorganic salt. Further, a gas 'X' with characteristic odour was released when the formed white precipitate was dissolved in dilute HCl. The anion present in the inorganic salt is

(A) I^-

(B) SO_3^{2-}

(C) S^{2-}

(D) NO_2^-

Answer (B)

Sol. Anion is SO_3^{-2}

$$BaSO_3 \xrightarrow{\text{dil HCl}} \underset{X(gas)}{SO_2} \uparrow$$

Gas is released with smell of burning sulphur.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. A box contains 0.90 g of liquid water in equilibrium with water vapour at 27°C. The equilibrium vapour pressure of water at 27°C is 32.0 Torr. When the volume of the box is increased, some of the liquid water evaporates to maintain the equilibrium pressure. If all the liquid water evaporates, then the volume of the box must be ______ litre. [nearest integer]

(Given : R = 0.082 L atm K^{-1} mol^{-1}]

(Ignore the volume of the liquid water and assume water vapours behave as an ideal gas.

Answer (29)

Sol. $H_2O(l) \rightleftharpoons H_2O(g)$

$t = t_{eq} \quad \frac{0.90}{18} - x \qquad x$

PV = nRT

$\frac{32}{760} \times V = .082 \times (x) \times 300$

$x = \frac{0.90}{18}$

$V = .082 \times \frac{0.90}{18} \times \frac{300 \times 760}{32}$

≈ 29.21 L

≈ 29 L

2. 2.2 g of nitrous oxide (N_2O) gas is cooled at a constant pressure of 1 atm from 310 K to 270 K causing the compression of the gas from 217.1 mL to 167.75 mL. The change in internal energy of the process, ΔU is '–x' J. The value of 'x' is ____. [nearest integer]

(Given : atomic mass of N = 14 g mol^{-1} and of O = 16 g mol^{-1}

Molar heat capacity of N_2O is 100 J K^{-1} mol^{-1})

Answer (195)

Sol. $\Delta T = -40$ K

$\Delta U = q + w$

$= \frac{100 \times 2.2}{44}(-40) - (-49.39) \times 10^{-3} \times 101.325$

$= -200 + 5$

$= -195$ J

x = 195

3. Elevation in boiling point for 1.5 molal solution of glucose in water is 4 K. The depression in freezing point for 4.5 molal solution of glucose in water is 4 K. The ratio of molal elevation constant to molal depression constant (K_b/K_f) is ___.

Answer (3)

Sol. $\Delta T_b = i \times K_b \times m$

$\Delta T_f = i \times K_f \times m \qquad i = 1$

$4 = 1 \times K_b \times 1.5$

$4 = 1 \times K_f \times 4.5$

$\frac{K_b}{K_f} = 3$

4. The cell potential for the given cell at 298 K

Pt | H_2 (g, 1 bar) | H^+ (aq) || Cu^{2+} (aq) | Cu(s)

is 0.31 V. The pH of the acidic solution is found to be 3, whereas the concentration of Cu^{2+} is 10^{-x} M. The value of x is _____.

(Given: $E^{\ominus}_{Cu^{2+}/Cu} = 0.34$ V and $\frac{2.303\,RT}{F} = 0.06$ V)

Answer (7)

Sol. $Q = \frac{[H^+]^2}{[Cu^{+2}]pH_2} = \frac{10^{-6}}{C} \qquad pH_2 = 1$

$E = E^o_{cell} - \frac{0.06}{n}\log Q$

$0.31 = 0.34 - \frac{0.06}{2}\log\frac{10^{-6}}{C}$

$\log\frac{10^{-6}}{C} = 1$

$C = 10^{-7}$ M

x = 7

5. The equation

$k = (6.5 \times 10^{12} s^{-1})e^{-26000 K/T}$ is followed for the decomposition of compound A. The activation energy for the reaction is ______ kJ mol^{-1}. [nearest integer]

(Given : R = 8.314 J K^{-1} mol^{-1}]

Answer (216)

Sol. $k = A e^{\frac{-E_a}{RT}}$

$$\frac{E_a}{RT} = \frac{26000}{T}$$

E_a = 26000 × 8.314

= 216164 J

= 216 kJ

6. Spin only magnetic moment of $[MnBr_6]^{4-}$ is _____B.M. [round off to the closest integer]

Answer (6)

Sol. $[MnBr_6]^{-4}$

x – 6 = –4

x = + 2

Mn = $3d^5 4s^2$

$Mn^{+2} = 3d^5 4s^\circ$

n = 5

$$\mu = \sqrt{n(n+2)}$$

$$= \sqrt{35} \approx 6 \text{ B.M.}$$

7. For the reaction given below:

$$CoCl_3 \cdot xNH_3 + AgNO_3(aq) \rightarrow$$

If two equivalents of AgCl precipitate out, then the value of x will be____.

Answer (5)

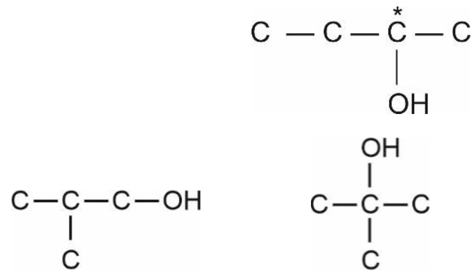
Sol. $[CoCl(NH_3)_5]Cl_2 \xrightarrow{AgNO_3} 2AgCl\downarrow$

x = 5

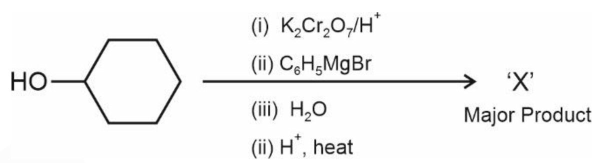
8. The number of chiral alcohol(s) with molecular formula $C_4H_{10}O$ is ____.

Answer (1)

Sol. C—C—C—C—OH



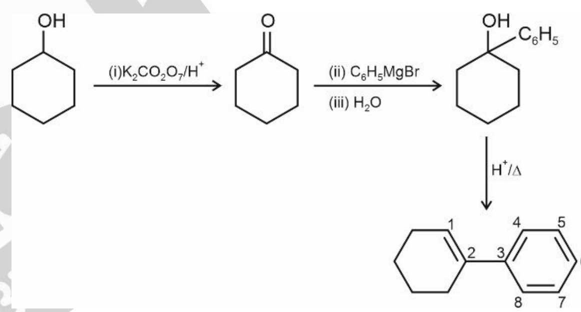
9. In the given reaction,



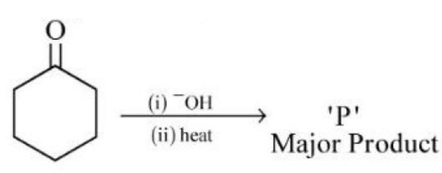
the number of sp^2 hybridised carbon(s) in compound 'X' is _____.

Answer (8)

Sol.



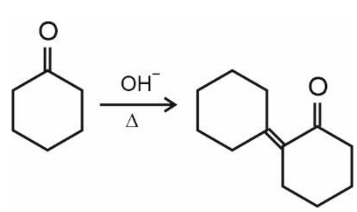
10. In the given reaction,



The number of π electrons present in the product 'P' is_____.

Answer (4)

Sol.



Total 4π electrons are there. Reaction is aldol condensation.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Let α be a root of the equation $1 + x^2 + x^4 = 0$. Then the value of $\alpha^{1011} + \alpha^{2022} - \alpha^{3033}$ is equal to

   (A) 1 (B) α

   (C) $1 + \alpha$ (D) $1 + 2\alpha$

Answer (A)

Sol. $1 + x^2 + x^4 = 0$

Root is ω(cube root of unity)

$\omega^{1011} + \omega^{2022} - \omega^{3033}$

$= (\omega^3)^{337} + (\omega^3)^{674} - (\omega^3)^{1011}$

$= 1 + 1 - 1 = 1$

2. Let arg(z) represent the principal argument of the complex number z.

   Then, $|z| = 3$ and $\arg(z - 1) - \arg(z + 1) = \frac{\pi}{4}$ intersect

   (A) exactly at one point

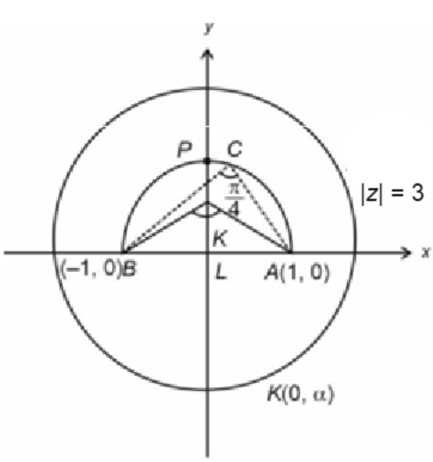
   (B) exactly at two points

   (C) nowhere

   (D) at infinitely many points

Answer (C)

Sol.



$|z| = 3$

$\arg(z - 1) - \arg(z + 1) = \frac{\pi}{4}$

$\angle AKL = \angle ACB = \frac{\pi}{4}$

$\Rightarrow LK = AL = \alpha = 1$

$K(0, 1)$

radius $= \sqrt{2}$

$PL = PK + KL = \sqrt{2} + 1$

$P(0, 1 + \sqrt{2})$

Number of intersection = 0

3. Let $A = \begin{pmatrix} 2 & -1 \\ 0 & 2 \end{pmatrix}$. If $B = I - {}^5C_1(\text{adj}A) + {}^5C_2(\text{adj}A)^2 - \ldots - {}^5C_5(\text{adj}A)^5$, then the sum of all elements of the matrix B is

   (A) –5 (B) –6

   (C) –7 (D) –8

Answer (C)

Sol. $A = \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \Rightarrow adj(A) \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$

$B = I - {}^5C_1(\text{adj}A) + {}^5C_2(\text{adj}A)^2 + \ldots + {}^5C_5(\text{adj}A)^5$

$= (I - \text{adj}A)^5 = \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}\right)^5 = \begin{bmatrix} -1 & -1 \\ 0 & 1 \end{bmatrix}^5$

Let $P = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix} \Rightarrow B = P^5$

$P^2 = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$P^3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & -3 \\ 0 & -1 \end{bmatrix}$

$P^4 = \begin{bmatrix} -1 & -3 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$

$P^5 = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & -5 \\ 0 & 1 \end{bmatrix} = B$

Sum of elements = –1 –5 – 1 + 0 = –7

4. The sum of the infinite series

$1+\frac{5}{6}+\frac{12}{6^2}+\frac{22}{6^3}+\frac{35}{6^4}+\frac{51}{6^5}+\frac{70}{6^6}+$ ..... is equal to

(A) $\frac{425}{216}$ (B) $\frac{429}{216}$

(C) $\frac{288}{125}$ (D) $\frac{280}{125}$

Answer (C)

$$S=1+\frac{5}{6}+\frac{12}{6^2}+\frac{22}{6^3}+....$$

$$\frac{1}{6}S=\frac{1}{6}+\frac{5}{6^2}+\frac{12}{6^3}+....$$

Sol. $\frac{5S}{6}=1+\frac{4}{6}+\frac{7}{6^2}+\frac{10}{6^3}+\frac{13}{6^4}+....$

$$\frac{5S}{36}=\frac{1}{6}+\frac{4}{6^2}+\frac{7}{6^3}+\frac{10}{6^4}+....$$

$$\frac{25S}{36}=1+\frac{3}{6}+\frac{3}{6^2}+\frac{3}{6^3}+\frac{3}{6^4}+....$$

$$\frac{25S}{36}=1+\frac{\frac{3}{6}}{1-\frac{1}{6}}$$

$$\frac{25S}{36}=\frac{8}{5}$$

$$S=\frac{288}{125}$$

5. The value of $\lim\limits_{x\to 1}\frac{(x^2-1)\sin^2(\pi x)}{x^4-2x^3+2x-1}$ is equal to

(A) $\frac{\pi^2}{6}$ (B) $\frac{\pi^2}{3}$

(C) $\frac{\pi^2}{2}$ (D) π^2

Answer (D)

Sol. $\lim\limits_{x\to 1}\frac{(x^2-1)\sin^2\pi x}{x^4-2x^3+2x-1}$

$$=\lim_{x\to 1}\frac{(x+1)(x-1)\sin^2\pi x}{(x-1)^3(x+1)}$$

Let $x - 1 = t$

$$\lim_{t\to 0}\frac{(2+t)t\sin^2\pi t}{t^3(t+2)}$$

$$=\lim_{t\to 0}=\frac{\sin^2\pi t}{\pi^2t^2}\cdot\pi^2$$

$$=\pi^2$$

6. Let $f : R \to R$ be a function defined by

$f(x)=(x-3)^{n_1}(x-5)^{n_2}$, $n_1, n_2 \in N$. Then, which of the following is NOT true?

(A) For $n_1 = 3$, $n_2 = 4$, there exists $\alpha\in(3, 5)$ where f attains local maxima.

(B) For $n_1 = 4$, $n_2 = 3$, there exists $\alpha\in(3, 5)$ where f attains local minima.

(C) For $n_1 = 3$, $n_2 = 5$, there exists $\alpha\in(3, 5)$ where f attains local maxima.

(D) For $n_1 = 4$, $n_2 = 6$, there exists $\alpha\in(3, 5)$ where f attains local maxima.

Answer (C)

Sol. For $n_2 \in$ odd, there will be local minima in (3, 5)

for $n_2 \in$ even, there will be local maxima in (3, 5)

7. Let f be a real valued continuous function on [0, 1] and $f(x)=x+\int_0^1(x-t)f(t)dt$. Then, which of the following points (x, y) lies on the curve $y=f(x)$?

(A) (2, 4) (B) (1, 2)

(C) (4, 17) (D) (6, 8)

Answer (D)

Sol. $f(x)=x\int_0^1(x-t)f(t)dt$

$$f(x)=x+x\int_0^1 f(t)dt-\int_0^1 tf(t)dt$$

$$f(x)=x\left(1+\int_0^1 f(t)dt\right)-\int_0^1 tf(t)dt$$

Let $1+\int_0^1 f(t)dt=a$ and $\int_0^1 tf(t)dt=1$

$$f(x)=ax-b$$

Now, $a = 1 + \int_0^1 (at - b)dt = 1 + \frac{a}{2} - b \Rightarrow \frac{a}{2} + b = 1$

$$b = \int_0^1 t(at - b)dt = \frac{a}{3} - \frac{b}{2} \Rightarrow \frac{3b}{2} = \frac{a}{3} \Rightarrow \boxed{b = \frac{2a}{9}}$$

$$\frac{a}{2} + \frac{2a}{9} = 1$$

$$\Rightarrow \boxed{a = \frac{18}{13}} \boxed{b = \frac{4}{13}}$$

$$f(x) = \frac{18x - 4}{13}$$

(6, 8) lies on $f(x)$ i.e. option (D)

8. If $\int_0^2 \left(\sqrt{2x} - \sqrt{2x - x^2}\right)dx = \int_0^1 \left(1 - \sqrt{1 - y^2} - \frac{y^2}{2}\right)dy + \int_1^2 \left(2 - \frac{y^2}{2}\right)dy + I$ then I equal is

(A) $\int_0^1 \left(1 + \sqrt{1 - y^2}\right)dy$

(B) $\int_0^1 \left(\frac{y^2}{2} - \sqrt{1 - y^2} + 1\right)dy$

(C) $\int_0^1 \left(1 - \sqrt{1 - y^2}\right)dy$

(D) $\int_0^1 \left(\frac{y^2}{2} + \sqrt{1 - y^2} + 1\right)dy$

Answer (C)

Sol. $\int_0^2 \sqrt{2x}dx - \int_0^2 \sqrt{1 - (x - 1)^2}dx = \int_0^2 \left(1 - \frac{y^2}{2}\right)dy - \int_0^1 \sqrt{1 - y^2}dy$

$+ 1 + I$

$$\Rightarrow \frac{8}{3} - 2\int_0^1 \sqrt{1 - y^2}dy = \frac{2}{3} + 1 - \int_0^1 \sqrt{1 - y^2}dy + I$$

$$\Rightarrow I = 1 - \int_0^1 \sqrt{1 - y^2}dy$$

$$\Rightarrow I = \int_0^1 \left(1 - \sqrt{1 - y^2}\right)dy$$

9. If $y = y(x)$ is the solution of the differential equation $\left(1 + e^{2x}\right)\frac{dy}{dx} + 2\left(1 + y^2\right)e^x = 0$ and $y(0) = 0$, then $6\left(y'(0) + \left(y\left(\log_e \sqrt{3}\right)\right)^2\right)$ is equal to

(A) 2

(B) –2

(C) –4

(D) –1

Answer (C)

Sol. $\left(1 + e^{2x}\right)\frac{dy}{dx} + 2\left(1 + y^2\right)e^x = 0$

$$\int \frac{dy}{1 + y^2} = -\int \frac{2e^x}{1 + e^{2x}}dx \qquad e^x = t, \quad e^x dx = dt$$

$$\tan^{-1} y = -2\int \frac{dt}{1 + t^2}$$

$$\tan^{-1} y = 2\tan^{-1}\left(e^x\right) + c$$

$$y(0) \Rightarrow c = \frac{\pi}{2}$$

$$\tan^{-1} y = -2\tan^{-1}\left(e^x\right) + \frac{\pi}{2}$$

$$y = \cot\left(2\tan^{-1} e^x\right)$$

$$\frac{dy}{dx} = -\text{cosec}^2\left(2\tan^{-1} e^x\right)\left(\frac{2e^x}{1 + e^{2x}}\right)$$

$$y'(0) = \left.\frac{dy}{dx}\right|_{x=0} = \frac{-2}{2} = -1$$

$$y = \cot\left(2\tan^{-1} e^x\right)$$

$$y\left(\ln\sqrt{3}\right) = \cot\left(2\tan^{-1} e^{\log_e \sqrt{3}}\right)$$

$$= \cot\left(2\tan^{-1}\sqrt{3}\right) = \cot\left(\frac{2\pi}{3}\right) = -\cot\frac{\pi}{3} = -\frac{1}{\sqrt{3}}$$

$$6\left(y'(0) + \left(y\left(\ln\sqrt{3}\right)\right)^2\right) = 6\left(-1 + \left(-\frac{1}{\sqrt{3}}\right)^2\right)$$

$$= 6\left(-1 + \frac{1}{3}\right) = -4$$

10. Let $P : y^2 = 4ax$, $a > 0$ be a parabola with focus S. Let the tangents to the parabola P make an angle of $\frac{\pi}{4}$ with the line $y = 3x + 5$ touch the parabola P at A and B. Then the value of a for which A, B and S are collinear is

(A) 8 only
(B) 2 only
(C) $\frac{1}{4}$ only
(D) any $a > 0$

Answer (D)

Sol. $P : y^2 = 4ax,\ a > 0 \qquad S(a,\ 0)$

Equation of tangent on parabola $y = mx + \frac{a}{m}$

$$y = 3x + 5$$

$$\tan\frac{\pi}{4} = \left|\frac{m-3}{1+3m}\right| \Rightarrow m - 3 = \pm(1 + 3m)$$

$$m - 3 = 1 + 3m$$
$$\boxed{m = -2}$$

$$m - 3 = -1 - 3m$$
$$\boxed{m = \frac{1}{2}}$$

Equation of one tangent : $y = -2x - \frac{a}{2}$

Equation of other tangent : $y = \frac{x}{2} + 2a$

Point of contact are

$$\left(\frac{a}{(-2)^2}, \frac{-2a}{(-2)}\right) \text{ and } \left(\frac{a}{\left(\frac{1}{2}\right)^2}, \frac{-2a}{\frac{1}{2}}\right)$$

$A\left(\frac{a}{4}, a\right)$ and $B(4a,\ -4a)$

Now or (ΔABS) = 0 [S is the focus]

$$\frac{1}{2}\begin{vmatrix} \frac{a}{4} & a & 1 \\ 4a & -4a & 1 \\ a & 0 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \frac{a}{4}(-4a - 0) - a(4a - a) + 1\left(0 - \left(-4a^2\right)\right) = 0$$

$$= -a^2 - 3a^2 + 4a^2 = 0$$

Always true

11. Let a triangle ABC be inscribed in the circle $x^2 - \sqrt{2}(x + y) + y^2 = 0$ such that $\angle BAC = \frac{\pi}{2}$. If the length of side AB is $\sqrt{2}$, then the area of the ΔABC is equal to :

(A) $\left(\sqrt{2} + \sqrt{6}\right)/3$

(B) $\left(\sqrt{6} + \sqrt{3}\right)/2$

(C) $\left(3 + \sqrt{3}\right)/4$

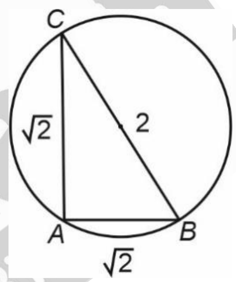
(D) $\left(\sqrt{6} + 2\sqrt{3}\right)/4$

Answer (Dropped)

Sol. $x^2 - \sqrt{2}x - \sqrt{2}y + y^2 = 0$

Centre $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

Radius = 1



BC is diameter

$$\text{ar}(\Delta ABC) = \frac{1}{2} \times \sqrt{2} \times \sqrt{2} = 1$$

12. Let $\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z+3}{-1}$ lie on the plane $px - qy + z = 5$, for some $p, q \in \mathbb{R}$. The shortest distance of the plane from the origin is :

(A) $\sqrt{\frac{3}{109}}$

(B) $\sqrt{\frac{5}{142}}$

(C) $\frac{5}{\sqrt{71}}$

(D) $\frac{1}{\sqrt{142}}$

Answer (B)

Sol. $\frac{x-2}{3}=\frac{y+1}{-2}=\frac{z+3}{-1}=\lambda$

$(3\lambda+2,-2\lambda-1,-\lambda-3)$ lies on plane $px-qy+z=5$

$p(3\lambda+2)-q(-2\lambda-1)+(-\lambda-3)=5$

$\lambda(3p+2q-1)+(2p+q-8)=0$

$\left.\begin{array}{l}3p+2q-1=0\\2p+q-8=0\end{array}\right\}\begin{array}{l}p=15\\q=-22\end{array}$

Equation of plane $15x+22y+z-5=0$

Shortest distance from origin = $\frac{|0+0+0-5|}{\sqrt{15^2+22^2+1}}$

$=\frac{5}{\sqrt{710}}$

$=\sqrt{\frac{5}{142}}$

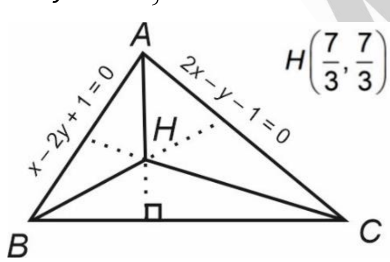
13. The distance of the origin from the centroid of the triangle whose two sides have the equations $x-2y+1=0$ and $2x-y-1=0$ and whose orthocenter is $\left(\frac{7}{3},\frac{7}{3}\right)$ is :

(A) $\sqrt{2}$ (B) 2

(C) $2\sqrt{2}$ (D) 4

Answer (C)

Sol. $\left.\begin{array}{l}AB: x-2y+1=0\\AC: 2x-y-1=0\end{array}\right\}A(1,1)$



Altitude from B is $BH = x+2y-7=0 \Rightarrow B(3,2)$

Altitude from C is $CH = 2x+y-7=0 \Rightarrow C(2,3)$

Centroid of $\Delta ABC = G(2,2)$ $OG=2\sqrt{2}$

14. Let Q be the mirror image of the point P(1, 2, 1) with respect to the plane $x+2y+2z=16$. Let T be a plane passing through the point Q and contains the line $\vec{r}=-\hat{k}+\lambda(\hat{i}+\hat{j}+2\hat{k}), \lambda\in\mathbb{R}$. Then, which of the following points lies on T?

(A) (2, 1, 0) (B) (1, 2, 1)

(C) (1, 2, 2) (D) (1, 3, 2)

Answer (B)

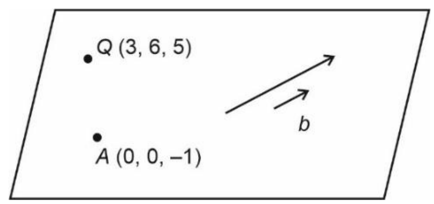
Sol. $P(1, 2, 1)$ image in plane $x+2y+2z=16$

$\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-1}{2}=\frac{-2(1+2\times2+2\times1-16)}{1^2+2^2+2^2}$

$\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-1}{2}=2$

$Q(3, 6, 5)$

$\vec{r}=-\hat{k}+\lambda(\hat{i}+\hat{j}+2\hat{k})$



$AQ=3\hat{i}+6\hat{j}+6\hat{k}$

$=3(\hat{i}+2\hat{j}+2\hat{k})$

$\vec{n}=(\hat{i}+2\hat{j}+2\hat{k})\times(\hat{i}+\hat{j}+2\hat{k})$

$\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&2&2\\1&1&2\end{vmatrix}$

$=2\hat{i}-0\hat{j}-\hat{k}$

Equation of plane $\equiv 2(x-0)+0(y-0)-1(z+1)=0$

$2x-z=1$

Point lying on plane from the option is (1, 2, 1) i.e., option (B)

15. Let A, B, C be three points whose position vectors respectively are

$\vec{a}=\hat{i}+4\hat{j}+3\hat{k}$

$\vec{b}=2\hat{i}+\alpha\hat{j}+4\hat{k}, \alpha\in\mathbb{R}$

$\vec{c}=3\hat{i}-2\hat{j}+5\hat{k}$

If α is the smallest positive integer for which $\vec{a},\vec{b},\vec{c}$ are non collinear, then the length of the median, in ΔABC, through A is:

(A) $\frac{\sqrt{82}}{2}$ (B) $\frac{\sqrt{62}}{2}$

(C) $\frac{\sqrt{69}}{2}$ (D) $\frac{\sqrt{66}}{2}$

Answer (A)

Sol. $\overrightarrow{AB} \parallel \overrightarrow{AC}$ if

$$\frac{1}{2} = \frac{\alpha - 4}{-6} = \frac{1}{2}$$

$\Rightarrow \quad \alpha = 1$

$\vec{a}, \vec{b}, \vec{c}$ are non-collinear for $\alpha = 2$ (smallest positive integer)

Mid point of $BC = M\left(\frac{5}{2}, 0, \frac{9}{2}\right)$

$$AM = \sqrt{\frac{9}{4} + 16 + \frac{9}{4}} = \frac{\sqrt{82}}{2}$$

16. The probability that a relation R from $\{x, y\}$ to $\{x, y\}$ is both symmetric and transitive, is equal to

(1) $\frac{5}{16}$ (2) $\frac{9}{16}$

(3) $\frac{11}{16}$ (4) $\frac{13}{16}$

Answer (A)

Sol. Total number of relations = $2^{2^2} = 2^4 = 16$

Relations which are symmetric as well as transitive are

ϕ, $\{(x, x)\}$, $\{(y, y)\}$, $\{(x, x), (x, y), (y, y), (y, x)\}$, $\{(x, x), (y, y)\}$

$\therefore$ favourable cases = 5

$\therefore \quad P_r = \frac{5}{16}$

17. The number of values of $a \in N$ such that the variance of 3, 7, 12, a, $43 - a$ is a natural number is :

(1) 0 (2) 2

(3) 5 (4) Infinite

Answer (A)

Sol. Mean = $\frac{3 + 12 + 7 + a + 43 - a}{5} = 13$

Variance =

$$\frac{9 + 49 + 144 + a^2 + (43 - a)^2}{5} - 13^2 \in \text{Natural number}$$

$$\frac{2a^2 - a + 1}{5} \in \text{Natural number}$$

$2a^2 - a + 1 = 5n \qquad [n \in N]$

$2a^2 - a + 1 - 5n = 0$

$D = 1 - 4(1 - 5n)2$

$\quad = 40n - 7$

D cannot be a perfect square as all perfect squares will be of the form of 4λ or $4\lambda + 1$

So, a cannot be natural number

$\therefore \quad$ Number of values = 0

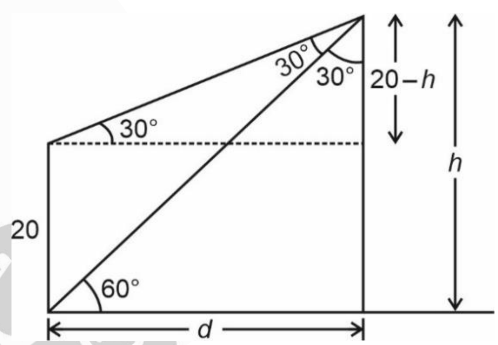
18. From the base of a pole of height 20 meter, the angle of elevation of the top of a tower is 60°. The pole subtends an angle 30° at the top of the tower. Then the height of the tower is :

(1) $15\sqrt{3}$ (2) $20\sqrt{3}$

(3) $20 + 10\sqrt{3}$ (4) 30

Answer (D)

Sol.



$$\tan 60° = \frac{h}{d} \quad \& \quad \tan 30° = \frac{20 - h}{d}$$

$$\Rightarrow \frac{20 - h}{h/\sqrt{3}} = \frac{1}{\sqrt{3}} \Rightarrow \frac{2h}{3} = 20 \Rightarrow h = 30 \text{ m}$$

19. Negation of the Boolean statement $(p \vee q) \Rightarrow ((\sim r) \vee p)$ is equivalent to

(A) $p \wedge (\sim q) \wedge r$

(B) $(\sim p) \wedge (\sim q) \wedge r$

(C) $(\sim p) \wedge q \wedge r$

(D) $p \wedge q \wedge (\sim r)$

Answer (C)

Sol. $p \vee q \Rightarrow (\sim r \vee p)$

$\equiv \sim (p \vee q) \vee (\sim r \vee p)$

$\equiv (\sim p \wedge \sim q) \vee (p \vee \sim r)$

$\equiv [(\sim p \vee p) \wedge (\sim q \vee p)] \vee \sim r$

$\equiv (\sim q \vee p) \vee \sim r$

Its negation is $\sim p \wedge q \wedge r$.

20. Let $n \geq 5$ be an integer. If $9^n - 8n - 1 = 64\alpha$ and $6^n - 5n - 1 = 25\beta$, then $\alpha - \beta$ is equal to

(A) $1+{}^nC_2(8-5)+{}^nC_3(8^2-5^2)+\ldots+{}^nC_n(8^{n-1}-5^{n-1})$

(B) $1+{}^nC_3(8-5)+{}^nC_4(8^2-5^2)+\ldots+{}^nC_n(8^{n-2}-5^{n-2})$

(C) ${}^nC_3(8-5)+{}^nC_4(8^2-5^2)+\ldots+{}^nC_n(8^{n-2}-5^{n-2})$

(D) ${}^nC_4(8-5)+{}^nC_5(8^2-5^2)+\ldots+{}^nC_n(8^{n-3}-5^{n-3})$

Answer (C)

Sol. $(1+8)^n - 8n - 1 = 64\alpha$

$\Rightarrow 1+8n+{}^nC_2 8^2+{}^nC_3 8^3+\ldots+{}^nC_n 8^n-8n-1 = 64\alpha$

$\Rightarrow \alpha = {}^nC_2+{}^nC_3 8+{}^nC_4 8^2+\ldots+{}^nC_n 8^{n-2}$ ...(1)

Similarly

$(1+5)^n - 5n - 1 = 25\beta$

$\Rightarrow 1+5n+{}^nC_2 5^2+{}^nC_3 5^3+\ldots+{}^nC_n 5^n-5n-1 = 25\beta$

$\Rightarrow \beta = {}^nC_2+{}^nC_3.5+{}^nC_4.5^2+\ldots+{}^nC_n 5^{n-2}$ ...(2)

$\alpha-\beta = {}^nC_3(8-5)+{}^nC_4(8^2-5^2)+\ldots+{}^nC_n(8^{n-2}-5^{n-2})$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse andw the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} + (\vec{b} \times \vec{c}) = \vec{0}$ and $\vec{b} \cdot \vec{c} = 5$. Then the value of $3(\vec{c} \cdot \vec{a})$ is equal to ________.

Answer (10*)

Sol. Data not correct

$\because \quad \vec{a}\cdot\vec{b} = 1-2+3 = 2 \quad \{\text{according to } \vec{a} \text{ and } \vec{b}\}$

but given that

$\vec{a} + (\vec{b}\times\vec{c}) = \vec{0}$

$\vec{a} = -(\vec{b}\times\vec{c})$

$\Rightarrow \vec{a} \perp \vec{b}$ and $\vec{a} \perp \vec{c}$ -

$\therefore \vec{a}\cdot\vec{b} = 0$ {Contradicts}

2. Let $y = y(x)$, $x > 1$, be the solution of the differential equation $(x-1)\frac{dy}{dx} + 2xy = \frac{1}{x-1}$, with $y(2) = \frac{1+e^4}{2e^4}$. If $y(3) = \frac{e^\alpha + 1}{\beta e^\alpha}$, then the value of $\alpha + \beta$ is equal to ____.

Answer (14)

Sol. $\frac{dy}{dx} + y\left(\frac{2x}{x-1}\right) = \frac{1}{(x-1)^2}$

I.F $= e^{\int \frac{2x}{x-1}dx}$

$= e^{2\int\left(\frac{x-1}{x-1}+\frac{1}{x-1}\right)dx}$

$= e^{2x+2\ln(x-1)}$

$= e^{2x}(x-1)^2$

$\Rightarrow \int d\left(y.e^{2x}(x-1)^2\right) = \int e^{2x}dx$

$\Rightarrow y.e^{2x}(x-1)^2 = \frac{e^{2x}}{2} + c$

$\downarrow y(2) = \frac{1+e^4}{2e^4}$

$\frac{1+e^4}{2e^4}\cdot e^4 = \frac{e^4}{2} + c$

$\Rightarrow c = \frac{e^4}{2}\left(\frac{1+e^4-e^4}{e^4}\right) = \frac{1}{2}$

$\Rightarrow y\cdot e^{2x}(x-1)^2 = \frac{e^{2x}+1}{2}$

$\downarrow y(3) = \frac{e^\alpha+1}{\beta e^\alpha}$

$\Rightarrow \frac{e^\alpha+1}{\beta e^\alpha}\cdot e^6\cdot 4 = \frac{e^6+1}{2}$

$\Rightarrow \alpha = 6$ and $\beta = 8 \Rightarrow \alpha + \beta = 14$

3. Let 3, 6, 9, 12, … upto 78 terms and 5, 9, 13, 17, … upto 59 terms be two series. Then, the sum of terms common to both the series is equal to _____.

Answer (2223)

Sol. S_1 : 3, 6, 9, 12,……78-terms

S_2 : 5, 9, 13, 17,…….59-terms

Common terms are 9, 21,……….

T_{78} of S_1 = 3 + (77).3 = 234

T_{59} of S_2 = 5 + (58) 4 = 237

So n^{th} common term ≤ 234

$\Rightarrow$ 9 + (n – 1) 12 ≤ 234

$\Rightarrow \ n < \dfrac{225}{12} + 1$

$\Rightarrow \ n < \dfrac{237}{12} \ \Rightarrow n = 19$

S_{19} of common terms

$= \dfrac{19}{2}\left[2(9) + 18.12\right]$

= 19 (9 + 108)

= 117 × 19 = 2223

4. The number of solutions of the equation $\sin x = \cos^2 x$ in the interval (0, 10) is _____.

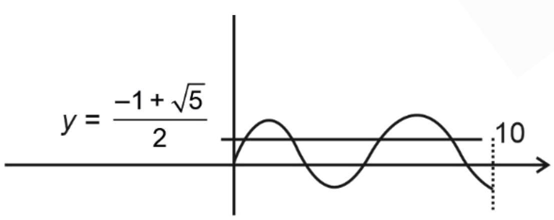
Answer (4)

Sol. $\sin x = 1 - \sin^2 x$

$\Rightarrow \sin^2 x + \sin x - 1 = 0$

$\Rightarrow \ \sin x = \dfrac{-1 \pm \sqrt{5}}{2}$

$\Rightarrow \ \sin x = \dfrac{-1 + \sqrt{5}}{2}$



4 solutions

5. For real number a, b (a > b > 0), let

$$\text{Area}\left\{(x, y) : x^2 + y^2 \le a^2 \text{ and } \frac{x^2}{a^2} + \frac{y^2}{b^2} \ge 1\right\} = 30\pi$$

and

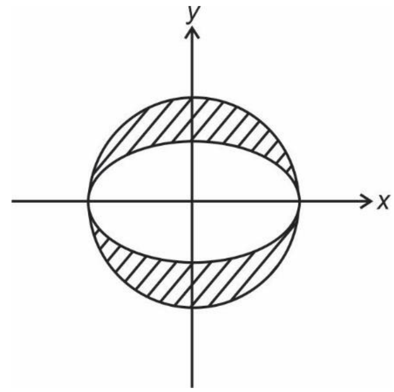
$$\text{Area}\left\{(x, y) : x^2 + y^2 \ge b^2 \text{ and } \frac{x^2}{a^2} + \frac{y^2}{b^2} \le 1\right\} = 18\pi$$

Then the value of $(a – b)^2$ is equal to _____.

Answer (12)

Sol. $x^2 + y^2 \le a^2$ is interior of circle

and $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} \ge 1$ is exterior of ellipse



$\therefore$ Area = $\pi a^2 - \pi ab = 30\pi$ …(1)

Similarly $x^2 + y^2 \ge b^2$ and $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} \le 1$ gives

$\pi ab - \pi b^2 = 18\pi$ …(2)

By (1) and (2), $\dfrac{a}{b} = \dfrac{5}{3} \Rightarrow a = \dfrac{5b}{3}$

$\Rightarrow \ \pi \cdot \dfrac{25b^2}{9} - \pi \cdot \dfrac{5b^2}{3} = 30\pi$

$\Rightarrow \ \left(\dfrac{25}{9} - \dfrac{5}{3}\right)b^2 = 30$

$\Rightarrow \ \dfrac{10}{9}b^2 = 30 \Rightarrow b^2 = 27$

and $a^2 = \dfrac{25}{9} \cdot 27 = 75$

$(a - b)^2 = \left(5\sqrt{3} - 3\sqrt{3}\right)^2 = 3 \cdot 4 = 12$

6. Let f and g be twice differentiable even functions on (–2, 2) such that $f\left(\frac{1}{4}\right) = 0,\ f\left(\frac{1}{2}\right) = 0,\ f(1) = 1$ and $g\left(\frac{3}{4}\right) = 0,\ g(1) = 2.$ Then, the minimum number of solutions of $f(x)g''(x) + f'(x)g'(x) = 0$ in (–2, 2) is equal to_____.

Answer (4)

Sol. Let $h(x) = f(x) \cdot g'(x)$

As f(x) is even $f\left(\frac{1}{2}\right) = \left(\frac{1}{4}\right) = 0$

$\Rightarrow \ f\left(-\frac{1}{2}\right) = f\left(-\frac{1}{4}\right) = 0$

and g(x) is even $\Rightarrow g'(x)$ is odd and $g(1) = 2$ ensures one root of $g'(x)$ is 0.

So, $h(x) = f(x) \cdot g'(x)$ has minimum five zeroes

$\therefore\ h'(x) = f'(x) \cdot g'(x) + f(x) \cdot g''(x) = 0$,

has minimum 4 zeroes

7. Let the coefficients of x^{-1} and x^{-3} in the expansion of $\left(2x^{\frac{1}{5}} - \frac{1}{x^{\frac{1}{5}}}\right)^{15}$, $x > 0$, be m and n respectively.

If r is a positive integer such that $mn^2 = {}^{15}C_r \cdot 2^r$, then the value of r is equal to ______.

Answer (5)

Sol. $T_{r+1} = {}^{15}C_r \left(2x^{1/5}\right)^{15-r}\left(\frac{-1}{x^{1/5}}\right)^r$

For coefficient of x^{-1}

$$\frac{15-r}{5} - \frac{r}{5} = -1 \Rightarrow 15 - 2r = -5 \Rightarrow r = 10$$

$\therefore\ m = {}^{15}C_{10}.2^5$

& for coefficient of x^{-3} $\quad 15 - 2r = -15 \Rightarrow r = 15$

$\therefore\ n = -{}^{15}C_{15}$

Given $mn^2 = {}^{15}C_r.2^r$

$\Rightarrow\ {}^{15}C_{10}.2^5 = {}^{15}C_r.2^r$

$\Rightarrow\ r = 5$

8. The total number of four digit numbers such that each of first three digits is divisible by the last digit, is equal to _______.

Answer (1086)

Sol. If unit digit is 1 then $\rightarrow 9 \times 10 \times 10 = 900$ numbers

If unit digit is 2 then $\rightarrow 4 \times 5 \times 5 = 100$ numbers

If unit digit is 3 then $\rightarrow 3 \times 4 \times 4 = 48$ numbers

If unit digit is 4 then $\rightarrow 2 \times 3 \times 3 = 18$ numbers

If unit digit is 5 then $\rightarrow 1 \times 2 \times 2 = 4$ numbers

If unit digit is 6 then $\rightarrow 1 \times 2 \times 2 = 4$ numbers

For 7, 8, 9 $\rightarrow 4 + 4 + 4 = 12$ Numbers

Total = 1086 Numbers

9. Let $M = \begin{bmatrix} 0 & -\alpha \\ \alpha & 0 \end{bmatrix}$, where α is a non-zero real number an $N = \sum_{k=1}^{49} M^{2k}$. If $(I - M^2)N = -2I$, then the positive integral value of α is ____ .

Answer (1)

Sol. $M = \begin{bmatrix} 0 & -\alpha \\ \alpha & 0 \end{bmatrix}$, $M^2 = \begin{bmatrix} -\alpha^2 & 0 \\ 0 & -\alpha^2 \end{bmatrix} = -\alpha^2 I$

$N = M^2 + M^4 + \ldots + M^{98}$

$= \left[-\alpha^2 + \alpha^4 - \alpha^6 + \ldots\right] I$

$= \frac{-\alpha^2\left(1 - (-\alpha^2)^{49}\right)}{1 + \alpha^2} \cdot I$

$I - M^2 = (1 + \alpha^2)I$

$(I - M^2)N = -\alpha^2(\alpha^{98} + 1) = -2$

$\therefore\ \alpha = 1$

10. Let $f(x)$ and $g(x)$ be two real polynomials of degree 2 and 1 respectively. If $f(g(x)) = 8x^2 - 2x$ and $g(f(x)) = 4x^2 + 6x + 1$, then the value of $f(2) + g(2)$ is ____________ .

Answer (18)

Sol. $f(g(x)) = 8x^2 - 2x$

$g(f(x)) = 4x^2 + 6x + 1$

let $f(x) = cx^2 + dx + e$

$g(x) = ax + b$

$f(g(x)) = c(ax + b)^2 + d(ax + b) + e \equiv 8x^2 - 2x$

$g(f(x)) = a(cx^2 + dx + e) + b \equiv 4x^2 + 6x + 1$

$\therefore\ ac = 4 \qquad ad = 6 \qquad ae + b = 1$

$a^2c = 8 \qquad 2abc + ad = -2 \qquad cb^2 + bd + e = 0$

By solving

$a = 2 \qquad b = -1$

$c = 2 \qquad d = 3 \qquad e = 1$

$\therefore\ f(x) = 2x^2 + 3x + 1$

$g(x) = 2x - 1$

$f(2) + g(2) = 2(2)^2 + 3(2) + 1 + 2(2) - 1$

$= 18$

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