29/06/2022
Morning

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Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2022 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Two balls *A* and *B* are placed at the top of 180 m tall tower. Ball *A* is released from the top at $t = 0$ s. Ball *B* is thrown vertically down with an initial velocity *u* at $t = 2$ s. After a certain time, both balls meet 100 m above the ground. Find the value of *u* in ms^{-1} [use $g = 10\ ms^{-2}$]

   (A) 10

   (B) 15

   (C) 20

   (D) 30

Answer (D)

Sol. Let us assume that they meet at $t = t_0$

$$A:\ 80 = \frac{1}{2}gt_0^2 \qquad \ldots(i)$$

$$B:\ 80 = u(t_0 - 2) + \frac{1}{2}g(t_0 - 2)^2 \qquad \ldots(ii)$$

From (i), $t_0 = 4$

$\Rightarrow$ $80 = 2u + 5(2)^2$

$\Rightarrow$ $u = 30$ m/s

2. A body of mass *M* at rest explodes into three pieces, in the ratio of masses 1 : 1 : 2. Two smaller pieces fly off perpendicular to each other with velocities of 30 ms^{-1} and 40 ms^{-1} respectively. The velocity of the third piece will be

   (A) 15 ms^{-1}

   (B) 25 ms^{-1}

   (C) 35 ms^{-1}

   (D) 50 ms^{-1}

Answer (B)

Sol. Conserving momentum:

$$m(30\hat{i}) + m(40\hat{j}) + 2m(\vec{v}) = \vec{0}$$

$\Rightarrow$ $\vec{v} = -15\hat{i} - 20\hat{j}$

$\Rightarrow$ $|\vec{v}| = 25$ m/s

3. The activity of a radioactive material is 2.56×10^{-3} Ci. If the half life of the material is 5 days, after how many days the activity will become 2×10^{-5} Ci?

   (A) 30 days

   (B) 35 days

   (C) 40 days

   (D) 25 days

Answer (B)

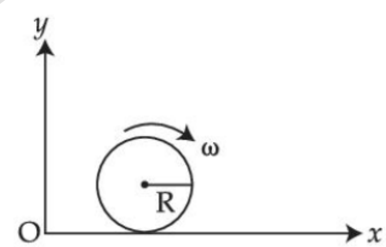
Sol. $A = A_0 e^{-\lambda t}$

$\Rightarrow$ $2 \times 10^{-5} = 2.56 \times 10^{-3}\, e^{-\lambda t}$

$\Rightarrow$ $e^{-\lambda t} = \frac{1}{128} = \left[\frac{1}{2}\right]^7$

$\Rightarrow$ $t = 7t_{\frac{1}{2}} = 35$ days

4. A spherical shell of 1 kg mass and radius *R* is rolling with angular speed ω on horizontal plane (as shown in figure). The magnitude of angular momentum of the shell about the origin *O* is $\frac{a}{3}R^2\omega$. The value of a will be



   (A) 2

   (B) 3

   (C) 5

   (D) 4

Answer (C)

Sol. $\vec{L}_0 = \vec{L}_{\text{of cm}} + \vec{L}_{\text{about cm}}$

$\Rightarrow$ $\frac{a}{3}R^2\omega = mvR + \frac{2}{3}mR^2\omega = \frac{5}{3}mR^2\omega$

$\Rightarrow$ $a = 5$

5. A cylinder of fixed capacity of 44.8 litres contains helium gas at standard temperature and pressure. The amount of heat needed to raise the temperature of gas in the cylinder by 20.0°C will be

(Given gas constant R = 8.3 JK^{-1}-mol^{-1})

(A) 249 J

(B) 415 J

(C) 498 J

(D) 830 J

Answer (C)

Sol. $\Delta Q = nC_V\Delta T$ (Isochoric process)

$$= 2 \times \frac{3R}{2} \times 20$$

= 498 J

6. A wire of length L is hanging from a fixed support. The length changes to L_1 and L_2 when masses 1 kg and 2 kg are suspended respectively from its free end. Then the value of L is equal to

(A) $\sqrt{L_1L_2}$

(B) $\frac{L_1 + L_2}{2}$

(C) $2L_1 - L_2$

(D) $3L_1 - 2L_2$

Answer (C)

Sol. $y = \frac{FL}{A\Delta L}$

$\Rightarrow \quad \Delta L = \frac{FL}{Ay}$

$\Rightarrow \quad L_1 = L + \frac{(1g)L}{Ay}$ ...(i)

and $L_2 = L + \frac{(2g)L}{Ay}$ ...(ii)

$\Rightarrow \quad L = 2L_1 - L_2$

7. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A : The photoelectric effect does not takes place, if the energy of the incident radiation is less than the work function of a metal.

Reason R : Kinetic energy of the photoelectrons is zero, if the energy of the incident radiation is equal to the work function of a metal.

In the light of the above statements, choose the **most appropriate** answer from the options given below.

(A) Both **A** and **R** are correct and **R** is the correct explanation of **A**

(B) Both **A** and **R** are correct but **R** is **not** the correct explanation of **A**

(C) **A** is correct but **R** is not correct

(D) **A** is not correct but **R** is correct

Answer (B)

Sol. When energy of incident radiation is equal to the work function of the metal, then the KE of photoelectrons would be zero. But this statement does not comment on the situation when energy is less than the work function.

8. A particle of mass 500 gm is moving in a straight line with velocity $v = bx^{5/2}$. The work done by the net force during its displacement from x = 0 to x = 4 m is : (Take b = 0.25 $m^{-3/2}s^{-1}$).

(A) 2 J (B) 4 J

(C) 8 J (D) 16 J

Answer (D)

Sol. $W_{\text{total}} = \Delta K$

$$= \frac{1}{2}\left(\frac{1}{2}\right)\left[\{b(4)^{5/2}\}^2 - 0\right]$$

$$= \frac{b^2}{4} \times 4^5$$

$\Rightarrow \quad W_{\text{total}}$ = 16 J

9. A charge particle moves along circular path in a uniform magnetic field in a cyclotron. The kinetic energy of the charge particle increases to 4 times its initial value. What will be the ratio of new radius to the original radius of circular path of the charge particle

(A) 1 : 1

(B) 1 : 2

(C) 2 : 1

(D) 1 : 4

Answer (C)

Sol. $R = \frac{mv}{Bq} = \frac{\sqrt{2mK}}{Bq}$

$\Rightarrow \quad R \propto \sqrt{K}$

$\Rightarrow$ ratio = 2 : 1

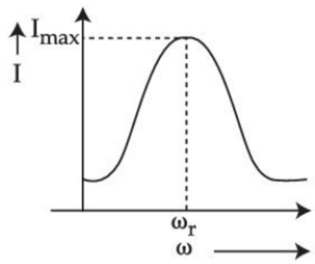
10. For a series LCR circuit, I vs ω curve is shown :

(a) To the left of ω_r, the circuit is mainly capacitive.

(b) To the left of ω_r, the circuit is mainly inductive.

(c) At ω_r, impedance of the circuit is equal to the resistance of the circuit.

(d) At ω_r, impedance of the circuit is 0.



Choose the **most appropriate** answer from the options given below

(A) (a) and (d) only (B) (b) and (d) only

(C) (a) and (c) only (D) (b) and (c) only

Answer (C)

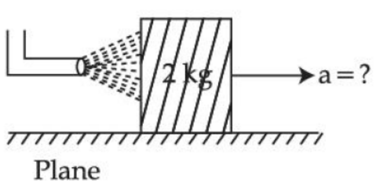
Sol. We know that $X_C = \frac{1}{\omega C}$ and $X_L = \omega L$

Also, at $\omega = \omega_r : X_L = X_C$

$\Rightarrow$ For $\omega < \omega_r$: capacitive

and At $\omega = \omega_r : z = \sqrt{R^2 + (X_L - X_C)^2} = R$

11. A block of metal weighing 2 kg is resting on a frictionless plane (as shown in figure). It is struck by a jet releasing water at a rate of 1 kgs^{-1} and at a speed of 10 ms^{-1}. Then, the initial acceleration of the block, in ms^{-2}, will be:



(A) 3

(B) 6

(C) 5

(D) 4

Answer (C)

Sol. $F = \rho v^2 a$

$\Rightarrow$ 10 × 1 = 2 × acceleration

$\Rightarrow$ Acc. = 5 m/s^2

12. In van der Wall equation $\left[P + \frac{a}{V^2}\right][V - b] = RT; P$ is pressure, V is volume, R is universal gas constant and T is temperature. The ratio of constants $\frac{a}{b}$ is dimensionally equal to:

(A) $\frac{P}{V}$

(B) $\frac{V}{P}$

(C) PV

(D) PV^3

Answer (C)

Sol. From the equation

$[a] \equiv [PV^2]$

$[b] \equiv [V]$

$\Rightarrow \left[\frac{a}{b}\right] \equiv [PV]$

13. Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. If magnitude of $\vec{A} + \vec{B}$ is equal to two times the magnitude of $\vec{A} - \vec{B}$, then the angle between $\vec{A}$ and $\vec{B}$ will be:

(A) $\sin^{-1}\left(\frac{3}{5}\right)$

(B) $\sin^{-1}\left(\frac{1}{3}\right)$

(C) $\cos^{-1}\left(\frac{3}{5}\right)$

(D) $\cos^{-1}\left(\frac{1}{3}\right)$

Answer (C)

Sol. $\sqrt{A^2 + A^2 + 2A^2\cos\theta} = 2\sqrt{A^2 + A^2 + 2A^2(-\cos\theta)}$

$\Rightarrow 2A^2 + 2A^2\cos\theta = 8A^2 + 8A^2(-\cos\theta)$

$\Rightarrow 5\cos\theta = 3$

$\Rightarrow \theta = \cos^{-1}\left(\frac{3}{5}\right)$

14. The escape velocity of a body on a planet '*A*' is 12 kms^{-1}. The escape velocity of the body on another planet '*B*', whose density is four times and radius is half of the planet '*A*', is:

(A) 12 kms^{-1}

(B) 24 kms^{-1}

(C) 36 kms^{-1}

(D) 6 kms^{-1}

Answer (A)

Sol. $v_{esc} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G}{R} \times \rho \times \frac{4}{3}\pi R^3}$

$\Rightarrow \quad v_{esc} \propto R\sqrt{\rho}$

$\Rightarrow \quad \frac{(v_{esc})_B}{(v_{esc})_A} = 1$

$\Rightarrow \quad (v_{esc})_B = 12$ km/s

15. At a certain place the angle of dip is 30° and the horizontal component of earth's magnetic field is 0.5 G. The earth's total magnetic field (in G), at that certain place, is :

(A) $\frac{1}{\sqrt{3}}$

(B) $\frac{1}{2}$

(C) $\sqrt{3}$

(D) 1

Answer (A)

Sol. $B_H = B\cos 30°$

$\Rightarrow \quad B = \frac{1}{\sqrt{3}} G$

16. A longitudinal wave is represented by $x = 10 \sin 2\pi \left(nt - \frac{x}{\lambda} \right)$ cm. The maximum particle velocity will be four times the wave velocity if the determined value of wavelength is equal to :

(A) 2π

(B) 5π

(C) π

(D) $\frac{5\pi}{2}$

Answer (B)

Sol. Particle velocity $= \frac{\partial x}{\partial t}$

$\Rightarrow$ Maximum particle velocity = $(2\pi n)$ (10)

$\Rightarrow \quad (2\pi n)(10) = (n\lambda)(4)$

$\Rightarrow \quad \lambda = 5\pi$

17. A parallel plate capacitor filled with a medium of dielectric constant 10, is connected across a battery and is charged. The dielectric slab is replaced by another slab of dielectric constant 15. Then the energy of capacitor will :

(A) increased by 50%

(B) decrease by 15%

(C) increase by 25%

(D) increase by 33%

Answer (A)

Sol. $U = \frac{1}{2}(kC_0)V^2$

$\Rightarrow \quad \frac{U'}{U} = 1.5$

$\Rightarrow$ Energy increases by 50%

18. A positive charge particle of 100 mg is thrown in opposite direction to a uniform electric field of strength 1 × 10^5 NC^{-1}. If the charge on the particle is 40 μC and the initial velocity is 200 ms^{-1}, how much distance it will travel before coming to the rest momentarily?

(A) 1 m

(B) 5 m

(C) 10 m

(D) 0.5 m

Answer (D)

Sol. $v^2 - u^2 = 2aS$

$\Rightarrow \quad 0^2 - 200^2 = 2\left(\frac{-qE}{m}\right)(S)$

$\Rightarrow \quad -200^2 = 2\left[\frac{-40 \times 10^{-6} \times 10^5}{100 \times 10^{-6}}\right][S]$

$\Rightarrow \quad S = \frac{4}{2 \times 4} \text{ m} = 0.5 \text{ m}$

19. Using Young's double slit experiment, a monochromatic light of wavelength 5000 Å produces fringes of fringe width 0.5 mm. If another monochromatic light of wavelength 6000 Å is used and the separation between the slits is doubled, then the new fringe width will be :

(A) 0.5 mm (B) 1.0 mm

(C) 0.6 mm (D) 0.3 mm

Answer (D)

Sol. Fringe width $= \frac{\lambda D}{d}$

$\Rightarrow$ Fringe width $\propto \frac{\lambda}{d}$

$\Rightarrow$ New fringe width = $0.5\text{ mm} \times \frac{1.2}{2} = 0.3\text{ mm}$

20. Only 2% of the optical source frequency is the available channel bandwidth for an optical communicating system operating at 1000 nm. If an audio signal requires a bandwidth of 8 kHz, how many channels can be accommodated for transmission?

(A) 375×10^7

(B) 75×10^7

(C) 375×10^8

(D) 75×10^9

Answer (B)

Sol. $v = f\lambda$

$$\Rightarrow f = \frac{v}{\lambda} = \frac{3\times10^8}{1000\times10^{-9}}\text{ Hz} = 3\times10^{14}\text{ Hz}$$

$$\Rightarrow \text{Channels} = \frac{\frac{2}{100}\times3\times10^{14}}{8\times10^3} = 75\times10^7$$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Two coils require 20 minutes and 60 minutes respectively to produce same amount of heat energy when connected separately to the same source. If they are connected in parallel arrangement to the same source; the time required to produce same amount of heat by the combination of coils, will be ___ min.

Answer (15)

Sol. $H = \frac{V^2}{R}\cdot\Delta t$

$$\Rightarrow H = \frac{V^2}{R_1}\cdot 20 = \frac{V^2}{R_2}\cdot 60 \quad \ldots\text{(i)}$$

Also, $$H = \frac{V^2}{\left[\frac{R_1R_2}{R_1+R_2}\right]}\cdot\Delta t$$

$$= \frac{4}{3}\cdot\frac{V^2}{R_1}\cdot\Delta t \qquad [\because R_2 = 3R_1]$$

$\Rightarrow \Delta t = 15$

2. The intensity of the light from a bulb incident on a surface is 0.22 W/m^2. The amplitude of the magnetic field in this light-wave is ___ × 10^{-9} T.

(Given : Permittivity of vacuum ε_0 = 8.85 × 10^{-12} C^2N^{-1} –m^{-2}, speed of light in vacuum c = 3 × 10^8 ms^{-1})

Answer (43)

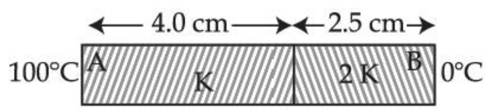
Sol. $I = \frac{1}{2}\varepsilon_0 E_0^2\cdot c = \frac{1}{2}\varepsilon_0(cB_0)^2 c$

$$\Rightarrow I = \frac{1}{2}\varepsilon_0 c^3 B_0^2$$

$$\Rightarrow 0.22 = \frac{1}{2}\left(8.85\times10^{-12}\right)\left(3\times10^8\right)^3 B_0^2$$

$$\Rightarrow B_0 \simeq 43\times10^{-9}T$$

3. As per the given figure, two plates *A* and *B* of thermal conductivity K and 2 K are joined together to form a compound plate. The thickness of plates are 4.0 cm and 2.5 cm respectively and the area of cross-section is 120 cm^2 for each plate. The equivalent thermal conductivity of the compound plate is $\left(1+\frac{5}{\alpha}\right)$ K, then the value of α will be ____.



Answer (21)

Sol. $\frac{L_1}{K_1A_1}+\frac{L_2}{K_2A_2}=\frac{L_1+L_2}{K_{eff}A_{eff}}$

$\Rightarrow \frac{4}{K}+\frac{2.5}{2K}=\frac{6.5}{K_{eff}}$

$\Rightarrow \frac{10.5}{2K}=\frac{6.5}{K_{eff}}$

$\Rightarrow K_{eff}=\frac{13K}{10.5}=\left(1+\frac{5}{21}\right)K$

$\Rightarrow \alpha = 21$

4. A body is performing simple harmonic with an amplitude of 10 cm. The velocity of the body was tripled by air Jet when it is at 5 cm from its mean position. The new amplitude of vibration is $\sqrt{x}$ cm. The value of x is ____.

Answer (700)

Sol. $v=\omega\sqrt{A^2-y^2}$

$\Rightarrow 3\omega\sqrt{10^2-5^2}=\omega\sqrt{(A')^2-5^2}$

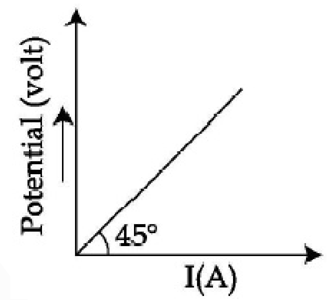
$\Rightarrow 9 \times 75 = (A')^2 - 25$

$\Rightarrow A'=\sqrt{28\times 25}$ cm

$\Rightarrow x = 700$

5. The variation of applied potential and current flowing through a given wire is shown in figure. The length of wire is 31.4 cm. The diameter of wire is measured as 2.4 cm. The resistivity of the given wire is measured as $x \times 10^{-3}\ \Omega$ cm. The value of *x* is____.

[Take π = 3.14]



Answer (144)

Sol. Resistance = tan45° = 1 Ω

$\Rightarrow 1=\frac{\rho l}{A}$

$\Rightarrow \rho=\frac{\pi(1.2\text{ cm})^2}{31.4\text{ cm}}=1.44\times 10^{-1}\ \Omega\text{cm}$

$\Rightarrow x = 144$

6. 300 cal. of heat is given to a heat engine and it rejects 225 cal. of heat. If source temperature is 227°C, then the temperature of sink will be _______°C.

Answer (102)

Sol. $\eta=\frac{W}{Q}=\frac{300-225}{300}$

$\Rightarrow \frac{75}{300}=1-\frac{T_L}{T_H}$

$\Rightarrow T_L=\frac{3}{4}\ T_H=\frac{3}{4}(500)$ = 375 K

$\Rightarrow T_L$ = 102°C

7. $\sqrt{d_1}$ and $\sqrt{d_2}$ are the impact parameters corresponding to scattering angles 60° and 90° respectively, when an α particle is approaching a gold nucleus. For $d_1 = x\, d_2$, the value of x will be ________.

Answer (3)

Sol. Impact parameter $\propto \cot\frac{\theta}{2}$

$$\Rightarrow \sqrt{\frac{d_1}{d_2}} = \frac{\sqrt{3}}{1}$$

$\Rightarrow d_1 = 3d_2$

$\Rightarrow x = 3$

8. A transistor is used in an amplifier circuit in common emitter mode. If the base current changes by 100 μA, it brings a change of 10 mA in collector current. If the load resistance is 2 kΩ and input resistance is 1 kΩ, the value of power gain is $x \times 10^4$. The value of x is ________.

Answer (2)

Sol. Power gain = $\left[\frac{\Delta i_C}{\Delta i_B}\right]^2 \times \frac{R_o}{R_i}$

$$= \left[\frac{10^{-2}}{10^{-4}}\right]^2 \times \frac{2}{1}$$

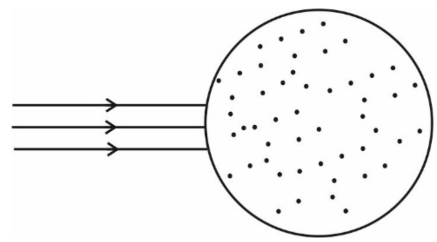
$= 2 \times 10^4$

$\Rightarrow x = 2$

9. A parallel beam of light is allowed to fall on a transparent spherical globe of diameter 30 cm and refractive index 1.5. The distance from the centre of the globe at which the beam of light can converge is________ mm.

Answer (225)

Sol.



1st refraction: $\frac{1.5}{v_1} - 0 = \frac{0.5}{15}$

$\Rightarrow v_1 = 45$ cm

2nd refraction: $\frac{1}{v_2} - \frac{1.5}{15} = \frac{-0.5}{-15}$

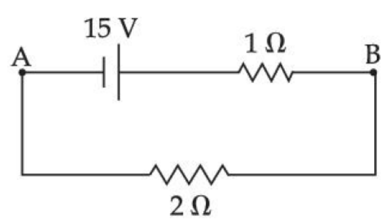
$$\Rightarrow \frac{1}{v_2} = \frac{1}{30} + \frac{1}{10}$$

$$= \frac{4}{30}$$

$\Rightarrow v_2 = +7.5$ cm

$\Rightarrow$ Distance from centre = 22.5 cm

10. For the network shown below, the value of $V_B - V_A$ is ________ V.



Answer (10)

Sol. $V_B - V_A = i \times 2$

$$= \frac{15}{1+2} \times 2$$

$\Rightarrow V_B - V_A = 10$ volts

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Production of iron in blast furnace follows the following equation

$$Fe_3O_4(s) + 4CO(g) \longrightarrow 3Fe(l) + 4CO_2(g)$$

when 4.640 kg of Fe_3O_4 and 2.520 kg of CO are allowed to react then the amount of iron (in g) produced is:

[Given : Molar Atomic mass (g mol^{-1}) : Fe = 56

Molar Atomic mass (g mol^{-1}) : O = 16

Molar Atomic mass (g mol^{-1}) : C = 12]

(A) 1400

(B) 2200

(C) 3360

(D) 4200

Answer (C)

Sol.

$$\underset{\substack{10^3\times\frac{4.64}{232}\text{moles} \\ 20\text{moles} \\ 0}}{Fe_3O_4(s)} + \underset{\substack{\frac{2.52\times10^3}{28}\text{moles} \\ 90\text{ moles} \\ 90-80}}{4CO(g)} \rightarrow \underset{\substack{60\text{ mole} = 60\times56 \\ =3360\text{g}}}{3Fe(l)} + 4CO_2(g)$$

2. Which of the following statements are **correct**?

(A) The electronic configuration of Cr is [Ar] $3d^5 4s^1$.

(B) The magnetic quantum number may have a negative value.

(C) In the ground state of an atom, the orbitals are filled in order of their increasing energies.

(D) The total number of nodes are given by n – 2.

Choose the **most appropriate** answer from the options given below :

(A) (A), (C) and (D) only

(B) (A) and (B) only

(C) (A) and (C) only

(D) (A), (B) and (C) only

Answer (D)

Sol. Cr = (Ar)$3d^5 4s^1$

M = +l to –l

As per Aufbau principle, orbitals are filled in increasing order of energy.

Total number of nodes = (n – 1)

3. Arrange the following in the decreasing order of their covalent character :

(A) LiCl

(B) NaCl

(C) KCl

(D) CsCl

Choose the **most appropriate** answer from the options given below :

(A) (A) > (C) > (B) > (D)

(B) (B) > (A) > (C) > (D)

(C) (A) > (B) > (C) > (D)

(D) (A) > (B) > (D) > (C)

Answer (C)

Sol. Covalent character $\propto$ polarising power of cation

Correct decreasing order of covalent character

LiCl > NaCl > KCl > CsCl

4. The solubility of AgCl will be maximum in which of the following?

(A) 0.01 M KCl

(B) 0.01 M HCl

(C) 0.01 M $AgNO_3$

(D) Deionised water

Answer (D)

Sol. Solubility decreases with increasing the concentration of common ion. Therefore, the maximum solubility of AgCl will be in deionized water.

5. Which of the following is a **correct** statement?

(A) Brownian motion destabilises sols.

(B) Any amount of dispersed phase can be added to emulsion without destabilising it.

(C) Mixing two oppositely charged sols in equal amount neutralises charges and stabilises colloids.

(D) Presence of equal and similar charges on colloidal particles provides stability to the colloidal solution.

Answer (D)

Sol. Presence of equal and similar charges on colloidal particle provides stability to the colloidal solution.

6. The electronic configuration of Pt(atomic number 78) is:

(A) [Xe] $4f^{14}$ $5d^9$ $6s^1$

(B) [Kr] $4f^{14}$ $5d^{10}$

(C) [Xe] $4f^{14}$ $5d^{10}$

(D) [Xe] $4f^{14}$ $5d^8$ $6s^2$

Answer (A)

Sol. Pt = [Xe] $4f^{14}$ $5d^9$ $6s^1$

7. In isolation of which one of the following metals from their ores, the use of cyanide salt is not commonly involved?

(A) Zinc (B) Gold

(C) Silver (D) Copper

Answer (D)

Sol. In the extraction of Silver and Gold, NaCN is used to leach the metal.

$$4Au(s) + 8C\bar{N}(aq) + 2H_2O + O_2(g) \rightarrow 4\left[Au(CN)_2\right]^-(aq) + 4OH^-(aq)$$

$$2[Au(CN)_2]^-(aq) + Zn \rightarrow 2Au(s) + [(Zn(CN)_4)]^{2-}(aq)$$

In case of ore containing ZnS and PbS, the depressant used is NaCN.

8. Which one of following reactions indicates the reducing ability of hydrogen peroxide in basic medium?

(A) $HOCl + H_2O_2 \rightarrow H_3O^+ + Cl^- + O_2$

(B) $PbS + 4H_2O_2 \rightarrow PbSO_4 + 4H_2O$

(C) $2MnO_4^- + 3H_2O_2 \rightarrow 2MnO_2 + 3O_2 + 2H_2O + 2OH^-$

(D) $Mn^{2+} + H_2O_2 \rightarrow Mn^{4+} + 2OH^-$

Answer (C)

Sol. $2\overset{+7}{Mn}O_4^- + 3H_2O_2 \rightarrow$

$$2\overset{+4}{Mn}O_2 + 3O_2 + 2H_2O + 2O\bar{H}$$

In basic medium MnO_4^- is reduced to MnO_2, whereas in acidic medium it is reduced to Mn^{+2}.

9. Match **List-I** with **List-II**.

List-I (Metal)	**List-II (Emitted light wavelength (nm))**
(A) Li	(I) 670.8
(B) Na	(II) 589.2
(C) Rb	(III) 780.0
(D) Cs	(IV) 455.5

Choose the **most appropriate** answer from the options given below:

(A) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

(B) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

(C) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)

(D) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

Answer (A)

Sol.

Metal	Li	Na	K	Rb	Cs
Colour	Crimson-red	Yellow	Violet	Red violet	Blue
λ/nm	670.8	589.2	766.5	780.0	455.5

10. Match **List-I** with **List-II**.

List-I (Metal)	**List-II (Application)**
(A) Cs	(I) High temperature thermometer
(B) Ga	(II) Water repellent sprays
(C) B	(III) Photoelectric cells
(D) Si	(IV) Bullet proof vest

Choose the **most appropriate** answer from the options given below:

(A) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(B) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(C) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

(D) (A)-(I), (B)-(IV), (C)-(II), (D)-(III)

Answer (A)

Sol.

Metal	**Application**
Cs	Photoelectric cells
Ga	High temperature thermometer
B	Bullet proof vest
Si	Water repellent sprays

11. The oxoacid of phosphorus that is easily obtained from a reaction of alkali and white phosphorus and has two P-H bonds, is:

(A) Phosphonic acid (B) Phosphinic acid

(C) Pyrophosphorus acid (D) Hypophosphoric acid

Answer (B)

Sol. White phosphorus + alkali $\rightarrow$ H_3PO_2

H_3PO_2 = phosphinic acid

12. The acid that is believed to be mainly responsible for the damage of Taj Mahal is

(A) sulfuric acid (B) hydrofluoric acid

(C) phosphoric acid (D) hydrochloric acid

Answer (A)

Sol. $CaCO_3 + H_2SO_4 \rightarrow CaSO_4 + H_2O + CO_2$

13. Two isomers 'A' and 'B' with molecular formula C_4H_8 give different products on oxidation with $KMnO_4/H^+$ results in effervescence of a gas and gives ketone. The compound 'A' is

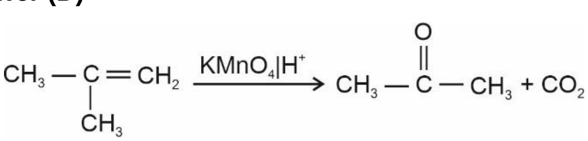
(A) But-1-ene

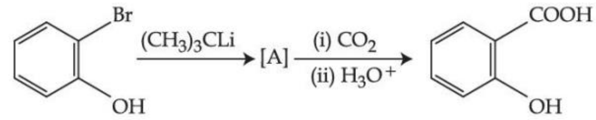
(B) cis-But-2-ene

(C) trans-But-2-ene

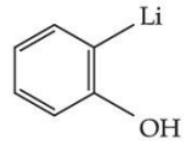
(D) 2-methyl propene

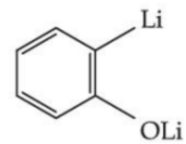
Answer (D)

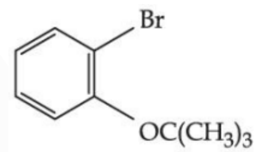
Sol. 

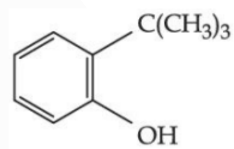
14. 

In the given conversion the compound A is:

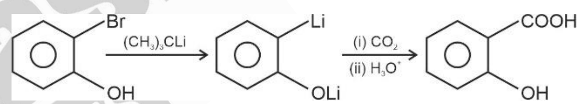
(A) 

(B) 

(C) 

(D) 

Answer (B)

Sol. 

15. Given below are two statements:

Statement I: The esterification of carboxylic acid with an alcohol is a nucleophilic acyl substitution.

Statement II: Electron withdrawing groups in the carboxylic acid will increase the rate of esterification reaction.

Choose the **most appropriate** option :

(A) Both **Statement I** and **Statement II** are correct.

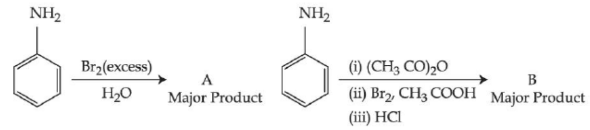
(B) Both **Statement I** and **Statement II** are incorrect.

(C) **Statement I** is correct but **Statement II** is incorrect.

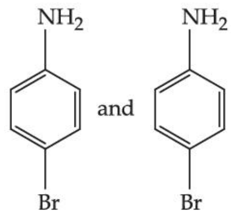
(D) **Statement I** is incorrect but **Statement II** is correct.

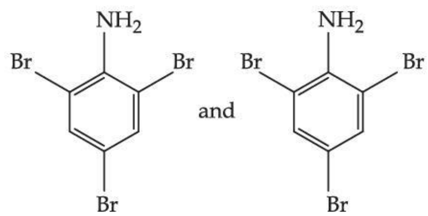
Answer (A)

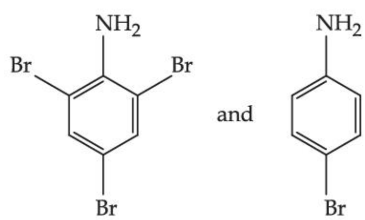
Sol. Esterification of carboxylic acid with an alcohol is nucleophilic acyl substitution and presence of electron withdrawing group in the carboxylic acid increases the rate of esterification reaction.

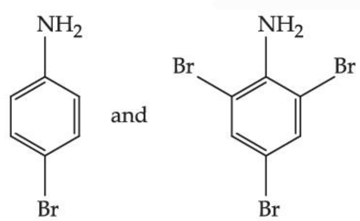
16. 

Consider the above reactions, the product A and product B respectively are

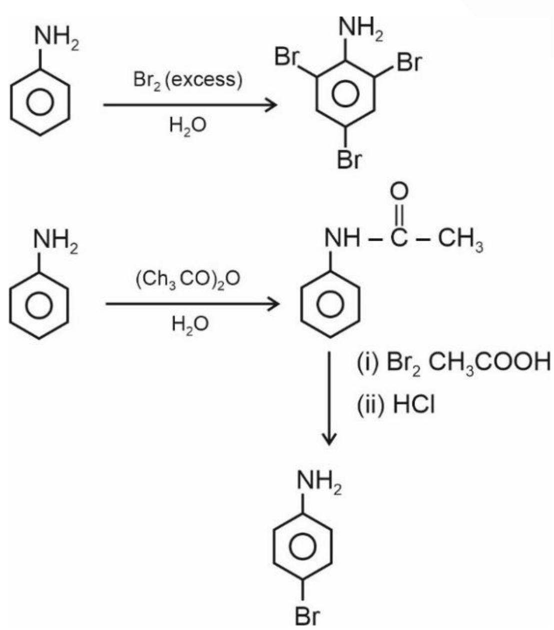
(A) 

(B) 

(C) 

(D) 

Answer (C)

Sol. 

17. The polymer, which can be stretched and retains its original status on releasing the force is

(A) Bakelite

(B) Nylon 6, 6

(C) Buna-N

(D) Terylene

Answer (C)

Sol. Buna – N is a synthetic rubber which can be stretched and retains its original status on releasing the force.

18. Sugar moiety in DNA and RNA molecules respectively are

(A) β-D-2-deoxyribose, β-D-deoxyribose

(B) β-D-2-deoxyribose, β-D-ribose

(C) β-D-ribose, β-D-2-deoxyribose

(D) β-D-deoxyribose, β-D-2-deoxyribose

Answer (B)

Sol. DNA consists of β-D-2-deoxyribose sugar whereas RNA consists of β-D-ribose.

19. Which of the following compound **does not** contain sulphur atom?

(A) Cimetidine

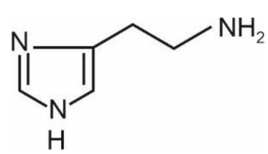
(B) Ranitidine

(C) Histamine

(D) Saccharin

Answer (C)

Sol. Histamine – does not contain sulphur.



20. Given below are two statements.

Statement I : Phenols are weakly acidic.

Statement II : Therefore they are freely soluble in NaOH solution and are weaker acids than alcohols and water.

Choose the **most appropriate** option.

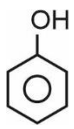
(A) Both **Statement I** and **Statement II** are correct.

(B) Both **Statement I** and **Statement II** are correct.

(C) **Statement I** is correct but **Statement II** is incorrect.

(D) **Statement I** is incorrect but **Statement II** is correct.

Answer (C)

Sol.  $pKa = 10$

Alcohol $pKa \approx 16$

H_2O $pKa \approx 15$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Geraniol, a volatile organic compound, is a component of rose oil. The density of the vapour is 0.46 gL^{-1} at 257°C and 100 mm Hg. The molar mass of geraniol is ____ g mol^{-1}. (Nearest Integer) [Given: R = 0.082 L atm K^{-1} mol^{-1}]

Answer (152)

Sol. $\frac{PM}{RT} = d \quad = \quad M = \frac{0.46 \times 0.082 \times 530 \times 760}{100}$

$\simeq 152 \text{ g mol}^{-1}$

2. 17.0 g of NH_3 completely vapourises at – 33.42°C and 1 bar pressure and the enthalpy change in the process is 23.4 kJ mol^{-1}. The enthalpy change for the vapourisation of 85 g of NH_3 under the same conditions is ____ kJ.

Answer (117)

Sol. Number of moles of NH_3 = 5

So, required $\Delta H = 5 \times 23.4$

$= 117$ kJ

3. 1.2 mL of acetic acid is dissolved in water to make 2.0 L of solution. The depression in freezing point observed for this strength of acid is 0.0198°C. The percentage of dissociation of the acid is ____ . (Nearest integer)

[Given: Density of acetic acid is 1.02 g mL^{-1}

Molar mass of acetic acid is 60 g mol^{-1}

$K_f(H_2O)$ = 1.85 K kg mol^{-1}]

Answer (5)

Sol. $\Delta T_b = i \times K_b \times m$

Moles of solute (acetic acid) = $\frac{1.2 \times 1.02}{60}$

As moles of solute are very less.

So, take molarity and molality same.

$0.0198 = i \times 1.85 \times \frac{1.2 \times 1.02}{60 \times 2}$

$i = 1.05$

$\alpha = \frac{i-1}{n-1} = \frac{0.05}{1} = 0.05$

4. A dilute solution of sulphuric acid is electrolysed using a current of 0.10 A for 2 hours to produce hydrogen and oxygen gas. The total volume of gases produced at STP is ____ cm^3. (Nearest integer)

[Given : Faraday constant F = 96500 C mol^{-1} at STP, molar volume of an ideal gas is 22.7 L mol^{-1}]

Answer (127 cm^3)

Sol. 2 F produces = $\frac{3}{2}$ mole of gas

0.10 ×2 × 3600 coulomb produces

$= \frac{\frac{3}{2} \times 0.1 \times 2 \times 3600}{2 \times 96500}$

= 0.0056 moles of gas

Volume of gas produced = 0.0056 × 22.7 L

$$\simeq 0.127 \text{ L}$$

$$= 127 \text{ mL}$$

5. The activation energy of one of the reactions in a biochemical process is 532611 J mol^{-1}. When the temperature falls from 310 K to 300 K, the change in rate constant observed is $k_{300} = x \times 10^{-3}\, k_{310}$. The value of x is ______.

[Given: ln10 = 2.3, R = 8.3 JK^{-1} mol^{-1}]

Answer (1)

Sol. $\log\left(\frac{K_{310}}{K_{300}}\right) = \frac{532611}{8.3}\left(\frac{1}{300} - \frac{1}{310}\right)$

$$\frac{K_{310}}{K_{300}} = 10^3 \;\Rightarrow\; K_{300} = 1\times 10^{-3} \times K_{310}$$

6. The number of terminal oxygen atoms present in the product B obtained from the following reaction is ________.

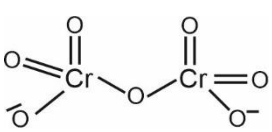
$FeCr_2O_4 + Na_2CO_3 + O_2 \rightarrow A + Fe_2O_3 + CO_2$

$A + H^+ \rightarrow B + H_2O + Na^+$

Answer (6)

Sol. $FeCr_2O_4 + Na_2CO_3 + O_2 \longrightarrow Fe_2O_3 + CO_2 + \underset{(A)}{Na_2CrO_4}$

$$Na_2CrO_4 + H^+ \longrightarrow Cr_2O_7^{-2} + H_2O + Na^+$$



7. An acidified manganate solution undergoes disproportionation reaction. The spin-only magnetic moment value of the product having manganese in higher oxidation state is ____ B.M. (Nearest integer)

Answer (00.00)

Sol. $MnO_4^{-2} \xrightarrow{H^+} MnO_2 + MnO_4^- + H_2O$

$Mn^{+7} = d^0$

Hence, magnetic moment = zero

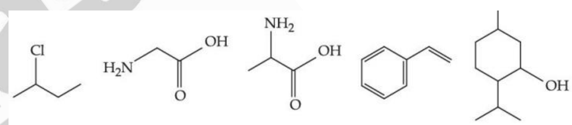
8. Kjeldahl's method was used for the estimation of nitrogen in an organic compound. The ammonia evolved from 0.55 g of the compound neutralised 12.5 mL of 1 M H_2SO_4 solution. The percentage of nitrogen in the compound is _____. (Nearest integer)

Answer (64)

Sol. $\%\,N = \frac{1.4 \times N \times V}{\text{Mass of organic compound}}$

$$= \frac{1.4 \times 2 \times 12.5}{0.55} = 63.63\% \simeq 64$$

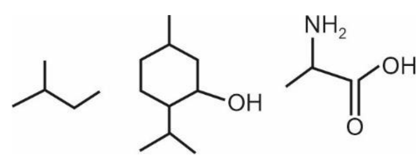
9. Observe structures of the following compounds



The total number of structures/compounds which possess asymmetric carbon atoms is _____.

Answer (3)

Sol.



10. $C_6H_{12}O_6 \xrightarrow{\text{Zymase}} A \xrightarrow[\Delta]{\text{NaOI}} B + CHI_3$

The number of carbon atoms present in the product B is ______.

Answer (1)

Sol. $C_6H_{12}O_6 \xrightarrow{\text{zymose}} C_2H_5OH \xrightarrow{\text{NaOI}} \underset{(B)}{HCOONa} + CHI_3$

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The probability that a randomly chosen 2 × 2 matrix with all the entries from the set of first 10 primes, is singular, is equal to :

(A) $\frac{133}{10^4}$ (B) $\frac{18}{10^3}$

(C) $\frac{19}{10^3}$ (D) $\frac{271}{10^4}$

Answer (C)

Sol. Let A = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29}

Let E be the event that matrix of order 2 × 2 is singular

Case-I

All entries are same example $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

$= {}^{10}C_1$

Case-II

Matrix with two prime numbers only example $\begin{bmatrix} 3 & 5 \\ 3 & 5 \end{bmatrix}$

$= {}^{10}C_2 \times 2! \times 2!$

$$P(E) = \frac{{}^{10}C_1 + {}^{10}C_2 \times 2! \times 2!}{10^4} = \frac{190}{10^4} = \frac{19}{10^3}$$

2. Let the solution curve of the differential equation $x\frac{dy}{dx} - y = \sqrt{y^2 + 16x^2}$, $y(1) = 3$ be $y = y(x)$. Then $y(2)$ is equal to :

(A) 15 (B) 11

(C) 13 (D) 17

Answer (A)

Sol. $x\frac{dy}{dx} - y = \sqrt{y^2 + 16x^2}$

$y = 4x \tan\theta$

$$\frac{dy}{dx} = 4\tan\theta + 4x\sec^2\theta\frac{d\theta}{dx}$$

$$4x\tan\theta + 4x^2\sec^2\theta\frac{d\theta}{dx} - 4x\tan\theta = 4x\sec\theta$$

$$\int \sec\theta\, d\theta = \int \frac{dx}{x}$$

$\log|\sec\theta + \tan\theta| = \log|x| + C$

$y(1) = 3 \Rightarrow 3 = 4\tan\theta$

$$= \tan\theta = \frac{3}{4} \Rightarrow \sec\theta = \frac{5}{4}$$

$$\ln\left|\frac{8}{4}\right| = \ln|1| + C$$

$\Rightarrow$ $C = \ln 2$

$\therefore$ $|\sec\theta + \tan\theta| = 2|x|$

To find $y(2)$ put $x = 2$

$$\Rightarrow \tan\theta = \frac{y}{8}$$

$(\sec\theta + \tan\theta)^2 = 16$

$\sec\theta + \tan\theta = \pm 4$

$\sec\theta - \tan\theta = \pm\frac{1}{4}$

$$2\tan\theta = \frac{15}{4} = 2\times\frac{y}{8}$$

$\Rightarrow$ $\boxed{y = 15}$

3. If the mirror image of the point (2, 4, 7) in the plane $3x - y + 4z = 2$ is (a, b, c), then $2a + b + 2c$ is equal to :

(A) 54 (B) 50

(C) –6 (D) –42

Answer (C)

Sol. Mirror image of (2, 4, 7) in $3x - y + 4z = 2$ is (a, b, c) then

$$\frac{a-2}{3} = \frac{b-4}{-1} = \frac{c-7}{4} = \frac{-2(6-4+28-2)}{3^2+(-1)^2+4^2}$$

$$\frac{a-2}{3} = \frac{b-4}{-1} = \frac{c-7}{4} = \frac{-28}{13}$$

$$a = \frac{-58}{13} \quad b = \frac{80}{13} \quad c = \frac{-21}{13}$$

$$2a + b + 2c = \frac{-116 + 80 - 42}{13}$$

$= -6$

4. Let $f : R \Rightarrow R$ be a function defined by :

$$f(x)=\begin{cases}\max\limits_{t\le x}\{t^3-3t\} & ; \quad x\le 2\\ x^2+2x-6 & ; \quad 2<x<3\\ [x-3]+9 & ; \quad 3\le x\le 5\\ 2x+1 & ; \quad x>5\end{cases}$$

where $[t]$ is the greatest integer less than or equal to t. Let m be the number of points where f is not differentiable and $I=\int_{-2}^{2} f(x)dx$. Then the ordered pair (m, I) is equal to :

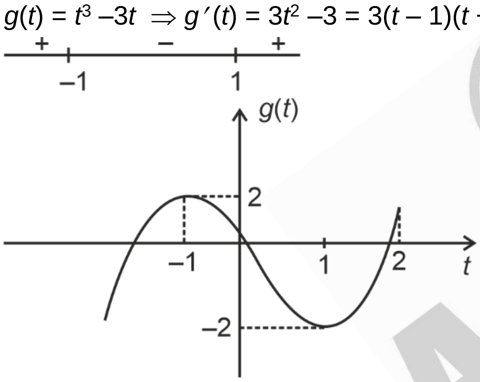
(A) $\left(3, \frac{27}{4}\right)$ (B) $\left(3, \frac{23}{4}\right)$

(C) $\left(4, \frac{27}{4}\right)$ (D) $\left(4, \frac{23}{4}\right)$

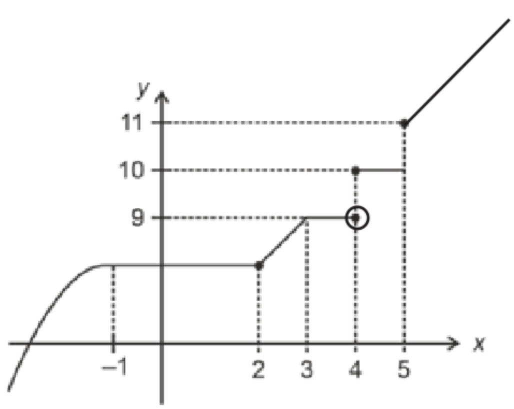
Answer (C)

Sol. $\max\limits_{t\le x}\{t^3-3t\}$; $x \le 2$

$g(t) = t^3 - 3t \Rightarrow g'(t) = 3t^2 - 3 = 3(t-1)(t+1)$



$$f(x)=\begin{cases}x^3-3x & x<-1\\ 2 & -1\le x\le 2\\ x^2+2x-6 & 2<x<3\\ 9 & 3\le x<4\\ 10 & 4\le x<5\\ 11 & x=5\\ 2x+1 & x>5\end{cases}$$



Points of non-differentiability = {2, 3, 4, 5}

$\Rightarrow \quad m = 4$

$$I=\int_{-2}^{2} f(x)dx=\int_{-2}^{-1}(x^3-3x)dx+\int_{-1}^{2} 2dx$$

$$=\left[\frac{x^4}{4}-\frac{3x^2}{2}\right]_{-2}^{-1}+2(2+1)=\left(\frac{1}{4}-\frac{3}{2}\right)-(4-6)+6$$

$$=\frac{27}{4}$$

5. Let $\vec{a}=\alpha\hat{i}+3\hat{j}-\hat{k}$, $\vec{b}=3\hat{i}-\beta\hat{j}+4\hat{k}$ and $\vec{c}=\hat{i}+2\hat{j}-2\hat{k}$ where $\alpha, \beta \in \mathbf{R}$, be three vectors. If the projection of $\vec{a}$ on $\vec{c}$ is $\frac{10}{3}$ and $\vec{b}\times\vec{c}=-6\hat{i}+10\hat{j}+7\hat{k}$, then the value of $\alpha + \beta$ is equal to :

(A) 3 (B) 4

(C) 5 (D) 6

Answer (A)

Sol. $\vec{a}=\alpha\hat{i}+3\hat{j}-\hat{k}$

$\vec{b}=3\hat{i}-\beta\hat{j}+4\hat{k}$

$\vec{c}=\hat{i}+2\hat{j}-2\hat{k}$

Projection of $\vec{a}$ on $\vec{c}$ is

$$\frac{\vec{a}\cdot\vec{c}}{|\vec{b}|}=\frac{10}{3}$$

$$\frac{\alpha+6+2}{\sqrt{1^2+2^2+(-2)^2}}=\frac{\alpha+8}{3}=\frac{10}{3}$$

$\therefore \boxed{\alpha = 2}$

$\vec{b}\times\vec{c}=-6\hat{i}+10\hat{j}+7\hat{k}$

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -\beta & 4\\ 1 & 2 & -2\end{vmatrix}=(2\beta-8)\hat{i}+10\hat{j}+(6+\beta)\hat{k}=-6\hat{i}+10\hat{j}+7\hat{k}$$

$2\beta - 8 = -6$ & $6 + \beta = 7$

$\therefore \boxed{\beta = 1}$

$\alpha + \beta = 2 + 1 = 3$

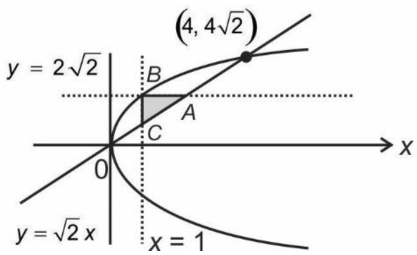
6. The area enclosed by $y^2 = 8x$ and $y = \sqrt{2}x$ that lies outside the triangle formed by $y = \sqrt{2}x,\ x = 1,\ y = 2\sqrt{2},$ is equal to :

(A) $\frac{16\sqrt{2}}{6}$ (B) $\frac{11\sqrt{2}}{6}$

(C) $\frac{13\sqrt{2}}{6}$ (C) $\frac{5\sqrt{2}}{6}$

Answer (C)

Sol.



$A\left(2, 2\sqrt{2}\right), B\left(1, 2\sqrt{2}\right), C\left(1, \sqrt{2}\right)$

$$\text{Area} = \int_0^{4\sqrt{2}} \left(\frac{y}{\sqrt{2}} - \frac{y^2}{8}\right) dy - \text{area}\,(\Delta BAC)$$

$$= \left[\frac{y^2}{2\sqrt{2}} - \frac{y^3}{24}\right]_0^{4\sqrt{2}} - \frac{1}{2} \times AB \times BC$$

$$= 8\sqrt{2} - \frac{32 \times 4\sqrt{2}}{24} - \frac{1}{2} \times 1 \times \sqrt{2}$$

$$= 8\sqrt{2} - \frac{16\sqrt{2}}{3} - \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{2}}{6}(48 - 32 - 3) = \frac{13\sqrt{2}}{6}$$

7. If the system of linear equations

$2x + y - z = 7$

$x - 3y + 2z = 1$

$x + 4y + \delta z = k$, where $\delta, k \in \mathbf{R}$

has infinitely many solutions, then $\delta + k$ is equal to:

(A) –3 (B) 3

(C) 6 (D) 9

Answer (B)

Sol. $2x + y - z = 7$

$x - 3y + 2z = 1$

$x + 4y + \delta z = k$

$$\Delta = \begin{vmatrix} 2 & 1 & -1 \\ 1 & -3 & 2 \\ 1 & 4 & \delta \end{vmatrix} = -7\delta - 21 = 0$$

$$\boxed{\delta = -3}$$

$$\Delta_1 = \begin{vmatrix} 7 & 1 & -1 \\ 1 & -3 & 2 \\ k & 4 & -3 \end{vmatrix}$$

$\Rightarrow\ 6 - k = 0 \Rightarrow k = 6$

$\delta + k = -3 + 6 = 3$

8. Let α and β be the roots of the equation $x^2 + (2i - 1) = 0$. Then, the value of $\left|\alpha^8 + \beta^8\right|$ is equal to:

(A) 50 (B) 250

(C) 1250 (D) 1500

Answer (A)

Sol. $x^2 + 2i - 1 = 0$

$\alpha^2 = \beta^2 = 1 - 2i$

$\alpha^4 = (1 - 2i)^2 = 1 + (2i)^2 - 4i = -3 - 4i$

$\alpha^8 = (-3 - 4i)^2 = 9 - 16 + 24i = -7 + 24i$

$$\left|\alpha^8 + \beta^8\right| = 2\left|-7 + 24i\right| = 2\sqrt{(-7)^2 + (24)^2} = 50$$

9. Let $\Delta \in \{\wedge, \vee, \Rightarrow, \Leftrightarrow\}$ be such that $(p \wedge q)\Delta((p \vee q) \Rightarrow q)$ is a tautology. Then Δ is equal to :

(A) $\wedge$ (B) $\vee$

(C) $\Rightarrow$ (D) $\Leftrightarrow$

Answer (C)

Sol. $(p \vee q) \Rightarrow q$

$\sim(p \vee q) \vee q$

$= (\sim p \wedge \sim q) \vee q$

$= (\sim p \vee q) \wedge (\sim q \vee q)$

$= (\sim p \vee q) \wedge T$

$= \sim p \vee q$

Now $(p \wedge q)\Delta(\sim p \vee q)$

p	q	$\sim p$	$p \wedge q$	$\sim p \vee q$	$(p \wedge q)\Delta(\sim p \vee q)$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	F	T	T
F	F	T	F	T	T

$\therefore\ \Delta = \Rightarrow$

10. Let $A = [a_{ij}]$ be a square matrix of order 3 such that $a_{ij} = 2^{j-i}$, for all $i, j = 1, 2, 3$. Then, the matrix $A^2 + A^3 + \ldots + A^{10}$ is equal to :

(A) $\left(\frac{3^{10}-3}{2}\right)A$ (B) $\left(\frac{3^{10}-1}{2}\right)A$

(C) $\left(\frac{3^{10}+1}{2}\right)A$ (D) $\left(\frac{3^{10}+3}{2}\right)A$

Answer (A)

Sol. $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 2^0 & 2^1 & 2^2 \\ 2^{-1} & 2^0 & 2^1 \\ 2^{-2} & 2^{-1} & 2^0 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 2 & 4 \\ \frac{1}{2} & 1 & 2 \\ \frac{1}{4} & \frac{1}{2} & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 4 \\ \frac{1}{2} & 1 & 2 \\ \frac{1}{4} & \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 12 \\ \frac{3}{2} & 3 & 6 \\ \frac{3}{4} & \frac{3}{2} & 3 \end{bmatrix} = 3A$$

$A^2 = 3A$

$A^3 = A.A^2 = A(3A) = 3A^2 = 3^2A$

$A^4 = 3^3A$

Now

$A^2 + A^3 + \ldots + A^{10}$

$A[3^1 + 3^2 + 3^3 + \ldots + 3^9]$

$$= \frac{3\left[3^9 - 1\right]}{3-1}A$$

$$= \frac{\left(3^{10} - 3\right)}{2}A$$

11. Let a set $A = A_1 \cup A_2 \cup \ldots \cup A_k$, where $A_i \cap A_j = \phi$ for $i \neq j$, $1 \leq i, j \leq k$. Define the relation R from A to A by $R = \{(x, y) : y \in A_i$ if and only if $x \in A_i, 1 \leq i \leq k\}$. Then, R is :

(A) reflexive, symmetric but not transitive

(B) reflexive, transitive but not symmetric

(C) reflexive but not symmetric and transitive

(D) an equivalence relation

Answer (D)

Sol. $R = \{(x, y) : y \in A_i$, iff $x \in A_i, 1 \leq i \leq k\}$

(1) Reflexive

$(a, a) \Rightarrow a \in A_i$ iff $a \in A_i$

(2) Symmetric

$(a, b) \Rightarrow a \in A_i$ iff $b \in A_i$

$(b, a) \in R$ as $b \in A_i$ iff $a \in A_i$

(3) Transitive

$(a, b) \in R$ & $(b, c) \in R$.

$\Rightarrow a \in A_i$ iff $b \in A_i$ & $b \in A_i$ iff $c \in A_i$

$\Rightarrow a \in A_i$ iff $c \in A_i$

$\Rightarrow (a, c) \in R.$

$\Rightarrow$ Relation is equivalence

12. Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $a_0 = a_1 = 0$ and $a_{n+2} = 2a_{n+1} - a_n + 1$ for all $n \geq 0$.

Then $\sum_{n=2}^{\infty} \frac{a_n}{7^n}$ is equal to :

(A) $\frac{6}{343}$ (B) $\frac{7}{216}$

(C) $\frac{8}{343}$ (D) $\frac{49}{216}$

Answer (B)

Sol. $a_{n+2} = 2a_{n+1} - a_n + 1$ & $a_0 = a_1 = 0$

$a_2 = 2a_1 - a_0 + 1 = 1$

$a_3 = 2a_2 - a_1 + 1 = 3$

$a_4 = 2a_3 - a_2 + 1 = 6$

$a_5 = 2a_4 - a_3 + 1 = 10$

$$\sum_{n=2}^{\infty} \frac{a_n}{7^n} = \frac{a_2}{7^2} + \frac{a_3}{7^3} + \frac{a_4}{7^4} + \ldots$$

$$s = \frac{1}{7^2} + \frac{3}{7^3} + \frac{6}{7^4} + \frac{10}{7^5} + \ldots$$

$$\frac{1}{7}s = \frac{1}{7^3} + \frac{3}{7^4} + \frac{6}{7^5} + \ldots$$

$$\frac{6s}{7} = \frac{1}{7^2} + \frac{2}{7^3} + \frac{3}{7^4} + \ldots$$

$$\frac{6s}{49} = \frac{1}{7^3} + \frac{2}{7^4} + \ldots$$

$$\frac{36s}{49} = \frac{1}{7^2} + \frac{1}{7^3} + \frac{1}{7^4} + \ldots$$

$$\frac{36s}{49} = \frac{\frac{1}{7^2}}{1-\frac{1}{7}}$$

$$\frac{36s}{49} = \frac{7}{49 \times 6}$$

$$\boxed{s = \frac{7}{216}}$$

13. The distance between the two points A and A' which lie on $y = 2$ such that both the line segments AB and $A'B$ (where B is the point (2, 3)) subtend angle $\frac{\pi}{4}$ at the origin, is equal to

(A) 10 (B) $\frac{48}{5}$

(C) $\frac{52}{5}$ (D) 3

Answer (C)

Sol. Let $A(\alpha, 2)$ Given $B(2, 3)$

$$m_{OA} = \frac{2}{\alpha} \;\; \& \;\; m_{OB} = \frac{3}{2}$$

$$\tan\frac{\pi}{4} = \left|\frac{\frac{2}{\alpha} - \frac{3}{2}}{1 + \frac{2}{\alpha}\cdot\frac{3}{2}}\right| \Rightarrow \frac{4-3\alpha}{2\alpha+6} = \pm 1$$

$4 - 3\alpha = 2\alpha + 6$ & $4 - 3\alpha = -2\alpha - 6$

$$\alpha = \frac{-2}{5} \;\; \& \;\; \alpha = 10$$

$$A\left(-\frac{2}{5}, 2\right) \;\; \& \;\; A'(10, 2) \text{ and } B(2, 3)$$

$$AA' = 10 + \frac{2}{5} = \frac{52}{5}$$

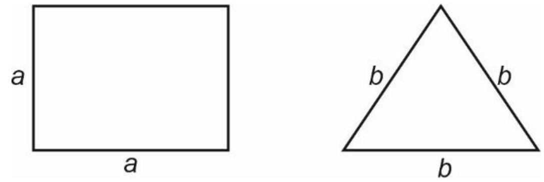
14. A wire of length 22 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into an equilateral triangle. Then, the length of the side of the equilateral triangle, so that the combined area of the square and the equilateral triangle is minimum, is

(A) $\frac{22}{9+4\sqrt{3}}$ (B) $\frac{66}{9+4\sqrt{3}}$

(C) $\frac{22}{4+9\sqrt{3}}$ (D) $\frac{66}{4+9\sqrt{3}}$

Answer (B)

Sol.



$4a + 3b = 22$

$$\text{Total area} = A = a^2 + \frac{\sqrt{3}}{4}b^2$$

$$A = \left(\frac{22-3b}{4}\right)^2 + \frac{\sqrt{3}}{4}b^2$$

$$\frac{dA}{dB} = 2\left(\frac{22-3b}{4}\right)\left(\frac{-3}{4}\right) + \frac{\sqrt{3}}{4}.2b = 0$$

$$\Rightarrow \frac{\sqrt{3}b}{2} = \frac{3}{8}(22-3b)$$

$$4\sqrt{3}b = 66 - 9b$$

$$\boxed{b = \frac{66}{9+4\sqrt{3}}}$$

15. The domain of the function $\cos^{-1}\left(\frac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi}\right)$ is :

(A) $\mathbf{R} - \left\{-\frac{1}{2}, \frac{1}{2}\right\}$

(B) $(-\infty, -1] \cup [1, \infty) \cup \{0\}$

(C) $\left(-\infty, \frac{-1}{2}\right) \cup \left(\frac{1}{2}, \infty\right) \cup \{0\}$

(D) $\left(-\infty, \frac{-1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, \infty\right) \cup \{0\}$

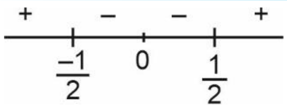
Answer (D)

Sol. $-1 \le \frac{2}{\pi}\sin^{-1}\left(\frac{1}{4x^2-1}\right) \le 1$

$$-\frac{\pi}{2} \le \sin^{-1}\frac{1}{4x^2-1} \le \frac{\pi}{2}$$

$$-1 \le \frac{1}{4x^2-1} \le 1$$

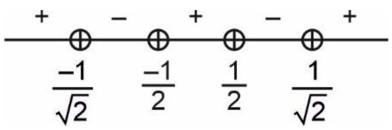
$$\frac{1}{4x^2-1} \ge -1 \Rightarrow \frac{4x^2}{(2n+1)(2x-1)} \ge 0$$



$x \in \left(-\infty, \frac{-1}{2}\right) \cup (0) \cup \left(\frac{1}{2}, \infty\right)$ ...(1)

$\frac{1}{4x^2-1} \le 1 \Rightarrow 1 - \frac{1}{4x^2-1} \ge 0 \Rightarrow \frac{4x^2-2}{4x^2-1} \ge 0$

$\Rightarrow \frac{\left(x+\frac{1}{\sqrt{2}}\right)\left(x-\frac{1}{\sqrt{2}}\right)}{\left(x+\frac{1}{2}\right)\left(x-\frac{1}{2}\right)} \ge 0$



$x \in \left(-\infty, \frac{-1}{\sqrt{2}}\right] \cup \left(\frac{-1}{2}, \frac{1}{2}\right) \cup \left(\frac{1}{\sqrt{2}}, \infty\right)$ ...(2)

From (1) & (2)

$x \in \left(-\infty, \frac{-1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, \infty\right) \cup \{0\}$

16. If the constant term in the expansion of $\left(3x^3 - 2x^2 + \frac{5}{x^5}\right)^{10}$ is $2^k \cdot l$, where l is an odd integer, then the value of k is equal to

(A) 6 (B) 7

(C) 8 (D) 9

Answer (D)

Sol. Constant term in

$\left(3x^3 - 2x^2 + \frac{5}{x^5}\right)^{10} \to x^0$

$\Rightarrow (3x^8 - 2x^7 + 5)^{10} \to x^{50}$

General term of $(3x^8 - 2x^7 + 5)^{10}$ is

$\frac{10!}{p!q!r!}(3x^8)^p(-2x^7)^q(5)^r$

Here $8p + 7q = 50$ and $p + q + r = 10$

$\Rightarrow p = 1, q = 6, r = 3$ is only valid solution

$\therefore \frac{10!}{1!6!r!} 3^1\, 2^6 \cdot 5^3 = 2^k \cdot l$

$\Rightarrow 5 \cdot 3 \cdot 7 \cdot 5^3 \cdot 3 \cdot 2^9 = 2^k\, l$

$\therefore k = 9$

17. $\int_0^5 \cos\left(\pi\left(x - \left[\frac{x}{2}\right]\right)\right)dx$, where $[t]$ denotes greatest integer less than or equal to t, is equal to

(A) –3 (B) –2

(C) 2 (D) 0

Answer (D)

Sol. $\int_0^5 \cos\left(\pi\left(x - \left[\frac{x}{2}\right]\right)\right)dx$

$= \int_0^2 \cos(\pi x)dx + \int_2^4 \cos(\pi(x-1))dx + \int_4^5 \cos(\pi(x-2))dx$

$\left.\frac{\sin \pi x}{\pi}\right|_0^2 + \left.\frac{\sin(\pi(x-1))}{\pi}\right|_2^4 + \left.\frac{\sin(\pi(x-2))}{\pi}\right|_4^5$

$= 0 + 0 + 0 = 0$

18. Let PQ be a focal chord of the parabola $y^2 = 4x$ such that it subtends an angle of $\frac{\pi}{2}$ at the point $(3, 0)$. Let the line segment PQ be also a focal chord of the ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a^2 > b^2$. If e is the eccentricity of the ellipse E, then the value of $\frac{1}{e^2}$ is equal to

(A) $1+\sqrt{2}$

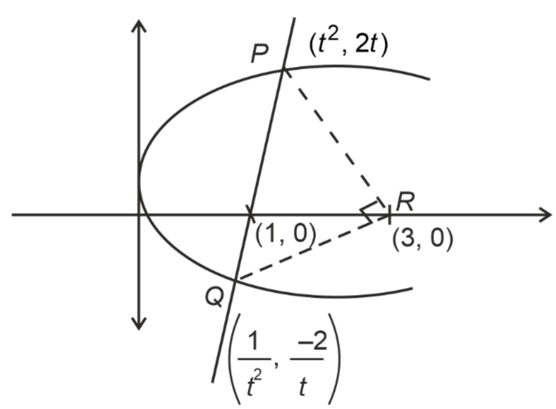
(B) $3+2\sqrt{2}$

(C) $1+2\sqrt{3}$

(D) $4+5\sqrt{3}$

Answer (B)

Sol.



As $\angle PRQ = \frac{\pi}{2}$

$$\left(\frac{\frac{2}{t}}{3-\frac{1}{t^2}}\right)\cdot\left(\frac{-2t}{3-t^2}\right) = -1$$

$\Rightarrow$ $t = \pm 1$

$\therefore$ $P \equiv (1, 2)$ & $Q(1, -2)$

$\therefore$ for ellipse $\frac{1}{a^2} + \frac{4}{b^2} = 1$ and $ae = 1$

$\Rightarrow$ $\frac{1}{a^2} + \frac{4}{a^2(1-e^2)} = 1$

$\Rightarrow$ $1 + \frac{4}{(1-e^2)} = \frac{1}{e^2}$

$\Rightarrow$ $(5 - e^2)e^2 = 1 - e^2$

$\Rightarrow$ $e^4 - 6e^2 + 1 = 0$

$\Rightarrow$ $e^2 = \frac{1}{3-2\sqrt{2}} \Rightarrow \frac{1}{e^2} = 3 + 2\sqrt{2}$

19. Let the tangent to the circle C_1: $x^2 + y^2 = 2$ at the point $M(-1, 1)$ intersect the circle C_2: $(x - 3)^2 + (y - 2)^2 = 5$, at two distinct points A and B. If the tangents to C_2 at the points A and B intersect at N, then the area of the triangle ANB is equal to

(A) $\frac{1}{2}$

(B) $\frac{2}{3}$

(C) $\frac{1}{6}$

(D) $\frac{5}{3}$

Answer (C)

Sol. Tangent to C_1 at M: $-x + y = 2 \equiv T$

Intersection of T with $C_2 \Rightarrow (x - 3)^2 + x^2 = 5$

$\Rightarrow$ $x = 1, 2$

$A(1, 3)$ and $B(2, 4)$

Let $N \equiv (\alpha, \beta)$

Then $-x + y = 2$ shall be chord of contact for $x^2 + y^2 - 6x - 4y + 8 = 0$

$\therefore$ $\alpha x + \beta y - 3x - 3\alpha - 2y - 2\beta + 8 = 0$ is same as $-x + y = 2$

$$\frac{\alpha - 3}{-1} = \frac{\beta - 2}{1} = \frac{3\alpha - 8 + 2\beta}{2}$$

$\Rightarrow$ $(\alpha, \beta) \equiv \left(\frac{4}{3}, \frac{11}{3}\right)$

$$\Delta = \frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 2 & 4 & 1 \\ 4/3 & 11/3 & 1 \end{vmatrix} = \frac{1}{6} \text{ units}$$

20. Let the mean and the variance of 5 observations x_1, x_2, x_3, x_4, x_5 be $\frac{24}{5}$ and $\frac{194}{25}$ respectively. If the mean and variance of the first 4 observation are $\frac{7}{2}$ and a respectively, then $(4a + x_5)$ is equal to

(A) 13

(B) 15

(C) 17

(D) 18

Answer (B)

Sol. $\sum_{i=1}^{5} x_i = 24$ ...(i)

$$\frac{\sum_{i=1}^{5} x_i^2}{5} - \left(\frac{24}{5}\right)^2 = \frac{194}{25}$$

$\Rightarrow$ $\sum x_i^2 = 1154$ ...(ii)

$\sum_{i=1}^{4} x_i = 14$

$\Rightarrow$ $x_5 = 10$

$$a = \frac{\sum_{i=1}^{4} x_i^2}{4} - \frac{49}{4} = \frac{54-49}{4} = \frac{5}{4}$$

$\Rightarrow$ $x_5 + 4a = 10 + 5 = 15$

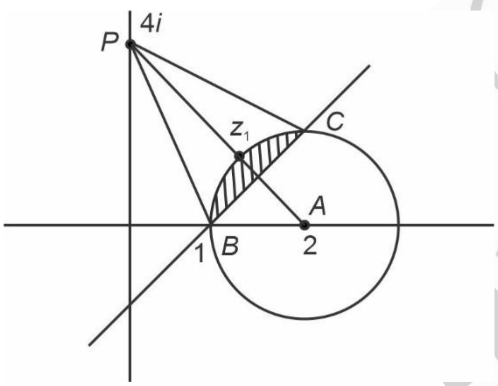
SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Let S = $\{z \in \mathbf{C} : |z-2| \le 1, z(1+i) + \bar{z}(1-i) \le 2\}$. Let $|z - 4i|$ attains minimum and maximum values, respectively, at $z_1 \in$ S and $z_2 \in$ S. If $5\left(|z_1|^2 + |z_2|^2\right) = \alpha + \beta\sqrt{5}$, where α and β are integers, then the value of $\alpha + \beta$ is equal to ______.

Answer (26)

Sol.



S represents the shaded region shown in the diagram.

Clearly z_1 will be the point of intersection of *PA* and given circle.

PA : $2x + y = 4$ and given circle has equation $(x-2)^2 + y^2 = 1$.

On solving we get

$$z_1 = \left(2 - \frac{1}{\sqrt{5}}\right) + \frac{2}{\sqrt{5}}i \Rightarrow |z_1|^2 = 5 - \frac{4}{\sqrt{5}}$$

z_2 will be either *B* or *C*.

$\because \quad PB = \sqrt{17}$ and $PC = \sqrt{13}$ hence $z_2 = 1$

So $5\left(|z_1|^2 + |z_2|^2\right) = 30 - 4\sqrt{5}$

Clearly $\alpha = 30$ and $\beta = -4 \Rightarrow \alpha + \beta = 26$

2. Let $y = y(x)$ be the solution of the differential equation $\dfrac{dy}{dx} + \dfrac{\sqrt{2}y}{2\cos^4 x - \cos 2x} = xe^{\tan^{-1}\left(\sqrt{2}\cot 2x\right)}$, $0 < x < \dfrac{\pi}{2}$ with $y\left(\dfrac{\pi}{4}\right) = \dfrac{\pi^2}{32}$. If $y\left(\dfrac{\pi}{3}\right) = \dfrac{\pi^2}{18}e^{-\tan^{-1}(\alpha)}$, then the value of $3\alpha^2$ is equal to ______.

Answer (2)

Sol. $\dfrac{dy}{dx} + \dfrac{2\sqrt{2}y}{1+\cos^2 2x} = xe^{\tan^{-1}\left(\sqrt{2}\cot 2x\right)}$

$$\text{I.F.} = e^{\int \frac{2\sqrt{2}dx}{1+\cos^2 2x}} = e^{\sqrt{2}\int \frac{2\sec^2 2x}{2+\tan^2 2x}dx}$$

$$= e^{\tan^{-1}\left(\frac{\tan 2x}{\sqrt{2}}\right)}$$

$$\Rightarrow \quad y \cdot e^{\tan^{-1}\left(\frac{\tan 2x}{\sqrt{2}}\right)} = \int xe^{\tan^{-1}\left(\sqrt{2}\cot 2x\right)} \cdot e^{\tan^{-1}\left(\frac{\tan 2x}{\sqrt{2}}\right)} dx + c$$

$$\Rightarrow \quad y \cdot e^{\tan^{-1}\left(\frac{\tan 2x}{\sqrt{2}}\right)} = e^{\frac{\pi}{2}} \cdot \frac{x^2}{2} + c$$

When $x = \dfrac{\pi}{4}, y = \dfrac{\pi^2}{32}$ gives $c = 0$

When $x = \dfrac{\pi}{3}, y = \dfrac{\pi^2}{18}e^{-\tan^{-1}\alpha}$

$$\text{So } \frac{\pi^2}{18}e^{-\tan^{-1}\alpha} \cdot e^{-\tan^{-1}\left(-\sqrt{\frac{3}{2}}\right)} = e^{\pi/2}\frac{\pi^2}{18}$$

$$\Rightarrow \quad -\tan^{-1}\alpha + \tan^{-1}\left(\sqrt{\frac{3}{2}}\right) = \frac{\pi}{2}$$

$$\Rightarrow \quad \tan^{-1}(-\alpha) = \tan^{-1}\left(\sqrt{\frac{2}{3}}\right)$$

$$\Rightarrow \quad \alpha = -\sqrt{\frac{2}{3}} \Rightarrow 3\alpha^2 = 2$$

3. Let *d* be the distance between the foot of perpendiculars of the point *P*(1, 2, –1) and *Q*(2, –1, 3) on the plane $-x + y + z = 1$. Then d^2 is equal to ____.

Answer (26)

Sol. Foot of perpendicular from P

$$\frac{x-1}{-1}=\frac{y-2}{1}=\frac{z+1}{1}=\frac{-(-1+2-1-1)}{3}$$

$$\Rightarrow \quad p' \equiv \left(\frac{2}{3},\frac{7}{3},\frac{-2}{3}\right)$$

and foot of perpendicular from Q

$$\frac{x-2}{-1}=\frac{y+1}{1}=\frac{z-3}{1}=\frac{-(-2-1+3-1)}{3}$$

$$\Rightarrow \quad Q' \equiv \left(\frac{5}{3},\frac{-2}{3},\frac{10}{3}\right)$$

$$P'Q'=\sqrt{(1)^2+(3)^2+(4)^2}=d=\sqrt{26}$$

$\Rightarrow \quad d^2 = 26$

4. The number of elements in the set

$S=\left\{\theta\in[-4\pi,\ 4\pi]:3\cos^2 2\theta+6\cos 2\theta-10\cos^2\theta+5=0\right\}$

is __________.

Answer (32)

Sol. $3\cos^2 2\theta+6\cos 2\theta-\dfrac{10(1+\cos 2\theta)}{2}+5=0$

$\Rightarrow \quad 3\cos^2 2\theta + \cos 2\theta = 0$

$\Rightarrow \quad \cos 2\theta = 0$ or $\cos 2\theta = \dfrac{-1}{3}$

As $\theta \in [0, \pi]$, $\cos 2\theta = \dfrac{-1}{3} \Rightarrow$ 2 times

$\Rightarrow \quad \theta \in [-4\pi, 4\pi]$, $\cos 2\theta = \dfrac{-1}{3} \Rightarrow$ 16 times

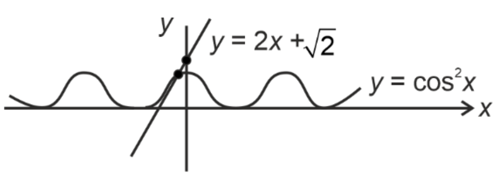
Similarly, $\cos 2\theta = 0 \Rightarrow$ 16 times

$\therefore$ Total 32 solutions

5. The number of solutions of the equation $2\theta-\cos^2\theta+\sqrt{2}=0$ in **R** is equal to __________.

Answer (1)

Sol. $\cos^2\theta = 2\theta+\sqrt{2}$



1 point of intersection = 1 solution.

6. $50\tan\left(3\tan^{-1}\left(\frac{1}{2}\right)+2\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)+4\sqrt{2}\tan\left(\frac{1}{2}\tan^{-1}\left(2\sqrt{2}\right)\right)$ is equal to __________.

Answer (29)

Sol. $50\tan\left(\tan^{-1}\frac{1}{2}+2\tan^{-1}\left(\frac{1}{2}\right)+2\tan^{-1}(2)\right)$

$$+4\sqrt{2}\tan\left(\frac{\tan^{-1}}{2}\left(2\sqrt{2}\right)\right)$$

$$\Rightarrow \quad 50\tan\left(\pi+\tan^{-1}\left(\frac{1}{2}\right)\right)+4\sqrt{2}\tan\left(\frac{1}{2}\tan^{-1}2\sqrt{2}\right)$$

$$\Rightarrow \quad 50\left(\frac{1}{2}\right)+4\sqrt{2}\tan\alpha$$

Where $2\alpha=\tan^{-1}2\sqrt{2}$

$$\Rightarrow \quad \frac{2\tan\alpha}{1-\tan^2\alpha}=2\sqrt{2} \qquad \ldots \text{(i)}$$

$$\Rightarrow \quad 2\sqrt{2}\tan^2\alpha+2\tan\alpha-2\sqrt{2}=0$$

$$\Rightarrow \quad 2\sqrt{2}\tan^2\alpha+4\tan\alpha-2\tan\alpha-2\sqrt{2}=0$$

$$\Rightarrow \quad \left(2\sqrt{2}\tan\alpha-2\right)\left(\tan\alpha-\sqrt{2}\right)=0$$

$$\Rightarrow \quad \tan\alpha=\sqrt{2} \text{ or } \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \tan\alpha=\frac{1}{\sqrt{2}}$$

$\left(\tan\alpha=\sqrt{2}\text{ doesn't satisfy (i)}\right)$

$$\Rightarrow \quad 25+4\sqrt{2}\frac{1}{\sqrt{2}}=29$$

7. Let $c, k \in \mathbf{R}$. If $f(x) = (c + 1)x^2 + (1 - c^2)x + 2k$ and $f(x + y) = f(x) + f(y) - xy$, for all $x, y \in \mathbf{R}$, then the value of $\left|2\left(f(1)+f(2)+f(3)+.....+f(20)\right)\right|$ is equal to __________.

Answer (3395)

Sol. $f(x)$ is polynomial

Put $y = 1/x$ in given functional equation we get

$$f\left(x+\frac{1}{x}\right)=f(x)+f\left(\frac{1}{x}\right)-1$$

$$\Rightarrow \quad (c+1)\left(x+\frac{1}{x}\right)^2+\left(1-c^2\right)\left(x+\frac{1}{x}\right)+2K$$

$$=(c+1)x^2+\left(1-c^2\right)x+2K$$

$$+(c+1)\frac{1}{x^2}+\left(1-c^2\right)\frac{1}{x}+2K-1$$

$\Rightarrow$ 2(c + 1) = 2K – 1 …(1)

and put $x = y = 0$ we get

$f(0) = 2 + f(0) - 0 \Rightarrow f(0) = 0 \Rightarrow k = 0$

$\therefore$ $k = 0$ and $2c = -3 \Rightarrow c = -3/2$

$f(x) = -\frac{x^2}{2} - \frac{5x}{4} = \frac{1}{4}(5x + 2x^2)$

$\left|2\sum_{i=1}^{20} f(i)\right| = \left|\frac{-2}{4}\left(\frac{5.20.21}{2} + \frac{2.20.21.41}{6}\right)\right|$

$= \left|\frac{-1}{2}(2730 + 5740)\right|$

$= \left|-\frac{6790}{2}\right| = 3395.$

8. Let H: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, a > 0, b > 0,$ be a hyperbola such that the sum of lengths of the transverse and the conjugate axes is $4(2\sqrt{2} + \sqrt{14})$. If the eccentricity H is $\frac{\sqrt{11}}{2}$, then the value of $a^2 + b^2$ is equal to ________.

Answer (88)

Sol. $2a + 2b = 4(2\sqrt{2} + \sqrt{14})$ …(1)

$1 + \frac{b^2}{a^2} = \frac{11}{4}$ ....(2)

$\Rightarrow \frac{b^2}{a^2} = \frac{7}{4}$ ...(3)

and $a + b = 4\sqrt{2} + 2\sqrt{14}$ …(4)

By (3) and (4)

$\Rightarrow a + \frac{\sqrt{7}}{2}a = 4\sqrt{2} + 2\sqrt{14}$

$\Rightarrow \frac{a(2+\sqrt{7})}{2} = 2\sqrt{2}(2+\sqrt{7})$

$\Rightarrow a = 4\sqrt{2} \Rightarrow a^2 = 32$ and $b^2 = 56$

$\Rightarrow a^2 + b^2 = 32 + 56 = 88$

9. Let $P_1 : \vec{r}\cdot(2\hat{i} + \hat{j} - 3\hat{k}) = 4$ be a plane. Let P_2 be another plane which passes through the points (2, –3, 2), (2, –2, –3) and (1, –4, 2). If the direction ratios of the line of intersection of P_1 and P_2 be 16, α, β, then the value of $\alpha + \beta$ is equal to ________.

Answer (28)

Sol. Direction ratio of normal to $P_1 \equiv$ < 2, 1, – 3 >

and that of $P_2 \equiv \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & -5 \\ -1 & -2 & 5 \end{vmatrix} = -5\hat{i} - \hat{j}(-5) + \hat{k}(1)$

i.e. < –5, 5, 1 >

d.r's of line of intersection are along vector

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ -5 & 5 & 1 \end{vmatrix} = \hat{i}(16) - \hat{j}(-13) + \hat{k}(15)$

i.e. < 16, 13, 15 >

$\therefore$ $\alpha + \beta = 13 + 15 = 28$

10. Let $b_1b_2b_3b_4$ be a 4-element permutation with $b_i \in$ {1, 2, 3,…..,100} for $1 \le i \le 4$ and $b_i \ne b_j$ for $i \ne j$, such that either b_1, b_2, b_3 are consecutive integers or b_2, b_3, b_4 are consecutive integers. Then the number of such permutations $b_1b_2b_3b_4$ is equal to ________.

Answer (18915)

Sol. There are 98 sets of three consecutive integer and 97 sets of four consecutive integers.

Using principle of inclusion and exclusion,

Number of permutations of $b_1b_2b_3b_4$ = Number of permutations when $b_1b_2b_3$ are consecutive + Number of permutations when $b_2b_3b_4$ are consecutive – Number of permutations when b_1b_2 b_3b_4 are consecutive

= 97 × 98 + 97 × 98 – 97 = 97 × 195 = 18915.

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