25/07/2022
Evening

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Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2022 (Online) Phase-2

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. In AM modulation, a signal is modulated on a carrier wave such that maximum and minimum amplitudes are found to be 6 V and 2 V respectively. The modulation index is

(A) 100% (B) 80%

(C) 60% (D) 50%

Answer (D)

Sol. $A_{max} = 6$ V

$A_{min} = 2$ V

$$\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} = \frac{6-2}{6+2} = 0.5$$

$\mu = 50$ %

2. The electric current in a circular coil of 2 turns produces a magnetic induction B_1 at its centre. The coil is unwound and is rewound into a circular coil of 5 turns and the same current produces a magnetic induction B_2 at its centre. The ratio of $\frac{B_2}{B_1}$ is

(A) $\frac{5}{2}$ (B) $\frac{25}{4}$

(C) $\frac{5}{4}$ (D) $\frac{25}{2}$

Answer (B)

Sol. $B = \frac{n\mu_0 I}{2R}$

$$B_1 = \frac{2\mu_0 I}{2R_1}$$

$$B_2 = \frac{5\mu_0 I}{2R_2}$$

$$R_2 = \frac{2R_1}{5}$$

$$\Rightarrow \frac{B_2}{B_1} = \frac{5}{2} \times \frac{R_1}{R_2} = \frac{25}{4}$$

3. A drop of liquid of density ρ is floating half immersed in a liquid of density σ and surface tension 7.5×10^{-4} N cm^{-1}. The radius of drop in cm will be ($g = 10$ ms^{-2})

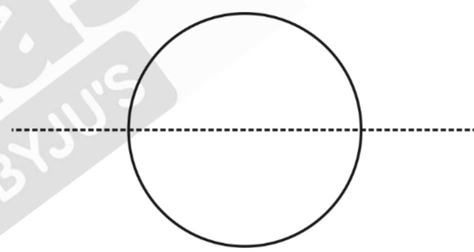
(A) $\frac{15}{\sqrt{(2\rho-\sigma)}}$ (B) $\frac{15}{\sqrt{(\rho-\sigma)}}$

(C) $\frac{3}{2\sqrt{(\rho-\sigma)}}$ (D) $\frac{3}{20\sqrt{(2\rho-\sigma)}}$

Answer (A)

Sol. Balancing the forces on drop

$$2\pi RT + \frac{4}{3}\pi R^3 \rho g = \frac{2}{3}\pi R^3 \sigma g$$



$$\Rightarrow 2T = \frac{2R^2}{3}(\sigma - 2\rho) \times 10$$

$$\Rightarrow \frac{15 \times 10^{-2} \times 3}{10(\sigma - 2\rho)2} = R^2$$

$$R = \frac{3}{2 \times 10}\sqrt{\frac{1}{(\sigma - 2\rho)}}$$

$$= \frac{3}{20}\sqrt{\frac{1}{\sigma - 2\rho}} \text{ (in m)}$$

$$(R) \text{ in cm} = \frac{3 \times 100}{20}\sqrt{\frac{1}{\sigma - 2\rho}} = 15 \times \frac{1}{\sqrt{\sigma - 2\rho}}$$

Now if $2\rho > \sigma (R_{\text{in cm}}) = \frac{15}{\sqrt{2\rho - \sigma}}$

4. Two billiard balls of mass 0.05 kg each moving in opposite directions with 10 ms^{-1} collide and rebound with the same speed. If the time duration of contact is t = 0.005 s, then what is the force exerted on the ball due to each other?

(A) 100 N (B) 200 N

(C) 300 N (D) 400 N

Answer (B)

Sol. Change in momentum of one ball

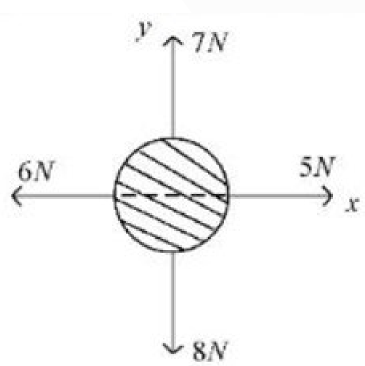
= 2 × (0.05)(10) kg m/s

= 1 kg m/s

$\Rightarrow \quad F_{avg} = \frac{1}{\Delta t} = \frac{1}{0.005}$ N

= 200 N

5. For a free body diagram shown in the figure, the four forces are applied in the 'x' and 'y' directions. What additional force must be applied and at what angle with positive x-axis so that net acceleration of body is zero?



(A) $\sqrt{2}$ N, 45° (B) $\sqrt{2}$ N, 135°

(C) $\frac{2}{\sqrt{3}}$ N, 30° (D) 2 N, 45°

Answer (A)

Sol. Resultant of already applied forces $= -\hat{i} - \hat{j}$

$\Rightarrow$ Force required to balance $= \hat{i} + \hat{j}$

$\Rightarrow$ Force required $= \sqrt{2}$ N in magnitude at angle 45° with +ve x-axis

6. Capacitance of an isolated conducting sphere of radius R_1 becomes n times when it is enclosed by a concentric conducting sphere of radius R_2 connected to earth.

The ratio of their radii $\left(\frac{R_2}{R_1}\right)$ is:

(A) $\frac{n}{n-1}$ (B) $\frac{2n}{2n+1}$

(C) $\frac{n+1}{n}$ (D) $\frac{2n+1}{n}$

Answer (A)

Sol. Initially = $C_0 = 4\pi\varepsilon_0 R_1$

finally $\frac{4\pi\varepsilon_0 R_1 R_2}{R_2 - R_1} = nC_0 = 4\pi\varepsilon_0 n R_1$

$$\frac{R_2}{R_2 - R_1} = n$$

$$1 - \frac{R_1}{R_2} = \frac{1}{n}$$

$$\frac{R_1}{R_2} = \frac{n-1}{n}$$

$$\frac{R_2}{R_1} = \frac{n}{n-1}$$

7. The ratio of wavelengths of proton and deuteron accelerated by potential V_p and V_d is $1 : \sqrt{2}$. Then, the ratio of V_p to V_d will be:

(A) 1 : 1 (B) $\sqrt{2} : 1$

(C) 2 : 1 (D) 4 : 1

Answer (D)

Sol. $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2meV}}$

so $\frac{\lambda_p}{\lambda_d} = \frac{\sqrt{m_d V_d}}{\sqrt{m_p V_p}} = \frac{1}{\sqrt{2}}$

$$\frac{2V_d}{V_p} = \frac{1}{2}$$

$$\frac{V_p}{V_d} = \frac{4}{1}$$

8. For an object placed at a distance 2.4 m from a lens, a sharp focused image is observed on a screen placed at a distance 12 cm form the lens. A glass plate of refractive index 1.5 and thickness 1 cm is introduced between lens and screen such that the glass plate plane faces parallel to the screen. By what distance should the object be shifted so that a sharp focused image is observed again on the screen?

(A) 0.8 m (B) 3.2 m

(C) 1.2 m (D) 5.6 m

Answer (B)

Sol. The shift produced by the glass plate is

$$d = t\left(1-\frac{1}{\mu}\right) = 1\times\left(1-\frac{1}{1.5}\right) = \frac{1}{3}\text{ cm}$$

So final image must be produced at $\left(12-\frac{1}{3}\right)\text{cm} = \frac{35}{3}$ cm from lens so that glass plate must shift it to produce image at screen. So

$$\frac{1}{12}-\frac{1}{-240} = \frac{1}{f} = \frac{1}{35/3}-\frac{1}{u}$$

$$\frac{1}{u} = \frac{3}{35}-\frac{1}{12}-\frac{1}{240}$$

or u = – 560 cm

so shift = 5.6 – 2.4 = 3.2 m

9. Light wave traveling in air along x-direction is given by $E_y = 540 \sin \pi \times 10^4 (x - ct)\text{Vm}^{-1}$. Then, the peak value of magnetic field of wave will be (Given $c = 3 \times 10^8\ \text{ms}^{-1}$)

(A) 18×10^{-7} T (B) 54×10^{-7} T

(C) 54×10^{-8} T (D) 18×10^{-8} T

Answer (A)

Sol. $c = 3 \times 10^8$ m/sec

$$B = \frac{E}{c} = \frac{540}{3\times10^8} = 18\times10^{-7}\text{ T}$$

10. When you walk through a metal detector carrying a metal object in your pocket, it raises an alarm. This phenomenon works on:

(A) Electromagnetic induction

(B) Resonance in ac circuits

(C) Mutual induction in ac circuits

(D) Interference of electromagnetic waves

Answer (B)

Sol. Metal detector works on the principle of resonance in ac circuits.

11. An electron with energy 0.1 keV moves at right angle to the earth's magnetic field of 1×10^{-4} Wbm^{-2}. The frequency of revolution of the electron will be

(Take mass of electron = 9.0×10^{-31} kg)

(A) 1.6×10^5 Hz (B) 5.6×10^5 Hz

(C) 2.8×10^6 Hz (D) 1.8×10^6 Hz

Answer (C)

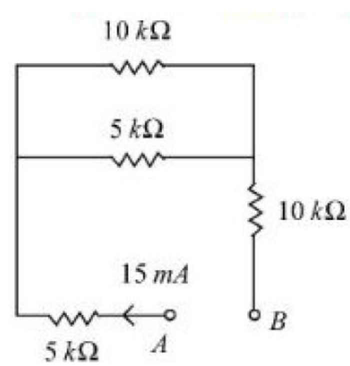
Sol. $T = \frac{2\pi m}{Bq}$

$\Rightarrow$ Frequency $f = \frac{Bq}{2\pi m}$

$$= \frac{10^{-4}\times1.6\times10^{-19}}{2\pi\times9\times10^{-31}}$$

$$\simeq 2.8\times10^6\text{ Hz}$$

12. A current of 15 mA flows in the circuit as shown in figure. The value of potential difference between the points A and B will be



(A) 50 V

(B) 75 V

(C) 150 V

(D) 275 V

Answer (D)

Sol. Effective $R = \left[5 + \frac{5 \times 10}{5 + 10} + 10\right]$ kΩ

$$= \frac{275}{15} \text{ k}\Omega$$

$$\Rightarrow \quad \Delta V_{AB} = 15 \text{ mA} \times \frac{275}{15} \text{ k}\Omega$$

$$= 275 \text{ V}$$

13. The length of a seconds pendulum at a height $h = 2R$ from earth surface will be

(Given R = Radius of earth and acceleration due to gravity at the surface of earth, $g = \pi^2$ ms^{-2})

(A) $\frac{2}{9}$ m (B) $\frac{4}{9}$ m

(C) $\frac{8}{9}$ m (D) $\frac{1}{9}$ m

Answer (D)

Sol. $g = \frac{GM}{(R+h)^2} = \frac{GM}{9R^2} = \frac{g_0}{9}$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{\ell}{\frac{g_0}{9}}}$$

$$\Rightarrow \quad 2 = 2\pi\sqrt{\frac{9\ell}{g_0}}$$

$$\Rightarrow \quad \ell = \frac{g_0}{9\pi^2} = \frac{1}{9} \text{ m}$$

14. Sound travels in a mixture of two moles of helium and n moles of hydrogen. If rms speed of gas molecules in the mixture is $\sqrt{2}$ times the speed of sound, then the value of n will be

(A) 1 (B) 2

(C) 3 (D) 4

Answer (B)

Sol. Molar mass $M = \frac{2 \times 4 + n \times 1}{2 + n}$ ...(i)

Also, $\gamma = \frac{n_1 C_{P_1} + n_2 C_{P_2}}{n_1 C_{V_1} + n_2 C_{V_2}} = \frac{2 \times 5R + n \times 7R}{2 \times 3R + n \times 5R}$

$$\Rightarrow \quad \gamma = \frac{10 + 7n}{6 + 5n} \qquad \text{...(ii)}$$

Given that $V_{rms} = \sqrt{2}\; V_{sound}$

$$\Rightarrow \quad \sqrt{\frac{3RT}{M}} = \sqrt{2}\sqrt{\frac{\gamma RT}{M}}$$

$$\Rightarrow \quad \gamma = \frac{3}{2}$$

$$\Rightarrow \quad n = 2$$

15. Let η_1 is the efficiency of an engine at T_1 = 447°C and T_2 = 147°C while η_2 is the efficiency at T_1 = 947°C and T_2 = 47°C. The ratio $\frac{\eta_1}{\eta_2}$ will be

(A) 0.41 (B) 0.56

(C) 0.73 (D) 0.70

Answer (B)

Sol. $\eta_1 = 1 - \frac{420}{720} = \frac{300}{720}$

And $\eta_2 = 1 - \frac{320}{1220} = \frac{900}{1220}$

$$\Rightarrow \quad \frac{\eta_1}{\eta_2} = \frac{300}{720} \times \frac{1220}{900}$$

$$\simeq 0.56$$

16. An object is taken to a height above the surface of earth at a distance $\frac{5}{4}R$ from the centre of the earth. Where radius of earth, R = 6400 km. The percentage decrease in the weight of the object will be

(A) 36% (B) 50%

(C) 64% (D) 25%

Answer (A)

Sol. $w = mg$

$$w' = \frac{mg}{\left(1 + \frac{h}{R}\right)^2} = \frac{mg}{\left(\frac{5}{4}\right)^2} = \frac{16}{25} mg$$

$\therefore$ % decrease in weight

$$= \left(1 - \frac{16}{25}\right) \times 100\%$$

$$= 36\%$$

17. A bag of sand of mass 9.8 kg is suspended by a rope. A bullet of 200 g travelling with speed 10 ms^{-1} gets embedded in it, then loss of kinetic energy will be

(A) 4.9 J (B) 9.8 J

(C) 14.7 J (D) 19.6 J

Answer (B)

Sol. Loss in $KE = \frac{1}{2}\times\frac{m_1m_2}{m_1+m_2}\times v^2$

$$= \frac{1}{2}\times\frac{9.8\times 0.2}{10}\times(10)^2$$

= 9.8 J

18. A ball is projected from the ground with a speed 15 ms^{-1} at an angle θ with horizontal so that its range and maximum height are equal, then 'tan θ' will be equal to

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$

(C) 2 (D) 4

Answer (D)

Sol. $\because\ \tan\theta = \frac{4H}{R}$

$\Rightarrow \tan\theta = 4 \times 1$

$\Rightarrow \tan\theta = 4$

19. The maximum error in the measurement of resistance, current and time for which current flows in an electrical circuit are 1%, 2% and 3% respectively. The maximum percentage error in the detection of the dissipated heat will be

(A) 2 (B) 4

(C) 6 (D) 8

Answer (D)

Sol. $\because\ H = i^2Rt$

$\because$ % error in H = 2 × 2% + 1% + 3%

= 8%

20. Hydrogen atom from excited state comes to the ground state by emitting a photon of wavelength λ. The value of principal quantum number 'n' of the excited state will be, (R: Rydberg constant)

(A) $\sqrt{\frac{\lambda R}{\lambda - 1}}$ (B) $\sqrt{\frac{\lambda R}{\lambda R - 1}}$

(C) $\sqrt{\frac{\lambda}{\lambda R - 1}}$ (D) $\sqrt{\frac{\lambda R^2}{\lambda R - 1}}$

Answer (B)

Sol. $\because\ \frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$

$$\Rightarrow \frac{1}{\lambda R} = 1 - \frac{1}{n^2}$$

$$\Rightarrow \frac{1}{n^2} = 1 - \frac{1}{\lambda R} = \frac{\lambda R - 1}{\lambda R}$$

$$\Rightarrow n = \sqrt{\frac{\lambda R}{\lambda R - 1}}$$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. A particle is moving in a straight line such that its velocity is increasing at 5 ms^{-1} per meter. The acceleration of the particle is ________ms^{-2} at a point where its velocity is 20 ms^{-1}.

Answer (100)

Sol. $\frac{dv}{dx} = 5\ \text{ms}^{-1}/\text{m}$

Acceleration of particle

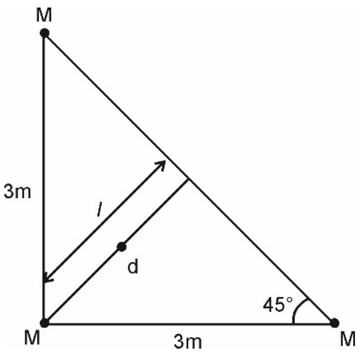
when v = 20 m/s

$a = v\frac{dv}{dx}$ = 20 (5) m/s^2 = 100 m/s^2

2. Three identical spheres each of mass M are placed at the corners of a right angled triangle with mutually perpendicular sides equal to 3 m each. Taking point of intersection of mutually perpendicular sides as origin, the magnitude of position vector of centre of mass of the system will be $\sqrt{x}$ m. The value of x is ________

Answer (2)

Sol.



$$d_{cm} = 3\sin 45° = \frac{3}{\sqrt{2}}$$

$$d_{cm} = \frac{2}{3} \times \frac{3}{\sqrt{2}} = \sqrt{2} = \sqrt{x}$$

$x = 2$

3. A block of ice of mass 120 g at temperature 0°C is put in 300 g of water at 25°C. The x g of ice melts as the temperature of the water reaches 0°C. The value of x is ________.

[Use specific heat capacity of water = 4200 $Jkg^{-1}K^{-1}$, Latent heat of ice = 3.5 ×10^5 Jkg^{-1}]

Answer (90)

Heat lost by water = Heat gained by ice

$0.3 \times 4200 \times 25 = x \times 3.5 \times 10^5$

$$x = \frac{0.3 \times 4200 \times 25}{3.5 \times 10^5}$$

= 90 × 100 × 10^5 × 10^3 gram = 90 gm

4. $\frac{x}{x+4}$ is the ratio of energies of photons produced due to transition of an electron of hydrogen atom from its

(i) Third permitted energy level to the second level and

(ii) The highest permitted energy level to the second permitted level.

The value of x will be _______.

Answer (5)

Sol. $E_n = -\frac{13.6}{n^2}$ eV

$$\frac{\frac{1}{2^2} - \frac{1}{3^2}}{\frac{1}{2^2}} = \frac{x}{x+4}$$

$$\Rightarrow \frac{9-4}{9 \times 4 \times \frac{1}{4}} = \frac{x}{x+4} = \frac{5}{9}$$

$x = 5$

5. In a potentiometer arrangement, a cell of emf 1.20 V gives a balance point at 36 cm length of wire. This cell is now replaced by another cell of emf 1.80 V. The difference in balancing length of potentiometer wire in above conditions will be ________ cm.

Answer (18)

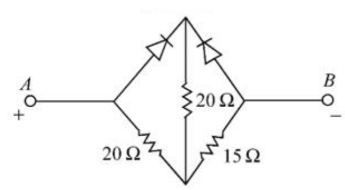
Sol. $E \propto l$

$$\frac{1.2}{1.8} = \frac{36}{l'}$$

$$l' = \frac{3}{2} \times 36 = 54 \text{ cm}$$

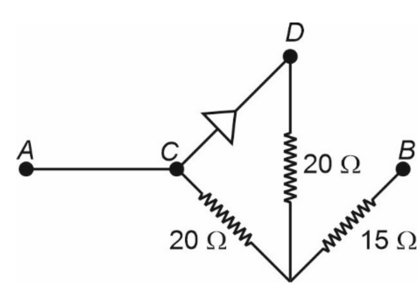
$\Delta l = l' - l = 54 - 36 = 18$ cm

6. Two ideal diodes are connected in the network as shown is figure. The equivalent resistance between A and B is ___ Ω.



Answer (25)

Sol.



$$R = \frac{20 \times 20}{40} + 15 = 25\ \Omega$$

7. Two waves executing simple harmonic motions travelling in the same direction with same amplitude and frequency are superimposed. The resultant amplitude is equal to the $\sqrt{3}$ times of amplitude of individual motions. The phase difference between the two motions is____ (degree).

Answer (60)

Sol. $A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$$\sqrt{3}A = \sqrt{A^2 + A^2 + 2A^2\cos\phi}$$

$$3A^2 = 2A^2 + 2A^2\cos\phi$$

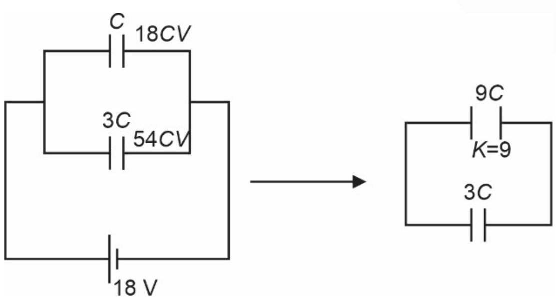
$$\cos\phi = \frac{1}{2}$$

$$\phi = 60^\circ$$

8. Two parallel plate capacitors of capacity C and $3C$ are connected in parallel combination and charged to a potential difference 18 V. The battery is then disconnected and the space between the plates of the capacitor of capacity C is completely filled with a material of dielectric constant 9. The final potential difference across the combination of capacitors will be ____ V.

Answer (6)

Sol.

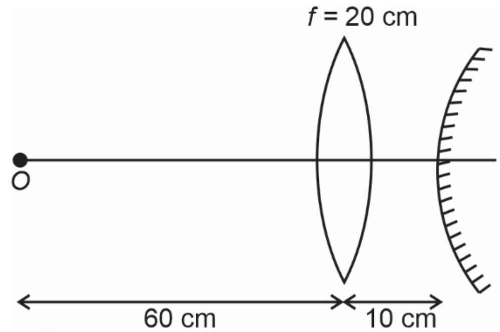


$$V_{common} = \frac{18CV + 54CV}{3C + 9C} = 6\text{ V}$$

9. A convex lens of focal length 20 cm is placed in front of a convex mirror with principal axis coinciding each other. The distance between the lens and mirror is 10 cm. A point object is placed on principal axis at a distance of 60 cm from the convex lens. The image formed by combination coincides the object itself. The focal length of the convex mirror is_______cm.

Answer (10)

Sol.



$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{-60} = \frac{1}{20}$$

$$\frac{1}{v} = -\frac{1}{60} + \frac{1}{20} = \frac{-1+3}{60} = \frac{2}{60}$$

$\Rightarrow v = +30$ cm

$\therefore$ Radius of curvature of mirror = 30 – 10 = 20 cm

$$\Rightarrow f_{mirror} = \frac{20}{2} = 10\text{ cm}$$

10. Magnetic flux (in weber) in a closed circuit of resistance 20 Ω varies with time t(s) as $\phi = 8t^2 - 9t + 5$. The magnitude of the induced current at $t = 0.25$ s will be _____ mA.

Answer (250)

Sol. $R = 20\ \Omega$

$$\phi = 8t^2 - 9t + 5$$

$$\varepsilon = \left|-\frac{d\phi}{dt}\right| = |16t - 9| = |16(0.25) - 9| = 5$$

$$i = \frac{\varepsilon}{R} = \frac{5}{20} = 0.25\text{ A} = \frac{0.25}{10^3}\times 10^3\text{ A} = 250\text{ mA}$$

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Match **List-I** with **List-II** :

List-I (Molecule)	**List-II (hybridization ; shape)**
A. XeO_3	I. sp^3d; linear
B. XeF_2	II. sp^3; pyramidal
C. $XeOF_4$	III. sp^3d^3; distorted octahedral
D. XeF_6	IV. sp^3d^2; square pyramidal

Choose the correct answer from the options given below:

(A) A-II, B-I, C-IV, D-III (B) A-II, B-IV, C-III, D-I

(C) A-IV, B-II, C-III, D-I (D) A-IV, B-II, C-I, D-III

Answer (A)

Sol. XeO_3 — sp^3, Pyramidal

XeF_2 — sp^3d, linear

$XeOF_4$ — sp^3d^2, Square Pyramidal

XeF_6 — sp^3d^3, distorted octahedral

2. Two solutions A and B are prepared by dissolving 1 g of non-volatile solutes X and Y, respectively in 1 kg of water. The ratio of depression in freezing points for A and B is found to be 1 : 4. The ratio of molar masses of X and Y is

(A) 1 : 4 (B) 1 : 0.25

(C) 1 : 0.20 (D) 1 : 5

Answer (B)

Sol. $\Delta T_f = i\,k_f \times m$

$$\frac{\Delta T_{f(A)}}{\Delta T_{f(B)}} = \frac{1}{4}$$

$$\frac{i \times K_f \times \frac{1}{M_A} \times 1}{i \times K_f \times \frac{1}{M_B} \times 1} = \frac{1}{4}$$

$$\frac{M_B}{M_A} = \frac{1}{4}$$

$M_A : M_B = 4 : 1$

3. K_{a_1}, K_{a_2} and K_{a_3} are the respective ionization constants for the following reactions (a), (b) and (c).

(a) $H_2C_2O_4 \rightleftharpoons H^+ + HC_2O_4^-$

(b) $HC_2O_4^- \rightleftharpoons H^+ + C_2O_4^{2-}$

(c) $H_2C_2O_4 \rightleftharpoons 2H^+ + C_2O_4^{2-}$

The relationship between K_{a_1}, K_{a_2} and K_{a_3} is given as

(A) $K_{a_3} = K_{a_1} + K_{a_2}$ (B) $K_{a_3} = K_{a_1} - K_{a_2}$

(C) $K_{a_3} = K_{a_1} / K_{a_2}$ (D) $K_{a_3} = K_{a_1} \times K_{a_2}$

Answer (D)

Sol. $H_2C_2O_4 \rightleftharpoons 2H^+ + C_2O_4^{2-}$ K_{a_3}

$H_2C_2O_4 \rightleftharpoons H^+ + HC_2O_4^-$ K_{a_1}

$HC_2O_4^- \rightleftharpoons H^+ + C_2O_4^{2-}$ K_{a_2}

$$K_{a_3} = \frac{[H^+]^2[C_2O_4^{2-}]}{[H_2C_2O_4]}$$

$$K_{a_1} = \frac{[H^+][HC_2O_4^-]}{[H_2C_2O_4]}, K_{a_2} = \frac{[H^+][C_2O_4^-]}{[HC_2O_4^-]}$$

$K_{a_3} = K_{a_1} \times K_{a_2}$

4. The molar conductivity of a conductivity cell filled with 10 moles of 20 mL NaCl solution is Λ_{m_1} and that of 20 moles another identical cell having 80 mL NaCl solution is Λ_{m_2}, The conductivities exhibited by these two cells are same. The relationship between Λ_{m_2} and Λ_{m_1} is

(A) $\Lambda_{m_2} = 2\Lambda_{m_1}$ (B) $\Lambda_{m_2} = \Lambda_{m_1}/2$

(C) $\Lambda_{m_2} = \Lambda_{m_1}$ (D) $\Lambda_{m_2} = 4\Lambda_{m_1}$

Answer (A)

Sol. $\Lambda_{m_1} = \frac{k_1 \times 1000}{M_1} = \frac{k \times 1000}{\frac{10}{0.02}}$

$$\Lambda_{m_2} = \frac{k_2 \times 1000}{\frac{20}{0.08}}$$

It is given that $k_1 = k_2$

$k_1 = \frac{\Lambda_{m_1}}{2}$ $k_2 = \frac{\Lambda_{m_2}}{4}$

Applying the given condition on conductivity.

$$\frac{\Lambda_{m_1}}{2} = \frac{\Lambda_{m_2}}{4}$$

$$\boxed{\Lambda_{m_2} = 2\Lambda_{m_1}}$$

5. For micelle formation, which of the following statements are correct?

A. Micelle formation is an exothermic process.

B. Micelle formation is an endothermic process.

C. The entropy change is positive

D. The entropy change is negative

(A) A and D only

(B) A and C only

(C) B and C only

(D) B and D only

Answer (C)

Sol. Micelle formation is an endothermic process with positive entropy change.

6. The first ionization enthalpies of Be, B, N and O follow the order

(A) O < N < B < Be

(B) Be < B < N < O

(C) B < Be < N < O

(D) B < Be < O < N

Answer (D)

Sol The first ionisation energy increase from left to right along 2nd period with the following exceptions

IE_1 : Be > B and N > O

This is due to stable configuration of Be in comparison to B and that of N in comparison to O.

Hence the correct order is N > O > Be > B

7. Given below are two statements.

Statement-I : Pig iron is obtained by heating cast iron with scrap iron.

Statement-II : Pig iron has a relatively lower carbon content than that of cast iron.

In the light of the above statements, choose the *correct* answer from the options given below.

(A) Both **Statement-I** and **Statement-II** are correct

(B) Both **Statement-I** and **Statement-II** are **not** correct.

(C) **Statement-I** is correct but **Statement-II** is **not** correct

(D) **Statement-I** is **not** correct but **Statement-II** is correct

Answer (B)

Sol Cast iron is made by melting pig iron with scrap iron and coke using hot air blast.

Hence Statement-I is incorrect

But Pig iron has relatively more carbon content

Hence statement-II is incorrect

8. High purity (>99.95%) dihydrogen is obtained by

(A) Reaction of zinc with aqueous alkali

(B) Electrolysis of acidified water using platinum electrodes

(C) Electrolysis of warm aqueous barium hydroxide solution between nickel electrodes

(D) Reaction of zinc with dilute acid

Answer (C)

Sol High purity (>99.95%) H_2 is obtained by electrolysing warm aqueous $Ba(OH)_2$ solution between nickel electrodes.

9. The correct order of density is

(A) Be > Mg > Ca > Sr (B) Sr > Ca > Mg > Be

(C) Sr > Be > Mg > Ca (D) Be > Sr > Mg > Ca

Answer (C)

Sol Density of Sr = 2.63 g/cm^3

Density of Be = 1.84 g/cm^3

Density of Mg = 1.74 g/cm^3

Density of Ca = 1.55 g/cm^3

10. The total number of acidic oxides from the following list is

NO, N_2O, B_2O_3, N_2O_5, CO, SO_3, P_4O_{10}

(A) 3 (B) 4

(C) 5 (D) 6

Answer (B)

Sol NO, N_2O, CO – neutral oxides

B_2O_3, N_2O_5, SO_3, P_4O_{10} – acidic oxides

11. The correct order of energy of absorption for the following metal complexes is

A : $[Ni(en)_3]^{2+}$, B : $[Ni(NH_3)_6]^{2+}$, C : $[Ni(H_2O)_6]^{2+}$

(A) C < B < A (B) B < C < A

(C) C < A < B (D) A < C < B

Answer (A)

Sol. Stronger is ligand attached to metal ion, greater will be the splitting between t_2g and eg (hence greater will be ΔU), $\therefore$ greater will be absorption of energy.

Hence correct order

$[Ni(en)_3]^{2+} > [Ni(NH_3)_6]^{2+} > [Ni(H_2O)_6]^{2+}$

12. Match **List I** with **List II**.

List I	List II
A. Sulphate	I. Pesticide
B. Fluoride	II. Bending of bones
C. Nicotine	III. Laxative effect
D. Sodium arsinite	IV. Herbicide

Choose the correct answer from the options given below:

(A) A-II, B-III, C-IV, D-I

(B) A-IV, B-III, C-II, D-I

(C) A-III, B-II, C-I, D-IV

(D) A-III, B-II, C-IV, D-I

Answer (C)

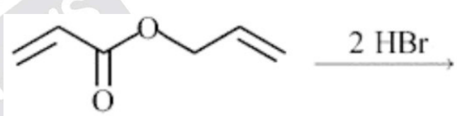
Sol. Sodium arsinite — Herbicide

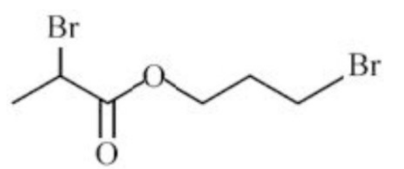
Nicotine — Pesticide

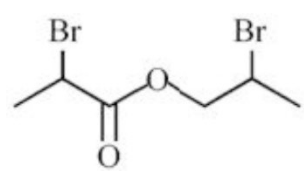
Sulphate — Laxative effect

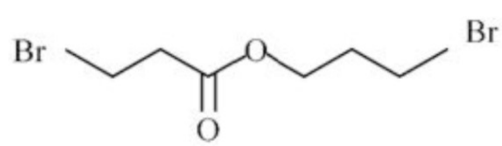
Fluoride — Bending of bones

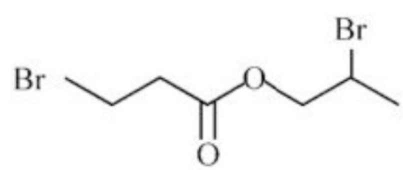
13. Major product of the following reaction is



(A) 

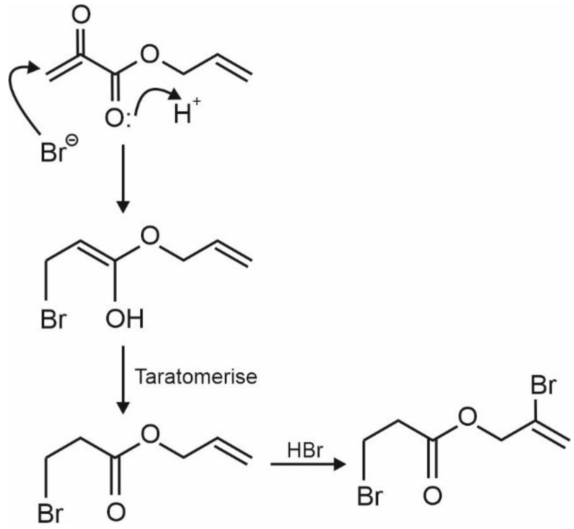
(B) 

(C) 

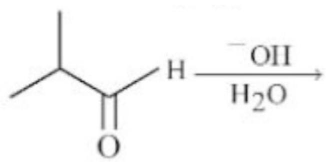
(D) 

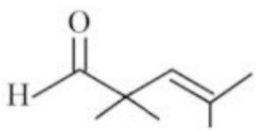
Answer (D)

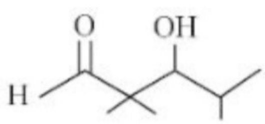
Sol.

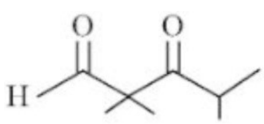


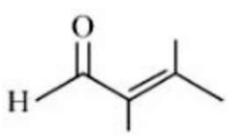
14. What is the major product of the following reaction?



(A) 

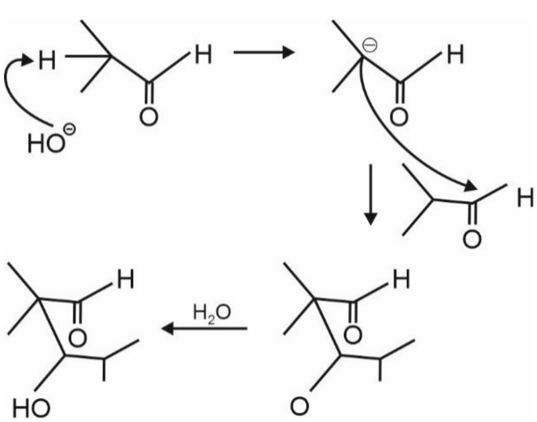
(B) 

(C) 

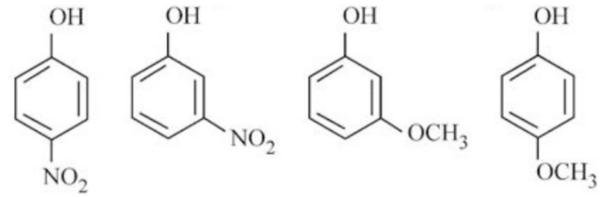
(D) 

Answer (B)

Sol.



15. Arrange the following in decreasing acidic strength



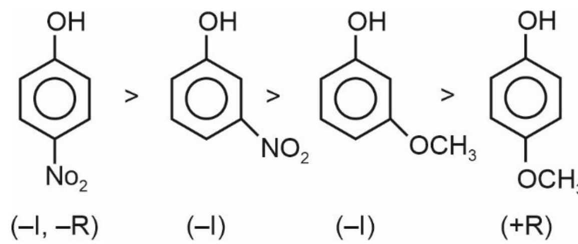
(A) (B) (C) (D)

(A) A > B > C > D (B) B > A > C > D

(C) D > C > A > B (D) D > C > B > A

Answer (A)

Sol.



16. $CH_3-CH_2-CN \xrightarrow[\text{Ether}]{CH_3MgBr} A$

$\xrightarrow{H_3O^+} B \xrightarrow[HCl]{Zn-Hg} C$

The correct structure of C is

(A) $CH_3-CH_2-CH_2-CH_3$

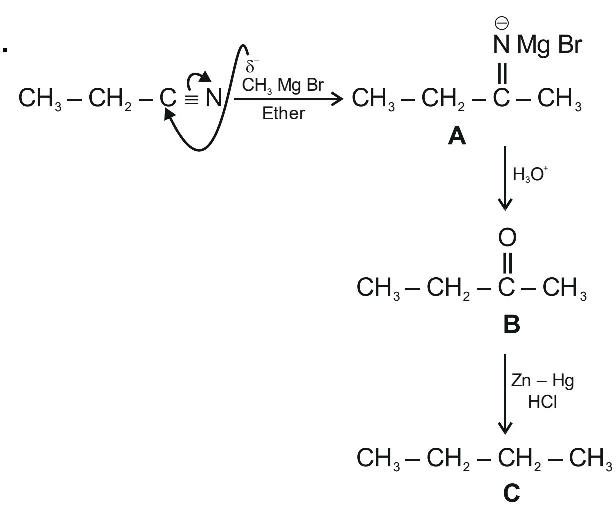
(B) $CH_3-CH_2-\overset{O}{\overset{||}{C}}-CH_3$

(C) $CH_3-CH_2-\overset{OH}{\overset{|}{CH}}-CH_3$

(D) $CH_3-CH_2-CH=CH_2$

Answer (A)

Sol.



17. Match List I with List II:

	List I Polymer		List II Used for items
A.	Nylon 6, 6	I	Buckets
B.	Low density polythene	II.	Non-stick utensils
C.	High density polythene	III.	Bristles of brushes
D.	Teflon	IV.	Toys

Choose the correct answer from the options given below:

(A) A-III, B-I, C-IV, D-II (B) A-III, B-IV, C-I, D-II

(C) A-II, B-I, C-IV, D-III (D) A-II, B-IV, C-I, D-III

Answer (B)

Sol. Nylon 6, 6 $\rightarrow$ used in making bristles of brushes

Low density polythene $\rightarrow$ used in making Toys

High density polythene $\rightarrow$ used in making Buckets

Teflon $\rightarrow$ used in making non-stick utensils

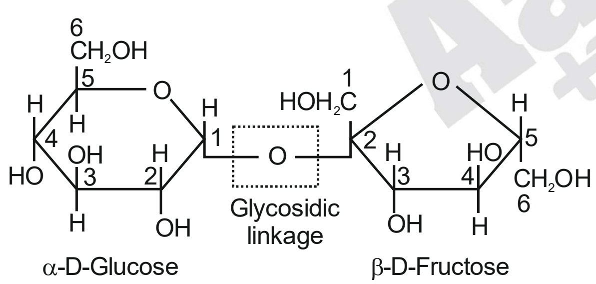
18. Glycosidic linkage between C1 of α-glucose and C2 of β-fructose is found in

(A) maltose (B) sucrose

(C) lactose (D) amylose

Answer (B)

Sol.



Sucrose

Hence in sucrose glycosidic linkage between C_1 of α-glucose and C_2 of β-D-fructose is found

Maltose $\Rightarrow$ Glycosidic linkage between C_1 and C_4

Lactose $\Rightarrow$ Glycosidic linkage between C_1 and C_4

Amylose $\Rightarrow$ Glycosidic linkage between C_1 and C_4

19. Some drugs bind to a site other than the active site of an enzyme. This site is known as

(A) non-active site (B) allosteric site

(C) competitive site (D) therapeutic site

Answer (B)

Sol. Some drugs do not bind to the enzyme's active site. These bind to a different site of enzyme which is called allosteric site.

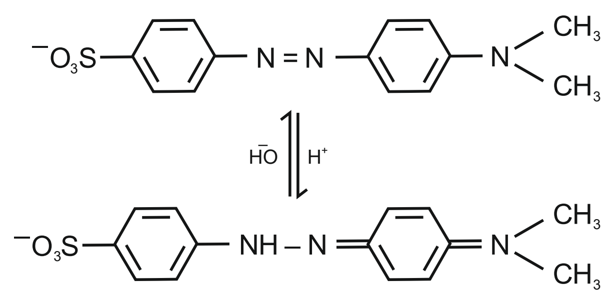
20. In base vs. acid titration, at the end point methyl orange is present as

(A) quinonoid form (B) heterocyclic form

(C) phenolic form (D) benzenoid form

Answer (A)

Sol.



(quinonoid form)

Hence at the end point methyl orange is present as quinonoid form.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. 56.0 L of nitrogen gas is mixed with excess of hydrogen gas and it is found that 20 L of ammonia gas is produced. The volume of unused nitrogen gas is found to be ____ L.

Answer (46)

Sol. $N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$

Since H_2 is in excess and 20 L of ammonia gas is produced.

Hence, 2 moles $NH_3 \equiv$ 1 mole N_2 ($v \propto n$)

20 L $NH_3 \equiv$ 10 L N_2

Volume of N_2 left = 56 – 10

= 46 L

2. A sealed flask with a capacity of 2 dm^3 contains 11 g of propane gas. The flask is so weak that it will burst if the pressure becomes 2 MPa. The minimum temperature at which the flask will burst is _______ °C. [Nearest integer]

(Given: R = 8.3 JK^{-1} mol^{-1}, Atomic masses of C and H are 12u and 1u, respectively.) (Assume that propane behaves as an ideal gas.)

Answer (1655)

Sol. From ideal gas equation,

$PV = nRT$

$P = 2 \times 10^6$ Pa

$V = 2\ dm^3 = 2 \times 10^{-3}\ m^3$

$R = 8.3\ JK^{-1}\ mol^{-1}$

$n = \frac{11}{44}$ mol

$2 \times 10^6 \times 2 \times 10^{-3} = \frac{11}{44} \times 8.3 \times T$

T = 1927.7 K

T (in °C) = 1927.7 – 273 ≃ 1655 °C

3. When the excited electron of a H atom from n = 5 drops to the ground state, the maximum number of emission lines observed are ____.

Answer (4)

Sol. Since there is a single hydrogen atom, so only $5 \to 4$, $4 \to 3$, $3 \to 2$, $2 \to 1$ lines are obtained.

4. While performing a thermodynamics experiment, a student made the following observations.

$HCl + NaOH \to NaCl + H_2O$; $\Delta H = -57.3$ kJ mol^{-1}

$CH_3COOH + NaOH \to CH_3COONa + H_2O$; $\Delta H = -55.3$ kJ mol^{-1}

The enthalpy of ionization of CH_3COOH as calculated by the student is _______ kJ mol^{-1}. [nearest integer]

Answer (2)

Sol. (I) $HCl + NaOH \to NaCl + H_2O$

$\Delta H_1 = -57.3$ KJ mol^{-1}

(II) $CH_3COOH + NaOH \to CH_3COONa + H_2O$

$\Delta H_2 = -55.3$ KJ mol^{-1}

Reaction (I) can be written as

(III) $NaCl + H_2O \to HCl + NaOH$

$\Delta H_3 = 57.3$ KJ mol^{-1}

By adding (II) and (III)

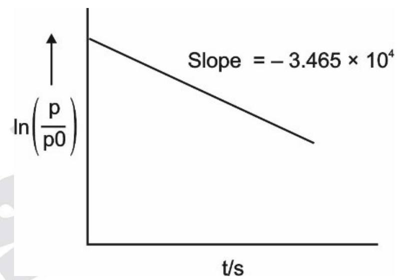
$CH_3COOH + NaCl \to CH_3COONa + HCl$ $\quad \Delta H_r$

$\Delta H_r = \Delta H_3 + \Delta H_2 = 57.3 - 55.3$

$= 2$ kJ mol^{-1}

5. For the decomposition of azomethane,

$CH_3N_2CH_3(g) \to CH_3CH_3(g) + N_2(g)$, a first order reaction, the variation in partial pressure with time at 600 K is given as



The half life of the reaction is _______ × 10^{-5}s. [Nearest integer]

Answer (2)

Sol. For first order reaction,

$\ln A = \ln A_0 - kt$

Hence Slope = –k

$-k = -3.465 \times 10^4$

$k = \frac{0.693}{t_{1/2}}$

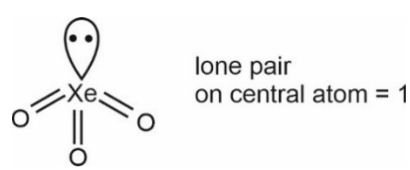
$3.465 \times 10^4 = \frac{0.693}{t_{1/2}}$

$t_{1/2} = 2 \times 10^{-5}$ s

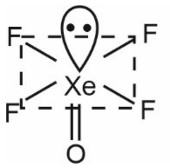
6. The sum of number of lone pairs of electrons present on the central atoms of XeO_3, $XeOF_4$ and XeF_6, is______

Answer (3)

Sol. $XeO_3 \Rightarrow$ S.N. (Steric number) = $\frac{1}{2}[8] = 4 \Rightarrow sp^3$

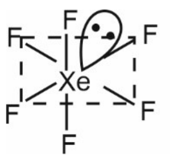


$XeOF_4 \Rightarrow$ S.N = $\frac{1}{2}$ [8 + 4] = 6 $\Rightarrow sp^3d^2$



lone pair on central atom = 1

$XeF_6 \Rightarrow$ S.N = $\frac{1}{2}$ [8 + 6] = 7 $\Rightarrow sp^3d^3$



lone pair on central atom = 1

Sum of lone pairs = 3

7. The spin-only magnetic moment value of M^{3+} ion (in gaseous state) from the pairs Cr^{3+}/Cr^{2+}, Mn^{3+}/Mn^{2+}, Fe^{3+}/Fe^{2+} and Co^{3+}/Co^{2+} that has negative standard electrode potential, is_____ B.M. [Nearest integer]

Answer (4)

Sol. Among the pairs given, Cr^{3+}/Cr^{2+} has negative reduction potential which is –0.41 V.

Cr (III) $\Rightarrow d^3$

Number of unpaired electrons = 3

$$\mu = \sqrt{3(3+2)} = \sqrt{15} \simeq 4 \text{ B.M.}$$

8. A sample of 4.5 mg of an unknown monohydric alcohol, R–OH was added to methylmagnesium iodide. A gas is evolved and is collected and its volume measured to be 3.1 mL. The molecular weight of the unknown alcohol is____ g/mol. [Nearest integer]

Answer (33)

Sol. $R-OH + CH_3MgI \Rightarrow R-OMgI + CH_4$

moles of alcohol (ROH) ≡ moles of CH_4

At STP, [Assuming STP]

1 mole corresponds to 22.7 L

Hence, 3.1 mL $\equiv \frac{3.1}{22700}$ mol

So, moles of alcohol = $\frac{3.1}{22700}$

$$\Rightarrow \frac{3.1}{22700} = \frac{4.5\times10^{-3}}{M}$$

M $\simeq$ 33 g/mol

9. The separation of two coloured substances was done by paper chromatography. The distances travelled by solvent front, substance A and substance B from the base line are 3.25 cm, 2.08 cm and 1.05 cm, respectively. The ratio of R_f values of A to B is_____.

Answer (2)

Sol. $R_f = \frac{\text{Distance travelled by the substance}}{\text{Distance travelled by the solvent front}}$

$$(R_f)_A = \frac{2.08}{3.25}$$

$$(R_f)_B = \frac{1.05}{3.25}$$

$$\frac{(R_f)_A}{(R_f)_B} \simeq 2$$

10. The total number of monobromo derivatives formed by the alkanes with molecular formula C_5H_{12} is (excluding stereo isomers)____.

Answer (8)

Sol. Total monobromo derivatives = 8

$CH_3-CH_2-CH_2-CH_2-CH_2-Br$,

$CH_3-CH_2-CH_2-CH(Br)-CH_3$,

$CH_3-CH_2-CH(Br)-CH_2-CH_3$,

$CH_3-CH_2-CH(CH_3)-CH_2-Br$,

$CH_3-CH_2-C(Br)(CH_3)-CH_3$, $CH_3-CH(Br)-CH(CH_3)-CH_3$,

$BrCH_2-CH_2-CH(CH_3)-CH_3$, $H_3C-C(CH_3)_2-CH_2Br$

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

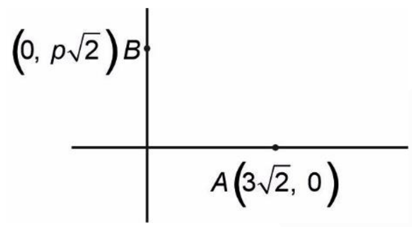
1. For $z \in \mathbb{C}$ if the minimum value of $\left(|z-3\sqrt{2}|+|z-p\sqrt{2}i|\right)$ is $5\sqrt{2}$, then a value of p is _________.

(A) 3 (B) $\frac{7}{2}$

(C) 4 (D) $\frac{9}{2}$

Answer (C)

Sol.



It is sum of distance of z from $(3\sqrt{2}, 0)$ and $(0, p\sqrt{2})$

For minimising, z should lie on AB and $AB = 5\sqrt{2}$

$$(AB)^2 = 18 + 2p^2$$

$$p = \pm 4$$

2. The number of real values of λ, such that the system of linear equations

$2x - 3y + 5z = 9$

$x + 3y - z = -18$

$3x - y + (\lambda^2 - |\lambda|)z = 16$

has no solutions, is

(A) 0 (B) 1

(C) 2 (D) 4

Answer (C)

Sol.
$$\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 1 & 3 & -1 \\ 3 & -1 & \lambda^2-|\lambda| \end{vmatrix} = 2\left(3\lambda^2 - 3|\lambda| - 1\right) + 3\left(\lambda^2 - |\lambda| + 3\right) + 5(-1-9)$$

$= 9\lambda^2 - 9|\lambda| - 43$

$= 9|\lambda|^2 - 9|\lambda| - 43$

$\Delta = 0$ for 2 values of $|\lambda|$ out of which one is –ve and other is +ve

So, 2 values of λ satisfy the system of equations to obtain no solution

3. The number of bijective functions $f : \{1, 3, 5, 7, \ldots, 99\} \rightarrow \{2, 4, 6, 8, \ldots, 100\}$ such that $f(3) \geq f(9) \geq f(15) \geq f(21) \geq \ldots \geq f(99)$, is_____.

(A) ${}^{50}P_{17}$ (B) ${}^{50}P_{33}$

(C) $33! \times 17!$ (D) $\frac{50!}{2}$

Answer (B)

Sol. As function is one-one and onto, out of 50 elements of domain set 17 elements are following restriction

$f(3) > f(9) > f(15) \ldots > f(99)$

So number of ways $= {}^{50}C_{17} \cdot 1 \cdot 33!$

$= {}^{50}P_{33}$

4. The remainder when $(11)^{1011} + (1011)^{11}$ is divided by 9 is

(A) 1 (B) 4

(C) 6 (D) 8

Answer (D)

Sol. $\text{Re}\left(\frac{(11)^{1011} + (1011)^{11}}{9}\right) = \text{Re}\left(\frac{2^{1011} + 3^{11}}{9}\right)$

For $\text{Re}\left(\frac{2^{1011}}{9}\right)$

$$2^{1011} = (9-1)^{337} = {}^{337}C_0 9^{337}(-1)^0 + {}^{337}C_1 9^{336}(-1)^1 + {}^{337}C_2 9^{335}(-1)^2 + \ldots + {}^{337}C_{337} 9^0 (-1)^{337}$$

so, remainder is 8

and $\text{Re}\left(\frac{3^{11}}{9}\right) = 0$

So, remainder is 8

5. The sum $\sum_{n=1}^{21}\frac{3}{(4n-1)(4n+3)}$ is equal to

(A) $\frac{7}{87}$ (B) $\frac{7}{29}$

(C) $\frac{14}{87}$ (D) $\frac{21}{29}$

Answer (B)

Sol. $\sum_{n=1}^{21}\frac{3}{(4n-1)(4n+3)}=\frac{3}{4}\sum_{n=1}^{21}\frac{1}{4n-1}-\frac{1}{4n+3}$

$$=\frac{3}{4}\left[\left(\frac{1}{3}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{11}\right)+.....+\left(\frac{1}{83}-\frac{1}{87}\right)\right]$$

$$=\frac{3}{4}\left[\frac{1}{3}-\frac{1}{87}\right]=\frac{3}{4}\cdot\frac{84}{3.87}=\frac{7}{29}$$

6. $\lim_{x\to\frac{\pi}{4}}\frac{8\sqrt{2}-(\cos x+\sin x)^7}{\sqrt{2}-\sqrt{2}\sin 2x}$ is equal to

(A) 14 (B) 7

(C) $14\sqrt{2}$ (D) $7\sqrt{2}$

Answer (A)

Sol. $\lim_{x\to\frac{\pi}{4}}\frac{8\sqrt{2}-(\cos x+\sin x)^7}{\sqrt{2}-\sqrt{2}\sin 2x}$ $\left(\frac{0}{0}\text{ form}\right)$

$=\lim_{x\to\frac{\pi}{4}}\frac{-7(\cos x+\sin x)^6(-\sin x+\cos x)}{-2\sqrt{2}\cos 2x}$ using L–H Rule

$=\lim_{x\to\frac{\pi}{4}}\frac{56(\cos x-\sin x)}{2\sqrt{2}\cos 2x}$ $\left(\frac{0}{0}\right)$

$=\lim_{x\to\frac{\pi}{4}}\frac{-56(\sin x+\cos x)}{-4\sqrt{2}\sin 2x}$ using L–H Rule

$=7\sqrt{2}\cdot\sqrt{2}=14$

7. $\lim_{n\to\infty}\frac{1}{2^n}\left(\frac{1}{\sqrt{1-\frac{1}{2^n}}}+\frac{1}{\sqrt{1-\frac{2}{2^n}}}+\frac{1}{\sqrt{1-\frac{3}{2^n}}}+.....+\frac{1}{\sqrt{1-\frac{2^n-1}{2^n}}}\right)$

is equal to

(A) $\frac{1}{2}$ (B) 1

(C) 2 (D) –2

Answer (C)

Sol. $I=\lim_{n\to\infty}\frac{1}{2^n}\left(\frac{1}{\sqrt{1-\frac{1}{2^n}}}+\frac{1}{\sqrt{1-\frac{2}{2^n}}}+\frac{1}{\sqrt{1-\frac{3}{2^n}}}+.....+\frac{1}{\sqrt{1-\frac{2^n-1}{2^n}}}\right)$

Let $2^n=t$ and if $n\to\infty$ then $t\to\infty$

$$I=\lim_{n\to\infty}\frac{1}{t}\left(\sum_{r=1}^{t-1}\frac{1}{\sqrt{1-\frac{r}{t}}}\right)$$

$$I=\int_0^1\frac{dx}{\sqrt{1-x}}=\int_0^1\frac{dx}{\sqrt{x}} \qquad \left(\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$$

$$=\left[2x^{\frac{1}{2}}\right]_0^1=2$$

8. If A and B are two events such that

$P(A)=\frac{1}{3},P(B)=\frac{1}{5}$ and $P(A\cup B)=\frac{1}{2}$, then

$P(A|B')+P(B|A')$ is equal to

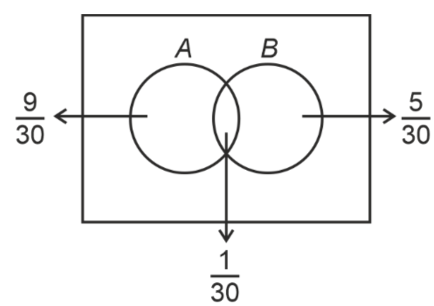
(A) $\frac{3}{4}$ (B) $\frac{5}{8}$

(C) $\frac{5}{4}$ (D) $\frac{7}{8}$

Answer (B)

Sol. $P(A)=\frac{1}{3},P(B)=\frac{1}{5}$ and $P(A\cup B)=\frac{1}{2}$

$\therefore\quad P(A\cap B)=\frac{1}{3}+\frac{1}{5}-\frac{1}{2}=\frac{1}{30}$



Now, $P(A|B')+P(B|A')=\frac{P(A\cap B')}{P(B')}+\frac{P(B\cap A')}{P(A')}$

$$=\frac{\frac{9}{30}}{\frac{4}{5}}+\frac{\frac{5}{30}}{\frac{2}{3}}=\frac{5}{8}$$

9. Let [t] denote the greatest integer less than or equal to t. Then the value of the integral $\int_{-3}^{101}\left(\left[\sin(\pi x)\right]+e^{\left[\cos(2\pi x)\right]}\right)dx$ is equal to

(A) $\frac{52(1-e)}{e}$ (B) $\frac{52}{e}$

(C) $\frac{52(2+e)}{e}$ (D) $\frac{104}{e}$

Answer (B)

Sol. $I=\int_{-3}^{101}\left(\left[\sin(\pi x)\right]+e^{\left[\cos(2\pi x)\right]}\right)dx$

$[\sin \pi x]$ is periodic with period 2 and $e^{\left[\cos(2\pi x)\right]}$ is periodic with period 1.

So,

$I=52\int_{0}^{2}\left([\sin \pi x]+e^{[\cos 2\pi x]}\right)dx$

$$=52\left\{\int_{1}^{2}-1\,dx+\int_{\frac{1}{4}}^{\frac{3}{4}}e^{-1}dx+\int_{\frac{5}{4}}^{\frac{7}{4}}e^{-1}\,dx+\int_{0}^{\frac{1}{4}}e^{0}\,dx+\int_{\frac{3}{4}}^{\frac{5}{4}}e^{0}\,dx+\int_{\frac{7}{4}}^{2}e^{0}\,dx\right\}$$

$=\frac{52}{e}$

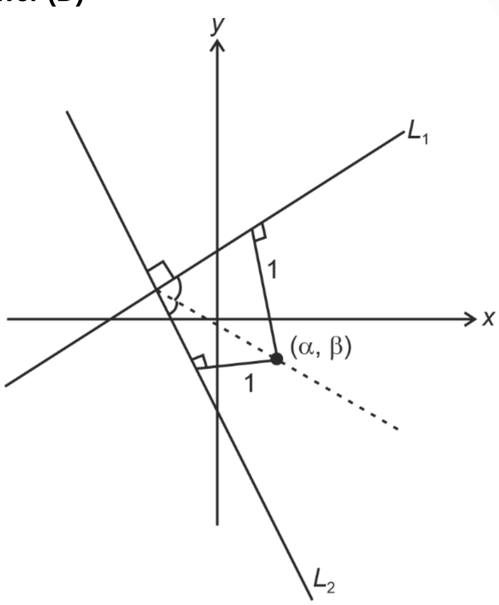
10. Let the point $P(\alpha,\beta)$ be at a unit distance from each of the two lines $L_1: 3x-4y+12=0$, and $L_2: 8x+6y+11=0$. If P lies below L_1 and above L_2, then $100(\alpha+\beta)$ is equal to

(A) –14 (B) 42

(C) –22 (D) 14

Answer (D)

Sol.



$L_1 : 3x - 4y + 12 = 0$

$L_2 : 8x + 6y + 11 = 0$

Equation of angle bisector of L_1 and L_2 of angle containing origin

$2(3x - 4y + 12) = 8x + 6y + 11$

$2x + 14y - 13 = 0$ …(i)

$\frac{3\alpha-4\beta+12}{5}=1$

$\Rightarrow\ 3\alpha-4\beta+7=0$ …(ii)

Solution of $2x + 14y - 13 = 0$ and $3x - 4y + 7 = 0$ gives the required point $P(\alpha, \beta)$, $\alpha=\frac{-23}{25}$, $\beta=\frac{53}{50}$

$100(\alpha+\beta)=14$

11. Let a smooth curve $y = f(x)$ be such that the slope of the tangent at any point (x, y) on it is directly proportional to $\left(\frac{-y}{x}\right)$. If the curve passes through the points (1, 2) and (8, 1), then $\left|y\left(\frac{1}{8}\right)\right|$ is equal to

(A) $2\log_e 2$ (B) 4

(C) 1 (D) $4\log_e 2$

Answer (B)

Sol. $\frac{dy}{dx}\propto\frac{-y}{x}$

$\frac{dy}{dx}=\frac{-ky}{x}\ \Rightarrow\ \int\frac{dy}{y}=-K\int\frac{dx}{x}$

$\ln|y|=-K\ln|x|+C$

If the above equation satisfy (1, 2) and (8, 1)

$\ln 2=-K\times 0+C\ \Rightarrow C=\ln 2$

$\ln 1=-K\ln 8+\ln 2\ \Rightarrow\ K=\frac{1}{3}$

So, at $x=\frac{1}{8}$

$\ln|y|=-\frac{1}{3}\ln\left(\frac{1}{8}\right)+\ln 2=2\ln 2$

$|y|=4$

12. If the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the line $\frac{x}{7}+\frac{y}{2\sqrt{6}}=1$ on the x-axis and the line $\frac{x}{7}-\frac{y}{2\sqrt{6}}=1$ on the y-axis, then the eccentricity of the ellipse is

(A) $\frac{5}{7}$ (B) $\frac{2\sqrt{6}}{7}$

(C) $\frac{3}{7}$ (D) $\frac{2\sqrt{5}}{7}$

Answer (A)

Sol. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the line $\frac{x}{7}+\frac{y}{2\sqrt{6}}=1$ on the x-axis

So, $a = 7$

and $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the line $\frac{x}{7}-\frac{y}{2\sqrt{6}}=1$ on the y-axis

So, $b = 2\sqrt{6}$

Therefore, $e^2 = 1-\frac{b^2}{a^2} = 1-\frac{24}{49}$

$e = \frac{5}{7}$

13. The tangents at the points $A(1, 3)$ and $B(1, -1)$ on the parabola $y^2 - 2x - 2y = 1$ meet at the point P. Then the area (in unit^2) of the triangle PAB is :

(A) 4 (B) 6

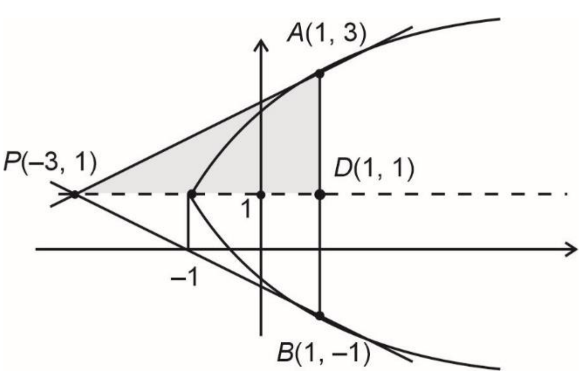
(C) 7 (D) 8

Answer (D)

Sol. Given curve : $y^2 - 2x - 2y = 1$.

Can be written as

$(y-1)^2 = 2(x+1)$



And, the given information

Can be plotted as shown in figure

Tangent at A : $2y - x - 5 = 0$ {using $T = 0$}

Intersection with $y = 1$ is $x = -3$

Hence, point P is $(-3, 1)$

Taking advantage of symmetry

Area of $\Delta PAB = 2\times\frac{1}{2}\times(1-(-3))\times(3-1)$

= 8 sq. units

14. Let the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{7}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{\alpha}=\frac{1}{25}$ coincide. Then the length of the latus rectum of the hyperbola is :

(A) $\frac{32}{9}$ (B) $\frac{18}{5}$

(C) $\frac{27}{4}$ (D) $\frac{27}{10}$

Answer (D)

Sol. Ellipse : $\frac{x^2}{16}+\frac{y^2}{7}=1$

Eccentricity = $\sqrt{1-\frac{7}{16}} = \frac{3}{4}$

Foci $\equiv (\pm a\,e, 0) \equiv (\pm 3,\ 0)$

Hyperbola : $\frac{x^2}{\left(\frac{144}{25}\right)}-\frac{y^2}{\left(\frac{\alpha}{25}\right)}=1$

Eccentricity = $\sqrt{1+\frac{\alpha}{144}} = \frac{1}{12}\sqrt{144+\alpha}$

Foci $\equiv (\pm ae, 0) \equiv \left(\pm\frac{12}{5}.\frac{1}{12}\sqrt{144+\alpha}, 0\right)$

If foci coincide then $3 = \frac{1}{5}\sqrt{144+\alpha} \Rightarrow \alpha = 81$

Hence, hyperbola is $\frac{x^2}{\left(\frac{12}{5}\right)^2}-\frac{y^2}{\left(\frac{9}{5}\right)^2}=1$

Length of latus rectum = $2\cdot\frac{81/25}{12/5} = \frac{27}{10}$

15. A plane E is perpendicular to the two planes $2x - 2y + z = 0$ and $x - y + 2z = 4$, and passes through the point $P(1, -1, 1)$. If the distance of the plane E from the point $Q(a, a, 2)$ is $3\sqrt{2}$, then $(PQ)^2$ is equal to

(A) 9
(B) 12
(C) 21
(D) 33

Answer (C)

Sol. First plane, $P_1 = 2x - 2y + z = 0$, normal vector $\equiv \bar{n}_1 = (2, -2, 1)$

Second plane, $P_2 \equiv x - y + 2z = 4$, normal vector $\equiv \bar{n}_2 = (1, -1, 2)$

Plane perpendicular to P_1 and P_2 will have normal vector $\bar{n}_3$

Where $\bar{n}_3 = (\bar{n}_1 \times \bar{n}_2)$

Hence, $\bar{n}_3 = (-3, -3, 0)$

Equation of plane E through $P(1, -1, 1)$ and $\bar{n}_3$ as normal vector

$(x-1, y+1, z-1)\cdot(-3, -3, 0) = 0$

$\Rightarrow \quad x + y = 0 \equiv E$

Distance of $PQ(a, a, 2)$ from $E = \left|\dfrac{2a}{\sqrt{2}}\right|$

as given, $\left|\dfrac{2a}{\sqrt{2}}\right| = 3\sqrt{2} \Rightarrow a = \pm 3$

Hence, $Q \equiv (\pm 3, \pm 3, 2)$

Distance $7Q = \sqrt{21} \Rightarrow (PQ)^2 = 21$

16. The shortest distance between the lines $\dfrac{x+7}{-6} = \dfrac{y-6}{7} = z$ and $\dfrac{7-x}{2} = y - 2 = z - 6$ is

(A) $2\sqrt{29}$
(B) 1
(C) $\sqrt{\dfrac{37}{29}}$
(D) $\dfrac{\sqrt{29}}{2}$

Answer (A)

Sol. $L_1 : \dfrac{x+7}{-6} = \dfrac{y-6}{7} = \dfrac{z-0}{1}$

Any point on it $\vec{a}_1(-7, 6, 0)$

and L_1 is parallel to $\vec{b}_1\,(-6, 7, 1)$

$L_2 : \dfrac{x-7}{-2} = \dfrac{y-2}{1} = \dfrac{z-6}{1}$

Any point on it, $\vec{a}_2(7, 2, 6)$

and L_2 is parallel to $\vec{b}_2\,(-2, 1, 1)$

Shortest distance between L_1 and L_2

$= \left|\dfrac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right| = \left|\dfrac{(-14, 4, -6)\cdot(3, 2, 4)}{\sqrt{9+4+16}}\right|$

$= 2\sqrt{29}.$

17. Let $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ and let $\vec{b}$ be a vector such that $\vec{a} \times \vec{b} = 2\hat{i} - \hat{k}$ and $\vec{a}\cdot\vec{b} = 3$. Then the projection of $\vec{b}$ on the vector $\vec{a} - \vec{b}$ is :

(A) $\dfrac{2}{\sqrt{21}}$
(B) $2\sqrt{\dfrac{3}{7}}$
(C) $\dfrac{2}{3}\sqrt{\dfrac{7}{3}}$
(D) $\dfrac{2}{3}$

Answer (A)

Sol. $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$

$\vec{a} \times \vec{b} = 2\hat{i} - \hat{k}$

$\vec{a}\cdot\vec{b} = 3$

$|\vec{a} \times \vec{b}|^2 + |\vec{a}\cdot\vec{b}|^2 = |\vec{a}|^2 \cdot |\vec{b}|^2$

$\Rightarrow \quad 5 + 9 = 6|\vec{b}|^2$

$\Rightarrow \quad |b|^2 = \dfrac{7}{3}$

$|\vec{a} - \vec{b}| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\cdot\vec{b}} = \sqrt{\dfrac{7}{3}}$

projection of $\vec{b}$ on $\vec{a} - \vec{b} = \dfrac{\vec{b}\cdot(\vec{a} - \vec{b})}{|\vec{a} - \vec{b}|}$

$= \dfrac{\vec{b}\cdot\vec{a} - |\vec{b}|^2}{|\vec{a} - \vec{b}|} = \dfrac{3 - \dfrac{7}{3}}{\sqrt{\dfrac{7}{3}}}$

$= \dfrac{2}{\sqrt{21}}$

18. If the mean deviation about median for the number 3, 5, 7, $2k$, 12, 16, 21, 24 arranged in the ascending order, is 6 then the median is

(A) 11.5
(B) 10.5
(C) 12
(D) 11

Answer (D)

Sol. Median = $\frac{2k+12}{2} = k+6$

Mean deviation = $\sum\frac{|x_i - M|}{n} = 6$

$$\Rightarrow \frac{(k+3)+(k+1)+(k-1)+(6-k)+(6-k)+(10-k)+(15-k)+(18-k)}{8}$$

$\therefore \frac{58-2k}{8} = 6$

$k = 5$

Median = $\frac{2\times5+12}{2} = 11$

19. $2\sin\left(\frac{\pi}{22}\right)\sin\left(\frac{3\pi}{22}\right)\sin\left(\frac{5\pi}{22}\right)\sin\left(\frac{7\pi}{22}\right)\sin\left(\frac{9\pi}{22}\right)$ is equal to :

(A) $\frac{3}{16}$

(B) $\frac{1}{16}$

(C) $\frac{1}{32}$

(D) $\frac{9}{32}$

Answer (B)

Sol. $2\sin\frac{\pi}{22}\sin\frac{3\pi}{22}\sin\frac{5\pi}{22}\sin\frac{7\pi}{22}\sin\frac{9\pi}{22}$

$$= 2\sin\left(\frac{11\pi-10\pi}{22}\right)\sin\left(\frac{11\pi-8\pi}{22}\right)\sin\left(\frac{11\pi-6\pi}{22}\right)\sin\left(\frac{11\pi-4\pi}{22}\right)\sin\left(\frac{11\pi-2\pi}{22}\right)$$

$$= 2\cos\frac{\pi}{11}\cos\frac{2\pi}{11}\cos\frac{3\pi}{11}\cos\frac{4\pi}{11}\cos\frac{5\pi}{11}$$

$$= \frac{2\sin\frac{32\pi}{11}}{2^5\sin\frac{\pi}{11}}$$

$$= \frac{1}{16}$$

20. Consider the following statements :

P : Ramu is intelligent.

Q : Ramu is rich.

R : Ramu is not honest.

The negation of the statement "Ramu is intelligent and honest if and only if Ramu is not rich" can be expressed as :

(A) $((P \wedge (\sim R)) \wedge Q) \wedge ((\sim Q) \wedge ((\sim P) \vee R))$

(B) $((P \wedge R) \wedge Q) \vee ((\sim Q) \wedge ((\sim P) \vee (\sim R)))$

(C) $((P \wedge R) \wedge Q) \wedge ((\sim Q) \wedge ((\sim P) \vee (\sim R)))$

(D) $((P \wedge (\sim R)) \wedge Q) \vee ((\sim Q) \wedge ((\sim P) \wedge R))$

Answer (D)

Sol. P : Ramu is intelligent

Q : Ramu is rich

R : Ramu is not honest

Given statement, "Ramu is intelligent and honest if and only if Ramu is not rich"

$= (P \wedge \sim R) \Leftrightarrow \sim Q$

So, negation of the statement is

$\sim [(P \wedge \sim R) \Leftrightarrow \sim Q]$

$= \sim [\{\sim (P \wedge \sim R) \vee \sim Q\} \wedge \{Q \vee (P \wedge \sim R)\}]$

$= ((P \wedge \sim R) \wedge Q) \vee (\sim Q \wedge (\sim P \vee R))$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$. Define $B = \{T \subseteq A : \text{either } 1 \notin T \text{ or } 2 \in T\}$ and $C = \{T \subseteq A : T$ the sum of all the elements of T is a prime number$\}$. Then the number of elements in the set $B \cup C$ is ______.

Answer (107)

Sol. $\because \quad (B \cup C)' = B' \cap C'$

B' is a set containing sub sets of A containing element 1 and not containing 2.

And C' is a set containing subsets of A whose sum of elements is not prime.

So, we need to calculate number of subsets of {3, 4, 5, 6, 7} whose sum of elements plus 1 is composite.

Number of such 5 elements subset = 1

Number of such 4 elements subset = 3 (except selecting 3 or 7)

Number of such 3 elements subset = 6 (except selecting {3, 4, 5}, {3, 6, 7}, {4, 5, 7} or {5, 6, 7})

Number of such 2 elements subset = 7 (except selecting {3, 7}, {4, 6}, {5, 7})

Number of such 1 elements subset = 3 (except selecting {4} or {6})

Number of such 0 elements subset = 1

$n(B' \cap C') = 21 \Rightarrow n(B \cup C) = 2^7 - 21 = 107$

2. Let $f(x)$ be a quadratic polynomial with leading coefficient 1 such that $f(0) = p$, $p \neq 0$, and $f(1) = \frac{1}{3}$.

If the equations $f(x) = 0$ and *fo fo fo* $f(x) = 0$ have a common real root, then $f(-3)$ is equal to ______.

Answer (25)

Sol. Let $f(x) = (x - \alpha)(x - \beta)$

It is given that $f(0) = p \Rightarrow \alpha\beta = p$

and $f(1) = \frac{1}{3} \quad \Rightarrow \quad (1-\alpha)(1-\beta) = \frac{1}{3}$

Now, let us assume that α is the common root of $f(x) = 0$ and $fofofof(x) = 0$

$fofofof(x) = 0$

$\Rightarrow \quad fofof(0) = 0$

$\Rightarrow \quad fof(p) = 0$

So, $f(p)$ is either α or β.

$(p - \alpha)(p - \beta) = \alpha$

$(\alpha\beta - \alpha)(\alpha\beta - \beta) = \alpha \Rightarrow (\beta - 1)(\alpha - 1)\beta = 1$

$(\because \alpha \neq 0)$

So, $\beta = 3$

$(1 - \alpha)(1 - 3) = \frac{1}{3}$

$\alpha = \frac{7}{6}$

$f(x) = \left(x - \frac{7}{6}\right)(x - 3)$

$f(-3) = \left(-3 - \frac{7}{6}\right)(3 - 3) = 25$

3. Let $A = \begin{bmatrix} 1 & a & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}, a, b \in \mathbb{R}$.

If for some $n \in N$, $A^n = \begin{bmatrix} 1 & 48 & 2160 \\ 0 & 1 & 96 \\ 0 & 0 & 1 \end{bmatrix}$ then

$n + a + b$ is equal to __________.

Answer (24)

Sol. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & a & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{bmatrix} = I + B$

$B^2 = \begin{bmatrix} 0 & a & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & a & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & ab \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$B^3 = 0$

$\therefore \; A^n = (1 + B)^n = {}^nC_0 I + {}^nC_1 B + {}^nC_2 B^2 + {}^nC_3 B^3 + \ldots$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & na & na \\ 0 & 0 & nb \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & \frac{n(n-1)ab}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & na & na + \frac{n(n-1)}{2}ab \\ 0 & 1 & nb \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 48 & 2160 \\ 0 & 1 & 48 \\ 0 & 0 & 1 \end{bmatrix}$$

On comparing we get $na = 48$, $nb = 96$ and

$na + \frac{n(n-1)}{2}ab = 2160$

$\Rightarrow a = 4, n = 12$ and $b = 8$

$n + a + b = 24$

4. The sum of the maximum and minimum values of the function $f(x) = |5x - 7| + [x^2 + 2x]$ in the interval $\left[\frac{5}{4}, 2\right]$, where $[t]$ is the greatest integer $\leq t$, is _____.

Answer (15)

Sol. $f(x) = |5x - 7| + [x^2 + 2x]$

$= |5x - 7| + [(x + 1)^2] - 1$

Critical points of

$f(x) = \frac{7}{5}, \sqrt{5} - 1, \sqrt{6} - 1, \sqrt{7} - 1, \sqrt{8} - 1, 2$

$\therefore$ Maximum or minimum value of $f(x)$ occur at critical points or boundary points

$\therefore \quad f\left(\frac{5}{4}\right) = \frac{3}{4} + 4 = \frac{19}{4}$

$f\left(\frac{7}{5}\right) = 0 + 4 = 4$

as both $|5x - 7|$ and $x^2 + 2x$ are increasing in nature after $x = \frac{7}{5}$

$\therefore \quad f(2) = 3 + 8 = 11$

$\therefore \quad f\left(\frac{7}{5}\right)_{min} = 4$ and $f(2)_{max} = 11$

Sum is 4 + 11 = 15

5. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = \frac{4y^3 + 2yx^2}{3xy^2 + x^3}$, $y(1) = 1$.

If for some $n \in N$, $y(2) \in [n - 1, n)$, then n is equal to ________.

Answer (3)

Sol. $\frac{dy}{dx} = \frac{y}{x}\frac{(4y^2 + 2x^2)}{(3y^2 + x^2)}$

Put $y = vx$ $\Rightarrow$ $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{v(4v^2 + 2)}{(3v^2 + 1)}$

$\Rightarrow x\frac{dv}{dx} = v\left(\frac{(4v^2 + 2 - 3v^2 - 1)}{3v^2 + 1}\right)$

$\Rightarrow \int (3v^2 + 1)\frac{dv}{v^3 + v} = \int \frac{dx}{x}$

$\Rightarrow \ln|v^3 + v| = \ln x + c$

$\Rightarrow \ln\left|\left(\frac{y}{x}\right)^3 + \left(\frac{y}{x}\right)\right| = \ln x + C$

$\downarrow y(1) = 1$

$\Rightarrow C = \ln 2$

$\therefore$ for $y(2)$

$\ln\left(\frac{y^3}{8} + \frac{y}{2}\right) = 2\ln 2 \Rightarrow \frac{y^3}{8} + \frac{y}{2} = 4$

$\Rightarrow [y(2)] = 2$

$\Rightarrow n = 3$

6. Let f be a twice differentiable function on R. If $f'(0) = 4$ and $f(x) + \int_0^x (x - t)f'(t)dt = \left(e^{2x} + e^{-2x}\right)\cos 2x + \frac{2}{a}x$, then $(2a + 1)^5 a^2$ is equal to ________.

Answer (8)

Sol. $\because f(x) + \int_0^x (x - t) f'(t)dt = \left(e^{2x} + e^{-2x}\right)\cos 2x + \frac{2x}{a}$ ...(i)

Here $f(0) = 2$ ...(ii)

On differentiating equation (i) w.r.t. x we get :

$f'(x) + \int_0^x f'(t)dt + xf'(x) - xf'(x) = 2\left(e^{2x} - e^{-2x}\right)\cos 2x - 2\left(e^{2x} + e^{-2x}\right)\sin 2x + \frac{2}{a}$

$\Rightarrow f'(x) + f(x) - f(0) = 2(e^{2x} - e^{-2x})\cos 2x - 2(e^{2x} + e^{-2x})\sin 2x + \frac{2}{a}$

Replace x by 0 we get :

$\Rightarrow 4 = \frac{2}{a} \Rightarrow a = \frac{1}{2}.$

$\therefore (2a + 1)^5 \cdot a^2 = 2^5 \cdot \frac{1}{2^2} = 2^3 = 8$

7. Let $a_n = \int_{-1}^{n}\left(1 + \frac{x}{2} + \frac{x^2}{3} + ... + \frac{x^{n-1}}{n}\right)dx$ for every $n \in N$. Then the sum of all the elements of the set $\{n \in N : a_n \in (2, 30)\}$ is ________.

Answer (5)

Sol. $\because a_n = \int_{-1}^{n}\left(1 + \frac{x}{2} + \frac{x^2}{3} + ... + \frac{x^{n-1}}{n}\right)dx$

$= \left[x + \frac{x^2}{2^2} + \frac{x^3}{3^2} + ... + \frac{x^n}{n^2}\right]_{-1}^{n}$

$a_n = \frac{n+1}{1^2} + \frac{n^2 - 1}{2^2} + \frac{n^3 + 1}{3^2} + \frac{n^4 - 1}{4^2} + ... + \frac{n^n + (-1)^{n+1}}{n^2}$

Here $a_1 = 2$, $a_2 = \frac{2+1}{1} + \frac{2^2 - 1}{2} = 3 + \frac{3}{2} = \frac{9}{2}$

$a_3 = 4 + 2 + \frac{28}{9} = \frac{100}{9}$

$a_4 = 5 + \frac{15}{4} + \frac{65}{9} + \frac{255}{16} > 31.$

$\therefore$ The required set is {2, 3}. $\because a_n \in (2, 30)$

$\therefore$ Sum of elements = 5.

8. If the circles $x^2 + y^2 + 6x + 8y + 16 = 0$ and $x^2 + y^2 + 2\left(3 - \sqrt{3}\right)x + 2\left(4 - \sqrt{6}\right)y = k + 6\sqrt{3} + 8\sqrt{6}$, $k > 0$, touch internally at the point $P(\alpha, \beta)$, then $\left(\alpha + \sqrt{3}\right)^2 + \left(\beta + \sqrt{6}\right)^2$ is equal to ______.

Answer (25)

Sol. The circle $x^2 + y^2 + 6x + 8y + 16 = 0$ has centre (–3, –4) and radius 3 units.

The circle $x^2+y^2+2\left(3-\sqrt{3}\right)x+2\left(4-\sqrt{6}\right)y = k+6\sqrt{3}+8\sqrt{6},\ k>0$ has centre $\left(\sqrt{3}-3,\ \sqrt{6}-4\right)$ and radius $\sqrt{k+34}$

$\because$ These two circles touch internally hence

$$\sqrt{3+6}=\left|\sqrt{k+34}-3\right|.$$

Here, $k = 2$ is only possible $(\because k > 0)$

Equation of common tangent to two circles is $2\sqrt{3}x+2\sqrt{6}y+16+6\sqrt{3}+8\sqrt{6}+k=0$

$\because$ $k = 2$ then equation is

$$x+\sqrt{2}y+3+4\sqrt{2}+3\sqrt{3}=0 \quad \ldots\text{(i)}$$

$\because$ (α, β) are foot of perpendicular from (–3, –4)

To line (i) then

$$\frac{\alpha+3}{1}=\frac{\beta+4}{\sqrt{2}}=\frac{-\left(-3-4\sqrt{2}+3+4\sqrt{2}+3\sqrt{3}\right)}{1+2}$$

$\therefore$ $\alpha+3=\dfrac{\beta+4}{\sqrt{2}}=-\sqrt{3}$

$\Rightarrow$ $\left(\alpha+\sqrt{3}\right)^2=9$ and $\left(\beta+\sqrt{6}\right)^2=16$

$\therefore$ $\left(\alpha+\sqrt{3}\right)^2+\left(\beta+\sqrt{6}\right)^2=25$

9. Let the area enclosed by the x-axis, and the tangent and normal drawn to the curve $4x^3 - 3xy^2 + 6x^2 - 5xy - 8y^2 + 9x + 14 = 0$ at the point (–2, 3) be A. Then $8A$ is equal to _______.

Answer (170)

Sol. $4x^3 - 3xy^2 + 6x^2 - 5xy - 8y^2 + 9x + 14 = 0$ differentiating both sides we get

$$12x^2-3y^2-6xyy'+12x-5y-5xy'-16yy'+9=0$$

$$\downarrow(-2,\ 3)$$

$\Rightarrow$ $48-27+36y'-24-15+10y'-48y'+9=0$

$\Rightarrow$ $2y'=-9$

$\Rightarrow$ $m_T=\dfrac{-9}{2}$ & $m_N=\dfrac{2}{9}$

$T\equiv y-3=\dfrac{-9}{2}(x+2)$ & $N\equiv y-3=\dfrac{2}{9}(x+2)$

$\downarrow y=0$ $\qquad$ $\downarrow y=0$

$x=\dfrac{-4}{3}$ $\qquad$ $x=\dfrac{-31}{2}$

$\therefore$ Area = $\dfrac{1}{2}\times$ Base × Height

$$A=\frac{1}{2}\times\left(\frac{-4}{3}+\frac{31}{2}\right)(3)=\frac{1}{2}\left(\frac{85}{6}\right)\cdot 3=\frac{85}{4}$$

= 8A = 170

10. Let $x = \sin(2\tan^{-1}\alpha)$ and $y=\sin\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right)$. If $S=\left\{\alpha\in\mathbb{R}: y^2=1-x\right\}$, then $\sum_{\alpha\in S}16\alpha^3$ is equal to _______.

Answer (130)

Sol. $\because$ $x=\sin\left(2\tan^{-1}\alpha\right)=\dfrac{2\alpha}{1+\alpha^2}$ $\quad\ldots$(i)

and $y=\sin\left(\dfrac{1}{2}\tan^{-1}\dfrac{4}{3}\right)=\sin\left(\sin^{-1}\dfrac{1}{\sqrt{5}}\right)=\dfrac{1}{\sqrt{5}}$

Now, $y^2=1-x$

$$\frac{1}{5}=1-\frac{2\alpha}{1+\alpha^2}$$

$\Rightarrow$ $1+\alpha^2=5+5\alpha^2-10\alpha$

$\Rightarrow$ $2\alpha^2-5\alpha+2=0$

$\therefore$ $\alpha=2,\dfrac{1}{2}$

$\therefore$ $\sum_{\alpha\in S}16\alpha^3=16\times 2^3+16\times\dfrac{1}{2^3}$

= 130

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