26/07/2022

Evening

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Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2022 (Online) Phase-2

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Two projectiles are thrown with same initial velocity making an angle of 45° and 30° with the horizontal respectively. The ratio of their respective ranges will be

(A) $1:\sqrt{2}$ (B) $\sqrt{2}:1$

(C) $2:\sqrt{3}$ (D) $\sqrt{3}:2$

Answer (C)

Sol. $R=\dfrac{u^2\sin 2\theta}{g}$

$$\Rightarrow \frac{R_1}{R_2}=\frac{\sin 2\theta_1}{\sin 2\theta_2}=\frac{1}{\frac{\sqrt{3}}{2}}=\frac{2}{\sqrt{3}}$$

2. In a Vernier Calipers, 10 divisions of Vernier scale is equal to the 9 divisions of main scale. When both jaws of Vernier calipers touch each other, the zero of the Vernier scale is shifted to the left of zero of the main scale and 4th Vernier scale division exactly coincides with the main scale reading. One main scale division is equal to 1 mm. While measuring diameter of a spherical body, the body is held between two jaws. It is now observed that zero of the Vernier scale lies between 30 and 31 divisions of main scale reading and 6th Vernier scale division exactly coincides with the main scale reading. The diameter of the spherical body will be

(A) 3.02 cm (B) 3.06 cm

(C) 3.10 cm (D) 3.20 cm

Answer (C)

Sol. 10 VSD = 9 MSD

$$\Rightarrow \text{LC}=\frac{1}{10}\text{MSD}=0.01\,\text{cm}$$

Negative error = (0.1 – 0.04) cm = 0.06 cm

Reading = (3.0 cm) + 6(0.01) cm + 0.06 cm

= 3.12 cm

Closer to 3.10 cm

3. A ball of mass 0.15 kg hits the wall with its initial speed of 12 ms^{-1} and bounces back without changing its initial speed. If the force applied by the wall on the ball during the contact is 100 N, calculate the time duration of the contact of ball with the wall.

(A) 0.018 s (B) 0.036 s

(C) 0.009 s (D) 0.072 s

Answer (B)

Sol. F = 100 N

ΔP = 2 × 0.15 × 12

= 3.6

$$\Rightarrow t=\frac{3.6}{100}=0.036\text{ s}$$

4. A body of mass 8 kg and another of mass 2 kg are moving with equal kinetic energy. The ratio of their respective momenta will be

(A) 1:1 (B) 2:1

(C) 1:4 (D) 4:1

Answer (B)

Sol. $P=\sqrt{2m\,KE}$

$$\Rightarrow \frac{P_1}{P_2}=\sqrt{\frac{m_1}{m_2}}$$

$$=\sqrt{\frac{8}{2}}=\frac{2}{1}$$

5. Two uniformly charged spherical conductors *A* and *B* of radii 5 mm and 10 mm are separated by a distance of 2 cm. If the spheres are connected by a conducting wire, then in equilibrium condition, the ratio of the magnitude of the electric fields at the surface of the sphere *A* and *B* will be

(A) 1:2 (B) 2:1

(C) 1:1 (D) 1:4

Answer (B)

Sol. After connection

$\sigma_1 R_1 = \sigma_2 R_2$

Now $E = \frac{\sigma}{\varepsilon_0}$

$$\Rightarrow \frac{E_1}{E_2} = \frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}$$

$$= \frac{2}{1}$$

6. The oscillating magnetic field in a plane electromagnetic wave is given by

$B_y = 5 \times 10^{-6} \sin 1000\pi(5x - 4 \times 10^8 t)$T.

The amplitude of electric field will be:

(A) 15×10^2 Vm^{-1} (B) 5×10^{-6} Vm^{-1}

(C) 16×10^{12}Vm^{-1} (D) 4×10^2Vm^{-1}

Answer (D)

Sol. Speed of light $c = \frac{\omega}{k} = \frac{4\times10^8}{5} = 0.8\times10^8$ m/sec

so $E_0 = cB_0$

$= 0.8 \times 10^8 \times 5 \times 10^{-6}$

$= 400$ V/m

$= 4 \times 10^{-2}$

7. Light travels in two media M_1 and M_2 with speeds 1.5×10^8 ms^{-1} and 2.0×10^8 ms^{-1} respectively. The critical angle between them is:

(A) $\tan^{-1}\left(\frac{3}{\sqrt{7}}\right)$ (B) $\tan^{-1}\left(\frac{2}{3}\right)$

(C) $\cos^{-1}\left(\frac{3}{4}\right)$ (D) $\sin^{-1}\left(\frac{2}{3}\right)$

Answer (A)

Sol. Critical angle between them

$$\sin i_c = \frac{\mu_2}{\mu_1} = \frac{v_1}{v_2}$$

$$\sin i_c = \frac{3}{4}$$

$$\Rightarrow \tan i_c = \frac{3}{\sqrt{7}}$$

$$i_c = \tan^{-1}\frac{3}{\sqrt{7}}$$

8. A body is projected vertically upwards from the surface of earth with a velocity equal to one third of escape velocity. The maximum height attained by the body will be:

(Take radius of earth = 6400 km and $g = 10$ ms^{-2})

(A) 800 km (B) 1600 km

(C) 2133 km (D) 4800 km

Answer (A)

Sol. Applying conservation of energy

$$-\frac{GM_e m}{R_e} + \frac{1}{2}m\left(\frac{1}{3}\sqrt{\frac{2Gm_e}{R_e}}\right)^2 = -\frac{GM_e m}{R_e + h}$$

$$-\frac{GM_e m}{R_e} + \frac{GM_e m}{9R_e} = -\frac{GM_e m}{R_e + h}$$

$$\frac{8}{9R_e} = \frac{1}{R_e + h}$$

$$\Rightarrow h = \frac{R_e}{8} = \frac{6400}{8} = 800 \text{ km}$$

9. The maximum and minimum voltage of an amplitude modulated signal are 60 V and 20 V respectively. The percentage modulation index will be:

(A) 0.5% (B) 50%

(C) 2% (D) 30%

Answer (B)

Sol. Percentage modulation

$$\mu = \frac{V_{max} - V_{min}}{V_{max} + V_{min}} \times 100$$

$$= \frac{60 - 20}{60 + 20} \times 100$$

= 50%

10. A nucleus of mass M at rest splits into two parts having masses $\frac{M'}{3}$ and $\frac{2M'}{3}$ $(M' < M)$. The ratio of de Broglie wavelength of two parts will be:

(A) 1 : 2

(B) 2 : 1

(C) 1 : 1

(D) 2 : 3

Answer (C)

Sol. Linear momentum is conserved

so $p_{M'/3} = p_{2M'/3}$

so $\frac{\lambda_{M'/3}}{\lambda_{2M'/3}} = \frac{1}{1}$

11. An ice cube of dimensions 60 cm × 50 cm × 20 cm is placed in an insulation box of wall thickness 1 cm. The box keeping the ice cube at 0°C of temperature is brought to a room of temperature 40°C. The rate of melting of ice is approximately.

(Latent heat of fusion of ice is 3.4×10^5 J kg^{-1} and thermal conducting of insulation wall is 0.05 Wm^{-1}°C^{-1})

(A) 61×10^{-3} kg s^{-1}

(B) 61×10^{-5} kg s^{-1}

(C) 208 kg s^{-1}

(D) 30×10^{-5} kg s^{-1}

Answer (B)

Sol. $\frac{\Delta Q}{\Delta t} = \frac{kA(T_1 - T_2)}{\ell}$

$\Rightarrow \frac{mL}{\Delta t} = \frac{kA(T_1 - T_2)}{\ell}$

$\Rightarrow \frac{m}{\Delta t} = \frac{kA(T_1 - T_2)}{L\ell}$

$\simeq 61.1 \times 10^{-5}$ kg/s

12. A gas has n degrees of freedom. The ratio of specific heat of gas at constant volume to the specific heat of gas at constant pressure will be

(A) $\frac{n}{n+2}$

(B) $\frac{n+2}{n}$

(C) $\frac{n}{2n+2}$

(D) $\frac{n}{n-2}$

Answer (A)

Sol. $C_V = \frac{nR}{2}$

And $C_P = \frac{nR}{2} + R$

$\Rightarrow \frac{C_V}{C_P} = \frac{\frac{nR}{2}}{\frac{nR}{2} + R} = \frac{n}{n+2}$

13. A transverse wave is represented by

$y = 2\sin(\omega t - kx)$ cm. The value of wavelength (in cm) for which the wave velocity becomes equal to the maximum particle velocity, will be

(A) 4π

(B) 2π

(C) π

(D) 2

Answer (A)

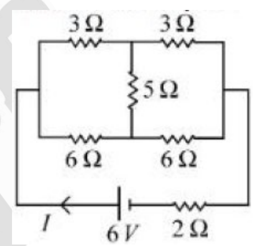
Sol. $\frac{\omega}{k} = A\omega$

$\Rightarrow k = \frac{1}{A} = \frac{1}{2\text{ cm}}$

$\Rightarrow \frac{2\pi}{\lambda} = \frac{1}{2\text{ cm}}$

$\Rightarrow \lambda = 4\pi$ cm

14. A battery of 6 V is connected to the circuit as shown below. The current I drawn from the battery is



(A) 1 A

(B) 2 A

(C) $\frac{6}{11}$ A

(D) $\frac{4}{3}$ A

Answer (A)

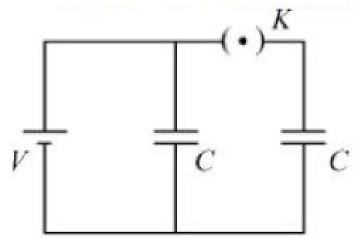
Sol. Balanced wheatstone

$\Rightarrow R_{eff} = \frac{3 \times 6}{3+6} \times 2 + 2$

$= 6\ \Omega$

$\Rightarrow I = \frac{V}{R} = 1$ A

15. A source of potential difference V is connected to the combination of two identical capacitors as shown in the figure. When key 'K' is closed, the total energy stored across the combination is E_1. Now key 'K' is opened and dielectric of dielectric constant 5 is introduced between the plates of the capacitors. The total energy stored across the combination is now E_2. The ratio E_1/E_2 will be



(A) $\frac{1}{10}$ (B) $\frac{2}{5}$

(C) $\frac{5}{13}$ (D) $\frac{5}{26}$

Answer (C)

Sol. $E_1 = \frac{1}{2}(2C)V^2$

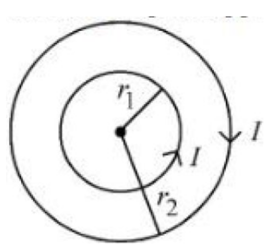
$\Rightarrow E_1 = CV^2$ ...(i)

$E_2 = \frac{1}{2}(5C)V^2 + \frac{1}{2}\frac{(CV)^2}{5C}$

$= \frac{13}{5}CV^2$

$\Rightarrow \frac{E_1}{E_2} = \frac{5}{13}$

16. Two concentric circular loops of radii r_1 = 30 cm and r_2 = 50 cm are placed in X-Y plane as shown in the figure. A current I = 7 A is flowing through them in the direction as shown in figure. The net magnetic moment of this system of two circular loops is approximately



(A) $\frac{7}{2}\hat{k}$ Am^2 (B) $-\frac{7}{2}\hat{k}$ Am^2

(C) $7\hat{k}$ Am^2 (D) $-7\hat{k}$ Am^2

Answer (B)

Sol. $\mu_1 = \pi r_1^2 \times I_1$

$\mu_2 = \pi r_2^2 \times I_2$

$\therefore \mu_{net} = (\mu_2 - \mu_1)(-\hat{k})$

$= \pi(r_2^2 - r_1^2)I(-\hat{k})$

$= 3.142 \times (0.5^2 - 0.3^2) \times 7(-\hat{k})$

$= -\frac{7}{2}\hat{k}$ Am^2

17. A velocity selector consists of electric field $\vec{E} = E\hat{k}$ and magnetic field $\vec{B} = B\hat{j}$ with B = 12 mT. The value of E required for an electron of energy 728 eV moving along the positive x-axis to pass undeflected is

(Given, mass of electron = 9.1 × 10^{-31} kg)

(A) 192 kVm^{-1} (B) 192 mVm^{-1}

(C) 9600 kVm^{-1} (D) 16 kVm^{-1}

Answer (A)

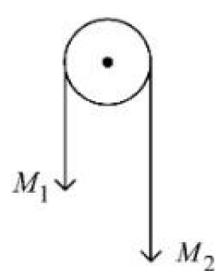
Sol. $v = \frac{E}{B}$ and $K = \frac{1}{2}mv^2$

$\Rightarrow \sqrt{\frac{2K}{m}} \times B = E$

$\Rightarrow E = \sqrt{\frac{2 \times 728 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}} \times 12 \times 10^{-3}$

= 192000 V/m

18. Two masses M_1 and M_2 are tied together at the two ends of a light inextensible string that passes over a frictionless pulley. When the mass M_2 is twice that of M_1, the acceleration of the system is a_1. When the mass M_2 is thrice that of M_1, the acceleration of the system is a_2. The ratio $\frac{a_1}{a_2}$ will be



(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $\frac{3}{2}$ (D) $\frac{1}{2}$

Answer (B)

Sol. $a_1 = \frac{M_2 - M_1}{M_2 + M_1} \times g = \frac{2M_1 - M_1}{3M_1} \times g$

$= \frac{g}{3}$

And, $a_2 = \frac{3M_1 - M_1}{4M_1} \times g = \frac{g}{2}$

$\therefore \frac{a_1}{a_2} = \frac{g/3}{g/2} = \frac{2}{3}$

19. Mass numbers of two nuclei are in the ratio of 4 : 3. Their nuclear densities will be in the ratio of

(A) 4 : 3

(B) $\left(\frac{3}{4}\right)^{\frac{1}{3}}$

(C) 1 : 1

(D) $\left(\frac{4}{3}\right)^{\frac{1}{3}}$

Answer (C)

Sol. $\therefore R = R_0 A^{\frac{1}{3}}$

$\Rightarrow \frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{\frac{1}{3}} = \left(\frac{4}{3}\right)^{\frac{1}{3}}$

$\therefore$ Density ratio, $\frac{\rho_1}{\rho_2} = \frac{A_1/V_1}{A_2/V_2}$

$= \left(\frac{A_1}{A_2}\right) \times \left(\frac{R_2}{R_1}\right)^3$

= 1 : 1

20. The area of cross section of the rope used to lift a load by a crane is 2.5 × 10^{-4} m^2. The maximum lifting capacity of the crane is 10 metric tons. To increase the lifting capacity of the crane to 25 metric tons, The required area of cross section of the rope should be

(take g = 10 ms^{-2})

(A) 6.25 × 10^{-4} m^2

(B) 10 × 10^{-4} m^2

(C) 1 × 10^{-4} m^2

(D) 1.67 × 10^{-4} m^2

Answer (A)

Sol. $\frac{W_1}{A_1} = \frac{W_2}{A_2}$

$\Rightarrow A_2 = \frac{W_2}{W_1} A_1$

$= \frac{25}{10} \times 2.5 \times 10^{-4}$

= 6.25 × 10^{-4} m^2

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. If $\vec{A} = \left(2\hat{i} + 3\hat{j} - \hat{k}\right)$ m and $\vec{B} = \left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$ m. The magnitude of component of vector $\vec{A}$ along vector $\vec{B}$ will be ______m

Answer (2)

Sol. $A\cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{B}|} = \frac{2+6-2}{3} = \frac{6}{3} = 2$

2. The radius of gyration of a cylindrical rod about an axis of rotation perpendicular to its length and passing through the center will be_________m.

Given the length of the rod is $10\sqrt{3}$ m.

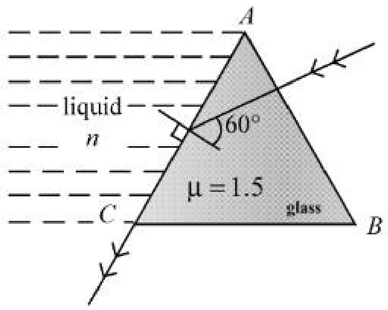
Answer (5)

Sol. $I = \frac{ML^2}{12} = MK^2$

$K = \frac{L}{\sqrt{12}} = \frac{10\sqrt{3}}{\sqrt{12}} = 5$ m

3. In the given figure, the face AC of the equilateral prism is immersed in a liquid of refractive index 'n'. For incident angle 60° at the side AC the refracted light beam just grazes along face AC. The refractive index of the liquid $n = \frac{\sqrt{x}}{4}$. The value of x is _______.

(Given refractive index of glass = 1.5)



Answer (27)

Sol. 1.5 sin60° = 4 sin90°

$$1.5\times\frac{\sqrt{3}}{2}=\frac{3\sqrt{3}}{4}=\frac{\sqrt{27}}{4}=\frac{\sqrt{x}}{4}$$

x = 27

4. Two lighter nuclei combine to from a comparatively heavier nucleus by the relation given blow:

$${}_1^2X+{}_1^2X={}_2^4Y$$

The binding energies per nucleon for ${}_1^2X$ and ${}_2^4Y$ are 1.1 MeV and 7.6 MeV respectively. The energy released in the process is ______ MeV

Answer (26)

Sol. Energy released = Change in B.E.

(7.6 × 4) – [4 × 1.1] = 26 MeV

5. A uniform heavy rod of mass 20 kg, cross sectional area 0.4 m^2 and length 20 m is hanging from a fixed support. Neglecting the lateral contraction, the elongation in the rod due to its own weight is $x \times 10^{-9}$ m. The value of x is _____

(Given Young's modulus $Y = 2 \times 10^{11}$ Nm^{-2} and $g = 10$ ms^{-2})

Answer (25)

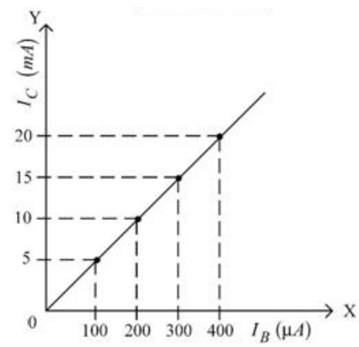
Sol. $$\frac{\frac{F}{A}}{\frac{\Delta L}{L}}=Y$$

$$\Delta L=\frac{FL}{AY}=\frac{T_{avg}L}{AY}=\frac{MgL}{2AY}$$

$$=\frac{20\times10\times20}{2\times0.4\times2\times10^{11}}=\frac{4\times10^3\times10^{-11}}{4\times0.4}$$

= 2.5 × 10^{-8} = 25 × 10^{-9}

6. The typical transfer characteristics of a transistor in CE configuration is shown in figure. A load resistor of 2 kΩ is connected in the collector branch of the circuit used. The input resistance of the transistor is 0.50 kΩ. The voltage gain of the transistor is ________.



Answer (200)

Sol. $V_{gain}=\text{Current gain}\times\frac{R_L}{R_i}$

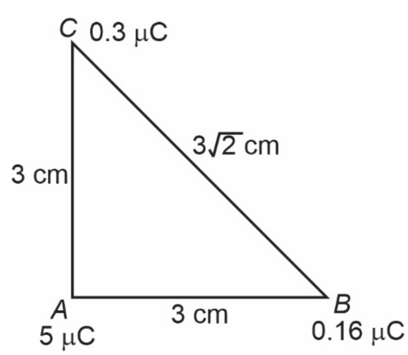
$$=\frac{\Delta I_C}{\Delta I_B}\times\frac{R_L}{R_i}$$

$$=\frac{5\times10^{-3}}{100\times10^{-6}}\times\frac{2\times10^3}{0.5\times10^3}=\frac{10}{0.5}\times10=200$$

7. Three point charges of magnitude 5 μC, 0.16 μC and 0.3 μC are located at the vertices *A*, *B*, *C* of a right angled triangle whose sides are *AB* = 3 cm, *BC* = $3\sqrt{2}$ cm and *CA* = 3 cm and point *A* is the right angle corner. Charge at point *A*, experiences ______N of electrostatic force due to the other two charges.

Answer (17)

Sol.



$$F_{AC}=\frac{k(5\times0.3)\times10^{-12}}{9\times10^{-4}}$$

$$F_{AB} = \frac{k(5\times0.16)\times10^{-12}}{9\times10^{-4}}$$

$$F_{net} = \frac{k\times10^{-12}}{9\times10^{-4}}\sqrt{1.5^2+(0.8)^2}$$

$$= \frac{10^9\times10^{-12}}{10^{-4}}\times1.7 = 17$$

8. In a coil of resistance 8 Ω, the magnetic flux due to an external magnetic field varies with time as $\phi = \frac{2}{3}\left(9-t^2\right)$. The value of total heat produced in the coil, till the flux becomes zero, will be _______ J.

Answer (2)

Sol. $R = 8\ \Omega$

$$\phi = \frac{2}{3}(9-t^2)$$

At $t = 3$, $\phi = 0$

$$\varepsilon = \left|-\frac{d\phi}{dt}\right| = \frac{4}{3}t$$

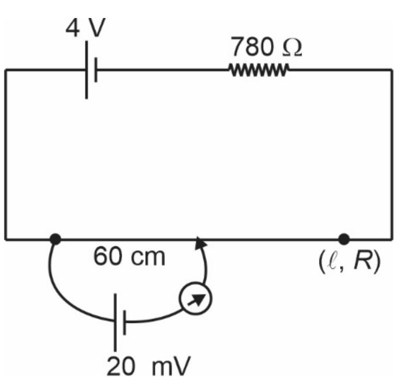
$$H = \int_0^3 \frac{V^2}{R}dt = \int_0^3 \frac{1}{8}\times\frac{16}{9}t^2dt$$

$$= \frac{2}{9}\times\left(\frac{t^3}{3}\right)_0^3 = \frac{2}{9\times3}\times27 = 2\text{ J}$$

9. A potentiometer wire of length 300 cm is connected in series with a resistance 780 Ω and a standard cell of emf 4 V. A constant current flows through potentiometer wire. The length of the null point for cell of emf 20 mV is found to be 60 cm. The resistance of the potentiometer wire is _____ Ω.

Answer (20)

Sol.



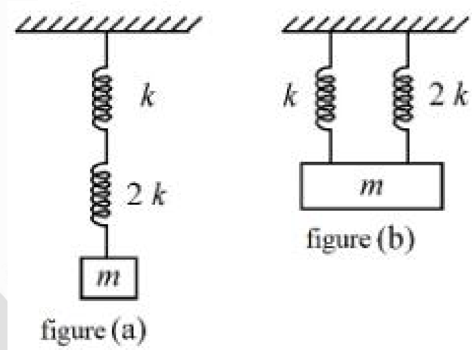
$\ell = 300$ cm

$\varepsilon = Kx$

$$20\times10^{-3} = \left(\frac{4\times R}{780+R}\times\frac{1}{300}\right)60$$

$$\boxed{R = 20}$$

10. As per given figures, two springs of spring constants k and $2k$ are connected to mass m. If the period of oscillation in figure (a) is 3 s, then the period of oscillation in figure (b) will be $\sqrt{x}$ s. The value of x is _______



Answer (2)

Sol. For case (a),

$$K_{eq} = \frac{2K}{3}$$

For case (b),

$$K_{eq} = 3K$$

$$\because T = 2\pi\sqrt{\frac{m}{K}}$$

$$\therefore\ \frac{T_a}{T_b} = \sqrt{\frac{K_b}{K_a}}$$

$$\frac{3}{T_b} = \sqrt{\frac{3K\times3}{2k}} = \frac{3}{\sqrt{2}}$$

$$T_b = \sqrt{2}$$

$x = 2$

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Haemoglobin contains 0.34% of iron by mass. The number of Fe atoms in 3.3 g of haemoglobin is

   (Given : Atomic mass of Fe is 56 u, $N_A = 6.022 \times 10^{23}$ mol^{-1})

   (A) 1.21×10^5 (B) 12.0×10^{16}

   (C) 1.21×10^{20} (D) 3.4×10^{22}

Answer (C)

Sol. According to the question,

100 g of haemoglobin contains 0.34 g of iron

3.3 g of haemoglobin contains $\frac{0.34}{100} \times 3.3$ g of iron

moles of Fe = $\frac{0.34 \times 3.3}{100 \times 56} = \frac{N}{N_A}$

$$N = \frac{0.34 \times 3.3 \times 6.022 \times 10^{23}}{100 \times 56}$$

$= 1.21 \times 10^{20}$

2. Arrange the following in increasing order of their covalent character.

   A. CaF_2

   B. $CaCl_2$

   C. $CaBr_2$

   D. CaI_2

   Choose the correct answer from the option given below.

   (A) B < A < C < D (B) A < B < C < D

   (C) A < B < D < C (D) A < C < B < D

Answer (B)

Sol. From Fajan's rule, for a given metal ion, as the size of anion increases, polarizability of anion increases and hence covalent character of the given ionic compound increases.

Hence, the increasing order of covalent character is

$CaF_2 < CaCl_2 < CaBr_2 < CaI_2$

3. Class XII students were asked to prepare one litre of buffer solution of pH 8.26 by their Chemistry teacher. The amount of ammonium chloride to be dissolved by the student in 0.2 M ammonia solution to make one litre of the buffer is

   (Given : pK_b (NH_3) = 4.74, Molar mass of NH_3 = 17 g mol^{-1}, Molar mass of NH_4Cl = 53.5 g mol^{-1})

   (A) 53.5 g (B) 72.3 g

   (C) 107.0 g (D) 126.0 g

Answer (C)

Sol. For basic Buffer, $pOH = pK_b + \log\frac{[salt]}{[Base]}$

pOH = 14 – 8.26 = 5.74

$$5.74 = 4.74 + \log\frac{[NH_4Cl]}{0.2}$$

$[NH_4Cl] = 2$ M

Moles of $NH_4Cl = 2 \times 1 = 2$ moles

Weight of $NH_4Cl = 2 \times 53.5 = 107$ g

4. At 30ºC, the half life for the decomposition of AB_2 is 200 s and is independent of the initial concentration of AB_2. The time required for 80% of the AB_2 to decompose is

   (Given : log 2 = 0.30, log 3 = 0.48)

   (A) 200 s (B) 323 s

   (C) 467 s (D) 532 s

Answer (C)

Sol. Since, half life is independent of the initial concentration of AB_2. Hence, reaction is "First Order".

$$k = \frac{2.303 \log 2}{t_{1/2}}$$

$$\frac{2.303 \log 2}{t_{1/2}} = \frac{2.303}{t} \log \frac{100}{(100-80)}$$

$$\frac{2.303 \times 0.3}{200} = \frac{2.303}{t} \log 5$$

t = 467 s

5. Given below are two statements: one is labelled as **Assertion A** and other is labelled as **Reason R**.

Assertion A : Finest gold is red in colour, as the size of the particles increases, it appears purple then blue and finally gold.

Reason R : The colour of the colloidal solution depends on the wavelength of light scattered by the dispersed particles.

In the light of the above statements, choose the ***most appropriate*** answer from the options given below.

(A) Both **A** and **R** are true and R is the correct explanation of **A**

(B) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

(C) **A** is true but **R** is false

(D) **A** is false but **R** is true

Answer (A)

Sol. Finest gold sol is red in colour; as the size of particles increases, it appears purple, then blue and finally golden.

The colour of colloidal solution depends on the wavelength of light scattered by the dispersed particles. The wavelength of light further depends on size and nature of the particles.

Hence, Both A and R are true and R is the correct explanation of A

6. The metal that has very low melting point and its periodic position is closer to a metalloid is

(A) Al (B) Ga

(C) Se (D) In

Answer (B)

Sol Among the given elements, Gallium has the lowest melting point, Gallium is also close to a metalloid

7. The metal that is not extracted from its sulfide ore is

(A) Aluminium

(B) Iron

(C) Lead

(D) Zinc

Answer (A)

Sol Aluminium is not extracted from sulphide ore. It is usually extracted from bauxite ore, leaching of bauxite ore is done followed by electrolytic reduction.

8. The products obtained from a reaction of hydrogen peroxide and acidified potassium permanganate are

(A) Mn^{4+}, H_2O only

(B) Mn^{2+}, H_2O only

(C) Mn^{4+}, H_2O, O_2 only

(D) Mn^{2+}, H_2O, O_2 only

Answer (D)

Sol $2MnO_4^- + 6H^+ + 5H_2O_2 \longrightarrow 2Mn^{2+} + 8H_2O + 5O_2$

This reaction shows reducing action of H_2O_2 in acidic medium.

The products formed are Mn^{2+}, H_2O and O_2

9. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : LiF is sparingly soluble in water.

Reason R : The ionic radius of Li^+ ion is smallest among its group members, hence has least hydration enthalpy.

In the light of the above statements, choose the ***most appropriate*** answer from the options given below.

(A) Both **A** and **R** are true and **R** is the correct explanation of **A**

(B) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

(C) **A** is true but **R** is false.

(D) **A** is false but **R** is true.

Answer (C)

Sol LiF is sparingly soluble in water.

The low solubility of LiF in water is due to its high lattice enthalpy (Since Li^+ and F^- are small in size). Also, due to small size of Li^+, its hydration enthalpy is high.

Hence, Assertion is true but Reason is false

10. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A: Boric acid is a weak acid

Reason R: Boric acid is not able to release H^+ ion on its own. It receives OH^- ion from water and releases H^+ ion.

In the light of the above statements, choose the ***most appropriate*** answer from the options given below.

(A) Both **A** and **R** are correct and **R** is the correct explanation of **A**.

(B) Both **A** and **R** are correct but **R** is NOT the correct explanation of **A**

(C) **A** is correct but **R** is not correct

(D) **A** is not correct but **R** is correct

Answer (A)

Sol Boric acid is a weak acid

$$H_3BO_3 + H_2O \rightleftharpoons [B(OH)_4]^{\ominus} + H^{\oplus}$$

Boric acid is not able to release H^+ ion on its own. It receives OH^- ion from water and releases H^+ ion as shown in the above reaction.

Hence, Both A and R are correct and R is the correct explanation of A.

11. The metal complex that is diamagnetic is (Atomic number : Fe, 26; Cu, 29)

(A) $K_3[Cu(CN)_4]$ (B) $K_2[Cu(CN)_4]$

(C) $K_3[Fe(CN)_4]$ (D) $K_4[FeCl_6]$

Answer (A)

Sol. $\Rightarrow$ $K_3[Cu(CN)_4]$ is diamagnetic

Cu(I) $\Rightarrow$ d^{10} configuration $\Rightarrow$ No unpaired electrons.

$\Rightarrow$ $K_2[Cu(CN)_4]$, $K_3[Fe(CN)_4]$ and $K_4[FeCl_6]$ are paramagnetic in nature

12. Match **List I** with **List II**.

List I **Pollutant**	**List II** **Source**
A. Microorganisms	I. Strip mining
B. Plant nutrients	II. Domestic sewage
C. Toxic heavy metals	III. Chemical fertilizer
D. Sediment	IV. Chemical factory

Choose the correct answer from the options given below:

(A) A-II, B-III, C-IV, D-I

(B) A-II, B-I, C-IV, D-III

(C) A-I, B-IV, C-II, D-III

(D) A-I, B-IV, C-III, D-II

Answer (A)

Sol. **Pollutant** **Source**

Microorganisms → Domestic sewage

Plant nutrients → Chemical fertilizers

Toxic heavy metals → Chemical factory

Sediment → Strip mining

13. The correct decreasing order of priority of functional groups in naming an organic compound as per IUPAC system of nomenclature is

(A) $-COOH > -CONH_2 > -COCl > -CHO$

(B) $-SO_3H > -COCl > -CONH_2 > -CN$

(C) $-COOR > -COCl > -NH_2 > \rangle C=O$

(D) $-COOH > -COOR > -CONH_2 > -COCl$

Answer (B)

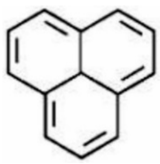
Sol. The order of decreasing priority for functional group is

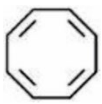
–COOH > –SO_3H > – COOR > –COCl > – $CONH_2$ > –CN > –CHO > >C = O > –NH_2

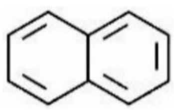
Hence correct order is

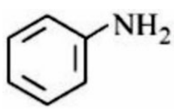
–SO_3H > –COCl > –$CONH_2$ > –CN

14. Which of the following is not an example of benzenoid compound?

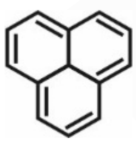
(A) 

(B) 

(C) 

(D) 

Answer (A) and (B)

Sol.  and  are not benzenoid compounds, since benzenoid compound contains benzene ring.

15. Hydrolysis of which compound will give carbolic acid?

(A) Cumene

(B) Benzenediazonium chloride

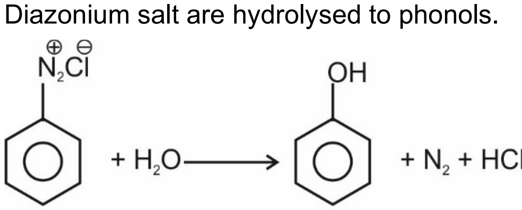
(C) Benzal chloride

(D) Ethylene glycol ketal

Answer (B)

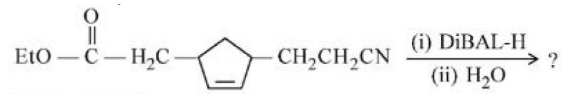
Sol. Phenol, is known as Carbolic acid.

Diazonium salt are hydrolysed to phonols.



(Carbolic acid)

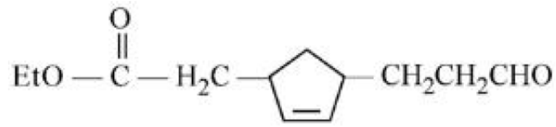
Benzal chloride on hydrolysis gives benzaldehyde

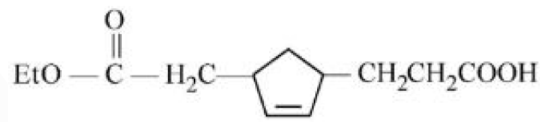
16. 

[Et is –C_2H_5]

Consider the above reaction and predict the major product.

(A) 

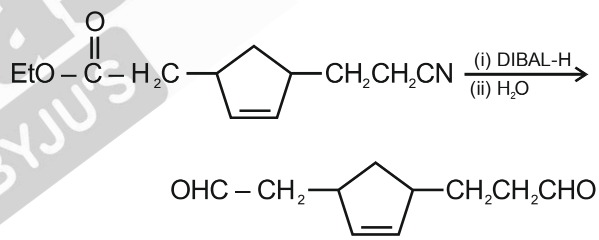
(B) 

(C) 

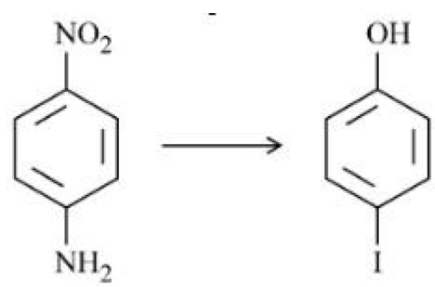
(D) 

Answer (A)

Sol. DIBAL-H reduces both the cyanides and esters to aldehydes.



17. The correct sequential order of the reagents for the given reaction is



(A) HNO_2, Fe/H^+, HNO_2, KI, H_2O/H^+

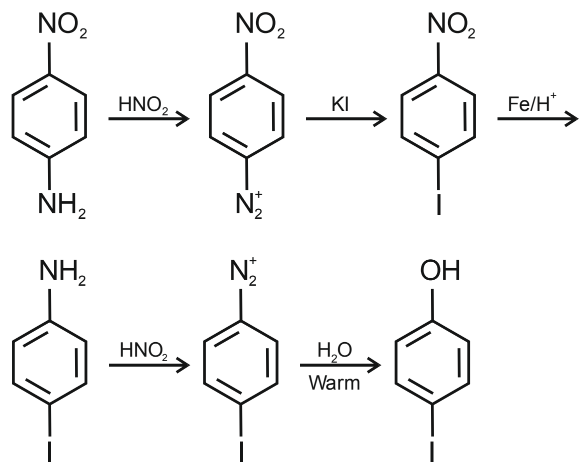
(B) HNO_2, KI, Fe/H^+, HNO_2, H_2O/warm

(C) HNO_2, KI, HNO_2, Fe/H^+, H_2O/H^+

(D) HNO_2, Fe/H^+, KI, HNO_2, H_2O/warm

Answer (B)

Sol.



18. Vulcanization of rubber is carried out by heating a mixture of

(A) isoprene and styrene

(B) neoprene and sulphur

(C) isoprene and sulphur

(D) neoprene and styrene

Answer (C)

Sol. When a mixture of isoprene and sulphur is heated, isoprene gets polymerised to natural rubber and then vulcanization of natural rubber with sulphur takes place.

19. Animal starch is the other name of

(A) amylose

(B) maltose

(C) glycogen

(D) amylopectin

Answer (C)

Sol. Animal starch is the other name of glycogen because its structure is similar to amylopectin.

20. Given below are two statements: One is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A: Phenolphthalein is a pH dependent indicator, remains colourless in acidic solution and gives pink colour in basic medium.

Reason R: Phenolphthalein is a weak acid. It doesn't dissociate in basic medium. In the light of the above statements, choose the ***most appropriate*** answer from the options given below.

(A) Both **A** and **R** are true and **R** is the correct explanation of **A**

(B) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

(C) **A** is true but **R** is false

(D) **A** is false but **R** is true

Answer (C)

Sol. Phenolphthalein is a pH dependent indicator. It is a weak acid which is colourless in the acidic solution but gives pink colour in basic medium. The pink colour is due to its conjugate form. Therefore, assertion (A) is true but Reason (R) is false.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. A 10 g mixture of hydrogen and helium is contained in a vessel of capacity 0.0125 m^3 at 6 bar and 27°C. The mass of helium in the mixture is _______ g. (nearest integer)

Given : $R = 8.3\ J\ K^{-1}\ mol^{-1}$

(Atomic masses of H and He are 1 u and 4 u, respectively)

Answer (8)

Sol. Number of moles of mixture of H_2 and He

$$= \frac{PV}{RT}$$

$$= \frac{6\times10^5\times0.0125}{8.3\times300} = 3$$

Let the mass of He in 10 g mixture be x g

$$\therefore \quad \frac{x}{4}+\frac{10-x}{2}=3$$

On solving x = 8 g

$\therefore$ Mass of He in the mixture = 8 g

2. Consider an imaginary ion $^{48}_{22}X^{3-}$. The nucleus contains 'a'% more neutrons than the number of electrons in the ion. The value of 'a' is ________. [nearest integer]

Answer (4)

Sol. Number of electrons in $^{48}_{22}X^{3-}$ is 25.

Number of neutrons = 48 – 22 = 26.

% increase in the number of neutrons over electrons

$$= \left(\frac{26-25}{25}\right)100 = 4\%$$

$\therefore$ a = 4

3. For the reaction

$H_2F_2(g) \rightarrow H_2(g) + F_2(g)$

ΔU = –59.6 kJ mol^{-1} at 27°C.

The enthalpy change for the above reaction is (–) _______ kJ mol^{-1} [nearest integer]

Given : R = 8.314 J K^{-1} mol^{-1}.

Answer (57)

Sol. $H_2F_2(g) \longrightarrow H_2(g) + F_2(g)$

ΔU = –59.6 kJ mol^{-1} at 27°C

$\Delta H = \Delta U + \Delta n_g RT$

$$= -59.6 + \frac{1\times8.314\times300}{1000}$$

= –57.10 kJ mol^{-1}

4. The elevation in boiling point for 1 molal solution of non-volatile solute A is 3 K. The depression in freezing point for 2 molal solution of A in the same solvent is 6 K. The ratio of K_b and K_f i.e., K_b/K_f is 1 : X. The value of X is [nearest integer]

Answer (1)

Sol. Molality of a solution of non volatile solute (A) = 1

Elevation in boiling point is given by

$\Delta T_b = K_b\, m$

$3 = K_b \times 1$ ... (1)

Molality of (A) in the same solvent = 2

Depression in freezing point is given by

$\Delta T_f = K_f\, m$

$6 = K_f \times 2$ ... (2)

Dividing (1) by (2)

$$\frac{K_b}{K_f} = \frac{1}{X} = \frac{1}{1}$$

$\therefore$ X = 1

5. 20 mL of 0.02 M hypo solution is used for the titration of 10 mL of copper sulphate solution, in the presence of excess of KI using starch as an indicator. The molarity of Cu^{2+} is found to be ________ × 10^{-2} M. [nearest integer]

Given : $2\,Cu^{2+} + 4\,I^- \rightarrow Cu_2I_2 + I_2$

$I_2 + 2S_2O_3^{2-} \rightarrow 2I^- + S_4O_6^{2-}$

Answer (4)

Sol. $2Cu^{2+} + 4I^- \rightarrow Cu_2I_2 + I_2$

$I_2 + S_2O_3^{2-} \rightarrow 2I^- + S_4O_6^{2-}$

Milliequivalents of hypo solution = 0.02 × 20 = 0.4

Milliequivalents of Cu^{2+} in 10 mL solution =

Milliequivalents of I_2 = Milliequivalents of hypo

= 0.4

Millimoles of Cu^{2+} ions in 10 mL = 0.4

Molarity of Cu^{2+} ions = $\frac{0.4}{10} = 0.04$ M

$= 4 \times 10^{-2}$ M

6. The number of non-ionisable protons present in the product B obtained from the following reactions is_____.

$$C_2H_5OH + PCl_3 \rightarrow C_2H_5Cl + A$$
$$A + PCl_3 \rightarrow B$$

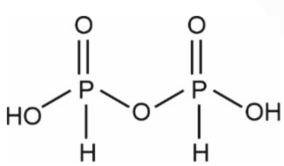
Answer (02.00)

Sol. $PCl_3 + C_2H_5OH \rightarrow C_2H_5Cl + H_3PO_3$ (A)

$H_3PO_3 + PCl_3 \rightarrow H_4P_2O_5$

(A) (B)

Structure of $H_4P_2O_5$



Total 2 non-ionizable protons are present

7. The spin-only magnetic moment value of the compound with strongest oxidizing ability among MnF_4, MnF_3 and MnF_2 is____ B.M. [nearest integer]

Answer (05.00)

Sol. MnF_3 has the strongest oxidising ability

$$\left[\begin{array}{l} E^{\circ}_{Mn^{+3}/Mn^{+2}} \simeq 1.57\text{ V} \\ \& \ E^{\circ}_{Mn^{+4}/Mn^{+2}} \simeq 1.2\text{ V} \end{array}\right]$$

So, spin only magnetic moment

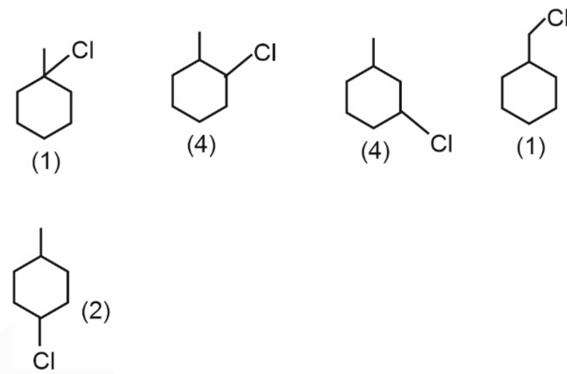
$= \sqrt{4(4+2)} = \sqrt{24}$ B.M.

$\simeq 5$

8. Total number of isomers (including stereoisomers) obtained on monochlorination of methylcyclohexane is____.

Answer (12.00)

Sol. Compounds formed on mono-chlorination of methylcyclohexane are :



$\therefore$ Total mono-chlorinated products formed = 12

9. A 100 mL solution of CH_3CH_2MgBr on treatment with methanol produces 2.24 mL of a gas at STP. The weight of gas produced is_____ mg. [nearest integer]

Answer (03.00)

Sol. $CH_3 - CH_2 - MgBr + CH_3OH \rightarrow CH_3 - CH_3 + MgBr(OCH_3)$

As 2.24 ml is formed at STP.

Number of moles of ethane gas produced

$= \frac{2.24X}{22.4}$

$= 10^{-4}$ ml

Mass of ethane produced = $10^{-4} \times 30 = 3 \times 10^{-} = 3$ mg

10. How many of the following drugs is/are examples(s) of broad-spectrum antibiotics?

Ofloxacin, Penicillin G, Terpineol, Salvarsan.

Answer (01.00)

Sol. Ofloxacin is the only broad spectrum antibiotic given in the question

Penicillin – G is a narrow spectrum antibiotic.

Salvarsan is mainly active against spirochete, a bacteria that causes syphilis

Terpineol is an antiseptic.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The minimum value of the sum of the squares of the roots of $x^2 + (3 - a)x + 1 = 2a$ is

(A) 4 (B) 5

(C) 6 (D) 8

Answer (C)

Sol. $x^2 + (3-a)x + 1 = 2a$ → α, β

$\alpha + \beta = a - 3,\ \alpha\beta = 1 - 2a$

$\Rightarrow\ \alpha^2 + \beta^2 = (a - 3)^2 - 2(1 - 2a)$

$= a^2 - 6a + 9 - 2 + 4a$

$= a^2 - 2a + 7$

$= (a - 1)^2 + 6$

So, $\alpha^2 + \beta^2 \geq 6$

2. If $z = x + iy$ satisfies $|z| - 2 = 0$ and $|z - i| - |z + 5i| = 0$, then

(A) $x + 2y - 4 = 0$ (B) $x^2 + y - 4 = 0$

(C) $x + 2y + 4 = 0$ (D) $x^2 - y + 3 = 0$

Answer (C)

Sol. $|z - i| = |z + 5i|$

So, z lies on $\perp^r$ bisector of (0, 1) and (0, –5)

i.e., line $y = -2$

as $|z| = 2$

$\Rightarrow\ z = -2i$

$x = 0$ and $y = -2$

so, $x + 2y + 4 = 0$

3. Let $A = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $B = \begin{bmatrix} 9^2 & -10^2 & 11^2 \\ 12^2 & 13^2 & -14^2 \\ -15^2 & 16^2 & 17^2 \end{bmatrix}$, then the value of $A'BA$ is

(A) 1224 (B) 1042

(C) 540 (D) 539

Answer (D)

Sol. $A'BA = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 9^2 & -10^2 & 11^2 \\ 12^2 & 13^2 & -14^2 \\ -15^2 & 16^2 & 17^2 \end{bmatrix} A$

$= \begin{bmatrix} 9^2 + 12^2 - 15^2 & -10^2 + 13^2 + 16^2 & 11^2 - 14^2 + 17^2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 9^2 + 12^2 - 15^2 - 10^2 + 13^2 + 16^2 + 11^2 - 14^2 + 17^2 \end{bmatrix}$

$= [(9^2 - 10^2) + (11^2 + 12^2) + (13^2 - 14^2) + (16^2 - 15^2) + 17^2]$

$= [-19 + 265 + (-27) + 31 + 289]$

$= [585 - 46] = [539]$

4. $\sum_{\substack{i,j=0 \\ i \neq j}}^{n} {}^nC_i\ {}^nC_j$ is equal to

(A) $2^{2n} - {}^{2n}C_n$ (B) $2^{2n-1} - {}^{2n-1}C_{n-1}$

(C) $2^{2n} - \frac{1}{2}\ {}^{2n}C_n$ (D) $2^{n-1} + {}^{2n-1}C_n$

Answer (A*)

Sol. $\sum_{\substack{i,j=0 \\ i \neq j}}^{n} {}^nC_i\ {}^nC_j = \sum_{i,j=0}^{n} {}^nC_i\ {}^nC_j - \sum_{i=j}^{n} {}^nC_i\ {}^nC_j$

$= \sum_{j=0}^{n} {}^nC_i \sum_{j=0}^{n} {}^nC_j - \sum_{i=0}^{n} {}^nC_i\ C_i$

$= 2^n \cdot 2^n - {}^{2n}C_n$

$= 2^{2n} - {}^{2n}C_n$

5. Let P and Q be any points on the curves $(x - 1)^2 + (y + 1)^2 = 1$ and $y = x^2$, respectively. The distance between P and Q is minimum for some value of the abscissa of P in the interval

(A) $\left(0, \frac{1}{4}\right)$ (B) $\left(\frac{1}{2}, \frac{3}{4}\right)$

(C) $\left(\frac{1}{4}, \frac{1}{2}\right)$ (D) $\left(\frac{3}{4}, 1\right)$

Answer (C)

Sol. $y = mx + 2a + \frac{1}{m^2}$ (Equation of normal to $x^2 = 4ay$ in slope form) through (1, –1).

$4m^3 + 6m^2 + 1 = 0$

$\Rightarrow\ m \simeq -1.6$

Slope of normal $\simeq \frac{-8}{5} = \tan\theta$

$\Rightarrow \cos\theta \simeq \frac{-5}{\sqrt{89}}, \sin\theta \simeq \frac{8}{\sqrt{89}}$

$x_p = 1 + \cos\theta \simeq 1 - \frac{5}{\sqrt{89}} \in \left(\frac{1}{4}, \frac{1}{2}\right)$

6. If the maximum value of a, for which the function $f_a(x) = \tan^{-1}2x - 3ax + 7$ is non-decreasing in $\left(-\frac{\pi}{6}, \frac{\pi}{6}\right)$, is $\bar{a}$, then $f_{\bar{a}}\left(\frac{\pi}{8}\right)$ is equal to

(A) $8 - \frac{9\pi}{4(9+\pi^2)}$

(B) $8 - \frac{4\pi}{9(4+\pi^2)}$

(C) $8\left(\frac{1+\pi^2}{9+\pi^2}\right)$

(D) $8 - \frac{\pi}{4}$

Answer (*)

Sol. $f_a(x) = \tan^{-1}2x - 3ax + 7$

$\because$ $f_a(x)$ is non-decreasing in $\left(-\frac{\pi}{6}, \frac{\pi}{6}\right)$

$\therefore$ $f_a'(x) \ge 0 \quad \Rightarrow \quad \frac{2}{1+4x^2} - 3a \ge 0$

$\Rightarrow \quad 3a \le \frac{2}{1+4x^2}$

So, $a_{max} = \frac{2}{3}\left(\frac{1}{1+4\times\frac{\pi^2}{36}}\right) = \frac{6}{9+\pi^2} = \bar{a}$

$\therefore$ $f_{\bar{a}}\left(\frac{\pi}{8}\right) = \tan^{-1}\frac{\pi}{4} - 3\frac{\pi}{8}\cdot\frac{6}{9+\pi^2} + 7$

7. Let $\beta = \lim_{x\to 0}\frac{\alpha x - (e^{3x}-1)}{\alpha x(e^{3x}-1)}$ for some $\alpha \in \mathbb{R}$. Then the value of $\alpha + \beta$ is

(A) $\frac{14}{5}$

(B) $\frac{3}{2}$

(C) $\frac{5}{2}$

(D) $\frac{7}{2}$

Answer (C)

Sol. $\beta = \lim_{x\to 0}\frac{\alpha x - (e^{3x}-1)}{\alpha x(e^{3x}-1)}, \ \alpha \in \mathbb{R}$

$= \lim_{x\to 0}\frac{\frac{\alpha}{3} - \left(\frac{e^{3x}-1}{3x}\right)}{\alpha x\left(\frac{e^{3x-1}}{3x}\right)}$

So, α = 3 (to make indeterminant form)

$\beta = \lim_{x\to 0}\frac{1 - \left(\frac{e^{3x}-1}{3x}\right)}{3x} = \frac{1 - \frac{\left(3x + \frac{9x^2}{2} + ...\right)}{3x}}{3x}$

$= \frac{-\left(\frac{9}{2}x^2 + \frac{(3x)^3}{31} + ...\right)}{9x^2} = \frac{-1}{2}$

$\therefore \quad \alpha + \beta = 3 - \frac{1}{2} = \frac{5}{2}$

8. The value of $\log_e 2\frac{d}{dx}(\log_{\cos x}\text{cosec}x)$ at $x = \frac{\pi}{4}$ is

(A) $-2\sqrt{2}$

(B) $2\sqrt{2}$

(C) –4

(D) 4

Answer (D)

Sol. Let $f(x) = \log_{\cos x}\text{cosec}x$

$= \frac{\log \text{cosec}x}{\log \cos x}$

$f'(x) = \frac{\log\cos x\cdot\sin x\cdot\left(-\text{cosec}x\cot x - \log\text{cosec}x\cdot\frac{1}{\cos x}\cdot - \sin x\right)}{(\log\cos x)^2}$

at $x = \frac{\pi}{4}$

$f'\left(\frac{\pi}{4}\right) = \frac{-\log\left(\frac{1}{\sqrt{2}}\right) + \log\sqrt{2}}{\left(\log\frac{1}{\sqrt{2}}\right)^2} = \frac{2}{\log\sqrt{2}}$

$\therefore \quad \log_e 2f'(x)$ at $x = \frac{\pi}{4} = 4$

9. $\int_0^{20\pi}\left(|\sin x| + |\cos x|\right)^2 dx$ is equal to

(A) $10(\pi+4)$

(B) $10(\pi+2)$

(C) $20(\pi-2)$

(D) $20(\pi+2)$

Answer (D)

Sol. $I = \int_0^{20\pi} \left(|\sin x| + |\cos x|\right)^2 dx$

$= 20\int_0^{\pi} \left(1 + |\sin 2x|\right) dx$

$= 40\int_0^{\frac{\pi}{2}} \left(1 + \sin 2x\right) dx$

$= 40\left(x - \frac{\cos 2x}{2}\right)\Big|_0^{\frac{\pi}{2}}$

$= 40\left(\frac{\pi}{2} + \frac{1}{2} + \frac{1}{2}\right) = 20(\pi + 2)$

10. Let the solution curve $y = f(x)$ of the differential equation $\frac{dy}{dx} + \frac{xy}{x^2 - 1} = \frac{x^4 + 2x}{\sqrt{1-x^2}}, x \in (-1, 1)$ pass through the origin. Then $\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x)dx$ is

(A) $\frac{\pi}{3} - \frac{1}{4}$ (B) $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$

(C) $\frac{\pi}{6} - \frac{\sqrt{3}}{4}$ (D) $\frac{\pi}{6} - \frac{\sqrt{3}}{2}$

Answer (B)

Sol. $\frac{dy}{dx} + \frac{xy}{x^2 - 1} = \frac{x^4 + 2x}{\sqrt{1-x^2}}$

which is first order linear differential equation.

Integrating factor (I.F.) $= e^{\int \frac{x}{x^2-1}dx}$

$= e^{\frac{1}{2}\ln|x^2-1|} = \sqrt{|x^2 - 1|}$

$= \sqrt{1-x^2} \quad \because x \in (-1, 1)$

Solution of differential equation

$y\sqrt{1-x^2} = \int \left(x^4 + 2x\right)dx = \frac{x^5}{5} + x^2 + c$

Curve is passing through origin, $c = 0$

$y = \frac{x^5 + 5x^2}{5\sqrt{1-x^2}}$

$\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} \frac{x^5 + 5x^2}{5\sqrt{1-x^2}}dx = 0 + 2\int_0^{\frac{\sqrt{3}}{2}} \frac{x^2}{\sqrt{1-x^2}}dx$

put $x = \sin\theta$

$dx = \cos\theta\, d\theta$

$I = 2\int_0^{\frac{\pi}{3}} \frac{\sin^2\theta \cdot \cos\theta d\theta}{\cos\theta}$

$= \int_0^{\frac{\pi}{3}} (1 - \cos 2\theta)d\theta$

$= \left(\theta - \frac{\sin 2\theta}{2}\right)\Big|_0^{\frac{\pi}{3}}$

$= \frac{\pi}{3} - \frac{\sqrt{3}}{4}$

11. The acute angle between the pair of tangents drawn to the ellipse $2x^2 + 3y^2 = 5$ from the point (1, 3) is

(A) $\tan^{-1}\left(\frac{16}{7\sqrt{5}}\right)$ (B) $\tan^{-1}\left(\frac{24}{7\sqrt{5}}\right)$

(C) $\tan^{-1}\left(\frac{32}{7\sqrt{5}}\right)$ (D) $\tan^{-1}\left(\frac{3 + 8\sqrt{5}}{35}\right)$

Answer (B)

Sol. $2x^2 + 3y^2 = 5$

Equation of tangent having slope m.

$y = mx \pm \sqrt{\frac{5}{2}m^2 + \frac{5}{3}}$

which passes through (1, 3)

$3 = m \pm \sqrt{\frac{5}{2}m^2 + \frac{5}{3}}$

$\frac{5}{2}m^2 + \frac{5}{3} = 9 + m^2 - 6m$

$\frac{3}{2}m^2 + 6m - \frac{22}{3} = 0$

$9m^2 + 36m - 44 = 0$

$m_1 + m_2 = -4, m_1 m_2 = -\frac{44}{9}$

$(m_1 - m_2)^2 = 16 + 4 \times \frac{44}{9} = \frac{320}{9}$

Acute angle between the tangents is given by

$$\alpha = \tan^{-1}\left|\frac{m_1 - m_2}{1+m_1m_2}\right|$$

$$= \tan^{-1}\left|\frac{\frac{8\sqrt{5}}{3}}{1-\frac{44}{9}}\right|$$

$$= \tan^{-1}\left(\frac{24\sqrt{5}}{35}\right)$$

$$\alpha = \tan^{-1}\left(\frac{24}{7\sqrt{5}}\right)$$

12. The equation of a common tangent to the parabolas $y = x^2$ and $y = -(x-2)^2$ is

(A) $y = 4(x-2)$ (B) $y = 4(x-1)$

(C) $y = 4(x+1)$ (D) $y = 4(x+2)$

Answer (B)

Sol. Equation of tangent of slope m to $y = x^2$

$$y = mx - \frac{1}{4}m^2$$

Equation of tangent of slope m to $y = -(x-2)^2$

$$y = m(x-2) + \frac{1}{4}m^2$$

If both equation represent the same line

$$\frac{1}{4}m^2 - 2m = -\frac{1}{4}m^2$$

$m = 0, 4$

So, equation of tangent

$y = 4x - 4$

13. Let the abscissae of the two points P and Q on a circle be the roots of $x^2 - 4x - 6 = 0$ and the ordinates of P and Q be the roots of $y^2 + 2y - 7 = 0$. If PQ is a diameter of the circle $x^2 + y^2 + 2ax + 2by + c = 0$, then the value of $(a + b - c)$ is

(A) 12 (B) 13

(C) 14 (D) 16

Answer (A)

Sol. Abscissae of PQ are roots of $x^2 - 4x - 6 = 0$

Ordinates of PQ are roots of $y^2 + 2y - 7 = 0$

and PQ is diameter

$\Rightarrow$ Equation of circle is

$$x^2 + y^2 - 4x + 2y - 13 = 0$$

But, given $x^2 + y^2 + 2ax + 2by + c = 0$

By comparison $a = -2$, $b = 1$, $c = -13$

$\Rightarrow$ $a + b - c = -2 + 1 + 13 = 12$

14. If the line $x - 1 = 0$ is a directrix of the hyperbola $kx^2 - y^2 = 6$, then the hyperbola passes through the point

(A) $(-2\sqrt{5}, 6)$ (B) $(-\sqrt{5}, 3)$

(C) $(\sqrt{5}, -2)$ (D) $(2\sqrt{5}, 3\sqrt{6})$

Answer (C)

Sol. Given hyperbola : $\frac{x^2}{6/k} - \frac{y^2}{6} = 1$

Eccentricity $= e = \sqrt{1+\frac{6}{6/k}} = \sqrt{1+k}$

Directrices : $x = \pm\frac{a}{e} \Rightarrow x = \pm\frac{\sqrt{6}}{\sqrt{k}\sqrt{k+1}}$

As given : $\frac{\sqrt{6}}{\sqrt{k}\sqrt{k+1}} = 1$

$\Rightarrow$ $k = 2$

Here hyperbola is $\frac{x^2}{3} - \frac{y^2}{6} = 1$

Checking the option gives $(\sqrt{5}, -2)$ satisfies it.

15. A vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i}, \hat{i}+\hat{j}$ and the plane determined by the vectors $\hat{i}-\hat{j}, \hat{i}+\hat{k}$. The obtuse angle between $\vec{a}$ and the vector $\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$ is

(A) $\frac{3\pi}{4}$ (B) $\frac{2\pi}{3}$

(C) $\frac{4\pi}{5}$ (D) $\frac{5\pi}{6}$

Answer (A)

Sol. If $\vec{n}_1$ is a vector normal to the plane determined by $\hat{i}$ and $\hat{i}+\hat{j}$ then

$$\vec{n}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{vmatrix} = \hat{k}$$

If $\vec{n}_2$ is a vector normal to the plane determined by $\hat{i}-\hat{j}$ and $\hat{i}+\hat{k}$ then

$$\vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = -\hat{i} - \hat{j} + \hat{k}$$

Vector $\vec{a}$ is parallel to $\vec{n}_1 \times \vec{n}_2$

i.e. $\vec{a}$ is parallel to $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ -1 & -1 & 1 \end{vmatrix} = \hat{i} - \hat{j}$

Given $\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$

Cosine of acute angle between

$\vec{a}$ and $\vec{b} = \left| \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|} \right| = \frac{1}{\sqrt{2}}$

Obtuse angle between $\vec{a}$ and $\vec{b} = \frac{3\pi}{4}$

16. If $0 < x < \frac{1}{\sqrt{2}}$ and $\frac{\sin^{-1} x}{\alpha} = \frac{\cos^{-1} x}{\beta}$, then a value of $\sin\left(\frac{2\pi\alpha}{\alpha+\beta}\right)$ is

(A) $4\sqrt{(1-x^2)}(1-2x^2)$ (B) $4x\sqrt{(1-x^2)}(1-2x^2)$

(C) $2x\sqrt{(1-x^2)}(1-4x^2)$ (D) $4\sqrt{(1-x^2)}(1-4x^2)$

Answer (B)

Sol. Let $\frac{\sin^{-1} x}{\alpha} = \frac{\cos^{-1} x}{\beta} = k \Rightarrow \sin^{-1} x + \cos^{-1} x = k(\alpha + \beta)$

$\Rightarrow \quad \alpha + \beta = \frac{\pi}{2k}$.

Now, $\frac{2\pi\,\alpha}{\alpha+\beta} = \frac{2\pi\,\alpha}{\frac{\pi}{2k}} = 4k\alpha = 4\sin^{-1} x$

Here $\sin\left(\frac{2\pi\,\alpha}{\alpha+\beta}\right) = \sin(4\sin^{-1} x)$

Let $\sin^{-1} x = \theta \quad \because \; x \in \left(0, \frac{1}{\sqrt{2}}\right) \Rightarrow \theta \in \left(0, \frac{\pi}{4}\right)$

$\Rightarrow \quad x = \sin\theta$

$\Rightarrow \quad \cos\theta = \sqrt{1-x^2}$

$\Rightarrow \quad \sin 2\theta = 2x \cdot \sqrt{1-x^2}$

$\Rightarrow \quad \cos 2\theta = \sqrt{1-4x^2(1-x^2)} = \sqrt{(2x^2-1)^2} = 1-2x^2$

$\left(\because \; \cos 2\theta > 0 \text{ as } 2\theta \in \left(0, \frac{\pi}{2}\right)\right)$

$\Rightarrow \quad \sin 4\theta = 2 \cdot 2x\sqrt{1-x^2}\,(1-2x^2)$

$= 4x\sqrt{1-x^2}\,(1-2x^2)$

17. Negation of the Boolean expression $p \Leftrightarrow (q \Rightarrow p)$ is

(A) $(\sim p) \wedge q$ (B) $p \wedge (\sim q)$

(C) $(\sim p) \vee (\sim q)$ (D) $(\sim p) \wedge (\sim q)$

Answer (D)

Sol. $p \Leftrightarrow (q \Rightarrow p)$

$\sim (p \Leftrightarrow (q \Leftrightarrow p))$

$\equiv p \Leftrightarrow \sim (q \Rightarrow p)$

$\equiv p \Leftrightarrow (q \wedge \sim p)$

$\equiv (p \Rightarrow (q \wedge \sim p)) \wedge ((q \wedge \sim p) \Rightarrow p))$

$\equiv (\sim p \vee (q \wedge \sim p)) \wedge ((\sim q \vee p) \vee p))$

$\equiv ((\sim p \vee q) \wedge \sim p) \wedge (\sim q \vee p)$

$\equiv \sim p \wedge (\sim q \vee p)$

$\equiv (\sim p \wedge \sim q) \vee (\sim p \wedge p)$

$\equiv (\sim p \wedge \sim q) \vee c$

$\equiv (\sim p \wedge \sim q)$

18. Let X be a binomially distributed random variable with mean 4 and variance $\frac{4}{3}$. Then, $54\,P(X \le 2)$ is equal to

(A) $\frac{73}{27}$ (B) $\frac{146}{27}$

(C) $\frac{146}{81}$ (D) $\frac{126}{81}$

Answer (B)

Sol. Mean = 4 = μ = np

Variance = $\sigma^2 = np(1-P) = \frac{4}{3}$

$4(1-P) = \frac{4}{3}$

$P = \frac{2}{3}$

$n \times \frac{2}{3} = 4$

$n = 6$

$P(X = k) = {}^nC_k\, P^k (1-P)^{n-k}$

$P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2)$

$= {}^6C_0\, P^0 (1-p)^6 + {}^6C_1\, P^1 (1-P)^5 + {}^6C_2\, P^2 (1-P)^4$

$= {}^6C_0 \left(\frac{1}{3}\right)^6 + {}^6C_1 \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^5 + {}^6C_2 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^4$

$= \left(\frac{1}{3}\right)^6 [1 + 12 + 60] = \frac{73}{3^6}$

$54P(X \le 2) = \frac{73}{3^6} \times 54 = \frac{146}{27}$

19. The integral $\int \frac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos x-\sin x)}{\left(1+\frac{2}{\sqrt{3}}\sin 2x\right)}dx$ is equal to

(A) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}\right|+C$

(B) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{x}{6}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{3}\right)}\right|+C$

(C) $\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}\right|+C$

(D) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}-\frac{\pi}{6}\right)}\right|+C$

Answer (A)

Sol. $=\int \frac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos x-\sin x)}{\left(1+\frac{2}{\sqrt{3}}\sin 2x\right)}dx$

$$=\int \frac{\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right)\sqrt{2}\sin\left(\frac{\pi}{4}-x\right)}{\left(\frac{2}{\sqrt{3}}\right)\left(\sin\frac{\pi}{3}+\sin 2x\right)}dx$$

$$=\int \frac{\frac{(\sqrt{3}-1)}{\sqrt{2}}\sin\left(\frac{\pi}{4}-x\right)}{\left(\sin\frac{\pi}{3}+\sin 2x\right)}dx$$

$$=\int \frac{\frac{\sqrt{3}-1}{2\sqrt{2}}\sin\left(\frac{\pi}{4}-x\right)}{\sin\left(\frac{\pi}{6}+x\right)\cos\left(\frac{\pi}{6}-x\right)}dx$$

$$=\frac{1}{2}\int \frac{2\sin\frac{\pi}{12}\sin\left(\frac{\pi}{4}-x\right)}{\sin\left(\frac{\pi}{6}+x\right)\cos\left(\frac{\pi}{6}-x\right)}dx$$

$$=\frac{1}{2}\int \frac{\cos\left(\frac{\pi}{6}-x\right)-\cos\left(\frac{\pi}{3}-x\right)}{\sin\left(\frac{\pi}{6}+x\right)\cos\left(\frac{\pi}{6}-x\right)}dx$$

$$=\frac{1}{2}\left[\int \operatorname{cosec}\left(\frac{\pi}{6}+x\right)dx-\int \sec\left(\frac{\pi}{6}-x\right)dx\right]$$

$$=\frac{1}{2}\left[\ln\left|\tan\left(\frac{\pi}{12}+\frac{x}{2}\right)\right|-\int \operatorname{cosec}\left(\frac{\pi}{3}-x\right)dx\right]$$

$$=\frac{1}{2}\left[\ln\left|\tan\left(\frac{\pi}{12}+\frac{x}{2}\right)\right|-\ln\left|\frac{\pi}{6}+\frac{x}{2}\right|\right]+C$$

$$=\frac{1}{2}\ln\left|\frac{\tan\left(\frac{\pi}{12}+\frac{x}{2}\right)}{\tan\left(\frac{\pi}{6}+\frac{x}{2}\right)}\right|+C$$

20. The area bounded by the curves $y = |x^2 - 1|$ and $y = 1$ is

(A) $\frac{2}{3}\left(\sqrt{2}+1\right)$

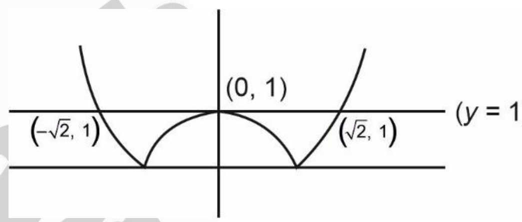
(B) $\frac{4}{3}\left(\sqrt{2}-1\right)$

(C) $2\left(\sqrt{2}-1\right)$

(D) $\frac{8}{3}\left(\sqrt{2}-1\right)$

Answer (D)

Sol.



$$\text{Area} = 2\int_0^{\sqrt{2}}(1-|x^2-1|)\,dx$$

$$2\left[\int_0^1\left(1-(1-x^2)\right)dx+\int_1^{\sqrt{2}}\left(2-x^2\right)dx\right]$$

$$=2\left[\left[\frac{x^3}{3}\right]_0^1+\left[2x-\frac{x^3}{3}\right]_1^{\sqrt{2}}\right]$$

$$=2\left(\frac{4\sqrt{2}-4}{3}\right)=\frac{8}{3}\left(\sqrt{2}-1\right)$$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{3, 6, 7, 9\}$. Then the number of elements in the set $\{C \subseteq A : C \cap B \neq \phi\}$ is ______________.

Answer (112)

Sol. As $C \cap B \neq \phi$, c must be not be formed by {1, 2, 4, 5}

$\therefore$ Number of subsets of $A = 2^7 = 128$

and number of subsets formed by {1, 2, 4, 5} = 16

$\therefore$ Required no. of subsets $= 2^7 - 2^4 = 128 - 16$

$= 112$

2. The largest value of a, for which the perpendicular distance of the plane containing the lines

$\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + a\hat{j} - \hat{k})$ and $\vec{r} = (\hat{i} + \hat{j}) + \mu(-\hat{i} + \hat{j} - a\hat{k})$

from the point (2, 1, 4) is $\sqrt{3}$, is ______________.

Answer (2*)

Sol. Normal to plane $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & a & -1 \\ -1 & 1 & -a \end{vmatrix}$

$= \hat{i}(1-a^2) - \hat{j}(-a-1) + \hat{k}(1+a)$

$= (1-a)\hat{i} + \hat{j} + \hat{k}$

$\therefore$ Plane $(1 - a)(x - 1) + (y - 1) + z = 0$

Distance from (2, 1, 4) is $\sqrt{3}$ i.e.

$\Rightarrow \left|\dfrac{(1-a)+0+4}{\sqrt{(1-a)^2+1+1}}\right| = \sqrt{3}$

$\Rightarrow 25 + a^2 - 10a = 3a^2 - 6a + 9$

$\Rightarrow 2a^2 + 4a - 16 = 0$

$\Rightarrow a^2 + 2a - 8 = 0$

$a = 2$ or -4

$\therefore a_{max} = 2$

3. Numbers are to be formed between 1000 and 3000, which are divisible by 4, using the digits 1, 2, 3, 4, 5 and 6 without repetition of digits. Then the total number of such numbers is ______________.

Answer (30)

Sol. Number must start by 1 or 2 and for divisibility by 4 last two digits shall be divisible by 4

$\therefore$ _ _ $\underline{1}\ \underline{2} \rightarrow$ 0 cases

$\underline{2}\ \underset{3}{\uparrow}\ \underline{1}\ \underline{6} \rightarrow$ 3 cases

$\underline{1}\ \underset{3}{\uparrow}\ \underline{2}\ \underline{4} \rightarrow$ 3 cases

$\underline{1}\ \underset{3}{\uparrow}\ \underline{3}\ \underline{2} \rightarrow$ 3 cases

$\underline{2}\ \underset{3}{\uparrow}\ \underline{3}\ \underline{6} \rightarrow$ 6 cases

$\underline{1}\ \underset{3}{\uparrow}\ \underline{5}\ \underline{2} \rightarrow$ 3 cases

$\underline{2}\ \underset{3}{\uparrow}\ \underline{5}\ \underline{6} \rightarrow$ 6 cases

$\underline{2}\ \underset{3}{\uparrow}\ \underline{6}\ \underline{4} \rightarrow$ 6 cases

$\Rightarrow$ Total 30 numbers

4. If $\displaystyle\sum_{k=1}^{10} \frac{k}{k^4+k^2+1} = \frac{m}{n}$, where m and n are co-prime, then $m + n$ is equal to

Answer (166)

Sol. $\displaystyle\sum_{k=1}^{10} \frac{k}{k^4+k^2+1}$

$= \dfrac{1}{2}\left[\displaystyle\sum_{k=1}^{10}\left(\frac{1}{k^2-k+1} - \frac{1}{k^2+k+1}\right)\right]$

$= \dfrac{1}{2}\left[1 - \dfrac{1}{3} + \dfrac{1}{3} - \dfrac{1}{7} + \dfrac{1}{7} - \dfrac{1}{13} + ... + \dfrac{1}{91} - \dfrac{1}{111}\right]$

$= \dfrac{1}{2}\left[1 - \dfrac{1}{111}\right] = \dfrac{110}{2.111} = \dfrac{55}{111} = \dfrac{m}{n}$

$\therefore m + n = 55 + 111 = 166$

5. If the sum of solutions of the system of equations $2\sin^2\theta - \cos 2\theta = 0$ and $2\cos^2\theta + 3\sin\theta = 0$ in the interval $[0, 2\pi]$ is $k\pi$, then k is equal to ________.

Answer (3)

Sol. Equation (1) $\quad 2\sin^2\theta = 1 - 2\sin^2\theta$

$\Rightarrow \sin^2\theta = \dfrac{1}{4}$

$\Rightarrow \sin\theta = \pm\dfrac{1}{2}$

$\Rightarrow \theta = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

Equation (2) $\quad 2\cos^2\theta + 3\sin\theta = 0$

$\Rightarrow 2\sin^2\theta - 3\sin\theta - 2 = 0$

$\Rightarrow$ $2\sin^2\theta - 4\sin\theta + \sin\theta - 2 = 0$

$\Rightarrow$ $(\sin\theta - 2)(2\sin\theta + 1) = 0$

$\Rightarrow \sin\theta = \frac{-1}{2} \qquad \Rightarrow \theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\therefore$ Common solutions = $\frac{7\pi}{6}; \frac{11\pi}{6}$

Sum of solutions = $\frac{7\pi + 11\pi}{6} = \frac{18\pi}{6} = 3\pi$

$\therefore$ $k = 3$

6. The mean and standard deviation of 40 observations are 30 and 5 respectively. It was noticed that two of these observations 12 and 10 were wrongly recorded. If σ is the standard deviation of the data after omitting the two wrong observations from the data, then $38\sigma^2$ is equal to ______.

Answer (238)

Sol. $\mu = \frac{\sum x_i}{40} = 30 \quad \Rightarrow \sum x_i = 1200$

$\sigma^2 = \frac{\sum x_i^2}{40} - (30)^2 = 25 \Rightarrow \sum x_i^2 = 37000$

After omitting two wrong observations

$\sum y_i = 1200 - 12 - 10 = 1178$

$\sum y_i^2 = 37000 - 144 - 100 = 36756$

Now $\sigma^2 = \frac{\sum y_i^2}{38} - \left(\frac{\sum y_i}{38}\right)^2$

$= \frac{36756}{38} - \left(\frac{1178}{38}\right)^2 = -31^2$

$38\sigma^2 = 36756 - 36518 = 238$

7. The plane passing through the line $L : lx - y + 3(1 - l)z = 1, x + 2y - z = 2$ and perpendicular to the plane $3x + 2y + z = 6$ is $3x - 8y + 7z = 4$. If θ is the acute angle between the line L and the y-axis, then $415\cos^2\theta$ is equal to ________.

Answer (125)

Sol. $L : lx - y + 3(1 - l)z = 1, x + 2y - z = 2$

and plane containing the line $p : 3x - 8y + 7z = 4$

Let $\vec{n}$ be the vector parallel to L.

then $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ l & -1 & 3(1-l) \\ 1 & 2 & -1 \end{vmatrix}$

$= (6l - 5)\hat{i} + (3 - 2l)\hat{j} + (2l + 1)\hat{k}$

$\because$ ***R*** containing L

$3(6l - 5) - 8(3 - 2l) + 7(2l + 1) = 0$

$18l + 16l + 14l - 15 - 24 + 7 = 0$

$\therefore \quad l = \frac{32}{48} = \frac{2}{3}$

Let θ be the acute angle between L and y-axis

$\therefore \quad \cos\theta = \frac{\frac{5}{3}}{\sqrt{1 + \frac{25}{9} + \frac{49}{9}}} = \frac{5}{\sqrt{83}}$

$\therefore \quad 415\cos^2\theta = 125$

8. Suppose $y = y(x)$ be the solution curve to the differential equation $\frac{dy}{dx} - y = 2 - e^{-x}$ such that $\lim_{x\to\infty} y(x)$ is finite. If a and b are respectively the x - and y - intercepts of the tangent to the curve at $x = 0$, then the value of $a - 4b$ is equal to _____.

Answer (3)

Sol. If = e^{-x}

$y \cdot e^{-x} = -2e^{-x} + \frac{e^{-2x}}{2} + C$

$\Rightarrow$ $y = -2 + e^{-x} + Ce^{x}$

$\lim_{x\to\infty} y(x)$ is finite so $C = 0$

$y = -2 + e^{-x}$

$\Rightarrow \frac{dy}{dx} = -e^{-x} \Rightarrow \left.\frac{dy}{dx}\right|_{x=0} = -1$

Equation of tangent

$y + 1 = -1(x - 0)$

or $y + x = -1$

So $a = -1$, $b = -1$

$\Rightarrow$ $a - 4b = 3$

9. Different A.P.'s are constructed with the first term 100, the last term 199, and integral common differences. The sum of the common differences of all such A.P.'s having at least 3 terms and at most 33 terms is ______.

Answer (53)

Sol. $d_1 = \frac{199-100}{2} \notin I$

$d_2 = \frac{199-100}{3} = 33$

$d_3 = \frac{199-100}{4} \notin I$

$d_n = \frac{199-100}{i+1} \in I$

d_i = 33 + 11, 9

Sum of CD's = 33 + 11 + 9

= 53

10. The number of matrices $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where *a*, *b*, *c*, *d* $\in$ {–1, 0, 1, 2, 3, ….., 10}, such that $A = A^{-1}$, is ______.

Answer (50)

Sol. $\because \quad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $A^2 = \begin{bmatrix} a^2+bc & b(a+d) \\ c(a+d) & bc+d^2 \end{bmatrix}$

For A^{-1} must exist $ad - bc \neq 0$ …(i)

and $A = A^{-1} \Rightarrow A^2 = I$

$\therefore \quad a^2 + bc = d^2 + bc = 1$ …(ii)

and $b(a + d) = c(a + d) = 0$ …(iii)

Case I : When $a = d = 0$, then possible values of (b, c) are (1, 1), (–1, 1) and (1, –1) and (–1, 1).

Total four matrices are possible.

Case II : When $a = -d$ then (a, d) be (1, –1) or (–1, 1).

Then total possible values of (b, c) are $(12 + 11) \times 2 = 46$.

$\therefore$ Total possible matrices = 46 + 4 = 50.

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