26/07/2022
Morning

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Answers & Solutions

Time : 3 hrs.

for

M.M. : 300

JEE (Main)-2022 (Online) Phase-2

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

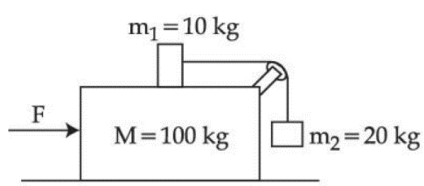
SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Three masses M = 100 kg, m_1 = 10 kg and m_2 = 20 kg are arranged in a system as shown in figure. All the surfaces are frictionless and strings are inextensible and weightless. The pulleys are also weightless and frictionless. A force F is applied on the system so that the mass m_2 moves upward with an acceleration of 2 ms^{-2}. The value of F is

   (Take g = 10 ms^{-2})

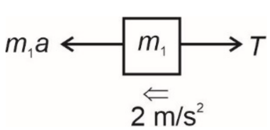


   (A) 3360 N (B) 3380 N

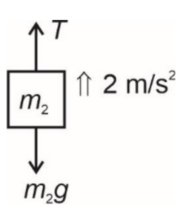
   (C) 3120 N (D) 3240 N

Answer (C)

Sol. In frame of block of mass M moving with acceleration a



$m_1a - T = 2m_1 \Rightarrow 10a - T = 20$ …(i)



$T - m_2g = m_22 \Rightarrow T - 200 = 40 \Rightarrow T = 240$ …(ii)

$\Rightarrow$ From equation 1 and 2 $10a = 260$ or $a = 26$ m/s^2

for block

$F = (M + m_2)a = 120 \times 26$

= 3120 N

2. A radio can tune to any station in 6 MHz to 10 MHz band. The value of corresponding wavelength bandwidth will be

   (A) 4 m

   (B) 20 m

   (C) 30 m

   (D) 50 m

Answer (B)

Sol. $\nu_1 = 6 \times 10^6$ Hz

$$\Rightarrow \lambda_1 = \frac{3\times10^8}{6\times10^6} = 50 \text{ m}$$

$\nu_2 = 10 \times 10^6$ Hz

$$\Rightarrow \lambda_2 = \frac{3\times10^8}{10\times10^6} = 30 \text{ m}$$

$\Rightarrow$ Wavelength band with

$= |\lambda_1 - \lambda_2| = 20$ m

3. The disintegration rate of a certain radioactive sample at any instant is 4250 disintegrations per minute. 10 minutes later, the rate becomes 2250 disintegrations per minute. The approximate decay constant is

   (Take $\log_{10}1.88 = 0.274$)

   (A) 0.02 min^{-1}

   (B) 2.7 min^{-1}

   (C) 0.063 min^{-1}

   (D) 6.3 min^{-1}

Answer (C)

Sol. $A_0 = 4250$

$A = 2250 = A_0e^{-\lambda t}$

$$\Rightarrow \frac{2250}{4250} = e^{-\lambda t}$$

$$\Rightarrow \lambda(10) = \ln\left(\frac{4250}{2250}\right)$$

$\lambda(10) = 0.636$

$\lambda = 0.063$

4. A parallel beam of light of wavelength 900 nm and intensity 100 Wm^{-2} is incident on a surface perpendicular to the beam. The number of photons crossing 1 cm^{-2} area perpendicular to the beam in one second is

(A) 3×10^{16} (B) 4.5×10^{16}

(C) 4.5×10^{17} (D) 4.5×10^{20}

Answer (B)

Sol. λ = 900 nm

I = 100 W/m^2

$A = 10^{-4}$

$\Rightarrow$ $P = 10^{-2}$ W

$\Rightarrow$ Number of photons incident per second

$$= \frac{10^{-2}\lambda}{hc}$$

$$= \frac{9\times10^{-11}\times10^{2}}{6.63\times10^{-34}\times3\times10^{8}} \simeq 4.5\times10^{16}$$

5. In Young's double slit experiment, the fringe width is 12 mm. If the entire arrangement is placed in water of refractive index $\frac{4}{3}$, then the fringe width becomes (in mm)

(A) 16 (B) 9

(C) 48 (D) 12

Answer (B)

Sol. $B = 12 \times 10^{-3}$

$$\beta' = \frac{\beta}{\mu} = \frac{12\times10^{-3}}{\frac{4}{3}}$$

$= 9 \times 10^{-3}$ m = 9 mm

6. The magnetic field of a plane electromagnetic wave is given by

$\vec{B} = 2\times10^{-8}\sin(0.5\times10^{3}x+1.5\times10^{11}t)\hat{j}\text{T}$.

The amplitude of the electric field would be

(A) 6 Vm^{-1} along x-axis

(B) 3 Vm^{-1} along z-axis

(C) 6 Vm^{-1} along z-axis

(D) 2×10^{-8} Vm^{-1} along z-axis

Answer (C)

Sol. Speed of light $c = \frac{\omega}{k} = \frac{1.5\times10^{11}}{0.5\times10^{3}} = 3\times10^{8}$ m/sec

So, $E_0 = B_0c$

$= 2 \times 10^{-8} \times 3 \times 10^{8}$

= 6 V/m

Direction will be along z-axis

7. In a series LR circuit $X_L = R$ and power factor of the circuit is P_1. When capacitor with capacitance C such that $X_L = X_C$ is put in series, the power factor becomes P_2. The ratio $\frac{P_1}{P_2}$ is

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$

(C) $\frac{\sqrt{3}}{\sqrt{2}}$ (D) 2 : 1

Answer (B)

Sol. $P_1 = \cos\phi = \frac{1}{\sqrt{2}}(X_L = R)$

$P_2 = \cos\phi' = 1$ (will become resonance circuit)

So, $\frac{P_1}{P_2} = \frac{1}{\sqrt{2}}$

8. A charge particle is moving in a uniform field $(2\hat{i}+3\hat{j})$T. If it has an acceleration of $(\alpha\hat{i}-4\hat{j})$m/s^2, then the value of α will be

(A) 3 (B) 6

(C) 12 (D) 2

Answer (B)

As magnetic force is perpendicular to magnetic field

So, $\vec{F}\cdot\vec{B}$ must be 0

So, $2\alpha - 12 = 0$

$\alpha = 6$

9. B_X and B_Y are the magnetic field at the centre of two coils X and Y respectively each carrying equal current. If coil X has 200 turns and 20 cm radius and coil Y has 400 turns and 20 cm radius, the ratio of B_X and B_Y is

(A) 1 : 1 (B) 1 : 2

(C) 2 : 1 (D) 4 : 1

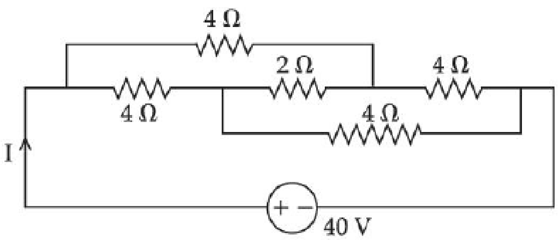
Answer (B)

Sol. $B = \dfrac{\mu_0 NI}{2R}$

$$\frac{B_X}{B_Y} = \frac{N_x R_y}{N_y R_x}$$

$$= \frac{200 \times 20}{400 \times 20} = \frac{1}{2}$$

10. The current I in the given circuit will be

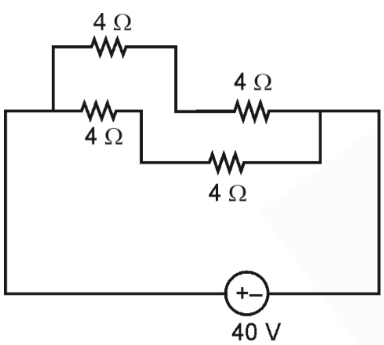


(A) 10 A (B) 20 A

(C) 4 A (D) 40 A

Answer (A)

Sol.



The grouping of resistance is a wheatstone bridge

So, R_{net} = 4 Ω

So, $i = \dfrac{V}{R_{net}} = 10$ A

11. The total charge on the system of capacitors C_1 = 1 μF, C_2 = 2 μF, C_3 = 4 μF and C_4 = 3 μF connected in parallel is :

(Assume a battery of 20 V is connected to the combination)

(A) 200 μC (B) 200 C

(C) 10 μC (D) 10 C

Answer (A)

Sol. Equivalent $C = \Sigma C_i$

= 10 μF

⇒ Charge $Q = CV$

= 200 μC

12. When a particle executes Simple Harmonic Motion, the nature of graph of velocity as a function of displacement will be :

(A) Circular (B) Elliptical

(C) Sinusoidal (D) Straight line

Answer (B)

Sol. Let $x = A\sin\omega t$

⇒ $v = A\omega\cos\omega t$

⇒ $v = \pm\omega\sqrt{A^2 - x^2}$

⇒ $\dfrac{v^2}{\omega^2} + x^2 = A^2$

⇒ Ellipse

13. 7 mol of a certain monoatomic ideal gas undergoes a temperature increase of 40 K at constant pressure. The increase in the internal energy of the gas in this process is :

(Given R = 8.3 JK^{-1} mol^{-1})

(A) 5810 J (B) 3486 J

(C) 11620 J (D) 6972 J

Answer (B)

Sol. $\Delta U = nC_v\Delta T$

$$= 7 \times \frac{3R}{2} \times 40$$

= 3486 J

14. A monoatomic gas at pressure P and volume V is suddenly compressed to one eighth of its original volume. The final pressure at constant entropy will be :

(A) P (B) $8P$

(C) $32P$ (D) $64P$

Answer (C)

Sol. PV^γ = constant

⇒ $PV^\gamma = (P')\left(\dfrac{V}{8}\right)^\gamma$ where $\gamma = 5/3$

⇒ $P' = 32P$

15. A water drop of radius 1 cm is broken into 729 equal droplets. If surface tension of water is 75 dyne/cm, then the gain in surface energy upto first decimal place will be :

(Given π = 3.14)

(A) 8.5×10^{-4} J (B) 8.2×10^{-4} J

(C) 7.5×10^{-4} J (D) 5.3×10^{-4} J

Answer (C)

Sol. $729 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

$\Rightarrow$ $R = 9r$ ...(1)

$\Delta U = S \times \Delta A$ ...(2)

$\Rightarrow$ $\Delta U = S \times \{-4\pi R^2 + 729 \times 4\pi r^2\}$

$= S \times 4\pi\{729r^2 - 81r^2\}$

$= 7.5 \times 10^{-4}$ J

16. The percentage decrease in the weight of a rocket, when taken to a height of 32 km above the surface of earth will, be:

(Radius of earth = 6400 km)

(A) 1% (B) 3%

(C) 4% (D) 0.5%

Answer (A)

Sol. $\because$ $g = \frac{GM}{r^2}$

$\Rightarrow$ $\frac{\Delta g}{g} = 2\frac{\Delta r}{r}$

$\Rightarrow$ $\frac{\Delta g}{g} \times 100 = 2 \times \frac{32}{6400} \times 100\% = 1\%$

$\Rightarrow$ % decrease in weight = 1%

17. As per the given figure, two blocks each of mass 250 g are connected to a spring of spring constant 2 Nm^{-1}. If both are given velocity v in opposite directions, then maximum elongation of the spring is:

(A) $\frac{v}{2\sqrt{2}}$ (B) $\frac{v}{2}$

(C) $\frac{v}{4}$ (D) $\frac{v}{\sqrt{2}}$

Answer (B)

Sol. $\because$ Loss in KE = Gain in spring energy

$\Rightarrow$ $\frac{1}{2}mv^2 \times 2 = \frac{1}{2}kx_m^2$

$\Rightarrow$ $2 \times \frac{1}{4} \times v^2 = 2 \times x_m^2$

$\Rightarrow$ $x_m = \sqrt{\frac{v^2}{4}} = \frac{v}{2}$

18. A monkey of mass 50 kg climbs on a rope which can withstand the tension (T) of 350 N. If monkey initially climbs down with an acceleration of 4 m/s^2 and then climbs up with an acceleration of 5 m/s^2. Choose the correct option (g = 10 m/s^2).

(A) T = 700 N while climbing upward

(B) T = 350 N while going downward

(C) Rope will break while climbing upward

(D) Rope will break while going downward

Answer (C)

Sol. $T_{down} = 50 \times (10 - 4)$

$= 50 \times 6$

$= 300$ N

$T_{up} = 50 \times (10 + 5)$

$= 50 \times 15$

$= 750$ N

$\Rightarrow$ Rope will break while climbing up.

19. Two projectiles thrown at 30° and 45° with the horizontal respectively, reach the maximum height in same time. The ratio of their initial velocities is :

(A) $1 : \sqrt{2}$ (B) 2 : 1

(C) $\sqrt{2} : 1$ (D) 1 : 2

Answer (C)

Sol. $\because$ $t_a = \frac{u \sin\theta}{g}$

$\Rightarrow$ $\frac{u_1 \sin(30°)}{g} = \frac{u_2 \sin(45°)}{g}$

$\Rightarrow$ $\frac{u_1}{u_2} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}} = \frac{\sqrt{2}}{1}$

20. A screw gauge of pitch 0.5 mm is used to measure the diameter of uniform wire of length 6.8 cm, the main scale reading is 1.5 mm and circular scale reading is 7. The calculated curved surface area of wire to appropriate significant figures is :

[Screw gauge has 50 divisions on its circular scale]

(A) 6.8 cm^2 (B) 3.4 cm^2

(C) 3.9 cm^2 (D) 2.4 cm^2

Answer (B)

Sol. Least count $= \frac{0.5}{50}$ mm $= 0.01$ mm

$\therefore$ Diameter, d = 1.5 mm + 7 × 0.01

= 1.57 mm

$\therefore$ Surface area = $(2\pi r) \times l$

$= \pi dl$

$= 3.142 \times \frac{1.57}{10} \times 6.8 \text{ cm}^2$

= 3.354 cm^2 = 3.4 cm^2

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. If the initial velocity in horizontal direction of a projectile is unit vector $\hat{i}$ and the equation of trajectory is $y = 5x(1 - x)$. The y component vector of the initial velocity is ___ $\hat{j}$.

(Take g = 10 m/s^2)

Answer (5)

Sol. $y = 5x - 5x^2$

$y = x \tan\theta - \frac{1}{2}\frac{gx^2}{v^2}$

$\tan\theta = 5 = \frac{u_y}{u_x}$

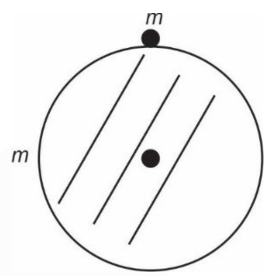
$\Rightarrow u_y = 5$

2. A disc of mass 1 kg and radius R is free to rotate about a horizontal axis passing through its centre and perpendicular to the plane of disc. A body of same mass as that of disc of fixed at the highest point of the disc. Now the system is released, when the body comes to the lowest position, it angular speed will be $4\sqrt{\frac{x}{3R}}$ rad s^{-1} where x = ___.

(g = 10 ms^{-2})

Answer (5)

Sol.



Loss in P.E = Gain in K.E.

$2mgR = \frac{1}{2}\left[\frac{1}{2}mR^2 + mR^2\right]w^2$

$2mg\,R = \frac{1}{2} \times \frac{3}{2}mR^2\ w^2$

$w^2 = \frac{8g}{3R}$

$w = \sqrt{\frac{8g}{3R}} = 4\sqrt{\frac{g}{2 \times 3R}}$

$\Rightarrow x = \frac{g}{2} = 5$

3. In an experiment of determine the Young's modulus of wire of a length exactly 1 m, the extension in the length of the wire is measured as 0.4 mm with an uncertainty of ± 0.02 mm when a load of 1 kg is applied. The diameter of the wire is measured as 0.4 mm with an uncertainty of ± 0.02 mm when a load of 1 kg is applied. The diameter of the wire is measured as 0.4 mm with an uncertainty of ± 0.01 mm. The error in the measurement of Young's modulus (ΔY) is found to be $x \times 10^{10}$ Nm^{-2}. The value of x is ___.

(Take g = 10 ms^{-2})

Answer (2)

Sol. $\dfrac{F/A}{l/L} = Y, A = \pi D^2$

$$\frac{\Delta Y}{Y} = \frac{\Delta F}{F} + \frac{2\Delta D}{D} + \frac{\Delta l}{e} + \frac{\Delta L}{L}$$

$$= 2\times\frac{0.01}{0.4} + \frac{0.02}{0.4}$$

$$= \frac{0.04}{0.4} = \frac{1}{10}$$

$$Y = \frac{Fl}{A\Delta l}$$

$$= \frac{10\times 1}{\pi(0.1\text{ mm})^2 \times 0.4\text{ mm}}$$

$= 1.988 \times 10^{11}$

$\approx 2 \times 10^{11}$

$$\frac{\Delta y}{y} = \frac{1}{10}$$

$$\Delta y = \frac{y}{10} = 2\times 10^{10}$$

4. When a car is approaching the observer, the frequency of horn is 100 Hz. After passing the observer, it is 50 Hz. If the observer moves with the car, the frequency will be $\dfrac{x}{3}$ Hz where x = ___.

Answer (200)

Sol. $100 = v_0 \dfrac{v}{v - v_C}$

$$50 = v_0 \frac{v}{v + v_c}$$

$$2 = \frac{v + v_c}{v - v_c}$$

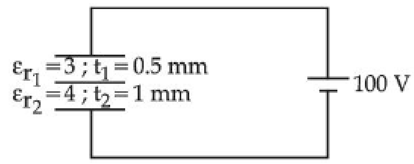
$2v - 2v_c = v + v_c$

$$v_c = \frac{v}{3}$$

$$100 = v_0 \frac{v\times 3}{2v} \Rightarrow v_0 = \frac{200}{3} = \frac{x}{3}$$

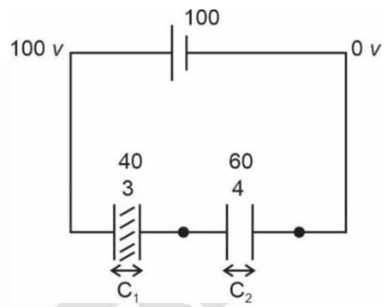
$\Rightarrow x = 200$

5. A composite parallel plate capacitor is made up of two different dielectric materials with different thickness (t_1 and t_2) as shown in figure. The two different dielectric materials are separated by a conducting foil F. The voltage of the conducting foil is ___V.



Answer (60)

Sol.

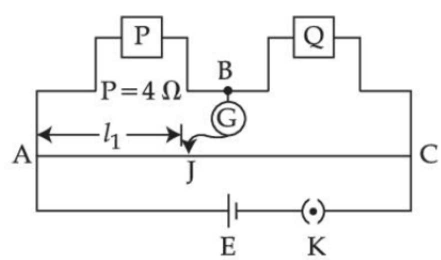


$$\frac{C_1}{C_2} = \frac{3\times t_2}{t_1 \times 4} = \frac{3}{2}$$

$$\frac{q}{C_1} = V_1, \frac{q}{C_2} = V_2$$

$$\frac{V_1}{V_2} = \frac{C_2}{C_1} = \frac{2}{3}$$

6. Resistances are connected in a meter bridge circuit as shown in the figure. The balancing length l_1 is 40 cm. Now an unknown resistance x is connected in series with P and new balancing length is found to be 80 cm measured from the same end. Then the value of x will be ________ Ω.



Answer (20)

Sol. $\frac{P}{40} = \frac{Q}{60}$ ...(1)

$\frac{P+x}{80} = \frac{Q}{20}$ ...(2)

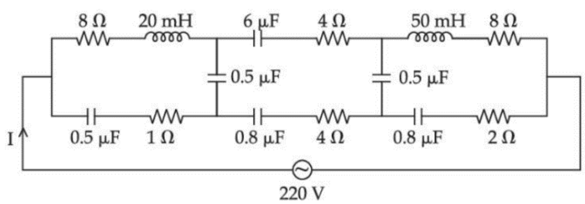
$\frac{P}{P+x} \times \frac{80}{40} = \frac{20}{60}$

$\frac{4}{4+x} \times 2 = \frac{1}{3}$

$24 = 4 + x$

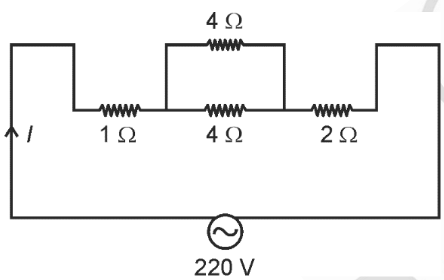
$x = 20$

7. The effective current I in the given circuit at very high frequencies will be ________ A.



Answer (44)

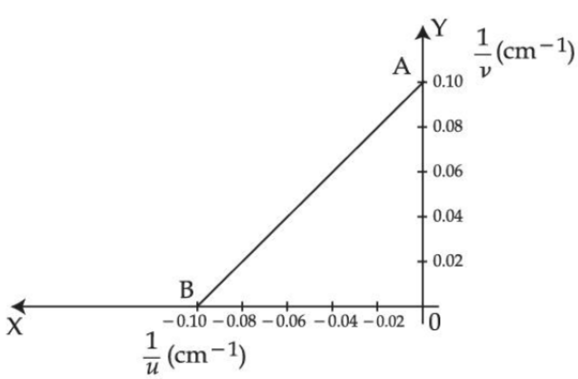
Sol. Equivalent circuit will be



$I = \frac{220}{5} = 44$ A

8. The graph between $\frac{1}{u}$ and $\frac{1}{v}$ for a thin convex lens in order to determine its focal length is plotted as shown in the figure. The refractive index of lens is 1.5 and its both the surfaces have same radius of curvature R. The value of R will be ______ cm.

(where u = object distance, v = image distance)



Answer (10)

Sol. $f = 10$ cm

$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{-R}\right)$

$\frac{1}{10} = \frac{1.5-1}{1} \times \frac{2}{R}$

$\frac{1}{10} = \frac{1}{R}$

$R = 10$ cm

9. In the hydrogen spectrum, λ be the wavelength of first transition line of Lyman series. The wavelength difference will be "$a\lambda$" between the wavelength of 3rd transition line of Paschen series and that of 2nd transition line of Balmer series where a = _______.

Answer (5)

Sol. $\frac{1}{\lambda} = R_H\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$

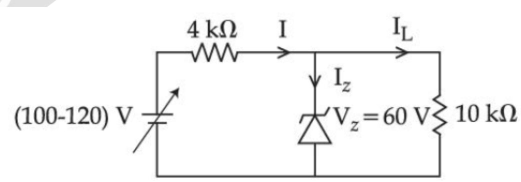
$\frac{1}{\lambda_3} = R_H\left(\frac{1}{3^2} - \frac{1}{6^2}\right)$

$\frac{1}{\lambda_2} = R_H\left(\frac{1}{2^2} - \frac{1}{4^2}\right)$

$\therefore\ \lambda_3 - \lambda_2 = a\lambda$

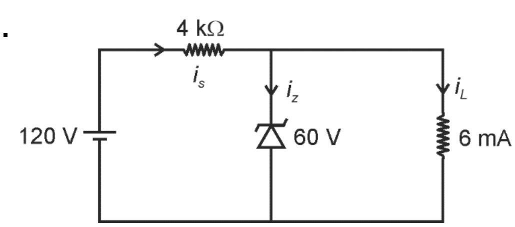
$a = 5$

10. In the circuit shown below, maximum Zener diode current will be ________ mA.



Answer (9)

Sol.



$i_s = \frac{60}{4 \times 10^3} = 15 \times 10^{-3}$ = 15 mA

$i_L = \frac{60}{10 \times 10^3}$ = 6 mA

$I_z = i_s - i_L = 9$ mA

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Match **List-I** with **List-II.**

List-I (Compound)	**List-II (Shape)**
(A) BrF_5	(I) bent
(B) $[CrF_6]^{3-}$	(II) square pyramidal
(C) O_3	(III) trigonal bipyramidal
(D) PCl_5	(IV) octahedral

Choose the **correct** answer from the options given below :

(A) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

(B) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(C) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

(D) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Answer (C)

Sol. (A) BrF_5 – square pyramidal

(B) $[CrF_6]^{3-}$ – octahedral

(C) O_3 – bent

(D) PCl_5 – trigonal bipyramidal

2. Match **List-I** with **List-II.**

List-I (Processes/ Reactions)	**List-II (Catalyst)**
(A) $2SO_2(g) + O_2(g) \rightarrow 2SO_3(g)$	(I) Fe(s)
(B) $4NH_3(g) + 5O_2(g) \rightarrow 4NO(g) + 6H_2O(g)$	(II) Pt(s) – Rh(s)
(C) $N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$	(III) V_2O_5
(D) Vegetable oil(l) + H_2 $\rightarrow$ Vegetable ghee(s)	(IV) Ni(s)

Choose the **correct** answer from the options given below :

(A) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)

(B) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

(C) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

(D) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

Answer (B)

Sol. (A) $2SO_2(g) + O_2(g) \xrightarrow{V_2O_5} 2SO_3$

(B) $4NH_3(g) + 5O_2(g) \xrightarrow{Pt(s)-Rh(s)} 4NO(g) + 6H_2O(g)$

(C) $N_2(g) + 3H_2(g) \xrightarrow{Fe(s)} 2NH_3(g)$

(D) Vegetable oil(l) + H_2 $\xrightarrow{Ni(s)}$ Vegetable ghee(s)

3. Given two statements below:

Statement I : In Cl_2 molecule the covalent radius is double of the atomic radius of chlorine.

Statement II : Radius of anionic species is always greater than their parent atomic radius.

Choose the **most appropriate** answer from options given below:

(A) Both **Statement I** and **Statement II** are correct.

(B) Both **Statement I** and **Statement II** are incorrect.

(C) **Statement I** is correct but **Statement II** is incorrect.

(D) **Statement I** is incorrect but **Statement II** is correct.

Answer (D)

Sol. • Covalent radius is not double of atomic radius.

• Radius of anionic species is always greater than their parent atomic radius as nuclear charge decreases in anionic counterpart.

4. Refining using liquation method is the most suitable for metals with:

(A) Low melting point

(B) High boiling point

(C) High electrical conductivity

(D) Less tendency to be soluble in melts than impurities

Answer (A)

Sol. Refining using liquation method is the most suitable for metals with low melting point.

5. Which of the following can be used to prevent the decomposition of H_2O_2?

(A) Urea (B) Formaldehyde

(C) Formic acid (D) Ethanol

Answer (A)

Sol. Urea is used as a stabilizer for the storage of H_2O_2.

6. Reaction of $BeCl_2$ with $LiAlH_4$ gives :

(A) $AlCl_3$

(B) BeH_2

(C) LiH

(D) LiCl

(E) $BeAlH_4$

Choose the **correct** answer from options given below :

(A) (A), (D) and (E) (B) (A), (B) and (D)

(C) (D) and (E) (D) (B), (C) and (D)

Answer (B)

Sol. $2BeCl_2 + LiAlH_4 \rightarrow 2BeH_2 + LiCl + AlCl_3$

7. Borazine, also known as inorganic benzene, can be prepared by the reaction of 3-equivalents of "X" with 6-equivalents of "Y". "X" and "Y", respectively are:

(A) $B(OH)_3$ and NH_3

(B) B_2H_6 and NH_3

(C) B_2H_6 and HN_3

(D) NH_3 and B_2O_3

Answer (B)

Sol. $3B_2H_6 + 6NH_3 \rightarrow \underset{\text{(Borazine)}}{2B_3N_3H_6}$

8. Which of the given reactions is not an example of disproportionation reaction?

(A) $2H_2O_2 \rightarrow 2H_2O + O_2$

(B) $2NO_2 + H_2O \rightarrow HNO_3 + HNO_2$

(C) $MnO_4^- + 4H^+ + 3e^- \rightarrow MnO_2 + 2H_2O$

(D) $3MnO_4^{2-} + 4H^+ \rightarrow 2MnO_4^- + MnO_2 + 2H_2O$

Answer (C)

Sol. $\overset{+7}{Mn}O_4^- + 4H^+ + 3e^- \longrightarrow \overset{+4}{Mn}O_2 + 2H_2O$

The above reaction involves the reduction of MnO_4^- to MnO_2.

9. The dark purple colour of $KMnO_4$ disappears in the titration with oxalic acid in acidic medium. The overall change in the oxidation number of manganese in the reaction is :

(A) 5 (B) 1

(C) 7 (D) 2

Answer (A)

Sol. $2K\overset{+7}{Mn}O_4 + 5H_2C_2O_4 + 3H_2SO_4 \rightarrow$

$K_2SO_4 + 2\overset{+2}{Mn}SO_4 + 10CO_2 + 8H_2O$

Change is oxidation state Mn is 5.

10. $\dot{C}l + CH_4 \rightarrow A + B$

A and B in the above atmospheric reaction step are:

(A) C_2H_6 and Cl_2 (B) $\dot{C}HCl_2$ and H_2

(C) $\dot{C}H_3$ and HCl (D) C_2H_6 and HCl

Answer (C)

Sol. $\dot{C}l + CH_4 \rightarrow \underset{(A)}{\dot{C}H_3} + \underset{(B)}{HCl}$

11. Which technique among the following, is most appropriate in separation of a mixture of 100 mg of *p*-nitrophenol and picric acid?

(A) Steam distillation

(B) 2-5 ft long column of silica gel

(C) Sublimation

(D) Preparative TLC (Thin Layer Chromatography)

Answer (D)

Sol. Thin layer chromatography is a technique used to isolate non-volatile mixtures.

Hence, mixture of p-nitrophenol and Picric acid is separated by TLC.

12. The difference in the reaction of phenol with bromine in chloroform and bromine in water medium is due to:

(A) Hyperconjugation in substrate

(B) Polarity of solvent

(C) Free radical formation

(D) Electromeric effect of substrate

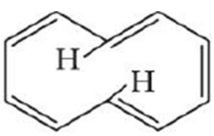
Answer (B)

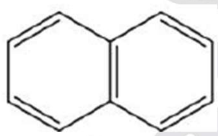
Sol. Phenol gives different products with bromine in chloroform and water medium due to the polarity difference between chloroform and water acting as solvent

13. Which of the following compounds is **not** aromatic?

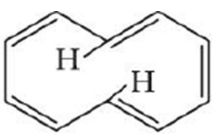
(A) 

(B) 

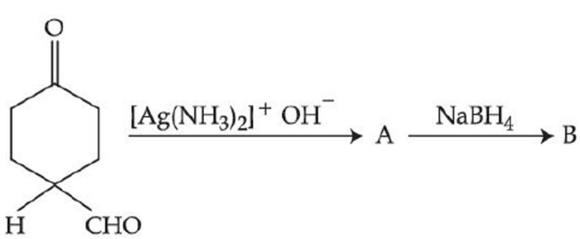
(C) 

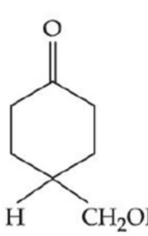
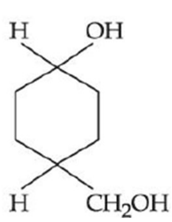
(D) 

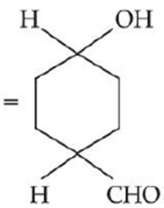
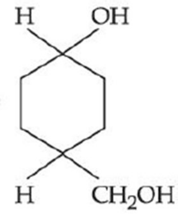
Answer (C)

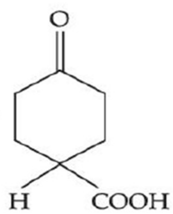
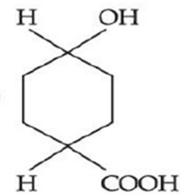
Sol.  is a non-planar compound, hence it is not aromatic.

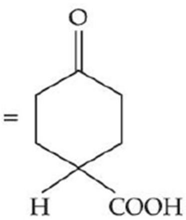
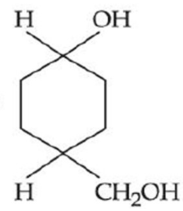
14. The products formed in the following reaction, **A** and **B** are



(A) A =  , B = 

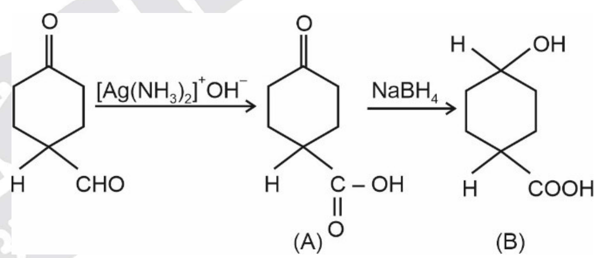
(B) A =  , B = 

(C) A =  , B = 

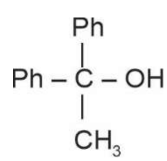
(D) A =  , B = 

Answer (C)

Sol.



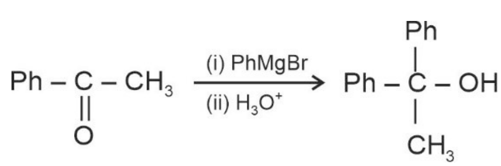
15. Which reactant will give the following alcohol on reaction with one mole of phenyl magnesium bromide (PhMgBr) followed by acidic hydrolysis?



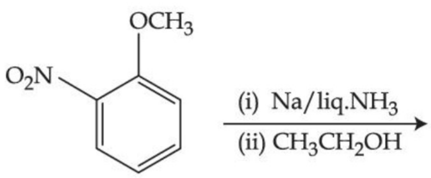
(A) $CH_3-C\equiv N$ (B) $Ph-C\equiv N$

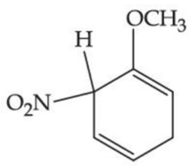
(C) $CH_3-\overset{\overset{O}{||}}{C}-O-Ph$ (D) $Ph-\underset{\underset{O}{||}}{C}-CH_3$

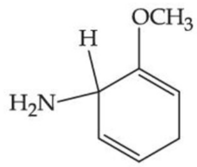
Answer (D)

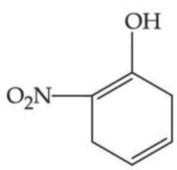
Sol. 

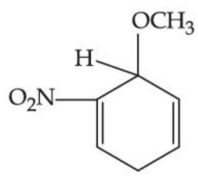
16. The major product of the following reaction is



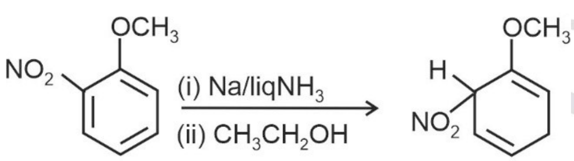
(A) 

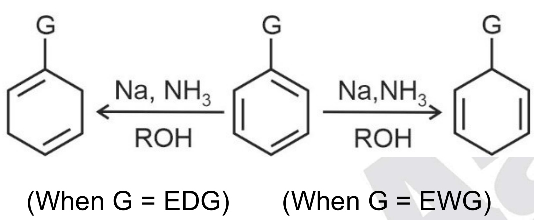
(B) 

(C) 

(D) 

Answer (A)

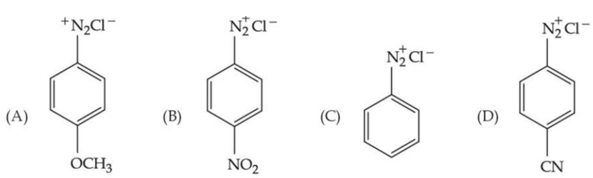
Sol. 



EDG → Electron donating group

EWG → Electron withdrawing group

17. The correct stability order of the following diazonium salt is



(A) (A) > (B) > (C) > (D)

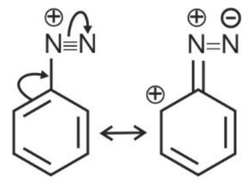
(B) (A) > (C) > (D) > (B)

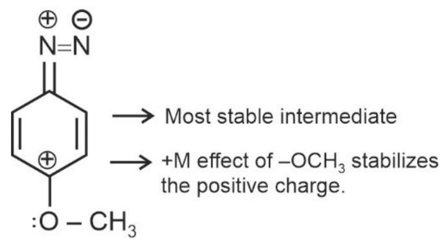
(C) (C) > (A) > (D) > (B)

(D) (C) > (D) > (B) > (A)

Answer (B)

Sol. Diazonium salt containing aryl group directly linked to electron donating group is most stable due to resonance. The +M effect stabilizes the intermediate whereas Electron withdrawing group on benzene destabilizes the intermediate at para position.





Order will be A > C > D > B.

18. Stearic acid and polyethylene glycol react to form which one of the following soap/s detergents?

(A) Cationic detergent (B) Soap

(C) Anionic detergent (D) Non-ionic detergent

Answer (D)

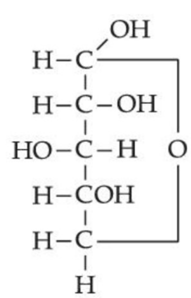
Sol. $CH_3(CH_2)_{16}COOH + HO(CH_2CH_2O)_nCH_2CH_2OH$

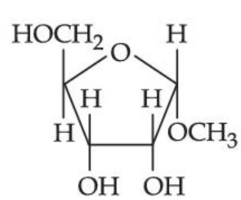
(Stearic acid) $\xrightarrow{-H_2O}$ (Polyethylene glycol)

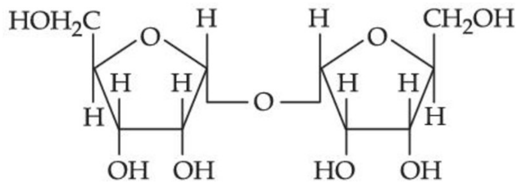
$CH_3(CH_2)_{16}COO(CH_2CH_2O)_nCH_2CH_2OH$

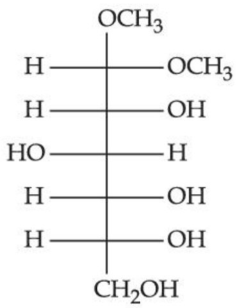
The product do not contain any ion in their constitution hence it is a non-ionic detergent.

19. Which one of the following is a reducing sugar?

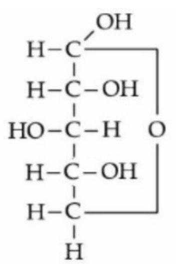
(A) 

(B) 

(C) 

(D) 

Answer (A)

Sol. 

The sugar gives +ve Tollen's test hence it's a reducing sugar.

20. Given below are two statements: one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Experimental reaction of CH_3Cl with aniline and anhydrous $AlCl_3$ does not give *o* and *p*-methylaniline.

Reason (R) : The $-NH_2$ group of aniline becomes deactivating because of salt formation with anhydrous $AlCl_3$ and hence yields *m*-methyl aniline as the product.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

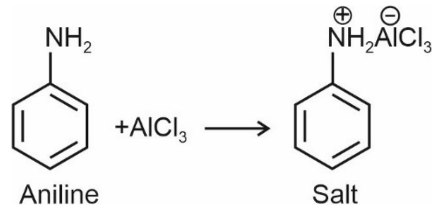
(A) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**.

(B) Both **(A)** and **(R)** are true but **(R)** is not the correct explanation of **(A)**.

(C) **(A)** is true, but **(R)** is false.

(D) **(A)** is false, but **(R)** is true.

Answer (C)

Sol. 

Aniline does not undergo Friedel Craft reaction because the reagent $AlCl_3$ being electron deficient acts as a Lewis acid.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Chlorophyll extracted from the crushed green leaves was dissolved in water to make 2 L solution of Mg of concentration 48 ppm. The number of atoms of Mg in this solution is $x \times 10^{20}$ atoms. The value of x is ________. (Nearest integer)

(Given : Atomic mass of Mg is 24 g mol^{-1}; $N_A = 6.02 \times 10^{23}\ mol^{-1}$)

Answer (24)

Sol. In 2L → 96 mg of Mg

$$\text{Number of atoms of Mg} = \frac{96\times10^{-3}}{24}\times N_A$$

$$= 4\times10^{-3}\times6\times10^{23}$$

$$= 24 \times 10^{20}$$

x = 24

2. A mixture of hydrogen and oxygen contains 40% hydrogen by mass when the pressure is 2.2 bar. The partial pressure of hydrogen is ________ bar. (Nearest integer)

Answer (2)

Sol. 40% w/w hydrogen gas is given in mixture of H_2 and oxygen.

Wt. of H_2 = 40 g

Wt. of O_2 = 60 g

$$\chi_{H_2} = \frac{n_{H_2}}{n_{H_2} + n_{O_2}}$$

$$= \frac{\frac{40}{2}}{\frac{40}{2} + \frac{60}{32}}$$

$$= \frac{20}{20 + 1.875}$$

$$= \frac{20}{21.875} = 0.914$$

$$P_{H_2} = \chi_{H_2} \times P_T$$

= 0.914 × 2.2

= 2.01 ≃ 2 bar

3. The wavelength of an electron and a neutron will become equal when the velocity of the electron is x times the velocity of neutron. The value of x is _______. (Nearest integer)

(Mass of electron is 9.1×10^{-31} kg and mass of neutron is 1.6×10^{-27} kg)

Answer (1758)

Sol. $\lambda_e = \frac{h}{m_e \times V_e}$, $\lambda_N = \frac{h}{m_N \times V_N}$

$\lambda_e = \lambda_N$ When $V_e = xV_N$

$$\frac{1}{m_e V_e} = \frac{1}{m_N \times V_N}$$

$$\frac{m_N}{m_e} = \frac{V_e}{V_N} = x$$

$$x = \frac{1.6 \times 10^{-27}}{9.1 \times 10^{-31}}$$

= 0.17582×10^4

≃ 1758

4. 2.4 g coal is burnt in a bomb calorimeter in excess of oxygen at 298 K and 1 atm pressure. The temperature of the calorimeter rises from 298 K to 300 K. The enthalpy change during the combustion of coal is –x kJ mol^{-1}. The value of x is _______. (Nearest integer)

(Given : Heat capacity of bomb calorimeter is 20.0 kJ K^{-1}. Assume coal to be pure carbon)

Answer (200)

Sol. $Q \text{ (Heat evolved)} = -\frac{C_{system}\ \Delta T}{n}$

$$n_{coal} = \frac{2.4}{12}$$

$$Q = \frac{-20(300-298)}{0.2}$$

$$Q = -200 \text{ kJ / mol}$$

x = 200

5. When 800 mL of 0.5 M nitric acid is heated in a beaker, its volume is reduced to half and 11.5 g of nitric acid is evaporated. The molarity of the remaining nitric acid solution is $x \times 10^{-2}$ M. (Nearest integer)

(Molar mass of nitric acid is 63 g mol^{-1})

Answer (54)

Sol. m moles of HNO_3 = 800 × 0.5

Moles of HNO_3 = 400×10^{-3}

= 0.4 moles

Weight of HNO_3 = 0.4 × 63 g

= 25.2 g

Remaining acid = 25.2 – 11.5

= 13.7 g

$$M = \frac{13.7 \times 1000}{400 \times 63}$$

$$= \frac{137}{252} = 0.54$$

= 54×10^{-2}

6. At 298 K, the equilibrium constant is 2×10^{15} for the reaction:

$$Cu(s) + 2Ag^+(aq) \rightleftharpoons Cu^{2+}(aq) + 2Ag(s)$$

The equilibrium constant for the reaction

$$\frac{1}{2}Cu^{2+}(aq) + Ag(s) \rightleftharpoons \frac{1}{2}Cu(s) + Ag^+(aq)$$

is $x \times 10^{-8}$. The value of x is ________ (Nearest integer)

Answer (2)

Sol. $Cu(s) + 2Ag^+(aq) \rightleftharpoons Cu^{2+}(aq) + 2Ag(s)$

$k = 2 \times 10^{15}$

$$\frac{1}{2}Cu(s) + Ag^+(aq) \rightleftharpoons Cu^{+2}(aq) + 2Ag(s)$$

$$K' = \frac{1}{(K)^{1/2}} = \frac{1}{(2\times10^{15})^{1/2}}$$

$$= 2.23 \times 10^{-8}$$

$x \simeq 2$

7. The amount of charge in F(Faraday) required to obtain one mole of iron from Fe_3O_4 is ________. (Nearest integer)

Answer (3)

Sol. For Fe_3O_4,

$$x = \frac{+8}{3}$$

where x is oxidation state of Fe.

$$Fe_3O_4 + 8H^+ + 8e^- \longrightarrow 3Fe + 4H_2O$$

Charge required = $\frac{8}{3} \times F = \frac{8F}{3} \simeq 3F$

8. For a reaction $A \rightarrow 2B + C$ the half lives are 100 s and 50 s when the concentration of reactant A is 0.5 and 1.0 mol L^{-1} respectively. The order of the reaction is ________. (Nearest integer)

Answer (2)

Sol. $t_{1/2} \propto \frac{1}{(a_0)^{n-1}}$

$t_{1/2} = 100$ sec $a_0 = 0.5$

$t_{1/2} = 50$ sec $a_0 = 1$

$$\frac{100}{50} = \left(\frac{1}{0.5}\right)^{n-1}$$

$(2) = (2)^{n-1}$

$n - 1 = 1$

$n = 2$

9. The difference between spin only magnetic moment value of $[Co(H_2O)_6]Cl_2$ and $[Cr(H_2O)_6]Cl_3$ is ________.

Answer (0)

Sol. $Co \rightarrow 4s^2\ 3d^7$

H_2O is weak field ligand.

$Co^{+2} \rightarrow 3d^7$

$n = 3$ $\mu_1 = \sqrt{n(n+2)}$

$= \sqrt{15}$ B.M.

$Cr \rightarrow 4s^1\ 3d^5$

$Co^{+3} \rightarrow 3d^3$

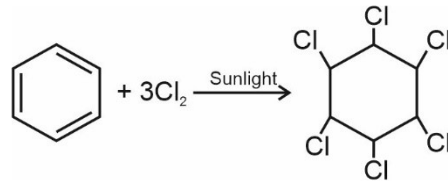
$n = 3$ $\mu_2 = \sqrt{15}$ B.M.

$\mu_1 - \mu_2 = 0$

10. In the presence of sunlight, benzene reacts with Cl_2 to give product X. The number of hydrogens in X is ________.

Answer (6)

Sol.



Total number of hydrogens are 6.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Let $f : \boldsymbol{R} \to \boldsymbol{R}$ be a continuous function such that $f(3x) - f(x) = x$. If $f(8) = 7$, then $f(14)$ is equal to

(A) 4 (B) 10

(C) 11 (D) 16

Answer (B)

Sol. $f(3x) - f(x) = x$ ...(1)

$$x \to \frac{x}{3}$$

$$f(x) - f\left(\frac{x}{3}\right) = \frac{x}{3} \quad ...(2)$$

Again $x \to \frac{x}{3}$

$$f\left(\frac{x}{3}\right) - f\left(\frac{x}{9}\right) = \frac{x}{3^2} \quad ...(3)$$

Similarly

$$f\left(\frac{x}{3^{n-2}}\right) - f\left(\frac{x}{3^{n-1}}\right) = \frac{x}{3^{n-1}} ..... (n)$$

Adding all these and applying $n \to \infty$

$$\lim_{n\to\infty}\left(f(3x) - f\left(\frac{x}{3^{n-1}}\right)\right) = x\left(1 + \frac{1}{3} + \frac{1}{3^2} + ...\right)$$

$$f(3x) - f(0) = \frac{3x}{2}$$

Putting $x = \frac{8}{3}$

$f(8) - f(0) = 4$

$\Rightarrow f(0) = 3$

Putting $x = \frac{14}{3}$

$f(14) - 3 = 7 \Rightarrow f(14) = 0$

2. Let O be the origin and A be the point $z_1 = 1 + 2i$. If B is the point z_2, $\text{Re}(z_2) < 0$, such that OAB is a right angled isosceles triangle with OB as hypotenuse, then which of the following is NOT true?

(A) $\arg z_2 = \pi - \tan^{-1}3$

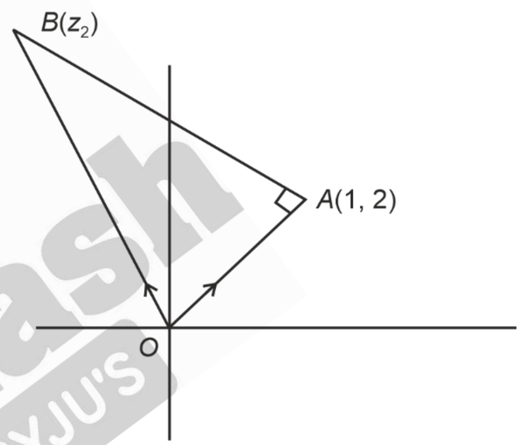
(B) $\arg (z_1 - 2z_2) = -\tan^{-1}\frac{4}{3}$

(C) $|z_2| = \sqrt{10}$

(D) $|2z_1 - z_2| = 5$

Answer (D)

Sol.



$$\frac{z_2 - 0}{(1+2i) - 0} = \frac{|OB|}{|OA|}e^{\frac{i\pi}{4}}$$

$$\Rightarrow \frac{z_2}{1+2i} = \sqrt{2}e^{\frac{i\pi}{4}}$$

OR $z_2 = (1 + 2i)(1 + i)$

$= -1 + 3i$

$\arg z_2 = \pi - \tan^{-1}3$

$|z_2| = \sqrt{10}$

$z_1 - 2z_2 = (1 + 2i) + 2 - 6i = 3 - 4i$

$$\arg(z_1 - 2z_2) = -\tan^{-1}\frac{4}{3}$$

$$|2z_1 - z_2| = |2 + 4i + 1 - 3i| = |3 + i|$$

$$= \sqrt{10}$$

3. If the system of linear equations.

$8x + y + 4z = -2$

$x + y + z = 0$

$\lambda x - 3y = \mu$

has infinitely many solutions, then the distance of the point $\left(\lambda, \mu, -\frac{1}{2}\right)$ from the plane $8x + y + 4z + 2 = 0$ is

(A) $3\sqrt{5}$ (B) 4

(C) $\frac{26}{9}$ (D) $\frac{10}{3}$

Answer (D)

Sol. $\Delta = \begin{vmatrix} 8 & 1 & 4 \\ 1 & 1 & 1 \\ \lambda & -3 & 0 \end{vmatrix}$

$= 8(3) - 1(-\lambda) + 4(-3 - \lambda)$

$= 24 + \lambda - 12 - 4\lambda$

$= 12 - 3\lambda$

So for $\lambda = 4$, it is having infinitely many solutions.

$\Delta_x = \begin{vmatrix} -2 & 1 & 4 \\ 0 & 1 & 1 \\ \mu & -3 & 0 \end{vmatrix}$

$= -2(3) - 1(-\mu) + 4(-\mu)$

$= -6 - 3\mu = 0$

For $\mu = -2$

Distance of $(4, -2, \frac{-1}{2})$ from $8x + y + 4z + 2 = 0$

$$= \frac{32 - 2 - 2 + 2}{\sqrt{64 + 1 + 16}} = \frac{10}{3} \text{ units}$$

4. Let A be a 2×2 matrix with $\det(A) = -1$ and $\det((A + I)(\text{Adj}(A) + I)) = 4$. Then the sum of the diagonal elements of A can be

(A) -1 (B) 2

(C) 1 (D) $-\sqrt{2}$

Answer (B)

Sol. $|(A + I)(\text{adj } A + I)| = 4$

$\Rightarrow |A \text{ adj } A + A + \text{adj } A + I| = 4$

$\Rightarrow |(A)I + A + \text{adj } A + I| = 4$

$|A| = -1 \Rightarrow |A + \text{adj } A| = 4$

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \text{adj } A = \begin{bmatrix} a & -b \\ -c & d \end{bmatrix}$

$\Rightarrow \begin{vmatrix} (a+d) & 0 \\ 0 & (a+d) \end{vmatrix} = 4$

$\Rightarrow a + d = \pm 2$

5. The odd natural number a, such that the area of the region bounded by $y = 1$, $y = 3$, $x = 0$, $x = y^a$ is $\frac{364}{3}$, is equal to

(A) 3 (B) 5

(C) 7 (D) 9

Answer (B)

Sol. a is a odd natural number and

$$\left|\int_1^3 y^a dy\right| = \frac{364}{3}$$

$$\Rightarrow \left|\frac{1}{a+1}\left(y^{a+1}\right)_1^3\right| = \frac{364}{3}$$

$$\Rightarrow \frac{3^{a+1} - 1}{a+1} = \pm\frac{364}{3}$$

Solving with (–) sign,

$$\frac{3^{a+1} - 1}{a+1} = \frac{364}{3} \Rightarrow (a = 5)$$

Solving with (+) sign,

$$\frac{3^{a+1} - 1}{a+1} = \frac{-364}{3}, \text{ No } a \text{ exist}$$

$\therefore (a = 5)$

6. Consider two G.Ps. $2, 2^2, 2^3, \ldots$ and $4, 4^2, 4^3, \ldots$ of 60 and n terms respectively. If the geometric mean of all the $60 + n$ terms is $(2)^{\frac{225}{8}}$, then $\sum_{k=1}^{n} k(n-k)$ is equal to

(A) 560 (B) 1540

(C) 1330 (D) 2600

Answer (C)

Sol. Given G.P's $2, 2^2, 2^3, \ldots$ 60 terms

$4, 4^2, \ldots n$ terms

Now, $\text{G.M} = 2^{\frac{225}{8}}$

$$\left(2.2^2 \ldots 4.4^2 \ldots\right)^{\frac{1}{60+n}} = 2^{\frac{225}{8}}$$

$$\left(2^{\frac{n^2+n+1830}{60+n}}\right)=2^{\frac{225}{8}}$$

$$\Rightarrow \frac{n^2+n+1830}{60+n}=\frac{225}{8}$$

$\Rightarrow$ $8n^2 - 217n + 1140 = 0$

$$n=\frac{57}{8}, 20, \text{ so } n=20$$

$$\therefore \sum_{k=1}^{20} k(20-k)=20\times\frac{20\times21}{2}-\frac{20\times21\times41}{6}$$

$$=\frac{20\times21}{2}\left[20-\frac{41}{3}\right]=1330$$

7. If the function

$$f(x)=\begin{cases}\dfrac{\log_e(1-x+x^2)+\log_e(1+x+x^2)}{\sec x-\cos x}, & x\in\left(\dfrac{-\pi}{2},\dfrac{\pi}{2}\right)-\{0\}\\ k &, x=0\end{cases}$$

is continuous at $x = 0$, then k is equal to

(A) 1

(B) –1

(C) e

(D) 0

Answer (A)

Sol. $$f(x)=\begin{cases}\dfrac{\log_e(1-x+x^2)+\log_e(1+x+x^2)}{\sec x-\cos x}, & x\in\left(\dfrac{-\pi}{2},\dfrac{\pi}{2}\right)-\{0\}\\ k &, x=0\end{cases}$$

for continuity at $x = 0$

$$\lim_{x\to0} f(x)=k$$

$$\therefore\ k=\lim_{x\to0}\frac{\log_e(x^4+x^2+1)}{\sec x-\cos x}\left(\frac{0}{0}\text{ form}\right)$$

$$=\lim_{x\to0}\frac{\cos x\log_e(x^4+x^2+1)}{\sin^2 x}$$

$$=\lim_{x\to0}\frac{\log_e(x^4+x^2+1)}{x^2}$$

$$=\lim_{x\to0}\frac{\ln(1+x^2+x^4)}{x^2+x^4}\cdot\frac{x^2+x^4}{x^2}$$

$= 1$

8. If

$$f(x)=\begin{cases}x+a &, x\le0\\ |x-4| &, x>0\end{cases} \text{ and } g(x)=\begin{cases}x+1 &, x<0\\ (x-4)^2+b &, x\ge0\end{cases}$$

are continuous on ***R***, then $(gof)(2) + (fog)(-2)$ is equal to

(A) –10 (B) 10

(C) 8 (D) –8

Answer (D)

Sol. $$f(x)=\begin{cases}x+a &, x\le0\\ |x-4| &, x>0\end{cases} \text{ and } g(x)=\begin{cases}x+1 &, x<0\\ (x-4)^2+b, & x\ge0\end{cases}$$

$\because$ $f(x)$ and $g(x)$ are continuous on R

$\therefore$ $a = 4$ and $b = 1 - 16 = -15$

then $(gof)(2) + (fog)(-2)$

$= g(2) + f(-1)$

$= -11 + 3 = -8$

9. Let $f(x)=\begin{cases}x^3-x^2+10x-7, & x\le1\\ -2x+\log_2(b^2-4), & x>1\end{cases}$.

Then the set of all values of b, for which $f(x)$ has maximum value at $x = 1$, is

(A) (–6, –2)

(B) (2, 6)

(C) [–6, –2) ∪ (2, 6]

(D) $\left[-\sqrt{6},-2\right)\cup\left(2,\sqrt{6}\right]$

Answer (C)

Sol. $$f(x)=\begin{cases}x^3-x^2+10x-7, & x\le1\\ -2x+\log_2(b^2-4), & x>1\end{cases}$$

If $f(x)$ has maximum value at $x = 1$ then $f(1+) \le f(1)$

$-2 + \log_2(b^2 - 4) \le 1 - 1 + 10 - 7$

$\log_2(b^2 - 4) \le 5$

$0 < b^2 - 4 \le 32$

(i) $b^2 - 4 > 0 \Rightarrow b \in (-\infty, -2) \cup (2, \infty)$

(ii) $b^2 - 36 \le 0 \Rightarrow b \in [-6, 6]$

Intersection of above two sets

$b \in [-6, -2) \cup (2, 6]$

10. If $a = \lim_{n\to\infty}\sum_{k=1}^{n}\frac{2n}{n^2+k^2}$ and

$f(x) = \sqrt{\frac{1-\cos x}{1+\cos x}}, x \in (0, 1)$, then

(A) $2\sqrt{2}f\left(\frac{a}{2}\right) = f'\left(\frac{a}{2}\right)$ (B) $f\left(\frac{a}{2}\right)f'\left(\frac{a}{2}\right) = \sqrt{2}$

(C) $\sqrt{2}\,f\left(\frac{a}{2}\right) = f'\left(\frac{a}{2}\right)$ (D) $f\left(\frac{a}{2}\right) = \sqrt{2}f'\left(\frac{a}{2}\right)$

Answer (C)

Sol. $a = \lim_{n\to\infty}\sum_{k=1}^{n}\frac{2n}{n^2+k^2}$

$$= \lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{2}{1+\left(\frac{k}{n}\right)^2}$$

$$a = \int_0^1 \frac{2}{1+x^2}dx = 2\tan^{-1}x\Big|_0^1 = \frac{\pi}{2}$$

$$f(x) = \sqrt{\frac{1-\cos x}{1+\cos x}},\ x \in (0, 1)$$

$$f(x) = \frac{1-\cos x}{\sin x} = \operatorname{cosec} x - \cot x$$

$$f'(x) = \operatorname{cosec}^2 x - \operatorname{cosec} x \cot x$$

$$\left.\begin{aligned} f\left(\frac{a}{2}\right) &= f\left(\frac{\pi}{4}\right) = \sqrt{2}-1 \\ f'\left(\frac{a}{2}\right) &= f'\left(\frac{\pi}{4}\right) = 2-\sqrt{2} \end{aligned}\right\} f'\left(\frac{a}{2}\right) = \sqrt{2}.f\left(\frac{a}{2}\right)$$

11. If $\frac{dy}{dx} + 2y\tan x = \sin x, 0 < x < \frac{\pi}{2}$ and $y\left(\frac{\pi}{3}\right) = 0$, then the maximum value of $y(x)$ is:

(A) $\frac{1}{8}$ (B) $\frac{3}{4}$

(C) $\frac{1}{4}$ (D) $\frac{3}{8}$

Answer (A)

Sol. $\frac{dy}{dx} + 2y\tan x = \sin x$

which is a first order linear differential equation.

Integrating factor (I. F.) $= e^{\int 2\tan x\,dx}$

$$= e^{2\ln|\sec x|} = \sec^2 x$$

Solution of differential equation can be written as

$$y\cdot\sec^2 x = \int \sin x\cdot\sec^2 x\,dx = \int \sec x\cdot\tan x\,dx$$

$$y\sec^2 x = \sec x + C$$

$$y\left(\frac{\pi}{3}\right) = 0, 0 = \sec\frac{\pi}{3} + C \Rightarrow \quad C = -2$$

$$y = \frac{\sec x - 2}{\sec^2 x} = \cos x - 2\cos^2 x$$

$$= \frac{1}{8} - 2\left(\cos x - \frac{1}{4}\right)^2$$

$$y_{\max} = \frac{1}{8}$$

12. A point P moves so that the sum of squares of its distances from the points (1, 2) and (–2, 1) is 14. Let $f(x, y) = 0$ be the locus of P, which intersects the x-axis at the points A, B and the y-axis at the points C, D. Then the area of the quadrilateral $ACBD$ is equal to

(A) $\frac{9}{2}$ (B) $\frac{3\sqrt{17}}{2}$

(C) $\frac{3\sqrt{17}}{4}$ (D) 9

Answer (B)

Sol. Let point $P : (h, k)$

$(h-1)^2 + (k-2)^2 + (h+2)^2 + (k-1)^2 = 14$

$2h^2 + 2k^2 + 2h - 6k - 4 = 0$

Locus of $P : x^2 + y^2 + x - 3y - 2 = 0$

Intersection with x-axis,

$x^2 + x - 2 = 0$

$\Rightarrow \quad x = -2, 1$

Intersection with y-axis,

$y^2 - 3y - 2 = 0$

$$\Rightarrow \quad y = \frac{3\pm\sqrt{17}}{2}$$

Area of the quadrilateral $ACBD$ is

$$= \frac{1}{2}(|x_1|+|x_2|)(|y_1|+|y_2|)$$

$$= \frac{1}{2}\times 3\times\sqrt{17} = \frac{3\sqrt{17}}{2}$$

13. Let the tangent drawn to the parabola $y^2 = 24x$ at the point (α, β) is perpendicular to the line $2x + 2y = 5$. Then the normal to the hyperbola $\frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1$ at the point $(\alpha + 4, \beta + 4)$ does **NOT** pass through the point

(A) (25, 10) (B) (20, 12)

(C) (30, 8) (D) (15, 13)

Answer (D)

Sol. Any tangent to $y^2 = 24x$ at (α, β)

$\beta y = 12(x + \alpha)$

Slope $= \frac{12}{\beta}$ and perpendicular to $2x + 2y = 5$

$\Rightarrow \frac{12}{\beta} = 1 \quad \Rightarrow \quad \beta = 12, \alpha = 6$

Hence hyperbola is $\frac{x^2}{36} - \frac{y^2}{144} = 1$ and normal is drawn at (10, 16)

Equation of normal $\frac{36 \cdot x}{10} + \frac{144 \cdot y}{16} = 36 + 144$

$\Rightarrow \frac{x}{50} + \frac{y}{20} = 1$

This does not pass though (15, 13) out of given option

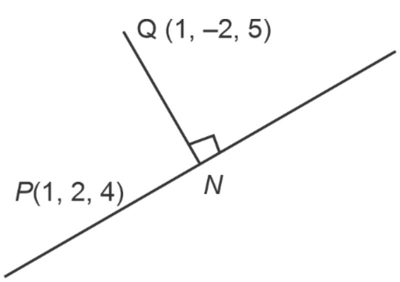
14. The length of the perpendicular from the point (1, –2, 5) on the line passing through (1, 2, 4) and parallel to the line $x + y - z = 0 = x - 2y + 3z - 5$ is

(A) $\sqrt{\frac{21}{2}}$ (B) $\sqrt{\frac{9}{2}}$

(C) $\sqrt{\frac{73}{2}}$ (D) 1

Answer (A)

Sol.



The line $x + y - z = 0 = x - 2y + 3z - 5$ is parallel to the vector

$$\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & -2 & 3 \end{vmatrix} = (1, 4, -3)$$

Equation of line through $P(1, 2, 4)$ and parallel to $\vec{b}$

$$\frac{x-1}{1} = \frac{y-2}{-4} = \frac{z-4}{-3}$$

Let $N \equiv (\lambda + 1, -4\lambda + 2, -3\lambda + 4)$

$\overrightarrow{QN} = (\lambda, -4\lambda + 4, -3\lambda - 1)$

$\overrightarrow{QN}$ is perpendicular to $\vec{b}$

$\Rightarrow (\lambda, -4\lambda + 4, -3\lambda - 1) \cdot (1, 4, -3) = 0$

$\Rightarrow \lambda = \frac{1}{2}$

Hence $\overrightarrow{QN} = \left(\frac{1}{2}, 2, \frac{-5}{2}\right)$ and $\left|\overrightarrow{QN}\right| = \sqrt{\frac{21}{2}}$

15. Let $\vec{a} = \alpha\hat{i} + \hat{j} - k$ and $\vec{b} = 2\hat{i} + \hat{j} - \alpha k, \alpha > 0$. If the projection of $\vec{a} \times \vec{b}$ on the vector $-\hat{i} + 2\hat{j} - 2k$ is 30, then α is equal to

(A) $\frac{15}{2}$ (B) 8

(C) $\frac{13}{2}$ (D) 7

Answer (D)

Sol. Given : $\vec{a} = (\alpha, 1, -1)$ and $\vec{b} = (2, 1, -\alpha)$

$$\vec{c} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 1 & -1 \\ 2 & 1 & -\alpha \end{vmatrix}$$

$$= (-\alpha + 1)\hat{i} + \left(\alpha^2 - 2\right)\hat{j} + (\alpha - 2)\hat{k}$$

Projection of $\vec{c}$ on $\vec{d} = -\hat{i} + 2\hat{j} - 2\hat{k}$

$$= \left|\vec{c} \cdot \frac{\vec{d}}{|\vec{d}|}\right| = 30 \text{ \{Given\}}$$

$$\Rightarrow \quad = \left|\frac{\alpha - 1 - 4 + 2\alpha^2 - 2\alpha + 4}{\sqrt{1 + 4 + 4}}\right| = 30$$

On solving $\alpha = \frac{-13}{2}$ (Rejected as $\alpha > 0$)

and $\alpha = 7$

16. The mean and variance of a binomial distribution are α and $\frac{\alpha}{3}$ respectively. If $P(X=1)=\frac{4}{243}$, then $P(X = 4 \text{ or } 5)$ is equal to :

(A) $\frac{5}{9}$ (B) $\frac{64}{81}$

(C) $\frac{16}{27}$ (D) $\frac{145}{243}$

Answer (C)

Sol. Given, mean = $np = \alpha$.

and variance $= npq = \frac{\alpha}{3}$

$\Rightarrow q=\frac{1}{3}$ and $p=\frac{2}{3}$

$P(X=1)=n.p^1.q^{n-1}=\frac{4}{243}$

$\Rightarrow n.\frac{2}{3}.\left(\frac{1}{3}\right)^{n-1}=\frac{4}{243}$

$\Rightarrow n = 6$

$P(X=4 \text{ or } 5)={}^6C_4.\left(\frac{2}{3}\right)^4.\left(\frac{1}{3}\right)^2+{}^6C_5.\left(\frac{2}{5}\right)^5.\frac{1}{3}$

$=\frac{16}{27}$

17. Let E_1, E_2, E_3 be three mutually exclusive events such that $P(E_1)=\frac{2+3p}{6}, P(E_2)=\frac{2-p}{8}$ and $P(E_3)=\frac{1-p}{2}$. If the maximum and minimum values of p are p_1 and p_2, then $(p_1 + p_2)$ is equal to :

(A) $\frac{2}{3}$ (B) $\frac{5}{3}$

(C) $\frac{5}{4}$ (D) 1

Answer (B)

Sol. $0\le\frac{2+3P}{6}\le1 \Rightarrow P\in\left[-\frac{2}{3},\frac{4}{3}\right]$

$0\le\frac{2-P}{8}\le1 \Rightarrow P\in[-6,2]$

$0\le\frac{1-P}{2}\le1 \Rightarrow P\in[-1,1]$

$0 < P(E_1) + P(E_2) + P(E_3) \le 1$

$0<\frac{13}{12}-\frac{P}{8}\le1$

$P\in\left[\frac{2}{3},\frac{26}{3}\right]$

Taking intersection of all

$P\in\left[\frac{2}{3},1\right)$

$P_1+P_2=\frac{5}{3}$

18. Let $S=\left\{\theta\in[0,2\pi]:8^{2\sin^2\theta}+8^{2\cos^2\theta}=16\right\}$. Then $n(S)+\sum_{\theta\in S}\left(\sec\left(\frac{\pi}{4}+2\theta\right)\text{cosec}\left(\frac{\pi}{4}+2\theta\right)\right)$ is equal to :

(A) 0 (B) – 2

(C) – 4 (D) 12

Answer (C)

Sol. $S=\left\{\theta\in[0,2\pi]:8^{2\sin^2\theta}+8^{2\cos^2\theta}=16\right\}$

Now apply AM ≥ GM for $8^{2\sin^2\theta}, 8^{2\cos^2\theta}$

$\frac{8^{2\sin^2\theta}+8^{2\cos^2\theta}}{2}\ge\left(8^{2\sin^2\theta+2\cos^2\theta}\right)^{\frac{1}{2}}$

$8\ge8$

$\Rightarrow 8^{2\sin^2\theta}=8^{2\cos^2\theta}$

or $\sin^2\theta = \cos^2\theta$

$\therefore \theta=\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}$

$n(S)+\sum_{\theta\in S}\sec\left(\frac{\pi}{4}+2\theta\right)\text{cosec}\left(\frac{\pi}{4}+2\theta\right)$

$4+\sum_{\theta\in S}\frac{2}{2\sin\left(\frac{\pi}{4}+2\theta\right)\cos\left(\frac{\pi}{4}+2\theta\right)}$

$=4+\sum_{\theta\in S}\frac{2}{\sin\left(\frac{\pi}{2}+4\theta\right)}=4+2\sum_{\theta\in S}\text{cosec}\left(\frac{\pi}{2}+4\theta\right)$

$=4+2\left[\text{cosec}\left(\frac{\pi}{2}+\pi\right)+\text{cosec}\left(\frac{\pi}{2}+3\pi\right)+\text{cosec}\left(\frac{\pi}{2}+5\pi\right)+\text{cosec}\left(\frac{\pi}{2}+7\pi\right)\right]$

$$= 4 + 2\left[-\operatorname{cosec}\frac{\pi}{2} - \operatorname{cosec}\frac{\pi}{2} - \operatorname{cosec}\frac{\pi}{2} - \operatorname{cosec}\frac{\pi}{2}\right]$$

$= 4 – 2(4)$

$= 4 – 8$

$= – 4$

19. $\tan\left(2\tan^{-1}\frac{1}{5} + \sec^{-1}\frac{\sqrt{5}}{2} + 2\tan^{-1}\frac{1}{8}\right)$ is equal to :

(A) 1

(B) 2

(C) $\frac{1}{4}$

(D) $\frac{5}{4}$

Answer (B)

Sol. $\tan\left(2\tan^{-1}\frac{1}{5} + \sec^{-1}\frac{\sqrt{5}}{2} + 2\tan^{-1}\frac{1}{8}\right)$

$$= \tan\left(2\tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{8}}{1-\frac{1}{5}\cdot\frac{1}{8}}\right) + \sec^{-1}\frac{\sqrt{5}}{2}\right)$$

$$= \tan\left[2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}\right]$$

$$= \tan\left[\tan^{-1}\frac{\frac{2}{3}}{1-\frac{1}{9}} + \tan^{-1}\frac{1}{2}\right]$$

$$= \tan\left[\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{1}{2}\right]$$

$$= \tan\left[\tan^{-1}\frac{\frac{3}{4}+\frac{1}{2}}{1-\frac{3}{8}}\right] = \tan\left[\tan^{-1}\frac{\frac{5}{4}}{\frac{5}{8}}\right]$$

$$= \tan\left[\tan^{-1}2\right] = 2$$

20. The statement $(\sim(p \Leftrightarrow \sim q)) \wedge q$ is :

(A) a tautology

(B) a contradiction

(C) equivalent to $(p \Rightarrow q) \wedge q$

(D) equivalent to $(p \Rightarrow q) \wedge p$

Answer (D)

Sol. $\sim(p \Leftrightarrow \sim q) \wedge q$

$= (p \Leftrightarrow q) \wedge q$

p	q	$p\leftrightarrow q$	$(p\leftrightarrow q)\wedge q$	$(p\rightarrow q)$	$(p\rightarrow q)\wedge q$	$(p\rightarrow q)\wedge p$
T	T	T	T	T	T	T
T	F	F	F	F	F	F
F	T	F	F	T	T	F
F	F	T	F	T	F	F

$\therefore$ $(\sim(p \Leftrightarrow \sim q)) \wedge q$ is equivalent to $(p \Rightarrow q) \wedge p$.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. If for some $p, q, r \in \boldsymbol{R}$, not all have same sign, one of the roots of the equation $(p^2 + q^2)x^2 – 2q(p + r)x + q^2 + r^2 = 0$ is also a root of the equation $x^2 + 2x – 8 = 0$, then $\frac{q^2+r^2}{p^2}$ is equal to ___________.

Answer (272)

Sol. Let roots of $(p^2 + q^2)x^2 – 2q(p + r)x + q^2 + r^2 = 0$ with roots α, β

$\therefore$ $\alpha + \beta > 0$ and $\alpha\beta > 0$

Also, it has a common root with $x^2 + 2x – 8 = 0$

$\therefore$ The common root between above two equations is 4.

$\Rightarrow$ $16(p^2 + q^2) – 8q(p + r) + q^2 + r^2 = 0$

$\Rightarrow$ $(16p^2 – 8pq + q^2) + (16q^2 – 8qr + r^2) = 0$

$\Rightarrow$ $(4p – q)^2 + (4q – r)^2 = 0$

$\Rightarrow$ $q = 4p$ and $r = 16p$

$$\therefore \quad \frac{q^2+r^2}{p^2} = \frac{16p^2+256p^2}{p^2} = 272$$

2. The number of 5-digit natural numbers, such that the product of their digits is 36, is __________.

Answer (180)

Sol. Factors of $36 = 2^2 \cdot 3^2 \cdot 1$

Five-digit combinations can be

(1, 2, 2, 3, 3) (1, 4, 3, 3, 1), (1, 9, 2, 2, 1)

(1, 4, 9, 11) (1, 2, 3, 6, 1) (1, 6, 6, 1, 1)

i.e., total numbers

$$\frac{5!}{2!2!}+\frac{5!}{2!2!}+\frac{5!}{2!2!}+\frac{5!}{3!}+\frac{5!}{2!}+\frac{5!}{3!2!}$$

$= (30 \times 3) + 20 + 60 + 10 = 180.$

3. The series of positive multiples of 3 is divided into sets: {3}, {6, 9, 12}, {15, 18, 21, 24, 27},…… Then the sum of the elements in the 11th set is equal to ________.

Answer (6993)

Sol. Given series

$$\underbrace{\{3\times1\}}_{\text{1–term}},\underbrace{\{3\times2,3\times3,3\times4\}}_{\text{3–terms}},\underbrace{\{3\times5,3\times6,3\times7,3\times8,3\times9\}}_{\text{5–terms}},....$$

$\therefore$ 11th set will have 1 + (10)2 = 21 term

Also upto 10th set total 3 × k type terms will be 1 + 3 + 5 +……+19 = 100 – term

$\therefore$ Set 11 = {3 × 101, 3 × 102,……3 × 121}

$\therefore$ Sum of elements = 3 × (101 + 102 +…+121)

$$=\frac{3\times222\times21}{2}=6993$$

4. The number of distinct real roots of the equation $x^5(x^3 - x^2 - x + 1) + x(3x^3 - 4x^2 - 2x + 4) - 1 = 0$ is ________.

Answer (3)

Sol. $x^8 - x^7 - x^6 + x^5 + 3x^4 - 4x^3 - 2x^2 + 4x - 1 = 0$

$\Rightarrow x^7(x-1) - x^5(x-1) + 3x^3(x-1) - x(x^2-1) + 2x(1-x) + (x-1) = 0$

$\Rightarrow (x-1)(x^7 - x^5 + 3x^3 - x(x+1) - 2x + 1) = 0$

$\Rightarrow (x-1)(x^7 - x^5 + 3x^3 - x^2 - 3x + 1) = 0$

$\Rightarrow (x-1)(x^5(x^2-1) + 3x(x^2-1) - 1(x^2-1)) = 0$

$\Rightarrow (x-1)(x^2-1)(x^5 + 3x - 1) = 0$

$\therefore$ $x = \pm 1$ are roots of above equation and $x^5 + 3x - 1$ is a monotonic term hence vanishes at exactly one value of x other then 1 or – 1.

$\therefore$ 3 real roots.

5. If the coefficients of x and x^2 – in the expansion of $(1 + x)^p (1 - x)^q$, p, $q \le 15$, are – 3 and – 5 respectively, then coefficient of x^3 is equal to ______.

Answer (23)

Sol. Coefficient of x in $(1 + x)^p (1 - x)^q$

$$-{}^pC_0\,{}^qC_1+{}^pC_1\,{}^qC_0=-3\Rightarrow\boxed{p-q=-3}$$

Coefficient of x^2 in $(1 + x)^p (1 - x)^q$

$${}^pC_0\,{}^qC_2-{}^pC_1{}^qC_1+{}^pC_2\,{}^qC_0=-5$$

$$\frac{q(q-1)}{2}-pq+\frac{p(p-1)}{2}=-5$$

$$\frac{q^2-q}{2}-(q-3)q+\frac{(q-3)(q-4)}{2}=-5$$

$\Rightarrow q = 11, p = 8$

Coefficient of x^3 in $(1 + x)^8 (1 - x)^{11}$ is

$$=-{}^{11}C_3+{}^8C_1\,{}^{11}C_2-{}^8C_2\,{}^{11}C_1+{}^8C_3=23$$

6. If $n(2n+1)\int_0^1\left(1-x^n\right)^{2n}dx=1177\int_0^1(1-x^n)^{2n+1}dx$, then $n\in\boldsymbol{N}$ is equal to ______.

Answer (24)

Sol. $\int_0^1(1-x^n)^{2n+1}dx=\int_0^1 1\cdot(1-x^n)^{2n+1}dx$

$$=\left[(1-x^n)^{2n+1}\cdot x\right]_0^1-\int_0^1 x\cdot(2n+1)(1-x^n)^{2n}\cdot -nx^{n-1}dx$$

$$=n(2n+1)\int_0^1\Big(1-(1-x^n)\Big)(1-x^n)^{2n}dx$$

$$=n(2n+1)\int_0^1(1-x^n)^{2n}dx-n(2n+1)\int_0^1(1-x^n)^{2n+1}dx$$

$$(1+n(2n+1))\int_0^1(1-x^n)^{2n+1}dx=n(2n+1)\int_0^1(1-x^n)^{2n}dx$$

$$(2n^2+n+1)\int_0^1(1-x^n)^{2n+1}dx=1177\int_0^1(1-x^n)^{2n+1}dx$$

$\therefore$ $2n^2 + n + 1 = 1177$

$2n^2 + n - 1176 = 0$

$\therefore$ $n = 24$ or $-\frac{49}{2}$

$\therefore$ $n = 24$

7. Let a curve $y = y(x)$ pass through the point (3, 3) and the area of the region under this curve, above the x-axis and between the abscissae 3 and $x(>3)$ be $\left(\frac{y}{x}\right)^3$. If this curve also passes through the point $\left(\alpha, 6\sqrt{10}\right)$ in the first quadrant, then α is equal to ________.

Answer (6)

Sol. $\int_3^x f(x)\,dx = \left(\frac{f(x)}{x}\right)^3$

$x^3 \cdot \int_3^x f(x)dx = f^3(x)$

Differentiate w.r.t. x

$x^3 f(x) + 3x^2 \cdot \frac{f^3(x)}{x^3} = 3f^2(x)f'(x)$

$\Rightarrow \quad 3y^2 \frac{dy}{dx} = x^3 y + \frac{3y^3}{x}$

$3xy\frac{dy}{dx} = x^4 + 3y^2$

Let $y^2 = t$

$\frac{3}{2}\frac{dt}{dx} = x^3 + \frac{3t}{x}$

$\frac{dt}{dx} - \frac{2t}{x} = \frac{2x^3}{3}$

$I.F. = \cdot e^{\int -\frac{2}{x}dx} = \frac{1}{x^2}$

Solution of differential equation

$t \cdot \frac{1}{x^2} = \int \frac{2}{3} x\, dx$

$\frac{y^2}{x^2} = \frac{x^2}{3} + C$

$y^2 = \frac{x^4}{3} + Cx^2$

Curve passes through (3, 3) $\Rightarrow C = -2$

$y^2 = \frac{x^4}{3} - 2x^2$

Which passes through $\left(\alpha, 6\sqrt{10}\right)$

$\frac{\alpha^4 - 6\alpha^2}{3} = 360$

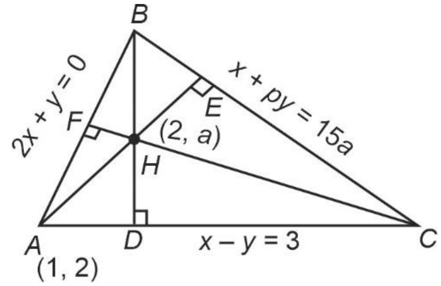
$\alpha^4 - 6\alpha^2 - 1080 = 0$

$\alpha = 6$

8. The equations of the sides AB, BC and CA of a triangle ABC are $2x + y = 0$, $x + py = 15a$ and $x - y = 3$ respectively. If its orthocentre is $(2, a), -\frac{1}{2} < a < 2$, then p is equal to _______.

Answer (3)

Sol.



Slope of $AH = \frac{a+2}{1}$

Sloe of $BC = -\frac{1}{p}$

$\therefore \quad p = a + 2 \qquad \ldots(i)$

Coordinate of $C = \left(\frac{18p-30}{p+1}, \frac{15p-33}{p+1}\right)$

Slope of HC

$= \dfrac{\frac{15p-33}{p+1} - a}{\frac{18p-30}{p+1} - 2} = \frac{15p - 33 - (p-2)(p+1)}{18p - 30 - 2p - 2}$

$= \frac{16p - p^2 - 31}{16p - 32}$

$\because \quad \frac{16p - p^2 - 31}{16p - 32} \times -2 = -1$

$\therefore \quad p^2 - 8p + 15 = 0$

$\therefore \quad p = 3$ or 5

But if $p = 5$ then $a = 3$ not acceptable

$\therefore \quad p = 3$

9. Let the function $f(x) = 2x^2 - \log_e x$, $x > 0$, be decreasing in (0, a) and increasing in (a, 4). A tangent to the parabola $y^2 = 4ax$ at a point P on it passes through the point (8a, 8a –1) but does not pass through the point $\left(-\frac{1}{a}, 0\right)$. If the equation of the normal at P is $\frac{x}{\alpha} + \frac{y}{\beta} = 1$, then $\alpha + \beta$ is equal to _________.

Answer (45)

Sol. $\delta'(x) = \dfrac{4x^2 - 1}{x}$ so f(x) is decreasing in $\left(0, \dfrac{1}{2}\right)$ and increasing in $\left(\dfrac{1}{2}, \infty\right) \Rightarrow a = \dfrac{1}{2}$

Tangent at $y^2 = 2x \Rightarrow y = mx + \dfrac{1}{2m}$

It is passing through (4, 3)

$3 = 4m + \dfrac{1}{2m} \Rightarrow m = \dfrac{1}{2} \text{or } \dfrac{1}{4}$

So tangent may be

$y = \dfrac{1}{2}x + 1 \text{ or } y = \dfrac{1}{4}x + 2$

But $y = \dfrac{1}{2}x + 1$ passes through (–2, 0) so rejected.

Equation of Normal

$y = -4x - 2\left(\dfrac{1}{2}\right)(-4) - \dfrac{1}{2}(-4)^3$

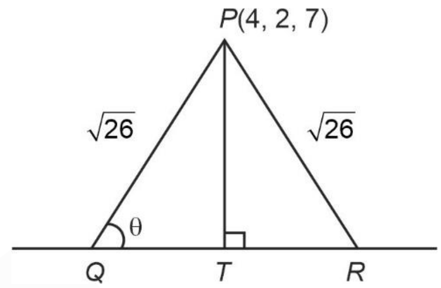
or $y = -4x + 4 + 32$

or $\dfrac{x}{9} + \dfrac{y}{36} = 1$

10. Let Q and R be two points on the line $\dfrac{x+1}{2} = \dfrac{y+2}{3} = \dfrac{z-1}{2}$ at a distance $\sqrt{26}$ from the point P(4, 2, 7). Then the square of the area of the triangle PQR is ________.

Answer (153)

Sol. $L : \dfrac{x+1}{2} = \dfrac{y+2}{3} = \dfrac{2-1}{2}$



Let $T(2t - 1, 3t - 2, 2t + 1)$

$\because$ $PT \perp^r QR$

$\therefore$ $2(2t - 5) + 3(3t - 4) + 2(2t - 6) = 0$

$17t = 34$ $\therefore$ $\boxed{t = 2}$ So $T(3, 4, 5)$

$\therefore$ $PT = \sqrt{1+4+4} = 3$

$\therefore$ $QT = \sqrt{26-9} = \sqrt{17}$

$\therefore$ Area of $\Delta PQR = \dfrac{1}{2} \times 2\sqrt{17} \times 3 = 3\sqrt{17}$

$\therefore$ Square of ar(ΔPQR) = 153.

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