27/07/2022
Morning

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Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2022 (Online) Phase-2

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. A torque meter is calibrated to reference standards of mass, length and time each with 5% accuracy. After calibration, the measured torque with this torque meter will have net accuracy of

(A) 15% (B) 25%

(C) 75% (D) 5%

Answer (B)

Sol. $[\tau] = [M^1L^2T^{-2}]$

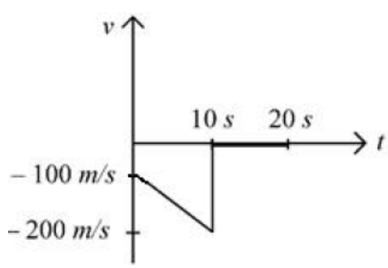
$$\Rightarrow \frac{\Delta\tau}{\tau} = \frac{\Delta M}{M} + 2\frac{\Delta L}{L} + 2\frac{\Delta T}{T}$$

= 5 × 5% = 25%

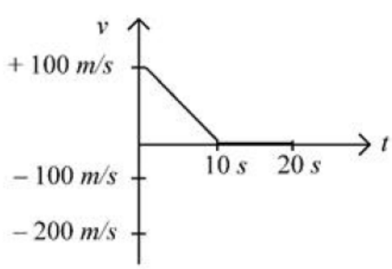
2. A bullet is shot vertically downwards with an initial velocity of 100 m/s from a certain height. Within 10s, the bullet reaches the ground and instantaneously comes to rest due to the perfectly inelastic collision. The velocity-time curve for total time t = - 20s will be

(Take g = 10 m/s²)

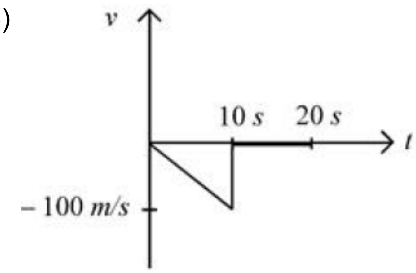
(A)



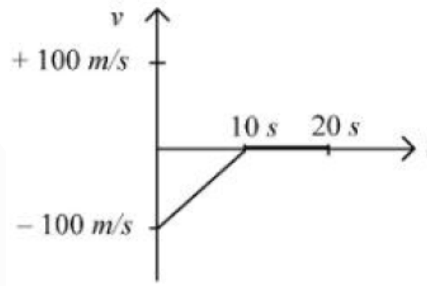
(B)



(C)



(D)



Answer (A)

Sol. $|v_{10}|$ = (100 + 10 × 10) m/s

v_{10} = – 200 m/s and v_0 = – 100 m/s

from 10s to 20s velocity remains zero

⇒ from t = 0 s to 10 s velocity increases in magnitude linearly.

⇒ graph given in option A fits correctly

3. Sand is being dropped from a stationary dropper at a rate of 0.5 kgs^{-1} on a conveyor belt moving with a velocity of 5 ms^{-1}. The power needed to keep the belt moving with the same velocity will be

(A) 1.25 W (B) 2.5 W

(C) 6.25 W (D) 12.5 W

Answer (D)

Sol. $\frac{dm}{dt} = 0.5$ kg/s

v = 5 m/s

$$F = \frac{vdm}{dt} = 2.5 \text{ kg m/s}^2$$

$$P = \bar{F}\cdot\bar{v} = (2.5)(5) \text{ W}$$

= 12.5 W

4. A bag is gently dropped on a conveyor belt moving at a speed of 2 m/s. The coefficient of friction between the conveyor belt and bag is 0.4. Initially the bag slips on the belt before it stops due to friction. The distance travelled by the bag on the belt during slipping motion, is

[Take g = 10 m/s^2]

(A) 2 m (B) 0.5 m

(C) 3.2 m (D) 0.8 ms

Answer (B)

Sol. v = 2 m/s

$\mu = 0.4$

$a = +(0.4)(g)$

$= +4$ m/s^2

$v^2 - u^2 = 2as$

$\Rightarrow$ (4) = 2 × (4) (s)

s = 0.5 m

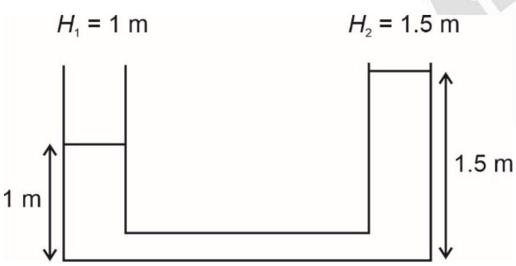
5. Two cylindrical vessels of equal cross-sectional area 16 cm^2 contain water upto heights 100 cm and 150 cm respectively. The vessels are interconnected so that the water levels in them become equal. The work done by the force of gravity during the process, is [Take, density of water = 10^3 kg/m^3 and g = 10 ms^{-2}]

(A) 0.25 J (B) 1 J

(C) 8 J (D) 12 J

Answer (B)

Sol. $A = 16 \times 10^{-4}$ m^2



$$E_{in} = m_1 g\frac{H_1}{2} + m_2 g\frac{H_2}{2}$$

$$= \rho g\frac{A}{2}\left(H_1^2 + H_2^2\right) = \rho g\frac{A}{2}\left(1^2 + 1.5^2\right)$$

$$E_{fin} = \rho g\frac{A}{2}(2H^2) = \rho g\frac{A}{2}(2\times 1.25^2)$$

$$W = \rho g\frac{A}{2}(3.25 - 3.125)$$

= 1 J

6. Two satellites A and B, having masses in the ratio 4 : 3, are revolving in circular orbits of radii 3r and 4r respectively around the earth. The ratio of total mechanical energy of A to B is

(A) 9 : 16 (B) 16 : 9

(C) 1 : 1 (D) 4 : 3

Answer (B)

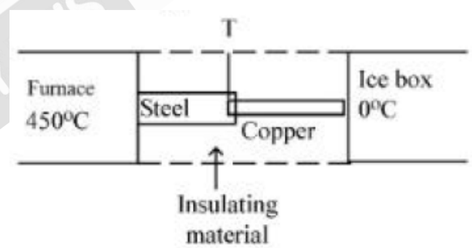
Sol. $U = -\dfrac{GM_e m}{2r}$

So, $\dfrac{U_A}{U_B} = \dfrac{m_A}{m_B}\times\dfrac{r_B}{r_A}$

$$= \frac{4}{3}\times\frac{4}{3} = \frac{16}{9}$$

7. If K_1 and K_2 are the thermal conductivities, L_1 and L_2 are the lengths and A_1 and A_2 are the cross sectional areas of steel and copper rods respectively such that $\dfrac{K_2}{K_1} = 9, \dfrac{A_1}{A_2} = 2, \dfrac{L_1}{L_2} = 2$.

Then, for the arrangement as shown in the figure, the value of temperature T of the steel-copper junction in the steady state will be



(A) 18°C (B) 14°C

(C) 45°C (D) 150°C

Answer (C)

Sol. $450 - T = \dfrac{dQ}{dt}\times\dfrac{l_1}{K_1A_1}$

$$T - 0 = \frac{dQ}{dt}\times\frac{l_2}{K_2A_2}$$

So, $\dfrac{450 - T}{T} = \dfrac{K_2A_2l_1}{K_1A_1l_2} = 9\times\dfrac{1}{2}\times 2 = 9$

$450 - T = 9T$

$\Rightarrow$ T = 45°C

8. Read the following statements:

A. When small temperature difference between a liquid and its surrounding is doubled, the rate of loss of heat of the liquid becomes twice.

B. Two bodies P and Q having equal surface areas are maintained at temperature 10°C and 20°C. The thermal radiation emitted in a given time by P and Q are in the ratio 1 : 1.15.

C. A carnot Engine working between 100 K and 400 K has an efficiency of 75%.

D. When small temperature difference between a liquid and its surrounding is quadrupled, the rate of loss of heat of the liquid becomes twice.

Choose the correct answer from the options given below

(A) A, B, C only (B) A, B only

(C) A, C only (D) B, C, D only

Answer (A)

Sol. From Newton's cooling law $\frac{dQ}{dt} = -k(T - T_s)$ the statement A is correct

For *B*

$U = \sigma e A T^4$

So, $\frac{U_1}{U_2} = \left(\frac{283}{293}\right)^4 \simeq \frac{1}{1.15}$

Statement B is correct

For *C*

$$\eta = 1 - \frac{T_1}{T_2} = 1 - \frac{100}{400} = \frac{3}{4}$$

So, efficiency is 75% C is correct

For *D*

From Newton's law of cooling $\frac{dQ}{dt} = -k(T - T_s)$

The statement is wrong

9. Same gas is filled in two vessels of the same volume at the same temperature. If the ratio of the number of molecules is 1 : 4, then

A. The r.m.s. velocity of gas molecules in two vessels will be the same.

B. The ratio of pressure in these vessels will be 1 : 4.

C. The ratio of pressure will be 1 : 1.

D. The r.m.s. velocity of gas molecules in two vessels will be in the ratio of 1 : 4.

Choose the correct answer from the options given below

(A) A and C only (B) B and D only

(C) A and B only (D) C and D only

Answer (C)

Sol. $v_{rms} = \sqrt{\frac{3RT}{M_0}}$ because T is same

v_{rms} will be same so, A is correct D is incorrect

$$\frac{P_1}{P_2} = \frac{n_1RT_1/V_1}{n_2RT_2/V_2} = \frac{n_1}{n_2} = \frac{1}{4}$$

B is correct

C is incorrect

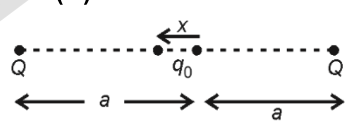
10. Two identical positive charges Q each are fixed at a distance of '$2a$' apart from each other. Another point charge q_0 with mass 'm' is placed at midpoint between two fixed charges. For a small displacement along the line joining the fixed charges, the charge q_0 executes SHM. The time period of oscillation of charge q_0 will be

(A) $\sqrt{\frac{4\pi^3\varepsilon_0 ma^3}{q_0Q}}$ (B) $\sqrt{\frac{q_0Q}{4\pi^3\varepsilon_0 ma^3}}$

(C) $\sqrt{\frac{2\pi^2\varepsilon_0 ma^3}{q_0Q}}$ (D) $\sqrt{\frac{8\pi^3\varepsilon_0 ma^3}{q_0Q}}$

Answer (A)

Sol.



(x << a) (α is acceleration)

$$F_{net} = -\left(\frac{kq_0Q}{(a-x)^2} - \frac{kQq_0}{(a+x)^2}\right)$$

$$m\alpha = -\frac{kq_0Q}{a^4}4ax$$

$$\Rightarrow \quad \alpha = -\frac{4kq_0Q}{ma^3}x$$

So, $T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 ma^3}{4q_0Q}}$

or $T = \sqrt{\frac{4\pi^3\varepsilon_0 ma^3}{q_0Q}}$

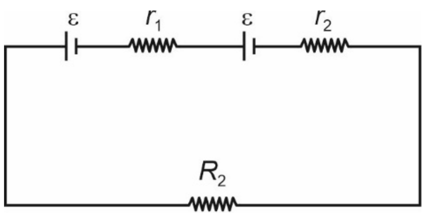
11. Two sources of equal emfs are connected in series. This combination is connected to an external resistance R. The internal resistances of the two sources are r_1 and r_2 ($r_1 > r_2$). If the potential difference across the source of internal resistance r_1 is zero, then the value of R will be :

(A) $r_1 - r_2$ (B) $\frac{r_1 r_2}{r_1 + r_2}$

(C) $\frac{r_1 + r_2}{2}$ (D) $r_2 - r_1$

Answer (A)

Sol.



$$\Delta V = 0 \Rightarrow \frac{2\varepsilon}{r_1 + r_2 + R} r_1 = \varepsilon$$

$$\Rightarrow R = r_1 - r_2$$

12. Two bar magnets oscillate in a horizontal plane in earth's magnetic field with time periods of 3 s and 4 s respectively. If their moments of inertia are in the ratio of 3 : 2, then the ratio of their magnetic moments will be :

(A) 2 : 1 (B) 8 : 3

(C) 1 : 3 (D) 27 : 16

Answer (B)

Sol. $T = 2\pi\sqrt{\frac{I}{MB_H}}$

$$\Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{I_1}{I_2}}\sqrt{\frac{M_2}{M_1}}$$

$$\Rightarrow \frac{3}{4} = \sqrt{\frac{3}{2}}\sqrt{\frac{M_2}{M_1}}$$

$$\Rightarrow \frac{M_1}{M_2} = \frac{3}{2} \times \frac{16}{9} = \frac{8}{3}$$

13. A magnet hung at 45° with magnetic meridian makes an angle of 60° with the horizontal. The actual value of the angle of dip is

(A) $\tan^{-1}\left(\sqrt{\frac{3}{2}}\right)$ (B) $\tan^{-1}(\sqrt{6})$

(C) $\tan^{-1}\left(\sqrt{\frac{2}{3}}\right)$ (D) $\tan^{-1}\left(\sqrt{\frac{1}{2}}\right)$

Answer (A)

Sol. $\tan 60^\circ = \frac{B_0 \sin\delta}{B_0 \cos\delta \cos 45^\circ}$

$$\Rightarrow \tan\delta = \sqrt{\frac{3}{2}}$$

$$\Rightarrow \delta = \tan^{-1}\left(\sqrt{\frac{3}{2}}\right)$$

14. A direct current of 4 A and an alternating current of peak value 4 A flow through resistance of 3 Ω and 2 Ω respectively. The ratio of heat produced in the two resistances in same interval of time will be :

(A) 3 : 2

(B) 3 : 1

(C) 3 : 4

(D) 4 : 3

Answer (B)

Sol. Ratio $= \frac{i_1^2 R_1}{\left(\frac{i_2}{\sqrt{2}}\right)^2 R_2} = \frac{4^2 \times 3}{\left(\frac{4}{\sqrt{2}}\right)^2 \times 2}$

$\Rightarrow$ Ratio = 3 : 1

15. A beam of light travelling along X-axis is described by the electric field $E_y = 900\sin \omega(t - x/c)$. The ratio of electric force to magnetic force on a charge q moving along Y-axis with a speed of 3×10^7 ms^{-1} will be :

(Given speed of light = 3×10^8 ms^{-1})

(A) 1 : 1

(B) 1 : 10

(C) 10 : 1

(D) 1 : 2

Answer (C)

Sol. Ratio $= \frac{|q\vec{E}|}{|q\vec{v} \times \vec{B}|}$

$$= \frac{E}{vB} = \frac{v_{wave}}{v}$$

$$\Rightarrow \text{Ratio} = \frac{3 \times 10^8}{3 \times 10^7} = 10$$

16. A microscope was initially placed in air (refractive index 1). It is then immersed in oil (refractive index 2). For a light whose wavelength in air is λ, calculate the change of microscope's resolving power due to oil and choose the correct option.

(A) Resolving power will be $\frac{1}{4}$ in the oil than it was in the air.

(B) Resolving power will be twice in the oil than it was in the air.

(C) Resolving power will be four times in the oil than it was in the air.

(D) Resolving power will be $\frac{1}{2}$ in the oil than it was in the air.

Answer (C)

Sol. $\because$ Resolving power = $\frac{2\mu\sin\theta}{1.22\lambda}$

$$\frac{P_1}{P_2} = \frac{\mu_1}{\mu_2} \times \frac{\mu_1}{\mu_2}$$

$$= \left(\frac{\mu_1}{\mu_2}\right)^2$$

$$\Rightarrow \frac{P_1}{P_2} = \frac{1}{4}$$

$$\Rightarrow P_2 = 4P_1$$

17. An electron (mass m) with an initial velocity $\vec{v} = v_0\hat{i}\,(v_0 > 0)$ is moving in an electric field $\vec{E} = E_0\hat{i}\,(E_0 > 0)$ where E_0 is constant. If at $t = 0$ de Broglie wavelength is $\lambda_0 = \frac{h}{mv_0}$, then its de Broglie wavelength after time t is given by

(A) λ_0

(B) $\lambda_0\left(1+\frac{eE_0t}{mv_0}\right)$

(C) $\lambda_0 t$

(D) $\frac{\lambda_0}{\left(1+\frac{eE_0t}{mv_0}\right)}$

Answer (D)

Sol. $E_0 \longleftarrow$

$\bullet\!\longrightarrow V_0$

$$\therefore\ a_x = \frac{eE_0}{m}\hat{i}$$

$$\therefore\ v(t) = V_0 + \frac{eE_0}{m}t$$

$$\therefore\ \frac{\lambda_0}{\lambda_2} = \frac{mv}{mV_0} = \left(1+\frac{eE_0t}{mV_0}\right)$$

$$\Rightarrow \lambda_2 = \frac{\lambda_0}{\left(1+\frac{eE_0t}{mV_0}\right)}$$

18. What is the half-life period of a radioactive material if its activity drops to $\frac{1}{16}$th of its initial value in 30 years?

(A) 9.5 years
(B) 8.5 years
(C) 7.5 years
(D) 10.5 years

Answer (C)

Sol. $\because\ A = \frac{A_0}{2^{\frac{t}{T_{1/2}}}}$

$$\Rightarrow 2^{\frac{t}{T_{1/2}}} = \frac{A_0}{A} = 16$$

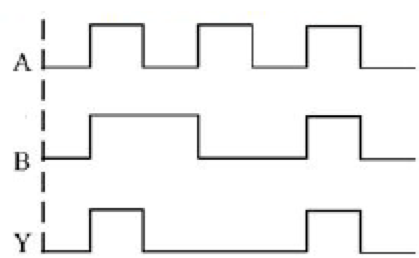
$$\Rightarrow \frac{t}{T_{1/2}} = 4$$

$$\Rightarrow \frac{30}{T_{1/2}} = 4$$

$$\Rightarrow T_{1/2} = \frac{30}{4}$$

= 7.5 years

19. A logic gate circuit has two inputs A and B and output Y. The voltage waveforms of A, B and Y are shown below.



The logic gate circuit is :

(A) AND gate
(B) OR gate
(C) NOR gate
(D) NAND gate

Answer (A)

Sol. From waveforms, it is an AND gate.

20. At a particular station, the TV transmission tower has a height of 100 m. To triple its coverage range, height of the tower should be increased to

(A) 200 m (B) 300 m

(C) 600 m (D) 900 m

Answer (D)

Sol. $\therefore \quad r_m = \sqrt{2Rh}$

$$\Rightarrow \frac{r_1}{r_2} = \sqrt{\frac{h_1}{h_2}}$$

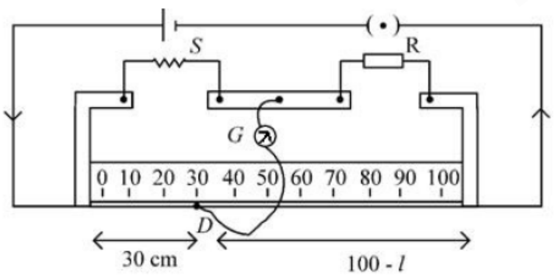
$$\Rightarrow \frac{1}{3} = \sqrt{\frac{100}{h_2}}$$

$\Rightarrow h_2 = 900$ m

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. In a meter bridge experiment, for measuring unknown resistance ‘S’, the null point is obtained at a distance 30 cm from the left side as shown at point D. If R is 5.6 kΩ, then the value of unknown resistance ‘S’ will be ___Ω.



Answer (2400)

Sol. $\frac{R}{S} = \frac{70}{30}$

$$S = \frac{3}{7} \times 5.6 \times 10^3 = 2.4 \times 10^3 \Omega$$

= 2400 Ω

2. The one division of main scale of Vernier callipers reads 1mm and 10 divisions of Vernier scale is equal to the 9 division on main scale. When the two jaws of the instrument touch each other, the zero of the Vernier lies to the right of zero of the main scale and its fourth division coincides with a main scale division. When a spherical bob is tightly placed between the two jaws, the zero of the Vernier scale lies in between 4.1 cm and 4.2 cm and 6th Vernier division coincides with a main scale division. The diameter of the bob will be ___ × 10^{-2} cm.

Answer (412)

Sol. 1 MSD = 1 mm

10 VSD = 9 MSD

$$LC = \frac{1}{10} mm$$

$$0 + 4\left(\frac{1}{10}\right) \text{mm} = 0.4 \text{ mm}$$

$$\text{Reading} = 41 + 6\left(\frac{1}{10}\right)$$

= 41 + 0.6

= 41.6 mm

True reading = 41.2 mm

= 412 × 10^{-2} cm

3. Two beams of light having intensities I and $4I$ interfere to produce a fringe pattern on a screen. The phase difference between the two beams are $\pi/2$ and $\pi/3$ at points A and B respectively. The difference between the resultant intensities at the two points is xI. The value of x will be ___.

Answer (2)

Sol. $I_{R_1} = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\phi$

$$I_A = I + 4I + 2\sqrt{I.4I} \cos 90°$$

$= 5I$

$$I_B = I + 4I + 2\sqrt{I.4I} \cos 60°$$

$= 7I$

$I_B - I_A = 2I$

4. To light, a *W*, 100 *V* lamp is connected, in series with a capacitor of capacitance $\frac{50}{\pi\sqrt{x}}\mu F$, with 200 V, 50 Hz AC source. The value of *x* will be ___.

Answer (3)

Sol. $X_C = \frac{1}{wc} = \frac{\pi\sqrt{x}}{2\pi \times 50 \times 50} \times 10^6$

$v_R^2 + v_C^2 = (200)^2$

$v_C^2 = 200^2 - 100^2$

$v_C = 100\sqrt{3}\ V$

$v_R = 100\ V$

$P = \frac{V^2}{R}$

$R = \frac{100 \times 100}{50} = 200\Omega$

$i_{rm} = \frac{1}{2}A$

$\frac{1}{2} \times x_C = 100\sqrt{3} \Rightarrow 10^{-6} \times \frac{\sqrt{x}}{5000} \times \frac{1}{2} = 100\sqrt{3}$

$\frac{10^{-6}\sqrt{x}}{10000 \times 100} = \sqrt{3}$

$\sqrt{x} = \sqrt{3}$

$x = 3$

5. A 1 m long copper wire carries a current of 1 *A*. If the cross section of the wire is 2.0 mm^2 and the resistivity of copper is 1.7 × 10^{-8} Ωm, the force experienced by moving electron in the wire is ___ × 10^{-23}N.

(Charge on electron = 1.6 × 10^{-19}C)

Answer (136)

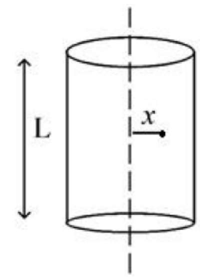
Sol. $I = nev_dA$

$J = \frac{E}{\rho}$

$F = eE = \frac{1.7 \times 1.6 \times 10^{-19} \times 10^{-8}}{2 \times 10^{-6}}$

= 136 × 10^{-23} N

6. A long cylindrical volume contains a uniformly distributed charge of density ρ Cm^{-3}. The electric field inside the cylindrical volume at a distance $x = \frac{2\varepsilon_0}{\rho}$ m from its axis is _______ Vm^{-1}



Answer (1)

Sol. $E = \frac{\rho r}{2\varepsilon_0}$

at $r = \frac{2\varepsilon_0}{\rho}$

$E = \frac{\rho}{2\varepsilon_0}\left(\frac{2\varepsilon_0}{\rho}\right)$

= 1

7. A mass 0.9 kg, attached a horizontal spring, executes SHM with an amplitude A_1. When this mass passes through its mean position, then a smaller mass of 124 g is placed over it and both masses move together with amplitude A_2. If the ratio $\frac{A_1}{A_2}$ is $\frac{\alpha}{\alpha - 1}$, then the value of α will be _____.

Answer (16)

Sol. $(0.9)A_1\sqrt{\frac{K}{0.9}} = (0.9 + 0.124)A_2\sqrt{\frac{K}{0.9 + 0.124}}$

$\frac{A_1}{A_2} = \sqrt{\frac{0.9 + 0.124}{0.9}}$

$= \sqrt{\frac{1.024}{0.9}}$

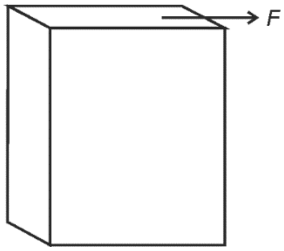
$= \frac{\alpha}{\alpha - 1}$

α = 16

8. A square aluminium (shear modulus is 25×10^9 Nm^{-2}) slab of side 60 cm and thickness 15 cm is subjected to a shearing force (on its narrow face) of 18.0×10^4 N. The lower edge is riveted to the floor. The displacement of the upper edge is ______ μm.

Answer (48)

Sol.



$$Y = \frac{Fl}{A\Delta l}$$

$$\Delta l = \frac{Fl}{YA}$$

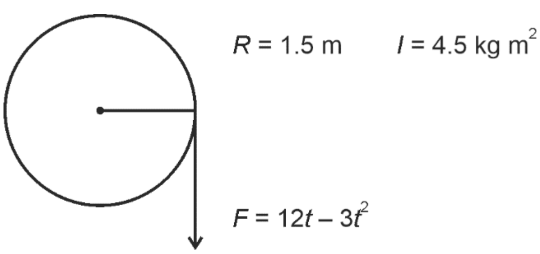
$$= \frac{18 \times 10^4 \times 60 \times 10^{-2}}{25 \times 10^9 \times 60 \times 15 \times 10^{-4}}$$

$= 48 \times 10^{-6}$ m

9. A pulley of radius 1.5 m is rotated about its axis by a force $F = (12t - 3t^2)$ N applied tangentially (while t is measured in seconds). If moment of inertia of the pulley about its axis of rotation is 4.5 kg m², the number of rotations made by the pulley before its direction of motion is reversed, will be $\frac{K}{\pi}$. The value of K is ________.

Answer (18)

Sol.



$FR = I\alpha$

$$\alpha = \frac{(12t - 3t^2) \times 1.5}{4.5} = 4t - t^2$$

$$w = \int \alpha dt = 2t^2 - \frac{t^3}{3}$$

$w = 0$

$$\Rightarrow t^2\left[2 - \frac{t}{3}\right] = 0$$

$t = 6$ sec

$$\theta = \int_0^6 \left[2t^2 - \frac{t^3}{3}\right] dt = \left[\frac{2t^3}{3} - \frac{t^4}{12}\right]\Bigg|_0^6$$

$$= \left[\frac{2}{3} \times 6^3 - \frac{6^4}{12}\right] = 36$$

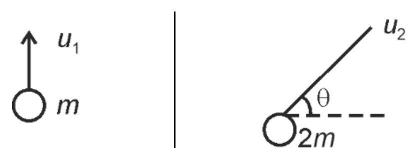
$$n = \frac{36}{2\pi}$$

$$= \frac{18}{\pi}$$

10. A ball of mass m is thrown vertically upward. Another ball of mass 2 m is thrown at an angle θ with the vertical. Both the balls stay in air for the same period of time. The ratio of the heights attained by the two balls respectively is $\frac{1}{x}$. The value of x is ______.

Answer (1)

Sol.



$$T_1 = \frac{2u_1}{g} \qquad T_2 = \frac{2u_2 \sin\theta}{g}$$

$\therefore$ $u_1 = u_2 \sin\theta$

$$\frac{H_1}{H_2} = \frac{\frac{u_1^2}{2g}}{\frac{u_2^2 \sin^2\theta}{2g}}$$

$$= \left(\frac{u_1}{u_2 \sin\theta}\right)^2$$

$= 1$

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. 250 g solution of D-glucose in water contains 10.8% of carbon by weight. The molality of the solution is nearest to (Given: Atomic Weights are, H, 1 u; C, 12 u; O, 16 u)

(A) 1.03 (B) 2.06

(C) 3.09 (D) 5.40

Answer (B)

Sol. Weight of D-glucose in water = 250 g

$\therefore$ Weight of carbon in D-glucose = $\frac{250}{180}\times 72$

= 100 g

% of carbon in the aqueous solution of glucose is = 10.8%

$\therefore$ Weight of the solution is = 925.93

$\therefore$ Molality of D-glucose is = $\frac{\frac{250}{180}}{(925.93-250)}\times 1000$

$= \frac{250}{180\times 675.93}\times 1000$

= 2.06

2. Given below are two statements.

Statement I: O_2, Cu^{2+}, and Fe^{3+} are weakly attracted by magnetic field and are magnetized in the same direction as magnetic field.

Statement II: NaCl and H_2O are weakly magnetized in opposite direction to magnetic field.

In the light of the above statements, choose the ***most appropriate*** answer from the options given below.

(A) Both **Statement I** and **Statement II** are correct.

(B) Both **Statement I** and **Statement II** are incorrect.

(C) **Statement I** is correct but **Statement II** is incorrect.

(D) **Statement I** is incorrect but **Statement II** is correct.

Answer (A)

Sol. O_2, Cu^{2+} and Fe^{3+} have 2, 1 and 5 unpaired electrons respectively, so these are the paramagnetic species. Hence, they are attracted by magnetic field.

NaCl and H_2O are the diamagnetic species so they are repelled by the magnetic field.

3. Given below are two statements. One is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A: Energy of 2s orbital of hydrogen atom is greater than that of 2s orbital of lithium.

Reason R : Energies of the orbitals in the same subshell decrease with increase in the atomic number.

In the light of the above statements, choose the ***correct*** answer from the options given below.

(A) Both **A** and **R** are true and **R** is the correct explanation of **A**.

(B) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**.

(C) **A** is true but **R** is false.

(D) **A** is false but **R** is true.

Answer (A)

Sol. As the atomic number increases then the potential energy of electrons present in same shell becomes more and more negative. And therefore total energy also becomes more negative.

$$E_{total} = -13.6\frac{z^2}{n^2}\ eV$$

$\therefore$ Energies of the orbitals in the same subshell decreases with increase in atomic number.

4. Given below are two statements. One is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A: Activated charcoal adsorbs SO_2 more efficiently than CH_4.

Reason R: Gases with lower critical temperatures are readily adsorbed by activated charcoal.

In the light of the above statements, choose the ***correct*** answer from the options given below.

(A) Both **A** and **R** are correct and **R** is the correct explanation of **A**.

(B) Both **A** and **R** are correct but **R** is NOT the correct explanation of **A**.

(C) **A** is correct but **R** is not correct.

(D) **A** is not correct but **R** is correct.

Answer (C)

Sol. More polar gases easily adsorbs on activated charcoal.

And more polar gases has more (higher) critical temperature as compared to non-polar or less polar gases.

$\therefore$ Gases with higher critical temperature adsorbed more.

5. Boiling point of a 2% aqueous solution of a non-volatile solute A is equal to the boiling point of 8% aqueous solution of a non-volatile solute B. The relation between molecular weights of A and B is

(A) $M_A = 4M_B$

(B) $M_B = 4M_A$

(C) $M_A = 8M_B$

(D) $M_B = 8M_A$

Answer (B)

Sol. $(\Delta T_b)_A = (\Delta T_b)_B$

$K_b \cdot M_A = K_b \cdot M_B$

$\Rightarrow M_A = M_B$

$$\Rightarrow \frac{\frac{2}{M_A}}{100}\times 1000 = \frac{\frac{8}{M_B}}{100}\times 1000$$

$\Rightarrow M_B = 4M_A$

6. The **incorrect** statement is

(A) The first ionization enthalpy of K is less than that of Na and Li.

(B) Xe does not have the lowest first ionization enthalpy in its group.

(C) The first ionization enthalpy of element with atomic number 37 is lower than that of the element with atomic number 38.

(D) The first ionization enthalpy of Ga is higher than that of the d-block element with atomic number 30.

Answer (D)

Sol. On moving down in a group ionisation energy decrease

$\therefore$ 1^{st} ionisation enthalpy order is Li > Na > K

Zn has more ionisation energy as compared to Ga because of their pseudo inert gas configuration.

7. Which of the following methods are not used to refine any metal?

A. Liquation

B. Calcination

C. Electrolysis

D. Leaching

E. Distillation

Choose the **correct** answer from the options given below :

(A) B and D only

(B) A, B, D and E only

(C) B, D and E only

(D) A, C and E only

Answer (A)

Sol. Leaching and calcination are the processes which are involved in the extraction of the metals.

Liquation, Electrolytic refining, Distillation are used in the refining or purification of metal.

8. Given below are two statements.

Statement I : Hydrogen peroxide can act as an oxidizing agent in both acidic and basic conditions.

Statement II : Density of hydrogen peroxide at 298 K is lower than that of D_2O.

In the light of the above statements, choose the ***correct*** answer from the options given below :

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Answer (C)

Sol. Density of H_2O_2 is more as compared to D_2O

$d_{H_2O_2} = 1.44$ g/cc

$d_{D_2O} = 1.106$ g/cc

And hydrogen peroxide acts as an oxidising as well as reducing agent in both acidic and basic medium.

$\therefore$ Statement I is correct.

9. Given below are two statements.

Statement I : The chlorides of Be and Al have Cl-bridged structure. Both are soluble in organic solvents and act as Lewis bases.

Statement II : Hydroxides of Be and Al dissolve in excess alkali to give beryllate and aluminate ions.

In the light of the above statements, choose the ***correct*** answer from the options given below.

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Answer (D)

Sol. Chlorides of Be and Al are

$BeCl_2$ and $AlCl_3$ have electron deficiency at central atom and behave as the Lewis acids.

$$BeCl_2 + H_2O \rightarrow \underset{\text{beryllate ion}}{[Be(OH)_4]^{2-}}$$

$$AlCl_3 + H_2O \rightarrow \underset{\text{aluminate ion}}{[Al(OH)_4]^{-}}$$

10. Which oxoacid of phosphorous has the highest number of oxygen atoms present in its chemical formula?

(A) Pyrophosphorus acid

(B) Hypophosphoric acid

(C) Phosphoric acid

(D) Pyrophosphoric acid

Answer (D)

Sol. Pyrophosphorus acid $\rightarrow$ $H_4P_2O_5$

Hypophosphoric acid $\rightarrow$ $H_4P_2O_6$

Phosphoric acid $\rightarrow$ H_3PO_4

Pyrophosphoric acid $\rightarrow$ $H_4P_2O_7$

11. Given below are two statements.

Statement I: Iron (III) catalyst, acidified $K_2Cr_2O_7$ and neutral $KMnO_4$ have the ability to oxidise I^- to I_2 independently.

Statement II: Manganate ion is paramagnetic in nature and involves $p\pi - p\pi$ bonding.

In the light of the above statements, choose the **correct** answer from the options given below.

(A) Both Statement I and Statement II are true

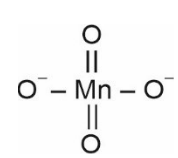
(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Answer (B)

Sol. Manganate ion MnO_4^{2-} has tetrahedral structure



has only $d\pi$ - $p\pi$ π-bonds.

Fe^{3+} is not used as a catalyst in the conversion of I^- to I_2 by $K_2Cr_2O_7$. $K_2Cr_2O_7$ oxidise I^- in acidic medium easily

12. The total number of Mn=O bonds in Mn_2O_7 is ____.

(A) 4

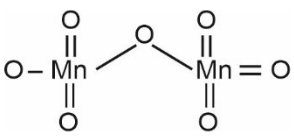
(B) 5

(C) 6

(D) 3

Answer (C)

Sol. Structure of Mn_2O_7 is as :



∴ There are total 6 M = O bonds are present in Mn_2O_7 compound.

13. Match **List I** with **List II**.

List I Pollutant	List II Disease/ sickness
A. Sulphate (> 500 ppm)	I. Methemoglobinemia
B. Nitrate (> 50 ppm)	II. Brown mottling of teeth
C. Lead (> 50 ppb)	III. Laxative effect
D. Fluoride (> 2ppm)	IV. Kidney damage

Choose, the coned answer from the options given below:

(A) A-IV, B-I, C-II, D-III

(B) A-III, B-I, C-IV, D-II

(C) A-II, B-IV, C-I, D-III

(D) A-II, B-IV, C-III, D-I

Answer (B)

Sol. The correct match of pollutants and disease because of the excess of these pollutants are:

Sulphate ⟶ Laxative effect

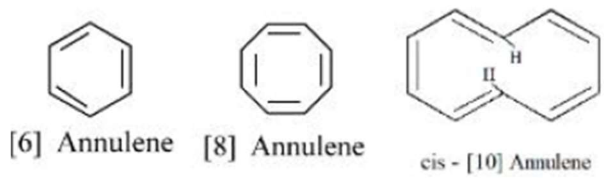
Nitrate ⟶ Methemoglobinemia

Lead ⟶ Kidney damage

Fluoride → Brown mottling of teeth

14. Given below are two statements: one is labelled as **Assertion A** and, the other is labelled as **Reason R**.

Assertion A: [6] Annulene, [8] Annulene and cis-[10] Annulene, are respectively aromatic, not-aromatic and aromatic.



Reason R: Planarity is one, of the requirements of aromatic systems.

In the light of the above statements, choose the **most appropriate** answer from the options given below.

(A) Both **A** and **R** are correct and **R** is the correct explanation of **A**

(B) Both **A** and **R** are correct but **R** is NOT the correct explanation of **A**

(C) **A** is correct but **R** is not correct

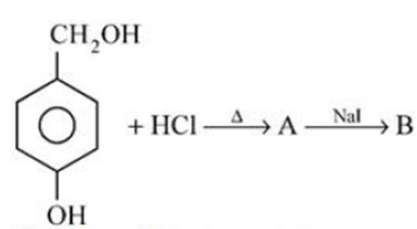
(D) **A** is not correct but **R** is correct

Answer (D)

Sol. [6] Annulene is aromatic because it is planar.

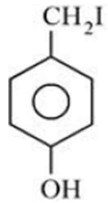
[8] Annulene and [10] Annulene are both not aromatic because they are not planar. So, Assertion (A) is not correct.

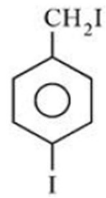
Reason (R) is correct because planarity is one of the requirements of aromatic system.

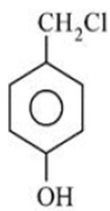
15. 

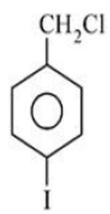
In the above reaction product B is:

Product B is

(A) 

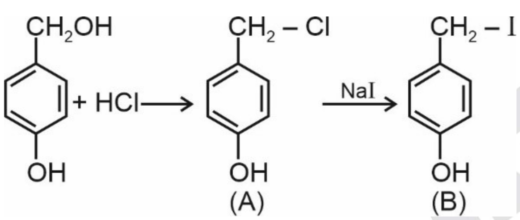
(B) 

(C) 

(D) 

Answer (A)

Sol.



Product B is 4-iodomethylphenol.

16. Match List-I with List-II.

List-1 Polymers	List II Commercial names
A. Phenol-formaldehyde resin	I. Glyptal
B. Copolymer of 1,3-butadiene and styrene	II. Novolac
C. Polyester of glycol and phthalic acid	III. Buna-S
D. Polyester of glycol and terephthalic acid	IV. Dacron

Choose the correct answer from the option give below:

(A) A-II, B-III, C-IV, D-I (B) A-II, B-III, C-I, D-IV

(C) A-II, B-I, C-III, D-IV (D) A-III, B-II, C-IV, D-I

Answer (B)

Sol.

Polymers	Commercial names
A. Phenol-formaldehyde resin	Novolac
B. Copolymer of 1,3-butadiene and styrene	Buna-S
C. Polyester of glycol and phthalic acid	Glyptal
D. Polyester of glycol and terephthalic acid	Dacron

∴ The Correct match is

A – II; B – III, C – I ; D - IV

17. A sugar 'X' dehydrates very slowly under acidic condition to give furfural which on further reaction with resorcinol gives the coloured product after sometime. Sugar 'X' is

(A) Aldopentose

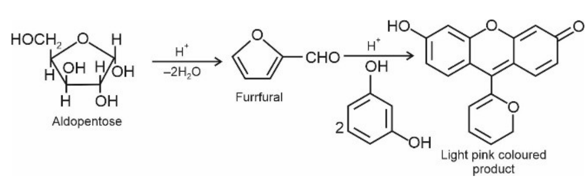
(B) Aldotetrose

(C) Oxalic acid

(D) Ketotetrose

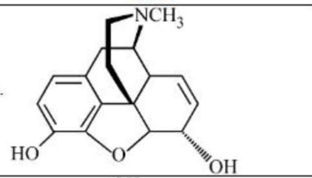
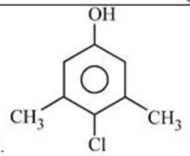
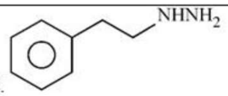
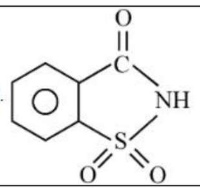
Answer (A)

Sol.



This is based on Seliwamoff's test which is used to distinguish between aldoses and Kotoses. Ketoses give this test more rapidly than aldoses because they are more rapidly dehydrated than aldoses.

18. Match List I and List II.

List I	List II
A. 	I. Anti-depressant
B. 	II. 550 times sweeter than cane sugar.
C. 	III. Narcotic analgesic
D. 	IV. Antiseptic

Choose the correct answer from the options given below:

(A) A-IV, B-III, C-II, D-I

(B) A-III, B-I, C-II, D-IV

(C) A-III, B-IV, C-I, D-II

(D) A-III, B-I, C-IV, D-II

Answer (C)

Sol.
- A is morphine which is a narcotic analgesic.
- B is chloroxylenol, an antiseptic.
- C is Nardil, an antidepressant.
- D is saccharin, which is around 550 times sweeter than cane sugar.

19. In Carius method of estimation of halogen, 0.45 g of an organic compound gave 0.36 g of AgBr. Find out the percentage of bromine in the compound.

(Molar masses: AgBr = 188 g mol^{-1}; Br = 80 g mol^{-1})

(A) 34.04%

(B) 40.04%

(C) 36.03%

(D) 38.04%

Answer (A)

Sol. 188 g of AgBr = 80 g of Br

$$0.36 \text{ g of AgBr} = \frac{80}{188}\times 0.36$$

% of Br in given organic compound

$$= \frac{80\times 0.36}{188\times 0.45}\times 100$$

$$= 34.04\ \%$$

20. Match List I with List II.

List I	List II
A. Benzenesulphonyl chloride	I. Test for primary amines
B. Hoffmann bromamide reaction	II. Anti Saytzeff
C. Carbylamine reaction	III. Hinsberg reagent
D. Hoffmann orientation	IV. Known reaction of Isocyanates.

Choose the correct answer from the options given below:

(A) A-IV, B-III, C-II, D-I

(B) A-IV, B-II, C-I, D-II

(C) A-III, B-IV, C-I, D-II

(D) A-IV, B-III, C-I, D-II

Answer (C)

Sol. (A) Benzene sulphonyl chloride is also known as Hinsberg reagent.

(B) Hoffmann bromamide reaction involves conversion of amide to amine having one C-atom less. This reaction involves isocyanate as intermediate.

(C) Carbylamine reaction is a test given by all primary amines.

(D) Hoffmann orientation refers to the addition of molecules to unsymmetrical alkenes according to anti Saytzeff's rule.

Correct match is

A – III; B – IV; C – I; D – II

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. 20 mL of 0.02 M $K_2Cr_2O_7$ solution is used for the titration of 10 mL of Fe^{2+} solution in the acidic medium. The molarity of Fe^{2+} solution is ______$\times 10^{-2}$ M. (Nearest integer)

Answer (24)

Sol. Applying the law of equivalence,

milliequivalents of Fe^{2+} = milliequivalents of $K_2Cr_2O_7$

$10 \times 1 \times M = 20 \times 6 \times .02$

$M = 24 \times 10^{-2}$ M

$\therefore$ Answer will be 24

2. $2NO + 2H_2 \rightarrow N_2 + 2H_2O$

The above reaction has been studied at 800°C. The related date are given in the table below

Reaction serial number	Initial Pressure of H_2/kPa	Initial Pressure of NO/kPa	Initial rate $\left(\frac{-dp}{dt}\right)$ /(kPa/s)
1	65.6	40.0	0.135
2	65.6	20.1	0.033
3	38.6	65.6	0.214
4	19.2	65.6	0.106

The order of the reaction with respect to NO is ___.

Answer (2)

Sol. Let the rate of reaction (r) is as

$r = K[NO]^n[H_2]^m$

From 1st data

$0.135 = K[40]^n \cdot (65.6)^m$ …(1)

From 2nd data

$0.033 = K(20.1)^n \cdot (65.6)^m$ …(2)

On dividing equation (1) by equation (2)

$$\frac{0.135}{0.033} = \left(\frac{40}{20.1}\right)^n$$

$4 = (2)^n$

$\therefore$ n = 2

$\therefore$ Order of reaction w.r.t. NO is 2.

3. Amongst the following, the number of oxide(s) which are paramagnetic in nature is

Na_2O, KO_2, NO_2, N_2O, ClO_2, NO, SO_2, Cl_2O

Answer (4)

Sol. Paramagnetic species: KO_2, NO_2, ClO_2, NO

Diamagnetic species are : Na_2O , N_2O, SO_2, Cl_2O

$\therefore$ There are total 4 paramagnetic molecules.

4. The molar heat capacity for an ideal gas at constant pressure is 20.785 J K^{-1} mol^{-1}. The change in internal energy is 5000 J upon heating it from 300 K to 500 K. The number of moles of the gas at constant volume is _____. (Nearest integer)
(Given : R = 8.314 J K^{-1} mol^{-1})

Answer (2)

Sol. $C_p = 20.785$ J K^{-1} mol^{-1}

and $\Delta U = nC_v \Delta T$

$$\therefore \quad nC_V = \frac{5000}{200} = 25$$

and we know that

$C_p - C_v = R$

$$20.785 - \frac{25}{n} = 8.314$$

$$n = \frac{25}{(20.785 - 8.314)} = 2$$

5. According to MO theory, number of species/ions from the following having identical bond order is ____.

$CN^-, NO^+, O_2, O_2^+, O_2^{2+}$

Answer (3)

Sol. CN^-, NO^+ and O_2^{2+} have bond order of '3'

O_2 has bond order of 2,

O_2^+ has bond order of 2.5

∴ 3 species have similar bond order.

6. At 310 K, the solubility of CaF_2 in water is 2.34×10^{-3} g/100 mL. The solubility product of CaF_2 is _____ $\times 10^{-8}$ (mol/L)^3.

(Given molar mass : CaF_2 = 78 g mol^{-1})

Answer (0)

Sol. $CaF_2 \underset{}{\overset{s}{\rightleftharpoons}} \underset{s}{Ca^{2+}} + \underset{2s}{2F^-}$

$K_{sp} = s(2s)^2$

$= 4s^3$

Solubility(s) = $2{\cdot}34 \times 10^{-3}$ g/100 mL

$= \dfrac{2\cdot34 \times 10^{-3} \times 10}{78}$ mole / lit

$= 3\times10^{-4}$ mole/lit

∴ $K_{sp} = 4 \times (3 \times 10^{-4})^3$

$= 108 \times 10^{-12}$

$= 0{\cdot}0108 \times 10^{-8}$ (mole/lit)^3

∴ $x \approx 0$

7. The conductivity of a solution of complex with formula $CoCl_3(NH_3)_4$ corresponds to 1 : 1 electrolyte, then the primary valency of central metal ion is ______

Answer (3)

Sol. In 1 : 1 type of electrolyte the ions have +1 and –1 charge on them

∴ Possible compound is → $[Co(NH_3)_4Cl_2]^+Cl^-$

Oxidation state of central atom represents the total number of primary valency

∴ Primary valency will be 3.

8. In the titration of $KMnO_4$ and oxalic acid in acidic medium, the change in oxidation number of carbon at the end point is _____

Answer (1)

Sol. $16H^+ + 2MnO_4^- + 5C_2O_4^{2-} \rightarrow 10CO_2 + 2Mn^{2+} + 8H_2O$

During titration of oxalic acid by $KMnO_4$, oxalic acid converts into CO_2.

∴ Change in oxidation state of carbon = 1

9. Optical activity of an enantiomeric mixture is +12.6° and the specific rotation of (+) isomer is +30°. The optical purity is _______%.

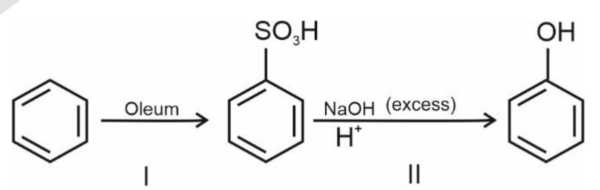
Answer (42)

Sol. Optical purity = $\dfrac{\text{Total rotation}}{\text{Specific rotation}} \times 100$

$= \dfrac{12\cdot6}{30} \times 100$

= 42%

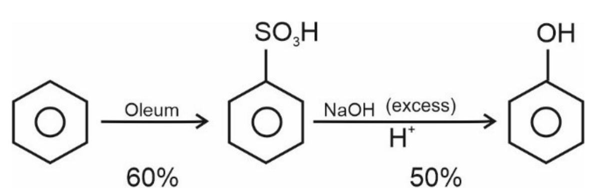
10. In the following reaction,



the % yield for reaction **I** is 60% and that of reaction **II** is 50%. The overall yield of the complete reaction is _______%. [Nearest integer]

Answer (30)

Sol.



The % yield of the complete reaction is

⇒ $0{\cdot}6 \times 0{\cdot}5 \times 100 = 30\%$

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Let R_1 and R_2 be two relations defined on $\mathbb{R}$ by a $R_1\, b \Leftrightarrow ab \geq 0$ and $a\, R_2\, b \Leftrightarrow a \geq b$. Then,

   (A) R_1 is an equivalence relation but not R_2

   (B) R_2 is an equivalence relation but not R_1

   (C) Both R_1 and R_2 are equivalence relations

   (D) Neither R_1 nor R_2 is an equivalence relation

Answer (D)

Sol. $a\, R_1\, b \Leftrightarrow ab \geq 0$

So, definitely $(a, a) \in R_1$ as $a^2 \geq 0$

If $(a, b) \in R_1 \Rightarrow (b, a) \in R_1$

But if $(a, b) \in R_1$, $(b, c) \in R_1$

$\Rightarrow$ Then (a, c) may or may not belong to R_1

{Consider $a = -5$, $b = 0$, $c = 5$ so (a, b) and $(b, c) \in R_1$ but $ac < 0$}

So, R_1 is not equivalence relation

$a\, R_2\, b \Leftrightarrow a \geq b$

$(a, a) \in R_2 \Rightarrow$ so reflexive relation

If $(a, b) \in R_2$ then (b, a) may or may not belong to R_2

$\Rightarrow$ So not symmetric

Hence it is not equivalence relation

2. Let $f, g : \mathbb{N} - \{1\} \to \mathbb{N}$ be functions defined by $f(a) = \alpha$, where α is the maximum of the powers of those primes p such that p^α divides a, and $g(a) = a + 1$, for all $a \in \mathbb{N} - \{1\}$. Then, the function $f + g$ is

   (A) one-one but not onto

   (B) onto but not one-one

   (C) both one-one and onto

   (D) neither one-one nor onto

Answer (D)

Sol. $f, g : N - \{1\} \to N$ defined as

$f(a) = \alpha$, where α is the maximum power of those primes p such that p^α divides a.

$g(a) = a + 1$,

Now, $f(2) = 1$, $g(2) = 3 \Rightarrow (f + g)(2) = 4$

$f(3) = 1$, $g(3) = 4 \Rightarrow (f + g)(3) = 5$

$f(4) = 2$, $g(4) = 5 \Rightarrow (f + g)(4) = 7$

$f(5) = 1$, $g(5) = 6 \Rightarrow (f + g)(5) = 7$

$\because \quad (f + g)(5) = (f + g)(4)$

$\therefore \quad f + g$ is not one-one

Now, $\because f_{min} = 1$, $g_{min} = 3$

So, there does not exist any $x \in N - \{1\}$ such that $(f + g)(x) = 1, 2, 3$

$\therefore$ $f + g$ is not onto

3. Let the minimum value v_0 of $v = |z|^2 + |z - 3|^2 + |z - 6i|^2$, $z \in \mathbb{C}$ is attained at $z = z_0$. Then $\left|2z_0^2 - \bar{z}_0^3 + 3\right|^2 + v_0^2$ is equal to

   (A) 1000

   (B) 1024

   (C) 1105

   (D) 1196

Answer (A)

Sol. Let $z = x + iy$

$v = x^2 + y^2 + (x - 3)^2 + y^2 + x^2 + (y - 6)^2$

$= (3x^2 - 6x + 9) + (3y^2 - 12y + 36)$

$= 3(x^2 + y^2 - 2x - 4y + 15)$

$= 3[(x - 1)^2 + (y - 2)^2 + 10]$

v_{min} at $z = 1 + 2i = z_0$ and $v_0 = 30$

so $|2(1 + 2i)^2 - (1 - 2i)^3 + 3|^2 + 900$

$= |2(-3 + 4i) - (1 - 8i^3 - 6i(1 - 2i) + 3|^2 + 900$

$= |-6 + 8i - (1 + 8i - 6i - 12) + 3|^2 + 900$

$= |8 + 6i|^2 + 900$

$= 1000$

4. Let $A = \begin{pmatrix} 1 & 2 \\ -2 & -5 \end{pmatrix}$. Let $\alpha, \beta, \in \mathbb{R}$ be such that $\alpha A^2 + \beta A = 2I$. Then $\alpha + \beta$ is equal to

(A) –10 (B) –6

(C) 6 (D) 10

Answer (D)

Sol. $A^2 = \begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix} = \begin{bmatrix} -3 & -8 \\ 8 & 21 \end{bmatrix}$

$$\alpha A^2 + \beta A = \begin{bmatrix} -3\alpha & -8\alpha \\ 8\alpha & 21\alpha \end{bmatrix} + \begin{bmatrix} \beta & 2\beta \\ -2\beta & -5\beta \end{bmatrix}$$

$$= \begin{bmatrix} -3\alpha + \beta & -8\alpha + 2\beta \\ 8\alpha - 2\beta & 21\alpha - 5\beta \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

On Comparing

$8\alpha = 2\beta$, $-3\alpha + \beta = 2$, $21\alpha - 5\beta = 2$

$\Rightarrow \alpha = 2, \beta = 8$

So, $\alpha + \beta = 10$

5. The remainder when $(2021)^{2022} + (2022)^{2021}$ is divided by 7 is

(A) 0 (B) 1

(C) 2 (D) 6

Answer (A)

Sol. $(2021)^{2022} + (2022)^{2021}$

$= (7k - 2)^{2022} + (7k_1 - 1)^{2021}$

$= \left[(7k-2)^3\right]^{674} + (7k_1)^{2021} - 2021(7k_1)^{2020} + \ldots.. - 1$

$= (7k_2 - 1)^{674} + (7m - 1)$

$= (7n + 1) + (7m - 1) = 7(m + n)$ (multiple of 7)

$\therefore$ Remainder = 0

6. Suppose $a_1, a_2, \ldots a_n, \ldots$ be an arithmetic progression of natural numbers. If the ration of the sum of first five terms to the sum of first nine terms of the progression is 5 : 17 and $110 < a_{15} < 120$, then the sum of the first ten terms of the progression is equal to

(A) 290

(B) 380

(C) 460

(D) 510

Answer (B)

Sol. $\because$ $a_1, a_2, \ldots a_n \ldots$ be an A.P of natural numbers and

$$\frac{S_5}{S_9} = \frac{5}{17} \Rightarrow \frac{\frac{5}{2}[2a_1 + 4d]}{\frac{9}{2}[2a_1 + 8d]} = \frac{5}{17}$$

$\Rightarrow 34a_1 + 68d = 18a_1 + 72d$

$\Rightarrow 16a_1 = 4d$

$\therefore$ $\boxed{d = 4a_1}$

And $110 < a_{15} < 120$

$\therefore$ $110 < a_1 + 14d < 120 \Rightarrow 110 < 57a_1 < 120$

$\therefore$ $a_1 = 2$ ($\because a_i \in N$)

$d = 8$

$\therefore$ $S_{10} = 5\,[4 + 9 \times 8] = 380$

7. Let $\mathbb{R} \to \mathbb{R}$ be function defined as $f(x) = a\sin\left(\frac{\pi[x]}{2}\right) + [2 - x], a \in \mathbb{R}$, where $[t]$ is the greatest integer less than or equal to t. If $\lim\limits_{x \to -1} f(x)$ exists, then the value of $\int_0^4 f(x)dx$ is equal to

(A) –1 (B) –2

(C) 1 (D) 2

Answer (B)

Sol. $f(x) = a\sin\left(\frac{\pi[x]}{2}\right) + [2 - x]\ a \in R$

Now,

$\because \lim\limits_{x \to -1} f(x)$ exist

$\therefore \lim\limits_{x \to -1^-} f(x) = \lim\limits_{x \to -1^+} f(x)$

$\Rightarrow a\sin\left(\frac{-2\pi}{2}\right) + 3 = a\sin\left(\frac{-\pi}{2}\right) + 2$

$\Rightarrow -a = 1 \Rightarrow \boxed{a = -1}$

Now, $\int_0^4 f(x)dx = \int_0^4 \left(-\sin\left(\frac{\pi[x]}{2}\right) + [2 - x]\right)dx$

$= \int_0^1 1dx + \int_1^2 -1dx + \int_2^3 -1dx + \int_3^4 (1 - 2)dx$

$= 1 - 1 - 1 - 1 = -2$

8. Let $I = \int_{\pi/4}^{\pi/3} \left(\frac{8\sin x - \sin 2x}{x} \right) dx$. Then

(A) $\frac{\pi}{2} < I < \frac{3\pi}{4}$ (B) $\frac{\pi}{5} < I < \frac{5\pi}{12}$

(C) $\frac{5\pi}{12} < I < \frac{\sqrt{2}}{3}\pi$ (D) $\frac{3\pi}{4} < I < \pi$

Answer (*)

Sol. I comes out around 1.536 which is not satisfied by any given options.

$$\int_{\pi/4}^{\pi/3} \frac{8x - 2x}{x} dx > I > \int_{\pi/4}^{\pi/3} \frac{8\sin x - 2x}{x} dx$$

$$\frac{\pi}{2} > I > \int_{\pi/4}^{\pi/3} \left(\frac{8\sin x}{x} - 2 \right) dx$$

$\frac{\sin x}{x}$ is decreasing in $\left(\frac{\pi}{4}, \frac{\pi}{3} \right)$ so it attains maximum at $x = \frac{x}{4}$

$$I > \int_{\pi/4}^{\pi/3} \left(\frac{8\sin \pi/3}{\pi/3} - 2 \right) dx$$

$$I > \sqrt{3} - \frac{\pi}{6}$$

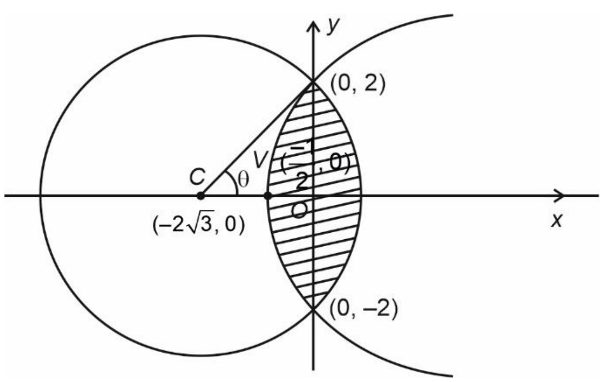
9. The area of the smaller region enclosed by the curves $y^2 = 8x + 4$ and $x^2 + y^2 + + 4\sqrt{3}x - 4 = 0$ is equal to

(A) $\frac{1}{3}\left(2 - 12\sqrt{3} + 8\pi\right)$ (B) $\frac{1}{3}\left(2 - 12\sqrt{3} + 6\pi\right)$

(C) $\frac{1}{3}\left(4 - 12\sqrt{3} + 8\pi\right)$ (D) $\frac{1}{3}\left(4 - 12\sqrt{3} + 6\pi\right)$

Answer (C)

Sol.



$$\cos\theta = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30^\circ$$

Area of the required region

$$= \frac{2}{3}\left(4 \times \frac{1}{2}\right) + 4^2 \times \frac{\pi}{6} - \frac{1}{2} \times 4 \times 2\sqrt{3}$$

$$= \frac{4}{3} + \frac{8\pi}{3} - 4\sqrt{3} = \frac{1}{3}\left\{4 - 12\sqrt{3} + 8\pi\right\}$$

10. Let $y = y_1(x)$ and $y = y_2(x)$ be two distinct solution of the differential equation $\frac{dy}{dx} = x + y$, with $y_1(0) = 0$ and $y_2(0) = 1$ respectively. Then, the number of points of intersection of $y = y_1(x)$ and $y = y_2(x)$ is

(A) 0 (B) 1

(C) 2 (D) 3

Answer (A)

Sol. $\frac{dy}{dx} = x + y$

Let $x + y = t$

$$1 + \frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dt}{dx} - 1 = t \Rightarrow \int \frac{dt}{t+1} = \int dx$$

$$\ln|t + 1| = x + C'$$

$$|t + 1| = Ce^x$$

$$|x + y + 1| = Ce^x$$

For $y_1(x)$, $y_1(0) = 0 \Rightarrow C = 1$

For $y_2(x)$, $y_2(0) = 1 \Rightarrow C = 2$

$y_1(x)$ is given by $|x + y + 1| = e^x$

$y_2(x)$ is given by $|x + y + 1| = 2e^x$

At point of intersection

$e^x = 2e^x$

No solution

So, there is no point of intersection of $y_1(x)$ and $y_2(x)$.

11. Let $P(a, b)$ be a point on the parabola $y^2 = 8x$ such that the tangent at P passes through the centre of the circle $x^2 + y^2 - 10x - 14y + 65 = 0$. Let A be the product of all possible values of a and B be the product of all possible values of b. Then the value of $A + B$ is equal to

(A) 0 (B) 25

(C) 40 (D) 65

Answer (D)

Sol. Centre of circle $x^2 + y^2 - 10x - 14y + 65 = 0$ is at (5, 7).

Let the equation of tangent to $y^2 = 8x$ is

$yt = x + 2t^2$

which passes through (5, 7)

$7t = 5 + 2t^2$

$\Rightarrow\ 2t^2 - 7t + 5 = 0$

$t = 1, \frac{5}{2}$

$A = 2\times 1^2 \times 2 \times \left(\frac{5}{2}\right)^2 = 25$

$B = 2\times 2\times 1\times 2\times 2\times \frac{5}{2} = 40$

$A + B = 65$

12. Let $\vec{a} = \alpha\hat{i} + \hat{j} + \beta\hat{k}$ and $\vec{b} = 3\hat{i} + 5\hat{j} + 4\hat{k}$ be two vectors, such that $\vec{a}\times\vec{b} = -\hat{i} + 9\hat{i} + 12\hat{k}$. Then the projection of $\vec{b} - 2\vec{a}$ on $\vec{b} + \vec{a}$ is equal to

(A) 2 (B) $\frac{39}{5}$

(C) 9 (D) $\frac{46}{5}$

Answer (D)

Sol. $\vec{a} = \alpha\hat{i} + \hat{j} + \beta\hat{k}, \vec{b} = 3\hat{i} - 5\hat{j} + 4\hat{k}$

$\vec{a}\times\vec{b} = -\hat{i} + 9\hat{j} + 12\hat{k}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 1 & \beta \\ 3 & -5 & 4 \end{vmatrix} = -\hat{i} + 9\hat{j} + 12\hat{k}$$

$4 + 5\beta = -1 \Rightarrow \beta = -1$

$-5\alpha - 3 = 12 \Rightarrow \alpha = -3$

$\vec{b} - 2\vec{a} = 3\hat{i} - 5\hat{j} + 4\hat{k} - 2\left(-3\hat{i} + \hat{j} - \hat{k}\right)$

$\vec{b} - 2\vec{a} = 9\hat{i} - 7\hat{j} + 6\hat{k}$

$\vec{b} + \vec{a} = \left(3\hat{i} - 5\hat{j} + 4\hat{k}\right) + \left(-3\hat{i} + \hat{j} - \hat{k}\right)$

$\vec{b} + \vec{a} = -4\hat{j} + 3\hat{k}$

Projection of $\vec{b} - 2\vec{a}$ on $\vec{b} + \vec{a}$ is $= \frac{\left(\vec{b} - 2\vec{a}\right)\cdot\left(\vec{b} + \vec{a}\right)}{\left|\vec{b} + \vec{a}\right|}$

$= \frac{28 + 18}{5} = \frac{46}{5}$

13. Let $\vec{a} = 2\hat{i} - \hat{j} + 5\hat{k}$ and $\vec{b} = \alpha\hat{i} + \beta\hat{j} + 2\hat{k}$. If $\left(\left(\vec{a}\times\vec{b}\right)\times\hat{i}\right)\cdot\hat{k} = \frac{23}{2}$, then $\left|\vec{b}\times 2\hat{j}\right|$ is equal to

(A) 4 (B) 5

(C) $\sqrt{21}$ (D) $\sqrt{17}$

Answer (B)

Sol. Given, $\vec{a} = 2\hat{i} - \hat{j} + 5\hat{k}$ and $\vec{b} = \alpha\hat{i} + \beta\hat{j} + 2\hat{k}$

Also, $\left(\left(\vec{a}\times\vec{b}\right)\times i\right)\cdot\hat{k} = \frac{23}{2}$

$\Rightarrow\ \left(\left(\vec{a}\cdot\hat{i}\right)\vec{b} - \left(\vec{b}\cdot\hat{i}\right)\cdot\vec{a}\right)\cdot\hat{k} = \frac{23}{2}$

$\Rightarrow\ \left(2\cdot\vec{b} - \alpha\cdot\vec{a}\right)\cdot\hat{k} = \frac{23}{2}$

$\Rightarrow\ 2\cdot 2 - 5\alpha = \frac{23}{2} \Rightarrow \alpha = \frac{-3}{2}$

Now, $\left|\vec{b}\times 2\hat{j}\right| = \left|\left(\alpha\hat{i} + \beta\hat{j} + 2\hat{k}\right)\times 2\hat{j}\right|$

$= \left|2\alpha\hat{k} + 0 - 4\hat{i}\right|$

$= \sqrt{4\alpha^2 + 16}$

$= \sqrt{4\left(\frac{-3}{2}\right)^2 + 16}$

$= 5$

14. Let S be the sample space of all five digit numbers. It p is the probability that a randomly selected number from S, is multiple of 7 but not divisible by 5, then $9p$ is equal to

(A) 1.0146 (B) 1.2085

(C) 1.0285 (D) 1.1521

Answer (C)

Sol. Among the 5 digit numbers,

First number divisible by 7 is 10003 and last is 99995.

$\Rightarrow$ Number of numbers divisible by 7.

$= \frac{99995 - 10003}{7} + 1$

$= 12857$

First number divisible by 35 is 10010 and last is 99995.

$\Rightarrow$ Number of numbers divisible by

$$35 = \frac{99995 - 10010}{35} + 1$$

$= 2572$

Hence number of number divisible by 7 but not by 5

$= 12857 - 2572$

$= 10285$

$$9P. = \frac{10285}{90000} \times 9$$

$= 1.0285$

15. Let a vertical tower AB of height $2h$ stands on a horizontal ground. Let from a point P on the ground a man can see upto height h of the tower with an angle of elevation 2α. When from P, he moves a distance d in the direction of $\overrightarrow{AP}$, he can see the top B of the tower with an angle of elevation α. if $d = \sqrt{7}h$, then $\tan\alpha$ is equal to

(A) $\sqrt{5} - 2$

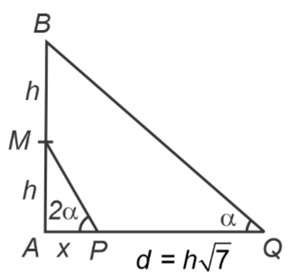
(B) $\sqrt{3} - 1$

(C) $\sqrt{7} - 2$

(D) $\sqrt{7} - \sqrt{3}$

Answer (C)

Sol.



ΔAPM gives

$$\tan 2\alpha = \frac{h}{x} \quad \ldots\text{(i)}$$

ΔAQB gives

$$\tan\alpha = \frac{2h}{x+d} = \frac{2h}{x + h\sqrt{7}} \quad \ldots\text{(ii)}$$

From (i) and (ii)

$$\tan\alpha = \frac{2 \cdot \tan 2\alpha}{1 + \sqrt{7} \cdot \tan 2\alpha}$$

Let $t = \tan\alpha$

$$\Rightarrow \quad t = \frac{2\dfrac{2t}{1-t^2}}{1 + \sqrt{7} \cdot \dfrac{2t}{1-t^2}}$$

$$\Rightarrow \quad t^2 - 2\sqrt{7}t + 3 = 0$$

$$t = \sqrt{7} - 2$$

16. $(p \wedge r) \Leftrightarrow (p \wedge (\sim q))$ is equivalent to $(\sim p)$ when r is

(A) p (B) $\sim p$

(C) q (D) $\sim q$

Answer (C)

Sol. The truth table

p	q	$\sim p$	$\sim q$	$p \wedge q$	$p \wedge \sim q$	$p \wedge q \Leftrightarrow p \wedge \sim q$
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	F	T
F	F	T	T	F	F	T

Clearly $p \wedge q \Leftrightarrow p \wedge \sim q \equiv \sim p$

$\therefore \quad r = q$

17. If the plane P passes through the intersection of two mutually perpendicular planes $2x + ky - 5z = 1$ and $3kx - ky + z = 5$, $k < 3$ and intercepts a unit length on positive x-axis, then the intercept made by the plane P on the y-axis is

(A) $\dfrac{1}{11}$ (B) $\dfrac{5}{11}$

(C) 6 (D) 7

Answer (D)

Sol. $P_1 : 2x + ky - 5z = 1$

$P_2 : 3kx - ky + z = 5$

$\because \quad P_1 \perp P_2 \;\Rightarrow\; 6k - k^2 + 5 = 0$

$\Rightarrow \quad k = 1, 5$

$\because \quad k < 3$

$\therefore \quad k = 1$

$P_1 : 2x + y - 5z = 1$

$P_2 : 3x - y + z = 5$

$P:(2x+y-5z-1)+\lambda(3x-y+z-5)=0$

Positive x-axis intercept = 1

$\Rightarrow \quad \dfrac{1+5\lambda}{2+3\lambda}=1$

$\Rightarrow \quad \lambda=\dfrac{1}{2}$

$\therefore \quad P: 7x+y-4z=7$

y intercept = 7.

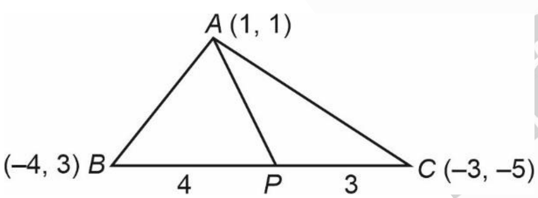
18. Let $A(1, 1)$, $B(-4, 3)$, $C(-2, -5)$ be vertices of a triangle ABC, P be a point on side BC, and Δ_1 and Δ_2 be the areas of triangles APB and ABC, respectively. If $\Delta_1 : \Delta_2 = 4 : 7$, then the area enclosed by the lines AP, AC and the x-axis is

(A) $\dfrac{1}{4}$ (B) $\dfrac{3}{4}$

(C) $\dfrac{1}{2}$ (D) 1

Answer (C)

Sol. $\dfrac{\Delta_1}{\Delta_2}=\dfrac{\frac{1}{2}\times BP\times AH}{\frac{1}{2}\times BC\times AH}=\dfrac{4}{7}$



$P\left(\dfrac{-20}{7},\dfrac{-11}{7}\right)$

Line $AC: y-1=2(x-1)$

Intersection with x-axis $=\left(\dfrac{1}{2},0\right)$

Line $AP: y-1=\dfrac{2}{3}(x-1)$

Intersection with x-axis $\left(\dfrac{-1}{2},0\right)$

Vertices are $(1,1),\left(\dfrac{1}{2},0\right)$ and $\left(\dfrac{-1}{2},0\right)$

Area $=\dfrac{1}{2}$ sq. unit

19. If the circle $x^2+y^2-2gx+6y-19c=0$, $g, c\in\mathbb{R}$ passes through the point (6, 1) and its centre lies on the line $x-2cy=8$, then the length of intercept made by the circle on x-axis is

(A) $\sqrt{11}$ (B) 4

(C) 3 (D) $2\sqrt{23}$

Answer (D)

Sol. Circle : $x^2+y^2-2gx+6y-19c=0$

It passes through $h(6, 1)$

$\Rightarrow \quad 36+1-12g+6-19c=0$

$= 12g+19c=43 \quad \ldots(1)$

Line $x-2cy=8$ passes though centre

$\Rightarrow \quad g+6c=8 \quad \ldots(2)$

From (1) & (2)

$g=2, c=1$

$C: x^2+y^2-4x+6y-19=0$

$x\,\text{int}=2\sqrt{g^2-C}$

$=2\sqrt{4+19}$

$=2\sqrt{23}$

20. Let a function $f:\mathbb{R}\to\mathbb{R}$ be defined as :

$$f(x)=\begin{cases}\displaystyle\int_0^x(5-|t-3|)dt, & x>4\\ x^2+bx, & x\le 4\end{cases}$$

where $b\in\mathbb{R}$. If f is continuous at $x = 4$ then which of the following statements is **NOT** true?

(A) f is not differentiable at $x = 4$

(B) $f'(3)+f'(5)=\dfrac{35}{4}$

(C) f is increasing in $\left(-\infty,\dfrac{1}{8}\right)\cup(8,\infty)$

(D) f has a local minima at $x=\dfrac{1}{8}$

Answer (C)

Sol. $\because \quad f(x)$ is continuous at $x = 4$

$\Rightarrow \quad f(4^-)=f(4^+)$

$\Rightarrow \quad 16+4b=\displaystyle\int_0^4(5-|t-3|)dt$

$$= \int_0^3 (2+t)dt + \int_3^4 (8-t)dt$$

$$= 2t + \frac{t^2}{2}\Big)_0^3 + 8t - \frac{t^2}{3}\Big]_3^4$$

$$= 6 + \frac{9}{2} - 0 + (32-8) - \left(24 - \frac{9}{2}\right)$$

16 + 4b = 15

$$\Rightarrow \; b = \frac{-1}{4}$$

$$\Rightarrow \; f(x) = \begin{cases} \int_0^x 5 - |t-3|\,dt & x > 4 \\ x^2 - \frac{x}{4} & x \le 4 \end{cases}$$

$$\Rightarrow \; f'(x) = \begin{cases} 5 - |x-3| & x > 4 \\ 2x - \frac{1}{4} & x \le 4 \end{cases}$$

$$\Rightarrow \; f'(x) = \begin{cases} 8 - x & x > 4 \\ 2x - \frac{1}{4} & x \le 4 \end{cases}$$

$$f'(x) < 0 \;\Rightarrow\; x \in \left(-\infty, \frac{1}{8}\right) \cup (8, \infty)$$

$$f'(3) + f'(5) = 6 - \frac{1}{4} = \frac{35}{4}$$

$f'(x) = 0 \;\Rightarrow\; x = \frac{1}{8}$ have local minima

$\therefore$ (C) is only incorrect option.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. For $k \in R$, let the solution of the equation

$$\cos\left(\sin^{-1}\left(x\cot\left(\tan^{-1}\left(\cos\left(\sin^{-1}\right)\right)\right)\right)\right)$$

$$= k, 0 < |x| < \frac{1}{\sqrt{2}}$$

Inverse trigonometric functions take only principal values. If the solutions of the equation $x^2 - bx - 5 = 0$ are $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ and $\frac{\alpha}{\beta}$, then $\frac{b}{k^2}$ is equal to ______.

Answer (12)

Sol. $\cos\left(\sin^{-1}\left(x\cot\left(\tan^{-1}\left(\cos\left(\sin^{-1}\right)\right)\right)\right)\right) = k$

$$\Rightarrow \; \cos\left(\sin^{-1}\left(x\cot\left(\tan^{-1}\sqrt{1-x^2}\right)\right)\right) = k$$

$$\Rightarrow \; \cos\left(\sin^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)\right) = k$$

$$\Rightarrow \; \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}} = k$$

$$\Rightarrow \; \frac{1-2x^2}{1-x^2} = k^2$$

$\Rightarrow$ 1 – 2x^2 = k^2 – k^2x^2

$$\therefore \; x^2 - \left(\frac{k^2-1}{k^2-2}\right) = 0 \;\; \begin{matrix}\alpha \\ \beta\end{matrix}$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = 2\left(\frac{k^2-2}{k^2-1}\right) \qquad \ldots(1)$$

$$\text{and } \frac{\alpha}{\beta} = -1 \qquad \ldots(2)$$

$$\therefore \; 2\left(\frac{k^2-2}{k^2-1}\right)(-1) = -5$$

$$\Rightarrow \; k^2 = \frac{1}{3}$$

and b = S.R = $2\left(\frac{k^2-2}{k^2-1}\right) - 1 = 4$

$$\therefore \; \frac{b}{k^2} = \frac{4}{\frac{1}{3}} = 12$$

2. The mean and variance of 10 observation were calculated as 15 and 15 respectively by a student who took by mistake 25 instead of 15 for one observation. Then, the correct standard deviation is ________.

Answer (2)

Sol. Given $\frac{\sum_{i=1}^{10} x_i}{10} = 15$ ...(1) $\Rightarrow \sum_{i=1}^{10} x_i = 150$

and $\frac{\sum_{i=1}^{10} x_i^2}{10} - 15^2 = 15 \quad \Rightarrow \sum_{i=1}^{10} x_i^2 = 2400$

Replacing 25 by 15 we get

$\sum_{i=1}^{9} x_i + 25 = 150 \qquad \Rightarrow \sum_{i=1}^{9} x_i = 125$

$\therefore$ Correct mean $= \frac{\sum_{i=1}^{9} x_i + 15}{10} = \frac{125+15}{10} = 14$

Similarly, $\sum_{i=1}^{2} x_i^2 = 2400 - 25^2 = 1775$

$\therefore$ correct variance $= \frac{\sum_{i=1}^{9} x_i^2 + 15^2}{10} - 14^2$

$= \frac{1775+225}{10} - 14^2 = 4$

$\therefore$ correct S.D = $\sqrt{4} = 2$.

3. Let the line $\frac{x-3}{7} = \frac{y-2}{-1} = \frac{z-3}{-4}$ intersect the plane containing the lines $\frac{x-4}{1} = \frac{y+1}{-2} = \frac{z}{1}$ and $4ax - y + 5z - 7a = 0 = 2x - 5y - z - 3$, $a \in \mathbb{R}$ at the point $P(\alpha, \beta, \gamma)$. Then the value of $\alpha + \beta + \gamma$ equals _____.

Answer (12)

Sol. Equation of plane containing the line

$4ax - y + 5z - 7a = 0 = 2x - 5y - z - 3$ can be written as

$4ax - y + 5z - 7a + \lambda(2x - 5y - z - 3) = 0$

$(4a + 2\lambda)\,x - (1 + 5\lambda)\,y + (5 - \lambda)\,z - (7a + 3\lambda) = 0$

Which is coplanar with the line

$\frac{x-4}{1} = \frac{y+1}{-2} = \frac{z}{1}$

$4(4a + 2\lambda) + (1 + 5\lambda) - (7a + 3\lambda) = 0$

$9a + 10\lambda + 1 = 0 \qquad$ ...(1)

$(4a + 2\lambda)1 + (1 + 5\lambda)\,2 + 5 - \lambda = 0$

$4a + 11\lambda + 7 = 0 \qquad$ ...(2)

$a = 1, \lambda = -1$

Equation of plane is $x + 2y + 3z - 2 = 0$

Intersection with the line

$\frac{x-3}{7} = \frac{y-2}{-1} = \frac{z-3}{-4}$

$(7t + 3) + 2(-t + 2) + 3(-4t + 3) - 2 = 0$

$-7t + 14 = 0$

$t = 2$

So, the required point is (17, 0, –5)

$\alpha + \beta + \gamma = 12$

4. An ellipse $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through the vertices of the hyperbola $H: \frac{x^2}{49} - \frac{y^2}{64} = -1$. Let the major and minor axes of the ellipse E coincide with the transverse and conjugate axes of the hyperbola H, respectively. Let the product of the eccentricities of E and H be $\frac{1}{2}$. If the length of the latus rectum of the ellipse E, then the value of $113l$ is equal to _____.

Answer (1552)

Sol. Vertices of hyperbola = (0, ± 8)

As ellipse pass through it i.e.,

$0 + \frac{64}{b^2} = 1 \Rightarrow b^2 = 64 \qquad$ ...(1)

As major axis of ellipse coincide with transverse axis of hyperbola we have $b > a$ i.e.

$e_E = \sqrt{1 - \frac{a^2}{64}} = \frac{\sqrt{64 - a^2}}{8}$

and $e_H = \sqrt{1 + \frac{49}{64}} = \frac{\sqrt{113}}{8}$

$\therefore \quad e_E \cdot e_H = \frac{1}{2} = \frac{\sqrt{64-a^2}\sqrt{113}}{64}$

$\Rightarrow \quad (64 - a^2)(113) = 32^2$

$\Rightarrow \quad a^2 = 64 - \frac{1024}{113}$

L.R of ellipse $= \frac{2a^2}{b} = \frac{2}{8}\left(\frac{113\times 64 - 1024}{113}\right)$

$= l = \frac{1552}{113}$

$\therefore \quad 113l = 1552$

5. Let $y = y(x)$ be the solution curve of the differential equation

$\sin(2x^2)\log_e(\tan x^2)dy$
$+\left(4xy - 4\sqrt{2}x\sin\left(x^2 - \frac{\pi}{4}\right)\right)dx = 0$,

$0 < x < \sqrt{\frac{\pi}{2}}$, which passes through the point $\left(\sqrt{\frac{\pi}{6}}, 1\right)$. Then $\left|y\left(\sqrt{\frac{\pi}{3}}\right)\right|$ is equal to ________.

Answer (1)

Sol. $\frac{dy}{dx} + y\left(\frac{4x}{\sin(2x^2)\ln(\tan x^2)}\right) = \frac{4\sqrt{2}\,x\sin\left(x^2 - \frac{\pi}{4}\right)}{\sin(2x^2)\ln(\tan x^2)}$

I.F $= e^{\int \frac{4x}{\sin(2x^2)} \frac{dx}{\ln(\tan x^2)}}$

$= e^{\ln|\ln(\tan x^2)|} = \ln(\tan x^2)$

$\therefore \quad \int d\left(y.\ln(\tan x^2)\right) = \int \frac{4\sqrt{2}\,x\sin\left(x^2 - \frac{\pi}{4}\right)}{\sin(2x^2)}dx$

$\Rightarrow \quad y\ln(\tan x^2) = \ln\left|\frac{\sec x^2 + \tan x^2}{\text{cosec } x^2 - \cot x^2}\right| + C$

$\ln\left(\frac{1}{\sqrt{3}}\right) = \ln\left(\frac{\frac{3}{\sqrt{3}}}{2-\sqrt{3}}\right) + C$

$e = \ln\left(\frac{1}{\sqrt{3}}\right) - \ln\left(\frac{\sqrt{3}}{2-\sqrt{3}}\right)$

For $y\left(\sqrt{\frac{\pi}{3}}\right)$

$y\ln(\sqrt{3}) = \ln\left|\frac{2+\sqrt{3}}{\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}}\right| + \ln\left(\frac{1}{\sqrt{3}}\right) - \ln\left(\frac{\sqrt{3}}{2\sqrt{3}}\right)$

$= \ln(2+\sqrt{3}) + \ln\left(\frac{1}{\sqrt{3}}\right) + \ln\left(\frac{1}{\sqrt{3}}\right) - \ln\left(\frac{\sqrt{3}}{2-\sqrt{3}}\right)$

$\Rightarrow \quad y\ln\sqrt{3} = \ln\left(\frac{1}{\sqrt{3}}\right)$

$\Rightarrow \quad \frac{y}{2}\ln 3 = -\frac{1}{2}\ln 3$

$\Rightarrow \quad y = 1$

$\therefore \quad \left|y\left(\sqrt{\frac{\pi}{3}}\right)\right| = 1.$

6. Let M and N be the number of points on the curve $y^5 - 9xy + 2x = 0$, where the tangents to the curve are parallel to x-axis and y-axis, respectively. Then the value of $M + N$ equals ________.

Answer (2)

Sol. Here equation of curve is

$y^5 - 9xy + 2x = 0$ …(i)

On differentiating: $5y^4\frac{dy}{dx} - 9y - 9x\frac{dy}{dx} + 2 = 0$

$\therefore \quad \frac{dy}{dx} = \frac{9y-2}{5y^4 - 9x}$

When tangents are parallel to x axis then $9y - 2 = 0$

$\therefore \quad M = 1.$

For tangent perpendicular to x-axis

$5y^4 - 9x = 0$ …(ii)

From equation (1) and equation (2) we get only one point.

$\therefore \quad N = 1.$

$\therefore \quad M + N = 2.$

7. Let $f(x) = 2x^2 - x - 1$ and $S = \{n \in \mathbb{Z} : |f(n)| \le 800\}$. Then, the value of $\sum_{n \in S} f(n)$ is equal to ________.

Answer (10620)

Sol. $\because \quad |f(n)| \le 800$

$\Rightarrow \quad -800 \le 2n^2 - n - 1 \le 800$

$\Rightarrow \quad 2n^2 - n - 801 \le 0$

$\therefore \quad n \in \left[\dfrac{-\sqrt{6409}+1}{4}, \dfrac{\sqrt{6409}+1}{4}\right]$ and $n \in z$.

$\therefore \quad n = -19, -18, -17, \ldots\ldots\ldots, 19, 20.$

$\therefore \quad \sum(2x^2 - x - 1) = 2\sum x^2 - \sum x - \sum 1.$

$= 2 \cdot 2 \cdot (1^2 + 2^2 + \ldots + 19^2) + 2.20^2 - 20 - 40$

$= 10620$

8. Let S be the set containing all 3×3 matrices with entries from $\{-1, 0, 1\}$. The total number of matrices $A \in S$ such that the sum of all the diagonal elements of $A^T A$ is 6 is ________.

Answer (5376)

Sol. Sum of all diagonal elements is equal to sum of square of each element of the matrix.

i.e., $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

then $t_r\ (A \cdot A^T)$

$= a_1^2 + a_2^2 + a_3^2 + b_1^2 + b_2^2 + b_3^2 + c_1^2 + c_2^2 + c_3^2$

$\because \quad a_i, b_i, c_i \in \{-1, 0, 1\}$ for $i = 1, 2, 3$

$\therefore$ Exactly three of them are zero and rest are 1 or -1.

Total number of possible matrices ${}^9C_3 \times 2^6$

$= \dfrac{9 \times 8 \times 7}{6} \times 64$

$= 5376$

9. If the length of the latus rectum of the ellipse $x^2 + 4y^2 + 2x + 8y - \lambda = 0$ is 4, and l is the length of its major axis, then $\lambda + l$ is equal to ________.

Answer (75)

Sol. Equation of ellipse is: $x^2 + 4y^2 + 2x + 8y - \lambda = 0$

$(x + 1)^2 + 4(y + 1)^2 = \lambda + 5$

$\dfrac{(x+1)^2}{\lambda+5} + \dfrac{(y+1)^2}{\left(\dfrac{\lambda+5}{4}\right)} = 1$

Length of latus rectum = $\dfrac{2 \cdot \left(\dfrac{\lambda+5}{4}\right)}{\sqrt{\lambda+5}} = 4.$

$\therefore \quad \lambda = 59.$

Length of major axis = $2 \cdot \sqrt{\lambda+5} = 16 = l$

$\therefore \quad \lambda + l = 75.$

10. Let $S = \{z \in \mathbb{C} : z^2 + \bar{z} = 0\}$. Then $\sum_{z \in S} (\mathrm{Re}(z) + \mathrm{Im}(z))$ is equal to ________.

Answer (0)

Sol. $\because \quad z^2 + \bar{z} = 0$ Let $z = x + iy$

$\therefore \quad x^2 - y^2 + 2ixy + x - iy = 0$

$(x^2 - y^2 + x) + i(2xy - y) = 0$

$\therefore \quad x^2 + y^2 = 0$ and $(2x - 1)y = 0$

if $x = +\dfrac{1}{2}$ then $y = \pm\dfrac{\sqrt{3}}{2}$

And if $y = 0$ then $x = 0, -1$

$\therefore \quad z = 0 + 0i, -1 + 0i, \dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i, \dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i$

$\therefore \quad \sum(R_e(z) + m(z)) = 0$

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