28/07/2022
Evening

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Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2022 (Online) Phase-2

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Consider the efficiency of carnot's engine is given by $\eta = \frac{\alpha\beta}{\sin\theta}\log_e\frac{\beta x}{kT}$, where α and β are constants. If T is temperature, k is Boltzmann constant, θ is angular displacement and x has the dimensions of length. Then, choose the incorrect option

   (A) Dimensions of β is same as that of force.

   (B) Dimensions of $\alpha^{-1}x$ is same as that of energy.

   (C) Dimensions of $\eta^{-1}\sin\theta$ is same as that of $\alpha\beta$.

   (D) Dimensions of α is same as that of β.

Answer (D)

Sol. (A) $[\beta] = \left[\frac{kT}{x}\right] = \left[\frac{E}{x}\right] = [MLT^{-2}]$

$= [F]$

(B) $[\alpha\beta] = [M^0L^0T^0]$

$[\alpha]^{-1} = [\beta] = \left[\frac{kT}{x}\right]$

So $[\alpha]^{-1}[x] = [kT] = [ML^2T^{-2}]$

(C) $\eta\sin\theta = \alpha\beta$

So $[\eta\sin\theta] = [\alpha\beta]$

$[\eta] = [M^0L^0T^0]$ it is dimensionless quantity

(D) $[\alpha] \neq [\beta]$

2. At time $t = 0$ a particle starts travelling from a height $7\hat{z}$ cm in a plane keeping z coordinate constant. At any instant of time it's position along the $\hat{x}$ and $\hat{y}$ directions are defined at $3t$ and $5t^3$ respectively. At $t = 1$ s acceleration of the particle will be

   (A) $-30\hat{y}$ (B) $30\hat{y}$

   (C) $3\hat{x}+15\hat{y}$ (D) $3\hat{x}+15\hat{y}+7\hat{z}$

Answer (B)

Sol. $x = 3t \Rightarrow a_x = 0$

$y = 5t^3 \Rightarrow a_y = 30t$

$\vec{a}(t=1) = 30\hat{y}$

3. A pressure-pump has a horizontal tube of cross sectional area 10 cm^2 for the outflow of water at a speed of 20 m/s. The force exerted on the vertical wall just in front of the tube which stops water horizontally flowing out of the tube, is

   [given: density of water = 1000 kg/m^3]

   (A) 300 N

   (B) 500 N

   (C) 250 N

   (D) 400 N

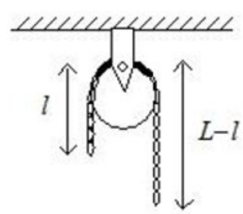
Answer (D)

Sol. $F_w = \rho A v^2$

$= 10^3 \times 10 \times 10^{-4} \times 20 \times 20$

$= 400$ N

4. A uniform metal chain of mass m and length 'L' passes over a massless and frictionless pully. It is released from rest with a part of its length 'l' is hanging on one side and rest of its length '$L - l$' is hanging on the other side of the pully. At a certain point of time, when $l = \frac{L}{x}$, the acceleration of the chain is $\frac{g}{2}$. The value of x is ______.



   (A) 6

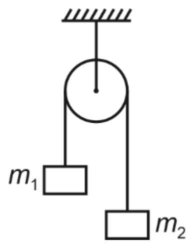
   (B) 2

   (C) 1.5

   (D) 4

Answer (D)

Sol.



$$a = \frac{m_2 - m_1}{m_1 + m} g = \frac{g}{2}$$

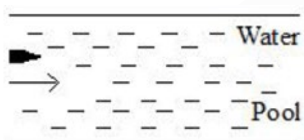
$\Rightarrow$ $m_2 = 3m_1$

$(L - l) = 3l$

$L = 4l$

$$l = \frac{L}{4}$$

5. A bullet of mass 200 g having initial kinetic energy 90 J is shot inside a long swimming pool as shown in the figure. If it's kinetic energy reduces to 40 J within 1 s, the minimum length of the pool, the bullet has to travel so that it completely comes to rest is



(A) 45 m
(B) 90 m
(C) 125 m
(D) 25 m

Answer (A)

Sol. $\frac{1}{2}mx^2 = 90$

$\Rightarrow$ $\frac{1}{2} \times 0.2 \times x^2 = 90$,

$x^2 = 900$

$x = 30$ m/s

$\frac{1}{2}mv^2 = 40 \Rightarrow v = \frac{2}{3} \times 30 = 20$ m/s

$20 = 30 - a \times 1 \Rightarrow a = -10$ m/s^2

$0 - x^2 = 2as$

$$s = \frac{x^2}{-2a} = \frac{30 \times 30}{2 \times 10}$$

$= 45$ m

6. Assume there are two identical simple pendulum clocks. Clock – 1 is placed on the earth and Clock - 2 is placed on a space station located at a height h above the earth surface. Clock – 1 and Clock – 2 operate at time periods 4 s and 6 s respectively. Then the value of h is-

(consider radius of earth R_E = 6400 km and g on earth 10 m/s^2)

(A) 1200 km
(B) 1600 km
(C) 3200 km
(D) 4800 km

Answer (C)

$T \propto \sqrt{1/g}$

$$\Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}} = \frac{R}{R+h}$$

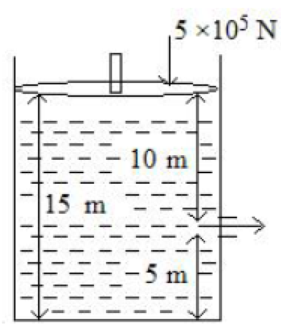
$$\frac{4}{6} = \frac{R}{R+h}$$

$\Rightarrow h = R/2$

$= 3200$ km

7. Consider a cylindrical tank of radius 1 m is filled with water. The top surface of water is at 15 m from the bottom of the cylinder. There is a hole on the wall of cylinder at a height of 5 m from the bottom. A force of 5×10^5 N is applied an the top surface of water using a piston. The speed of iflux from the hole will be:

(given atmosphere pressure $P_A = 1.01 \times 10^5$ Pa, density of water ρ_w = 1000 kg/m^3 and gravitational acceleration g = 10 m/s^2)



(A) 11.6 m/s

(B) 10.8 m/s

(C) 17.8 m/s

(D) 14.4 m/s

Answer (C)

Sol. By Bernoulli's theorem,

$$\frac{5\times10^5}{\pi(1)^2}+\rho g(10)=1.01\times10^5+\frac{1}{2}\rho(v)^2$$

$$\Rightarrow v^2=200+\frac{10^6}{1000\pi}-202$$

$$\Rightarrow v \simeq 17.8 \text{ m/s}$$

8. A vessel contains 14 g of nitrogen gas at a temperature of 27°C. The amount of heat to be transferred to the gas to double the r.m.s speed of its molecules will be:

Take R = 8.32 J mol^{-1} K^{-1}.

(A) 2229 J (B) 5616 J

(C) 9360 J (D) 13,104 J

Answer (C)

Sol. n = 0.5

T = 300

For v_{rms} to be doubled T' = 4 × 300 = 1200

$\Rightarrow$ Heat transferred

$$=(0.5)\left(\frac{5}{2}\right)(8.32)(900)$$

= 9360 J

9. A slab of dielectric constant K has the same cross-sectional area as the plates of a parallel plate capacitor and thickness $\frac{3}{4}d$, where d is the separation of the plates. The capacitance of the capacitor when the slab is inserted between the plates will be:

(Given C_0 = capacitance of capacitor with air as medium between plates.)

(A) $\frac{4KC_0}{3+K}$ (B) $\frac{3KC_0}{3+K}$

(C) $\frac{3+K}{4KC_0}$ (D) $\frac{K}{4+K}$

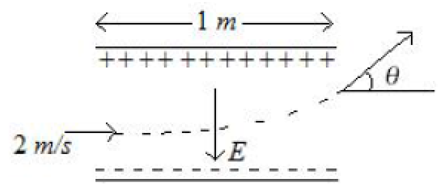
Answer (A)

Sol. $C_0=\frac{\varepsilon_0 A}{d}$

$$C=\frac{\varepsilon_0 A}{d-\frac{3d}{4}+\frac{3d}{4K}}=\frac{4\varepsilon_0 AK}{3d+Kd}$$

$$=\frac{4KC_o}{3+K}$$

10. A uniform electric field E = (8m/e) V/m is created between two parallel plates of length 1 m as shown in figure, (where m = mass of electron and e = charge of electron). An electron enters the field symmetrically between the plates with a speed of 2 m/s. The angle of the deviation (θ) of the path of the electron as it comes out of the field will be________.



(A) $\tan^{-1}(4)$ (B) $\tan^{-1}(2)$

(C) $\tan^{-1}\left(\frac{1}{3}\right)$ (D) $\tan^{-1}(3)$

Answer (B)

Sol. $E=\frac{8\,m}{e}$ V/m

l = 1 m

v_x = 2 m/s

a_y = – 8 m/s^2

$t=\frac{l}{v_x}=\frac{1}{2}$ s

$\Rightarrow |v_y|$ = 4 m/s

$\Rightarrow$ angle of deviation = θ

$$\tan\theta=\frac{v_y}{v_x}$$

$$\theta=\tan^{-1}\left(\frac{4}{2}\right)=\tan^{-1}(2)$$

11. Given below are two statements:

Statement I : A uniform wire of resistance 80 Ω is cut into four equal parts. These parts are now connected in parallel. The equivalent resistance of the combination will be 5 Ω.

Statement II : Two resistances $2R$ and $3R$ are connected in parallel in a electric circuit. The value of thermal energy developed in $3R$ and $2R$ will be in the ratio 3 : 2.

In the light of the above statements, choose the **most appropriate** answer from the option given below.

(A) Both statement I and statement II are correct

(B) Both statement I and statement II are incorrect

(C) Statement I is correct but statement II is incorrect

(D) Statement I is incorrect but statement II is correct

Answer (C)

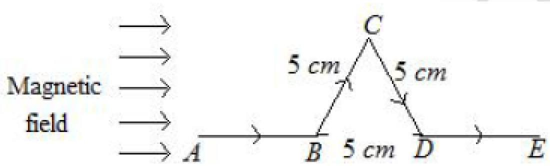
Sol. Statement I : $R_{1\text{ part}} = \frac{80}{4} = 20\ \Omega$

$\Rightarrow \quad R_{\text{eff}} = \frac{20}{4} = 5\ \Omega$

Statement II : Ratio $= \frac{\frac{(\Delta V)^2}{3R}}{\frac{(\Delta V)^2}{2R}}$

$= \frac{2}{3}$

12. A triangular shaped wire carrying 10 A current is placed in a uniform magnetic field of 0.5 T, as shown in figure. The magnetic force on segment *CD* is (Given *BC* = *CD* = *BD* = 5 cm).



(A) 0.126 N (B) 0.312 N

(C) 0.216 N (D) 0.245 N

Answer (C)

Sol. $\vec{F} = i\vec{\ell} \times \vec{B}$

$= i\ell B\sin 60°$

$= 10 \times \frac{5}{100} \times 0.5 \times \frac{\sqrt{3}}{2}$

= 0.2165 N

13. The magnetic field at the center of current carrying circular loop is B_1. The magnetic field at a distance of $\sqrt{3}$ times radius of the given circular loop from the center on its axis is B_2. The value of B_1/B_2 will be

(A) 9 : 4 (B) $12 : \sqrt{5}$

(C) 8 : 1 (D) $5 : \sqrt{3}$

Answer (C)

Sol. $B_1 = \frac{\mu_0 i}{2R}$

$$B_2 = \frac{\mu_0 i R^2}{2(R^2 + x^2)^{\frac{3}{2}}}$$

$$\Rightarrow \quad \frac{B_1}{B_2} = \frac{1}{R^3}(R^2 + x^2)^{\frac{3}{2}}$$

$$= \frac{1}{R^3}(8R^3)$$

= 8

14. A transformer operating at primary voltage 8 kV and secondary voltage 160 V serves a load of 80 kW. Assuming the transformer to be ideal with purely resistive load and working on unity power factor, the loads in the primary and secondary circuit would be

(A) 800 Ω and 1.06 Ω (B) 10 Ω and 500 Ω

(C) 800 Ω and 0.32 Ω (D) 1.06 Ω and 500 Ω

Answer (C)

Sol. $V_1 i_1 = V_2 i_2 = 80\ \text{kW}$

$\Rightarrow \quad i_1 = 10$ A and $i_2 = \frac{80 \times 1000}{160} = 500$ A

$\Rightarrow \quad R_1 = \frac{V_1}{i_1} = 800\ \Omega$ and $R_2 = \frac{160}{500} = 0.32\ \Omega$

15. Sun light falls normally on a surface of area 36 cm^2 and exerts an average force of 7.2×10^{-9} N within a time period of 20 minutes. Considering a case of complete absorption, the energy flux of incident light is

(A) 25.92×10^2 W/cm^2

(B) 8.64×10^{-6} W/cm^2

(C) 6.0 W/cm^2

(D) 0.06 W/cm^2

Answer (D)

Sol. Pressure $= \frac{I}{c}$

$\Rightarrow \quad \frac{F}{A} = \frac{I}{c}$

$\Rightarrow \quad I = \frac{7.2 \times 10^{-9} \times 3 \times 10^8}{36 \times 10^{-4}}$ W/m^2

= 600 W/m^2

$\Rightarrow \quad I = 0.06\ \text{W/cm}^2$

16. The power of a lens (biconvex) is 1.25 m^{-1} in particular medium. Refractive index of the lens is 1.5 and radii of curvature are 20 cm and 40 cm respectively. The refractive index of surrounding medium

(A) 1.0
(B) $\frac{9}{7}$
(C) $\frac{3}{2}$
(D) $\frac{4}{3}$

Answer (B)

Sol. $\because \frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$$\Rightarrow \frac{1.25}{100}=\left(\frac{1.5}{\mu_1}-1\right)\left(\frac{1}{20}+\frac{1}{40}\right)$$

$$\Rightarrow \frac{1}{80}=\left(\frac{1.5}{\mu_1}-1\right)\times\frac{(4+2)}{80}$$

$$\Rightarrow \frac{1.5}{\mu_1}-1=\frac{1}{6}$$

$$\Rightarrow \frac{1.5}{\mu_1}=\frac{7}{6}$$

$$\Rightarrow \mu_1=\frac{1.5\times 6}{7}=\frac{9}{7}$$

17. Two streams of photons, possessing energies equal to five and ten times the work function of metal are incident on the metal surface successively. The ratio of maximum velocities of the photoelectron emitted, in the two cases respectively, will be

(A) 1 : 2
(B) 1 : 3
(C) 2 : 3
(D) 3 : 2

Answer (C)

Sol. $\frac{1}{2}mv_1^2=5\phi-\phi$

And, $\frac{1}{2}mv_2^2=10\phi-\phi$

$$\Rightarrow \left(\frac{v_1}{v_2}\right)^2=\frac{4}{9}$$

$$\Rightarrow \frac{v_1}{v_2}=\frac{2}{3}$$

18. A radioactive sample decays $\frac{7}{8}$ times its original quantity in 15 minutes. The half-life of the sample is

(A) 5 min
(B) 7.5 min
(C) 15 min
(D) 30 min

Answer (A)

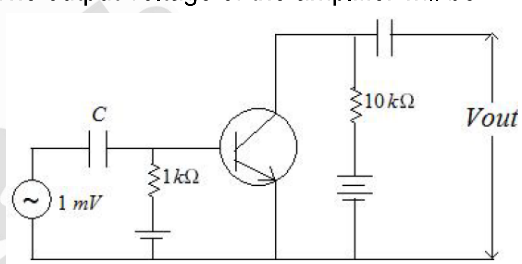
Sol. $N=\frac{N_0}{2^{\frac{t}{T_{1/2}}}}$

$$\Rightarrow 2^{\frac{t}{T_{1/2}}}=\frac{N_0}{N}=\frac{N_0}{\left(\frac{N_0}{8}\right)}=8$$

$$\Rightarrow \frac{t}{T_{1/2}}=3$$

$$\Rightarrow T_{1/2}=\frac{15}{3}=5\text{ min}$$

19. An n.p.n. transistor with current gain β = 100 in common emitter configuration is shown in figure. The output voltage of the amplifier will be



(A) 0.1 V
(B) 1.0 V
(C) 10 V
(D) 100 V

Answer (B)

Sol. $\frac{V_o}{V_i}=\beta\times\left(\frac{R_C}{R_B}\right)$

$$\Rightarrow V_o=100\times\left(\frac{10}{1}\right)\times 10^{-3}$$

= 1.0 V

20. A FM Broad cast transmitter, using modulating signal of frequency 20 kHz has a deviation ratio of 10. The Bandwidth required for transmission is

(A) 220 kHz
(B) 180 kHz
(C) 360 kHz
(D) 440 kHz

Answer (D)

Sol. Band width of FM wave = $2(\Delta f + f_m)$

$\frac{\Delta f}{f_m}=10$ (Given)

$\Delta f = f_m(10) = 20 \times 10 = 200$ kHz

BW = 2(200 + 20) kHz

= 440 kHz

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. A ball is thrown vertically upwards with a velocity of 19.6 ms^{-1} from the top of a tower. The ball strikes the ground after 6 s. The height from the ground up to which the ball can rise will be $\left(\frac{k}{5}\right)$ m . The value of k is ______ (use g = 9.8 m/s^2)

Answer (392)

Sol. v = 19.6 m/s

t = 6s

Time taken in upward motion above tower = 2s

$\Rightarrow$ Time taken from top most point to ground = 4s

$$\Rightarrow \sqrt{\frac{2h}{g}} = 4$$

$$h = \frac{16 \times 9.8}{2} = 8 \times 9.8$$

$\Rightarrow$ $k = 8 \times 9.8 \times 5 = 392$

2. The distance of centre of mass from end A of a one dimensional rod (AB) having mass density $\rho = \rho_0\left(1-\frac{x^2}{L^2}\right)$ kg/m and length L (in meter) is $\frac{3L}{\alpha}$ m. The value of α is ______. (where x is the distance from end A)

Answer (8)

Sol. $\rho = \rho_0\left(1-\frac{x^2}{L^2}\right)$ kg/m

$$x_{cm} = \frac{A\int_0^L \rho_0\left(1-\frac{x^2}{L^2}\right) x\,dx}{A\int_0^L \rho_0\left(1-\frac{x^2}{L^2}\right) dx}$$

$$x_{cm} = \frac{\frac{L^2}{2}-\frac{L^2}{4}}{L-\frac{L}{3}} = \frac{\frac{L^2}{4}}{\frac{2L}{3}} = \frac{3L}{8}$$

$\Rightarrow$ $\alpha = 8$

3. A string of area of cross-section 4 mm^2 and length 0.5 m is connected with a rigid body of mass 2 kg. The body is rotated in a vertical circular path of radius 0.5 m. The body acquires a speed of 5 m/s at the bottom of the circular path. Strain produced in the string when the body is at the bottom of the circle is ______$\times 10^{-5}$.

(use young's modulus 10^{11} N/m^2 and g = 10 m/s^2)

Answer (30)

Sol. $A = 4 \times 10^{-6}$ m^2

l = 0.5 m

m = 2 kg

v_b = 5 m/s

$$T_b = mg + m\left(\frac{V_b^2}{l}\right)$$

$$= 20 + 2 \times \frac{25}{\frac{1}{2}} = 120\text{ N}$$

$$\frac{\Delta l}{l} = \frac{T_b}{A} \times \frac{1}{Y} = \frac{120}{4 \times 10^{-6}} \times 10^{-11} = 30 \times 10^{-5}$$

4. At a certain temperature, the degrees of freedom per molecule for gas is 8. The gas performs 150 J of work when it expands under constant pressure. The amount of heat absorbed by the gas will be ______ J.

Answer (750)

Sol. f = 8

$W = P\,dV = 150$

$Q = W + \Delta U$

$$= P\,dV + \frac{f}{2} PdV$$

$Q = 5 \times 150 = 750$ J

5. The potential energy of a particle of mass 4 kg in motion along the x-axis is given by $U = 4\ (1–\cos 4x)$ J. The time period of the particle for small oscillation $(\sin\theta \simeq \theta)$ is $\left(\frac{\pi}{K}\right)$s. The value of K is _______.

Answer (2)

Sol. $U = 4(1 – \cos 4x)$

$$\Rightarrow \quad F = -\frac{dU}{dx} = -(4)\,(4\sin 4x)$$

$$= –16 \sin 4x$$

as small x

$F = –16(4x) = – 64x \equiv – kx$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{4}{64}} = \frac{\pi}{2}$$

$\Rightarrow \quad K = 2$

6. An electrical bulb rated 220 V, 100 W, is connected in series with another bulb rated 220 V, 60 W. If the voltage across combination is 220 V, the power consumed by the 100 W bulb will be about ____ W.

Answer (14)

Sol. $P_{100} = \frac{V^2}{R_{100}} \Rightarrow R_{100} = \frac{V^2}{P_{100}}$

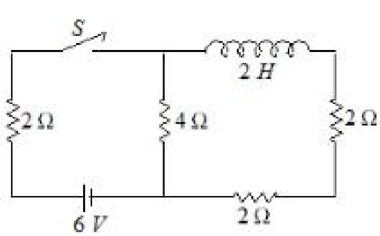
$$P_{60} = \frac{V^2}{R_{60}} \Rightarrow R_{60} = \frac{V^2}{P_{60}}$$

$$P_{net} = \frac{V^2}{R_{60} + R_{100}} = \frac{P_{60}P_{100}}{P_{60} + P_{100}} = \frac{60 \times 100}{160} = 37.5$$

This power developed is proportional to resistance.

So, $P'_{60} = P_{net} \times \frac{60}{160} = 37.5 \times \frac{60}{160} \simeq 14$ W

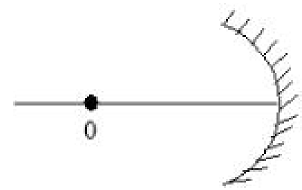
7. For the given circuit the current through battery of 6 V just after closing the switch S will be _____ A.



Answer (1)

Sol. Just after closing the switch, $i = \frac{6}{4+2} = 1$ A

8. An object O is placed at a distance of 100 cm in front of a concave mirror of radius of curvature 200 cm as shown in the figure. The object starts moving towards the mirror at a speed 2 cm/s. The position of the image from the mirror after 10 s will be at ____ cm.



Answer (400)

Sol. The object after 10 second will be at u = –80 cm.

So $\frac{1}{v} - \frac{1}{80} = -\frac{1}{100} \Rightarrow v = \frac{8000}{+20} = 400$ cm

9. In an experiment with a convex lens, the plot of the image distance (ν') against the object distance (μ') measured from the focus gives a curve $\nu'\mu'$ = 225. If all the distances are measured in cm. The magnitude of the focal length of the lens is _______ cm.

Answer (15)

Sol. Using Newton's formula for lenses,

$\nu'\mu' = f^2 = 225 \Rightarrow f = 15$

10. In an experiment to find acceleration due to gravity (g) using simple pendulum, time period of 0.5 s is measured from time of 100 oscillations with a watch of 1 s resolution. If measured value of length is 10 cm known to 1 mm accuracy, the accuracy in the determination of g is found to be x%. The value of x is _________

Answer (5)

Sol. $T = 2\pi\sqrt{\frac{l}{g}}$

$$\frac{dg}{g} \times 100 = \frac{2dT}{T} \times 100 + \frac{d\ell}{\ell} \times 100$$

$$= 2 \times \frac{1}{50} \times 100 + \frac{1}{100} \times 100 = 5\%$$

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Given below are two statements: One is labelled as **Assertion A** and the other is labelled as **Reason R**

   Assertion A: Zero orbital overlap is an out of phase overlap.

   Reason R: It results due to different orientation/direction of approach of orbitals.

   In the light of the above statements, choose the **correct** answer from the options given below

   (A) Both A and R are true and R is the correct explanation of A

   (B) Both A and R are true but R is NOT the correct explanation of A

   (C) A is true but R is false

   (D) A is false but R is true

Answer (A)

Zero overlapping is something in which there is no overlapping between two orbitals. The first condition is that the two orbitals should not be symmetrical and the second condition is that both orbitals should be in different planes.

2. The correct decreasing order for metallic character is

   (A) Na > Mg > Be > Si > P

   (B) P > Si > Be > Mg > Na

   (C) Si > P > Be > Na > Mg

   (D) Be > Na > Mg > Si > P

Answer (A)

Metallic character increases top to bottom in group and decreases left to right in a period.

Mg is from second group it will be less metallic than Na. Be comes above Mg hence less metallic than Mg. Si is more metallic than phosphorous.

3. Given below are two statements: One is labelled as **Assertion A** and the other is labelled as **Reason R**

   Assertion A: The reduction of a metal oxide is easier if the metal formed is in liquid state than solid state.

   Reason R: The value of $\Delta G^{\ominus}$ becomes more on negative side as entropy is higher in liquid state than solid state.

   In the light of the above statements, choose the most appropriate answer from the options given below

   (A) Both A and R are correct and R is the correct explanation of A

   (B) Both A and R are correct but R is not the correct explanation of A

   (C) A is correct but R is not correct

   (D) A is not correct but R is correct

Answer (A)

Reduction of a metal oxide is easier if the metal is formed in a liquid state at the temperature of reduction because the entropy is higher if the metal is in a liquid state.

4. The products obtained during treatment of hard water using Clark's method are:

   (A) $CaCO_3$ and $MgCO_3$

   (B) $Ca(OH)_2$ and $Mg(OH)_2$

   (C) $CaCO_3$ and $Mg(OH)_2$

   (D) $Ca(OH)_2$ and $MgCO_3$

Answer (C)

Clark's method :

$$Mg(HCO_3)_2 + 2Ca(OH)_2 \longrightarrow 2CaCO_3\downarrow + Mg(OH_2)\downarrow + 2H_2O$$

5. **Statement-I**: An alloy of lithium and magnesium is used to make aircraft plates.

Statement-II: The magnesium ions are important for cell-membrane integrity.

In the light of the above statements, choose the **correct** answer from the options given below:

(A) Both Statement-I and Statement-II are true

(B) Both Statement-I and Statement-II are false

(C) Statement-I is true but Statement-II is false

(D) Statement-I is false but Statement-II is true

Answer (B)

Magnesium alloys are used to make body of aircraft which is lightweight and resistant to corrosion. Calcium is responsible for cell membrane integrity.

6. White phosphorus reacts with thionyl chloride to give

(A) PCl_5, SO_2 and S_2Cl_2

(B) PCl_3, SO_2 and S_2Cl_2

(C) PCl_3, SO_2 and Cl_2

(D) PCl_5, SO_2 and Cl_2

Answer (B)

Sol $\underset{\text{White phosphorous}}{P_4} + \underset{\text{Thionyl chloride}}{8SOCl_2} \longrightarrow 4PCl_3 + 4SO_2 + 2S_2Cl_2$

7. Concentrated HNO_3 reacts with Iodine to give

(A) HI, NO_2 and H_2O (B) HIO_2, N_2O and H_2O

(C) HIO_3, NO_2 and H_2O (D) HIO_4, N_2O and H_2O

Answer (C)

Sol $I_2 + 10HNO_3 \rightarrow 2HIO_3 + 10NO_2 + 4H_2O$

8. Which of the following pair is not isoelectronic species?

(At. no. Sm, 62; Er, 68; Yb, 70; Lu, 71; Eu, 63; Tb, 65; Tm, 69)

(A) Sm^{2+} and Er^{3+} (B) Yb^{2+} and Lu^{3+}

(C) Eu^{2+} and Tb^{4+} (D) Tb^{2+} and Tm^{4+}

Answer (A, D)

Sol Species having same number of electrons are isoelectronic

$$\left.\begin{array}{ll} Tb \longrightarrow 65 & Tb^{+2} \longrightarrow 63 \text{ electrons} \\ Tm \longrightarrow 69 & Tm^{+4} \longrightarrow 65 \text{ electrons} \end{array}\right\} \text{not isoelectronic}$$

$$\left.\begin{array}{l} Sm^{+2} \longrightarrow 60 \text{ electrons} \\ Er^{+3} \longrightarrow 65 \text{ electrons} \end{array}\right\} \text{not isoelectronic}$$

9. Given below are two statements: One is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A: Permanganate titrations are not performed in presence of hydrochloric acid.

Reason R: Chlorine is formed as a consequence of oxidation of hydrochloric acid.

In the light of the above statements, choose the ***correct*** answer from the options given below.

(A) Both **A** and **R** are true and **R** is the correct explanation of **A**

(B) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

(C) **A** is true but **R** is false

(D) **A** is false but **R** is true

Answer (A)

Sol HCl is not used in the process of titration because it reacts with the ($KMnO_4$) that is used in the process and gets oxidized.

10. Match **List-I** with **List-II**

List I (Complex)	List II (Hybridization)
A. $Ni(CO)_4$	I. sp^3
B. $[Ni(CN)_4]^{2-}$	II. sp^3d^2
C. $[Co(CN)_6]^{3-}$	III. d^2sp^3
D. $[CoF_6]^{3-}$	IV. dsp^2

Choose the correct answer from the options given below:

(A) A-IV, B-I, C-III, D-II

(B) A-I, B-IV, C-III, D-II

(C) A-I, B-IV, C-II, D-III

(D) A-IV, B-I, C-II, D-III

Answer (B)

Sol A. $Ni(CO)_4 \rightarrow sp^3$

B $[Ni(CN)_4]^{-2} \rightarrow dsp^2$

C. $[Co(CN)_6]^{-3} \rightarrow d^2sp^3$

D. $[CoF_6]^{-3} \rightarrow sp^3d^2$

11. Dinitrogen and dioxygen, the main constituents of air do not react with each other in atmosphere to form oxides of nitrogen because

(A) N_2 is unreactive in the condition of atmosphere

(B) Oxides of nitrogen are unstable

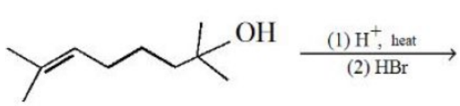
(C) Reaction between them can occur in the presence of a catalyst

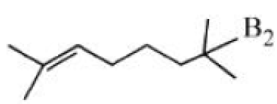
(D) The reaction is endothermic and require very high temperature

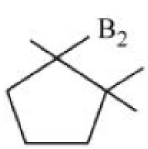
Answer (D)

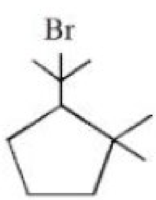
Sol. N_2 is unreactive, its reaction with oxides is endothermic and require very high temperature.

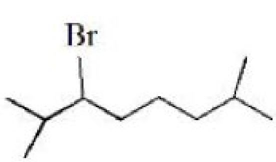
12. The major product in the given reaction is



(A) 

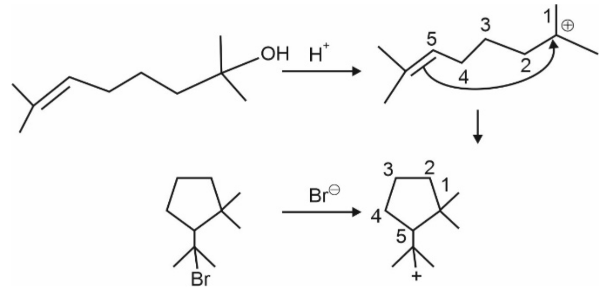
(B) 

(C) 

(D) 

Answer (C)

Sol.



13. Arrange the following in increasing order of reactivity towards nitration

A. p-xylene

B. bromobenzene

C. mesitylene

D. nitrobenzene

E. benzene

(A) C < D < E < A < B

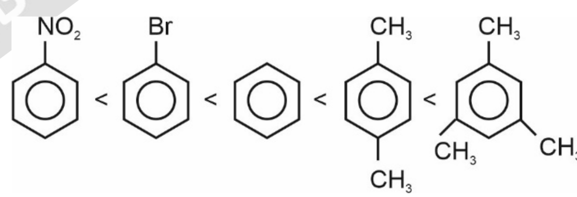
(B) D < B < E < A < C

(C) D < C < E < A < B

(D) C < D < E < B < A

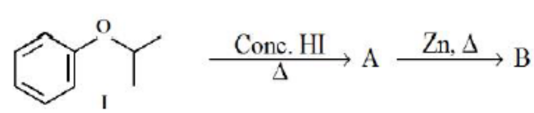
Answer (B)

Sol. The correct order of reactivity towards nitration is



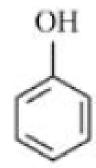
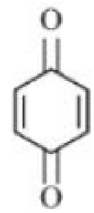
as electron releasing groups on benzene ring facilitate the nitration at benzene ring.

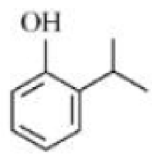
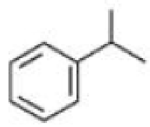
14. Compound I is heated with Conc. HI to give a hydroxy compound A which is further heated with Zn dust to give compound B. Identify A and B.

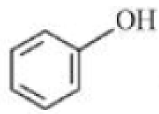


(A) A = 

, B = 

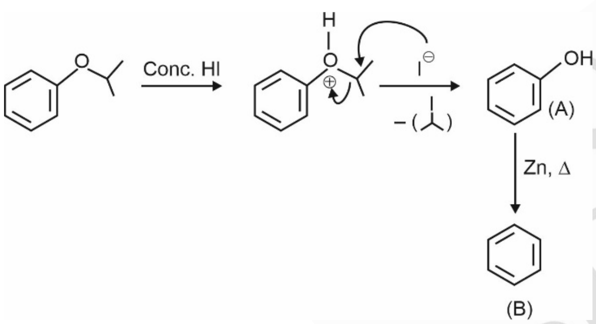
(B) A =  , B = 

(C) A =  , B = 

(D) A =  , B = 

Answer (D)

Sol.



15. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : Aniline on nitration yields ortho, meta & para nitro derivatives of aniline.

Reason R : Nitrating mixture is a strong acidic mixture.

In the light of the above statements, choose the correct answer from the options given below.

(A) Both **A** and **R** are true and **R** is the correct explanation of **A**

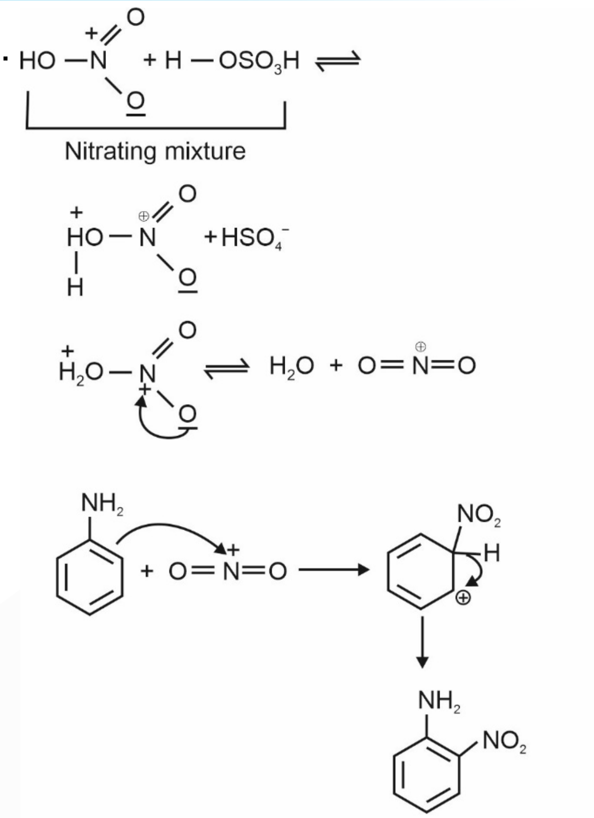
(B) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

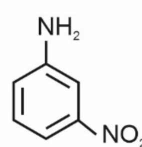
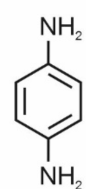
(C) **A** is true but **R** is false

(D) **A** is false but **R** is true

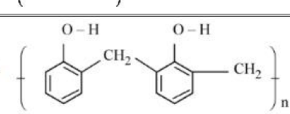
Answer (A)

Sol.



Similarly  and  are formed

16. Match List I with List II

List I (Polymer)	List II (Nature)
A. $\left[CH_2 - \underset{Cl}{C} = CH - CH_2 \right]_n$	I. Thermosetting polymer
B. $\left[\overset{H}{N} - (CH_2)_6 - \overset{H}{N} - \overset{O}{\overset{\Vert}{C}} - (CH_2)_4 - \overset{O}{\overset{\Vert}{C}} \right]_n$	II. Fibers
C. $\left[CH_2 - \overset{Cl}{CH} \right]_n$	III. Elastomer
D. 	IV. Thermoplastic polymer

Choose the correct answer from the options given below:

(A) A-II, B-III, C-IV, D-I (B) A-III, B-II, C-IV, D-I

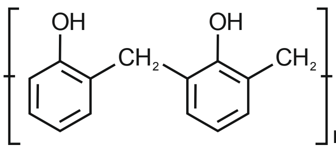
(C) A-III, B-I, C-IV, D-II (D) A-I, B-III, C-IV, D-II

Answer (B)

Sol. (A) $\left[CH_2 - \underset{\displaystyle Cl}{\underset{|}{C}} = CH - CH_2 \right]_n$ — Elastomer

(B) $\left[\overset{\displaystyle H}{\overset{|}{N}} - (CH_2)_6 - \overset{\displaystyle H}{\overset{|}{N}} - \overset{\displaystyle O}{\overset{||}{C}} - (CH_2)_4 - \overset{\displaystyle O}{\overset{||}{C}} \right]_n$ — Fibers

(C) $\left[CH_2 - \overset{\displaystyle Cl}{\overset{|}{CH}} \right]_n$ — Thermoplastic polymer

(D) 

— Thermosetting polymer

17. Two statements in respect of drug-enzyme interaction are given below

Statement I: Action of an enzyme can be blocked only when an inhibitor blocks the active site of the enzyme.

Statement II: An inhibitor can form a strong covalent bond with the enzyme.

In the light of the above statements, choose the correct answer from the options given below

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Answer (D)

Sol. Action of an enzyme can be altered by a number of factors like temperature, pH, presence of activators and coenzymes and presence of inhibitors and poisons.

Inhibitors or poisons interact with the active functional groups on the enzyme surface and often reduce or completely destroy the catalytic activity of the enzymes.

18. Given below are two statements: One is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A: Thin layer chromatography is an adsorption chromatography.

Reason R: A thin layer of silica gel is spread over a glass plate of suitable size in thin layer chromatography which acts as an adsorbent.

In the light of the above statements, choose the **correct** answer from the options given below

(A) Both **A** and **R** are true and **R** is the correct explanation of **A**.

(B) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

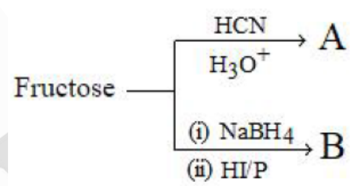
(C) **A** is true but **R** is false

(D) **A** is false but **R** is true

Answer (A)

Sol. Thin layer chromatography is an adsorption chromatography. A thin layer of silica gel is spread over a glass plate of suitable size and act as an adsorbent.

19. The formulas of A and B for the following reaction sequence



are

(A) A = $C_7H_{14}O_8$, B = C_6H_{14}

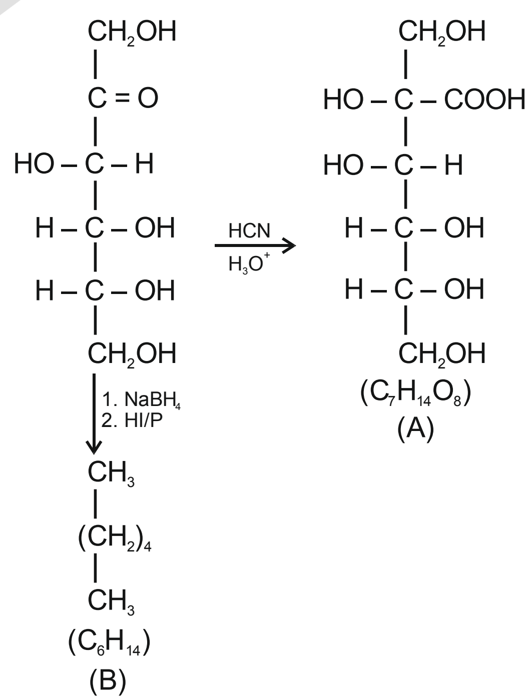
(B) A = $C_7H_{13}O_7$, B = $C_7H_{14}O$

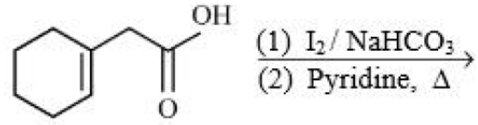
(C) A = $C_7H_{12}O_8$, B = C_6H_{14}

(D) A = $C_7H_{14}O_8$, B = $C_6H_{14}O_6$

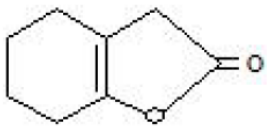
Answer (A)

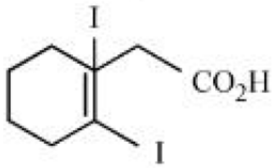
Sol.

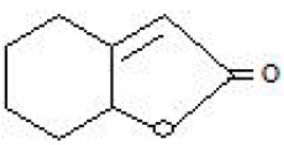


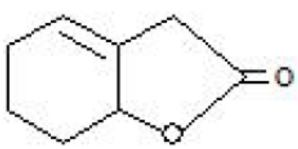
20. 

Find out the major product for the above reaction.

(A) 

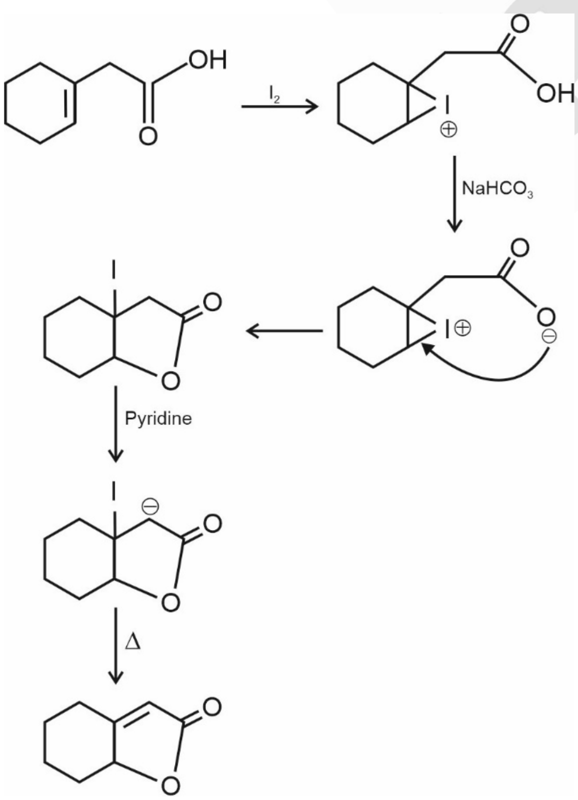
(B) 

(C) 

(D) 

Answer (C)

Sol.



SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. 2 L of 0.2 M H_2SO_4 is reacted with 2 L of 0.1 M NaOH solution, the molarity of the resulting product Na_2SO_4 in the solution is _______ millimolar. (Nearest integer)

Answer (25)

Sol.

$$H_2SO_4 + 2NaOH \longrightarrow Na_2SO_4 + 2H_2O$$

	H_2SO_4	$2NaOH$	Na_2SO_4
at intial	0.4 mole	0.2 mole	–
	0.3	0	0.1

Molarity of $Na_2SO_4 = \frac{0.1}{4} = 0.025$ M

$= 25$ millimolar.

2. Metal M crystallizes into a fcc lattice with the edge length of 4.0×10^{-8} cm. The atomic mass of the metal is _______ g/mol. (Nearest integer)

(Use : $N_A = 6.02 \times 10^{23}$ mol^{-1}, density of metal, M = 9.03 g cm^{-3})

Answer (87)

Sol. $\rho = \frac{ZM}{N_A a^3} \Rightarrow M = \frac{9.03 \times 6.02 \times 10^{23} \times (4 \times 10^{-8})^3}{4}$

$$= \frac{9.03 \times 6.02 \times 64 \times 10^{-1}}{4}$$

$= 86.9$ g mol^{-1}

≈ 87 g mol^{-1}

3. If the wavelength for an electron emitted from H-atom is 3.3×10^{-10} m, then energy absorbed by the electron in its ground state compared to minimum energy required for its escape from the atom, is _______ times. (Nearest integer)

[Given : $h = 6.626 \times 10^{-34}$ J s]

[Mass of electron = 9.1×10^{-31} kg]

Answer (2)

Sol. $\lambda = \dfrac{h}{mv}$

$$\Rightarrow \quad mv = \frac{h}{\lambda} = \frac{6.626\times10^{-34}\ \text{kg}\dfrac{m^2}{\sec^2}\times\sec}{3.3\times10^{-10}\,m}$$

$$mv = \frac{6.626\times10^{-24}}{3.3} = 2\times10^{-24}\ \text{kg m sec}^{-1}$$

$$\text{Kinetic energy} = \frac{1}{2}mv^2$$

$$= \frac{(mv)^2}{2m}$$

$$= \frac{\left(2\times10^{-24}\right)^2}{2\times9.1\times10^{-31}\ \text{kg}}$$

$= 2.18 \times 10^{-18}$ J

$= 21.8 \times 10^{-19}$ J

Total energy absorbed = Ionization energy + Kinetic energy

$= (21.76 + 21.8) \times 10^{-19}$

$= 43.56 \times 10^{-19}$ J

≈ 2 times of 21.76×10^{-19} J

4. A gaseous mixture of two substances A and B, under a total pressure of 0.8 atm is in equilibrium with an ideal liquid solution. The mole fraction of substance A is 0.5 in the vapour phase and 0.2 in the liquid phase. The vapour pressure of pure liquid A is _______ atm. (Nearest integer)

Answer (2)

Sol. Given that $X_A = 0.2$, $Y_A = 0.5$, $P_T = 0.8$ atm

We know that $P_A = Y_A \times P_T$

$P_A = 0.5 \times 0.8 = 0.4$

Now $P_A = X_A \times P_A^o \Rightarrow P_A^o = \dfrac{0.4}{0.2} = 2$ atm

5. At 600 K, 2 mol of NO are mixed with 1 mol of O_2.

$$2NO(g) + O_2(g) \rightleftharpoons 2NO_2(g)$$

The reaction occurring as above comes to equilibrium under a total pressure of 1 atm. Analysis of the system shows that 0.6 mol of oxygen are present at equilibrium. The equilibrium constant for the reaction is _______. (Nearest integer)

Answer (2)

Sol.

	$2NO(g)$ +	$O_2(g)$ $\rightleftharpoons$	$2NO_2(g)$
at intial	2	1	0
at equilibrium	2 – 0.8	0.6	0.8

Partial pressure of NO(g) = $\dfrac{1.2}{2.6}\times1$

Partial pressure of O_2(g) = $\dfrac{0.6}{2.6}$

Partial pressure of NO_2(g) = $\dfrac{0.8}{2.6}$

$$K_p = \frac{\left(P_{NO_2}\right)^2}{\left(P_{NO}\right)^2\left(P_{O_2}\right)} = \frac{0.8\times0.8\times2.6}{1.2\times1.2\times0.6}$$

$= 1.925$

≈ 2

6. A sample of 0.125 g of an organic compound when analysed by Duma's method yields 22.78 mL of nitrogen gas collected over KOH solution at 280 K and 759 mm Hg. The percentage of nitrogen in the given organic compound is_____.(Nearest integer)

Given :

(a) The vapour pressure of water of 280 K is 14.2 mm Hg.

(b) R = 0.082 L atm K^{-1} mol^{-1}

Answer (22)

Sol. $P_{actual} = 759 - 14.2 = 744.8$ mmHg

$$n_{N_2} = \frac{744.8 \times 22.78}{760 \times 0.0821 \times 280 \times 1000}$$

= 0.000971 mol

Mass of N_2 = 0.02719 gm

Percentage of nitrogen

$$= \frac{0.0271}{0.125} \times 100 = 21.75 \simeq 22$$

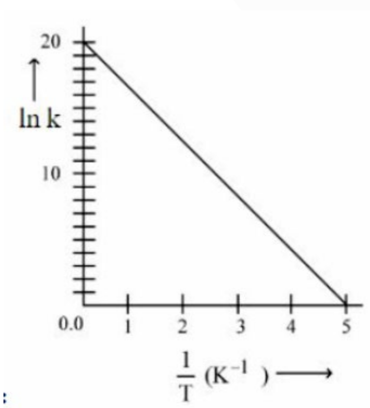
7. On reaction with stronger oxidizing agent like KIO_4, hydrogen peroxide oxidizes with the evolution of O_2. The oxidation number of I in KIO_4 changes to____.

Answer (5)

Sol. $$\overset{+7}{KIO_4} + H_2O_2 \longrightarrow \overset{+5}{KIO_3} + H_2O + O_2$$

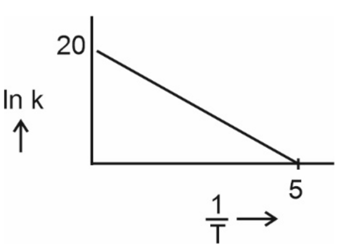
8. For a reaction, given below is the graph of ln k vs $\frac{1}{T}$. The activation energy for the reaction is equal to____ cal mol^{-1}. (Nearest integer)

(Given: R = 2 cal K^{-1} mol^{-1})



Answer (8)

Sol.

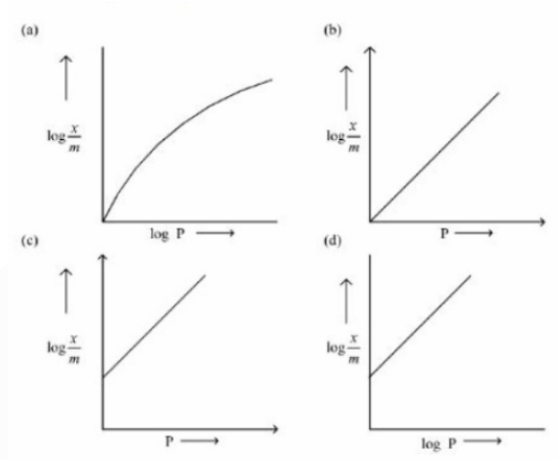


$$\text{Slope} = \frac{-20}{5}$$

$$\ln k = \ln A - \frac{E_a}{RT}$$

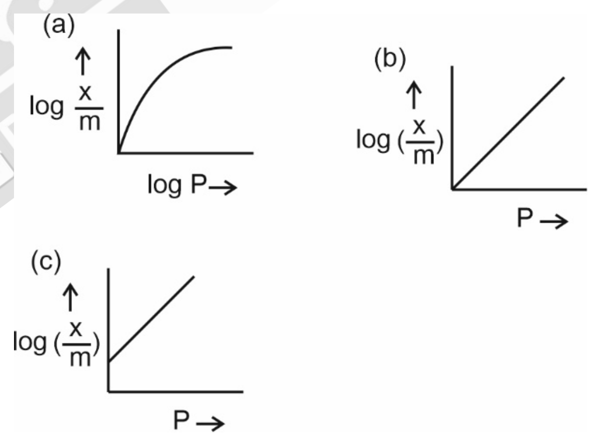
$$\therefore \frac{E_a}{R} = \frac{20}{5} \Rightarrow E = \frac{20\,R}{5} = 8 \text{ cal mol}^{-1}$$

9. Among the following the number of curves not in accordance with Freundlich adsorption isotherm is____.



Answer (3)

Sol. The following curves are not in accordance with Freundlich adsorption isotherm.



10. Among the following the number of state variables is____.

Internal energy (U)

Volume (V)

Heat (q)

Enthalpy (H)

Answer (3)

Sol. State variables are internal energy (U), Volume (V) and Enthalpy (H).

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Let $S=\left\{x\in[-6,\ 3]-\{-2,\ 2\}:\frac{|x+3|-1}{|x|-2}\ge 0\right\}$ and $T=\left\{x\in\mathbb{Z}:x^2-7|x|+9\le 0\right\}$. Then the number of elements in $S\cap T$ is

(A) 7 (B) 5

(C) 4 (D) 3

Answer (D)

Sol. $|x|^2-7|x|+9\le 0$

$$\Rightarrow\ |x|\in\left[\frac{7-\sqrt{13}}{2},\ \frac{7+\sqrt{13}}{2}\right]$$

As $x\in Z$

So, x can be ±2, ±3, ±4, ±5

Out of these values of x,

x = 3, –4, –5

satisfy S as well

$n(S\cap T)=3$

2. Let α, β be the roots of the equation $x^2-\sqrt{2}x+\sqrt{6}=0$ and $\frac{1}{\alpha^2}+1,\ \frac{1}{\beta^2}+1$, $\frac{1}{\beta^2}+1$ be the roots of the equation $x^2+ax+b=0$. Then the roots of the equation $x^2-(a+b-2)x+(a+b+2)=0$ are

(A) non-real complex number

(B) real and both negative

(C) real and both positive

(D) real and exactly one of them is positive

Answer (B)

Sol. $\alpha+\beta=\sqrt{2},\ \alpha\beta=\sqrt{6}$

$$\frac{1}{\alpha^2}+1+\frac{1}{\beta^2}+1=2+\frac{\alpha^2+\beta^2}{6}$$

$$=2+\frac{2-2\sqrt{6}}{6}=-a$$

$$\left(\frac{1}{\alpha^2}+1\right)\left(\frac{1}{\beta^2}+1\right)=1+\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\alpha^2\beta^2}$$

$$=\frac{7}{6}+\frac{2-2\sqrt{6}}{6}=b$$

$$\Rightarrow\quad a+b=\frac{-5}{6}$$

So, equation is $x^2+\frac{17x}{6}+\frac{7}{6}=0$

OR $6x^2+17x+7=0$

Both roots of equation are –ve and distinct

3. Let A and B be any two 3 × 3 symmetric and skew symmetric matrices respectively. Then Which of the following is **NOT** true?

(A) A^4-B^4 is a symmetric matrix

(B) $AB-BA$ is a symmetric matrix

(C) B^5-A^5 is a skew-symmetric matrix

(D) $AB+BA$ is a skew-symmetric matrix

Answer (C)

Sol. (A) $M=A^4-B^4$

$M^T=(A^4-B^4)^T=(A^T)^4-(B^T)^4$

$=A^4-(-B)^4=A^4-B^4=M$

(B) $M=AB-BA$

$M^T=(AB-BA)^T=(AB)^T-(BA)^T$

$=B^TA^T-A^TB^T$

$=-BA-A(-B)$

$=AB-BA=M$

(C) $M=B^5-A^5$

$M^T=(B^T)^5-(A^T)^5=-(B^5+A^5)\ne -M$

(D) $M=AB+BA$

$M^T=(AB)^T+(BA)^T$

$=B^TA^T+A^TB^T=-BA-AB=-M$

4. Let $f(x)=ax^2+bx+c$ be such that $f(1)=3$, $f(-2)=\lambda$ and $f(3)=4$. If $f(0)+f(1)+f(-2)+f(3)=14$, then λ is equal to

(A) –4 (B) $\frac{13}{2}$

(C) $\frac{23}{2}$ (D) 4

Answer (D)

Sol. $f(1) = a + b + c = 3$ …(i)

$f(3) = 9a + 3b + c = 4$ …(ii)

$f(0) + f(1) + f(-2) + f(3) = 14$

OR $c + 3 + (4a - 2b + c) + 4 = 14$

OR $4a - 2b + 2c = 7$ …(iii)

From (i) and (ii) $8a + 2b = 1$ …(iv)

From (iii) – (2) × (i)

$\Rightarrow$ $2a - 4b = 1$ …(v)

From (iv) and (v) $a = \frac{1}{6}$, $b = \frac{-1}{6}$ and $c = 3$

$f(-2) = 4a - 2b + c$

$= \frac{4}{6} + \frac{2}{6} + 3 = 4$

5. The function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \lim\limits_{n\to\infty} \frac{\cos(2\pi x) - x^{2n}\sin(x-1)}{1 + x^{2n+1} - x^{2n}}$ is continuous for all x in

(A) $\mathbb{R} - \{-1\}$

(B) $\mathbb{R} - \{-1, 1\}$

(C) $\mathbb{R} - \{1\}$

(D) $\mathbb{R} - \{0\}$

Answer (B)

Sol. $f(x) = \lim\limits_{n\to\infty} \frac{\cos(2\pi x) - x^{2n}\sin(x-1)}{1 + x^{2n+1} - x^{2n}}$

For $|x| < 1$, $f(x) = \cos 2\pi x$, continuous function

$|x| > 1$, $f(x) = \lim\limits_{n\to\infty} \frac{\frac{1}{x^{2n}}\cos 2\pi x - \sin(x-1)}{\frac{1}{x^{2n}} + x - 1}$

$= \frac{-\sin(x-1)}{x-1}$, continuous

For $|x| = 1$, $f(x) = \begin{cases} 1 & \text{if } x = 1 \\ -(1+\sin 2) & \text{if } x = -1 \end{cases}$

Now,

$\lim\limits_{x\to 1^+} f(x) = -1$, $\lim\limits_{x\to 1^-} f(x) = 1$, so

discontinuous at $x = 1$

$\lim\limits_{x\to -1^+} f(x) = 1$, $\lim\limits_{x\to -1^-} f(x) = -\frac{\sin 2}{2}$, so

discontinuous at $x = -1$

$\therefore$ $f(x)$ is continuous for all $x \in R - \{-1, 1\}$

6. The function $f(x) = xe^{x(1-x)}, x \in \mathbb{R}$, is

(A) Increasing in $\left(-\frac{1}{2}, 1\right)$

(B) Decreasing in $\left(\frac{1}{2}, 2\right)$

(C) Increasing in $\left(-1, -\frac{1}{2}\right)$

(D) Decreasing in $\left(-\frac{1}{2}, \frac{1}{2}\right)$

Answer (A)

Sol. $f(x) = xe^{x(1-x)}, x \in \mathbb{R}$

$f'(x) = xe^{x(1-x)} \cdot (1-2x) + e^{x(1-x)}$

$= e^{x(1-x)}\left[x - 2x^2 + 1\right]$

$= -e^{x(1-x)}\left[2x^2 - x - 1\right]$

$= -e^{x(1-x)}(2x+1)(x-1)$

$\therefore$ $f(x)$ is increasing in $\left(-\frac{1}{2}, 1\right)$ and decreasing in $\left(-\infty, -\frac{1}{2}\right) \cup (1, \infty)$

7. The sum of the absolute maximum and absolute minimum values of the function $f(x) = \tan^{-1}(\sin x - \cos x)$ in the interval $[0, \pi]$ is

(A) 0

(B) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right) - \frac{\pi}{4}$

(C) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right) - \frac{\pi}{4}$

(D) $\frac{-\pi}{12}$

Answer (C)

Sol. $f(x) = \tan^{-1}(\sin x - \cos x)$, $[0, \pi]$

Let $g(x) = \sin x - \cos x$

$= \sqrt{2}\sin\left(x - \frac{\pi}{4}\right)$ and $x - \frac{\pi}{4} \in \left[\frac{-\pi}{4}, \frac{3\pi}{4}\right]$

$\therefore$ $g(x) \in \left[-1, \sqrt{2}\right]$

and $\tan^{-1}x$ is an increasing function

$\therefore \quad f(x) \in \left[\tan^{-1}(-1), \tan^{-1}\sqrt{2}\right]$

$$\in \left[-\frac{\pi}{4}, \tan^{-1}\sqrt{2}\right]$$

$\therefore \quad$ Sum of f_{max} and $f_{min} = \tan^{-1}\sqrt{2} - \frac{\pi}{4}$

$$= \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) - \frac{\pi}{4}$$

8. Let $x(t) = 2\sqrt{2}\cos t\sqrt{\sin 2t}$ and $y(t) = 2\sqrt{2}\sin t\sqrt{\sin 2t}$, $t \in \left(0, \frac{\pi}{2}\right)$. Then $\dfrac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}$ at $t = \frac{\pi}{4}$ is equal to

(A) $\frac{-2\sqrt{2}}{3}$ (B) $\frac{2}{3}$

(C) $\frac{1}{3}$ (D) $\frac{-2}{3}$

Answer (D)

Sol. $x = 2\sqrt{2}\cos t\sqrt{\sin 2t}$, $y = 2\sqrt{2}\sin t\sqrt{\sin 2t}$

$$\therefore \quad \frac{dx}{dt} = \frac{2\sqrt{2}\cos 3t}{\sqrt{\sin 2t}}, \quad \frac{dy}{dt} = \frac{2\sqrt{2}\sin 3t}{\sqrt{\sin 2t}}$$

$$\therefore \quad \frac{dy}{dx} = \tan 3t, \quad \left(\text{at } t = \frac{\pi}{4}, \ \frac{dy}{dx} = -1\right)$$

and $\dfrac{d^2y}{dx^2} = 3\sec^2 3t \cdot \dfrac{dt}{dx} = \dfrac{3\sec^2 3t \cdot \sqrt{\sin 2t}}{2\sqrt{2}\cos 3t}$

$$\left(\text{At } t = \frac{\pi}{4}, \ \frac{d^2y}{dx^2} = -3\right)$$

$$\therefore \quad \frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}} = \frac{2}{-3} = \frac{-2}{3}$$

9. Let $I_n(x) = \int_0^x \frac{1}{(t^2+5)^n}dt$, $n = 1, 2, 3, \ldots\ldots$ Then

(A) $50I_6 - 9I_5 = xI_5'$ (B) $50I_6 - 11I_5 = xI_5'$

(C) $50I_6 - 9I_5 = I_5'$ (D) $50I_6 - 11I_5 = I_5'$

Answer (A)

Sol. $I_n(x) = \displaystyle\int_0^x \frac{1}{(t^2+5)^n}dt$

$$= \int_0^x \underbrace{\frac{1}{(t^2+5)^n}}_{I} \times \underset{II}{1}\, dt$$

$$= \left.\frac{t}{(t^2+5)^n}\right|_0^x - \int_0^x \frac{-2nt}{(t^2+5)^{n+1}} \times t\, dt$$

$$= \frac{x}{(x^2+5)^n} + \int_0^x 2n\left(\frac{t^2+5-5}{(t^2+5)^{n+1}}\right)dt$$

$$I_n(x) = \frac{x}{(x^2+5)^n} + 2n\, I_n(x) - 10n\, I_{n+1}(x)$$

$$10n\, I_{n+1}(x) - (2n-1)\, I_n(x) = xI_n'(x)$$

For $n = 5$

$$50I_6(x) - 9I_5(x) = xI_5'(x)$$

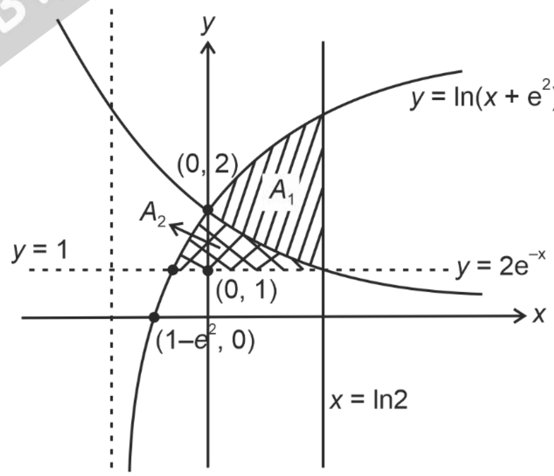
10. The area enclosed by the curves $y = \log_e\left(x+e^2\right)$, $x = \log_e\left(\frac{2}{y}\right)$ and $x = \log_e 2$, above the line $y = 1$ is

(A) $2 + e - \log_e 2$ (B) $1 + e - \log_e 2$

(C) $e - \log_e 2$ (D) $1 + \log_e 2$

Answer (B*)

Sol.



According to NTA, the required region A_2 which is shaded in crossed lines and comes out to be

$$A_2 = \int_1^2 \left(\ln\frac{2}{y} - e^y + e^2\right)dy = 1 + e - \ln 2$$

But according to us the required region A_1 comes out to be shaded in parallel lines, which can be obtained as

$$A_1 = \int_0^{\ln 2} \left(\ln\left(x+e^2\right) - 2e^{-x}\right)dx$$

$$= \left\{\left(x+e^2\right)\ln\left(x+e^2\right) - x + 2e^{-x}\right\}\Big|_0^{\ln 2}$$

$$= \left(\ln 2 + e^2\right)\ln\left(\ln 2 + e^2\right) - \ln 2 + 1$$

$$-2e^2 - 2$$

$$= \left(\ln 2 + e^2\right)\ln\left(\ln 2 + e^2\right) - \ln 2 - 2e^2 - 1$$

Not given in any option.

The region asked in the question is bounded by three curves

$$y = \ln\left(x+e^2\right)$$

$$x = \ln\left(\frac{2}{y}\right)$$

$$x = \ln 2$$

There is only one region which satisfies above requirement and which also lies above line $y = 1$

Line $y = 1$ may or may not be the boundary of the region.

11. Let $y = y(x)$ be the solution curve of the differential equation $\dfrac{dy}{dx} + \dfrac{1}{x^2-1}y = \left(\dfrac{x-1}{x+1}\right)^{1/2}, x > 1$ passing through the point $\left(2, \sqrt{\dfrac{1}{3}}\right)$. Then $\sqrt{7}\, y(8)$ is

(A) $11 + 6 \log_e 3$ (B) 19

(C) $12 - 2 \log_e 3$ (D) $19 - 6 \log_e 3$

Answer (D)

Sol. $\dfrac{dy}{dx} + \dfrac{1}{x^2-1}y = \sqrt{\dfrac{x-1}{x+1}}, x > 1$

Integrating factor $\text{I.F.} = e^{\int \frac{1}{x^2-1}dx} = e^{\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|}$

$$= \sqrt{\frac{x-1}{x+1}}$$

Solution of differential equation

$$y\sqrt{\frac{x-1}{x+1}} = \int \frac{x-1}{x+1}dx = \int\left(1 - \frac{2}{x+1}\right)dx$$

$$y\sqrt{\frac{x-1}{x+1}} = x - 2\ln|x+1| + C$$

Curve passes through $\left(2, \sqrt{\dfrac{1}{3}}\right)$

$$\frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = 2 - 2\ln 3 + C$$

$$C = 2\ln 3 - \frac{5}{3}$$

$$y(8) \times \frac{\sqrt{7}}{3} = 8 - 2\ln 9 + 2\ln 3 - \frac{5}{3}$$

$$\sqrt{7}\cdot y(8) = 19 - 6\ln 3$$

12. The differential equation of the family of circles passing through the points (0, 2) and (0, –2) is

(A) $2xy\dfrac{dy}{dx} + (x^2 - y^2 + 4) = 0$

(B) $2xy\dfrac{dy}{dx} + (x^2 + y^2 - 4) = 0$

(C) $2xy\dfrac{dy}{dx} + (y^2 - x^2 + 4) = 0$

(D) $2xy\dfrac{dy}{dx} - (x^2 - y^2 + 4) = 0$

Answer (A)

Sol. Family of circles passing through the points (0, 2) and (0, –2)

$x^2 + (y - 2)(y + 2) + \lambda x = 0,\ \lambda \in \mathbb{R}$

$x^2 + y^2 + \lambda x - 4 = 0$ ...(1)

Differentiate w.r.t x

$$2x + 2y\frac{dy}{dx} + \lambda = 0 \qquad ...(2)$$

Using (1) and (2), eliminate λ

$$x^2 + y^2 - \left(2x + 2y\frac{dy}{dx}\right)x - 4 = 0$$

$$2xy\frac{dy}{dx} + x^2 - y^2 + 4 = 0$$

13. Let the tangents at two points A and B on the circle $x^2 + y^2 - 4x + 3 = 0$ meet at origin $O(0, 0)$. Then the area of the triangle OAB is

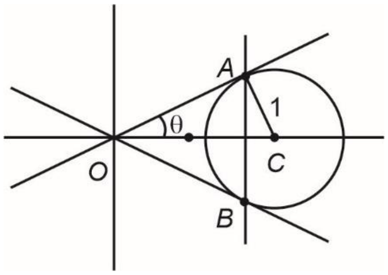
(A) $\dfrac{3\sqrt{3}}{2}$ (B) $\dfrac{3\sqrt{3}}{4}$

(C) $\dfrac{3}{2\sqrt{3}}$ (D) $\dfrac{3}{4\sqrt{3}}$

Answer (B)

Sol. $x^2 + y^2 - 4x + 3 = 0$

$\Rightarrow \quad (x-2)^2 + y^2 = 1$



$AO = \sqrt{(OC)^2 - (AC)^2}$

$= \sqrt{4-1} = \sqrt{3}$

$\sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$

Also, $AO = BO$

Area of $\Delta OAB = \frac{1}{2} \cdot OA \cdot OB \sin 60°$

$= \frac{1}{2} \times \sqrt{3} \cdot \sqrt{3} \cdot \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{4}$

14. Let the hyperbola $H : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ pass through the point $(2\sqrt{2}, -2\sqrt{2})$. A parabola is drawn whose focus is same as the focus of H with positive abscissa and the directrix of the parabola passes through the other focus of H. If the length of the latus rectum of the parabola is e times the length of the latus rectum of H, where e is the eccentricity of H, then which of the following points lies on the parabola?

(A) $(2\sqrt{3}, 3\sqrt{2})$ (B) $(3\sqrt{3}, -6\sqrt{2})$

(C) $(\sqrt{3}, -\sqrt{6})$ (D) $(3\sqrt{6}, 6\sqrt{2})$

Answer (B)

Sol. $H : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Focus of parabola: $(ae, 0)$

Directrix: $x = -ae$.

Equation of parabola $\equiv y^2 = 4aex$

Length of latus rectum of parabola = $4ae$

Length of latus rectum of hyperbola $= \frac{2.b^2}{a}$

as given, $4ae = \frac{2b^2}{a} \cdot e$

$2 = \frac{b^2}{a^2}$ ...(i)

$\because$ H passes through $(2\sqrt{2}, -2\sqrt{2}) \Rightarrow \frac{8}{a^2} - \frac{8}{b^2} = 1$

...(ii)

From (i) and (ii) $a^2 = 4$ and $b^2 = 8 \Rightarrow e = \sqrt{3}$

$\Rightarrow$ Equation of parabola is $y^2 = 8\sqrt{3}x$.

15. Let the lines $\frac{x-1}{\lambda} = \frac{y-2}{1} = \frac{z-3}{2}$ and $\frac{x+26}{-2} = \frac{y+18}{3} = \frac{z+28}{\lambda}$ be coplanar and P be the plane containing these two lines. Then which of the following points does **NOT** lie on P?

(A) (0, –2, –2) (B) (–5, 0, –1)

(C) (3, –1, 0) (D) (0, 4, 5)

Answer (D)

Sol. $L_1 : \frac{x-1}{\lambda} = \frac{y-2}{1} = \frac{z-3}{2}$,

through a point $\vec{a}_1 \equiv (1, 2, 3)$

parallel to $\vec{b}_1 \equiv (\lambda, 1, 2)$

$L_2 : \frac{x+26}{-2} = \frac{y+18}{3} = \frac{z+28}{\lambda}$

through a point $\vec{a}_2 = (-26, -18, -28)$

parallel to $\vec{b}_2 = (-2, 3, 1)$

If lines are coplanar then, $(\vec{a}_2 - \vec{a}_1) \cdot \vec{b}_1 \times \vec{b}_2 = 0$

$$\Rightarrow \begin{vmatrix} 27 & 20 & 31 \\ \lambda & 1 & 2 \\ -2 & 3 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda = 3$$

Vector normal to the required plane $\vec{n} = \vec{b}_1 \times \vec{b}_2$

$$\Rightarrow \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ -2 & 3 & 3 \end{vmatrix} = -3\hat{i} - 13\hat{j} + 11\hat{k}$$

Equation of plane

$\equiv ((x-1), (y-2), (z-3)) \cdot (-3, -13, 11) = 0$

$\Rightarrow \quad 3x + 13y - 11z + 4 = 0$

Checking the option gives (0, 4, 5) does not lie on the plane.

16. A plane P is parallel to two lines whose direction rations are –2, 1, –3 and –1, 2, –2 and it contains the point (2, 2, –2). Let P intersect the co-ordinate axes at the points A, B, C making the intercepts α, β, γ. If V is the volume of the tetrahedron $OABC$, where O is the origin and $p = \alpha + \beta + \gamma$, then the ordered pair (V, p) is equal to :

(A) (48, –13) (B) (24, –13)

(C) (48, 11) (D) (24, –5)

Answer (B)

Sol. Let $\vec{a}_1 = (-2, 1, -3)$ and $\vec{a}_2 = (-1, 2, -2)$

Vector normal to plane $\bar{n} = \vec{a}_1 \times \vec{a}_2$

$\bar{n} = (4, -1, -3)$

Plane through (2, 2, – 2) and normal to $\bar{n}$

$(x - 2, y - 2, z + 2) \cdot (4, -1, -3) = 0$

$\Rightarrow\ 4x - y - 3z = 12$

$\Rightarrow\ \dfrac{x}{3} + \dfrac{y}{-12} + \dfrac{z}{-4} = 1$

Intercepts α, β, γ are 3, –12, –4

$P = \alpha + \beta + \gamma = -13$

$V = \dfrac{1}{6} \times 3 \times 12 \times 4 = 24$

17. Let S be the set of all $a \in$ R for which the angle between the vectors $\vec{u} = a(\log_e b)\hat{i} - 6\hat{j} + 3\hat{k}$ and $\vec{v} = (\log_e b)\hat{i} + 2\hat{j} + 2a\,(\log_e b)\,\hat{k}, (b > 1)$ is acute. Then S is equal to :

(A) $\left(-\infty, -\dfrac{4}{3}\right)$ (B) Φ

(C) $\left(-\dfrac{4}{3}, 0\right)$ (D) $\left(\dfrac{12}{7}, \infty\right)$

Answer (B)

Sol. $\vec{u} = a(\log_e b)\hat{i} - 6\hat{j} + 3\hat{k}$

$\vec{v} = (\log_e b)\hat{i} + 2\hat{j} + 2a(\log_e b)\hat{k}$

For acute angle $\vec{u} \cdot \vec{v} > 0$

$\Rightarrow\ a(\log_e b)^2 - 12 + 6a(\log_e b) > 0$

$\because\ b > 1$

Let $\log_e b = t\ \Rightarrow\ t > 0$ as $b > 1$

$at^2 + 6at - 12 > 0 \qquad \forall\, t > 0$

$\Rightarrow\ a \in \phi$

18. A horizontal park is in the shape of a triangle OAB with $AB = 16$. A vertical lamp post OP is erected at the point O such that $\angle PAO = \angle PBO = 15°$ and $\angle PCO = 45°$, where C is the midpoint of AB. Then $(OP)^2$ is equal to

(A) $\dfrac{32}{\sqrt{3}}\left(\sqrt{3} - 1\right)$

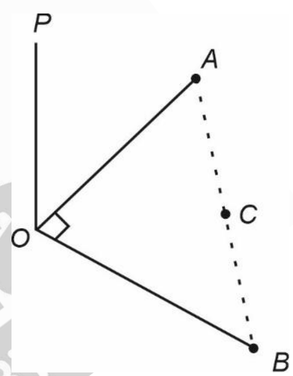
(B) $\dfrac{32}{\sqrt{3}}\left(2 - \sqrt{3}\right)$

(C) $\dfrac{16}{\sqrt{3}}\left(\sqrt{3} - 1\right)$

(D) $\dfrac{16}{\sqrt{3}}\left(2 - \sqrt{3}\right)$

Answer (B)

Sol.



$OP = OA \tan 15 = OB \tan 15$ …(i)

$OP = OC \tan 45 \Rightarrow OP = OC$ …(ii)

$OA = OB$ …(iii)

$OC^2 + 8^2 = OA^2$

$OP^2 + 64 = OP^2\left(\dfrac{\sqrt{3}+1}{\sqrt{3}-1}\right)^2$

$64 = OP^2\left[\dfrac{\left(\sqrt{3}+1\right)^2 - \left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}-1\right)^2}\right]$

$= OP^2\left(\dfrac{4\sqrt{3}}{\left(\sqrt{3}-1\right)^2}\right)$

$OP^2 = \dfrac{64\left(\sqrt{3}-1\right)^2}{4\sqrt{3}} = \dfrac{32}{\sqrt{3}}\left(2 - \sqrt{3}\right)$

19. Let A and B be two events such that $P(B \setminus A)\frac{2}{5}, P(A \setminus B)=\frac{1}{7}$ and $P(A \cap B)=\frac{1}{9}$. Consider

$(S1)\ P(A' \cup B)=\frac{5}{6}$,

$(S2)\ P(A' \cap B')=\frac{1}{18}$. Then

(A) Both ($S1$) and ($S2$) are true

(B) Both ($S1$) and ($S2$) are false

(C) Only ($S1$) is true

(D) Only ($S2$) is true

Answer (A)

Sol. $P(A/B)=\frac{1}{7} \Rightarrow \frac{P(A\cap B)}{P(B)}=\frac{1}{7}$

$\Rightarrow \quad P(B)=\frac{7}{9}$

$P(B/A)=\frac{2}{5} \Rightarrow \frac{P(A\cap B)}{P(A)}=\frac{2}{5}$

$P(A)=\frac{5}{2}\cdot\frac{1}{9}=\frac{5}{18}$

$S2:\ P(A' \cap B')=\frac{1}{18}$

$S1:$ and $P(A' \cup B)=\frac{1}{9}+\frac{6}{9}+\frac{1}{18}=\frac{5}{6}.$

20. Let

p : Ramesh listens to music.

q : Ramesh is out of his village.

r : It is Sunday.

s : It is Saturday.

Then the statement "Ramesh listens to music only if he is in his village and it is Sunday or Saturday" can be expressed as

(A) $((\sim q) \wedge (r \vee s)) \Rightarrow p$

(B) $(q \wedge (r \vee s)) \Rightarrow p$

(C) $p \Rightarrow (q \wedge (r \vee s))$

(D) $p \Rightarrow ((\sim q) \wedge (r \vee s)$

Answer (D)

Sol. p : Ramesh listens to music

q : Ramesh is out of his village

r : It is Sunday

s : It is Saturday

$p \rightarrow q$ conveys the same p only if q

Statement "Ramesh listens to music only if he is in his village and it is Sunday or Saturday"

$p \Rightarrow ((\sim q) \wedge (r \vee s))$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Let the coefficients of the middle terms in the expansion of $\left(\frac{1}{\sqrt{6}}+\beta x\right)^4$, $(1 - 3\beta x)^2$ and $\left(1-\frac{\beta}{2}x\right)^6$, $\beta > 0$, respectively form the first three terms of an A.P. If d is the common difference of this A.P., then $50-\frac{2d}{\beta^2}$ is equal to _______.

Answer (57)

Sol. Coefficients of middle terms of given expansions are ${}^4C_2\frac{1}{6}\beta^2$, ${}^2C_1(-3\beta)$, ${}^6C_3\left(\frac{-\beta}{2}\right)^3$ form an A.P.

$\therefore \quad 2.2(-3\beta)=\beta^2-\frac{5\beta^3}{2}$

$\Rightarrow -24 = 2\beta - 5\beta^2$

$\Rightarrow 5\beta^2 - 2\beta - 24 = 0$

$\Rightarrow 5\beta^2 - 12\beta + 10\beta - 24 = 0$

$\Rightarrow \beta(5\beta - 12) + 2(5\beta - 12) = 0$

$\beta=\frac{12}{5}$

$d = -6\beta - \beta^2$

$\therefore \quad 50-\frac{2d}{\beta^2}=50-2\frac{(-6\beta-\beta^2)}{\beta^2}=50+\frac{12}{\beta}+2=57$

2. A class contains b boys and g girls. If the number of ways of selecting 3 boys and 2 girls from the class is 168, then $b + 3g$ is equal to _______.

Answer (17)

Sol. ${}^bC_3 \cdot {}^gC_2 = 168$

$\Rightarrow \frac{b(b-1)(b-2)}{6}\cdot\frac{g(g-1)}{2}=168$

$\Rightarrow b(b-1)(b-2)\ \ g(g-1) = 2^5 \cdot 3^2 \cdot 7$

$\Rightarrow b(b-1)(b-2)\ \ g(g-1) = 6. 7. 8. 3. 2$

$\therefore \quad b = 8$ and $g = 3$

$\therefore \quad b + 3g = 17$

3. Let the tangents at the points P and Q on the ellipse $\frac{x^2}{2}+\frac{y^2}{4}=1$ meet at the point $R\left(\sqrt{2}, 2\sqrt{2}-2\right)$. If S is the focus of the ellipse on its negative major axis, then $SP^2 + SQ^2$ is equal to ________.

Answer (13)

Sol. $E \equiv \frac{x^2}{2}+\frac{y^2}{4}=1$

$T \equiv y = mx \pm \sqrt{2m^2+4}$

$\downarrow \left(\sqrt{2}, 2\sqrt{2}-2\right)$

$\Rightarrow \left(2\sqrt{2}-2-m\sqrt{2}\right) = \pm\sqrt{2m^2+4}$

$\Rightarrow 2m^2 - 2m\sqrt{2}\left(2\sqrt{2}-2\right)+4(3-2\sqrt{2}) = 2m^2+4$

$\Rightarrow -2\sqrt{2}\,m\,(2\sqrt{2}-2) = 4-12+8\sqrt{2}$

$\Rightarrow -4\sqrt{2}\,m\,(\sqrt{2}-1) = 8(\sqrt{2}-1)$

$\Rightarrow m = -\sqrt{2}$ and $m \to \infty$

$\therefore$ Tangents are $x = \sqrt{2}$ and $y = -\sqrt{2}x+\sqrt{8}$

$\therefore$ $P(\sqrt{2}, 0)$ and $Q(1, \sqrt{2})$

and $S = (0, -\sqrt{2})$

$\therefore$ $(PS)^2 + (QS)^2 = 4 + 9 = 13$

4. If $1 + (2 + {}^{49}C_1 + {}^{49}C_2 + \ldots {}^{49}C_{49})$ $({}^{50}C_2 + {}^{50}C_4 + \ldots {}^{50}C_{50})$ is equal to $2^n \cdot m$, where m is odd, then $n + m$ is equal to _______.

Answer (99)

Sol. $I = 1 + (1 + {}^{49}C_0 + {}^{49}C_1 + \ldots + {}^{49}C_{49})\,({}^{50}C_2 + {}^{50}C_4 + \ldots + {}^{50}C_{50})$

As ${}^{49}C_0 + {}^{49}C_1 + \ldots + {}^{49}C_{49} = 2^{49}$

and ${}^{50}C_0 + {}^{50}C_2 + \ldots + {}^{50}C_{50} = 2^{49}$

$\Rightarrow$ ${}^{50}C_2 + {}^{50}C_4 + \ldots + {}^{50}C_{50} = 2^{49} - 1$

$\therefore$ $I = 1 + (2^{49} + 1)(2^{49} - 1)$

$= 2^{98}$

$\therefore$ $m = 1$ and $n = 98$

$m + n = 99$

5. Two tangent lines l_1 and l_2 are drawn from the point $(2, 0)$ to the parabola $2y^2 = -x$. If the lines l_1 and l_2 are also tangent to the circle $(x-5)^2 + y^2 = r$, then $17r$ is equal to _________.

Answer (9)

Sol. Given: $y^2 = \frac{-x}{2}$

$T \equiv y = mx - \frac{1}{8m}$

$\downarrow (2,0)$

$\Rightarrow m^2 = \frac{1}{16} \Rightarrow m = \pm\frac{1}{4}$

Tangents are $y = \frac{1}{4}x - \frac{1}{2},\ y = \frac{-x}{4}+\frac{1}{2}$

$4y = x - 2$ and $4y + x = 2$

If these are also tangent to circle then

$d_c = r$

$\Rightarrow \left|\frac{5-2}{\sqrt{17}}\right| = \sqrt{r} \Rightarrow r = \left(\frac{3}{\sqrt{17}}\right)^2$

$\Rightarrow 17r = 17\cdot\frac{9}{17} = 9$

6. If $\frac{6}{3^{12}}+\frac{10}{3^{11}}+\frac{20}{3^{10}}+\frac{40}{3^9}+\ldots+\frac{10240}{3} = 2^n \cdot m$, where m is odd, then $m\cdot n$ is equal to ________.

Answer (12)

Sol. $\frac{1}{3^{12}}+5\left(\frac{2^0}{3^{12}}+\frac{2^1}{3^{11}}+\frac{2^2}{3^{10}}+\ldots\ldots+\frac{2^{11}}{3}\right) = 2^n \cdot m$

$\Rightarrow \frac{1}{3^{12}}+5\left(\frac{1}{3^{12}}\frac{\left((6)^2-1\right)}{(6-1)}\right) = 2^n \cdot m$

$\Rightarrow \frac{1}{3^{12}}+\frac{5}{5}\left(\frac{1}{3^{12}}\cdot 2^{12}\cdot 3^{12}-\frac{1}{3^{12}}\right) = 2^n \cdot m$

$\Rightarrow \frac{1}{3^{12}}+2^{12}-\frac{1}{3^{12}} = 2^n \cdot m$

$\Rightarrow$ $2^n \cdot m = 2^{12}$

$\Rightarrow$ $m = 1$ and $n = 12$

$m \cdot n = 12$

7. Let $S = \left[-\pi, \frac{\pi}{2}\right) - \left[-\frac{\pi}{2}, -\frac{\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}\right]$. Then the number of elements in the set $A = \left\{\theta \in S : \tan\theta\left(1+\sqrt{5}\tan(2\theta)\right) = \sqrt{5}-\tan(2\theta)\right\}$ is ________.

Answer (5)

Sol. Let $\tan\alpha = \sqrt{5}$

$\therefore \quad \tan\theta = \dfrac{\tan\alpha - \tan 2\theta}{1 + \tan\alpha\tan 2\theta}$

$\therefore \quad \tan\theta = \tan(\alpha - 2\theta)$

$\alpha - 2\theta = n\pi + \theta$

$\Rightarrow \quad 3\theta = \alpha - n\pi$

$\Rightarrow \quad \theta = \dfrac{\alpha}{3} - \dfrac{n\pi}{3} \quad ; n \in Z$

If $\theta \in [-\pi, \pi/2)$ then

$n = 0, 1, 2, 3, 4$ are acceptable

$\therefore$ 5 solutions.

8. Let $z = a + ib$, $b \neq 0$ be complex numbers satisfying $z^2 = \bar{z}\cdot 2^{1-|z|}$. Then the least value of $n \in N$, such that $z^n = (z+1)^n$, is equal to _____.

Answer (6)

Sol. $\because \quad z^2 = \bar{z}\cdot 2^{1-|z|} \quad ...(1)$

$\Rightarrow \quad |z|^2 = |\bar{z}|\cdot 2^{1-|z|}$

$\Rightarrow \quad |z| = 2^{1-|z|}, \quad \because b \neq 0 \Rightarrow |z| \neq 0$

$\therefore \quad |z| = 1 \quad ...(2)$

$\because \quad z = a + ib$ then $\sqrt{a^2 + b^2} = 1 \quad ...(3)$

Now again from equation (1), equation (2), equation (3) we get :

$a^2 - b^2 + i2ab = (a - ib)\, 2^0$

$\therefore \quad a^2 - b^2 = a$ and $2ab = -b$

$\therefore \quad a = -\dfrac{1}{2}$ and $b = \pm\dfrac{\sqrt{3}}{2}$

$\therefore \quad z = -\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i$ or $z = -\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i$

$$z^n = (z+1)^n \Rightarrow \left(\frac{z+1}{z}\right)^n = 1$$

$$\left(1 + \frac{1}{z}\right)^n = 1$$

$\left(\dfrac{1+\sqrt{3}i}{2}\right)^n = 1$, then minimum value of n is 6.

9. A bag contains 4 white and 6 black balls. Three balls are drawn at random from the bag. Let X be the number of white balls, among the drawn balls. If σ^2 is the variance of X, then 100 σ^2 is equal to ____.

Answer (56)

Sol. X = Number of white ball drawn

$$P(X=0) = \frac{{}^6C_3}{{}^{10}C_3} = \frac{1}{6}$$

$$P(X=1) = \frac{{}^6C_2 \times {}^4C_1}{{}^{10}C_3} = \frac{1}{2},$$

$$P(X=2) = \frac{{}^6C_1 \times {}^4C_2}{{}^{10}C_3} = \frac{3}{10}$$

and $P(X=3) = \dfrac{{}^6C_0 \times {}^4C_3}{{}^{10}C_3} = \dfrac{1}{30}$

Variance $= \sigma^2 = \sum P_i X_i^2 - \left(\sum P_i X_i\right)^2$

$$\sigma^2 = \frac{1}{2} + \frac{12}{10} + \frac{3}{10} - \left(\frac{1}{2} + \frac{6}{10} + \frac{1}{10}\right)^2$$

$$= \frac{56}{100}$$

$100\sigma^2 = 56.$

10. The value of the integral $\displaystyle\int_0^{\frac{\pi}{2}} 60\frac{\sin(6x)}{\sin x}dx$ is equal to _____.

Answer (104)

Sol. $I = \displaystyle\int_0^{\frac{\pi}{2}} 60.\frac{\sin 6x}{\sin x}dx$

$$= 60.2\int_0^{\frac{\pi}{2}} (3 - 4\sin^2 x)(4\cos^2 x - 3)\cos x\,dx$$

$$= 120\int_0^{\frac{\pi}{2}} (3 - 4\sin^2 x)(1 - 4\sin^2 x)\cos x\,dx$$

Let $\sin x = t \Rightarrow \cos x dx = dt$

$$= 120\int_0^1 (3 - 4t^2)(1 - 4t^2)dt$$

$$= 120\int_0^1 (3 - 16t^2 + 16t^4)dt$$

$$= 120\left[3t - \frac{16t^3}{3} + \frac{16t^5}{5}\right]_0^1$$

$= 104$

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