

28/07/2022

Morning



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## Answers & Solutions

Time : 3 hrs.

*for*

M.M. : 300

## JEE (Main)-2022 (Online) Phase-2

(Physics, Chemistry and Mathematics)

### IMPORTANT INSTRUCTIONS:

- (1) The test is of **3 hours** duration.
- (2) The Test Booklet consists of 90 questions. The maximum marks are 300.
- (3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.
  - (i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **-1 mark** for wrong answer.
  - (ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **-1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

**PHYSICS**

**SECTION - A**

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer :**

1. The dimensions of  $\left(\frac{B^2}{\mu_0}\right)$  will be

(if  $\mu_0$  : permeability of free space and  $B$  : magnetic field)

- (A)  $[ML^2T^{-2}]$                       (B)  $[MLT^{-2}]$   
(C)  $[ML^{-1}T^{-2}]$                     (D)  $[ML^2T^{-2}A^{-1}]$

**Answer (C)**

**Sol.**  $\left[\frac{B^2}{\mu_0}\right] = [\text{Energy density}]$   
$$= \frac{ML^2T^{-2}}{L^3} = ML^{-1}T^{-2}$$

2. A NCC parade is going at a uniform speed of 9 km/h under a mango tree on which a monkey is sitting at a height of 19.6 m. At any particular instant, the monkey drops a mango. A cadet will receive the mango whose distance from the tree at time of drop is (Given  $g = 9.8 \text{ m/s}^2$ )

- (A) 5 m                                      (B) 10 m  
(C) 19.8 m                                (D) 24.5 m

**Answer (A)**

**Sol.**  $H = \frac{1}{2}gt^2$   
 $19.6 = 4.9 t^2$   
 $t = 2 \text{ sec}$   
 $D = 9 \times \frac{5}{18} \times 2 = 5 \text{ m}$

3. In two different experiments, an object of mass 5 kg moving with a speed of  $25 \text{ ms}^{-1}$  hits two different walls and comes to rest within (i) 3 second, (ii) 5 seconds, respectively.

Choose the **correct** option out of the following:

- (A) Impulse and average force acting on the object will be same for both the cases.  
(B) Impulse will be same for both the cases but the average force will be different.  
(C) Average force will be same for both the cases but the impulse will be different.  
(D) Average force and impulse will be different for both the cases.

**Answer (B)**

**Sol.**  $\Delta P = \text{impulse} = \text{same}$  since acceleration is different force acting will be different.

4. A balloon has mass of 10 g in air. The air escapes from the balloon at a uniform rate with velocity 4.5 cm/s. If the balloon shrinks in 5 s completely. Then, the average force acting on that balloon will be (in dyne).

- (A) 3    (B) 9  
(C) 12    (D) 18

**Answer (B)**

**Sol.**  $F_{\text{avg}} = \mu \times v_{\text{rel}}$   
$$= \frac{10}{5} \times 4.5 = 9$$

5. If the radius of earth shrinks by 2% while its mass remains same. The acceleration due to gravity on the earth's surface will approximately

- (A) Decrease by 2%                      (B) Decrease by 4%  
(C) Increase by 2%                      (D) Increase by 4%

**Answer (D)**

**Sol.**  $g = \frac{GM}{R^2}$   
 $\frac{\Delta R}{R} \times 100 = -2$   
 $\frac{\Delta g}{g} = -\frac{2\Delta R}{R} = 4\%$   
Increase by 4%

6. The force required to stretch a wire of cross-section  $1 \text{ cm}^2$  to double its length will be:

(Given Young's modulus of the wire =  $2 \times 10^{11} \text{ N/m}^2$ )

- (A)  $1 \times 10^7 \text{ N}$                       (B)  $1.5 \times 10^7 \text{ N}$   
 (C)  $2 \times 10^7 \text{ N}$                       (D)  $2.5 \times 10^7 \text{ N}$

**Answer (C)**

**Sol.**  $A = 1 \text{ cm}^2$

$$Y = \frac{F l}{A \Delta l}$$

$$F = \frac{Y A \Delta l}{l} = \frac{2 \times 10^{11} \times 10^{-4} \times l}{l}$$

$$= 2 \times 10^7 \text{ N}$$

7. A Carnot engine has efficiency of 50%. If the temperature of sink is reduced by  $40^\circ\text{C}$ , its efficiency increases by 30%. The temperature of the source will be:

- (A) 166.7 K                      (B) 255.1 K  
 (C) 266.7 K                      (D) 367.7 K

**Answer (C)**

**Sol.**  $1 - \frac{T_L}{T_H} = 0.5$                       ....(1)

$1 - \frac{T_L - 40}{T_H} = 0.65$                       ....(2)

$\Rightarrow T_H = \frac{800}{3} \text{ K} \approx 266.7 \text{ K}$

8. Given below are two statements :

**Statement I :** The average momentum of a molecule in a sample of an ideal gas depends on temperature.

**Statement II :** The rms speed of oxygen molecules in a gas is  $v$ . If the temperature is doubled and the oxygen molecules dissociate into oxygen atoms, the rms speed will become  $2v$ .

In the light of the above statements, choose the **correct** answer from the options given below:

- (A) Both **Statement I** and **Statement II** are true  
 (B) Both **Statement I** and **Statement II** are false  
 (C) **Statement I** is true but **Statement II** is false  
 (D) **Statement I** is false but **Statement II** is true

**Answer (D)**

**Sol.** Average momentum =  $\langle \vec{P} \rangle = 0$

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

If temperature is doubled and oxygen atoms are used then

$$v'_{\text{rms}} = \sqrt{\frac{3R(2T)}{M/2}} = 4v_{\text{rms}}$$

9. In the wave equation

$$y = 0.5 \sin \frac{2\pi}{\lambda} (400t - x) \text{ m}$$

the velocity of the wave will be :

- (A) 200 m/s                      (B)  $200\sqrt{2}$  m/s  
 (C) 400 m/s                      (D)  $400\sqrt{2}$  m/s

**Answer (C)**

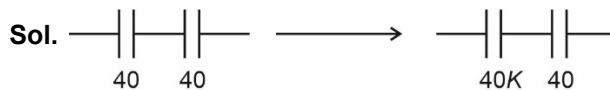
**Sol.**  $v_{\text{wave}} = \left| \frac{\text{coefficient of } t}{\text{coefficient of } x} \right|$

$$= \frac{400}{1} = 400 \text{ m/s}$$

10. Two capacitors, each having capacitance  $40 \mu\text{F}$  are connected in series. The space between one of the capacitors is filled with dielectric material of dielectric constant  $K$  such that the equivalence capacitance of the system became  $24 \mu\text{F}$ . The value of  $K$  will be :

- (A) 1.5                                      (B) 2.5  
 (C) 1.2                                      (D) 3

**Answer (A)**



$$\frac{40K \times 40}{40K + 40} = 24$$

$$40K = 24 (K + 1)$$

$$40K = 24K + 24$$

$$16K = 24$$

$$K = \frac{24}{16} = \frac{3}{2} = 1.5$$

11. A wire of resistance  $R_1$  is drawn out so that its length is increased by twice of its original length. The ratio of new resistance to original resistance is :

- (A) 9 : 1                      (B) 1 : 9  
(C) 4 : 1                      (D) 3 : 1

**Answer (A)**

**Sol.**  $R = \frac{\rho l}{A}$  ... (1)

$Al = \text{constant}$  ... (2)

$\Rightarrow R \propto l^2$

$\Rightarrow \text{Ratio} = 3^2 = 9$

12. The current sensitivity of a galvanometer can be increased by :

- (A) Decreasing the number of turns  
(B) Increasing the magnetic field  
(C) Decreasing the area of the coil  
(D) Decreasing the torsional constant of the spring

Choose the **most appropriate** answer from the options given below :

- (A) (B) and (C) only      (B) (C) and (D) only  
(C) (A) and (C) only      (D) (B) and (D) only

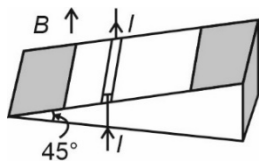
**Answer (D)**

**Sol.**  $NiAB = k\theta$

$\Rightarrow \frac{\theta}{i} = \frac{NAB}{k}$

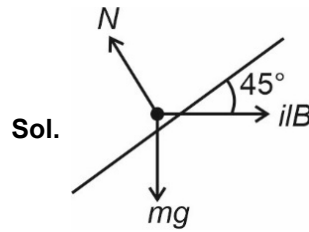
$\Rightarrow$  Sensitivity increases if  $B \uparrow$  and  $k \downarrow$

13. As shown in the figure, a metallic rod of linear density  $0.45 \text{ kg m}^{-1}$  is lying horizontally on a smooth inclined plane which makes an angle of  $45^\circ$  with the horizontal. The minimum current flowing in the rod required to keep it stationary, when  $0.15 \text{ T}$  magnetic field is acting on it in the vertical upward direction, will be : {Use  $g = 10 \text{ m/s}^2$ }



- (A) 30 A                      (B) 15 A  
(C) 10 A                      (D) 3 A

**Answer (A)**



$mg \times \frac{1}{\sqrt{2}} = \frac{ilB}{\sqrt{2}}$

$\Rightarrow i = \frac{mg}{Bl}$

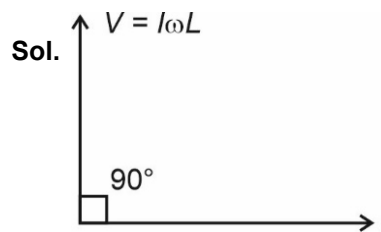
$= \frac{0.45 \times 10}{0.15} = 30 \text{ A}$

14. The equation of current in a purely inductive circuit is  $5\sin(49\pi t - 30^\circ)$ . If the inductance is  $30 \text{ mH}$  then the equation for the voltage across the inductor, will be :

{ Let  $\pi = \frac{22}{7}$  }

- (A)  $1.47\sin(49\pi t - 30^\circ)$   
(B)  $1.47\sin(49\pi t + 60^\circ)$   
(C)  $23.1\sin(49\pi t - 30^\circ)$   
(D)  $23.1\sin(49\pi t + 60^\circ)$

**Answer (D)**



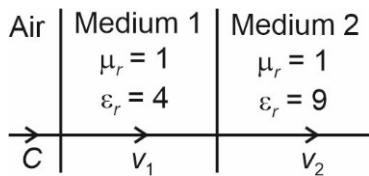
$V(t) = I\omega L \sin(49\pi t - 30^\circ + 90^\circ)$

$= 5 \times 49\pi \times \frac{30}{1000} \sin(49\pi t + 60^\circ)$

$= 23.1\sin(49\pi t + 60^\circ)$

15. As shown in the figure, after passing through the medium 1. The speed of light  $v_2$  in medium 2 will be :

(Given  $c = 3 \times 10^8 \text{ ms}^{-1}$ )



- (A)  $1.0 \times 10^8 \text{ ms}^{-1}$
- (B)  $0.5 \times 10^8 \text{ ms}^{-1}$
- (C)  $1.5 \times 10^8 \text{ ms}^{-1}$
- (D)  $3.0 \times 10^8 \text{ ms}^{-1}$

**Answer (A)**

**Sol.**  $V = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_r \epsilon_r \mu_0 \epsilon_0}}$

$\Rightarrow v_2 = \frac{c}{\sqrt{9}} = 10^8 \text{ m/s}$

16. In normal adjustment, for a refracting telescope, the distance between objective and eye piece is 30 cm. The focal length of the objective, when the angular magnification of the telescope is 2, will be:

- (A) 20 cm
- (B) 30 cm
- (C) 10 cm
- (D) 15 cm

**Answer (A)**

**Sol.**  $\therefore m = \frac{f_o}{f_e}$

$\Rightarrow 2 = \frac{f_o}{f_e} \dots(i)$

and,  $l = f_o + f_e$

$\Rightarrow 30 = f_o + f_e \dots(ii)$

$\Rightarrow 30 = f_o + \frac{f_o}{2}$

$\Rightarrow 30 \times \frac{2}{3} = f_o$

$\Rightarrow f_o = 20 \text{ cm}$

17. The equation  $\lambda = \frac{1.227}{x} \text{ nm}$  can be used to find the de-Broglie wavelength of an electron.

In this equation x stands for:

Where  $m$  = mass of electron

$P$  = momentum of electron

$K$  = Kinetic energy of electron

$V$  = Accelerating potential in volts for electron

- (A)  $\sqrt{mK}$
- (B)  $\sqrt{P}$
- (C)  $\sqrt{K}$
- (D)  $\sqrt{V}$

**Answer (D)**

**Sol.**  $\therefore \lambda = \frac{1.227}{\sqrt{V}} \text{ nm}$

$\Rightarrow x = \sqrt{V}$

18. The half life period of a radioactive substance is 60 days. The time taken for  $\frac{7}{8}$ th of its original mass to disintegrate will be :

- (A) 120 days
- (B) 130 days
- (C) 180 days
- (D) 20 days

**Answer (C)**

**Sol.**  $\therefore N = \frac{N_0}{2^{\frac{t}{T_{1/2}}}}$

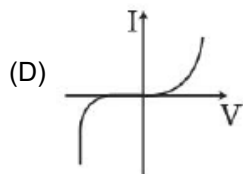
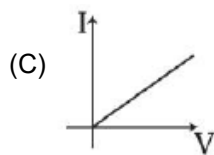
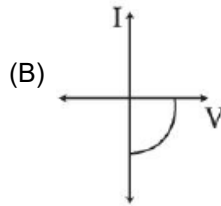
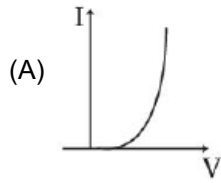
$\Rightarrow 2^{\frac{t}{T_{1/2}}} = \frac{N_0}{N} = \frac{N_0}{\left(\frac{N_0}{8}\right)}$

$\Rightarrow 2^{\frac{t}{T_{1/2}}} = 8 = 2^3$

$\Rightarrow t = 3 \times T_{1/2} = 3 \times 60$

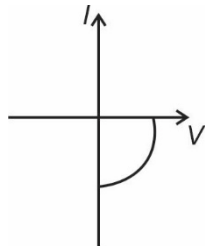
$= 180 \text{ days}$

19. Identify the solar cell characteristics from the following options :



**Answer (B)**

**Sol.** Solar cell characteristics



20. In the case of amplitude modulation to avoid distortion the modulation index ( $\mu$ ) should be :

- (A)  $\mu \leq 1$
- (B)  $\mu \geq 1$
- (C)  $\mu = 2$
- (D)  $\mu = 0$

**Answer (A)**

**Sol.** For effective modulation,

$$\mu \leq 1$$

**SECTION - B**

**Numerical Value Type Questions:** This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. projection of  $2\hat{i} + 4\hat{j} - 2\hat{k}$  on  $\hat{i} + 2\hat{j} - \alpha\hat{k}$  is zero.

Then, the value of  $\alpha$  will be \_\_\_\_\_.

**Answer (5)**

**Sol.**  $\vec{A} = 2\hat{i} + 4\hat{j} - 2\hat{k}$

$$\vec{B} = \hat{i} + 2\hat{j} + \alpha\hat{k}$$

$$\vec{A} \cdot \vec{B} = 0, \text{ as } \vec{A} \text{ should be perpendicular to } \vec{B}$$

$$\Rightarrow 2 + 8 - 2\alpha = 0$$

$$\alpha = 5$$

2. A freshly prepared radioactive source of half life 2 hours 30 minutes emits radiation which is 64 times the permissible safe level. The minimum time, after which it would be possible to work safely with source, will be \_\_\_\_\_ hours.

**Answer (15)**

**Sol.**  $T_{1/2} = 150$  minutes

$$A_0 = 64x, \text{ where } x \text{ is safe limit}$$

$$x = 64x \times 2^{-\frac{n}{T_{1/2}}}$$

$$\Rightarrow \frac{1}{64} = 2^{-\frac{n}{T_{1/2}}}$$

$$\text{or } \frac{n}{T_{1/2}} = 6$$

$$\Rightarrow n = 6 \times 150 \text{ minutes} \\ = 15 \text{ hours}$$

3. In a Young's double slit experiment, a laser light of 560 nm produces an interference pattern with consecutive bright fringes' separation of 7.2 mm. Now another light is used to produce an interference pattern with consecutive bright fringes' separation of 8.1 mm. The wavelength of second light is \_\_\_\_\_ nm.

**Answer (630)**

**Sol.**  $\lambda = 560 \times 10^{-9}$

$$B_1 = 7.2 \times 10^{-3}$$

$$B_2 = 8.1 \times 10^{-3}$$

$$\frac{B_1}{B_2} = \frac{\lambda_1}{\lambda_2}$$

$$\begin{aligned} \Rightarrow \lambda_2 &= \frac{560 \times 10^{-9} \times 8.1 \times 10^{-3}}{7.2 \times 10^{-3}} \\ &= 6.3 \times 10^{-7} \text{ m} \\ &= 630 \text{ nm} \end{aligned}$$

4. The frequencies at which the current amplitude in an LCR series circuit becomes  $\frac{1}{\sqrt{2}}$  times its maximum value, are 212 rad s<sup>-1</sup> and 232 rad s<sup>-1</sup>. The value of resistance in the Question: circuit is  $R = 5 \Omega$ . The self-inductance in the circuit is \_\_\_\_\_ mH.

**Answer (250)**

$$\frac{i}{i_{\max}} = \frac{1}{\sqrt{2}}$$

**Sol.**

$$\begin{aligned} &= \frac{V_0/Z}{V_0/R} \end{aligned}$$

$$\Rightarrow \frac{R}{Z} = \frac{1}{\sqrt{2}}$$

$$\text{and } \frac{1}{212C} - 212L = 232L - \frac{1}{232C}$$

$$\text{so } 212L = \frac{1}{232C}$$

$$\text{so } \frac{R}{\sqrt{R^2 + \left(232L + \frac{1}{232C}\right)^2}} = \frac{1}{\sqrt{2}}$$

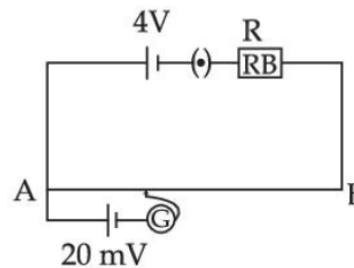
$$\frac{R^2}{R^2 + (20L)^2} = \frac{1}{2}$$

$$400 L^2 = R^2$$

$$L = \frac{5}{20}$$

$$\begin{aligned} H &= \frac{5}{20} \times 1000 \text{ mH} \\ &= 250 \text{ mH} \end{aligned}$$

5. As shown in the figure, a potentiometer wire of resistance  $20 \Omega$  and length 300 cm is connected with resistance box (R.B.) and a standard cell of emf 4 V. For a resistance 'R' of resistance box introduced into the circuit, the null point for a cell of 20 mV is found to be 60 cm. The value of 'R' is  $\Omega$ .



**Answer (780)**

**Sol.**  $I = 3\text{m}, R_W = 20 \Omega$

$$\epsilon_0 = 4\text{V}$$

$$\frac{4 \times 20}{20 + R} \times \frac{60}{300} = 20 \times 10^{-3}$$

$$\frac{4}{20 + R} = 5 \times 10^{-3}$$

$$20 + R = 800$$

$$R = 780 \Omega$$

6. Two electric dipoles of dipole moments  $1.2 \times 10^{-30}$  C-m and  $2.4 \times 10^{-30}$  C-m are placed in two different uniform electric fields of strength  $5 \times 10^4$  NC<sup>-1</sup> and  $15 \times 10^4$  NC<sup>-1</sup> respectively. The ratio of maximum torque experienced by the electric dipoles will be  $\frac{1}{x}$ . The value of x is \_\_\_\_\_.

**Answer (6)**

**Sol.**  $\frac{\rho_1}{\rho_2} = \frac{\mu_1 B_1 \sin 90}{\mu_2 B_2 \sin 90}$

$$= \frac{1.2 \times 10^{-30} \times 5 \times 10^4}{2.4 \times 10^{-30} \times 15 \times 10^4}$$

$$= \frac{1}{6}$$

7. The frequency of echo will be \_\_\_\_\_ Hz if the train blowing a whistle of frequency 320 Hz is moving with a velocity of 36 km/h towards a hill from which an echo is heard by the train driver. Velocity of sound in air is 330 m/s.

**Answer (340)**

**Sol.**  $v_s = 36 \times \frac{5}{18} = 10 \text{ m/sec}$

$$f = \frac{v + v_s}{v - v_s} f_0$$

$$= \frac{340}{320} \times 320$$

$$= 340 \text{ Hz}$$

8. The diameter of an air bubble which was initially 2 mm, rises steadily through a solution of density  $1750 \text{ kg m}^{-3}$  at the rate of  $0.35 \text{ cms}^{-1}$ . The coefficient of viscosity of the solution is \_\_\_\_\_ poise (in nearest integer). (The density of air is negligible).

**Answer (11)**

**Sol.**  $F = 6\pi\eta r v$

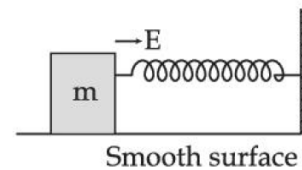
$$\frac{4}{3}\pi r^3 \rho_l g = 6\pi\eta r v$$

$$\eta = \frac{2r^2 \rho_l g}{9v}$$

$$= \frac{2 \times (2 \times 10^{-3})^2 \times 1750 \times 10}{9 \times 3.5 \times 10^{-3} \times 4}$$

$$= 11 \text{ poise}$$

9. A block of mass 'm' (as shown in figure) moving with kinetic energy  $E$  compresses a spring through a distance 25 cm when, its speed is halved. The value of spring constant of used spring will be  $nE \text{ Nm}^{-1}$  for  $n =$  \_\_\_\_\_.



**Answer (24)**

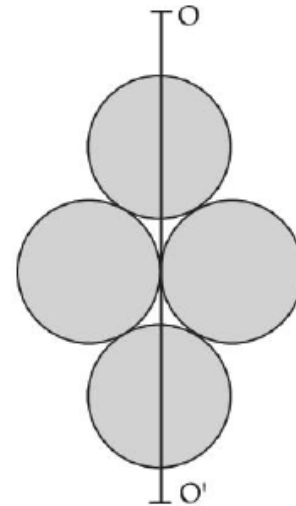
**Sol.**  $\Delta KE = W_{\text{all}}$

$$\text{So } \frac{E}{4} - E = -\frac{1}{2} K \times (0.25)^2$$

$$\frac{3E}{4} = \frac{1}{2} K \times \frac{1}{16}$$

$$K = 24E$$

10. Four identical discs each of mass ' $M$ ' and diameter ' $a$ ' are arranged in a small plane as shown in figure. If the moment of inertia of the system about  $OO'$  is  $\frac{x}{4} Ma^2$ . Then, the value of  $x$  will be \_\_\_\_\_.



**Answer (3)**

**Sol.**  $I = 2 \times \left( \frac{M \left( \frac{a}{2} \right)^2}{4} \right) + 2 \times \left( \frac{M \left( \frac{a}{2} \right)^2}{4} + M \left( \frac{a}{2} \right)^2 \right)$

$$= \frac{Ma^2}{8} + \frac{5Ma^2}{8}$$

$$= \frac{6Ma^2}{8} = \frac{3}{4} Ma^2$$



**CHEMISTRY**

**SECTION - A**

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer :**

- Identify the incorrect statement from the following.
  - A circular path around the nucleus in which an electron moves is proposed as Bohr's orbit.
  - An orbital is the one electron wave function ( $\Psi$ ) in an atom.
  - The existence of Bohr's orbits is supported by hydrogen spectrum.
  - Atomic orbital is characterised by the quantum numbers  $n$  and  $l$  only.

**Answer (D)**

**Sol.** Atomic orbital is characterised by the quantum numbers  $n$ ,  $l$  and  $m$ .

Hence option D is incorrect.

- Which of the following relation is not correct?

- $\Delta H = \Delta U - P\Delta V$
- $\Delta U = q + W$
- $\Delta S_{\text{sys}} + \Delta S_{\text{surr}} \geq 0$
- $\Delta G = \Delta H - T\Delta S$

**Answer (A)**

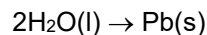
**Sol.**  $\Delta H = \Delta U + P\Delta V$

Hence option A is incorrect.

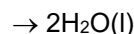
- Match **List-I** with **List-II** :

List-I	List-II
(A) $\text{Cd(s)} + 2\text{Ni(OH)}_3\text{(s)}$ $\rightarrow \text{CdO(s)}$ $+ 2\text{Ni(OH)}_2\text{(s)}$ $+ \text{H}_2\text{O(l)}$	(I) Primary battery
(B) $\text{Zn(Hg)} + \text{HgO(s)}$ $\rightarrow \text{ZnO(s)} + \text{Hg(l)}$	(II) Discharging of secondary battery

- (C)  $2\text{PbSO}_4\text{(s)} +$  (III) Fuel cell



- (D)  $2\text{H}_2\text{(g)} + \text{O}_2\text{(g)}$  (IV) Charging of



secondary battery

Choose the **correct** answer from the options given below:

- (A) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

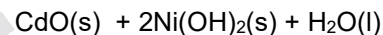
- (B) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

- (C) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

- (D) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)

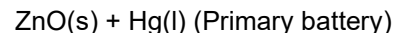
**Answer (C)**

**Sol.** (A)  $\text{Cd(s)} + 2\text{Ni(OH)}_3\text{(s)} \rightarrow$

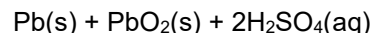


(Discharging of secondary battery)

- (B)  $\text{Zn(Hg)} + \text{HgO(s)} \rightarrow$



- (C)  $2\text{PbSO}_4\text{(s)} + 2\text{H}_2\text{O(l)} \rightarrow$



(Charging of secondary battery)

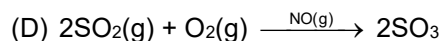
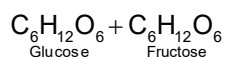
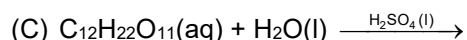
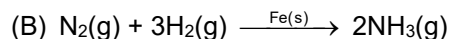
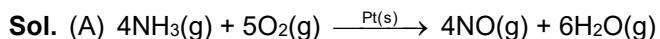
- (D)  $2\text{H}_2\text{(g)} + \text{O}_2\text{(g)} \rightarrow 2\text{H}_2\text{O(l)}$  (Fuel cell)

- Match **List-I** with **List-II**.

List-I Reaction	List-II Catalyst
(A) $4\text{NH}_3\text{(g)} + 5\text{O}_2\text{(g)}$ $\rightarrow 4\text{NO(g)} + 6\text{H}_2\text{O(g)}$	(I) $\text{NO(g)}$
(B) $\text{N}_2\text{(g)} + 3\text{H}_2\text{(g)}$ $\rightarrow 2\text{NH}_3\text{(g)}$	(II) $\text{H}_2\text{SO}_4\text{(l)}$
(C) $\text{C}_{12}\text{H}_{22}\text{O}_{11}\text{(aq)}$ $+ \text{H}_2\text{O(l)}$ $\rightarrow \underset{\text{Glucose}}{\text{C}_6\text{H}_{12}\text{O}_6} + \underset{\text{Fructose}}{\text{C}_6\text{H}_{12}\text{O}_6}$	(III) $\text{Pt(s)}$
(D) $2\text{SO}_2\text{(g)} + \text{O}_2\text{(g)}$ $\rightarrow 2\text{SO}_3\text{(g)}$	(IV) $\text{Fe(s)}$

Choose the **correct** answer from the options given below:

- (A) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)  
 (B) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)  
 (C) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)  
 (D) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

**Answer (C)**

5. In which of the following pairs, electron gain enthalpies of constituent elements are nearly the same or identical?

- (A) Rb and Cs  
 (B) Na and K  
 (C) Ar and Kr  
 (D) I and At

Choose the **correct** answer from the options given below:

- (A) (A) and (B) only      (B) (B) and (C) only  
 (C) (A) and (C) only      (D) (C) and (D) only

**Answer (C)**

**Sol.** Element                      Electron gain enthalpy (kJ mol<sup>-1</sup>)

Rb                                      -47

Cs                                      -46

Electron gain enthalpy of noble gases is almost zero.

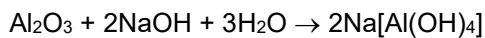
Hence the correct option is (C).

6. Which of the reaction is suitable for concentrating ore by leaching process?

- (A)  $2\text{Cu}_2\text{S} + 3\text{O}_2 \rightarrow 2\text{Cu}_2\text{O} + 2\text{SO}_2$   
 (B)  $\text{Fe}_3\text{O}_4 + \text{CO} \rightarrow 3\text{FeO} + \text{CO}_2$   
 (C)  $\text{Al}_2\text{O}_3 + 2\text{NaOH} + 3\text{H}_2\text{O} \rightarrow 2\text{Na}[\text{Al}(\text{OH})_4]$   
 (D)  $\text{Al}_2\text{O}_3 + 6\text{Mg} \rightarrow 6\text{MgO} + 4\text{Al}$

**Answer (C)**

**Sol.** Leaching involves the treatment of ore with a suitable reagent so as it make it soluble while impurities remain insoluble.



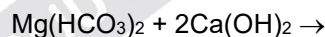
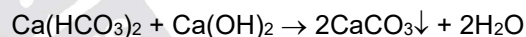
Soluble complex

7. The metal salts formed during softening of hardwater using Clark's method are :

- (A)  $\text{Ca}(\text{OH})_2$  and  $\text{Mg}(\text{OH})_2$   
 (B)  $\text{CaCO}_3$  and  $\text{Mg}(\text{OH})_2$   
 (C)  $\text{Ca}(\text{OH})_2$  and  $\text{MgCO}_3$   
 (D)  $\text{CaCO}_3$  and  $\text{MgCO}_3$

**Answer (B)**

**Sol.** In Clark's method, calculated amount of lime is added to hard water. It precipitates out calcium carbonate and magnesium hydroxide which can filtered off.



8. Which of the following statement is incorrect?

- (A) Low solubility of LiF in water is due to its small hydration enthalpy.  
 (B)  $\text{KO}_2$  is paramagnetic.  
 (C) Solution of sodium in liquid ammonia is conducting in nature.  
 (D) Sodium metal has higher density than potassium metal.

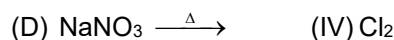
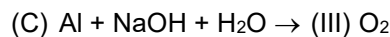
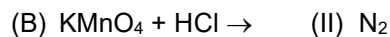
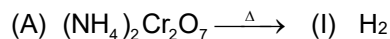
**Answer (A)**

**Sol.** Low solubility of LiF in water is due to the fact that though Li<sup>+</sup> is having high hydration enthalpy but it has higher lattice enthalpy when present in LiF. Due to higher lattice enthalpy its solubility is less.

9. Match **List-I** with **List-II**, match the gas evolved during each reaction.

**List-I**

**List-II**



Choose the **correct** answer from the options given below:

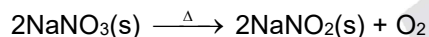
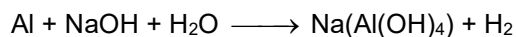
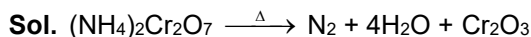
(A) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

(B) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(C) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

(D) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

**Answer (C)**



10. Which of the following has least tendency to liberate  $\text{H}_2$  from mineral acids?

(A) Cu

(B) Mn

(C) Ni

(D) Zn

**Answer (A)**

**Sol.** The metal atom whose oxidation potential is less than that of hydrogen can release  $\text{H}_2$  from mineral acids.

$E^\circ_{\text{Zn}/\text{Zn}^{+2}} = 0.76$

$E^\circ_{\text{Ni}/\text{Ni}^{+2}} = 0.25$

$E^\circ_{\text{Mn}/\text{Mn}^{+2}} = 1.18$

$E^\circ_{\text{Cu}/\text{Cu}^{+2}} = -0.34$

11. Given below are two statements:

**Statement I:** In polluted water values of both dissolved oxygen and BOD are very low.

**Statement II:** Eutrophication results in decrease in the amount of dissolved oxygen.

In the light of the above statements, choose the most appropriate answer from the options given below:

(A) Both **Statement I** and **Statement II** are true

(B) Both **Statement I** and **Statement II** are false

(C) **Statement I** is true but **Statement II** is false

(D) **Statement I** is false but **Statement II** is true

**Answer (D)**

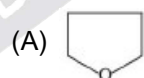
**Sol.** • Clean water would have BOD value of less than 5 ppm whereas highly polluted water could have BOD value of 17 ppm or more.

• Eutrophication results in decrease in the amount of dissolved oxygen.

12. Match List – I with List – II

**List-I**

**List-II**



(i) Spiro compound

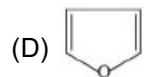


(ii) Aromatic compound



(iii) Non-planar

Heterocyclic compound



(iv) Bicyclo compound

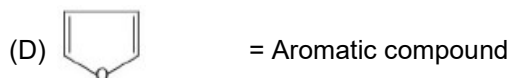
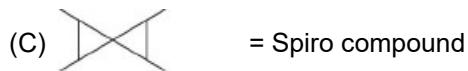
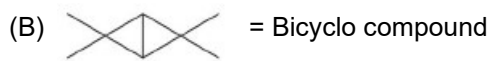
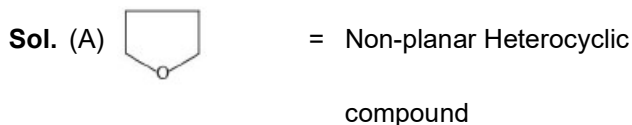
(A) (A) – (II), (B) – (I), (C) – (IV), (D) – (III)

(B) (A) – (IV), (B) – (III), (C) – (I), (D) – (II)

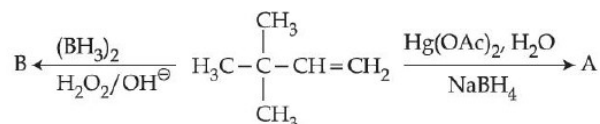
(C) (A) – (III), (B) – (IV), (C) – (I), (D) – (II)

(D) (A) – (IV), (B) – (III), (C) – (II), (D) – (I)

Answer (C)



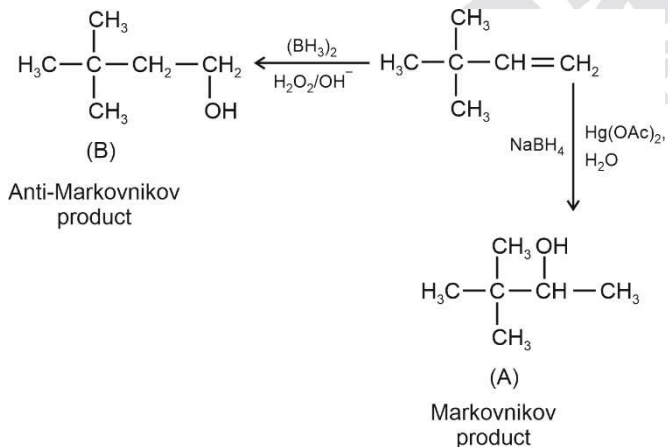
13. Choose the correct option for the following reactions.



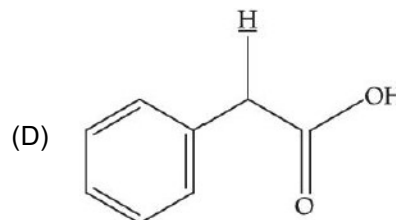
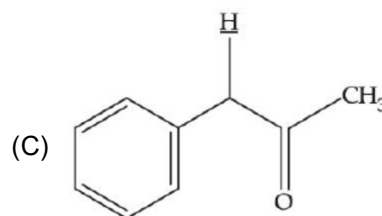
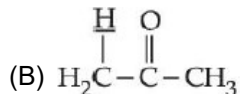
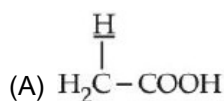
- (A) 'A' and 'B' are both Markovnikov addition products  
 (B) 'A' is Markovnikov product and 'B' is anti-Markovnikov product  
 (C) 'A' and 'B' are both anti-Markovnikov products  
 (D) 'B' is Markovnikov and 'A' is anti-Markovnikov product

Answer (B)

Sol.

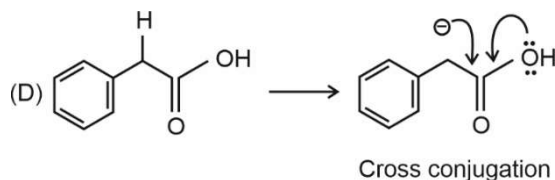
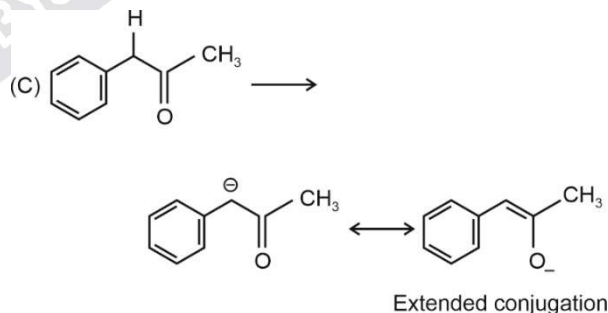
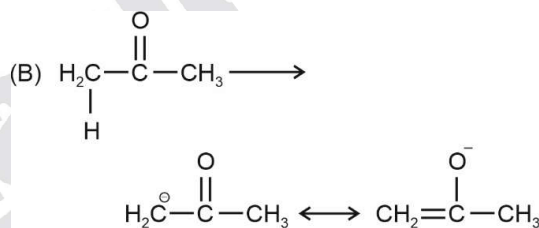
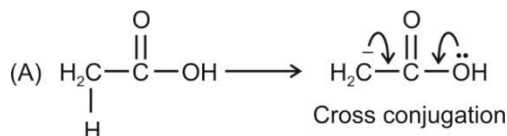


14. Among the following marked proton of which compound shows lowest  $pK_a$  value?



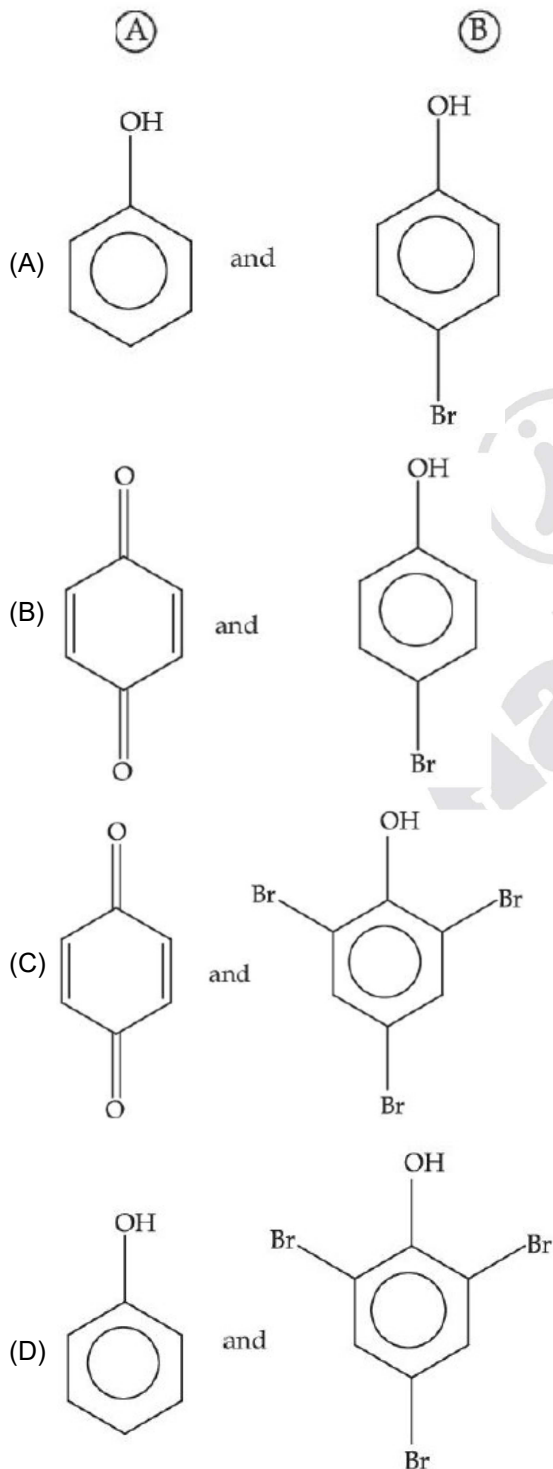
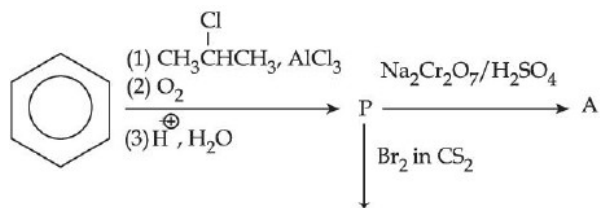
Answer (C)

Sol.



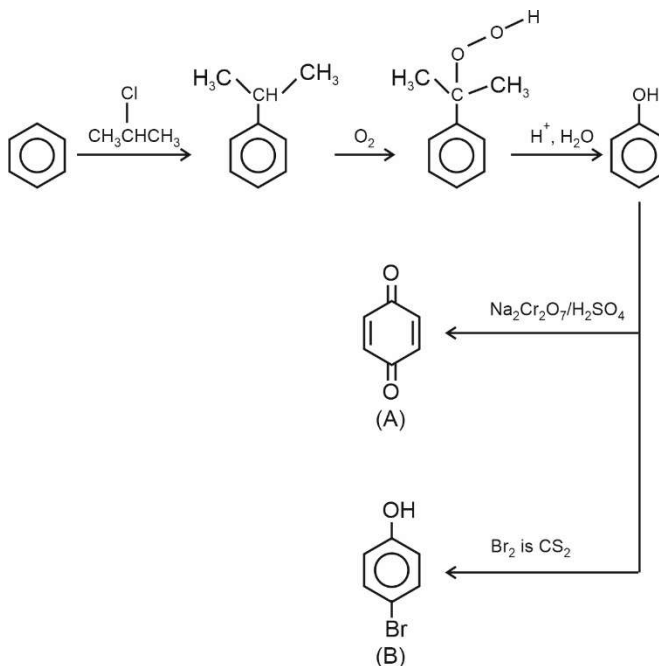
The conjugate base of compound (C) is stabilized by extended conjugation. Hence the indicated proton of compound C is most acidic i.e. will have lowest  $pK_a$ .

15. Identify the major products A and B for the below given reaction sequence.

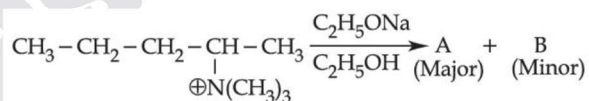


**Answer (B)**

**Sol.**

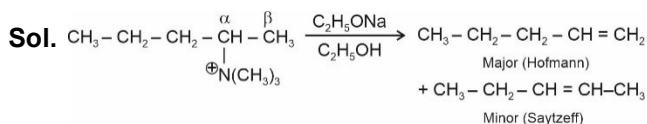


16. Identify the correct statement for the below given transformation.



- (A) A-CH<sub>3</sub>CH<sub>2</sub>CH=CH-CH<sub>3</sub>, B-CH<sub>3</sub>CH<sub>2</sub>CH<sub>2</sub>CH=CH<sub>2</sub>, Saytzeff products
- (B) A-CH<sub>3</sub>CH<sub>2</sub>CH=CH-CH<sub>3</sub>, B-CH<sub>3</sub>CH<sub>2</sub>CH<sub>2</sub>CH=CH<sub>2</sub>, Hofmann products
- (C) A-CH<sub>3</sub>CH<sub>2</sub>CH<sub>2</sub>CH=CH<sub>2</sub>, B-CH<sub>3</sub>CH<sub>2</sub>CH=CHCH<sub>2</sub>, Hofmann products
- (D) A-CH<sub>3</sub>CH<sub>2</sub>CH<sub>2</sub>CH=CH<sub>2</sub>, B-CH<sub>3</sub>CH<sub>2</sub>CH=CHCH<sub>3</sub>, Saytzeff products

**Answer (C)**

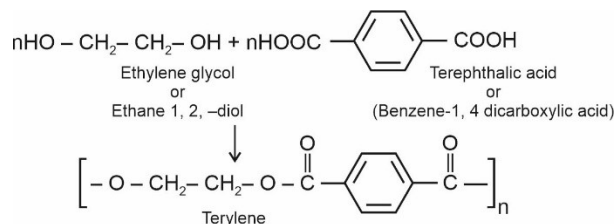


17. Terylene polymer is obtained by condensation of:
- (A) Ethane-1, 2-diol and Benzene-1, 3 dicarboxylic acid
- (B) Propane-1, 2-diol and Benzene-1, 4 dicarboxylic acid

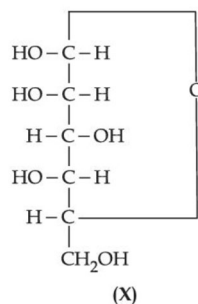
- (C) Ethane-1, 2-diol and Benzene-1, 4 dicarboxylic acid  
 (D) Ethane-1, 2-diol and Benzene-1, 2 dicarboxylic acid

**Answer (C)**

**Sol.**



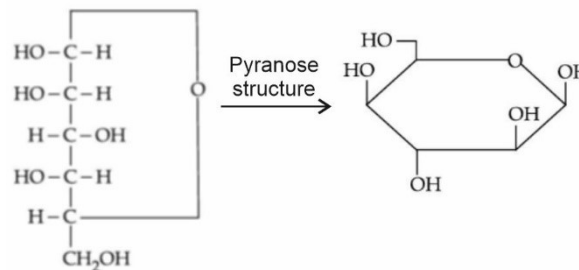
18. For the below given cyclic hemiacetal (X), the correct pyranose structure is :



- (A)
- (B)
- (C)
- (D)

**Answer (D)**

**Sol.**



-OH on right side will point downwards

-OH on left side will point upwards

19. Statements about Enzyme Inhibitor Drugs are given below :

- (A) There are Competitive and Non-competitive inhibitor drugs.  
 (B) These can bind at the active sites and allosteric sites.  
 (C) Competitive Drugs are allosteric site blocking drugs.  
 (D) Non-competitive Drugs are active site blocking drugs.

Choose the correct answer from the options given below:

- (A) (A), (D) only  
 (B) (A), (C) only  
 (C) (A), (B) only  
 (D) (A), (B), (C) only

**Answer (C)**

**Sol.** Drugs can inhibit the attachment of substrate on active site of Enzyme in two ways.

- (1) Competitive, (2) Non-competitive

Competitive inhibitors bind on the active site of Enzymes. Non-Competitive inhibitors bind on allosteric site.

20. For kinetic study of the reaction of iodide ion with  $\text{H}_2\text{O}_2$  at room temperature :

- (A) Always use freshly prepared starch solution.  
 (B) Always keep the concentration of sodium thiosulphate solution less than that of KI solution.  
 (C) Record the time immediately after the appearance of blue colour.  
 (D) Record the time immediately before the appearance of blue colour.  
 (E) Always keep the concentration of sodium thiosulphate solution more than that of KI solution.

Choose the **correct** answer from the options given below :

- (A) (A), (B), (C) only  
 (B) (A), (D), (E) only  
 (C) (D), (E) only  
 (D) (A), (B), (E) only

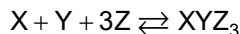
**Answer (A)**

**Sol.** To minimize contamination, use freshly prepared starch solution to determine end point. As KI is used in excess to consume all the  $\text{H}_2\text{O}_2$  the concentration of sodium thiosulphate solution is less than KI solution. After appearance of blue colour record the time immediately.

**SECTION - B**

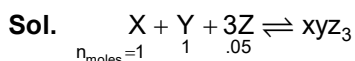
**Numerical Value Type Questions:** This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. In the given reaction,

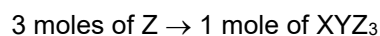


if one mole of each of X and Y with 0.05 mol of Z gives compound  $XYZ_3$ . (Given : Atomic masses of X, Y and Z are 10, 20 and 30 amu, respectively.) the yield of  $XYZ_3$  is \_\_\_\_\_ g. (Nearest integer)

**Answer (2)**



Limiting reagent is  $Z = \frac{.05}{3} = .016$



.05 mole of Z  $\rightarrow \frac{1}{3} \times .05$  mole of  $XYZ_3$

M.wt. of  $XYZ_3 = 10 + 20 + 90$   
 $= 120$  amu

Wt. of  $XYZ_3 = \frac{.05}{3} \times 120$   
 $= 2$  g

2. An element M crystallises in a body centred cubic unit cell with a cell edge of 300 pm. The density of the element is  $6.0 \text{ g cm}^{-3}$ . The number of atoms present in 180 g of the element is \_\_\_\_\_  $\times 10^{23}$ . (Nearest integer)

**Answer (22)**

**Sol.**  $a = 300$  pm  
 $= 300 \times 10^{-12}$  m  
 $= 300 \times 10^{-10}$  cm.  
 $= 3 \times 10^{-8}$  cm.  
 $\rho = 6 \text{ g/cm}^3$   
 wt = 180 gm  
 Volume =  $a^3$   
 $= (3 \times 10^{-8})^3$

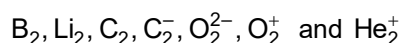
$$\text{Volume occupied per atom} = \frac{27 \times 10^{-24}}{2}$$

$$\rho = 6 \quad \text{wt} = 180 \text{ g}$$

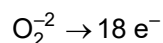
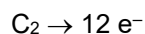
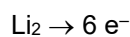
$$\text{Volume} = \frac{180}{6} = 30$$

$$\begin{aligned} \text{Number of atoms} &= \frac{30 \times 2}{27 \times 10^{-24}} \\ &= \frac{60}{27} \times 10^{24} \\ &= 22.2 \times 10^{23} \end{aligned}$$

3. The number of paramagnetic species among the following is \_\_\_\_\_.



**Answer (4)**

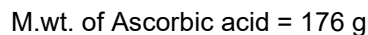


Species with odd number of electrons are paramagnetic except boron and oxygen.

4. 150 g of acetic acid was contaminated with 10.2 g ascorbic acid ( $\text{C}_6\text{H}_8\text{O}_6$ ) to lower down its freezing point by  $(x \times 10^{-1})^\circ\text{C}$ . The value of x is \_\_\_\_\_. (Nearest integer)

(Given :  $K_f = 3.9 \text{ K kg mol}^{-1}$ ; molar mass of ascorbic acid =  $176 \text{ g mol}^{-1}$ )

**Answer (15)**



$$\Delta T_f = K_f m$$

$$\Delta T_f = \frac{3.9 \times 10.2 \times 1000}{176 \times 150}$$

$$\Delta T_f = 1.506$$

$$= 15.06 \times 10^{-1}$$

$$= 15$$

5.  $K_a$  for butyric acid ( $\text{C}_3\text{H}_7\text{COOH}$ ) is  $2 \times 10^{-5}$ . The pH of 0.2 M solution of butyric acid is \_\_\_\_\_  $\times 10^{-1}$ . (Nearest integer) (Given  $\log 2 = 0.30$ )

**Answer (27)**



$$\alpha = \sqrt{\frac{K_a}{C}} \quad K_a = 2 \times 10^{-5}$$

$$= \sqrt{\frac{2 \times 10^{-5}}{2 \times 10^{-1}}}$$

$$= 10^{-2}$$

$$[\text{H}^+] = C\alpha$$

$$= 0.2 \times 10^{-2}$$

$$= 2 \times 10^{-3}$$

$$\text{pH} = 3 - \log 2$$

$$= 3 - 0.30$$

$$= 2.7$$

$$\text{pH} = 27 \times 10^{-1}$$

6. For the given first order reaction  $\text{A} \rightarrow \text{B}$ , the half-life of the reaction is 0.3010 min. The ratio of the initial concentration of reactant to the concentration of reactant at time 2.0 min will be equal to \_\_\_\_\_. (Nearest integer)

**Answer (100)**



$$K = \frac{0.693}{0.3010}$$

$$K = 2.30$$



$$K = \frac{2.303}{t} \log \frac{(A_0)}{(A_t)}$$

$A_0 \rightarrow$  initial concentration of reactant

$A_t \rightarrow$  concentration of reactant at time  $t$

$$2.303 = \frac{2.303}{2} \log \frac{(A_0)}{(A_t)}$$

$$2 = \log \frac{(A_0)}{(A_t)}$$

$$100 = \frac{A_0}{A_t}$$

7. The number of interhalogens from the following having square pyramidal structure is :

$\text{ClF}_3$ ,  $\text{IF}_7$ ,  $\text{BrF}_5$ ,  $\text{BrF}_3$ ,  $\text{I}_2\text{Cl}_6$ ,  $\text{IF}_5$ ,  $\text{ClF}$ ,  $\text{ClF}_5$

**Answer (3)**

**Sol.**  $\text{ClF}_3 \rightarrow 3 \sigma$  bond + 2 lone pair

$\text{IF}_7 \rightarrow 7 \sigma$  bond + 0 lone pair

$\text{BrF}_5 \rightarrow 5 \sigma$  bond + 1 lone pair  $\rightarrow$  Square pyramidal

$\text{BrF}_3 \rightarrow 3 \sigma$  bond + 2 lone pair

$\text{I}_2\text{Cl}_6 \rightarrow 4 \sigma$  bond + 2 lone pair

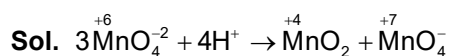
$\text{IF}_5 \rightarrow 5 \sigma$  bond + 1 lone pair  $\rightarrow$  Square pyramidal

$\text{ClF} \rightarrow 1 \sigma$  bond + 3 lone pair

$\text{ClF}_5 \rightarrow 5 \sigma$  bond + 1 lone pair  $\rightarrow$  Square pyramidal

8. The disproportionation of  $\text{MnO}_4^{2-}$  in acidic medium resulted in the formation of two manganese compounds A and B. If the oxidation state of Mn in B is smaller than that of A, then the spin-only magnetic moment ( $\mu$ ) value of B in BM is \_\_\_\_\_. (Nearest integer)

**Answer (4)**



$\text{Mn} \rightarrow 4s^2 3d^5$

$\text{Mn}^{+4} \rightarrow 3d^3$

$n = 3$

$$\mu = \sqrt{n(n+2)}$$

$$= \sqrt{3(5)}$$

$$= \sqrt{15}$$

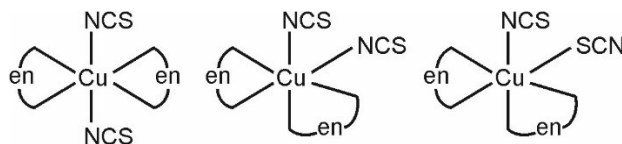
$$= 3.87 \approx 4 \text{ B.M.}$$

9. Total number of relatively more stable isomer(s) possible for octahedral complex  $[\text{Cu}(\text{en})_2(\text{SCN})_2]$  will be \_\_\_\_\_.

**Answer (3)**

**Sol.**  $[\text{Cu}(\text{en})_2(\text{SCN})_2]$

Total isomers



10. On complete combustion of 0.492 g of an organic compound containing C, H and O, 0.7938 g of  $\text{CO}_2$  and 0.4428 g of  $\text{H}_2\text{O}$  was produced. The % composition of oxygen in the compound is \_\_\_\_\_.

**Answer (46)**

**Sol.** % of H =  $\frac{2}{18} \times \frac{\text{wt. of H}_2\text{O}}{\text{wt. of organic compound}} \times 100$

$$= \frac{2}{18} \times \frac{0.4428}{0.492} \times 100$$

$$= 0.11 \times 0.9 \times 100$$

$$= .099 \times 100 = 9.9$$

$$\% \text{ of C} = \frac{12}{44} \times \frac{0.7938}{0.492} \times 100$$

$$= 0.27 \times 1.61 \times 100$$

$$= 43.47$$

$$\% \text{ Oxygen} = 100 - (43.47 + 9.9)$$

$$= 100 - 53.37$$

$$\approx 46$$

MATHEMATICS

SECTION - A

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer :**

1. Let the solution curve of the differential equation  $xdy = (\sqrt{x^2 + y^2} + y)dx$ ,  $x > 0$ , intersect the line  $x = 1$  at  $y = 0$  and the line  $x = 2$  at  $y = \alpha$ . Then the value of  $\alpha$  is

- (A)  $\frac{1}{2}$  (B)  $\frac{3}{2}$   
(C)  $-\frac{3}{2}$  (D)  $\frac{5}{2}$

**Answer (B)**

**Sol.**  $\frac{xdy - ydx}{\sqrt{x^2 + y^2}} = dx$

$$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{x^2 + y^2}}{x} + \frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx} = \sqrt{1 + \frac{y^2}{x^2}} + \frac{y}{x}$$

Let  $\frac{y}{x} = v$

$$\Rightarrow v + x \frac{dv}{dx} = \sqrt{1 + v^2} + v$$

$$\Rightarrow \frac{dv}{\sqrt{1 + v^2}} = \frac{dx}{x}$$

OR  $\ln(v + \sqrt{1 + v^2}) = \ln x + C$

at  $x = 1, y = 0$

$$\Rightarrow C = 0$$

$$\frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}} = x$$

At  $x = 2,$

$$\frac{y}{2} + \sqrt{1 + \frac{y^2}{4}} = 2$$

$$\Rightarrow 1 + \frac{y^2}{4} = 4 + \frac{y^2}{4} - 2y$$

OR  $y = \frac{3}{2}$

2. Considering only the principal values of the inverse trigonometric functions, the domain of the function

$$f(x) = \cos^{-1}\left(\frac{x^2 - 4x + 2}{x^2 + 3}\right) \text{ is}$$

- (A)  $\left(-\infty, \frac{1}{4}\right]$  (B)  $\left[-\frac{1}{4}, \infty\right)$   
(C)  $\left(\frac{-1}{3}, \infty\right)$  (D)  $\left(-\infty, \frac{1}{3}\right]$

**Answer (B)**

**Sol.**  $-1 \leq \frac{x^2 - 4x + 2}{x^2 + 3} \leq 1$

$$\Rightarrow -x^2 - 3 \leq x^2 - 4x + 2 \leq x^2 + 3$$

$$\Rightarrow 2x^2 - 4x + 5 \geq 0 \quad \& \quad -4x \leq 1$$

$$x \in \mathbb{R} \quad \& \quad x \geq -\frac{1}{4}$$

So domain is  $\left[-\frac{1}{4}, \infty\right)$

3. Let the vectors  $\vec{a} = (1+t)\hat{i} + (1-t)\hat{j} + \hat{k}$ ,  $\vec{b} = (1-t)\hat{i} + (1+t)\hat{j} + 2\hat{k}$  and  $\vec{c} = t\hat{i} - t\hat{j} + \hat{k}$ ,  $t \in \mathbb{R}$  be such that for  $\alpha, \beta, \gamma \in \mathbb{R}$ ,  $\alpha\vec{a} + \beta\vec{b} + \gamma\vec{c} = \vec{0}$   
 $\Rightarrow \alpha = \beta = \gamma = 0$ . Then, the set of all values of  $t$  is

- (A) A non-empty finite set  
(B) Equal to  $\mathbb{N}$   
(C) Equal to  $\mathbb{R} - \{0\}$   
(D) Equal to  $\mathbb{R}$

**Answer (C)**

**Sol.** Clearly  $\vec{a}, \vec{b}, \vec{c}$  are non-coplanar

$$\begin{vmatrix} 1+t & 1-t & 1 \\ 1-t & 1+t & 2 \\ t & -t & 1 \end{vmatrix} \neq 0$$

$$\Rightarrow (1+t)(1+t+2t) - (1-t)(1-t-2t) + 1(t^2 - t - t^2) \neq 0$$

$$\Rightarrow (3t^2 + 4t + 1) - (1-t)(1-3t) - 2t \neq 0$$

$$\Rightarrow (3t^2 + 4t + 1) - (3t^2 - 4t + 1) - 2t \neq 0$$

$$\Rightarrow t \neq 0$$

4. Considering the principal values of the inverse trigonometric functions, the sum of all the solutions of the equation  $\cos^{-1}(x) - 2\sin^{-1}(x) = \cos^{-1}(2x)$  is equal to

- (A) 0 (B) 1  
(C)  $\frac{1}{2}$  (D)  $-\frac{1}{2}$

**Answer (A)**

**Sol.**  $\cos^{-1}x - 2\sin^{-1}x = \cos^{-1}2x$

**For Domain :**  $x \in \left[ \frac{-1}{2}, \frac{1}{2} \right]$

$$\cos^{-1}x - 2\left(\frac{\pi}{2} - \cos^{-1}x\right) = \cos^{-1}(2x)$$

$$\Rightarrow \cos^{-1}x + 2\cos^{-1}x = \pi + \cos^{-1}2x$$

$$\Rightarrow \cos(3\cos^{-1}x) = -\cos(\cos^{-1}2x)$$

$$\Rightarrow 4x^3 = x$$

$$\Rightarrow x = 0, \pm \frac{1}{2}$$

5. Let the operations  $*$ ,  $\odot \in \{\wedge, \vee\}$ . If  $(p * q) \odot (p \odot \sim q)$  is a tautology, then the ordered pair  $(*, \odot)$  is

- (A)  $(\vee, \wedge)$  (B)  $(\vee, \vee)$   
(C)  $(\wedge, \wedge)$  (D)  $(\wedge, \vee)$

**Answer (B)**

**Sol.**  $*, \odot \in \{\wedge, \vee\}$

Now for  $(p * q) \odot (p \odot \sim q)$  is tautology

(A)  $(\vee, \wedge) : (p \vee q) \wedge (p \wedge \sim q)$  not a tautology

(B)  $(\vee, \vee) : (p \vee q) \vee (p \vee \sim q)$   
 $= P \vee T$  is tautology

(C)  $(\wedge, \wedge) : (p \wedge q) \wedge (p \wedge \sim q)$   
 $= (p \wedge p) \wedge (q \wedge \sim q) = p \wedge F$  not a tautology  
(Fallacy)

(D)  $(\wedge, \vee) : (p \wedge q) \vee (p \vee \sim q)$  not a tautology

6. Let a vector  $\vec{a}$  has magnitude 9. Let a vector  $\vec{b}$  be such that for every  $(x, y) \in \mathbf{R} \times \mathbf{R} - \{(0, 0)\}$ , the vector  $(x\vec{a} + y\vec{b})$  is perpendicular to the vector  $(6y\vec{a} - 18x\vec{b})$ . Then the value of  $|\vec{a} \times \vec{b}|$  is equal to

- (A)  $9\sqrt{3}$  (B)  $27\sqrt{3}$   
(C) 9 (D) 81

**Answer (B)**

**Sol.**  $(x\vec{a} + y\vec{b}) \cdot (6y\vec{a} - 18x\vec{b}) = 0$

$$\Rightarrow (6xy|\vec{a}|^2 - 18xy|\vec{b}|^2) + (6y^2 - 18x^2)\vec{a} \cdot \vec{b} = 0$$

As given equation is identity

Coefficient of  $x^2 =$  coefficient of  $y^2 =$  coefficient of  $xy = 0$

$$\Rightarrow |\vec{a}|^2 = 3|\vec{b}|^2 \Rightarrow |\vec{b}| = 3\sqrt{3}$$

and  $\vec{a} \cdot \vec{b} = 0$

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta$$

$$= 9 \cdot 3\sqrt{3} \cdot 1 = 27\sqrt{3}$$

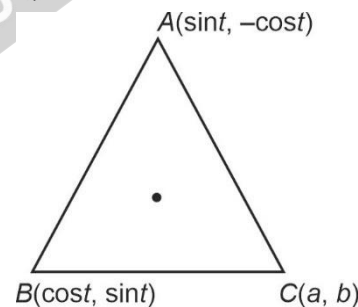
7. For  $t \in (0, 2\pi)$ , if  $ABC$  is an equilateral triangle with vertices  $A(\sin t, -\cos t)$ ,  $B(\cos t, \sin t)$  and  $C(a, b)$  such that its orthocentre lies on a circle with centre

$\left(1, \frac{1}{3}\right)$ , then  $(a^2 - b^2)$  is equal to

- (A)  $\frac{8}{3}$  (B) 8  
(C)  $\frac{77}{9}$  (D)  $\frac{80}{9}$

**Answer (B)**

**Sol.** Let  $P(h, k)$  be the orthocentre of  $\triangle ABC$



Then

$$h = \frac{\sin t + \cos t + a}{3}, k = \frac{-\cos t + \sin t + b}{3}$$

(orthocentre coincide with centroid)

$$\therefore (3h - a)^2 + (3k - b)^2 = 2$$

$$\therefore \left(h - \frac{a}{3}\right)^2 + \left(k - \frac{b}{3}\right)^2 = \frac{2}{9}$$

$\therefore$  orthocentre lies on circle with centre  $\left(1, \frac{1}{3}\right)$

$$\therefore a = 3, b = 1$$

$$\therefore a^2 - b^2 = 8$$

8. For  $\alpha \in \mathbf{N}$ , consider a relation  $R$  on  $\mathbf{N}$  given by  $R = \{(x, y) : 3x + \alpha y \text{ is a multiple of } 7\}$ . The relation  $R$  is an equivalence relation if and only if
- (A)  $\alpha = 14$   
 (B)  $\alpha$  is a multiple of 4  
 (C) 4 is the remainder when  $\alpha$  is divided by 10  
 (D) 4 is the remainder when  $\alpha$  is divided by 7

**Answer (D)**

**Sol.**  $R = \{(x, y) : 3x + \alpha y \text{ is multiple of } 7\}$ , Now  $R$  to be an equivalence relation

(1)  $R$  should be reflexive :  $(a, a) \in R \forall a \in \mathbf{N}$

$$\therefore 3a + a\alpha = 7k$$

$$\therefore (3 + \alpha) a = 7k$$

$$\therefore 3 + \alpha = 7k_1 \Rightarrow \alpha = 7k_1 - 3 = 7k_1 + 4$$

(2)  $R$  should be symmetric :  $aRb \Leftrightarrow bRa$

$$aRb : 3a + (7k - 3)b = 7m$$

$$\Rightarrow 3(a - b) + 7kb = 7m$$

$$\Rightarrow 3(b - a) + 7ka = 7m$$

$$\text{So, } aRb \Rightarrow bRa$$

$$\therefore R \text{ will be symmetric for } a = 7k_1 - 3$$

(3) Transitive : Let  $(a, b) \in R, (b, c) \in R$

$$\Rightarrow 3a + (7k - 3)b = 7k_1 \text{ and}$$

$$3b + (7k_2 - 3)c = 7k_3$$

$$\text{Adding } 3a + 7kb + (7k_2 - 3)c = 7(k_1 + k_3)$$

$$3a + (7k_2 - 3)c = 7m$$

$$\therefore (a, c) \in R$$

$$\therefore R \text{ is transitive}$$

$$\therefore \alpha = 7k - 3 = 7k + 4$$

9. Out of 60% female and 40% male candidates appearing in an exam, 60% candidates qualify it. The number of females qualifying the exam is twice the number of males qualifying it. A candidate is randomly chosen from the qualified candidates. The probability, that the chosen candidate is a female, is

(A)  $\frac{3}{4}$

(B)  $\frac{11}{16}$

(C)  $\frac{23}{32}$

(D)  $\frac{13}{16}$

**Answer (\*) None of the given option is correct**

**Sol.**  $P(\text{Female}) = \frac{60}{100} = \frac{3}{5}$

$$P(\text{Male}) = \frac{2}{5}$$

$$P(\text{Female/Qualified}) = \frac{40}{60} = \frac{2}{3}$$

$$P(\text{Male / Qualified}) = \frac{20}{60} = \frac{1}{3}$$

10. If  $y = y(x), x \in \left(0, \frac{\pi}{2}\right)$  be the solution curve of the differential equation  $(\sin^2 2x) \frac{dy}{dx} + (8\sin^2 2x + 2\sin 4x)y = 2e^{-4x}(2\sin 2x + \cos 2x)$ , with  $y\left(\frac{\pi}{4}\right) = e^{-\pi}$ ,

then  $y\left(\frac{\pi}{6}\right)$  is equal to

(A)  $\frac{2}{\sqrt{3}} e^{-2\pi/3}$

(B)  $\frac{2}{\sqrt{3}} e^{2\pi/3}$

(C)  $\frac{1}{\sqrt{3}} e^{-2\pi/3}$

(D)  $\frac{1}{\sqrt{3}} e^{2\pi/3}$

**Answer (A)**

**Sol.**  $(\sin^2 2x) \frac{dy}{dx} + (8\sin^2 2x + 2\sin 4x)y = 2e^{-4x}(2\sin 2x + \cos 2x)$

$$= 2e^{-4x}(2\sin 2x + \cos 2x)$$

$$\frac{dy}{dx} + (8 + 4\cot 2x)y = 2e^{-4x} \left( \frac{2\sin 2x + \cos 2x}{\sin^2 2x} \right)$$

Integrating factor

$$(I.F.) = e^{\int (8+4\cot 2x) dx}$$

$$= e^{8x+2\ln \sin 2x}$$

Solution of differential equation

$$y \cdot e^{8x+2\ln \sin 2x}$$

$$= \int 2e^{(4x+2\ln \sin 2x)} \frac{(2\sin 2x + \cos 2x)}{\sin^2 2x} dx$$

$$= 2 \int e^{4x} (2\sin 2x + \cos 2x) dx$$

$$y \cdot e^{8x+2\ln \sin 2x} = e^{4x} \sin 2x + c$$

$$y\left(\frac{\pi}{4}\right) = e^{-\pi}$$

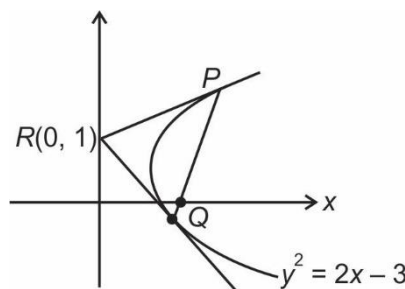
$$e^{-\pi} \cdot e^{2\pi} = e^{\pi} + c \Rightarrow c = 0$$

$$y\left(\frac{\pi}{6}\right) = \frac{e^{\frac{2\pi}{3}} \frac{\sqrt{3}}{2}}{e^{\left(\frac{4\pi}{3} + 2\ln\frac{\sqrt{3}}{2}\right)}} = e^{-\frac{2\pi}{3}} \cdot \frac{2}{\sqrt{3}}$$

11. If the tangents drawn at the points  $P$  and  $Q$  on the parabola  $y^2 = 2x - 3$  intersect at the point  $R(0, 1)$ , then the orthocentre of the triangle  $PQR$  is :

- (A) (0, 1)                      (B) (2, -1)  
 (C) (6, 3)                      (D) (2, 1)

**Answer (B)**



**Sol.**

Equation of chord  $PQ$

$$\Rightarrow y \times 1 = x - 3$$

$$\Rightarrow x - y = 3$$

For point  $P$  &  $Q$

Intersection of  $PQ$  with parabola  $P : (6, 3)$   $Q : (2, -1)$

Slope of  $RQ = -1$  & Slope of  $PQ = 1$

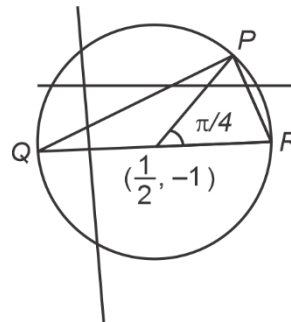
Therefore  $\angle PQR = 90^\circ \Rightarrow$  Orthocentre is at  $Q : (2, -1)$

12. Let  $C$  be the centre of the circle  $x^2 + y^2 - x + 2y = \frac{11}{4}$  and  $P$  be a point on the circle. A line passes through the point  $C$ , makes an angle of  $\frac{\pi}{4}$  with the line  $CP$  and intersects the circle at the  $Q$  and  $R$ . Then the area of the triangle  $PQR$  (in unit<sup>2</sup>) is :

- (A) 2  
 (B)  $2\sqrt{2}$   
 (C)  $8\sin\left(\frac{\pi}{8}\right)$   
 (D)  $8\cos\left(\frac{\pi}{8}\right)$

**Answer (B)**

**Sol.**



$$QR = 2r = 4$$

$$P = \left(\frac{1}{2} + 2\cos\frac{\pi}{4}, -1 + 2\sin\frac{\pi}{4}\right)$$

$$= \left(\frac{1}{2} + \sqrt{2}, -1 + \sqrt{2}\right)$$

$$\text{Area of } \triangle PQR = \frac{1}{2} \times 4 \times \sqrt{2}$$

$$= 2\sqrt{2} \text{ sq. units}$$

13. The remainder when  $7^{2022} + 3^{2022}$  is divided by 5 is:

- (A) 0                              (B) 2  
 (C) 3                              (D) 4

**Answer (C)**

**Sol.** Let  $E = 7^{2022} + 3^{2022}$

$$= (15 - 1)^{1011} + (10 - 1)^{1011}$$

$$= -1 + (\text{multiple of } 15) - 1 + \text{multiple of } 10$$

$$= -2 + (\text{multiple of } 5)$$

Hence remainder on dividing  $E$  by 5 is 3.

14. Let the matrix  $A = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix}$  and the matrix  $B_0$

$= A^{49} + 2A^{98}$ . If  $B_n = \text{Adj}(B_{n-1})$  for all  $n \geq 1$ , then  $\det(B_4)$  is equal to :

- (A)  $3^{28}$                               (B)  $3^{30}$   
 (C)  $3^{32}$                               (D)  $3^{36}$

**Answer (C)**

**Sol.**  $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$$\Rightarrow A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$$\text{Now } B_0 = A^{49} + 2A^{98} = (A^3)^{16} \cdot A + 2(A^3)^{32} \cdot A^2$$

$$B_0 = A + 2A^2 =$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 2 \\ 2 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix}$$

$$|B_0| = 9$$

$$\text{Since, } B_n = \text{Adj } |B_{n-1}| \Rightarrow |B_n| = |B_{n-1}|^2$$

$$\text{Hence } |B_4| = |B_3|^2 = |B_2|^4 = |B_1|^8 = |B_0|^{16} = |3^2|^{16} = 3^{32}$$

15. Let  $S_1 = \left\{ z_1 \in \mathbb{C} : |z_1 - 3| = \frac{1}{2} \right\}$  and

$S_2 = \left\{ z_2 \in \mathbb{C} : |z_2 - |z_2 + 1|| = |z_2 + |z_2 - 1|| \right\}$ . Then, for  $z_1 \in S_1$  and  $z_2 \in S_2$ , the least value of  $|z_2 - z_1|$  is :

(A) 0 (B)  $\frac{1}{2}$

(C)  $\frac{3}{2}$  (D)  $\frac{5}{2}$

**Answer (C)**

**Sol.**  $\therefore |z_2 + |z_2 - 1||^2 = |z_2 - |z_2 + 1||^2$

$$\Rightarrow (z_2 + |z_2 - 1|)(\bar{z}_2 + |z_2 - 1|) = (z_2 - |z_2 + 1|)(\bar{z}_2 - |z_2 + 1|)$$

$$\Rightarrow z_2(|z_2 - 1| + |z_2 + 1|) + \bar{z}_2(|z_2 - 1| + |z_2 + 1|) = |z_2 + 1|^2 - |z_2 - 1|^2$$

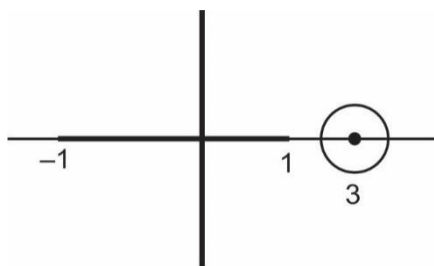
$$\Rightarrow (z_2 + \bar{z}_2)(|z_2 + 1| + |z_2 - 1|) = 2(z_2 + \bar{z}_2)$$

$$\Rightarrow \text{Either } z_2 + \bar{z}_2 = 0 \text{ or } |z_2 + 1| + |z_2 - 1| = 2$$

So,  $z_2$  lies on imaginary axis or on real axis within  $[-1, 1]$

Also  $|z_1 - 3| = \frac{1}{2} \Rightarrow z_1$  lies on the circle having

center 3 and radius  $\frac{1}{2}$ .



Clearly  $|z_1 - z_2|_{\min} = \frac{3}{2}$

16. The foot of the perpendicular from a point on the circle  $x^2 + y^2 = 1, z = 0$  to the plane  $2x + 3y + z = 6$  lies on which one of the following curves?

(A)  $(6x + 5y - 12)^2 + 4(3x + 7y - 8)^2 = 1, z = 6 - 2x - 3y$

(B)  $(5x + 6y - 12)^2 + 4(3x + 5y - 9)^2 = 1, z = 6 - 2x - 3y$

(C)  $(6x + 5y - 14)^2 + 9(3x + 5y - 7)^2 = 1, z = 6 - 2x - 3y$

(D)  $(5x + 6y - 14)^2 + 9(3x + 7y - 8)^2 = 1, z = 6 - 2x - 3y$

**Answer (B)**

**Sol.** Any point on  $x^2 + y^2 = 1, z = 0$  is  $p(\cos\theta, \sin\theta, 0)$

If foot of perpendicular of  $p$  on the plane  $2x + 3y + z = 6$  is  $(h, k, l)$  then

$$\frac{h - \cos\theta}{2} = \frac{k - \sin\theta}{3} = \frac{l - 0}{1}$$

$$= - \left( \frac{2\cos\theta + 3\sin\theta + 0 - 6}{2^2 + 3^2 + 1^2} \right) = r(\text{let})$$

$$h = 2r + \cos\theta, k = 3r + \sin\theta, l = r$$

$$\text{Hence, } h - 2l = \cos\theta \text{ and } k - 3l = \sin\theta$$

$$\text{Hence } (h - 2l)^2 + (k - 3l)^2 = 1$$

$$\text{When } l = 6 - 2h - 3k$$

Hence required locus is

$$(x - 2(6 - 2x - 3y))^2 + (y - 3(6 - 2x - 3y))^2 = 1$$

$$\Rightarrow (5x + 6y - 12)^2 + 4(3x + 5y - 9)^2 = 1, z = 6 - 2x - 3y$$

17. If the minimum value of  $f(x) = \frac{5x^2}{2} + \frac{\alpha}{x^5}, x > 0$ , is

14, then the value of  $\alpha$  is equal to

(A) 32 (B) 64

(C) 128 (D) 256

**Answer (C)**

**Sol.**  $f(x) = \frac{5x^2}{2} + \frac{\alpha}{x^5} \{x > 0\}$

$$f'(x) = 5x - \frac{5\alpha}{x^6} = 0$$

$$\Rightarrow x = (\alpha)^{\frac{1}{7}}$$

$$f(x)_{\min} = \frac{5(\alpha)^{\frac{2}{7}}}{2} + \frac{\alpha}{\alpha^{\frac{5}{7}}} = 14$$

$$\frac{5}{2}\alpha^{\frac{2}{7}} + \alpha^{\frac{2}{7}} = 14$$

$$\frac{7}{2}\alpha^{\frac{2}{7}} = 14$$

$$\alpha = 128$$

18. Let  $\alpha, \beta$  and  $\gamma$  be three positive real numbers. Let  $f(x) = \alpha x^5 + \beta x^3 + \gamma x, x \in \mathbb{R}$  and  $g: \mathbb{R} \rightarrow \mathbb{R}$  be such that  $g(f(x)) = x$  for all  $x \in \mathbb{R}$ . If  $a_1, a_2, a_3, \dots, a_n$  be in arithmetic progression with mean zero, then the

value of  $f\left(g\left(\frac{1}{n}\sum_{i=1}^n f(a_i)\right)\right)$  is equal to

- (A) 0 (B) 3  
(C) 9 (D) 27

**Answer (A)**

**Sol.**  $f\left(g\left(\frac{1}{n}\sum_{i=1}^n f(a_i)\right)\right)$

$$\frac{a_1 + a_2 + a_3 + \dots + a_n}{n} = 0$$

$\therefore$  First and last term, second and second last and so on are equal in magnitude but opposite in sign.

$$f(x) = \alpha x^5 + \beta x^3 + \gamma x$$

$$\sum_{i=1}^n f(a_i) = \alpha(a_1^5 + a_2^5 + a_3^5 + \dots + a_n^5)$$

$$+ \beta(a_1^3 + a_2^3 + \dots + a_n^3)$$

$$+ \gamma(a_1 + a_2 + \dots + a_n)$$

$$= 0\alpha + 0\beta + 0\gamma$$

$$= 0$$

$$\therefore f\left(g\left(\frac{1}{n}\sum_{i=1}^n f(a_i)\right)\right) = \frac{1}{n}\sum_{i=1}^n f(a_i) = 0$$

19. Consider the sequence  $a_1, a_2, a_3, \dots$  such that

$$a_1 = 1, a_2 = 2 \text{ and } a_{n+2} = \frac{2}{a_{n+1}} + a_n \text{ for } n = 1, 2, 3,$$

$$\dots \text{ if } \left(\frac{a_1 + \frac{1}{a_2}}{a_3}\right) \cdot \left(\frac{a_2 + \frac{1}{a_3}}{a_4}\right) \cdot \left(\frac{a_3 + \frac{1}{a_4}}{a_5}\right) \dots \left(\frac{a_{30} + \frac{1}{a_{31}}}{a_{32}}\right)$$

$= 2^\alpha ({}^{61}C_{31})$ , then  $\alpha$  is equal to

- (A) -30 (B) -31  
(C) -60 (D) -61

**Answer (C)**

**Sol.**  $a_{n+2} = \frac{2}{a_{n+1}} + a_n$

$$\Rightarrow a_n a_{n+1} + 1 = a_{n+1} a_{n+2} - 1$$

$$\Rightarrow a_{n+2} a_{n+1} - a_n \cdot a_{n+1} = 2$$

For  $n = 1$   $a_3 a_2 - a_1 a_2 = 2$

$n = 2$   $a_4 a_3 - a_2 a_3 = 2$

$n = 3$   $a_5 a_4 - a_3 a_4 = 2$

$\vdots$

$n = n$   $a_{n+2} a_{n+1} - a_n a_{n+1} = 2$

---


$$a_{n+2} a_{n+1} = 2n + a_1 a_2$$

Now,

$$\frac{(a_1 a_2 + 1)}{a_2 a_3} \cdot \frac{(a_2 a_3 + 1)}{a_3 a_4} \cdot \frac{(a_3 a_4 + 1)}{a_4 a_5} \cdot \dots \cdot \frac{(a_{30} a_{31} + 1)}{a_{31} a_{32}}$$

$$= \frac{3}{4} \cdot \frac{5}{6} \cdot \frac{7}{8} \cdot \dots \cdot \frac{61}{62}$$

$$= 2^{-60} ({}^{61}C_{31})$$

20. The minimum value of the twice differentiable

function  $f(x) = \int_0^x e^{x-t} f'(t) dt - (x^2 - x + 1)e^x, x \in \mathbb{R}$ ,

is

(A)  $-\frac{2}{\sqrt{e}}$  (B)  $-2\sqrt{e}$

(C)  $-\sqrt{e}$  (D)  $\frac{2}{\sqrt{e}}$

**Answer (A)**

**Sol.**  $f(x) = \int_0^x e^{x-t} f'(t) dt - (x^2 - x + 1)e^x$

$$f(x) = e^x \int_0^x e^{-t} f'(t) dt - (x^2 - x + 1)e^x$$

$$e^{-x} f(x) = \int_0^x e^{-t} f'(t) dt - (x^2 - x + 1)$$

Differentiate on both side

$$e^{-x} f'(x) + (-f(x)e^{-x}) = e^{-x} f'(x) - 2x + 1$$

$$f(x) = e^x (2x - 1)$$

$$f'(x) = e^x(2) + e^x(2x-1)$$

$$= e^x(2x+1)$$

$$x = -\frac{1}{2}$$

$$f''(x) = e^x(2) + (2x+1)e^x$$

$$= e^x(2x+3)$$

$$\text{For } x = -\frac{1}{2} \quad f''(x) > 0$$

⇒ Maxima

$$\therefore \text{Max.} = e^{-\frac{1}{2}}(-1-1)$$

$$\therefore -\frac{2}{\sqrt{e}}$$

**SECTION - B**

**Numerical Value Type Questions:** This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Let  $S$  be the set of all passwords which are six to eight characters long, where each character is either an alphabet from  $\{A, B, C, D, E\}$  or a number from  $\{1, 2, 3, 4, 5\}$  with the repetition of characters allowed. If the number of passwords in  $S$  whose at least one character is a number from  $\{1, 2, 3, 4, 5\}$  is  $\alpha \times 5^6$ , then  $\alpha$  is equal to \_\_\_\_\_.

**Answer (7073)**

**Sol.** If password is 6 character long, then

$$\text{Total number of ways having atleast one number} = 10^6 - 5^6$$

$$\text{Similarly, if 7 character long} = 10^7 - 5^7$$

$$\text{and if 8-character long} = 10^8 - 5^8$$

$$\text{Number of password} = (10^6 + 10^7 + 10^8) - (5^6 + 5^7 + 5^8)$$

$$= 5^6(2^6 + 5 \cdot 2^7 + 25 \cdot 2^8 - 1 - 5 - 25)$$

$$= 5^6(64 + 640 + 6400 - 31)$$

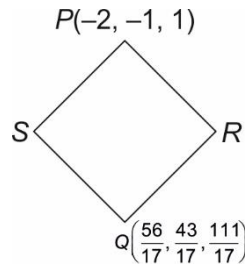
$$= 7073 \times 5^6$$

$$\therefore \alpha = 7073.$$

2. Let  $P(-2, -1, 1)$  and  $Q\left(\frac{56}{17}, \frac{43}{17}, \frac{111}{17}\right)$  be the vertices of the rhombus  $PRQS$ . If the direction ratios of the diagonal  $RS$  are  $\alpha, -1, \beta$ , where both  $\alpha$  and  $\beta$  are integers of minimum absolute values, then  $\alpha^2 + \beta^2$  is equal to \_\_\_\_\_.

**Answer (450)**

**Sol.**



$$\text{d.r.'s of } RS = \langle \alpha, -1, \beta \rangle$$

$$\text{d.r.'s of } PQ = \langle \frac{90}{17}, \frac{60}{17}, \frac{94}{17} \rangle = \langle 45, 30, 47 \rangle$$

as  $PQ$  and  $RS$  are diagonals of rhombus

$$\alpha(45) + 30(-1) + 47(\beta) = 0$$

$$\Rightarrow 45\alpha + 47\beta = 30$$

$$\text{i.e., } \alpha = \frac{30 - 47\beta}{45}$$

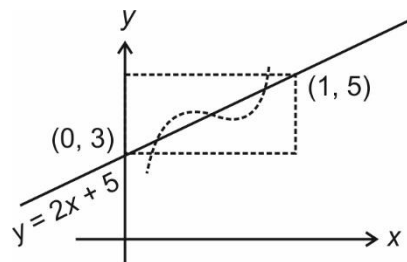
for minimum integral value  $\alpha = -15$  and  $\beta = 15$

$$\Rightarrow \alpha^2 + \beta^2 = 450.$$

3. Let  $f : [0, 1] \rightarrow \mathbf{R}$  be a twice differentiable function in  $(0, 1)$  such that  $f(0) = 3$  and  $f(1) = 5$ . If the line  $y = 2x + 3$  intersects the graph of  $f$  at only two distinct points in  $(0, 1)$  then the least number of points  $x \in (0, 1)$  at which  $f'(x) = 0$ , is \_\_\_\_\_.

**Answer (2)**

**Sol.**



If a graph cuts  $y = 2x + 5$  in  $(0, 1)$  twice then its concavity changes twice

$$\therefore f'(x) = 0 \text{ at atleast two points.}$$



4. If  $\int_0^{\sqrt{3}} \frac{15x^3}{\sqrt{1+x^2} + \sqrt{(1+x^2)^3}} dx = \alpha\sqrt{2} + \beta\sqrt{3}$ , where  $\alpha,$

$\beta$  are integers, then  $\alpha + \beta$  is equal to

**Answer (10)**

**Sol.** Put  $x = \tan \theta \Rightarrow dx = \sec^2 \theta d\theta$

$$\Rightarrow I = \int_0^{\frac{\pi}{3}} \frac{15 \tan^3 \theta \cdot \sec^2 \theta d\theta}{\sqrt{1 + \tan^2 \theta} + \sqrt{\sec^6 \theta}}$$

$$\Rightarrow I = \int_0^{\frac{\pi}{3}} \frac{15 \tan^2 \theta \sec^2 \theta d\theta}{\sec \theta \sqrt{1 + \sec \theta}}$$

$$\Rightarrow I = \int_0^{\frac{\pi}{3}} \frac{15(\sec^2 \theta - 1) \sec \theta \tan \theta d\theta}{(\sqrt{1 + \sec \theta})}$$

Now put  $1 + \sec \theta = t^2$

$$\Rightarrow \sec \theta \tan \theta d\theta = 2t dt$$

$$\Rightarrow I = \int_{\sqrt{2}}^{\sqrt{3}} \frac{15 \left( (t^2 - 1)^2 - 1 \right) 2t dt}{t}$$

$$\Rightarrow I = 30 \int_{\sqrt{2}}^{\sqrt{3}} (t^4 - 2t^2 + 1 - 1) dt$$

$$\Rightarrow I = 30 \int_{\sqrt{2}}^{\sqrt{3}} (t^4 - 2t^2) dt$$

$$\Rightarrow I = 30 \left( \frac{t^5}{5} - \frac{2t^3}{3} \right) \Big|_{\sqrt{2}}^{\sqrt{3}}$$

$$= 30 \left[ \left( \frac{9\sqrt{3}}{5} - 2\sqrt{3} \right) - \left( \frac{4\sqrt{2}}{5} - \frac{4\sqrt{2}}{3} \right) \right]$$

$$= (54\sqrt{3} - 60\sqrt{3}) - (24\sqrt{2} - 40\sqrt{2})$$

$$= 16\sqrt{2} - 6\sqrt{3}$$

$$\therefore \alpha = 16 \text{ and } \beta = -6$$

$$\alpha + \beta = 10.$$

5. Let  $A = \begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix}$  and  $B = \begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix}$ ,  $\alpha, \beta \in \mathbf{R}$ . Let  $\alpha_1$  be the value of  $\alpha$  which satisfies  $(A+B)^2 = A^2 + \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$  and  $\alpha_2$  be the value of  $\alpha$  which satisfies  $(A+B)^2 = B^2$ . Then  $|\alpha_1 - \alpha_2|$  is equal to \_\_\_\_\_.

**Answer (2)**

**Sol.**  $(A+B)^2 = A^2 + B^2 + AB + BA$

$$= A^2 + \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$\therefore B^2 + AB + BA = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \dots(1)$$

$$AB = \begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix} \begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \beta - 1 & 1 \\ \alpha + 2\beta & 2 \end{bmatrix}$$

$$BA = \begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix} = \begin{bmatrix} \beta + 2 & \alpha - \beta \\ 1 & -1 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \beta^2 + 1 & \beta \\ \beta & 1 \end{bmatrix}$$

By (1) we get

$$\begin{bmatrix} \beta^2 + 2\beta + 2 & \alpha + 1 \\ \alpha + 3\beta + 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$\therefore \alpha = 1 \text{ } \beta = 0 \Rightarrow \alpha_1 = 1$$

Similarly If  $A^2 + AB + BA = 0$  then

$$\left( A^2 = \begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix} = \begin{bmatrix} -1 & -1 - \alpha \\ 2 + 2\alpha & \alpha^2 - 2 \end{bmatrix} \right)$$

$$\begin{bmatrix} 2\beta & \alpha - \beta + 1 - 1 - \alpha \\ \alpha + 2\beta + 1 + 2 + 2\alpha & \alpha^2 - 2 + 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \beta = 0 \text{ and } \alpha = -1 \Rightarrow \alpha_2 = -1$$

$$\therefore |\alpha_1 - \alpha_2| = |2| = 2.$$

6. For  $p, q, \in \mathbf{R}$ , consider the real valued function  $f(x) = (x-p)^2 - q$ ,  $x \in \mathbf{R}$  and  $q > 0$ . Let  $a_1, a_2, a_3$  and  $a_4$  be in an arithmetic progression with mean  $p$  and positive common difference. If  $|f(a_j)| = 500$  for all  $i = 1, 2, 3, 4$ , then the absolute difference between the roots of  $f(x) = 0$  is

**Answer (50)**

**Sol.**  $\therefore a_1, a_2, a_3, a_4$  are in A.P and its mean is  $p$ .

$$\therefore a_1 = p - 3d, a_2 = p - d, a_3 = p + d \text{ and } a_4 = p + 3d$$

Where  $d > 0$

$$\therefore |f(a_i)| = 500$$

$$\Rightarrow |9d^2 - q| = 500$$

$$\text{and } |d^2 - q| = 500 \quad \dots(i)$$

either  $9d^2 - q = d^2 - q$

$\Rightarrow d = 0$  not acceptable

$$\therefore 9d^2 - q = q - d^2$$

$$\therefore 5d^2 - q = 0 \quad \dots(ii)$$

Roots of  $f(x) = 0$  are  $p + \sqrt{q}$  and  $p - \sqrt{q}$

$$\therefore \text{absolute difference between roots} = |2\sqrt{q}| = 50$$

7. For the hyperbola H:  $x^2 - y^2 = 1$  and the ellipse

$$E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b > 0, \text{ let the}$$

(1) eccentricity of E be reciprocal of the eccentricity of H, and

(2) the line  $y = \sqrt{\frac{5}{2}}x + K$  be a common tangent of

E and H.

Then  $4(a^2 + b^2)$  is equal to \_\_\_\_\_.

**Answer (03.00)**

**Sol.** The equation of tangent to hyperbola  $x^2 - y^2 = 1$  within slope  $m$  is equal to  $y = mx \pm \sqrt{m^2 - 1}$  ... (i)

And for same slope  $m$ , equation of tangent to ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ is } y = mx \pm \sqrt{a^2 m^2 + b^2} \quad \dots(ii)$$

$\therefore$  Equation (i) and (ii) are identical

$$\therefore a^2 m^2 + b^2 = m^2 - 1$$

$$\therefore m^2 = \frac{1 + b^2}{1 - a^2}$$

But equation of common tangent is  $y = \sqrt{\frac{5}{2}}x + k$

$$\therefore m = \sqrt{\frac{5}{2}} \Rightarrow \frac{5}{2} = \frac{1 + b^2}{1 - a^2}$$

$$\therefore 5a^2 + 2b^2 = 3 \quad \dots(i)$$

$$\text{eccentricity of ellipse} = \frac{1}{\sqrt{2}}$$

$$\therefore 1 - \frac{b^2}{a^2} = \frac{1}{2}$$

$$\Rightarrow a^2 = 2b^2 \quad \dots(ii)$$

From equation (i) and (ii):  $a^2 = \frac{1}{2}, b^2 = \frac{1}{4}$

$$\therefore 4(a^2 + b^2) = 3$$

8. Let  $x_1, x_2, x_3, \dots, x_{20}$  be in geometric progression

with  $x_1 = 3$  and the common ratio  $\frac{1}{2}$ . A new data is

constructed replacing each  $x_i$  by  $(x_i - i)^2$ . If  $\bar{x}$  is the mean of new data, then the greatest integer less than or equal to  $\bar{x}$  is \_\_\_\_\_.

**Answer (142)**

**Sol.**  $x_1, x_2, x_3, \dots, x_{20}$  are in G.P.

$$x_1 = 3, r = \frac{1}{2}$$

$$\bar{x} = \frac{\sum x_i^2 - 2\sum x_i i + i^2}{20}$$

$$= \frac{1}{20} \left[ 12 \left( 1 - \frac{1}{2^{40}} \right) - 6 \left( 4 - \frac{11}{2^{18}} \right) + 70 \times 41 \right]$$

$$\left\{ \begin{aligned} S &= 1 + 2 \cdot \frac{1}{2} + 3 \cdot \frac{1}{2^2} + \dots \\ \frac{S}{2} &= \frac{1}{2} + \frac{2}{2^2} + \dots \end{aligned} \right.$$

$$\frac{S}{2} = 2 \left( 1 - \frac{1}{2^{20}} \right) - \frac{20}{2^{20}} = 4 - \frac{11}{2^{18}}$$

$$\therefore [\bar{x}] = \left[ \frac{2858}{20} - \left( \frac{12}{240} - \frac{66}{2^{18}} \right) \cdot \frac{1}{20} \right]$$

$$= 142$$

$$9. \lim_{x \rightarrow 0} \left( \frac{(x + 2 \cos x)^3 + 2(x + 2 \cos x)^2 + 3 \sin(x + 2 \cos x)}{(x + 2)^3 + 2(x + 2)^2 + 3 \sin(x + 2)} \right)^{\frac{100}{x}}$$

is equal to \_\_\_\_\_.

**Answer (01)**

**Sol.** Let  $x + 2 \cos x = a$

$$x + 2 = b$$

as  $x \rightarrow 0, a \rightarrow 2$  and  $b \rightarrow 2$

$$\lim_{x \rightarrow 0} \left( \frac{a^3 + 2a^2 + 3 \sin a}{b^3 + 2b^2 + 3 \sin b} \right)^{\frac{100}{x}}$$

$$= e^{\lim_{x \rightarrow 0} \frac{100}{x} \cdot \frac{(a^3 - b^3) + 2(a^2 - b^2) + 3(\sin a - \sin b)}{b^3 + 2b^2 + 3 \sin b}}$$

$$\therefore \lim_{x \rightarrow 0} \frac{a - b}{x} = \lim_{x \rightarrow 0} \frac{2(\cos x - 1)}{x} = 0$$

$$= e^0$$

$$= 1$$

10. The sum of all real value of  $x$  for which

$$\frac{3x^2 - 9x + 17}{x^2 + 3x + 10} = \frac{5x^2 - 7x + 19}{3x^2 + 5x + 12} \text{ is equal to}$$

\_\_\_\_\_.

**Answer (06)**

**Sol.**  $\frac{3x^2 - 9x + 17}{x^2 + 3x + 10} = \frac{5x^2 - 7x + 19}{3x^2 + 5x + 12}$

$$\Rightarrow \frac{3x^2 - 9x + 17}{5x^2 - 7x + 19} = \frac{x^2 + 3x + 10}{3x^2 + 5x + 12}$$

$$\frac{-2x^2 - 2x - 2}{5x^2 - 7x + 19} = \frac{-2x^2 - 2x - 2}{3x^2 + 5x + 12}$$

$$\Rightarrow \begin{array}{l} \text{Either } x^2 + x + 1 = 0 \text{ or} \\ \text{No real roots} \end{array} \left\{ \begin{array}{l} 5x^2 - 7x + 19 \\ = 3x^2 + 5x + 12 \\ 2x^2 - 12x + 7 = 0 \\ \text{sum of roots} = 6 \end{array} \right.$$

