29/07/2022
Evening

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Answers & Solutions

Time : 3 hrs. *for* **M.M. : 300**

JEE (Main)-2022 (Online) Phase-2

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

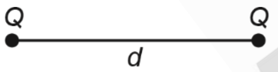
1. Two identical metallic spheres *A* and *B* when placed at certain distance in air repel each other with a force of *F*. Another identical uncharged sphere *C* is first placed in contact with *A* and then in contact with *B* and finally placed at midpoint between spheres *A* and *B*. The force experienced by sphere *C* will be

(A) 3*F*/2 (B) 3*F*/4

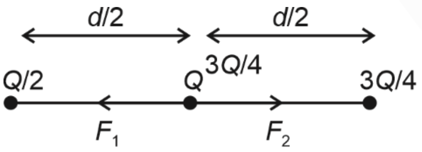
(C) *F* (D) 2*F*

Answer (B)

Sol. When two identical sphere come in contact with each other, the total charge on them is equally distribute.



$$\frac{kQ^2}{d^2} = F$$



$$F' = \frac{k9Q^2}{16\times\frac{d^2}{4}} - \frac{k3Q^2}{8\times\frac{d^2}{4}}$$

$$= \frac{9kQ^2}{4d^2} - \frac{3kQ^2}{2d^2}$$

$$= \frac{kQ^2}{d^2}\left[\frac{9}{4} - \frac{3}{2}\right]$$

$$= \frac{6}{8}F = \frac{3}{4}F$$

2. Match List I with List II.

	List I		List II
A.	Torque	I.	Nms^{-1}
B.	Stress	II.	Jkg^{-1}
C.	Latent Heat	III.	Nm
D.	Power	IV	Nm^{-2}

Choose the correct answer from the options given below:

(A) A-III, B-II, C-I, D-IV (B) A-III, B-IV, C-II, D-I

(C) A-IV, B-I, C-III, D-II (D) A-II, B-III, C-I, D-IV

Answer (B)

Sol. Torque → Nm

Stress → N/m^2

Latent heat → J/kg

Power → N m/s

A-III, B-IV, C-II, D-I

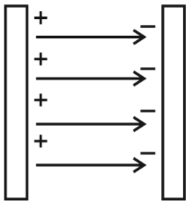
3. Two identical thin metal plates has charge q_1 and q_2 respectively such that $q_1 > q_2$. The plates were brought close to each other to form a parallel plate capacitor of capacitance *C*. The potential difference between them is

(A) $\frac{(q_1+q_2)}{C}$ (B) $\frac{(q_1-q_2)}{C}$

(C) $\frac{(q_1-q_2)}{2C}$ (D) $\frac{2(q_1-q_2)}{C}$

Answer (C)

Sol. $\frac{q_1-q_2}{2}$ $-\frac{(q_1-q_2)}{2}$



$$E = \frac{q_1 - q_2}{2\varepsilon_0 A}$$

$$v = \frac{(q_1 - q_2)d}{2\varepsilon_0 A}$$

$$= \frac{q_1 - q_2}{2C}$$

4. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A: Alloys such as constantan and manganin are used in making standard resistance coils.

Reason R: Constantan and manganin have very small value of temperature coefficient of resistance.

In the light of the above statements, choose the correct answer from the options given below.

(A) Both **A** and **R** are true and **R** is the correct explanation of **A**.

(B) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**.

(C) **A** is true but **R** is false.

(D) **A** is false but **R** is true.

Answer (A)

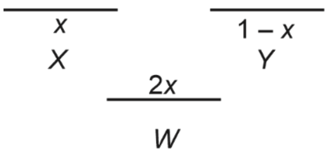
Sol. Since they have low temperature coefficient of resistance, their resistance remains almost constant.

5. A 1 m long wire is broken into two unequal parts *X* and *Y*. The *X* part of the wire is stretched into another wire *W*. Length of *W* is twice the length of *X* and the resistance of *W* is twice that of *Y*. Find the ratio of length of *X* and *Y*.

(A) 1:4	(B) 1:2
(C) 4:1	(D) 2:1

Answer (B)

Sol.



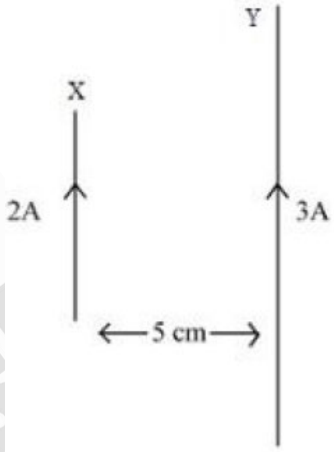
$R_w = 2R_y$

$$\rho\frac{2x}{\frac{A}{2}} = \frac{2\rho(1-x)}{A}$$

$4x = 2(1 - x)$

$$\frac{x}{1-x} = \frac{1}{2}$$

6. A wire *X* of length 50 cm carrying a current of 2 A is placed parallel to a long wire *Y* of length 5 m. The wire *Y* carries a current of 3 A. The distance between two wires is 5 cm and currents flow in the same direction. The force acting on the wire *Y* is



(A) 1.2×10^{-5} N directed towards wire *X*

(B) 1.2×10^{-4} N directed away from wire *X*

(C) 1.2×10^{-4} N directed towards wire *X*

(D) 2.4×10^{-5} N directed towards wire *X*

Answer (A)

Sol. $F_{XY} = F_{YX} = F$

$$F = \frac{\mu_0 I_2}{2\pi r} I_1(l)$$

$$= \frac{4\pi \times 10^{-7} \times 3 \times 2 \times \left[50 \times 10^{-2}\right]}{2\pi\left(5 \times 10^{-2}\right)}$$

$= 1.2 \times 10^{-5}$ N

7. A juggler throws balls vertically upwards with same initial velocity in air. When the first ball reaches its highest position, he throws the next ball. Assuming the juggler throws *n* balls per second, the maximum height the balls can reach is

(A) $g/2n$	(B) g/n
(C) $2gn$	(D) $g/2n^2$

Answer (D)

Sol. $t = \frac{u}{g} = \frac{1}{n}$

$$u = \frac{g}{n}$$

$$H_{\max} = \frac{u^2}{2g} = \frac{g}{2n^2}$$

8. A circuit element X when connected to an a.c. supply of peak voltage 100 V gives a peak current of 5 A which is in phase with the voltage. A second element Y when connected to the same a.c. supply also gives the same value of peak current which lags behind the voltage by $\frac{\pi}{2}$. If X and Y are connected in series to the same supply, what will be the rms value of the current in ampere?

(A) $\frac{10}{\sqrt{2}}$ (B) $\frac{5}{\sqrt{2}}$

(C) $5\sqrt{2}$ (D) $\frac{5}{2}$

Answer (D)

Sol. $R = \frac{100}{5} = 20\ \Omega$

$$X_L = \frac{100}{5} = 20\ \Omega$$

When in series

$$z = \sqrt{20^2 + 20^2} = 20\sqrt{2}\ \Omega$$

$$i = \frac{100}{z} = \frac{100}{20\sqrt{2}} = \frac{5}{\sqrt{2}}$$

$$i_{\text{rms}} = \frac{1}{\sqrt{2}} i$$

$$= \frac{5}{2}$$

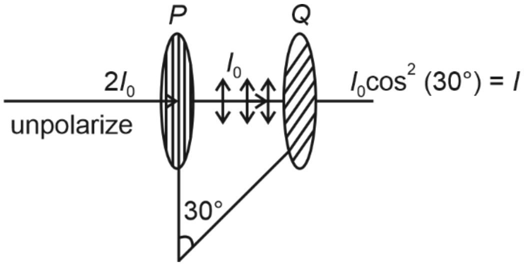
9. An unpolarised light beam of intensity $2I_0$ is passed through a polaroid P and then through another polaroid Q which is oriented in such a way that its passing axis makes an angle of 30° relative to that of P. The intensity of the emergent light is

(A) $\frac{I_0}{4}$ (B) $\frac{I_0}{2}$

(C) $\frac{3I_0}{4}$ (D) $\frac{3I_0}{2}$

Answer (C)

Sol.



$$I = I_0 \times \frac{3}{4}$$

10. An α particle and a proton are accelerated from rest through the same potential difference. The ratio of linear momenta acquired by above two particles will be:

(A) $\sqrt{2} : 1$ (B) $2\sqrt{2} : 1$

(C) $4\sqrt{2} : 1$ (D) $8 : 1$

Answer (B)

Sol. $\frac{p_\alpha}{p_p} = \frac{\sqrt{2(4m)(2eV)}}{\sqrt{2(m)(eV)}}$

$$= \frac{\sqrt{16}}{\sqrt{2}}$$

$$= \frac{4}{\sqrt{2}} = \frac{2\sqrt{2}}{1}$$

11. Read the following statements:

(A) Volume of the nucleus is directly proportional to the mass number.

(B) Volume of the nucleus is independent of mass number.

(C) Density of the nucleus is directly proportional to the mass number.

(D) Density of the nucleus is directly proportional to the cube root of the mass number.

(E) Density of the nucleus is independent of the mass number.

Choose the correct option from the following options

(A) (A) and (D) only (B) (A) and (E) only

(C) (B) and (E) only (D) (A) and (C) only

Answer (B)

Sol. $R = R_0 A^{\frac{1}{3}}$

$$\Rightarrow \quad V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A$$

$$\Rightarrow \quad \rho = \frac{M}{V} \propto \frac{A}{A} \propto A^0$$

12. An object of mass 1 kg is taken to a height from the surface of earth which is equal to three times the radius of earth. The gain in potential energy of the object will be

[If, g = 10 ms^{-2} and radius of earth = 6400 km]

(A) 48 MJ (B) 24 MJ

(C) 36 MJ (D) 12 MJ

Answer (A)

Sol. $\Delta U = U_f - U_i$

$$= -\frac{GMm}{4R} + \frac{GMm}{R}$$

$$= \frac{3GMm}{4R} = \frac{3}{4}mgR$$

= 48 MJ

13. A ball is released from a height h. If t_1 and t_2 be the time required to complete first half and second half of the distance respectively. Then, choose the correct relation between t_1 and t_2.

(A) $t_1 = \left(\sqrt{2}\right)t_2$

(B) $t_1 = \left(\sqrt{2}-1\right)t_2$

(C) $t_2 = \left(\sqrt{2}+1\right)t_1$

(D) $t_2 = \left(\sqrt{2}-1\right)t_1$

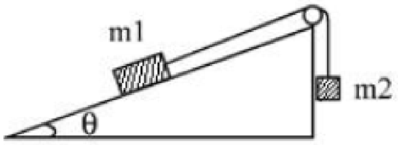
Answer (D)

Sol. $t_1 = \sqrt{\dfrac{2 \cdot \frac{H}{2}}{g}} = \sqrt{\dfrac{H}{g}}$

And $t_2 = \sqrt{\dfrac{2H}{g}} - t_1$

$$\Rightarrow \quad t_2 = \sqrt{\frac{2H}{g}} - \sqrt{\frac{H}{g}} = \sqrt{\frac{H}{g}}\left\{\sqrt{2}-1\right\}$$

14. Two bodies of masses m_1 = 5 kg and m_2 = 3 kg are connected by a light string going over a smooth light pulley on a smooth inclined plane as shown in the figure. The system is at rest. The force exerted by the inclined plane on the body of mass m_1 will be [Take g = 10 ms^{-2}]



(A) 30 N (B) 40 N

(C) 50 N (D) 60 N

Answer (B)

Sol. $m_2g = m_1g\sin\theta$ ...(i)

$N = m_1g\cos\theta$ ...(ii)

$$\Rightarrow \quad \frac{N}{m_2g} = \cot\theta$$

$$\Rightarrow \quad N = 3 \times 10 \times \cot\theta = 3 \times 10 \times \frac{4}{3} \quad \left(\because \ \sin\theta = \frac{3}{5}\right)$$

$\Rightarrow$ N = 40 Newtons

15. If momentum of a body is increased by 20%, then its kinetic energy increases by

(A) 36% (B) 40%

(C) 44% (D) 48%

Answer (C)

Sol. $K = \dfrac{p^2}{2m}$

$$K' = \frac{(1.2p)^2}{2m}$$

$$\Rightarrow \quad \frac{K'-K}{K} = (1.2)^2 - 1 = 0.44$$

$\Rightarrow$ 44% increase

16. The torque of a force $5\hat{i}+3\hat{j}-7\hat{k}$ about the origin is τ. If the force acts on a particle whose position vector is $2\hat{i}+2\hat{j}+\hat{k}$, then the value of τ will be

(A) $11\hat{i}+19\hat{j}-4\hat{k}$ (B) $-11\hat{i}+9\hat{j}-16\hat{k}$

(C) $-17\hat{i}+19\hat{j}-4\hat{k}$ (D) $17\hat{i}+9\hat{j}+16\hat{k}$

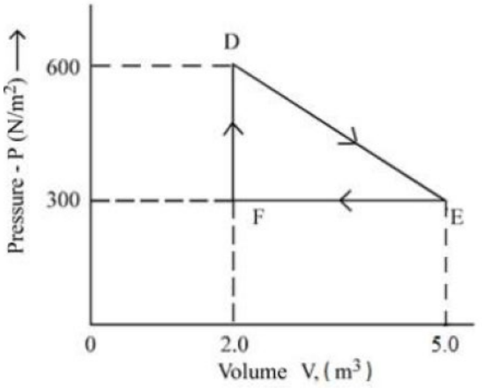
Answer (C)

Sol. $\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & 1 \\ 5 & 3 & -7 \end{vmatrix}$

$$= \hat{i}(-14-3) + \hat{j}(5+14) + \hat{k}(6-10)$$

$$= -17\hat{i} + 19\hat{j} - 4\hat{k}$$

17. A thermodynamic system is taken from an original state *D* to an intermediate state *E* by the linear process shown in the figure. Its volume is then reduced to the original volume from *E* to *F* by an isobaric process. The total work done by the gas from *D* to *E* to *F* will be



(A) –450 J
(B) 450 J
(C) 900 J
(D) 1350 J

Answer (B)

Sol. $W = \frac{1}{2} \times (5-2) \times (600-300)$ J

$$= \frac{1}{2} \times 3 \times 300$$

= 450 J

18. The vertical component of the earth's magnetic field is 6 × 10^{-5} T at any place where the angle of dip is 37°. The earth's resultant magnetic field at that place will be (Given $\tan 37° = \frac{3}{4}$)

(A) 8 × 10^{-5} T
(B) 6 × 10^{-5} T
(C) 5 × 10^{-4} T
(D) 1 × 10^{-4} T

Answer (D)

Sol. $B_v = B_0 \sin\theta$

$$B_0 = \frac{B_v}{\sin\theta} = \frac{6 \times 10^{-5}}{\sin 37°}$$

$$= \frac{6 \times 10^{-5}}{3} \times 5$$

= 1 × 10^{-4} T

19. The root mean square speed of smoke particles of mass 5 × 10^{-17} kg in their Brownian motion in air at NTP is approximately. [Given *k* = 1.38 × 10^{-23} JK^{-1}]

(A) 60 mm s^{-1}
(B) 12 mm s^{-1}
(C) 15 mm s^{-1}
(D) 36 mm s^{-1}

Answer (C)

Sol. At NTP, *T* = 298 K

$$\Rightarrow v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$= \sqrt{\frac{3kN_A \times 298}{5 \times 10^{-17} \times N_A}}$$

$\simeq$ 15 mm/s

20. Light enters from air into a given medium at an angle of 45° with interface of the air-medium surface. After refraction, the light ray is deviated through an angle of 15° from its original direction. The refractive index of the medium is

(A) 1.732
(B) 1.333
(C) 1.414
(D) 2.732

Answer (C)

Sol. 1 × sin45° = μ × sin30°

$$\Rightarrow \mu = \frac{1}{\sqrt{2}} \times \frac{2}{1}$$

$$\mu = \sqrt{2}$$

= 1.414

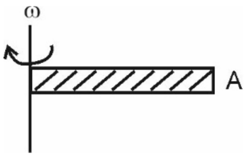
SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. A tube of length 50 cm is filled completely with an incompressible liquid of mass 250 g and closed at both ends. The tube is then rotated in horizontal plane about one of its ends with a uniform angular velocity $x\sqrt{F}$ rad s^{-1}. If F be the force exerted by the liquid at the other end then the value of x will be______

Answer (4)

Sol.



Applying $F_c = \dfrac{m\omega^2 l}{2}$

$$\frac{m\omega^2 l}{2} = F$$

$$\omega = \sqrt{\frac{2F}{\frac{1}{2}\times\frac{1}{4}}} = \sqrt{16\,F} = 4\sqrt{F}$$

2. Nearly 10% of the power of a 110 W light bulb is converted to visible radiation. The change in average intensities of visible radiation, at a distance of 1 m from the bulb to a distance of 5 m is $a \times 10^{-2}$ W.m^2. The value of 'a' will be _____

Answer (84)

Sol. $P_{\text{radiation}} = 0.1 \times 110 = 11$ W

$$\Delta I_{\text{radiation}_1} = I_{\text{radiation}_1} - I_{\text{radiation}_2}$$

$$= 11\left(\frac{1}{4\pi} - \frac{1}{4\pi\times 25}\right) = \frac{11\times 24}{4\pi\times 25}$$

$= 84 \times 10^{-2}$ W/m^2

3. A metal wire of length 0.5 m and cross-sectional area 10^{-4} m^2 has breaking stress 5×10^8 Nm^{-2}. A block of 10 kg is attached at one end of the string and is rotating in a horizontal circle. The maximum linear velocity of block will be _____ ms^{-1}.

Answer (50)

Sol. $A = 10^{-4}$ m^2

$$l = \frac{1}{2}\,\text{m}$$

$$\sigma = 5 \times 10^8$$

$$\frac{mv^2}{lA} = 5\times 10^8$$

$$v = \sqrt{\frac{5\times 10^8 \times \frac{1}{2}\times 10^{-4}}{10}} = 5 \times 10 = 50\ \text{m/s}$$

4. The velocity of a small ball of mass 0.3 g and density 8 g/cc when dropped in a container filled with glycerine becomes constant after some time. If the density of glycerine is 1.3 g/cc, then the value of viscous force acting on the ball will be $x \times 10^{-4}$ N. The value of x is _______. [use $g = 10$ m/s^2]

Answer (25)

Sol. $F_v = 6\pi\eta r v_T$

$F_v + F_B = mg$

$\Rightarrow F_v = mg - F_B$

$$= 10\times(8-1.3)\times\frac{0.3}{8}\times 10^{-3}$$

$= 2.5125 \times 10^{-3}$ N $\simeq 25\times 10^{-4}$ N

5. A modulating signal 2sin $(6.28 \times 10^6)\,t$ is added to the carrier signal 4sin$(12.56 \times 10^9)\,t$ for amplitude modulation. The combined signal is passed through a non-linear square law device. The output is then passed through a band pass filter. The bandwidth of the output signal of band pass filter will be ______MHz.

Answer (2)

Sol. $W_C = 12.56 \times 10^9$

$W_m = 6.25 \times 10^6$

After amplitude modulation

Bandwidth frequency

$$= \frac{2W_m}{2\pi} = \frac{2\times 6.28}{2\pi}\times 10^6 = 2\,\text{MHz}$$

6. The speed of a transverse wave passing through a string of length 50 cm and mass 10 g is 60 ms^{-1}. The area of cross-section of the wire is 2.0 mm^2 and its Young's modulus is 1.2×10^{11} Nm^{-2}. The extension of the wire over its natural length due to its tension will be $x \times 10^{-5}$ m. The value of x is _____.

Answer (15)

$$v = \sqrt{\frac{T}{\mu}}$$

So $T = 60^2 \times \frac{10 \times 10^{-3}}{0.5} = 72\text{ N}$

$$\Delta\ell = \frac{T\ell}{YA} = \frac{72 \times 0.5}{1.2 \times 10^{-11} \times 2 \times 10^{-6}} = 15 \times 10^{-5}\text{ m}$$

7. The metallic bob of simple pendulum has the relative density 5. The time period of this pendulum is 10 s. If the metallic bob is immersed in water, then the new time period becomes $5\sqrt{x}$ s. The value of x will be _____.

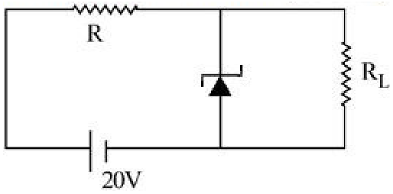
Answer (5)

$$T = 2\pi\sqrt{\frac{\ell}{g}} = 10$$

$$T' = 2\pi\sqrt{\frac{\ell}{g\left(1 - \frac{1}{\rho}\right)}}$$

$$= 2\pi\sqrt{\frac{\ell}{g} \times \frac{5}{4}} = 10\sqrt{\frac{5}{4}} = 5\sqrt{5}$$

8. A 8 V Zener diode along with a series resistance R is connected across a 20 V supply (as shown in the figure). If the maximum Zener current is 25 mA, then the minimum value of R will be ____ Ω.



Answer (480)

R will be minimum when R_L is infinitely large, so

$$R_{\text{Zener}} = \frac{8}{25 \times 10^{-3}} = 320\ \Omega$$

So $\frac{R}{R_{\text{Zener}}} = \frac{12}{8}$

$$R = \frac{12}{8} \times 320 = 480\ \Omega$$

9. Two radioactive materials A and B have decay constants 25λ and 16λ respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of B to that of A will be 'e' after a time $\frac{1}{a\lambda}$. The value of a is _____.

Answer (9)

$$N_A = N_0 e^{-25\lambda t}$$

$$N_B = N_0 e^{-16\lambda t}$$

$$\frac{N_B}{N_A} = e = e^{9\lambda t}$$

$$t = \frac{1}{9\lambda}$$

10. A capacitor of capacitance 500 μF is charged completely using a dc supply of 100 V. It is now connected to an inductor of inductance 50 mH to form an LC circuit. The maximum current in the LC circuit will be ______A.

Answer (10)

$q_0 = CV$

$= 500 \times 100 \times 10^{-6}$ C

$= 5 \times 10^{-2}$ C

For i_{max},

$$\frac{1}{2}Li_m^2 = \frac{1}{2}\frac{q_0^2}{C}$$

$$50 \times 10^{-3} \times i_m^2 = \frac{(5 \times 10^{-2})^2}{500 \times 10^{-6}}$$

$$\Rightarrow \quad i_m = \frac{5 \times 10^{-2}}{5 \times 10^{-3}} = 10\text{ A}$$

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Consider the reaction

$$4HNO_3(l) + 3KCl(s) \rightarrow Cl_2(g) + NOCl(g) + 2H_2O(g) + 3KNO_3(s)$$

The amount of HNO_3 required to produce 110.0 g of KNO_3 is

(Given : Atomic masses of H, O, N and K are 1, 16, 14 and 39 respectively.)

(A) 32.2 g (B) 69.4 g

(C) 91.5 g (D) 162.5 g

Answer (C)

$$4HNO_3(l) + 3KCl(s) \longrightarrow Cl_2(g) + NOCl(g) + 2H_2O(g) + 3KNO_3(s)$$

$\because$ 110 g of $KNO_3 \Rightarrow$ moles of $KNO_3 = \frac{110}{101}$

$= 1.089$ mol

As, 4 mole of HNO_3 produces 3 mol of kNO_3. Hence, the moles of HNO_3 required to produce 1.089 moles of $KNO_3 = \frac{4}{3} \times 1.089 = 1.452$ mol

Hence, mass of HNO_3 required is 1.452×63

$= 91.5$ g (approx.)

2. Given below are the quantum numbers for 4 electrons.

A. $n = 3, l = 2, m_l = 1, m_s = +1/2$

B. $n = 4, l = 1, m_l = 0, m_s = +1/2$

C. $n = 4, l = 2, m_l = -2, m_s = -1/2$

D. $n = 3, l = 1, m_l = -1, m_s = +1/2$

The correct order of increasing energy is

(A) D < B < A < C (B) D < A < B < C

(C) B < D < A < C (D) B < D < C < A

Answer (B)

Energy of the sub-shell is given by, (n + l) rule.

		(n + l)
For,	A	5
	B	5
	C	6
	D	4

Hence, the correct order of increasing energy is D < A < B < C

3. $C(s) + O_2(g) \rightarrow CO_2(g) + 400$ kJ

$C(s) + \frac{1}{2}O_2(g) \rightarrow CO(g) + 100$ kJ

When coal of purity 60% is allowed to burn in presence of insufficient oxygen, 60% of carbon is converted into 'CO' and the remaining is converted into 'CO_2'. The heat generated when 0.6 kg of coal is burnt is _______.

(A) 1600 kJ (B) 3200 kJ

(C) 4400 kJ (D) 6600 kJ

Answer (D)

Weight of coal = 0.6 kg = 600 gm

$\therefore$ 60% of it is carbon

So weight of carbon $= 600 \times \frac{60}{100} = 360$ g

$\therefore$ moles of carbon $= \frac{360}{12} = 30$ moles

$$\underset{12\text{ moles}}{C} + O_2 \longrightarrow CO_2$$

$$\underset{\substack{18\text{ moles}\\(60\%\text{ of total Carbon})}}{C} + \frac{1}{2}O_2 \longrightarrow CO$$

$\therefore$ Heat generated = 12 × 400 + 18 × 100 = 6600 kJ

4. 200 mL of 0.01 M HCl is mixed with 400 mL of 0.01 M H_2SO_4. The pH of the mixture is ___.

[Given log 2 = 0.30, log 3 = 0. 48, log 5 = 0.70, log7 = 0.84, log 11 = 1.04.]

(A) 1.14 (B) 1.78

(C) 2.34 (D) 3.02

Answer (B)

Molarity of resultant solution is given by

$$\begin{array}{ccccc} m_1v_1n_1 & + & m_2v_2n_2 & = & mv \\ \text{200 mL of 0.01 m HCl} & + & \text{400 mL of 0.01m } H_2SO_4 & & \\ 200\times0.01\times1 & + & 400\times0.01\times2 & = & m \times v \end{array}$$

$$\text{Molarity} = \frac{10}{600} \text{ of equivalents.}$$

$$\left[H^+\right] = \frac{10}{600}$$

$$pH = -\log[H^+]$$

$$pH = -\log\left[\frac{10}{600}\right] = 1.778$$

5. Given below are the critical temperatures of some of the gases :

Gas	**Critical temperature (K)**
He	5.2
CH_4	190.0
CO_2	304.2
NH_3	405.5

The gas showing least adsorption on a definite amount of charcoal is

(A) He (B) CH_4

(C) CO_2 (D) NH_3

Answer (A)

Extent of adsorption $\propto T_C$ (critical temperature)

$\because$ Lower the T_C, Lower will be the adsorption

Hence, Helium shows least adsorption on a definite amount of charcoal.

6. In liquation process used for tin (Sn), the metal

(A) is reacted with acid

(B) is dissolved in water

(C) is brought to molten form which is made to flow on a slope

(D) is fused with NaOH

Answer (C)

Sol In liquation method, a low melting metal like tin can be made to flow on a sloping surface.

7. Given below are two statements.

Statement-I: Stannane is an example of a molecular hydride.

Statement-II: Stannane is a planar molecule

In the light of the above statement, choose the ***most appropriate*** answer from the options given below.

(A) Both **Statement-I** and **Statement-II** are true

(B) Both **Statement-I** and **Statement-II** are false

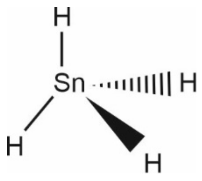
(C) **Statement-I** is true but **Statement-II** is false

(D) **Statement-I** is false but **Statement-II** is true

Answer (C)

Sol Stannane or tin hydride is an inorganic compound with formula SnH_4

Structure of SnH_4 is



$\because$ It is a non-planar molecule.

8. Portland cement contains 'X' to enhance the setting time. What is 'X'?

(A) $CaSO_4 \cdot \frac{1}{2}H_2O$

(B) $CaSO_4 \cdot 2H_2O$

(C) $CaSO_4$

(D) $CaCO_3$

Answer (B)

Sol Setting of cement: When mixed with water, the setting of cement takes place to give a hard mass.

This is due to the hydration of the molecule of the constituents and their rearrangement. The purpose of adding gypsum ($CaSO_4.2H_2O$) is only to slow down the process of setting of the cement so that it gets sufficiently hardened.

9. When borax is heated with CoO on a platinum loop, blue coloured bead formed is largely due to

(A) B_2O_3
(B) $Co(BO_2)_2$
(C) CoB_4O_7
(D) $Co[B_4O_5(OH)_4]$

Answer (B)

Sol $Na_2B_4O_7 \xrightarrow{\Delta} 2NaBO_2 + B_2O_3$

$B_2O_3 + CoO \rightarrow Co(BO_2)_2$

Cobalt metaborate

(blue coloured)

10. Which of the following 3d-metal ion will give the lowest enthalpy of hydration ($\Delta_{hyd}H$) when dissolved in water?

(A) Cr^{2+}
(B) Mn^{2+}
(C) Fe^{2+}
(D) Co^{2+}

Answer (B)

	Δ_{hyd} H (M^{+2})
Cr	–1925
Mn	–1862
Fe	–1560
Co	– 1640

Mn^{+2} has lowest $\Delta_{hyd}H$

11. Octahedral complexes of copper(II) undergo structural distortion (Jahn-Teller). Which one of the given copper(II) complexes will show the maximum structural distortion ?

(en – ethylenediamine; $H_2N\text{-}CH_2\text{-}CH_2\text{-}NH_2$)

(A) $[Cu(H_2O)_6]SO_4$
(B) $[Cu(en)(H_2O)_4]SO_4$
(C) cis-$[Cu(en)_2Cl_2]$
(D) trans-$[Cu(en)_2Cl_2]$

Answer (D)

Sol. John teller distortion: Any non-linear compound remove its degeneracy to attain the stability.

Extent of John teller distortion depends upon metal ion as well as nature of ligand.

Stronger the ligand, more will be the John Teller distortion and more will be the stability.

Hence Trans $[Cu(en)_2Cl_2]$ will exhibit maximum John Teller distortion.

12. Dinitrogen is a robust compound, but reacts at high altitudes to form oxides. The oxide of nitrogen that can damage plant leaves and retard photosynthesis is

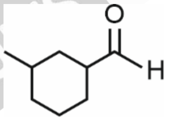
(A) NO
(B) NO_3^-
(C) NO_2
(D) NO_2^-

Answer (C)

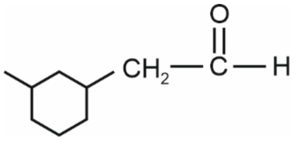
Sol. Higher concentration of NO_2 damages the leaves of plant and retards photosynthesis.

13. Correct structure of γ-methylcyclohexane carbaldehyde is

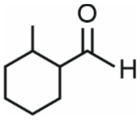
(A)



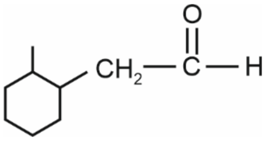
(B)



(C)

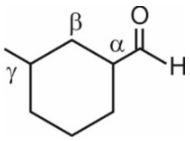


(D)



Answer (A)

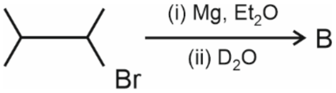
Sol.



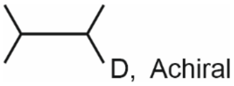
γ-Methyl cyclohexane carbaldehyde

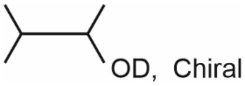
14. Compound ‘*A*’ undergoes following sequence of reactions to give compound ‘*B*’. The correct structure and chirality of compound ‘*B*’ is

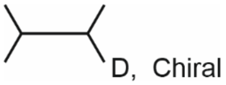
[where Et is $–C_2H_5$]

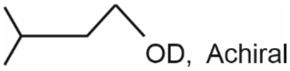


Compound ‘A’

(A) 

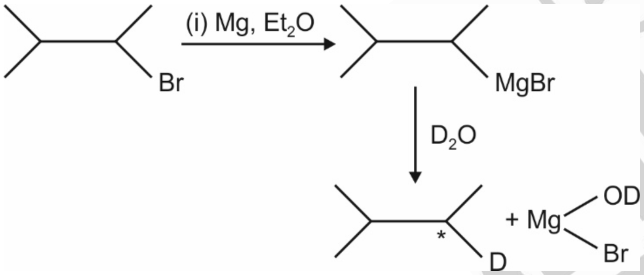
(B) 

(C) 

(D) 

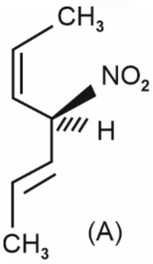
Answer (C)

Sol.



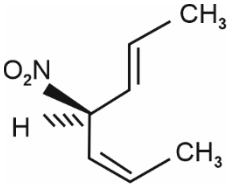
15. Given below are two statements.

Statement I: The compound



is optically active.

Statement II:



is mirror image of above compound A.

In the light of the above statement, choose the ***most appropriate*** answer from the options given below.

(A) Both **Statement I** and **Statement II** are correct.

(B) Both **Statement I** and **Statement II** are incorrect.

(C) **Statement I** is correct but **Statement II** is incorrect.

(D) **Statement I** is incorrect but **Statement II** is correct.

Answer (C)

Sol. Compound (A) in Statement-**I** and compound in Statement-**II** is not the mirror image of (**I**).

16. When ethanol is heated with conc. H_2SO_4, a gas is produced. The compound formed, when this gas is treated with cold dilute aqueous solution of Baeyer’s reagent, is

(A) Formaldehyde

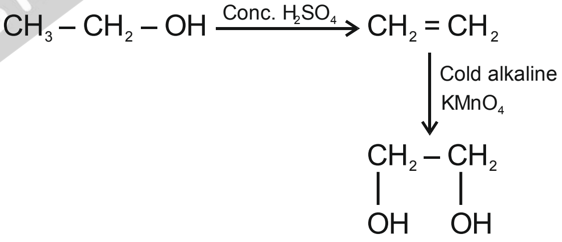
(B) Formic acid

(C) Glycol

(D) Ethanoic acid

Answer (C)

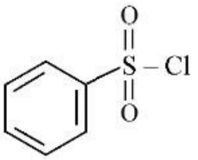
Sol.

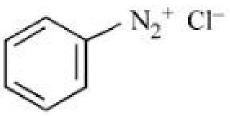


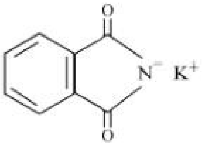
$$CH_3-CH_2-OH \xrightarrow{\text{Conc. } H_2SO_4} CH_2=CH_2 \xrightarrow{\text{Cold alkaline } KMnO_4} \underset{OH}{CH_2}-\underset{OH}{CH_2}$$

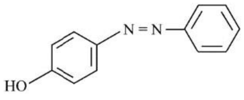
ethylene glycol

17. The Hinsberg reagent is

(A) 

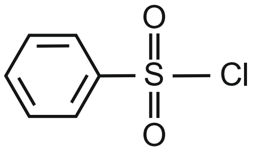
(B) 

(C) 

(D) 

Answer (A)

Sol. Hinsberg reagent is :



18. Which of the following is not a natural polymer?

(A) Protein
(B) Starch
(C) Rubber
(D) Rayon

Answer (D)

Sol. Rayon is not natural polymer. It is semi-synthetic, rest all are natural polymers

19. Given below are two statements. One is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : Amylose is insoluble in water.

Reason R : Amylose is a long linear molecule with more than 200 glucose units. In the light of the above statements, choose the correct answer from the options given below.

(A) Both A and R are correct and R is the correct explanation of A
(B) Both A and R are correct but R is NOT the correct explanation of A
(C) A is correct but R is not correct
(D) A is not correct but R is correct

Answer (D)

Sol. Amylose is a linear polymer formed by combination of α-D glucose through 1, 4- glycosidic linkage.

It is water soluble

So, assertion is incorrect

20. A compound 'X' is a weak acid and it exhibits colour change at pH close to the equivalence point during neutralization of NaOH with CH_3COOH. Compound 'X' exists in ionized from in basic medium. The compound 'X' is

(A) Methyl orange
(B) Methyl red
(C) Phenolphthalein
(D) Eriochrome Black T

Answer (C)

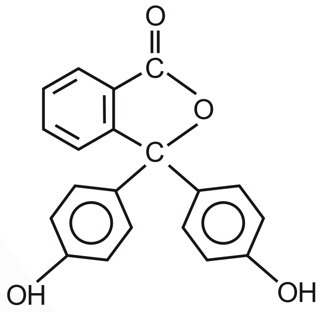
Sol.

$$HPh \rightleftharpoons H^{\oplus} + Ph^{\ominus}$$

(Phenolphthalein)
(unionised form)
(colourless)

(Ionised form)
(pink)

In basic medium, $[H^{\oplus}]$ decreases & therefore more of $(Ph^{\ominus})$ is produced



(Phenolphthalein)

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. 'x' g of molecular oxygen (O_2) is mixed with 200 g of neon (Ne). The total pressure of the non-reactive mixture of O_2 and Ne in the cylinder is 25 bar. The partial pressure of Ne is 20 bar at the same temperature and volume. The value of 'x' is ___.

[Given : Molar mass of O_2 = 32 g mol^{-1}.

Molar mass of Ne = 20 g mol^{-1}]

Answer (80)

Sol. $P_{O_2} = 25 - 20 = 5$ bar

$$P_{O_2} = x_{O_2} \times P_{Total}$$

$$\frac{5}{25} = \frac{n_{O_2}}{n_{O_2} + n_{Ne}}$$

$$\frac{1}{5}=\frac{x/32}{\frac{x}{32}+\frac{200}{20}} \Rightarrow \frac{x}{32}+10=\frac{5x}{32}$$

$$\Rightarrow \frac{x}{8}=10$$

x = 80 gm

2. Consider, PF_5, BrF_5, PCl_3, SF_6, $[ICl_4]^-$, ClF_3 and IF_5.

Amongst the above molecule(s)/ion(s), the number of molecule(s)/ion(s) having sp^3d^2 hybridisation is _____.

Answer (4)

Sol. Hybridisation of Central atom

$PF_5 \longrightarrow sp^3d$

$BrF_5 \longrightarrow sp^3d^2$

$PCl_3 \longrightarrow sp^3$

$SF_6 \longrightarrow sp^3d^2$

$ICl_4^{\ominus} \longrightarrow sp^3d^2$

$ClF_3 \longrightarrow sp^3d$

$IF_5 \longrightarrow sp^3d^2$

3. 1.80 g of solute A was dissolved in 62.5 cm^3 of ethanol and freezing point of the solution was found to be 155.1 K. The molar mass of solute A is _____ g mol^{-1}.

[Given : Freezing point of ethanol is 156.0 K.

Density of ethanol is 0.80 g cm^{-3}.

Freezing point depression constant of ethanol is 2.00 K kg mol^{-1}]

Answer (80)

Sol. $\Delta T_f = k_f\, m$

$$0.9 = \frac{2\times1.8\times1000}{62.5\times0.8\times M}$$

$$M = \frac{2\times1800}{62.5\times0.8\times0.9}$$

= 80 g/mol

4. For a cell, Cu(s) | Cu^{2+} (0.001M) || Ag^+ (0.01M) | Ag(s) the cell potential is found to be 0.43 V at 298 K. The magnitude of standard electrode potential for Cu^{2+}/Cu is _______ × 10^{-2} V.

$$\left[\text{Given}: E^{\ominus}_{Ag^+/Ag}=0.80\text{ V and }\frac{2.303RT}{F}=0.06\text{ V}\right]$$

Answer (34)

Sol. $$E = E^{\circ}-\frac{0.06}{2}\log\frac{\left[Cu^{+2}\right]}{\left[Ag^{\oplus}\right]^2}$$

$$= E^{\circ}-\frac{0.06}{2}\log\frac{0.001}{(0.01)^2}$$

0.43 = E° – 0.03

E° = 0.46 V

$$E^{o}_{Ag^{\oplus}/Ag}-E^{o}_{Cu^{+2}/Cu}=0.46$$

$$\therefore\ E^{o}_{Cu^{+2}/Cu}=0.8-0.46$$

= 0.34 V

= 34 × 10^{-2} V

5. Assuming 1μg of trace radioactive element X with a half life of 30 years is absorbed by a growing tree. The amount of X remaining in the tree after 100 years is _______ × 10^{-1} μg.

[Given : ln 10 = 2.303; log 2 = 0.30]

Answer (1)

Sol. $kt = \ln\frac{1}{1-X}$

$$\frac{0.693}{30}(100)=\ln\frac{1}{1-X}$$

$$2.303 = 2.303\log\frac{1}{1-X} \Rightarrow \frac{1}{1-X}=10$$

$\Rightarrow$ 1 = 10 – 10X

$$\Rightarrow X = \frac{9}{10}$$

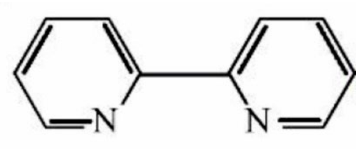
= 0.9 μg

Amount of X remaining = 1 – X

= 1 – 0.9 = 0.1 μg

= 1 × 10^{-1} μg

6. Sum of oxidation state (magnitude) and coordination number of cobalt in Na[Co(bpy)Cl$_4$] is____.

(Given : bpy = )

Answer (9)

Sol. Na [Co(bpy)Cl$_4$]

Oxidation state of cobalt = + 3

Coordination number of cobalt = 6

[As bpy is bidentate]

So, sum = 9

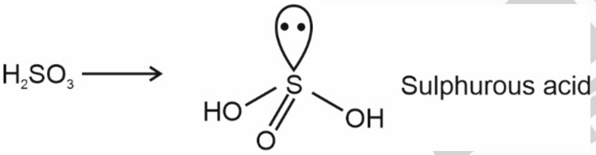
7. Consider the following sulphur based oxoacids.

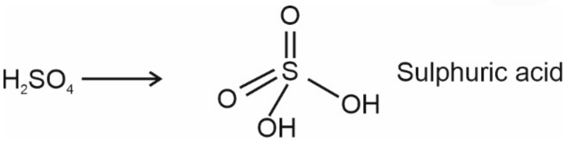
H_2SO_3, H_2SO_4, $H_2S_2O_8$ and $H_2S_2O_7$.

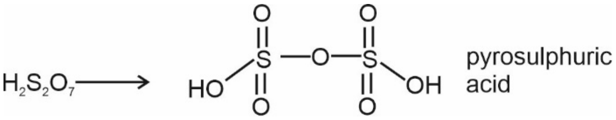
Amongst these oxoacids, the number of those with peroxo (O–O) bonds is_____.

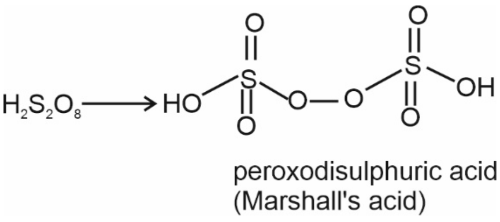
Answer (1)

Sol.









8. A 1.84 mg sample of polyhydric alcoholic compound 'X' of molar mass 92.0 g/mol gave 1.344 mL of H_2 gas at STP. The number of alcoholic hydrogens present in compound 'X' is____.

Answer (6)

Sol. Moles of H_2 produced at STP

$$= \frac{1.344 \times 10^{-3}}{22.4}$$

$= 6 \times 10^{-5}$ mole

∴ Moles of hydrogen atom produced

$= 12 \times 10^{-5}$ mol

Moles of organic compound

$$= \frac{1.84 \times 10^{-3}}{92}$$

$= 2 \times 10^{-5}$

∴ Number of alcoholic hydrogen present

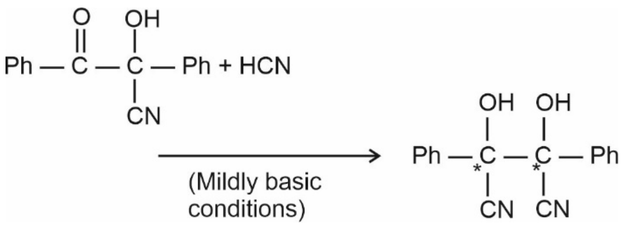
$$= \frac{12 \times 10^{-5}}{2 \times 10^{-5}} = 6$$

9. The number of stereoisomers formed in a reaction of (±) Ph (C = O) C (OH) (CN) Ph with HCN is_____.

[where Ph is $-C_6H_5$]

Answer (3)

Sol.

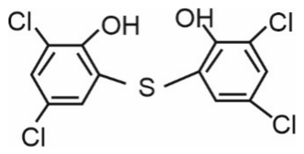


Number of stereoisomers = 3

10. The number of chlorine atoms in bithionol is______.

Answer (4)

Sol. Number of chlorine atoms in bithionol = 4



MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. If $z \ne 0$ be a complex number such that $\left|z-\frac{1}{z}\right|=2,$ then the maximum value of $|z|$ is

(A) $\sqrt{2}$ (B) 1

(C) $\sqrt{2}-1$ (D) $\sqrt{2}+1$

Answer (D)

Sol. $\left|z-\frac{1}{z}\right| \ge \left||z|-\frac{1}{z}\right|$

$\Rightarrow \left||z|-\frac{1}{|z|}\right| \le 2$

Let $|z| = r$

$\left|r-\frac{1}{r}\right| \le 2$

$-2 \le r-\frac{1}{r} \le 2$

$r-\frac{1}{r} \ge -2$ and $r-\frac{1}{r} \le 2$

$r^2+2r-1 \ge 0$ and $r^2-2r-1 \le 0$

$r \in \left[-\infty,-1-\sqrt{2}\right] \cup \left[-1+\sqrt{2},\ \infty\right]$ and

$r \in \left[1-\sqrt{2},\ 1+\sqrt{2}\right]$

Taking intersection $r \in \left[\sqrt{2}-1,\ \sqrt{2}+1\right]$

2. Which of the following matrices can **NOT** be obtained from the matrix $\begin{bmatrix}-1 & 2\\ 1 & -1\end{bmatrix}$ by a single elementary row operation?

(A) $\begin{bmatrix}0 & 1\\ 1 & -1\end{bmatrix}$ (B) $\begin{bmatrix}1 & -1\\ -1 & 2\end{bmatrix}$

(C) $\begin{bmatrix}-1 & 2\\ -2 & 7\end{bmatrix}$ (D) $\begin{bmatrix}-1 & 2\\ -1 & 3\end{bmatrix}$

Answer (C)

Sol. (1) By $R_1 \to R_1 + R_2$, $\begin{bmatrix}0 & 1\\ 1 & -1\end{bmatrix}$ is possible

(2) By $R_1 \leftrightarrow R_2$, $\begin{bmatrix}1 & -1\\ -1 & 2\end{bmatrix}$ is possible

(3) This matrix can't be obtained

(4) By $R_2 \to R_2 + 2R_1$, $\begin{bmatrix}-1 & 2\\ -1 & 3\end{bmatrix}$ is possible

3. If the system of equations

$x + y + z = 6$

$2x + 5y + \alpha z = \beta$

$x + 2y + 3z = 14$

has infinitely many solutions, then $\alpha + \beta$ is equal to

(A) 8 (B) 36

(C) 44 (D) 48

Answer (C)

Sol. $\Delta = \begin{vmatrix}1 & 1 & 1\\ 2 & 5 & \alpha\\ 1 & 2 & 3\end{vmatrix} = 1(15-2\alpha)-1(6-\alpha)+1(-1)$

$= 15 - 2\alpha - 6 + \alpha - 1$

$= 8 - \alpha$

For infinite solutions, $\Delta = 0 \Rightarrow \alpha = 8$

$\Delta_x = \begin{vmatrix}6 & 1 & 1\\ \beta & 5 & 8\\ 14 & 2 & 3\end{vmatrix} = 6(-1)-1(3\beta-112)+1(2\beta-70)$

$= -6 - 3\beta + 112 + 2\beta - 70$

$= 36 - \beta$

$\Delta_x = 0 \Rightarrow$ for $\beta = 36$

$\alpha + \beta = 44$

4. Let the function

$f(x)=\begin{cases}\dfrac{\log_e(1+5x)-\log_e(1+\alpha x)}{x} & ;\ \text{if } x \in 0\\ 10 & ;\ \text{if } x = 0\end{cases}$ be

continuous at $x = 0$. Then α is equal to

(A) 10 (B) –10

(C) 5 (D) –5

Answer (D)

Sol. $\underset{x\to 0}{\text{lt}} \dfrac{\ln(1+5x)-\ln(1+\alpha x)}{x}$

$= 5 - \alpha = 10$

$\Rightarrow \quad \alpha = -5$

5. If $[t]$ denotes the greatest integer $\le t$, then the value of $\int_0^1 \left[2x - |3x^2 - 5x + 2| + 1\right] dx$ is

(A) $\dfrac{\sqrt{37}+\sqrt{13}-4}{6}$

(B) $\dfrac{\sqrt{37}-\sqrt{13}-4}{6}$

(C) $\dfrac{-\sqrt{37}-\sqrt{13}+4}{6}$

(D) $\dfrac{-\sqrt{37}+\sqrt{13}+4}{6}$

Answer (A)

Sol. $I = \int_0^1 \left[2x - |3x^2 - 5x + 2| + 1\right] dx$

$$I = \int_0^{2/3}\left[\underbrace{-3x^2+7x-2}_{I_1}\right]dx + \int_{2/3}^{1}\left[\underbrace{3x^2-3x+2}_{I_2}\right]dx + 1$$

$$I_1 = \int_0^{t_1}(-2)dx + \int_{t_1}^{1/3}(-1)dx + \int_{1/3}^{t_2}0.dx + \int_{t_2}^{2/3}dx$$

$$= -t_1 - t_2 + \frac{1}{3}\text{, where } t_1 = \frac{7-\sqrt{37}}{6}, t_2 = \frac{7-\sqrt{13}}{6}$$

$$I_2 = \int_{2/3}^{1} 1dx = \frac{1}{3}$$

$$\therefore \quad I = \frac{1}{3} - t_1 - t_2 + \frac{1}{3} + 1 = \frac{5}{3} - \left[\frac{7-\sqrt{37}}{6} + \frac{7-\sqrt{13}}{6}\right]$$

$$= \frac{\sqrt{37}+\sqrt{13}-4}{6}$$

6. Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $a_0 = a_1 = 0$ and $a_{n+2} = 3a_{n+1} - 2a_n + 1,\ \forall\ n \ge 0.$

Then $a_{25}a_{23} - 2a_{25}a_{22} - 2a_{23}a_{24} + 4a_{22}a_{24}$ is equal to

(A) 483

(B) 528

(C) 575

(D) 624

Answer (B)

Sol. $a_{n+2} = 3a_{n+1} - 2a_n + 1,\ \forall\ n \ge 0 \quad (a_0 = a_1 = 0)$

$$(a_{n+2} - a_{n+1}) - 2(a_{n+1} - a_n) - 1 = 0$$

Put $n = 0$

$$(a_2 - a_1) - 2(a_1 - a_0) - 1 = 0$$

$n = 1$

$$(a_3 - a_2) - 2(a_2 - a_1) - 1 = 0$$

$n = 2$

$$(a_4 - a_3) - 2(a_3 - a_2) - 1 = 0$$

$$\vdots$$

$n = n$

$$(a_{n+2} - a_{n+1}) - 2(a_{n+1} - a_n) - 1 = 0$$

Adding,

$$(a_{n+2} - a_1) - 2(a_{n+1} - a_0) - (n+1) = 0$$

$$\therefore \quad a_{n+2} - 2a_{n+1} - (n+1) = 0$$

$n \to n - 2$

$$a_n - 2a_{n-1} - n + 1 = 0$$

Now, $a_{25}a_{23} - 2a_{25}a_{22} - 2a_{23}a_{24} + 4a_{22}a_{24}$

$$= a_{25}(a_{23} - 2a_{22}) - 2a_{24}(a_{23} - 2a_{22})$$

$$= (a_{25} - 2a_{24})(a_{23} - 2a_{22})$$

$$= 24 \cdot 22 = 528$$

7. $\sum_{r=1}^{20}(r^2+1)(r!)$ is equal to

(A) 22! – 21!

(B) 22! – 2(21!)

(C) 21! – 2(20!)

(D) 21! – 20!

Answer (B)

Sol. $\sum_{r=1}^{20}(r^2 + 1 + 2r - 2r)r! = \sum_{r=1}^{20}\left((r+1)^2 - 2r\right)r!$

$$= \sum_{r=1}^{20}\left[(r+1)(r+1)! - rr!\right] - \sum_{r=1}(r+1)r! = r!$$

$$= (2\cdot 2! - 1!) + (3\cdot 3! - 2\cdot 2!) + \ldots. + (21.21! - 20.20!)$$

$$-\left[(2! - 1!) + (3! - 2!) + \ldots. + (21! - 20!)\right]$$

$$= (21\cdot 21! - 1) - (21! - 1)$$

$$= 20.21! = (22 - 2)21!$$

$$= 22! - 2(21!)$$

8. For $I(x) = \int \dfrac{\sec^2 x - 2022}{\sin^{2022} x} dx$, if $I\left(\dfrac{\pi}{4}\right) = 2^{1011}$, then

(A) $3^{1010} I\left(\dfrac{\pi}{3}\right) - I\left(\dfrac{\pi}{6}\right) = 0$

(B) $3^{1010} I\left(\dfrac{\pi}{6}\right) - I\left(\dfrac{\pi}{3}\right) = 0$

(C) $3^{1011} I\left(\dfrac{\pi}{3}\right) - I\left(\dfrac{\pi}{6}\right) = 0$

(D) $3^{1011} I\left(\dfrac{\pi}{6}\right) - I\left(\dfrac{\pi}{3}\right) = 0$

Answer (A)

Sol. $I(x)=\int \frac{\sec^2 x - 2022}{\sin^{2022} x}\,dx$

$$=\int\left(\sec^2 x\cdot\sin^{-2022} x - 2022\sin^{-2022} x\right)dx$$

$$=\sin^{-2022} x\tan x + \int 2022\sin^{-2023} x\cos x\cdot\tan x\,dx$$

$$-\int 2022\sin^{-2022} x\,dx + c$$

$$I(x)=\sin^{-2022} x\ \tan x + c$$

$$\because\quad I\left(\frac{\pi}{4}\right)=2^{1011}\quad\Rightarrow\quad c=2^{1011}-2^{1011}=0$$

$$\therefore\quad I\left(\frac{\pi}{3}\right)=\left(\frac{2}{\sqrt{3}}\right)^{2022}\sqrt{3},\ I\left(\frac{\pi}{6}\right)=2^{2022}\frac{1}{\sqrt{3}}$$

So, option (A) : $\frac{3^{1010}2^{2022}}{3^{1011}}\cdot\sqrt{3}-\frac{2^{2022}}{\sqrt{3}}=0$

$\therefore$ Option (A) is correct

9. if the solution curve of the differential equation $\frac{dy}{dx}=\frac{x+y-2}{x-y}$ passes through the points (2, 1) and (k + 1, 2), k > 0, then

(A) $2\tan^{-1}\left(\frac{1}{k}\right)=\log_e\left(k^2+1\right)$

(B) $\tan^{-1}\left(\frac{1}{k}\right)=\log_e\left(k^2+1\right)$

(C) $2\tan^{-1}\left(\frac{1}{k+1}\right)=\log_e\left(k^2+2k+2\right)$

(D) $2\tan^{-1}\left(\frac{1}{k}\right)=\log_e\left(\frac{k^2+1}{k^2}\right)$

Answer (A)

Sol. $\frac{dy}{dx}=\frac{x+y-2}{x-y}=\frac{(x-1)+(y-1)}{(x-1)-(y-1)}$

Let $x-1=X$, $y-1=Y$

$$\frac{dY}{dX}=\frac{X+Y}{X-Y}$$

Let $Y=tX\Rightarrow\frac{dY}{dX}=t+X\frac{dt}{dX}$

$$t+X\frac{dt}{dX}=\frac{1+t}{1-t}$$

$$X\frac{dt}{dX}=\frac{1+t}{1-t}-t=\frac{1+t^2}{1-t}$$

$$\int\frac{1-t}{1+t^2}dt=\int\frac{dX}{X}$$

$$\tan^{-1}t-\frac{1}{2}\ln\left(1+t^2\right)=\ln|X|+c$$

$$\tan^{-1}\left(\frac{y-1}{x-1}\right)-\frac{1}{2}\ln\left(1+\left(\frac{y-1}{x-1}\right)^2\right)=\ln|x-1|+c$$

Curve passes through (2, 1)

0 – 0 = 0 + $c\Rightarrow c$ = 0

If (k + 1, 2) also satisfies the curve

$$\tan^{-1}\left(\frac{1}{k}\right)-\frac{1}{2}\ln\left(\frac{1+k^2}{k^2}\right)=\ln k$$

$$2\tan^{-1}\left(\frac{1}{k}\right)=\ln\left(1+k^2\right)$$

10. Let y = y(x) be the solution curve of the differential equation $\frac{dy}{dx}+\left(\frac{2x^2+11x+13}{x^3+6x^2+11x+6}\right)y=\frac{(x+3)}{x+1},\ x>-1,$ which passes through the point (0, 1). Then y(1) is equal to

(A) $\frac{1}{2}$ (B) $\frac{3}{2}$

(C) $\frac{5}{2}$ (D) $\frac{7}{2}$

Answer (B)

Sol. $\frac{dy}{dx}+\left(\frac{2x^2+11x+13}{x^3+6x^2+11x+6}\right)y=\frac{(x+3)}{x+1},\ x>-1,$

Integrating factor I.F. = $e^{\int\frac{2x^2+11x+13}{x^3+6x^2+11x+6}dx}$

Let $\frac{2x^2+11x+13}{(x+1)(x+2)(x+3)}=\frac{A}{x+1}+\frac{B}{x+2}+\frac{C}{x+3}$

$A=2,\ B=1,\ C=-1$

I.F. = $e^{(2\ln|x+1|+\ln|x+2|-\ln|x+3|)}$

$$=\frac{(x+1)^2(x+2)}{x+3}$$

Solution of differential equation

$$y\cdot\frac{(x+1)^2(x+2)}{x+3}=\int(x+1)(x+2)dx$$

$$y\frac{(x+1)^2(x+2)}{x+3}=\frac{x^3}{3}+\frac{3x^2}{2}+2x+c$$

Curve passes through (0, 1)

$$1\times\frac{1\times2}{3}=0+c\quad\Rightarrow\quad c=\frac{2}{3}$$

So, $y(1)=\frac{\frac{1}{3}+\frac{3}{2}+2+\frac{2}{3}}{\frac{\left(2^2\times3\right)}{4}}=\frac{3}{2}$

11. Let m_1, m_2 be the slopes of two adjacent sides of a square of side a such that $a^2 + 11a + 3(m_1^2 + m_2^2)$ = 220. If one vertex of the square is (10(cosα – sinα),10(sinα + cosα)), where $\alpha \in \left(0, \frac{\pi}{2}\right)$ and the equation of one diagonal is $(\cos\alpha - \sin\alpha)x + (\sin\alpha + \cos\alpha)y = 10$, then $72(\sin^4\alpha + \cos^4\alpha) + a^2 - 3a + 13$ is equal to :

(A) 119
(B) 128
(C) 145
(D) 155

Answer (B)

Sol. One vertex of square is

(10 (cosα – sinα), 10(sinα + cosα))

and one of the diagonal is

(cosα – sinα) x + (sinα + cosα)y = 10

So the other diagonal can be obtained as

(cosα + sinα)x – (cosα – sinα)y = 0

So, point of intersection of diagonal will be

(5(cosα – sinα), 5(cosα + sinα)).

Therefore, the vertex opposite to the given vertex is (0, 0).

So, the diagonal length = $10\sqrt{2}$

Side length (a) = 10

It is given that

$$a^2 + 11a + 3\left(m_1^2 + m_2^2\right) = 220$$

$$m_1^2 + m_2^2 = \frac{220 - 100 - 110}{3} = \frac{10}{3}$$

and $m_1 m_2 = -1$

Slopes of the sides are tanα and – cotα

$$\tan^2\alpha = 3 \text{ or } \frac{1}{3}$$

$72(\sin^4\alpha + \cos^4\alpha) + a^2 - 3a + 13$

$$= 72 \cdot \frac{\tan^4\alpha + 1}{\left(1 + \tan^2\alpha\right)^2} + a^2 - 3a + 13 = 128$$

12. The number of elements in the set

$$S = \left\{x \in \mathbb{R} : 2\cos\left(\frac{x^2 + x}{6}\right) = 4^x + 4^{-x}\right\} \text{ is :}$$

(A) 1
(B) 3
(C) 0
(D) infinite

Answer (A)

Sol. $S = \left\{x \in \mathbb{R} : 2\cos\left(\frac{x^2 + x}{6}\right) = 4^x + 4^{-x}\right\}$

LHS is less than or equal to 2 and RHS is greater than or equal to 2.

So equality holds only if LHS = RHS = 2

RHS is 2 when x = 0

and at x = 0, LHS is also 2.

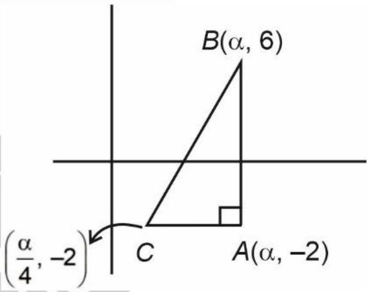
So, only one solution exist.

13. Let $A(\alpha, -2)$, $B(\alpha, 6)$ and $C\left(\frac{\alpha}{4}, -2\right)$ be vertices of a ΔABC. If $\left(5, \frac{\alpha}{4}\right)$ is the circumcentre of ΔABC, then which of the following is **NOT** correct about ΔABC.

(A) area is 24
(B) perimeter is 25
(C) circumradius is 5
(D) inradius is 2

Answer (B)

Sol.



Circumcentre of ΔABC

$$= \left(\frac{\alpha + \frac{\alpha}{4}}{2}, \frac{6-2}{2}\right)$$

$$= \left(\frac{5\alpha}{8}, 2\right)$$

$$= \left(5, \frac{\alpha}{4}\right)$$

$\Rightarrow \quad \alpha = 8$

$$\text{area}(\Delta ABC) = \frac{1}{2} \cdot \frac{3\alpha}{4} \times 8 = 24 \text{ sq. units}$$

$$\text{Perimeter} = 8 + \frac{3\alpha}{4} + \sqrt{8^2 + \left(\frac{3\alpha}{4}\right)^2}$$

$$= 8 + 6 + 10 = 24$$

$$\text{Circumradius} = \frac{10}{2} = 5$$

$$r = \frac{\Delta}{s} = \frac{24}{12} = 2$$

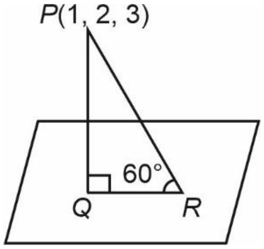
14. Let Q be the foot of perpendicular drawn from the point $P(1, 2, 3)$ to the plane $x + 2y + z = 14$. If R is a point on the plane such that $\angle PRQ = 60°$, then the area of ΔPQR is equal to :

(A) $\frac{\sqrt{3}}{2}$ (B) $\sqrt{3}$

(C) $2\sqrt{3}$ (D) 3

Answer (B)

Sol.



$$PQ = \left|\frac{1+4+3-14}{\sqrt{6}}\right| = \sqrt{6}$$

$$QR = \frac{PQ}{\tan 60°} = \frac{\sqrt{6}}{\sqrt{3}} = \sqrt{2}$$

Area $(\Delta PQR) = \frac{1}{2} \cdot PQ \cdot QR = \sqrt{3}$

15. If (2, 3, 9), (5, 2, 1), (1, λ, 8) and (λ, 2, 3) are coplanar, then the product of all possible values of λ is :

(A) $\frac{21}{2}$

(B) $\frac{59}{8}$

(C) $\frac{57}{8}$

(D) $\frac{95}{8}$

Answer (D)

Sol. $\because$ (2, 3, 9) (5, 2, 1), (1, λ, 8) and (λ, 2, 3) are coplanar.

$$\therefore \begin{vmatrix} \lambda-2 & -1 & -6 \\ -1 & \lambda-3 & -1 \\ 3 & -1 & -8 \end{vmatrix} = 0$$

$\therefore$ $8\lambda^2 - 67\lambda + 95 = 0$

$\therefore$ Product of all values of $\lambda = \frac{95}{8}$

16. Bag I contains 3 red, 4 black and 3 white balls and Bag II contains 2 red, 5 black and 2 white balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be black in colour. Then the probability, that the transferred ball is red, is :

(A) $\frac{4}{9}$

(B) $\frac{5}{18}$

(C) $\frac{1}{6}$

(D) $\frac{3}{10}$

Answer (B)

Sol. Let $E \rightarrow$ Ball drawn from Bag II is black.

$E_R \rightarrow$ Bag I to Bag II red ball transferred.

$E_B \rightarrow$ Bag I to Bag II black ball transferred.

$E_W \rightarrow$ Bag I to Bag II white ball transferred.

$$P\left(\frac{E_R}{E}\right) = \frac{P\left(\frac{E}{E_R}\right) \cdot P(E_R)}{P\left(\frac{E}{E_R}\right)P(E_R) + P\left(\frac{E}{E_B}\right)P(E_B) + P\left(\frac{E}{E_W}\right)P(E_W)}$$

Here,

$$P(E_R) = \frac{3}{10}, \quad P(E_B) = \frac{4}{10}, \quad P(E_W) = \frac{3}{10}$$

and

$$P\left(\frac{E}{E_R}\right) = \frac{5}{10}, \quad P\left(\frac{E}{E_B}\right) = \frac{6}{10}, \quad P\left(\frac{E}{E_W}\right) = \frac{5}{10}$$

$$\therefore \quad P\left(\frac{E_R}{E}\right) = \frac{\frac{15}{100}}{\frac{15}{100} + \frac{24}{100} + \frac{15}{100}}$$

$$= \frac{15}{54} = \frac{5}{18}$$

17. Let $S = \{z = x + iy : |z - 1 + i| \geq |z|, |z| < 2, |z + i| = |z - 1|\}$. Then the set of all values of x, for which $w = 2x + iy \in S$ for some $y \in \mathbb{R}$, is

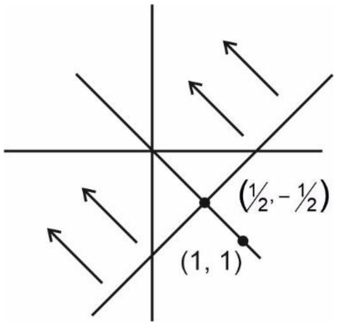
(A) $\left(-\sqrt{2}, \frac{1}{2\sqrt{2}}\right]$ (B) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{4}\right]$

(C) $\left(-\sqrt{2}, \frac{1}{2}\right]$ (D) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}\right]$

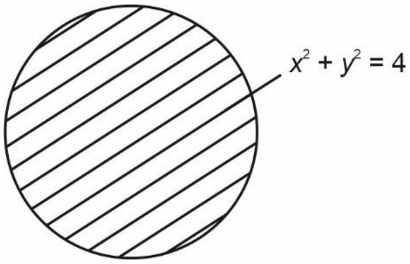
Answer (B)

Sol. $S : \{z = x + iy : |z - 1 + i| \geq |z|, |z| < 2, |z - i| = |z - 1|\}$

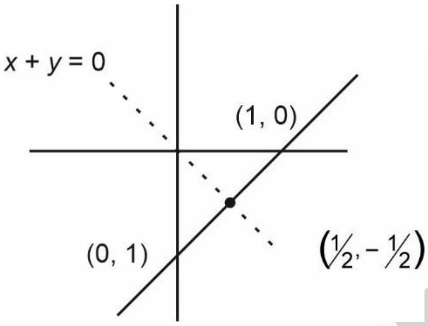
$|z - 1 + i| \geq |z|$

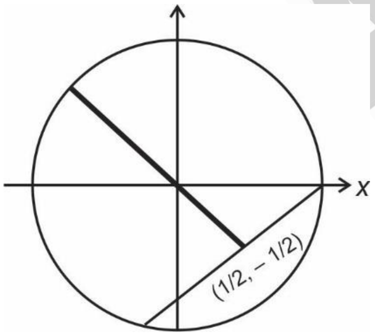


$|z| < 2$



$|z - i| = |z - 1|$





∵ $w \in S$ and $w = 2x + iy$

$2x < \frac{1}{2}$ ∴ $x < \frac{1}{4}$

$(2x)^2 + (-2x)^2 < 4$

$4x^2 + 4x^2 < 4$

$x^2 < \frac{1}{2} \Rightarrow x \in \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

∴ $x \in \left(-\frac{1}{2}, \frac{1}{4}\right]$

18. Let $\vec{a}, \vec{b}, \vec{c}$ be three coplanar concurrent vectors such that angles between any two of them is same. If the product of their magnitudes is 14 and $\left(\vec{a}\times\vec{b}\right)\cdot\left(\vec{b}\times\vec{c}\right)+\left(\vec{b}\times\vec{c}\right)\cdot(\vec{c}\times\vec{a})+(\vec{c}\times\vec{a})\cdot\left(\vec{a}\times\vec{b}\right)=168,$ then $|\vec{a}|+|\vec{b}|+|\vec{c}|$ is equal to :

(A) 10 (B) 14

(C) 16 (D) 18

Answer (C)

Sol. $|\vec{a}||\vec{b}||\vec{c}|=14$

$$\vec{a}\wedge\vec{b}=\vec{b}\wedge\vec{c}=\vec{c}\wedge\vec{a}=\theta=\frac{2\pi}{3}$$

$$\vec{a}\cdot\vec{b}=-\frac{1}{2}|\vec{a}||\vec{b}|$$

$$\vec{b}\cdot\vec{c}=-\frac{1}{2}|\vec{b}||\vec{c}|$$

$$\vec{c}\cdot\vec{a}=-\frac{1}{2}|\vec{c}||\vec{a}|$$

Now,

$$(\vec{a}\times\vec{b})\cdot(\vec{b}\times\vec{c})+(\vec{b}\times\vec{c})\cdot(\vec{c}\times\vec{a})+(\vec{c}\times\vec{a})\cdot(\vec{a}\times\vec{b})$$

$$= 168 \qquad \text{...(i)}$$

$$(\vec{a}\times\vec{b})\cdot(\vec{b}\times\vec{c})=(\vec{a}\cdot\vec{b})(\vec{b}\cdot\vec{c})-(\vec{a}\cdot\vec{c})|\vec{b}|^2$$

$$=\frac{1}{4}|\vec{b}|^2|\vec{a}||\vec{c}|+\frac{1}{2}|\vec{a}||\vec{b}|^2|\vec{c}|$$

$$=\frac{3}{4}|\vec{a}||\vec{b}|^2|\vec{c}| \qquad \text{...(ii)}$$

Similarly $(\vec{b}\times\vec{c})\cdot(\vec{c}\times\vec{a})=\frac{3}{4}|\vec{a}||\vec{b}||\vec{c}|^2$ ...(iii)

$$(\vec{c}\times\vec{a})\cdot(\vec{a}\times\vec{b})=\frac{3}{4}|\vec{a}|^2|\vec{b}||\vec{c}| \qquad \text{...(iv)}$$

Substitute (ii), (iii), (iv) in (i)

$$\frac{3}{4}|\vec{a}||\vec{b}||\vec{c}|\left[|\vec{a}|+|\vec{b}|+|\vec{c}|\right]=168$$

$$\frac{3}{4}\times 14\left[|\vec{a}|+|\vec{b}|+|\vec{c}|\right]=168$$

$$|\vec{a}|+|\vec{b}|+|\vec{c}|=16$$

19. The domain of the function $f(x)=\sin^{-1}\left(\frac{x^2-3x+2}{x^2+2x+7}\right)$ is :

(A) $[1, \infty)$ (B) $[-1, 2]$

(C) $[-1, \infty)$ (D) $(-\infty, 2]$

Answer (C)

Sol. $f(x) = \sin^{-1}\left(\frac{x^2 - 3x + 2}{x^2 + 2x + 7}\right)$

$$-1 \le \frac{x^2 - 3x + 2}{x^2 + 2x + 7} \le 1$$

$$\frac{x^2 - 3x + 2}{x^2 + 2x + 7} \le 1$$

$$x^2 - 3x + 2 \le x^2 + 2x + 7$$

$$5x \ge -5$$

$$x \ge -1 \qquad \text{… (i)}$$

$$\frac{x^2 - 3x + 2}{x^2 + 2x + 7} \ge -1$$

$$x^2 - 3x + 2 \ge -x^2 - 2x - 7$$

$$2x^2 - x + 9 \ge 0$$

$$x \in R \qquad \text{… (ii)}$$

(i) $\cap$ (ii)

Domain $\in [-1, \infty)$

20. The statement $(p \Rightarrow q) \vee (p \Rightarrow r)$ is **NOT** equivalent to

(A) $(p \wedge (\sim r)) \Rightarrow q$

(B) $(\sim q) \Rightarrow ((\sim r) \vee p)$

(C) $p \Rightarrow (q \vee r)$

(D) $(p \wedge (\sim q)) \Rightarrow r$

Answer (B)

Sol. (A) $(p \wedge (\sim r)) \Rightarrow q$

$\sim (p \wedge \sim r) \vee q$

$\equiv (\sim p \vee r) \vee q$

$\equiv \sim p \vee (r \vee q)$

$\equiv p \rightarrow (q \vee r)$

$\equiv (p \Rightarrow q) \vee (p \Rightarrow r)$

(C) $p \Rightarrow (q \vee r)$

$\equiv \sim p \vee (q \vee r)$

$\equiv (\sim p \vee q) \vee (\sim p \vee r)$

$\equiv (p \rightarrow q) \vee (p \rightarrow r)$

(D) $(p \wedge \sim q) \Rightarrow r$

$\equiv p \Rightarrow (q \vee r)$

$\equiv (p \Rightarrow q) \vee (p \Rightarrow r)$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. The sum and product of the mean and variance of a binomial distribution are 82.5 and 1350 respectively. Then the number of trials in the binomial distribution is _______.

Answer (96)

Sol. Given $np + npq = 82.5$ … (1)

and $np\,(npq) = 1350$ … (2)

$$\therefore \quad x^2 - 82.5x + 1350 = 0 \begin{array}{l} \nearrow \text{Mean} \\ \searrow \text{Variance} \end{array}$$

$$\Rightarrow \quad x^2 - 22.5\,x - 60x + 1350 = 0$$

$$\Rightarrow \quad x - (x - 22.5) - 60\,(x - 22.5) = 0$$

Mean = 60 and Variance = 22.5

$np = 60$, $npq = 22.5$

$$\Rightarrow \quad q = \frac{9}{24} = \frac{3}{8}, \quad p = \frac{5}{8}$$

$$\therefore \quad n\frac{5}{8} = 60 \qquad \Rightarrow n = 96$$

2. Let $\alpha, \beta(\alpha > \beta)$ be the roots of the quadratic equation $x^2 - x - 4 = 0$. If $P_n = \alpha^n - \beta^n$, $n \in \mathbb{N}$, then $\frac{P_{15}P_{16} - P_{14}P_{16} - P_{15}^2 + P_{14}P_{15}}{P_{13}P_{14}}$ is equal to _______.

Answer (16)

Sol. $x^2 - x - 4 = 0 \begin{array}{l} \nearrow \alpha \\ \searrow \beta \end{array}$ and $P_n = \alpha^n - \beta^n$

$$\therefore \quad I = \frac{(P_{15} - P_{14})\,P_{16} - P_{15}(P_{15} - P_{14})}{P_{13}\,P_{14}} = \frac{(P_{16} - P_{15})\,(P_{15} - P_{14})}{P_{13}\,P_{14}}$$

$$\Rightarrow \quad I = \frac{(\alpha^{16} - \beta^{16} - \alpha^{15} + \beta^{15})\,(\alpha^{15} - \beta^{15} - \alpha^{14} + \beta^{14})}{\left(\alpha^{13} - \beta^{13}\right)\left(\alpha^{14} - \beta^{14}\right)}$$

$$\Rightarrow \quad I = \frac{(\alpha^{15}(\alpha - 1) - \beta^{15}(\beta - 1))\,(\alpha^{14}(\alpha - 1) - \beta^{14}(\beta - 1))}{\left(\alpha^{13} - \beta^{13}\right)\left(\alpha^{14} - \beta^{14}\right)}$$

As $\alpha^2 - \alpha = 4 \quad \Rightarrow \alpha - 1 = \frac{4}{\alpha}$ and $\beta - 1 = \frac{4}{\beta}$

$$\Rightarrow \quad I = \frac{\left(\alpha^{15}\cdot\frac{4}{\alpha} - \beta^{15}\cdot\frac{4}{\beta}\right)\left(\alpha^{14}\cdot\frac{4}{\alpha} - \beta^{14}\cdot\frac{4}{\beta}\right)}{\left(\alpha^{13} - \beta^{13}\right)\left(\alpha^{14} - \beta^{14}\right)}$$

$$= \frac{16\left(\alpha^{14} - \beta^{14}\right)\left(\alpha^{13} - \beta^{13}\right)}{\left(\alpha^{14} - \beta^{14}\right)\left(\alpha^{13} - \beta^{13}\right)} = 16$$

3. Let $x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}$. For $k \in \mathbb{N}$, if

$X'A^kX = 33$, then k is equal to ________.

Answer (10*)

Sol. Given $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad A^4 = \begin{bmatrix} 1 & 0 & 12 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad A^k = \begin{bmatrix} 1 & 0 & 3k \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \quad X'A^kX = [111]\begin{bmatrix} 1 & 0 & 3k \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = [3k+3]$$

$\Rightarrow$ [3k + 3] = 33 (here it shall be [33] as matrix can't be equal to a scalar)

i.e. [3k + 3] = 33

3k + 3 = [33] $\Rightarrow k = 10$

If k is odd and apply above process, we don't get odd value of k

$\therefore$ $k = 10$

4. The number of natural numbers lying between 1012 and 23421 that can be formed using the digits 2, 3, 4, 5, 6 (repetition of digits is not allowed) and divisible by 55 is ________.

Answer (6)

Sol. Case-I When number is 4-digit number $\left(a\,b\,c\,d\right)$

here d is fixed as 5

So, (a, b, c) can be (6, 4, 3), (3, 4, 6), (2, 3, 6), (6, 3, 2), (3, 2, 4) or (4, 2, 3)

$\Rightarrow$ 6 numbers

Case-II No number possible

5. If $\sum_{k=1}^{10} K^2\left(10_{C_K}\right)^2 = 22000L$, then L is equal to ______.

Answer (221)

Sol. $\sum_{K=1}^{10} K^2\,({}^{10}C_K)^2 = 1^2\,{}^{10}C_1^2 + 2^2\,{}^{10}C_2^2 + \ldots + 10^2\,{}^{10}C_{10}$

Let $(1 + x)^{10} = {}^{10}C_0 + {}^{10}C_1 x + {}^{10}C_2 x^2 + \ldots.$

$+ {}^{10}C_{10}\,x^{10}$

$\Rightarrow 10(1 + x)^9 = {}^{10}C_1 + 2\cdot{}^{10}C_2 x + \ldots + 10\cdot{}^{10}C_{10}x^9 \ldots(1)$

Similarly, $10(x + 1)^9 = 10\cdot{}^{10}C_0\,x^9 + 9.{}^{10}C_1\,x^8 + \ldots + 1.{}^{10}C_9$

$100(1+x)^{18}$ has required term with coefficient of x^9

i.e. ${}^{18}C_9\,100 = 22000\,L$

$\Rightarrow$ $L = 221$

6. If [t] denotes the greatest integer $\le t$, then the number of points, at which the function $f(x) = 4|2x+3| + 9\left[x + \frac{1}{2}\right] - 12[x + 20]$ is not differentiable in the open interval (–20, 20), is ________.

Answer (79)

Sol. $f(x) = 4\,|\,2x+3\,| + 9\left[x + \frac{1}{2}\right] - 12[x + 20]$

$= 4\,|\,2x+3\,| + 9\left[x + \frac{1}{2}\right] - 12[x] - 240$

$f(x)$ is non differentiable at $x = -\frac{3}{2}$

and $f(x)$ is discontinuous at {–19, –18, ….., 18, 19}

as well as $\left\{-\frac{39}{2}, -\frac{37}{2}, \ldots, -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \ldots, \frac{39}{2}\right\}$,

at same point they are also non differentiable

$\therefore$ Total number of points of non differentiability

= 39 + 40

= 79

7. If the tangent to the curve $y = x^3 - x^2 + x$ at the point (a, b) is also tangent to the curve $y = 5x^2 + 2x - 25$ at the point (2, –1), then |2a + 9b| is equal to ________.

Answer (195)

Sol. Slope of tangent to curve $y = 5x^2 + 2x - 25$

$$= m = \left(\frac{dy}{dx}\right)_{\text{at}(2,-1)} = 22$$

$\therefore$ Equation of tangent : $y + 1 = 22(x - 2)$

$\therefore$ $y = 22x - 45$.

Slope of tangent to $y = x^3 - x^2 + x$ at point (a, b)

$= 3a^2 - 2a + 1$

$3a^2 - 2a + 1 = 22$

$3a^2 - 2a - 21 = 0$

$\therefore$ $a = 3$ or $-\frac{7}{3}$

Also $b = a^3 - a^2 + a$

Then $(a, b) = (3, 21)$ or $\left(-\frac{7}{3}, -\frac{151}{9}\right)$.

$\left(-\frac{7}{3}, -\frac{151}{9}\right)$ does not satisfy the equation of tangent

$\therefore$ $a = 3, b = 21$

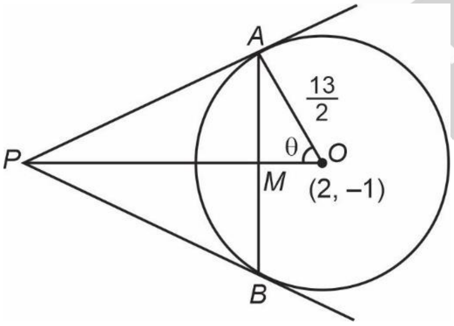
$\therefore$ $|2a + 9b| = 195$

8. Let AB be a chord of length 12 of the circle $(x-2)^2 + (y+1)^2 = \frac{169}{4}$. If tangents drawn to the circle at points A and B intersect at the point P, then five times the distance of point P from chord AB is equal to ________.

Answer (72)

Sol. Here $AM = BM = 6$

$$OM = \sqrt{\left(\frac{13}{2}\right)^2 - 6^2} = \frac{5}{2}$$



$$\sin\theta = \frac{12}{13}$$

In ΔPAO:

$$\frac{PO}{OA} = \sec\theta$$

$$PO = \frac{13}{2}\cdot\frac{13}{5} = \frac{169}{10}$$

$$\therefore \quad PM = \frac{169}{10} - \frac{5}{2} = \frac{144}{10} = \frac{72}{5}$$

$\therefore$ $5\,PM = 72$.

9. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + 2|\vec{b}|^2, \vec{a}\cdot\vec{b} = 3$ and $|\vec{a}\times\vec{b}|^2 = 75$.

Then $|\vec{a}|^2$ is equal to _____.

Answer (14)

Sol. $\because$ $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + 2|\vec{b}|^2$

or $|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} = |\vec{a}|^2 + 2|\vec{b}|^2$

$\therefore$ $|\vec{b}|^2 = 6$ ...(i)

Now $|\vec{a}\times\vec{b}|^2 = |\vec{a}|^2\,|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2$

$75 = |\vec{a}|^2 \cdot 6 - 9$

$\therefore$ $|\vec{a}|^2 = 14$

10. Let

$S = \{(x, y) \in \mathbb{N}\times\mathbb{N} : 9(x-3)^2 + 16(y-4)^2 \le 144\}$

and $T = \{(x, y) \in \mathbb{R}\times\mathbb{R} : (x-7)^2 + (y-4)^2 \le 36\}$.

Then $n(S \cap T)$ is equal to ______.

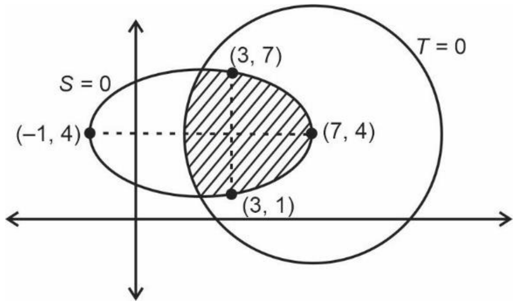
Answer (27)

Sol. $S = \left\{(x, y) \in \mathbb{N}\times\mathbb{N} : \frac{(x-3)^2}{16} + \frac{(y-4)^2}{9} \le 1\right\}$

represents all the integral points inside and on the ellipse $\frac{(x-3)^2}{16} + \frac{(y-4)^2}{9} = 1$, in first quadrant.

and $T = \{(x, y) \in \mathbb{R}\times\mathbb{R} : (x-7)^2 + (y-4)^2 \le 36\}$

represents all the points on and inside the circle $(x-7)^2 + (y-4)^2 = 36$.



$\therefore$ $n(S \cap T) = \{(3, 1), (2, 2), (3, 2), (4, 2), (5, 2), (2, 3), \ldots. (6, 5)\}$

Total number of points = 27

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