

24/02/2021
Morning



Aakash

Medical | IIT-JEE | Foundations

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Time : 3 hrs.

Answers & Solutions

M.M. : 300

for

JEE (MAIN)-2021 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS :

- (1) The test is of **3 hours** duration.
- (2) The Test Booklet consists of 90 questions. The maximum marks are 300.
- (3) There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry and Mathematics** having 30 questions in each part of equal weightage. Each part has two sections.
 - (i) Section-I : This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **-1 mark** for wrong answer.
 - (ii) Section-II : This section contains 10 questions. In Section-II, attempt any **five questions out of 10**. There will be **no negative marking for Section-II**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and there is no negative marking for wrong answer.

PART-A : PHYSICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. In a Young's double slit experiment, the width of the one of the slit is three times the other slit. The amplitude of the light coming from a slit is proportional to the slit-width. Find the ratio of the maximum to the minimum intensity in the interference pattern.

- (1) 4 : 1 (2) 2 : 1
(3) 3 : 1 (4) 1 : 4

Answer (1)

Sol. $I_1 = I_0$

$I_2 = 9I_0$ as $I \propto A^2$

$$\therefore \frac{I_{\max}}{I_{\min}} = \frac{(\sqrt{I_2} + \sqrt{I_1})^2}{(\sqrt{I_2} - \sqrt{I_1})^2} = \left(\frac{3+1}{3-1}\right)^2 = 4 : 1$$

2. Each side of a box made of metal sheet in cubic shape is 'a' at room temperature 'T', the coefficient of linear expansion of the metal sheet is ' α '. The metal sheet is heated uniformly, by a small temperature ΔT , so that its new temperature is $T + \Delta T$. Calculate the increase in the volume of the metal box.

- (1) $4\pi a^3 \alpha \Delta T$ (2) $3a^3 \alpha \Delta T$
(3) $\frac{4}{3}\pi a^3 \alpha \Delta T$ (4) $4a^3 \alpha \Delta T$

Answer (2)

Sol. $V = a^3, \gamma = 3\alpha$

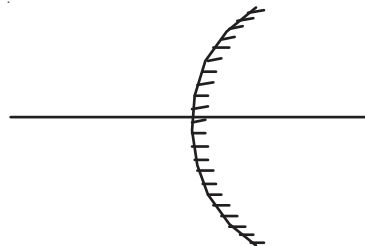
$$\begin{aligned} \therefore \Delta V &= V\gamma\Delta T \\ &= a^3 \times (3\alpha)\Delta T \\ &= 3a^3 \alpha \Delta T \end{aligned}$$

3. The focal length f is related to the radius of curvature r of the spherical convex mirror by

- (1) $f = r$ (2) $f = -\frac{1}{2}r$
(3) $f = +\frac{1}{2}r$ (4) $f = -r$

Answer (3)

Sol.



$$f = +\frac{r}{2}$$

as f is positive for convex mirror.

4. Two equal capacitors are first connected in series and then in parallel. The ratio of the equivalent capacities in the two cases will be

- (1) 1 : 2 (2) 2 : 1
(3) 1 : 4 (4) 4 : 1

Answer (3)

Sol. $C_1 = \frac{C}{2}$ for series

$C_2 = C + C = 2C$ for parallel

$$\therefore \frac{C_1}{C_2} = \frac{\frac{C}{2}}{2C} = \frac{1}{4}$$

5. If Y, K and η are the values of Young's modulus, bulk modulus and modulus of rigidity of any material respectively. Choose the correct relation for these parameters.

- (1) $Y = \frac{9K\eta}{3K - \eta} \text{ N/m}^2$
(2) $Y = \frac{9K\eta}{2\eta + 3K} \text{ N/m}^2$
(3) $K = \frac{Y\eta}{9\eta - 3Y} \text{ N/m}^2$
(4) $\eta = \frac{3YK}{9K + Y} \text{ N/m}^2$

Answer (3)

Sol. $\therefore Y = \frac{9K\eta}{\eta + 3K}$

$$\Rightarrow K = \frac{Y\eta}{9\eta - 3Y}$$

6. A current through a wire depends on time as

$$i = \alpha_0 t + \beta t^2$$

where $\alpha_0 = 20 \text{ A/s}$ and $\beta = 8 \text{ As}^{-2}$. Find the charge crossed through a section of the wire in 15 s.

- (1) 2100 C (2) 11250 C
 (3) 2250 C (4) 260 C

Answer (2)

Sol. $i = \alpha_0 t + \beta t^2$

$$\int dq = \int_0^{15} (20t + 8t^2) dt$$

$$\Rightarrow q = 20 \times \left(\frac{15^2 - 0^2}{2} \right) + \frac{8}{3} (15^3 - 0^3)$$

$$= 11250 \text{ C}$$

7. If an emitter current is changed by 4 mA, the collector current changes by 3.5 mA. The value of β will be :

- (1) 7 (2) 0.5
 (3) 0.875 (4) 3.5

Answer (1)

Sol. $\Delta I_E = 4 \text{ mA}$

$$\Delta I_C = 3.5 \text{ mA}$$

$$\Rightarrow \Delta I_B = (4 - 3.5) \text{ mA} = 0.5 \text{ mA}$$

$$\therefore \beta = \frac{\Delta I_C}{\Delta I_B} = \frac{3.5}{0.5} = 7$$

8. The work done by a gas molecule in an isolated system is given by, $W = \alpha\beta^2 e^{-\frac{x^2}{\alpha kT}}$, where x is the displacement, k is the Boltzmann constant and T is the temperature. α and β are constants. Then the dimensions of β will be :

- (1) $[M^2LT^2]$ (2) $[ML^2T^{-2}]$
 (3) $[MLT^{-2}]$ (4) $[M^0LT^0]$

Answer (3)

Sol. $W = \alpha\beta^2 e^{-\frac{x^2}{\alpha kT}}$

$$\left[\frac{x^2}{\alpha kT} \right] = \text{dimensionless}$$

$$\Rightarrow [\alpha] = \frac{L^2}{ML^2T^{-2}} = M^{-1}T^2$$

$$\text{and } [\alpha\beta^2] = ML^2T^{-2}$$

$$[\beta^2] = \frac{ML^2T^{-2}}{M^{-1}T^2} = M^2L^2T^{-4}$$

$$\Rightarrow [\beta] = MLT^{-2}$$

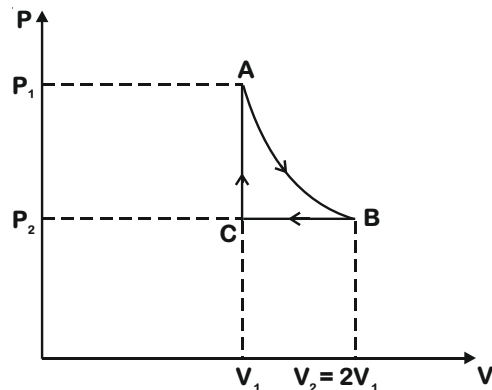
9. n mole of a perfect gas undergoes a cyclic process ABCA (see figure) consisting of the following processes.

A \rightarrow B : Isothermal expansion at temperature T so that the volume is doubled from V_1 to $V_2 = 2V_1$ and pressure changes from P_1 to P_2 .

B \rightarrow C : Isobaric compression at pressure P_2 to initial volume V_1 .

C \rightarrow A : Isochoric change leading to change of pressure from P_2 to P_1 .

Total workdone in the complete cycle ABCA is :



- (1) 0 (2) $nRT \left(\ln 2 + \frac{1}{2} \right)$
 (3) $nRT \left(\ln 2 - \frac{1}{2} \right)$ (4) $nRT \ln 2$

Answer (3)

Sol. $W_{AB} = nRT \ln \left(\frac{V_2}{V_1} \right)$

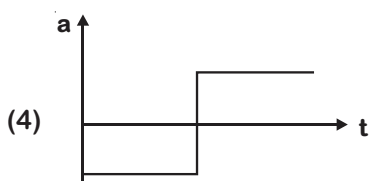
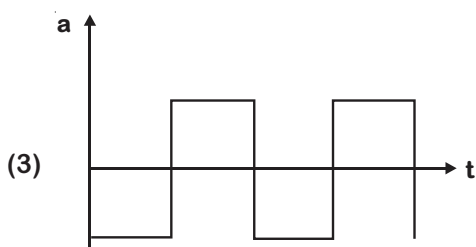
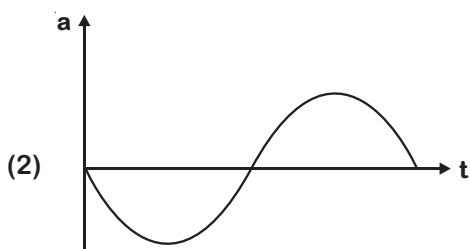
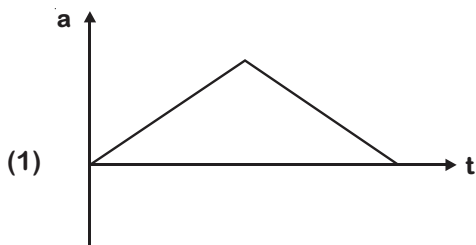
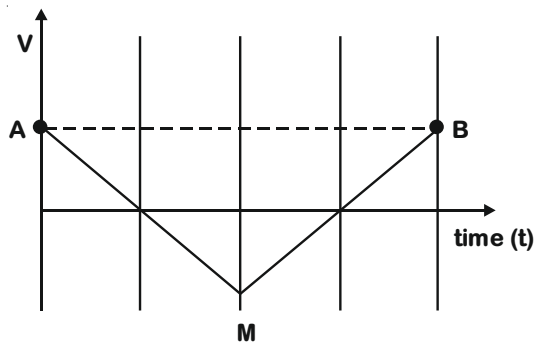
$$= nRT \ln (2)$$

$$W_{BC} = nR(T_C - T_B) = -\frac{nRT}{2}$$

$$W_{CA} = 0$$

$$\therefore W_{\text{total}} = nRT \ln 2 - \frac{nRT}{2} = nRT \left(\ln 2 - \frac{1}{2} \right)$$

10. If the velocity-time graph has the shape AMB, what would be the shape of the corresponding acceleration-time graph?



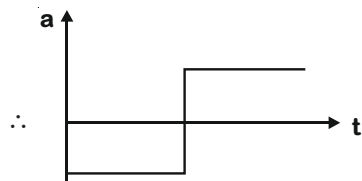
Answer (4)

Sol. From A to M,

Acceleration is negative and constant

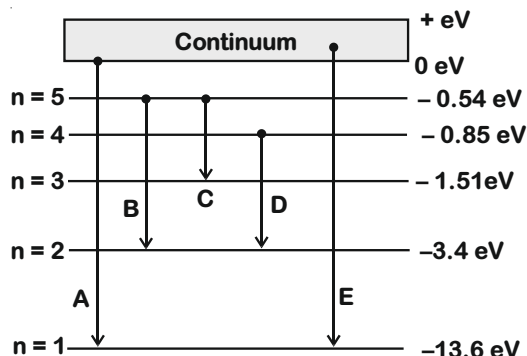
From M to B

Acceleration is positive and constant.



11. In the given figure, the energy levels of hydrogen atom have been shown along with some transitions marked A, B, C, D and E.

The transitions A, B and C respectively represent :



- (1) The ionization potential of hydrogen, second member of Balmer series and third member of Paschen series.
- (2) The series limit of Lyman series, third member of Balmer series and second member of Paschen series.
- (3) The series limit of Lyman series, second member of Balmer series and second member of Paschen series
- (4) The first member of the Lyman series, third member of Balmer series and second member of Paschen series.

Answer (2)

Sol. A corresponds to transition from ∞ to 1.

B corresponds to transition from $n = 5$ to $n = 2$.

C corresponds to transition from $n = 5$ to $n = 3$.

12. Consider two satellites S_1 and S_2 with periods of revolution 1 hr. and 8 hr. respectively revolving around a planet in circular orbits. The ratio of angular velocity of satellite S_1 to the angular velocity of satellite S_2 is :

- (1) 1 : 4
(2) 1 : 8
(3) 2 : 1
(4) 8 : 1

Answer (4)

Sol. $\frac{\omega_1}{\omega_2} = \left(\frac{2\pi}{T_1}\right) \times \left(\frac{T_2}{2\pi}\right) = \frac{T_2}{T_1} = \frac{8}{1}$

13. Moment of inertia (M. I.) of four bodies, having same mass and radius, are reported as;

I_1 = M.I. of thin circular ring about its diameter,

I_2 = M.I. of circular disc about an axis perpendicular to disc and going through the centre,

I_3 = M.I. of solid cylinder about its axis and

I_4 = M.I. of solid sphere about its diameter.

Then :

(1) $I_1 + I_3 < I_2 + I_4$ (2) $I_1 = I_2 = I_3 > I_4$

(3) $I_1 + I_2 = I_3 + \frac{5}{2}I_4$ (4) $I_1 = I_2 = I_3 < I_4$

Answer (2)

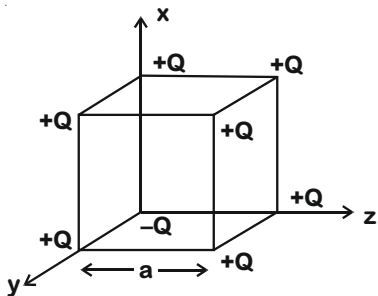
Sol. $I_1 = \frac{MR^2}{2}$

$I_2 = \frac{MR^2}{2}$

$I_3 = \frac{MR^2}{2}$

$I_4 = \frac{2}{5}MR^2$

14. A cube of side 'a' has point charges +Q located at each of its vertices except at the origin where the charge is -Q. The electric field at the centre of cube is :



(1) $\frac{-Q}{3\sqrt{3}\pi\epsilon_0 a^2}(\hat{x} + \hat{y} + \hat{z})$

(2) $\frac{Q}{3\sqrt{3}\pi\epsilon_0 a^2}(\hat{x} + \hat{y} + \hat{z})$

(3) $\frac{2Q}{3\sqrt{3}\pi\epsilon_0 a^2}(\hat{x} + \hat{y} + \hat{z})$

(4) $\frac{-2Q}{3\sqrt{3}\pi\epsilon_0 a^2}(\hat{x} + \hat{y} + \hat{z})$

Answer (4)

Sol. Field due to all charges will cancel out except two charges +Q and -Q placed along body diagonal.

$$\vec{E}_{-Q} = -\frac{Q}{4\pi\epsilon_0} \frac{3a^2}{4} \frac{(\hat{x} + \hat{y} + \hat{z})}{\sqrt{3}}$$

$$\vec{E}_{+Q} = \frac{-Q(\hat{x} + \hat{y} + \hat{z})}{3\pi\epsilon_0 a^2 \sqrt{3}}$$

$$\vec{E}_{\text{net}} = \frac{-2Q(\hat{x} + \hat{y} + \hat{z})}{3\sqrt{3}\pi\epsilon_0 a^2}$$

15. Four identical particles of equal masses 1 kg made to move along the circumference of a circle of radius 1 m under the action of their own mutual gravitational attraction. The speed of each particle will be :

(1) $\frac{\sqrt{(1+2\sqrt{2})G}}{2}$

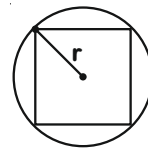
(2) $\sqrt{G(1+2\sqrt{2})}$

(3) $\sqrt{\frac{G}{2}(2\sqrt{2}-1)}$

(4) $\sqrt{\frac{G}{2}(1+2\sqrt{2})}$

Answer (1)

Sol. Centripetal force is being provided by gravitational force, F_g



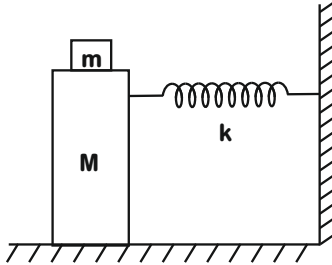
$$F_g = \frac{\sqrt{2} Gm^2}{(\sqrt{2}r)^2} + \frac{Gm^2}{4r^2}$$

$$\frac{mv^2}{r} = \frac{Gm^2}{\sqrt{2}r^2} + \frac{Gm^2}{4r^2}$$

$$v^2 = \frac{G}{\sqrt{2}} + \frac{G}{4}$$

$$v = \frac{\sqrt{(1+2\sqrt{2})G}}{2}$$

16. In the given figure, a mass M is attached to a horizontal spring which is fixed on one side to a rigid support. The spring constant of the spring is k . The mass oscillates on a frictionless surface with time period T and amplitude A . When the mass is in equilibrium position, as shown in the figure, another mass m is gently fixed upon it. The new amplitude of oscillation will be :



- (1) $A\sqrt{\frac{M-m}{M}}$ (2) $A\sqrt{\frac{M}{M-m}}$
 (3) $A\sqrt{\frac{M}{M+m}}$ (4) $A\sqrt{\frac{M}{M+m}}$

Answer (3)

On putting m on M Let velocity becomes V
 $(m + M) V = MV_0$

Now Kinetic Energy = $\frac{1}{2}(m + M)V^2$

$$K' = \frac{1}{2}(m + M) \frac{M^2 V_0^2}{(m + M)^2}$$

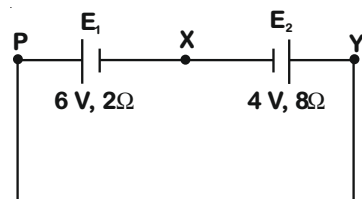
$$K' = \frac{1}{2} \frac{M^2 V_0^2}{(M + m)}$$

$$\frac{1}{2} M V_0^2 = \frac{1}{2} K A^2$$

$$K' = \frac{1}{2} K (A')^2$$

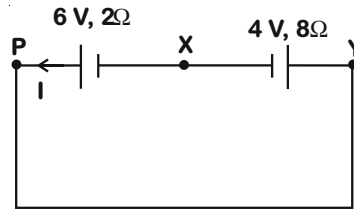
$$\text{Hence, } A' = \sqrt{\frac{M}{M + m}} A$$

17. A cell E_1 of emf 6 V and internal resistance 2Ω is connected with another cell E_2 of emf 4 V and internal resistance 8Ω (as shown in the figure). The potential difference across points X and Y is :



- (1) 3.6 V (2) 10.0 V
 (3) 5.6 V (4) 2.0 V

Answer (3)



$$I = \frac{6 - 4}{10} = 0.2 \text{ A}$$

$$V = 6 - 0.2 \times 2 = 5.6 \text{ V}$$

18. Given below are two statements:

Statement I : Two photons having equal linear momenta have equal wavelengths

Statement II : If the wavelength of photon is decreased, then the momentum and energy of a photon will also decrease

In the light of the above statements, choose the correct answer from the options given below

- (1) Statement I is true but Statement II is false
 (2) Both Statement I and Statement II are false
 (3) Statement I is false but Statement II is true
 (4) Both Statement I and Statement II are true

Answer (1)

$$E = \frac{hc}{\lambda}, p = \frac{h}{\lambda}$$

on decreasing wavelength photon and energy will decrease

19. Two stars of masses m and $2m$ at a distance d rotate about their common centre of mass in free space. The period of revolution is :

(1) $2\pi\sqrt{\frac{d^3}{3Gm}}$ (2) $2\pi\sqrt{\frac{3Gm}{d^3}}$

(3) $\frac{1}{2\pi}\sqrt{\frac{3Gm}{d^3}}$ (4) $\frac{1}{2\pi}\sqrt{\frac{d^3}{3Gm}}$

Answer (1)



$$m\omega^2 \left(\frac{2d}{3}\right) = \frac{2Gm^2}{d^2}$$

$$\omega^2 = \frac{3Gm}{d^3}$$

$$\omega = \sqrt{\frac{3Gm}{d^3}}$$

$$\frac{2\pi}{T} = \sqrt{\frac{3Gm}{d^3}}$$

$$T = 2\pi\sqrt{\frac{d^3}{3Gm}}$$

20. Match List I with List II

- | List I | List II |
|----------------|-------------------------------|
| (a) Isothermal | (i) Pressure constant |
| (b) Isochoric | (ii) Temperature constant |
| (c) Adiabatic | (iii) Volume constant |
| (d) Isobaric | (iv) Heat content is constant |

Choose the correct answer from the options given below :

- (1) (a) → (iii), (b) → (ii), (c) → (i), (d) → (iv)
- (2) (a) → (ii), (b) → (iv), (c) → (iii), (d) → (i)
- (3) (a) → (ii), (b) → (iii), (c) → (iv), (d) → (i)
- (4) (a) → (i), (b) → (iii), (c) → (ii), (d) → (iv)

Answer (3)

Isothermal → T = Constant

Isochoric → V = Constant

Adiabatic → Q = 0

Isobaric → P = Constant

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

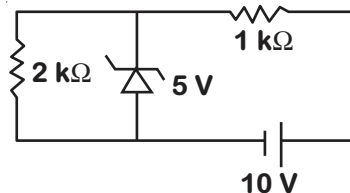
1. An electromagnetic wave of frequency 5 GHz, is travelling in a medium whose relative electric permittivity and relative magnetic permeability both are 2. Its velocity in this medium is _____ × 10⁷ m/s.

Answer (15)

Sol. $n = \sqrt{\mu_r \epsilon_r} = 2$

$$v = \frac{c}{n} = \frac{3 \times 10^8}{2} = 15 \times 10^7 \text{ m/s}$$

2. In connection with the circuit drawn below, the value of current flowing through 2 kΩ resistor is _____ × 10⁻⁴ A.



Answer (25)

Sol. $i = \frac{V_z}{R} = \frac{5}{2 \times 10^3}$
 $= 25 \times 10^{-4} \text{ A}$

3. An inclined plane is bent in such a way that the vertical cross-section is given by $y = \frac{x^2}{4}$ where y is in vertical and x in horizontal direction. If the upper surface of this curved plane is rough with coefficient of friction $\mu = 0.5$, the maximum height in cm at which a stationary block will not slip downward is _____ cm.

Answer (25)

Sol.

For no slipping

$$\tan \theta \leq \mu$$

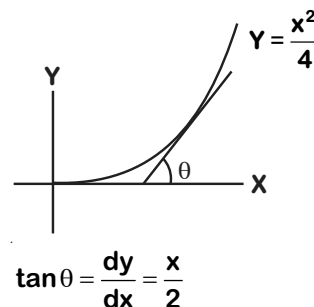
$$\frac{x}{2} \leq 0.5$$

$$x \leq 1$$

$$\sqrt{4Y} \leq 1$$

$$Y \leq \frac{1}{4} \text{ m}$$

$$Y_{\text{max}} = 25 \text{ cm}$$



4. A ball with a speed of 9 m/s collides with another identical ball at rest. After the collision, the direction of each ball makes an angle of 30° with the original direction. The ratio of velocities of the balls after collision is x : y, where x is _____.

Answer (1)

Sol.



Using conservation of linear momentum

In x-direction —

$$mv_1 \cos 30^\circ + mv_2 \cos 30^\circ = m \cdot 9$$

In y-direction —

$$mv_1 \sin 30^\circ - mv_2 \sin 30^\circ = 0$$

$$\frac{v_1}{v_2} = 1$$

5. An unpolarized light beam is incident on the polarizer of a polarization experiment and the intensity of light beam emerging from the analyzer is measured as 100 Lumens. Now, if the analyzer is rotated around the horizontal axis (direction of light) by 30° in clockwise direction, the intensity of emerging light will be _____ Lumens.

Answer (75)

Sol. Assuming initially transmission axes of polarizer and analyser are parallel.

$$I = \frac{I_0}{2} = 100 \text{ Lumen}$$

Now if analyser is rotated by angle of 30° , then —

$$I' = \frac{I_0}{2} \cos^2 \theta = 100 \times \frac{3}{4} = 75 \text{ Lumen}$$

6. A hydraulic press can lift 100 kg when a mass 'm' is placed on the smaller piston. It can lift _____ kg when the diameter of the larger piston is increased by 4 times and that of the smaller piston is decreased by 4 times keeping the same mass 'm' on the smaller piston.

Answer (25600)

Sol. Initially $\frac{100g}{A_1} = \frac{mg}{A_2} (A_2 < A_1)$

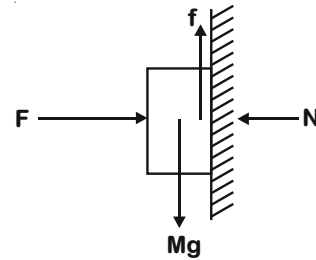
Later $\frac{Mg}{16A_1} = \frac{mg}{A_2/16}$

$$\Rightarrow M = 25600$$

7. The coefficient of static friction between a wooden block of mass 0.5 kg and a vertical rough wall is 0.2. The magnitude of horizontal force that should be applied on the block to keep it adhere to the wall will be _____ N. [$g = 10 \text{ ms}^{-2}$]

Answer (25)

Sol. For equilibrium of the block



$$N = F$$

$$f = Mg$$

$$\text{and } Mg \leq f_1$$

$$\Rightarrow F \geq \frac{Mg}{\mu}$$

$$F_{\max} = 25 \text{ N}$$

8. An audio signal $v_m = 20 \sin 2\pi(1500t)$ amplitude modulates a carrier $v_c = 80 \sin 2\pi(100,000t)$. The value of percent modulation is _____.

Answer (25)

Sol. $m = \frac{A_m}{A_c} = \frac{1}{4}$

$$\% \text{ modulation} = \frac{100}{4} = 25\%$$

9. A resonance circuit having inductance and resistance $2 \times 10^{-4} \text{ H}$ and 6.28Ω respectively oscillates at 10 MHz frequency. The value of quality factor of this resonator is _____.

$$[\pi = 3.14]$$

Answer (2000)

Sol. $Q = \frac{X_L}{R} = 2000$

10. A common transistor radio set requires 12 V (D.C.) for its operation. The D.C. source is constructed by using a transformer and a rectifier circuit, which are operated at 220 V (A.C.) on standard domestic A.C. supply. The number of turns of secondary coil are 24, then the number of turns of primary are _____.

Answer (440)

Sol. $\frac{V_s}{V_p} = \frac{N_s}{N_p}$

$$N_p = \frac{V_p}{V_s} \cdot N_s = \frac{220}{12} \times 24 = 440$$

PART-B : CHEMISTRY

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

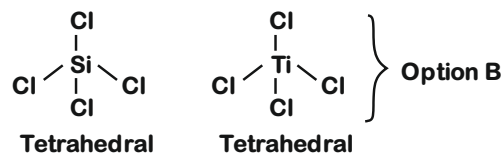
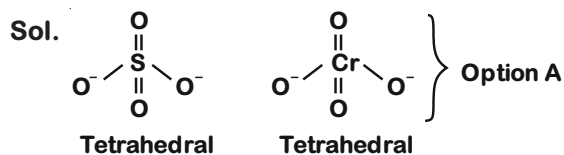
Choose the correct answer :

1. Which of the following are isostructural pairs?

- A. SO_4^{2-} and CrO_4^{2-}
- B. SiCl_4 and TiCl_4
- C. NH_3 and NO_3^-
- D. BCl_3 and BrCl_3

- (1) A and C only (2) B and C only
- (3) A and B only (4) C and D only

Answer (3)



Isostructural means same structure

So option-3 is the correct answer

2. Out of the following, which type of interaction is responsible for the stabilisation of α -helix structure of proteins?

- (1) Covalent bonding
- (2) Hydrogen bonding
- (3) Ionic bonding
- (4) vander Waals forces

Answer (2)

Sol. " α -Helix is one of the most common ways in which a polypeptide chain forms all possible hydrogen bonds by twisting into a right handed screw (helix) with the $-\text{NH}$ group of each amino acid residue hydrogen bonded to the >C=O of an adjacent turn of the helix"

3. The major components in "Gun Metal" are:

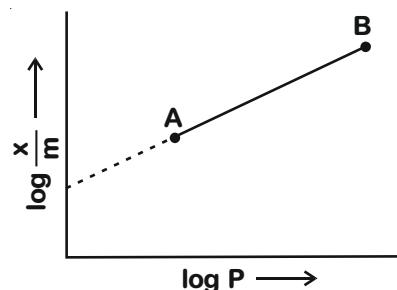
- (1) Cu, Ni and Fe (2) Cu, Sn and Zn
- (3) Al, Cu, Mg and Mn (4) Cu, Zn and Ni

Answer (2)

Sol. Gun metal has composition of Cu, Zn, Sn

- Cu – 87%
- Zn – 3 %
- Sn – 10%

4. In Freundlich adsorption isotherm, slope of AB line is:



- (1) n with (n, 0.1 to 0.5)
- (2) $\frac{1}{n}$ with $\left(\frac{1}{n} = 0 \text{ to } 1\right)$
- (3) log n with (n > 1)
- (4) $\log \frac{1}{n}$ with (n < 1)

Answer (2)

Sol. According to Freundlich adsorption isotherm

$$\frac{x}{m} = k.P^{1/n} (n > 1)$$

$$\log \frac{x}{m} = \log k + \frac{1}{n} \log P$$

so in the plot of $\log \frac{x}{m}$ vs log P the slope is $\frac{1}{n}$,

where $\frac{1}{n}$ varies from 0 to 1

5. Which of the following ore is concentrated using group 1 cyanide salt?

- (1) Sphalerite (2) Malachite
- (3) Siderite (4) Calamine

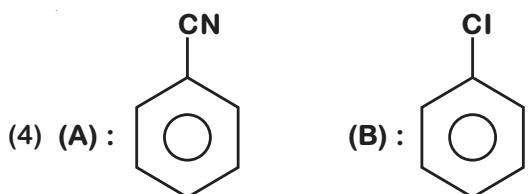
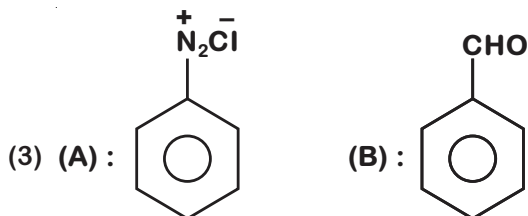
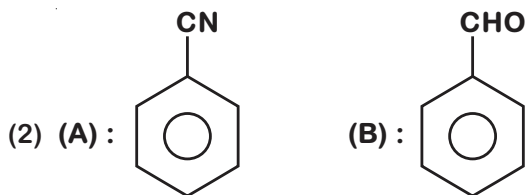
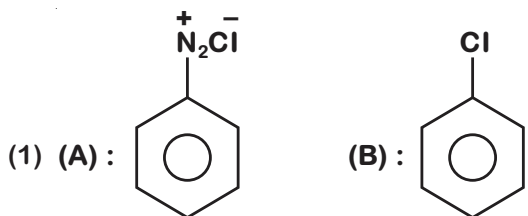
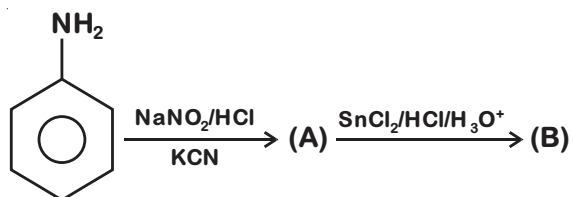
Answer (1)

Sol. Ore	Formula
Sphalerite	ZnS
Siderite	FeCO ₃
Malachite	Cu(OH) ₂ ·CuCO ₃
Calamine	ZnCO ₃

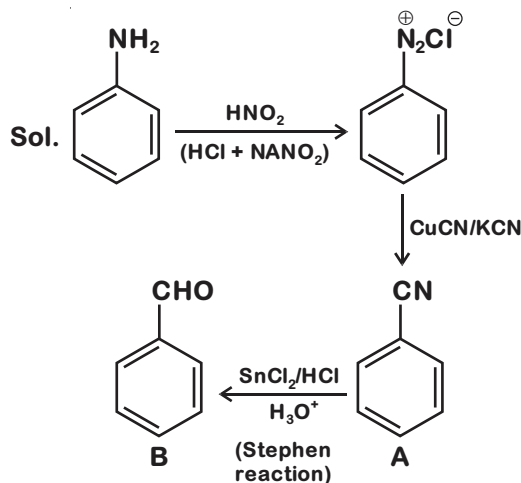
$ZnS + 4 NaCN \rightarrow [Zn(CN)_4]^{2-} + 4 Na^+ + S^{2-}$

the reagent NaCN/KCN is used to suppress the floating characteristics of ZnS by forming a soluble complex with KCN.

6. 'A' and 'B' in the following reactions are :



Answer (2)



7. Consider the elements Mg, Al, S, P and Si, the correct increasing order of their first ionization enthalpy is :

- (1) Mg < Al < Si < P < S
- (2) Mg < Al < Si < S < P
- (3) Al < Mg < S < Si < P
- (4) Al < Mg < Si < S < P

Answer (4)

Sol. Across the period, generally ionization enthalpy increases but half filled and fully filled configuration are stable and may change the regular trend.

P has more IE₁ than S because of half filled.

Al has lower IE₁ than Mg because of effective shielding of 3P electrons from the nucleus by 3s-electrons.

Finally order should be

P > S > Si > Mg > Al

8. The electrode potential of M²⁺ / M of 3d-series elements shows positive value for :

- (1) Cu
- (2) Zn
- (3) Co
- (4) Fe

Answer (1)

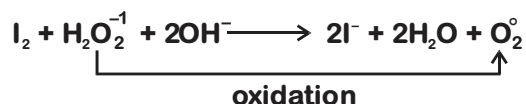
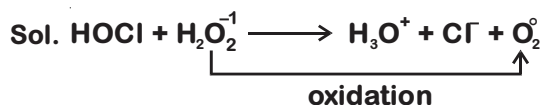
Sol. Only Cu²⁺/Cu has positive SRP among 3d-series metals.

- (A) HOCl + H₂O₂ → H₃O⁺ + Cl⁻ + O₂
- (B) I₂ + H₂O₂ + 2OH⁻ → 2I⁻ + 2H₂O + O₂

Choose the correct option.

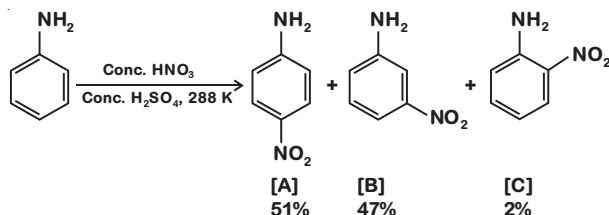
- (1) H₂O₂ acts as oxidising agent in equations (A) and (B).
- (2) H₂O₂ act as oxidizing and reducing agent respectively in equations (A) and (B).
- (3) H₂O₂ acts as reducing agent in equations (A) and (B).
- (4) H₂O₂ acts as reducing and oxidising agent respectively in equations (A) and (B).

Answer (3)



So, H_2O_2 is acting as reducing agent in both reactions.

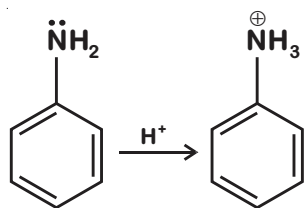
10. In the following reaction the reason why meta-nitro product also formed is :



- (1) Formation of anilinium ion
- (2) low temperature
- (3) $-\text{NO}_2$ substitution always takes place at meta-position
- (4) $-\text{NH}_2$ group is highly meta-directive

Answer (1)

Sol. Aniline itself is strong ortho/para director but on addition of acid it becomes anilinium ion which is a meta director.

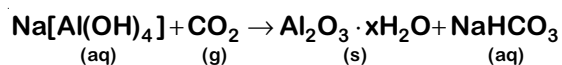
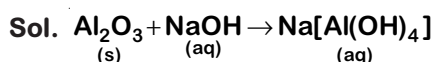


So the answer should be 1.

11. Al_2O_3 was leached with alkali to get X. The solution of X on passing of gas Y, forms Z. X, Y and Z respectively are:

- (1) $\text{X} = \text{Na}[\text{Al}(\text{OH})_4]$, $\text{Y} = \text{SO}_2$, $\text{Z} = \text{Al}_2\text{O}_3$
- (2) $\text{X} = \text{Al}(\text{OH})_3$, $\text{Y} = \text{SO}_2$, $\text{Z} = \text{Al}_2\text{O}_3 \cdot x\text{H}_2\text{O}$
- (3) $\text{X} = \text{Al}(\text{OH})_3$, $\text{Y} = \text{CO}_2$, $\text{Z} = \text{Al}_2\text{O}_3$
- (4) $\text{X} = \text{Na}[\text{Al}(\text{OH})_4]$, $\text{Y} = \text{CO}_2$, $\text{Z} = \text{Al}_2\text{O}_3 \cdot x\text{H}_2\text{O}$

Answer (4)



12. Given below are two statements:

Statement-I : Colourless cupric metaborate is reduced to cuprous metaborate in a luminous flame.

Statement-II : Cuprous metaborate is obtained by heating boric anhydride and copper sulphate in a non-luminous flame.

In the light of the above statements, choose the most appropriate answer from the options given below.

- (1) Statement I is false but statement II is true
- (2) Both statement I and statement II are true
- (3) Both statement I and statement II are false
- (4) Statement I is true but statement II is false

Answer (3)

Sol. Statement-I : Cupric metaborate is blue in colour

Hence statement-I is false



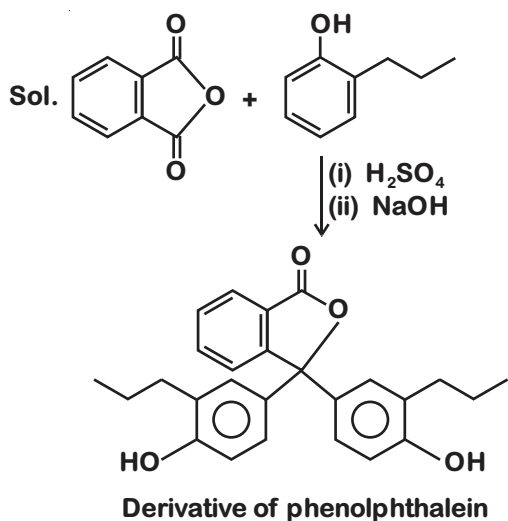
cupric metaborate is obtained instead of cuprous metaborate.

Hence statement-II is false

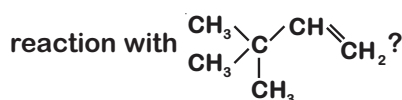
13. Which of the following compound gives pink colour on reaction with phthalic anhydride in conc. H_2SO_4 followed by treatment with NaOH ?

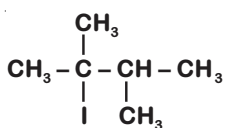
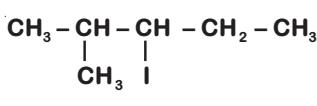
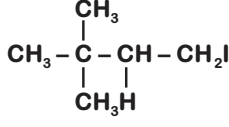
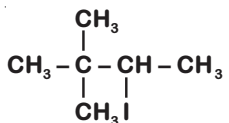
- (1)
- (2)
- (3)
- (4)

Answer (4)

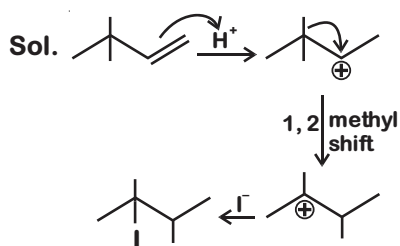


14. What is the major product formed by HI on

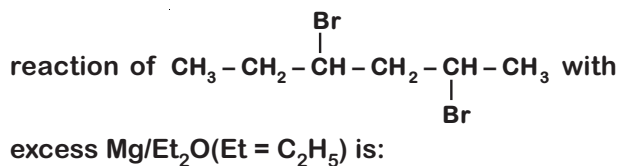


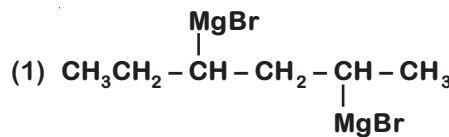
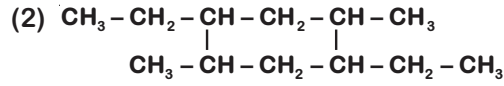
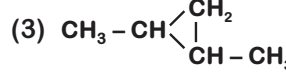
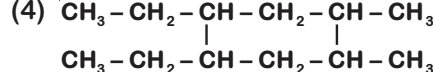
- (1) 
- (2) 
- (3) 
- (4) 

Answer (1)

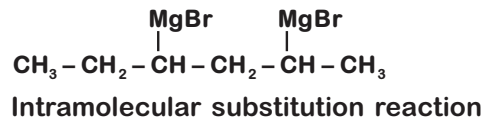
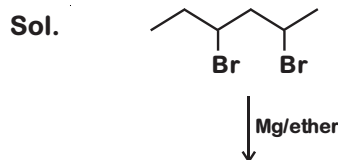


15. The product formed in the first step of the

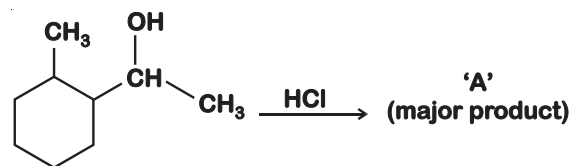


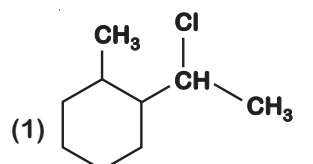
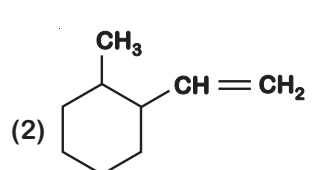
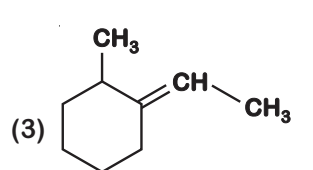
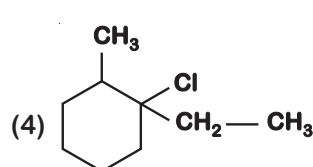
- (1) 
- (2) 
- (3) 
- (4) 

Answer (1)

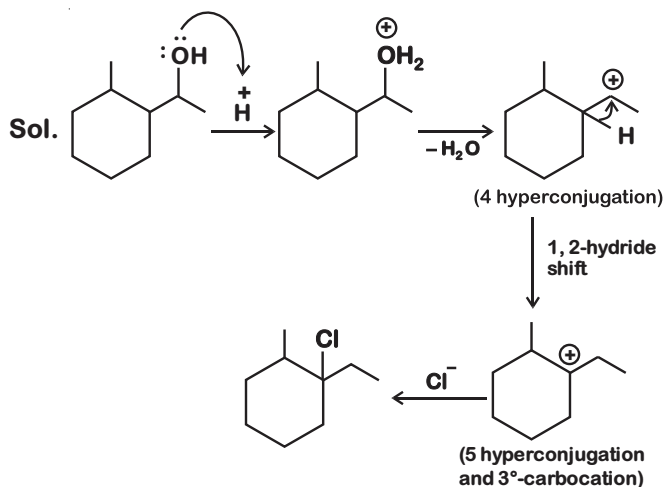


16. What is the final product (major) 'A' in the given reaction?

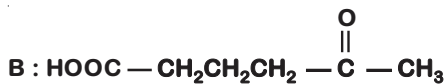
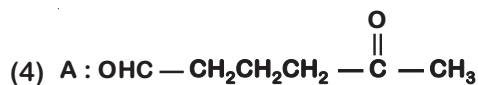
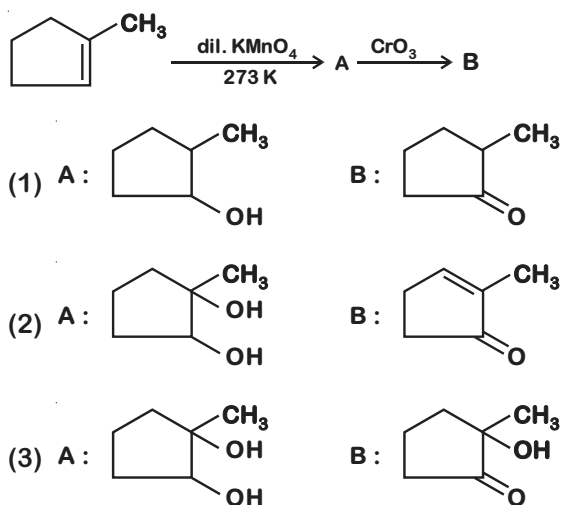


- (1) 
- (2) 
- (3) 
- (4) 

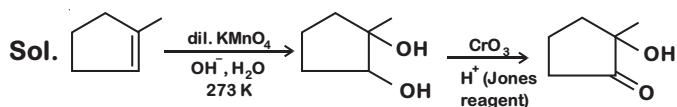
Answer (4)



17. Identify Products A and B.



Answer (3)



3°-alcohols do not undergo oxidation reaction easily.

18. The gas released during anaerobic degradation of vegetation may lead to :

- (1) Acid rain
- (2) Corrosion of metals
- (3) Ozone hole
- (4) Global warming and cancer

Answer (4)

Sol. During anaerobic degradation of vegetation CO_2 and CH_4 are released which may lead to cause global warming and cancer.

19. Match List I with List II.

List I (Monomer unit)	List II (Polymer)
(a) Caprolactum	(i) Natural rubber
(b) 2-Chloro-1, 3-butadiene	(ii) Buna-N
(c) Isoprene	(iii) Nylon 6
(d) Acrylonitrile	(iv) Neoprene

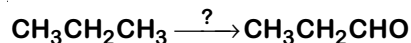
Choose the correct answer from the options given below:

- (1) (a) → (iv), (b) → (iii), (c) → (ii), (d) → (i)
- (2) (a) → (iii), (b) → (iv), (c) → (i), (d) → (ii)
- (3) (a) → (i), (b) → (ii), (c) → (iii), (d) → (iv)
- (4) (a) → (ii), (b) → (i), (c) → (iv), (d) → (iii)

Answer (2)

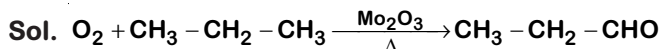
Sol. Monomer	Polymer
Isoprene	Natural rubber
2-Chloro-1, 3-butadiene	Neoprene
Caprolactum	Nylon 6
Acrylonitrile	Buna-N

20. Which of the following reagent is used for the following reaction?



- (1) Copper at high temperature and pressure
- (2) Manganese acetate
- (3) Molybdenum oxide
- (4) Potassium permanganate

Answer (3)



$\text{Mn}(\text{OAc})_2$ - manganese acetate oxidizes alkanes to carboxylic acids in presence of O_2 and heat.

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. A proton and a Li^{3+} nucleus are accelerated by the same potential. If λ_{Li} and λ_{p} denote the de Broglie wavelengths of Li^{3+} and proton

respectively, then the value of $\frac{\lambda_{\text{Li}}}{\lambda_{\text{p}}}$ is $x \times 10^{-1}$.

The value of x is _____. (Rounded off to the nearest integer)

[Mass of $\text{Li}^{3+} = 8.3$ mass of proton]

Answer (2)

Sol. de-Broglie wavelength $(\lambda) = \frac{h}{mV} = \frac{h}{\sqrt{2m(\text{K.E.})}}$

When a charge particle is accelerated by potential difference (V) then increase in K.E. = $q.V$

$$\lambda_{\text{Li}} = \frac{h}{\sqrt{2 \times m_{\text{p}} \times 8.3 \times 3 \times V}}$$

$$\lambda_{\text{p}} = \frac{h}{\sqrt{2 \times m_{\text{p}} \times 1 \times V}}$$

$$\therefore \frac{\lambda_{\text{Li}}}{\lambda_{\text{p}}} = \frac{h}{\sqrt{2 \times 24.9 \times m_{\text{p}} \times V}} \times \frac{\sqrt{2 \times m_{\text{p}} \times V}}{h} = \frac{1}{5}$$

$$= 2 \times 10^{-1}$$

2. Gaseous cyclobutene isomerizes to butadiene in a first order process which has a 'k' value of $3.3 \times 10^{-4} \text{ s}^{-1}$ at 153°C . The time in minutes it takes for the isomerization to proceed 40% to completion at this temperature is _____. (Rounded off to the nearest integer)

Answer (26)

Sol. For a 1st order reaction

$$kt = \ln \frac{a_0}{a_t}$$

$$t_{40\%} = \frac{1}{3.3 \times 10^{-4}} \ln \frac{a_0}{0.6a_0}$$

$$= \frac{10^4}{3.3} \ln \frac{10}{6} \text{ sec}$$

$$= \frac{10^4 \times 0.51}{3.3 \times 60} \text{ min}$$

$$= 25.8 \text{ min}$$

$$\approx 26$$

3. 4.5 g of compound A (MW = 90) was used to make 250 mL of its aqueous solution. The molarity of the solution in M is $x \times 10^{-1}$. The value of x is _____. (Rounded off to the nearest integer)

Answer (2)

Sol. Molarity = $\frac{\text{no. of moles of solute}}{\text{vol}^m \text{ of sol}^n \text{ (in L)}}$

$$= \frac{4.5 \times 1000}{90 \times 250}$$

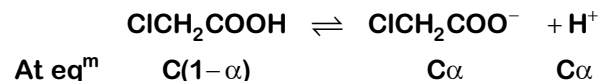
$$= 2 \times 10^{-1} \text{ M}$$

4. When 9.45 g of ClCH_2COOH is added to 500 mL of water, its freezing point drops by 0.5°C . The dissociation constant of ClCH_2COOH is $x \times 10^{-3}$. The value of x is _____. (Rounded off to the nearest integer)

$$[K_{\text{f}(\text{H}_2\text{O})} = 1.86 \text{ K kg mol}^{-1}]$$

Answer (35)

Sol. Moles of $\text{ClCH}_2\text{COOH} = \frac{9.45}{94.5} = 0.1 \text{ moles}$



van't Hoff factor (i) = $1 + \alpha$

$$C = \frac{0.1}{0.5} = 0.2 \text{ M}$$

Assuming molarity = molality

$$\Delta T_{\text{f}} = i k_{\text{f}} \cdot m = (1 + \alpha) 1.86 \times 0.2$$

$$\Rightarrow (1 + \alpha) = \frac{0.5}{0.2 \times 1.86} = 1.344 \approx 1.34$$

$$\Rightarrow \alpha = 0.34$$

$$K_{\text{a}} \text{ of } (\text{ClCH}_2\text{COOH}) = \frac{C\alpha \cdot C\alpha}{C(1-\alpha)}$$

$$= \frac{C\alpha^2}{1-\alpha} = \frac{0.2 \times (0.34)^2}{1-0.34}$$

$$= 35 \times 10^{-3}$$

5. The coordination number of an atom in a body-centered cubic structure is _____.

[Assume that the lattice is made up of atoms.]

Answer (8)

Sol. In body - centered cubic structure, atoms occupy all the corners of the cube as well as body centre position in a unit cell.

∴ Co-ordination number of atom in BCC structure = 8

6. At 1990 K and 1 atm pressure, there are equal number of Cl_2 molecules and Cl atoms in the reaction mixture. The value of K_p for the reaction $\text{Cl}_{2(g)} \rightleftharpoons 2\text{Cl}_{(g)}$ under the above conditions is $x \times 10^{-1}$. The value of x is _____. (Rounded off to the nearest integer)

Answer (5)

Sol. $\text{Cl}_{2(g)} \rightleftharpoons 2\text{Cl}_{(g)}$

∴ No. of atoms of Cl = no. of molecules of Cl_2

i.e $n_{\text{Cl}(g)} = n_{\text{Cl}_2(g)}$

$P_T = 1 \text{ atm}$

Using Dalton's law of partial pressure.

$P_{\text{Cl}(g)} = 0.5 \text{ atm} = P_{\text{Cl}_2(g)}$

∴ $K_p = \frac{(0.5)^2}{0.5} = 5 \times 10^{-1}$

7. For the reaction $\text{A}_{(g)} \rightarrow \text{B}_{(g)}$, the value of the equilibrium constant at 300 K and 1 atm is equal to 100.0. The value of $\Delta_r G$ for the reaction at 300 K and 1 atm in J mol^{-1} is $-xR$, where x is _____. (Rounded off to the nearest integer)

[$R = 8.31 \text{ J mol}^{-1}\text{K}^{-1}$ and $\ln 10 = 2.3$]

Answer (1380)

Sol. $\text{A}_{(g)} \rightarrow \text{B}_{(g)}$

$K_{\text{eq}} = 100$

$\Delta_r G^\circ = -RT \ln K_{\text{eq}}$

$= -R \times 300 \times \ln 100$

$= -R \times 300 \times 2 \times 2.3$

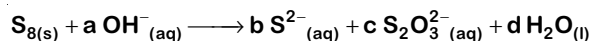
$= -1380 R$

$\Delta_r G$ of reaction at equilibrium will be zero.

$\Delta_r G^\circ = -1380 R$

Note : In this question, we have reported the value of $\Delta_r G^\circ$ which matches with the answer given by NTA.

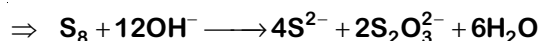
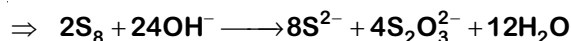
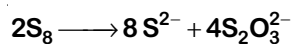
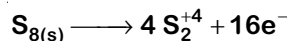
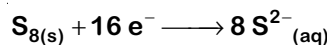
8. The reaction of sulphur in alkaline medium is given below:



The value of 'a' is _____. (Integer answer)

Answer (12)

Sol. $\text{S}_{8(s)} + a \text{OH}^-_{(aq)} \longrightarrow b \text{S}^{2-}_{(aq)} + c \text{S}_2\text{O}_3^{2-}_{(aq)} + d \text{H}_2\text{O}_{(l)}$



∴ $a = 12$

9. Number of amphoteric compounds among the following is ____

- (1) BeO (2) BaO
(3) Be(OH)₂ (4) Sr(OH)₂

Answer (2)

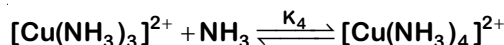
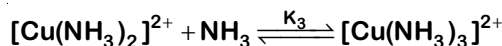
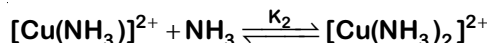
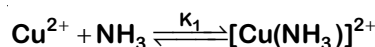
Sol. BeO – Amphoteric oxide

BaO – Basic oxide

Be(OH)₂ – Amphoteric hydroxide

Sr(OH)₂ – Basic hydroxide

10. The stepwise formation of $[\text{Cu}(\text{NH}_3)_4]^{2+}$ is given below:



The value of stability constants K_1 , K_2 , K_3 and K_4 are 10^4 , 1.58×10^3 , 5×10^2 and 10^2 respectively. The overall equilibrium constants for dissociation of $[\text{Cu}(\text{NH}_3)_4]^{2+}$ is $x \times 10^{-12}$. The value of x is _____. (Rounded off to the nearest integer)

Answer (1)

Sol. K_f of $[\text{Cu}(\text{NH}_3)_4]^{2+} = K_1 \cdot K_2 \cdot K_3 \cdot K_4$

∴ Dissociation constant of $[\text{Cu}(\text{NH}_3)_4]^{2+} = \frac{1}{K_f}$

$$= \frac{1}{10^4 \times 1.58 \times 10^3 \times 5 \times 10^2 \times 10^2}$$

$$= \frac{1}{7.9} \times 10^{-11}$$

$$= 1.26 \times 10^{-12}$$

$$\approx 1 \times 10^{-12}$$

PART-C : MATHEMATICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. If $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx = a \sin^{-1} \left(\frac{\sin x + \cos x}{b} \right) + c$, where c is a constant of integration, then the ordered pair (a, b) is equal to :
- (1) $(-1, 3)$ (2) $(3, 1)$
(3) $(1, -3)$ (4) $(1, 3)$

Answer (4)

Sol. $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx = \int \frac{\cos x - \sin x}{\sqrt{9 - (\sin x + \cos x)^2}} dx$

let $\sin x + \cos x = t$
 $(\cos x - \sin x)dx = dt$

$= \int \frac{dt}{\sqrt{9 - t^2}}$
 $= \sin^{-1} \left(\frac{t}{3} \right) + c$
 $= \sin^{-1} \left(\frac{\sin x + \cos x}{3} \right) + c$

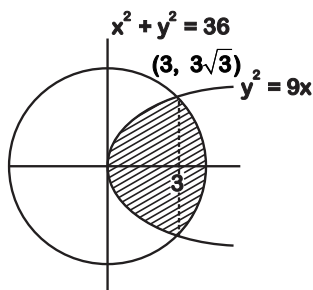
Hence $(a, b) = (1, 3)$

2. The area (in sq. units) of the part of the circle $x^2 + y^2 = 36$, which is outside the parabola $y^2 = 9x$, is :
- (1) $12\pi + 3\sqrt{3}$ (2) $24\pi + 3\sqrt{3}$
(3) $12\pi - 3\sqrt{3}$ (4) $24\pi - 3\sqrt{3}$

Answer (4)

Sol. Area of the shaded region

$= 2 \left[\int_0^3 3\sqrt{x} dx + \int_3^6 \sqrt{36 - x^2} dx \right]$



$= 2 \left[2x^{3/2} \Big|_0^3 + \left(\frac{1}{2} x \sqrt{36 - x^2} + 18 \sin^{-1} \left(\frac{x}{6} \right) \right) \Big|_3^6 \right]$
 $= 2 \left[6\sqrt{3} + 9\pi - \frac{9\sqrt{3}}{2} - 3\pi \right] = 3\sqrt{3} + 12\pi$

Required area $= 36\pi - (3\sqrt{3} + 12\pi)$
 $= 24\pi - 3\sqrt{3}$

3. If $e^{(\cos^2 x + \cos^4 x + \cos^6 x + \dots)} \log_e 2$ satisfies the equation $t^2 - 9t + 8 = 0$, then the value of $\frac{2 \sin x}{\sin x + \sqrt{3} \cos x}$ ($0 < x < \frac{\pi}{2}$) is :

- (1) $\frac{3}{2}$ (2) $\sqrt{3}$
(3) $2\sqrt{3}$ (4) $\frac{1}{2}$

Answer (4)

Sol. $e^{\left(\frac{\cos^2 x}{1 - \cos^2 x} \right) \ln 2} = 2^{\cot^2 x}$

$\therefore t = 1$ or 8

So, $2^{\cot^2 x} = 2^0$ or $2^3 \Rightarrow \cot^2 x = 0$ or 3

$\therefore x \in \left(0, \frac{\pi}{2} \right)$ then $\cot x = \sqrt{3} \Rightarrow x = \frac{\pi}{6}$

$\frac{2 \sin x}{\sin x + \sqrt{3} \cos x} = \frac{2 \left(\frac{1}{2} \right)}{\frac{1}{2} + \sqrt{3} \cdot \left(\frac{\sqrt{3}}{2} \right)} = \frac{1}{2}$

4. The population $P = P(t)$ at time 't' of a certain species follows the differential equation $\frac{dP}{dt} = 0.5P - 450$. If $P(0) = 850$, then the time at which population becomes zero is :

- (1) $\log_e 18$ (2) $\frac{1}{2} \log_e 18$
(3) $\log_e 9$ (4) $2 \log_e 18$

Answer (4)

Sol. $\therefore \frac{dP}{dt} = \frac{1}{2}(P - 900)$

$$\Rightarrow \frac{dP}{P-900} = \frac{1}{2} dt \Rightarrow \ln|P-900| = \frac{1}{2}t + c$$

When $t = 0$, $P = 850 \Rightarrow c = \ln 50$

When $P = 0$, $t = 2(\ln 900 - \ln 50) = 2\ln 18$

5. The statement among the following that is a tautology is :

- (1) $A \vee (A \wedge B)$ (2) $B \rightarrow [A \wedge (A \rightarrow B)]$
 (3) $[A \wedge (A \rightarrow B)] \rightarrow B$ (4) $A \wedge (A \vee B)$

Answer (3)

Sol. (1) $A \vee (A \wedge B) = A$

(2) $\because A \wedge (A \rightarrow B) = A \wedge (\sim A \vee B) = A \wedge B$

So, $B \rightarrow (A \wedge B) = \sim B \vee (A \wedge B) = \sim B \vee A$

(3) $(A \wedge (A \rightarrow B)) \rightarrow B = (A \wedge B) \rightarrow B = \sim (A \wedge B) \vee B = \sim A \vee \sim B \vee B$ (Tautology)

(4) $A \wedge (A \vee B) = A$

6. Let p and q be two positive numbers such that $p + q = 2$ and $p^4 + q^4 = 272$. Then p and q are roots of the equation :

- (1) $x^2 - 2x + 2 = 0$ (2) $x^2 - 2x + 8 = 0$
 (3) $x^2 - 2x + 136 = 0$ (4) $x^2 - 2x + 16 = 0$

Answer (4)

Sol. $\because p^4 + q^4 = (p + q)^4 - 4pq(p^2 + q^2) - 6p^2q^2$

$$\Rightarrow 272 = 16 - 4pq(4 - 2pq) - 6p^2q^2$$

$$\Rightarrow 2p^2q^2 - 16pq - 256 = 0$$

$$\Rightarrow pq = -8 \text{ or } 16$$

$$\because p, q > 0, \text{ so } pq = 16$$

Required quadratic equation is

$$x^2 - 2x + 16 = 0$$

7. The system of linear equations

$$3x - 2y - kz = 10$$

$$2x - 4y - 2z = 6$$

$$x + 2y - z = 5m$$

is inconsistent if :

(1) $k \neq 3, m \neq \frac{4}{5}$ (2) $k \neq 3, m \in \mathbb{R}$

(3) $k = 3, m = \frac{4}{5}$ (4) $k = 3, m \neq \frac{4}{5}$

Answer (4)

Sol. Here $\Delta = \begin{vmatrix} 3 & -2 & -k \\ 2 & -4 & -2 \\ 1 & 2 & -1 \end{vmatrix} = 0$

$$\Rightarrow 3(4 + 4) + 2(-2 + 2) - k(4 + 4) = 0$$

$$\Rightarrow 24 + 0 - 8k = 0 \Rightarrow \boxed{k = 3}$$

Now,

$$\Delta_1 = \begin{vmatrix} 10 & -2 & -3 \\ 6 & -4 & -2 \\ 5m & 2 & -1 \end{vmatrix} = 10(4+4) + 2(-6+10m) - 3(12+20m)$$

$$= 80 - 12 + 20m - 36 - 60m$$

$$= 32 - 40m$$

$$\Delta_2 = \begin{vmatrix} 3 & 10 & -3 \\ 2 & 6 & -2 \\ 1 & 5m & -1 \end{vmatrix} = 3(-6+10m) - 10(-2+2) - 3(10m-6)$$

$$= -18 + 30m + 0 - 30m + 18 = 0$$

$$\Delta_3 = \begin{vmatrix} 3 & -2 & 10 \\ 2 & -4 & 6 \\ 1 & 2 & 5m \end{vmatrix} = 3(-20m-12) + 2(10m-6) + 10(4+4)$$

$$= -60m - 36 + 20m - 12 + 80$$

$$= -40m + 32$$

For inconsistent we have $k = 3$, &

$$32 - 40m \neq 0 \Rightarrow m \neq \frac{4}{5}$$

8. A scientific committee is to be formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians. Then the number of ways, the committee can be formed, is :

- (1) 560 (2) 1050
 (3) 1625 (4) 575

Answer (3)

Sol. Indians = 6, Foreigners = 8

According to questions

The no. of ways to form the committee are

(2I, 4F) or (3I, 6F) or (4I, 8F)

$$\Rightarrow {}^6C_2 \times {}^8C_4 + {}^6C_3 \times {}^8C_6 + {}^6C_4 \times {}^8C_8$$

$$= 15 \times 70 + 20 \times 28 + 15 \times 1$$

$$= 1625$$

9. The equation of the plane passing through the point (1, 2, -3) and perpendicular to the planes $3x + y - 2z = 5$ and $2x - 5y - z = 7$, is:

- (1) $3x - 10y - 2z + 11 = 0$
 (2) $6x - 5y - 2z - 2 = 0$
 (3) $6x - 5y + 2z + 10 = 0$
 (4) $11x + y + 17z + 38 = 0$

Answer (4)

Sol. The given planes are $3x + y - 2z = 5$... (1)
 $2x - 5y - z = 7$... (2)
 Since the required plane passes through $(1, 2, -3)$
 So equation of this plane is
 $a(x - 1) + b(y - 2) + c(z + 3) = 0$... (3)
 Now this plane (3) is \perp to the planes (1) & (2)
 So $3a + b - 2c = 0$
 & $2a - 5b - c = 0$

$$\Rightarrow \frac{a}{-11} = \frac{b}{-1} = \frac{c}{-17}$$

So equation of plane is $11(x - 1) + (y - 2) + 17(2 + 3) = 0$

$$\Rightarrow 11x + y + 17z + 38 = 0$$

10. If the tangent to the curve $y = x^3$ at the point $P(t, t^3)$ meets the curve again at Q, then the ordinate of the point which divides PQ internally in the ratio 1 : 2 is :

- (1) $-2t^3$ (2) $2t^3$
 (3) 0 (4) $-t^3$

Answer (1)

Sol. Curve is $y = x^3$... (1)

So equation of tangent at (t, t^3)

$$(y - t^3) = 3t^2(x - t) \quad \dots (2)$$

\therefore It meets again the curve at Q

So solving (1) & (2) we get

$$x = -2t \quad \Rightarrow \quad Q = (-2t, -8t^3)$$

Now by section formula

$$\begin{aligned} \text{Ordinate} &= \frac{2t^3 - 8t^3}{1+2} \\ &= \frac{-6t^3}{3} \\ &= -2t^3 \end{aligned}$$

11. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = 2x - 1$ and

$$g: \mathbb{R} - \{1\} \rightarrow \mathbb{R} \text{ be defined as } g(x) = \frac{x-1}{x-1}$$

Then the composition function $f(g(x))$ is :

- (1) neither one-one nor onto
 (2) onto but not one-one
 (3) both one-one and onto
 (4) one-one but not onto

Answer (4)

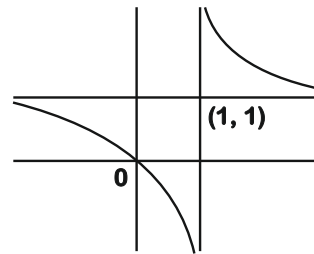
Sol. Here $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = 2x - 1$

$$\text{and } g: \mathbb{R} - \{1\} \rightarrow \mathbb{R} \quad g(x) = \frac{x-1}{x-1}$$

So, $f(g(x)) = 2g(x) - 1$

$$\begin{aligned} &= 2 \left(\frac{x-1}{x-1} \right) - 1 \\ &= \frac{2x-1-x+1}{x-1} = \frac{x-1+1}{x-1} \\ &= 1 + \frac{1}{x-1} \end{aligned}$$

So Clearly it is one-one but not onto



12. $\lim_{x \rightarrow 0} \frac{\int_0^{x^2} (\sin \sqrt{t}) dt}{x^3}$ is equal to :

- (1) 0
 (2) $\frac{1}{15}$
 (3) $\frac{2}{3}$
 (4) $\frac{3}{2}$

Answer (3)

Sol. $\lim_{x \rightarrow 0} \frac{\int_0^{x^2} (\sin \sqrt{t}) dt}{x^3} = \frac{0}{0}$ (form)

\Rightarrow By D, L Hospital rule

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{2x \sin x}{3x^2} &= \frac{2}{3} \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) \\ &= \frac{2}{3} \times 1 = \frac{2}{3} \end{aligned}$$

13. An ordinary dice is rolled for a certain number of times. If the probability of getting an odd number 2 times is equal to the probability of getting an even number 3 times, then the probability of getting an odd number for odd number of times is :

- (1) $\frac{3}{16}$ (2) $\frac{1}{2}$
(3) $\frac{1}{32}$ (4) $\frac{5}{16}$

Answer (2)

Sol. Let number of trials be 'n'
given

$${}^n C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{n-2} = {}^n C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{n-3}$$

$$\Rightarrow n = 5$$

Probability of getting odd number for odd number of times is

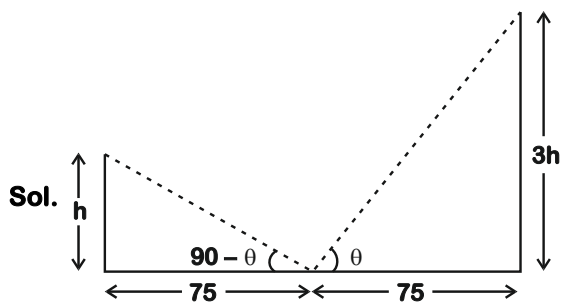
$$= ({}^5 C_1 + {}^5 C_3 + {}^5 C_5) \frac{1}{2^5}$$

$$= \frac{2^4}{2^5} = \frac{1}{2}$$

14. Two vertical poles are 150 m apart and the height of one is three times that of the other. If from the middle point of the line joining their feet, an observer finds the angles of elevation of their tops to be complementary, then the height of the shorter pole (in meters) is :

- (1) $25\sqrt{3}$ (2) 30
(3) 25 (4) $20\sqrt{3}$

Answer (1)



Sol. Given $\tan \theta = \frac{3h}{75}$... (i)

and $\tan(90 - \theta) = \frac{h}{75}$... (ii)

\Rightarrow Multiplying (i) and (ii) we get,

$$1 = \frac{3h^2}{(75)^2}$$

$$\Rightarrow h = 25\sqrt{3}$$

15. If $f : \mathbb{R} \rightarrow \mathbb{R}$ is a function defined by $f(x) = [x - 1]$

$$\cos\left(\frac{2x-1}{2}\right)\pi, \text{ where } [\cdot] \text{ denotes the greatest}$$

integer function, then f is :

- (1) discontinuous only at $x = 1$
(2) continuous for every real x
(3) discontinuous at all integral values of x except at $x = 1$
(4) continuous only at $x = 1$

Answer (2)

Sol. $f(x) = [x - 1] \cos\left(\frac{2x-1}{2}\right)\pi$

at $x = 1$

$$\lim_{x \rightarrow 1^-} [x - 1] \cos\left(\frac{2x-1}{2}\right)\pi = 0$$

$$\lim_{x \rightarrow 1^+} [x - 1] \cos\left(\frac{2x-1}{2}\right)\pi = 0$$

$$f(1) = 0$$

at any general integer $x = k$

$$\lim_{x \rightarrow k^-} [x - 1] \cos\left(\frac{2k-1}{2}\right)\pi = 0$$

$$\lim_{x \rightarrow k^+} [x - 1] \cos\left(\frac{2k-1}{2}\right)\pi = 0$$

$$f(k) = 0$$

$\therefore f(x)$ is continuous $\forall x \in \mathbb{R}$

16. The distance of the point (1, 1, 9) from the point

of intersection of the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$

and the plane $x + y + z = 17$ is:

- (1) 38 (2) $19\sqrt{2}$
(3) $2\sqrt{19}$ (4) $\sqrt{38}$

Answer (4)

Sol. Let a point $P(\lambda)$ on the line

$$\Rightarrow \frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2} = \lambda$$

$$\therefore P(\lambda + 3, 2\lambda + 4, 2\lambda + 5)$$

as P also satisfies the given plane

$$\lambda + 3 + 2\lambda + 4 + 2\lambda + 5 = 17$$

$$\Rightarrow 5\lambda = 5 \Rightarrow \lambda = 1$$

$$\therefore P \equiv (4, 6, 7)$$

Distance from (1, 1, 9) is

$$\sqrt{(4-1)^2 + (6-1)^2 + (7-9)^2}$$

$$= \sqrt{9+25+4} = \sqrt{38}$$

17. The locus of the mid-point of the line segment joining the focus of the parabola $y^2 = 4ax$ to a moving point of the parabola, is another parabola whose directrix is

(1) $x = a$

(2) $x = \frac{a}{2}$

(3) $x = 0$

(4) $x = -\frac{a}{2}$

Answer (3)

Sol. Let the moving point be $P(at^2, 2at)$

Focus of given parabola is $(a, 0)$

Let point of required locus (h, k)

$$\therefore \frac{at^2 + a}{2} = h \quad \dots(i)$$

$$\text{and } \frac{2at + 0}{2} = k \quad \dots(ii)$$

$$\Rightarrow \frac{a}{2}(t^2 + 1) = h \quad \dots(iii)$$

$$\text{and } t = \frac{k}{a} \quad \dots(iv)$$

By (iii) and (iv) we have

$$\frac{a}{2} \left(\frac{k^2}{a^2} + 1 \right) = h$$

Locus is $k^2 + a^2 = 2ah$

$$\Rightarrow y^2 = 2a \left(x - \frac{a}{2} \right)$$

$$\text{Equation of directrix } x - \frac{a}{2} + \frac{a}{2} = 0$$

$$\Rightarrow x = 0$$

18. A man is walking on a straight line. The arithmetic mean of the reciprocals of the intercepts of this line on the coordinate axes is

$\frac{1}{4}$. Three stones A, B and C are placed at the

points (1, 1), (2, 2) and (4, 4) respectively. Then which of these stones is/are on the path of the man?

(1) C only (2) B only

(3) All the three (4) A only

Answer (2)

Sol. Let line be $\frac{x}{a} + \frac{y}{b} = 1 \quad \dots(i)$

$$\text{given } \frac{\frac{1}{a} + \frac{1}{b}}{2} = \frac{1}{4}$$

$$\Rightarrow \frac{1}{a} + \frac{1}{b} = \frac{1}{2} \quad \dots(ii)$$

By (i) and (ii), we get

$$\frac{x}{a} + \left(\frac{1}{2} - \frac{1}{a} \right) y = 1$$

$$\Rightarrow \lambda(x - y) + \left(\frac{y}{2} - 1 \right) = 0$$

\therefore Represents family of line passing through (2, 2)

19. The function

$$f(x) = \frac{4x^3 - 3x^2}{6} - 2\sin x + (2x - 1)\cos x :$$

(1) Decreases in $\left[\frac{1}{2}, \infty \right)$

(2) Decreases in $\left(-\infty, \frac{1}{2} \right]$

(3) Increases in $\left(-\infty, \frac{1}{2} \right]$

(4) Increases in $\left[\frac{1}{2}, \infty \right)$

Answer (4)

Sol. $\therefore f(x) = \frac{4x^3 - 3x^2}{6} - 2\sin x + (2x - 1)\cos x$

On differentiating both sides w.r.t. x we get

$$f'(x) = \frac{12x^2 - 6x}{6} - 2\cos x - (2x - 1)\sin x + 2\cos x$$

$$f'(x) = 2x^2 - x - (2x - 1)\sin x$$

$$f'(x) = (2x - 1)(x - \sin x)$$

When $x > \frac{1}{2}$, $2x - 1 > 0$ and $x - \sin x > 0$.

$$\therefore f'(x) > 0 \text{ if } x > \frac{1}{2}$$

$f(x)$ is increasing in $\left[\frac{1}{2}, \infty\right)$

20. The value of

$${}^{-15}C_1 + 2 \cdot {}^{15}C_2 - 3 \cdot {}^{15}C_3 + \dots - 15 \cdot {}^{15}C_{15} \\ + {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + \dots + {}^{14}C_{11} \text{ is}$$

- (1) 2^{14} (2) $2^{16} - 1$
(3) $2^{13} - 13$ (4) $2^{13} - 14$

Answer (4)

Sol. $\therefore (1+x)^{14} = {}^{14}C_0 + {}^{14}C_1x + {}^{14}C_2x^2 + \dots + {}^{14}C_{14}x^{14}$... (i)

$$\therefore 2^{14} = {}^{14}C_0 + {}^{14}C_1 + {}^{14}C_2 + \dots + {}^{14}C_{14} \text{ ... (ii)}$$

$$0 = {}^{14}C_0 - {}^{14}C_1 + {}^{14}C_2 - \dots + {}^{14}C_{14} \text{ ... (iii)}$$

$$\therefore {}^{14}C_1 + {}^{14}C_3 + \dots + {}^{14}C_{13} = 2^{13} \text{ ... (iv)}$$

and $(1-x)^{15} = {}^{15}C_0 - {}^{15}C_1x + {}^{15}C_2x^2 - \dots + {}^{15}C_{15}x^{15}$

Differentiate w.r.t. x we get

$$-15(1-x)^{14} = -{}^{15}C_1 + 2 \cdot {}^{15}C_2x - \dots - 15 \cdot {}^{15}C_{15}x^{14}$$

Put $x = 1$, we get

$$-15 \cdot {}^{15}C_1 + 2 \cdot {}^{15}C_2 - 3 \cdot {}^{15}C_3 + \dots - 15 \cdot {}^{15}C_{15} = 0 \text{ ... (v)}$$

From equation (iv) + equation (v) we get

$$-15 \cdot {}^{15}C_1 + 2 \cdot {}^{15}C_2 + \dots - 15 \cdot {}^{15}C_{15} + {}^{14}C_1 \\ + {}^{14}C_3 + \dots + {}^{14}C_{11} \\ = 2^{13} - 14 \\ = 2^{13} - 14$$

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Let $A = \{n \in \mathbb{N} : n \text{ is a 3-digit number}\}$

$$B = \{9k + 2 : k \in \mathbb{N}\}$$

and $C = \{9k + l : k \in \mathbb{N}\}$ for some l ($0 < l < 9$)

If the sum of all the elements of the set $A \cap (B \cup C)$ is 274×400 , then l is equal to

_____.

Answer (5)

Sol. Sum of all elements of $A \cap (B \cup C)$ is

$$274 \times 400 = \sum_{k=0}^{99} \{(99 + 9k + l) + (99 + 9k + 2)\}$$

$$\Rightarrow 274 \times 400 = 200 \times 100 + 100l + 18 \cdot \left(\frac{99 \times 100}{2}\right)$$

$$\Rightarrow 274 \times 4 = 200 + l + 9 \times 99$$

$$\Rightarrow l = 5$$

2. If $\int_{-a}^a (|x| + |x-2|) dx = 22$, ($a > 2$) and $[x]$

denotes the greatest integer $\leq x$, then

$$\int_a^{-a} (x + [x]) dx \text{ is equal to } \underline{\hspace{2cm}}.$$

Answer (3)

Sol. $\int_{-a}^a (|x| + |x-2|) dx = \frac{1}{2}a^2 + \frac{1}{2}a^2 + \frac{1}{2}(a-2)^2$

$$+ \frac{1}{2}(a+2)^2$$

$$\Rightarrow 22 = 2a^2 + 4 \Rightarrow a = 3$$

Now,

$$\int_3^{-3} (x + [x]) dx = - \int_{-3}^3 [x] dx = - \int_{-3}^{-2} [x] dx - \int_{-2}^{-1} [x] dx - \int_{-1}^0 [x] dx - \int_0^1 [x] dx - \int_1^2 [x] dx - \int_2^3 [x] dx$$

3. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in \mathbb{R}$. Suppose

$Q = [q_{ij}]$ is a matrix satisfying $PQ = kI_3$ for some non-zero $k \in \mathbb{R}$. If $q_{23} = -\frac{k}{8}$ and $|Q| = \frac{k^2}{2}$, then $\alpha^2 + k^2$ is equal to _____.

Answer (17)

Sol. $\therefore Q = k.P^{-1}$ and $|P||Q| = k^3$, $|Q| = \frac{k^2}{2}$ then $|P| = 2k$

$\therefore q_{23} = \frac{kC_{32}}{|P|}$ (Where C_{ij} is co-factor of P_{ij} of P)

$-\frac{k}{8} = -\frac{(3\alpha+4)k}{2k} \Rightarrow 3\alpha+4 = \frac{k}{4} \dots(1)$

Also $|P| = 2k \Rightarrow 12\alpha + 20 = 2k \Rightarrow k = 6\alpha + 10 \dots(2)$

From (1) and (2) we get

$k = 4$ and $\alpha = -1$
then $k^2 + \alpha^2 = 17$

4. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord of another circle 'C', whose center is at (2, 1), then its radius is _____.

Answer (3)

Sol. Circle $x^2 + y^2 - 2x - 6y + 6 = 0$ has centre $O_1(1, 3)$ and radius $r_1 = 2$.

Let centre $O_2(2, 1)$ of required circle and its radius being R .

So $R^2 = O_1O_2^2 = r^2$
 $\Rightarrow R^2 = 5 + 4$
 $\Rightarrow R = 3$

5. Let B_i ($i = 1, 2, 3$) be three independent events in a sample space. The probability that only B_1 occurs is α , only B_2 occurs is β and only B_3 occurs is γ . Let p be the probability that none of the events B_i occurs and these 4 probabilities satisfy the equations $(\alpha - 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$ (All the probabilities are assumed to lie

in the interval (0, 1)). Then $\frac{P(B_1)}{P(B_3)}$ is equal to _____.

Answer (06)

Sol. Let $P(B_1) = x, P(B_2) = y, P(B_3) = z$

$\alpha = P(B_1 \cap \bar{B}_2 \cap \bar{B}_3) = P(B_1)P(\bar{B}_2)P(\bar{B}_3)$
 $\Rightarrow \alpha = x(1-y)(1-z) \dots(i)$
Similarly $\beta = (1-x)y(1-z) \dots(ii)$
 $\gamma = (1-x)(1-y)z \dots(iii)$
 $p = (1-x)(1-y)(1-z) \dots(iv)$

(i) & (iv) $\Rightarrow \frac{x}{1-x} = \frac{\alpha}{p} \Rightarrow x = \frac{\alpha}{\alpha+p}$

(iii) & (iv) $\Rightarrow \frac{z}{1-z} = \frac{\gamma}{p} \Rightarrow z = \frac{\gamma}{\gamma+p}$

$\frac{P(B_1)}{P(B_3)} = \frac{x}{z} = \frac{\frac{\alpha}{\alpha+p}}{\frac{\gamma}{\gamma+p}} = \frac{1+\frac{p}{\gamma}}{1+\frac{p}{\alpha}} \dots(v)$

Given that,

$(\alpha - 2\beta)p = \alpha\beta \Rightarrow \alpha p = (\alpha + 2\beta)\beta \dots(vi)$

$(\beta - 3\gamma)p = 2\beta\gamma \Rightarrow 3\gamma p = (p - 2\gamma)\beta \dots(vii)$

(vi) & (vii) $\Rightarrow \frac{\alpha}{3\gamma} = \frac{\alpha+2\beta}{p-2\gamma}$

$\Rightarrow p\alpha - 6\beta\gamma = 5\gamma\alpha$

$\frac{p}{\gamma} - \frac{6\beta}{\alpha} = 5$

$\frac{p}{\gamma} + 1 = 6\left(\frac{p}{\alpha} + 1\right) \dots(viii)$

(v) & (viii) $\Rightarrow \frac{P(B_1)}{P(B_3)} = 6$

6. If the least and the largest real values of α , for which the equation $z + \alpha|z - 1| + 2i = 0$ ($z \in \mathbb{C}$ and $i = \sqrt{-1}$) has a solution, are p and q respectively; then $4(p^2 + q^2)$ is equal to _____.

Answer (10*)

Sol. Let $z = x + iy, x, y \in \mathbb{R}$

$\therefore z + \alpha|z - 1| + 2i = 0.$

$\Rightarrow x + i(y + 2) + \alpha\sqrt{(x-1)^2 + y^2} = 0.$

$\therefore y = -2$ and $x + \alpha\sqrt{(x-1)^2 + 4} = 0.$

Now $\alpha\sqrt{(x-1)^2 + 4} = -x$ *

Squaring both sides : $x^2 = \alpha^2((x-1)^2 + 4)$

$\therefore (\alpha^2 - 1)x^2 - 2\alpha^2x + 5\alpha^2 = 0.$

For real x, $D \geq 0.$

$4\alpha^4 - 4 \cdot 5\alpha^2(\alpha^2 - 1) \geq 0.$

$\alpha^2(5 - 4\alpha^2) \geq 0.$

$\therefore \alpha \in \left[\frac{-\sqrt{5}}{2}, \frac{\sqrt{5}}{2} \right]$

$\therefore \alpha_{\min} = \frac{-\sqrt{5}}{2} = -p$ and $\alpha_{\max} = \frac{\sqrt{5}}{2} = q$

$\Rightarrow 4(p^2 + q^2) = 10.$

*The step is not permissible as squaring may include extraneous roots.

No maximum value of α exists as is shown in the below procedure.

Let $z = x + iy$

$x + \alpha\sqrt{(x-1)^2 + y^2} + i(y+2) = 0$

$\Rightarrow y = -2$ and $\alpha = \frac{-x}{\sqrt{x^2 - 2x + 5}}$

$\frac{d\alpha}{dx} = \frac{(x^2 - x) - (x^2 - 2x + 5)}{(x^2 - 2x + 5)^{3/2}} = \frac{x - 5}{(x^2 - 2x + 5)^{3/2}}$

So α is decreasing in $(-\infty, 5)$ and increasing in $(5, \infty)$

$\alpha_{\min} = -\frac{5}{\sqrt{20}} = 1\sqrt{\frac{5}{4}} = p$ (at $x = 5$)

and $\alpha_{\max} = \lim_{x \rightarrow -\infty} \frac{-x}{\sqrt{x^2 - 2x + 5}} = 1$ $q = 1$ (however this value is not achievable.)

7. The minimum value of α for which the equation

$\frac{4}{\sin x} + \frac{1}{1 - \sin x} = \alpha$ has at least one solution in

$\left(0, \frac{\pi}{2}\right)$ is _____.

Answer (9)

Sol. Let $f(x) = \frac{4}{\sin x} + \frac{1}{1 - \sin x}$ where $\sin x \in (0, 1)$

$\therefore \frac{\frac{2}{\sin x} + \frac{2}{\sin x} + \frac{1}{1 - \sin x}}{3} \geq \frac{3}{\frac{\sin x}{2} + \frac{\sin x}{2} + 1 - \sin x}$

$\Rightarrow f(x) \geq 9$

So least value of α is 9.

8. Let M be any 3×3 matrix with entries from the set $(0, 1, 2)$. The maximum number of such matrices, for which the sum of diagonal elements of $M^T M$ is seven is _____.

Answer (540)

Sol. Let $\{a_{ij}\}_{3 \times 3}$

$\text{Tr}(M^T \cdot M) = \sum_{i=1}^3 \sum_{j=1}^3 a_{ij}^2 = 7$

So there will be two cases.

Case I : Any seven a_{ij} s are 1 and remaining two elements are zero.

Number of such matrices $M = \frac{9}{7!2} = 36$

Case II : Any one elements is 2, any three elements are 1 and remaining elements are 0.

Number of such matrices = $\frac{9}{1!3!5} = 504$

Total number of possible matrices $M = 540.$

9. $\lim_{n \rightarrow \infty} \tan \left\{ \sum_{r=1}^n \tan^{-1} \left(\frac{1}{1+r+r^2} \right) \right\}$ is equal to _____.

Answer (1)

Sol. $\tan^{-1} \left(\frac{1}{1+r+r^2} \right) = \tan^{-1} \left(\frac{(r+1) - r}{1+(r+1)r} \right)$

$= \tan^{-1}(r+1) - \tan^{-1}r$

So $\sum_{r=1}^n \tan^{-1} \left(\frac{1}{1+r+r^2} \right) = (\tan^{-1}2 - \tan^{-1}1)$

$+ (\tan^{-1}3 - \tan^{-1}2) + \dots + (\tan^{-1}(n+1) - \tan^{-1}n)$

$$= \tan^{-1}(n + 1) - \tan^{-1}1$$

$$\Rightarrow \lim_{n \rightarrow \infty} \tan \left\{ \sum_{r=1}^n \tan^{-1} \left(\frac{1}{1+r+r^2} \right) \right\}$$

$$\lim_{n \rightarrow \infty} \tan \left(\tan^{-1}(n+1) - \tan^{-1}1 \right)$$

$$= \tan \left(\frac{\pi}{2} - \frac{\pi}{4} \right) = 1$$

10. Let three vectors \vec{a} , \vec{b} and \vec{c} be such that \vec{c} is coplanar with \vec{a} and \vec{b} , $\vec{a} \cdot \vec{c} = 7$ and \vec{b} is perpendicular to \vec{c} , where $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{k}$, then the value of $2|\vec{a} + \vec{b} + \vec{c}|^2$ is _____.

Answer (75)

Sol. Let $\vec{c} = \lambda\vec{a} + \mu\vec{b}$ ($\because \vec{a}, \vec{b}$ and \vec{c} are coplanar)

$$\therefore \vec{a} \cdot \vec{c} = 7, \vec{b} \cdot \vec{c} = 0 \text{ and } \vec{a} \cdot \vec{b} = -1$$

$$\text{So } 7 = \lambda|\vec{a}|^2 + \mu\vec{a} \cdot \vec{b} \Rightarrow 3\lambda - \mu = 7$$

$$\text{and } 0 = \lambda\vec{a} \cdot \vec{b} + \mu|\vec{b}|^2 \Rightarrow -\lambda + 5\mu = 0$$

$$\text{Clearly } \lambda = \frac{5}{2} \text{ and } \mu = \frac{1}{2} \Rightarrow \vec{c} = \frac{5\vec{a} + \vec{b}}{2}$$

$$\begin{aligned} \text{So } 2|\vec{a} + \vec{b} + \vec{c}|^2 &= 2 \left| \frac{7\vec{a} + 3\vec{b}}{2} \right|^2 \\ &= \frac{1}{2} |-\hat{i} + 7\hat{j} + 10\hat{k}|^2 \\ &= \frac{1 + 49 + 100}{2} = 75 \end{aligned}$$

