25/02/2021
Morning

Aakash
Medical | IIT-JEE | Foundations

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Time : 3 hrs. M.M. : 300

Answers & Solutions

for

JEE (MAIN)-2021 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS :

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part has two sections.

(i) Section-I : This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) Section-II : This section contains 10 questions. In Section-II, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and there is no negative marking for wrong answer.

PART–A : PHYSICS

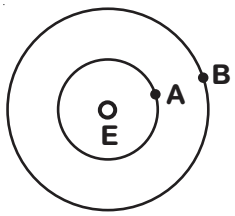
SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Two satellites A and B of masses 200 kg and 400 kg are revolving round the earth at height of 600 km and 1600 km respectively.

   If T_A and T_B are the time periods of A and B respectively then the value of $T_B - T_A$:



   [Given : radius of earth = 6400 km, mass of earth = 6×10^{24} kg]

   (1) 4.24×10^2 s (2) 1.33×10^3 s

   (3) 4.24×10^3 s (4) 3.33×10^2 s

Answer (2)

Sol. $T = 2\pi\sqrt{\frac{r^3}{GM}}$

$$\therefore \quad T_B - T_A = \frac{2\pi}{\sqrt{GM}}\left[(8\times10^6)^{3/2} - (7\times10^6)^{3/2}\right]$$

$$= \frac{2\pi}{\sqrt{6.67\times10^{-11}\times6\times10^{24}}}\times10^9\left[8^{3/2} - 7^{3/2}\right]$$

$$\approx 1300 \text{ s}$$

2. A student is performing the experiment of resonance column. The diameter of the column tube is 6 cm. The frequency of the tuning fork is 504 Hz. Speed of the sound at the given temperature is 336 m/s. The zero of the metre scale coincides with the top end of the resonance column tube. The reading of the water level in the column when the first resonance occurs is :

   (1) 13 cm (2) 16.6 cm

   (3) 14.8 cm (4) 18.4 cm

Answer (3)

Sol. $\lambda = \frac{v}{\nu} = \frac{336}{504}\,m = \frac{2}{3}m$

$$\therefore \quad \frac{\lambda}{4} = l + e$$

$$\Rightarrow \quad \frac{\frac{2}{3}\times100}{4} = l + 0.6\times3$$

$$\Rightarrow \quad l = 14.8 \text{ cm}$$

3. The coherent light sources having intensity in the ratio 2x produce an interference pattern. The ratio $\frac{I_{max} - I_{min}}{I_{max} + I_{min}}$ will be

   (1) $\frac{\sqrt{2x}}{2x+1}$ (2) $\frac{\sqrt{2x}}{x+1}$

   (3) $\frac{2\sqrt{2x}}{x+1}$ (4) $\frac{2\sqrt{2x}}{2x+1}$

Answer (4)

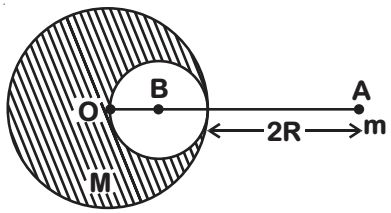
Sol. $\frac{I_2}{I_1} = 2X$

$$\therefore \quad \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{\left(\sqrt{\frac{I_2}{I_1}}+1\right)^2 - \left(\sqrt{\frac{I_2}{I_1}}-1\right)^2}{\left(\sqrt{\frac{I_2}{I_1}}+1\right)^2 + \left(\sqrt{\frac{I_2}{I_1}}-1\right)^2}$$

$$= \frac{(\sqrt{2x}+1)^2 - (\sqrt{2x}-1)^2}{(\sqrt{2x}+1)^2 + (\sqrt{2x}-1)^2}$$

$$= \frac{4\sqrt{2x}}{4x+2} = \frac{2\sqrt{2x}}{2x+1}$$

4. A solid sphere of radius R gravitationally attracts a particle placed at 3R from its centre with a force F_1. Now a spherical cavity of radius $\left(\frac{R}{2}\right)$ is made in the sphere (as shown in figure) and the force becomes F_2. The value of $F_1 : F_2$ is :



   (1) 36 : 25 (2) 50 : 41

   (3) 41 : 50 (4) 25 : 36

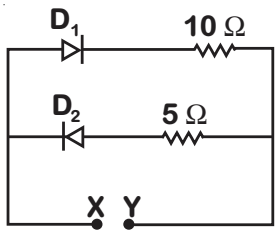
Answer (2)

Sol. $F_1 = \frac{GMm}{(3R)^2} = \frac{GMm}{9R^2}$

$$F_2 = \frac{GMm}{(3R)^2} - \frac{G\left(\frac{M}{8}\right)m}{\left(\frac{5R}{2}\right)^2} = \frac{41}{450}\frac{GMm}{R^2}$$

$$\therefore \quad \frac{F_1}{F_2} = \frac{1\times 450}{9\times 41} = \frac{50}{41}$$

5. A 5 V battery is connected across the points X and Y. Assume D_1 and D_2 to be normal silicon diodes. Find the current supplied by the battery if the +ve terminal of the battery is connected to point X.

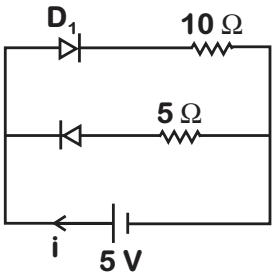


(1) ~ 1.5 A (2) ~ 0.5 A

(3) ~ 0.43 A (4) ~ 0.86 A

Answer (3)

Sol.



$$i \sim \frac{(5-0.7)}{10}$$

$$\Rightarrow \quad i \sim 0.43 \text{ A}$$

6. Match List-I with List-II :

List-I	List-II
(a) h (Planck's constant)	(i) $[ML^{-1}]$
(b) E (kinetic energy)	(ii) $[ML^2T^{-1}]$
(c) V (electric potential)	(iii) $[ML^2T^{-2}]$
(d) P (linear momentum)	(iv) $[ML^2I^{-1}T^{-3}]$

Choose the correct answer from the options given below :

(1) (a) → (i), (b) → (ii), (c) → (iv), (d) → (iii)

(2) (a) → (ii), (b) → (iii), (c) → (iv), (d) → (i)

(3) (a) → (iii), (b) → (ii), (c) → (iv), (d) → (i)

(4) (a) → (iii), (b) → (iv), (c) → (ii), (d) → (i)

Answer (2)

Sol. $h = [ET] = ML^2T^{-2} \times T$

$= ML^2T^{-1}$

$[E] = ML^2T^{-2}$

$$[V] = \left[\frac{U}{q}\right] = \frac{ML^2T^{-2}}{AI} = ML^2T^{-3}I^{-1}$$

$$[P] = M \times LT^{-1} = MLT^{-1}$$

$\therefore$ a → (ii), b → (iii), c → (iv), d → (i)

7. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: When a rod lying freely is heated, no thermal stress is developed in it.

Reason R: On heating, the length of the rod increases.

In the light of the above statements, choose the correct answer from the options given below:

(1) A is true but R is false

(2) A is false but R is true

(3) Both A and R are true but R is NOT the correct explanation of A

(4) Both A and R are true and R is the correct explanation of A

Answer (3)

Sol. In free expansion, thermal stress = 0

So, statement A is true.

Statement R is also true but it is not correct reason for A.

8. A diatomic gas, having $C_P = \frac{7}{2}R$ and $C_V = \frac{5}{2}R$, is heated at constant pressure. The ratio dU : dQ : dW :

(1) 5 : 7 : 2 (2) 3 : 5 : 2

(3) 3 : 7 : 2 (4) 5 : 7 : 3

Answer (1)

Sol. At constant pressure,

$$dQ = n \times \left(\frac{7R}{2}\right) \cdot \Delta T$$

$dU = n \times \left(\frac{5R}{2}\right) \cdot \Delta T$

$dW = n \times R \times \Delta T$

$\therefore \quad dU : dQ : dW = \frac{5}{2} : \frac{7}{2} : 1$

$= 5 : 7 : 2$

9. An α particle and a proton are accelerated from rest by a potential difference of 200 V. After this, de Broglie wavelengths are λ_α and λ_P respectively. The ratio $\frac{\lambda_P}{\lambda_\alpha}$ is :

(1) 8 (2) 2.8

(3) 3.8 (4) 7.8

Answer (2)

Sol. $\lambda_P = \frac{h}{\sqrt{2m_p \times (e \times V)}}$

$\lambda_\alpha = \frac{h}{\sqrt{2m_\alpha \times (2e \times V)}}$

$\therefore \quad \frac{\lambda_P}{\lambda_\alpha} = \sqrt{\frac{m_\alpha}{m_P} \times 2} = \sqrt{4 \times 2}$

$= 2\sqrt{2} = 2.8$

10. Two radioactive substance X and Y originally have N_1 and N_2 nuclei respectively. Half life of X is half of the half life of Y. After three half lives of Y, number of nuclei of both are equal.

The ratio $\frac{N_1}{N_2}$ will be equal to :

(1) $\frac{1}{3}$ (2) $\frac{8}{1}$

(3) $\frac{1}{8}$ (4) $\frac{3}{1}$

Answer (2)

Sol. $T_x = \frac{1}{2} \times T_y$

$\Rightarrow \quad 3T_y = 6T_x$

$\therefore \quad N_x = \frac{N_1}{2^6}$

and, $N_y = \frac{N_2}{2^3}$

$\because \quad N_x = N_y \Rightarrow \frac{N_1}{64} = \frac{N_2}{8}$

$\Rightarrow \quad \frac{N_1}{N_2} = \frac{8}{1}$

11. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : The escape velocities of planet A and B are same. But A and B are of unequal mass.

Reason R : The product of their mass and radius must be same. $M_1R_1 = M_2R_2$

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) A is not correct but R is correct

(2) Both A and R are correct but R is NOT the correct explanation of A

(3) A is correct but R is not correct

(4) Both A and R are correct and R is the correct explanation of A

Answer (3)

Sol. $V_{es} \propto \sqrt{\frac{GM}{R}}$

12. If the time period of a two meter long simple pendulum is 2 s, the acceleration due to gravity at the place where pendulum is executing S.H.M. is :

(1) 9.8 ms^{-2}

(2) $\pi^2 ms^{-2}$

(3) 16 m/s^2

(4) $2\pi^2 ms^{-2}$

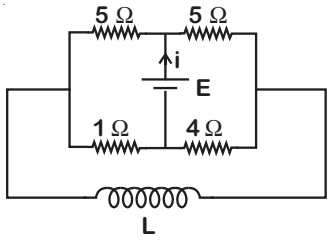
Answer (4)

Sol. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$2 = 2\pi\sqrt{\frac{2}{g}}$

$g = 2\pi^2$ m/s^2

13. The current (i) at time t = 0 and t = ∞ respectively for the given circuit is :



(1) $\frac{5E}{18}, \frac{10E}{33}$ (2) $\frac{18E}{55}, \frac{5E}{18}$

(3) $\frac{5E}{18}, \frac{18E}{55}$ (4) $\frac{10E}{33}, \frac{5E}{18}$

Answer (1)

Sol. $i(\text{at } t=0)=\frac{E}{6}+\frac{E}{9}=\frac{5E}{18}$

$$i(\text{at } t=\infty)=\frac{E}{\frac{5}{2}}+\frac{E}{\frac{4}{5}}=\frac{10E}{33}$$

14. Magnetic fields at two points on the axis of a circular coil at a distance of 0.05 m and 0.2 m from the centre are in the ratio 8 : 1. The radius of coil is ___

(1) 0.15 m

(2) 0.1 m

(3) 0.2 m

(4) 1.0 m

Answer (2)

Sol. $B \propto \frac{1}{(R^2+x^2)^{3/2}}$

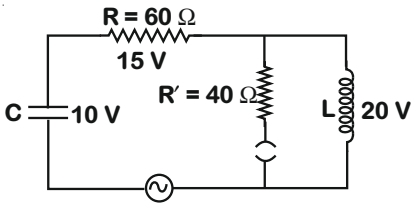
$$\frac{B_1}{B_2}=\frac{\left[R^2+(0.2)^2\right]^{3/2}}{\left[R^2+(0.05)^2\right]^{3/2}}=\frac{8}{1}$$

$$\frac{R^2+0.04}{R^2+0.0025}=\frac{4}{1}$$

$R^2 + 0.04 = 4R^2 + 0.01$

$R = 0.1$ m

15. The angular frequency of alternating current in a L-C-R circuit is 100 rad/s. The components connected are shown in the figure. Find the value of inductance of the coil and capacity of condenser.



(1) 1.33 H and 250 μF (2) 0.8 H and 150 μF

(3) 0.8 H and 250 μF (4) 1.33 H and 150 μF

Answer (3)

Sol. $\frac{X_L}{R}=\frac{4}{3}$

$X_L = 80\ \Omega$

$100 \times L = 80\ \Omega$

$L = 0.8$ H

$$\frac{1}{\omega C}=\frac{2}{3}\times 60$$

$$\frac{1}{100\times 40}=C$$

$C = 250\ \mu F$

16. A proton, a deuteron and an α particle are moving with same momentum in a uniform magnetic field. The ratio of magnetic forces acting on them is ____ and their speed is ____ in the ratio

(1) 1 : 2 : 4 and 1 : 1 : 2

(2) 1 : 2 : 4 and 2 : 1 : 1

(3) 4 : 2 : 1 and 2 : 1 : 1

(4) 2 : 1 : 1 and 4 : 2 : 1

Answer (4)

Sol.

$V=\frac{p}{m}$

$V_p : V_d : V_\alpha = 4 : 2 : 1$

$f_p : f_d : f_\alpha = q_pV_p : q_dV_d : q_\alpha V_\alpha$

$= 4 : 2 : 2$

$= 2 : 1 : 1$

17. An engine of a train, moving with uniform acceleration, passes the signal-post with velocity u and the last compartment with velocity v. The velocity with which middle point of the train passes the signal post is :

(1) $\sqrt{\frac{v^2-u^2}{2}}$ (2) $\frac{v-u}{2}$

(3) $\sqrt{\frac{v^2+u^2}{2}}$ (4) $\frac{u+v}{2}$

Answer (3)

Sol.

$v^2 - u^2 = 2ax$

Let velocity of middle point be v_1

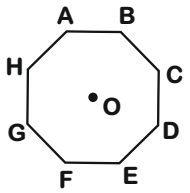
$v_1^2 - u^2 = 9x$

$v_1 = \sqrt{\frac{u^2+v^2}{2}}$

18. In an octagon ABCDEFGH of equal side, what is the sum of

$\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}+\overrightarrow{AG}+\overrightarrow{AH}$,

if $\overrightarrow{AO} = 2\hat{i}+3\hat{j}-4\hat{k}$



(1) $-16\hat{i}-24\hat{j}+32\hat{k}$ (2) $16\hat{i}+24\hat{j}+32\hat{k}$

(3) $16\hat{i}+24\hat{j}-32\hat{k}$ (4) $16\hat{i}-24\hat{j}+32\hat{k}$

Answer (3)

Sol. $\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}+\overrightarrow{AG}+\overrightarrow{AH}$

$= 8\overrightarrow{AO}$

19. Given below are two statements :

Statement I : A speech signal of 2 kHz is used to modulate a carrier signal of 1 MHz. The band width requirement for the signal is 4 kHz.

Statement II : The side band frequencies are 1002 kHz and 998 kHz.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both Statement I and Statement II are true

(2) Statement I is false but Statement II is true

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are false

Answer (1)

Sol. Side band frequency are = (1000 ± 2) kHz

Bandwidth = 4 kHz

20. The pitch of the screw gauge is 1 mm and there are 100 divisions on the circular scale. When nothing is put in between the jaws, the zero of the circular scale lies 8 divisions below the reference line. When a wire is placed between the jaws, the first linear scale division is clearly visible while 72^{nd} division on circular scale coincides with the reference line. The radius of the wire is :

(1) 0.82 mm (2) 1.64 mm

(3) 1.80 mm (4) 0.90 mm

Answer (1)

Sol. LC = 0.01 mm

Zero Error = 0.08 mm

Diameter = 1.72 – 0.08 = 1.64 mm

$\Rightarrow$ Radius = $\frac{1.64}{2}$ mm = 0.82 mm

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. A coil of inductance 2 H having negligible resistance is connected to a source of supply whose voltage is given by V = 3t volt. (where t is in second). If the voltage is applied when t = 0, then the energy stored in the coil after 4 s is _____ J.

Answer (144)

Sol. $V - L\frac{di}{dt} = 0$

$\int_0^i di = \frac{V}{L}\int_0^t dt$

$i = \frac{3}{4}t^2$

= 12 A at t = 4 second

$U = \frac{1}{2}Li^2$

= 144 J

2. The electric field in a region is given by $\vec{E}=\left(\frac{3}{5}E_0\hat{i}+\frac{4}{5}E_0\hat{j}\right)\frac{N}{C}$. The ratio of flux of reported field through the rectangular surface of area 0.2 m^2 (parallel to y – z plane) to that of the surface of area 0.3 m^2 (parallel to x – z plane) is a : b, where a = _____.

[Here, $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axes respectively]

Answer (1)

Sol. $\vec{E}=\frac{3}{5}E_0\,\hat{i}+\frac{4}{5}E_0\,\hat{j}\ \ \frac{N}{C}$

$$\phi_1=\vec{E}\cdot\vec{A}_1=\frac{6}{50}E_0$$

$$\phi_2=\vec{E}\cdot\vec{A}_2=\frac{12}{50}E_0$$

$$\frac{\phi_1}{\phi_2}=\frac{1}{2}$$

3. The same size images are formed by a convex lens when the object is placed at 20 cm or at 10 cm from the lens. The focal length of convex lens is _____ cm.

Answer (15)

Sol. Let magnification be m.

In case-I (Real Image)

$$\frac{1}{mV_1}-\frac{1}{-V_1}=\frac{1}{f}$$

In case-II (Virtual Image)

$$\frac{1}{-mV_2}-\frac{1}{-V_2}=\frac{1}{f}$$

$$\Rightarrow\quad -\frac{f}{-20+f}=+\frac{f}{-10+f}$$

f = 15 cm

4. In a certain thermodynamical process, the pressure of a gas depends on its volume as kV^3. The work done when the temperature changes from 100°C to 300°C will be _____nR, where n denotes number of moles of a gas.

Answer (50)

Sol. $P = KV^3$

PV^{-3} = constant

$$W=\frac{nR\Delta T}{1-m}$$

$$=\frac{nR\cdot 200}{1-(-3)}$$

= 50 nR

5. A small bob tied at one end of a thin string of length 1 m is describing a vertical circle so that the maximum and minimum tension in the string are in the ratio 5 : 1. The velocity of the bob at the highest position is _____ m/s. (Take g = 10 m/s^2)

Answer (5)

Sol. $\frac{T_{max}}{T_{min}}=5$

$T_{max} - T_{min} = 6$ mg

$$T_{min}=\frac{3}{2}mg$$

$$T_{min} + mg = \frac{mv^2}{l}$$

$$\Rightarrow\quad v=\sqrt{\frac{5}{2}gl}\ = 5\ m/s$$

6. The potential energy (U) of a diatomic molecule is a function dependent on r (interatomic distance) as

$$U=\frac{\alpha}{r^{10}}-\frac{\beta}{r^5}-3$$

where α and β are positive constants. The equilibrium distance between two atoms will be $\left(\frac{2\alpha}{\beta}\right)^{\frac{a}{b}}$, where a = _____.

Answer (1)

Sol. $$U=\frac{\alpha}{r^{10}}-\frac{\beta}{r^5}-3$$

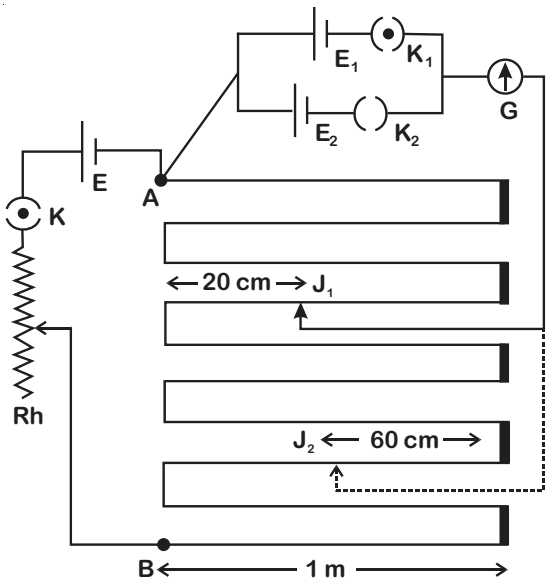
$$\frac{dU}{dr}=-\frac{10\alpha}{r^{11}}+\frac{5\beta}{r^6}$$

for equilibrium $\frac{dU}{dr}=0$

$$r=\left(\frac{2\alpha}{\beta}\right)^{\frac{1}{5}}$$

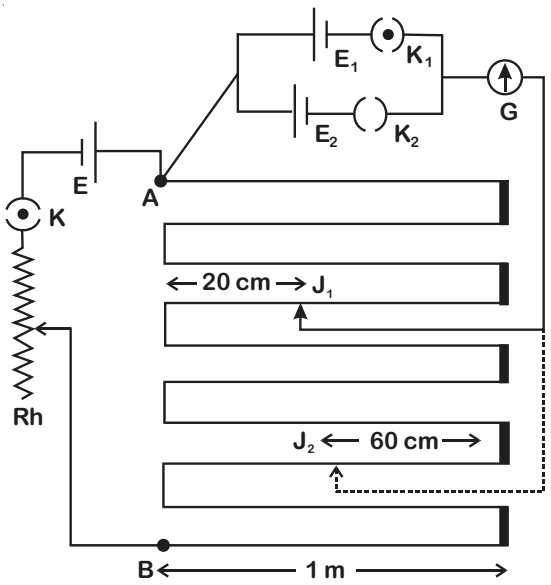
7. In the given circuit of potentiometer, the potential difference E across AB (10 m length) is larger than E_1 and E_2 as well. For key K_1 (closed), the jockey is adjusted to touch the wire at point J_1 so that there is no deflection in the galvanometer. Now the first battery (E_1) is replaced by second battery (E_2) for working by making K_1 open and K_2 closed. The galvanometer gives then null deflection at J_2.

The value of $\frac{E_1}{E_2}$ is $\frac{a}{b}$, where a = _____.



Answer (1)

Sol.



As per given circuit diagram, null deflection will not occur.

However if we reverse polarity of cell E, then as per given information.

$$\frac{E_1}{E_2}=\frac{l_1}{l_2}=\frac{3.8}{7.6}=\frac{1}{2}$$

8. A transmitting station releases waves of wavelength 960 m. A capacitor of 256 μF is used in the resonant circuit. The self inductance of coil necessary for resonance is _____ × 10^{-8} H.

Answer (10.00)

Sol. $f=\frac{C}{\lambda}=\frac{3\times10^8}{960}$

and $f=\frac{1}{2\pi\sqrt{LC}}$

$\Rightarrow L=\frac{1}{4\pi^2f^2C}$

$= 10.13 \times 10^{-8}$ H

$\sim 10.00\times10^{-8}$ H

9. 512 identical drops of mercury are charged to a potential of 2 V each. The drops are joined to form a single drop. The potential of this drop is _____ V.

Answer (128)

Sol. $\frac{Kq}{r}=V_0$

$V=\frac{KQ}{R}$, where Q = nq and $R=n^{\frac{1}{3}}r$

$V=n^{\frac{2}{3}}V_0$

$=128$ volts.

10. A monoatomic gas of mass 4.0 u is kept in an insulated container. Container is moving with velocity 30 m/s. If container is suddenly stopped then change in temperature of the gas (R = gas constant) is $\frac{x}{3R}$. Value of x is _____.

Answer (3600)

Sol. $nC_v\Delta T=\frac{1}{2}mv^2$

$\Delta T=\frac{1}{2}\left(\frac{m}{n}\right).\frac{2}{3R}(30)^2=\frac{3600}{3R}$

* Assuming mass per mole $=4\frac{kg}{mol}$.

PART–B : CHEMISTRY

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Which one of the following reactions will not form acetaldehyde?

(1) $CH_3CH_2OH \xrightarrow{CrO_3-H_2SO_4}$

(2) $CH_2 = CH_2 + O_2 \xrightarrow[H_2O]{Pd(II)/Cu(II)}$

(3) $CH_3CH_2OH \xrightarrow[573\ K]{Cu}$

(4) $CH_3CN \xrightarrow[(ii)\ H_2O]{(i)\ DIBAL\text{-}H}$

Answer (1)

Sol. $CH_3CH_2OH \xrightarrow[\text{Jones reagent}]{CrO_3-H_2SO_4} CH_3COOH$

In the rest of the options acetaldehyde will be formed.

2. Given below are two statements:

Statement-I: CeO_2 can be used for oxidation of aldehyde and ketones.

Statement-II: Aqueous solution of $EuSO_4$ is a strong reducing agent.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are false

(2) Both Statement I and Statement II are true

(3) Statement I is true but Statement II is false

(4) Statement I is false but Statement II is true

Answer (2)

Sol. Ce and Eu have stable oxidation state of +3.

So $\overset{+4}{Ce}O_2$ acts as oxidizing agent to get reduced to +3 and $\overset{+2}{Eu}SO_4$ acts as reducing agent to get oxidized to +3.

3. Given below are two statements:

Statement-I : An allotrope of oxygen is an important intermediate in the formation of reducing smog.

Statement-II: Gases such as oxides of nitrogen and sulphur present in troposphere contribute to the fomation of photochemical smog.

In the light of the above statements, choose the correct answer from the options given below:

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are false

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II are true

Answer (2)

Sol.
- Reducing smog is a mixture of smoke, fog and sulphur dioxide. It does not involve O_3 (allotrope of oxygen) during its formation.
- The main component of the photochemical smog result from the action of sunlight on unsaturated hydrocarbons and nitrogen oxides. No involvement of oxides of S.

So the answer should be, both statements false.

4. Which of the glycosidic linkage between galactose and glucose is present in lactose?

(1) C-1 of galactose and C-4 of glucose

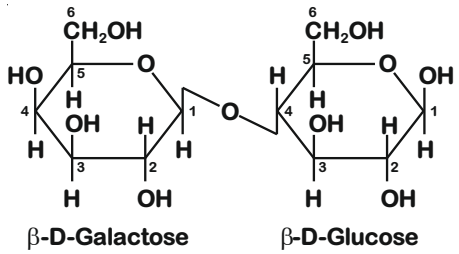
(2) C-1 of glucose and C-6 of galactose

(3) C-1 of galactose and C-6 of glucose

(4) C-1 of glucose and C-4 of galactose

Answer (1)

Sol.



Lactose

A glycosidic linkage is between C1 of β-D- galactose and C4 of β-D-glucose.

So option-1 is the correct answer.

5. In which of the following pairs, the outer most electronic configuration will be the same?

(1) Ni^{2+} and Cu^{+}

(2) Fe^{2+} and Co^{+}

(3) Cr^{+} and Mn^{2+}

(4) V^{2+} and Cr^{+}

Answer (3)

Sol. $Cr^{+} - 4s^{0}\ 3d^{5}$, $Mn^{2+} - 4s^{0}\ 3d^{5}$ } have some electronic configuration in the outer most shell

6. The major product of the following chemical reaction is:

$$CH_3CH_2CN \xrightarrow[3)\ Pd/BaSO_4, H_2]{1)\ H_3O^{+}, \Delta \ 2)\ SOCl_2} ?$$

(1) $CH_3CH_2CH_3$

(2) $CH_3CH_2CH_2OH$

(3) CH_3CH_2CHO

(4) $(CH_3CH_2CO)_2O$

Answer (3)

Sol. $CH_3CH_2CN \xrightarrow[\Delta]{H_3O^{+}} CH_3-CH_2-COOH$

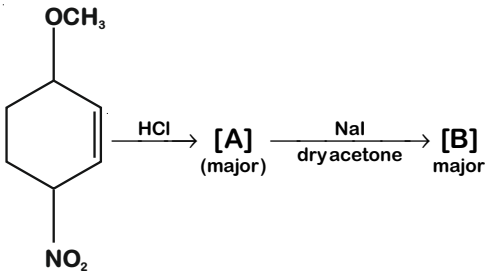
$\xrightarrow{SOCl_2} CH_3-CH_2-\overset{O}{\overset{\|}{C}}-Cl$

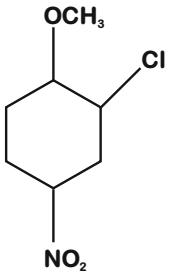
$\downarrow Pd/BaSO_4, H_2$

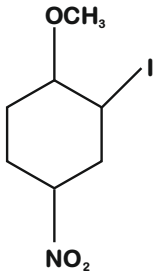
$CH_3-CH_2-\overset{O}{\overset{\|}{C}}-H$

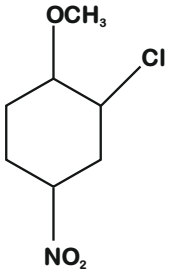
Correct option should be (3)

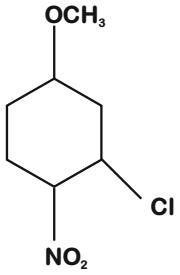
7. Identify A and B in the chemical reaction.

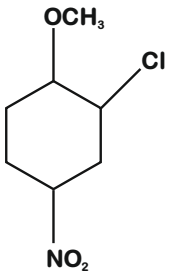


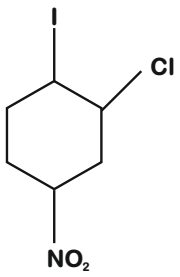
(1) A = 

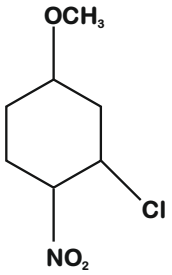
B = 

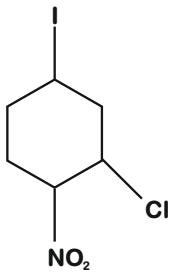
(2) A = 

B = 

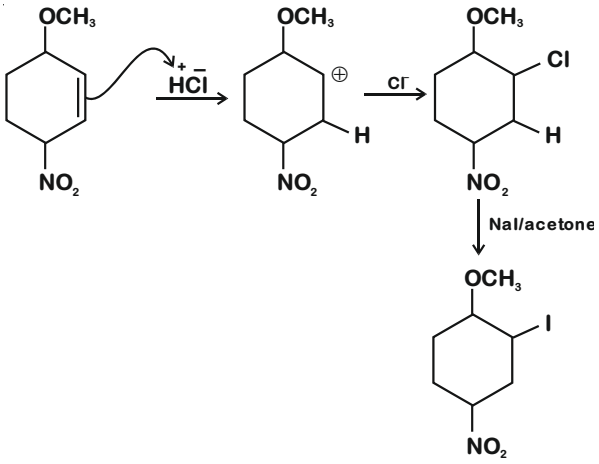
(3) A = 

B = 

(4) A = 

B = 

Answer (1)

Sol. 

Correct option should be (1)

8. Ellingham diagram is a graphical representation of :

(1) ΔH vs T

(2) ΔG vs P

(3) ΔG vs T

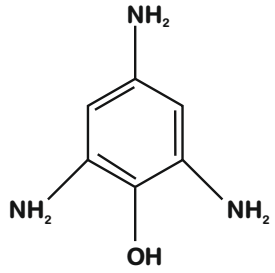
(4) $(\Delta G - T\Delta S)$ vs T

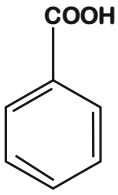
Answer (3)

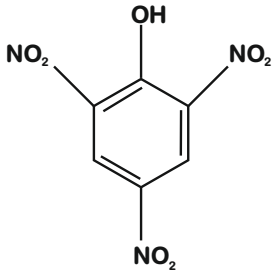
Sol. Ellingham diagram is a graphical representation of Gibbs energy ($\Delta G°$) vs T plots for the formation of the oxides.

Answer (3)

9. Compound(s) which will liberate carbon dioxide with sodium bicarbonate solution is/ are :

A = 

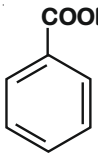
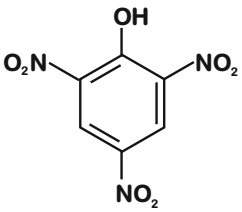
B = 

C = 

(1) B only (2) C only

(3) A and B only (4) B and C only

Answer (4)

Sol.  and  are acidic enough to liberate CO_2 with $NaHCO_3$ solution.

Answer (4)

10. In Freundlich adsorption isotherm at moderate pressure, the extent of adsorption $\left(\frac{x}{m}\right)$ is directly proportional to P^x. The value of x is:

(1) zero (2) 1

(3) ∞ (4) $\frac{1}{n}$

Answer (4)

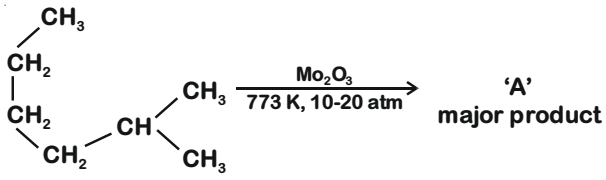
Sol. Freundlich adsorption isotherm can be plotted using

$$\frac{x}{m} = kP^{1/n}$$

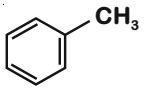
When pressure is moderate $\frac{x}{m} \propto P^{1/n}$

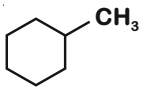
So, $x = \frac{1}{n}$

11. Identify A in the given chemical reaction.



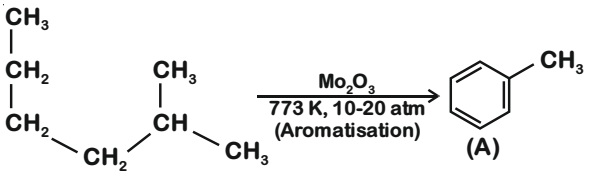
(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. 

12. According to molecular orbital theory, the species among the following that does not exist is

(1) He_2^-

(2) Be_2

(3) He_2^+

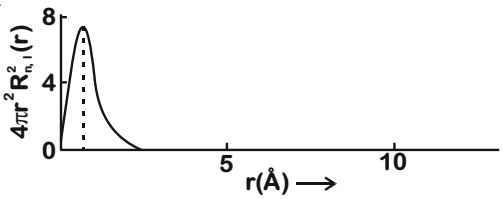
(4) O_2^{2-}

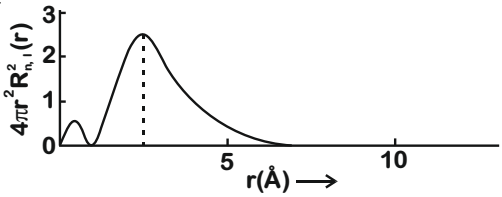
Answer (2)

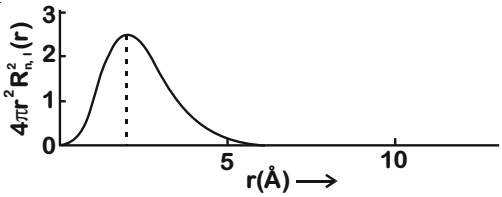
Sol. Species with bond order equal to zero will not exist.

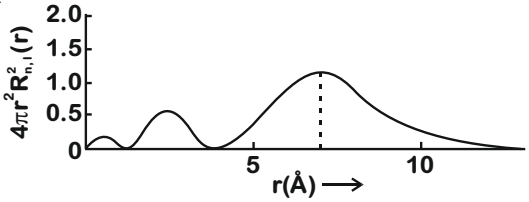
Species	Bond order
He_2^-	0.5
Be_2	0
He_2^+	0.5
O_2^{2-}	1

13. The plots of radial distribution functions for various orbitals of hydrogen atom against 'r' are given below

(A) 

(B) 

(C) 

(D) 

The correct plot for 3s orbital is

(1) (C)

(2) (D)

(3) (B)

(4) (A)

Answer (2)

Sol. 3s orbital has 2 radial nodes

Number of radial nodes = n – (l + 1)

$\therefore$ Graph (A) can be for 1s

Graph (B) can be for 2s

Graph (C) can be for 2p

Graph (D) can be for 3s

14. The solubility of AgCN in a buffer solution of pH = 3 is x. The value of x is :

[Assume : No cyano complex is formed; $K_{sp}(AgCN) = 2.2 \times 10^{-16}$ and $K_a(HCN) = 6.2 \times 10^{-10}$]

(1) 1.9×10^{-5}

(2) 1.6×10^{-6}

(3) 2.2×10^{-16}

(4) 0.625×10^{-6}

Answer (1)

Sol. $AgCN \rightleftharpoons Ag^+ + CN^-$

$CN^- + H_3O^+ \rightleftharpoons HCN + H_2O$

let solubility of AgCN = x molar

$$k_a = \frac{[H^+][CN^-]}{[HCN]}$$

$$\frac{[HCN]}{[CN^-]} = 1.6\times10^6$$

As each CN^- ion hydrolyses to give one HCN

$x = [Ag^+] = [CN^-] + [HCN]$

$\because [CN^-] << [HCN]$

$\therefore x = [Ag^+] \approx [HCN]$

$$[CN^-] = \frac{x}{1.6\times10^6}$$

$K_{sp} = [Ag^+][CN^-]$

$$2.2\times10^{-16} = \frac{x^2}{1.6\times10^6}$$

$x \approx 1.9 \times 10^{-5}$ M

15. The hybridization and magnetic nature of $[Mn(CN)_6]^{4-}$ and $[Fe(CN)_6]^{3-}$, respectively are

(1) d^2sp^3 and paramagnetic

(2) d^2sp^3 and diamagnetic

(3) sp^3d^2 and paramagnetic

(4) sp^3d^2 and diamagnetic

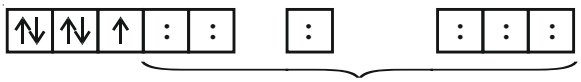
Answer (1)

Sol. $[Mn(CN)_6]^{4-}$

$Mn^{2+} = 3d^5\ 4s^0$

^-CN is a strong field ligand

$\therefore$ Pairing will occur



d^2sp^3

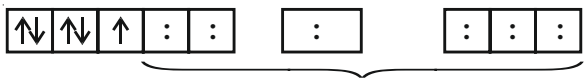
Paramagnetic

$[Fe(CN)_6]^{3-}$

$Fe^{3+} = 3d^5\ 4s^0$

CN^- is a strong field ligand

$\therefore$ Pairing will occur



d^2sp^3

Paramagnetic

16. Complete combustion of 1.80 g of an oxygen containing compound ($C_xH_yO_z$) gave 2.64 g of CO_2 and 1.08 g of H_2O. The percentage of oxygen in the organic compound is :

(1) 50.33

(2) 53.33

(3) 51.63

(4) 63.53

Answer (2)

Sol. $C_xH_yO_z + O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O$

2.64 g of CO_2 contains 0.72 g C.

1.08 g of H_2O contains 0.12 g H.

$\therefore$ mass of oxygen present = 1.80 – (0.72 +0.12) = 0.96 g

$$\% \text{ of O} = \frac{0.96}{1.80} \times 100 = 53.33\ \%$$

17. Which of the following equation depicts the oxidizing nature of H_2O_2?

(1) $2I^- + H_2O_2 + 2H^+ \rightarrow I_2 + 2H_2O$

(2) $KIO_4 + H_2O_2 \rightarrow KIO_3 + H_2O + O_2$

(3) $Cl_2 + H_2O_2 \rightarrow 2HCl + O_2$

(4) $I_2 + H_2O_2 + 2OH^- \rightarrow 2I^- + 2H_2O + O_2$

Answer (1)

Sol. $\overset{-1}{2I^-} + \overset{-1}{H_2O_2} + 2H^+ \rightarrow \overset{0}{I_2} + \overset{-2}{2H_2O}$

I is oxidised from –1 to 0 oxidation state.

18. Which statement is correct?

(1) Buna-S is a synthetic and linear thermosetting polymer.

(2) Buna-N is a natural polymer.

(3) Synthesis of Buna-S needs nascent oxygen.

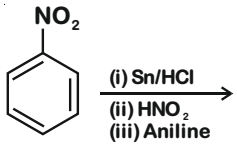
(4) Neoprene is an addition copolymer used in plastic bucket manufacturing.

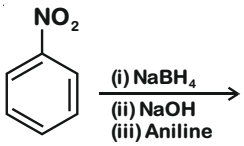
Answer (3)

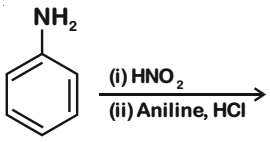
So.

- Buna-S is an elastomer
- Buna-N is a synthetic polymer
- Buna-S is polymerised by addition polymerisation method which needs radical initiator for chain propagation step. Nascent oxygen can be used as an Radical initiator.
- Neoprene is a synthetic rubber.

19. Which of the following reaction/s will not give p-aminoazobenzene?

A. 

B. 

C. 

(1) C only

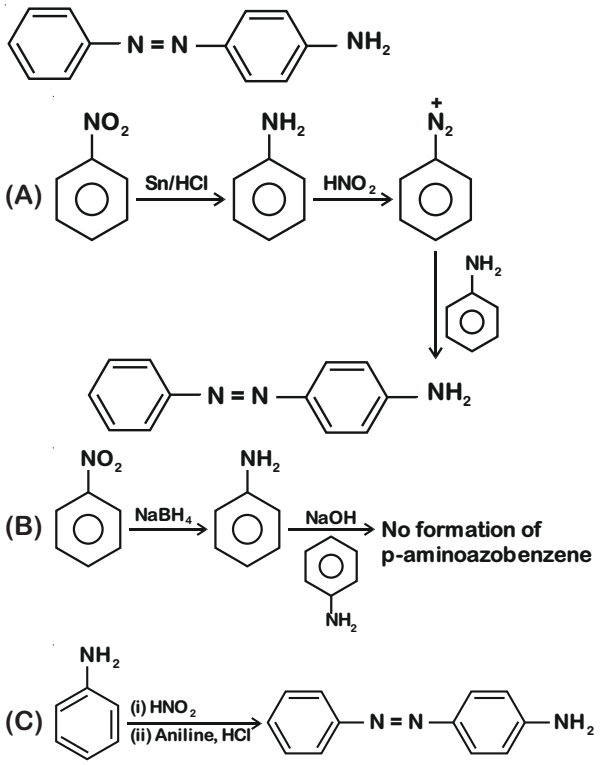
(2) B only

(3) A only

(4) A and B

Answer (2)

Sol. p-aminoazobenzene



20. The correct statement about B_2H_6 is :

(1) Terminal B – H bonds have less *p*-character when compared to bridging bonds.

(2) All B – H – B angles are of 120°.

(3) The two B – H – B bonds are not of same length.

(4) Its fragment, BH_3, behaves as a Lewis base.

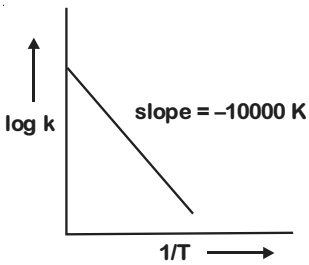
Answer (1)

Sol. Terminal B – H bonds are shorter than the bridging B – H bonds which shows that the terminal B – H bonds have greater s-character and less p-character.

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. For the reaction, aA + bB $\rightarrow$ cC +dD, the plot of log k vs $\frac{1}{T}$ is given below:



The temperature at which the rate constant of the reaction is $10^{-4}s^{-1}$ is _____ K.

(Rounded-off to the nearest integer)

[Given : The rate constant of the reaction is $10^{-5}s^{-1}$ at 500 K.]

Answer (526)

Sol. $k = Ae^{-Ea/RT}$

$$\log k = \log A - \frac{Ea}{2.303R} \times \frac{1}{T}$$

From given graph : slope = –10000 = $\frac{-Ea}{2.303R}$

$$\because \log\frac{k_2}{k_1} = \frac{Ea}{2.303R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$\log\frac{10^{-4}}{10^{-5}} = 10000\left[\frac{1}{500} - \frac{1}{T_2}\right]$$

$$\Rightarrow 1 = 20 - \frac{10000}{T_2}$$

$$\Rightarrow T_2 = \frac{10000}{19} = 526.3 \approx 526\ K$$

2. 0.4 g mixture of NaOH, Na_2CO_3 and some inert impurities was first titrated with $\frac{N}{10}$ HCl using phenolphthalein as an indicator, 17.5 mL of HCl was required at the end point. After this methyl orange was added and titrated. 1.5 mL of same HCl was required for the next end point. The weight percentage of Na_2CO_3 in the mixture is _____. (Rounded-off to the nearest integer)

Answer (4)

Sol. 0.4 g mixture of NaOH + Na_2CO_3 + inert impurity

Assume : no. of moles of NaOH = a m. moles

: no. of moles of Na_2CO_3 = b m. moles

When phenolphthalein is used as indicator:

NaOH will react with HCl and convert into NaCl and H_2O.

Na_2CO_3 will react with HCl and convert into $NaHCO_3$ and NaCl.

Using law of equivalence :

$$a \times 1 + b \times 1 = 17.5 \times \frac{1}{10} = 1.75$$

$$a + b = 1.75$$

When methyl orange is added as indicator in the same solution.

$NaHCO_3$ will convert into H_2CO_3 and NaCl

using law of equivalence

$$b \times 1 = 1.5 \times \frac{1}{10} = 0.15$$

$$W_{Na_2CO_3} \text{ in the mixture} = \frac{0.15}{1000} \times 106$$

$$\simeq 0.016\,g$$

$$\text{weight \% of } Na_2CO_3 = \frac{0.016}{0.4} \times 100 = 4\%$$

3. The reaction of cyanamide, $NH_2CN_{(s)}$ with oxygen was run in a bomb calorimeter and ΔU was found to be –742.24 kJ mol^{-1}. The magnitude of ΔH_{298} for the reaction

$$NH_2CN(s) + \frac{3}{2}O_2(g) \rightarrow N_2(g) + CO_2(g) + H_2O(l)$$

is _____ kJ. (Rounded off to the nearest integer)

[Assume ideal gases and R=8.314 J mol^{-1} K^{-1}]

Answer (741)

Sol. $NH_2CN(s) + \frac{3}{2}O_2(g) \rightarrow N_2(g) + CO_2(g) + H_2O(l)$

$$\because \Delta H = \Delta U + \Delta n_g RT$$

$$\Delta n_g = 2 - \frac{3}{2} = 0.5$$

Assuming that the ΔU is given at the same temperature.

$$\Delta H_{298} = -742.24 + \frac{0.5 \times 8.314 \times 298}{1000}$$

$$= -742.24 + 1.24$$

$$= -741 \text{ kJ}$$

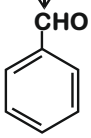
4. Consider the following chemical reaction.

$$CH \equiv CH \xrightarrow[2)\,CO,HCl,AlCl_3]{1)\,\text{Red hot Fe tube, 873K}} \text{Product}$$

The number of sp^2 hybridized carbon atom(s) present in the product is _____.

Answer (7)

Sol. $CH \equiv CH \xrightarrow[873\ K]{(1)\,\text{Red hot Fe tube}}$ 

$\xrightarrow{(2)\,CO,\,HCl,\,AlCl_3}$ 

All the 7-carbon-atoms in product are sp^2 hybridised.

5. In basic medium CrO_4^{2-} oxidises $S_2O_3^{2-}$ to form SO_4^{2-} and itself changes into $Cr(OH)_4^-$. The volume of 0.154 M CrO_4^{2-} required to react with 40 mL of 0.25 M $S_2O_3^{2-}$ is _____ mL. (Rounded-off to the nearest integer)

Answer (173)

Sol. $CrO_4^{2-} + S_2O_3^{2-} \rightarrow Cr(OH)_4^- + SO_4^{2-}$

using law of equivalence

m. equivalents of CrO_4^{2-} used = m. equivalents of $S_2O_3^{2-}$ used

n-factor of CrO_4^{2-} = 3

n-factor of $S_2O_3^{2-}$ = 4×2 = 8

$\therefore$ 0.154 × V × 3 = 0.25 × 40 × 8

$\Rightarrow V = 173.16$ mL

$\simeq 173$ mL

6. Among the following, the number of halide(s) which is/are inert to hydrolysis is _____.

(A) BF_3 (B) $SiCl_4$

(C) PCl_5 (D) SF_6

Answer (1)

Sol. BF_3 – Shows Partial hydrolysis

$SiCl_4$ – Undergoes hydrolysis readily

PCl_5 – Undergoes hydrolysis by addition–elimination mechanism.

SF_6 – Inert towards hydrolysis.

7. A car tyre is filled with nitrogen gas at 35 psi at 27°C. It will burst if pressure exceeds 40 psi. The temperature in °C at which the car tyre will burst is _____. (Rounded-off to the nearest integer)

Answer (70)

Sol. Assuming that no. of moles of N_2 and volume of tyre remains constant and pressure is changed by changing temperature.

Using : $\frac{P}{T} = \text{constant}$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\Rightarrow T_2 = \frac{40 \times 300}{35}$$

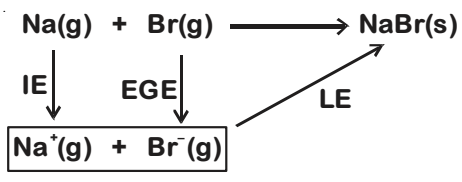
$= 342.86$ K

$T_2 = 69.86°C \approx 70°C$

8. The ionization enthalpy of Na^+ formation from $Na_{(g)}$ is 495.8 kJ mol^{-1}, while the electron gain enthalpy of Br is –325.0 kJ mol^{-1}. Given the lattice enthalpy of NaBr is –728.4 kJ mol^{-1}. The energy for the formation of NaBr ionic solid is (–) _____ $\times 10^{-1}$ kJ mol^{-1}.

Answer (5576)

Sol. From the data given :



$\Delta_r H = 495.8 - 325 - 728.4$

$= -557.6$ kJ/mol

$= -5576 \times 10^{-1}$ kJ/mol

Note : We have solved the question on the basis of information/data given. The final value obtained will not be the enthalpy of formation of NaBr(s). As for calculation of enthalpy of formation of NaBr(s), sublimation energy of Na(s), enthalpy of vapourisation of $Br_2(\ell)$, and bond energy of $Br_2(g)$ is also required.

9. Using the provided information in the following paper chromatogram :

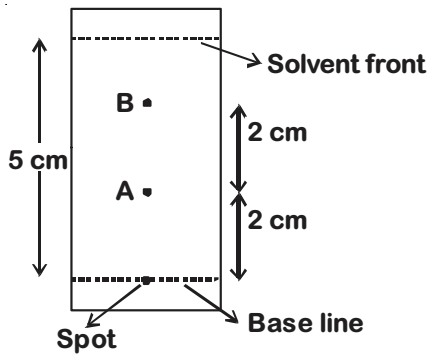


Fig : Paper chromatography for compounds A and B.

the calculated R_f value of A _____ $\times 10^{-1}$.

Answer (4)

Sol. Retardation factor (R_f)

$$= \frac{\text{Distance moved by the substance from base line}}{\text{Distance moved by the solvent from base line}}$$

$= \frac{2}{5}$

$= 0.4$

$= 4 \times 10^{-1}$

10. 1 molal aqueous solution of an electrolyte A_2B_3 is 60% ionised. The boiling point of the solution at 1 atm is _____ K. (Rounded-off to the nearest integer)

[Given K_b for $(H_2O) = 0.52$ K kg mol^{-1}]

Answer (375)

Sol.

$A_2B_3 \longrightarrow 2A^{3+} + 3B^{2-}$ (Assuming A – cation, B – anion)

$1 - 0.6 \quad 2 \times 0.6 \quad 3 \times 0.6$

$\therefore$ van't Hoff factor $= 1 - 0.6 + 1.2 + 1.8 = 3.4$

$\Delta T_b = ik_b \cdot m$

$= 3.4 \times 0.52 \times 1$

$= 1.768$

$T_b = 373 + 1.768$

$= 374.77$

≈ 375

PART–C : MATHEMATICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Let α be the angle between the lines whose direction cosines satisfy the equations $l+m-n=0$ and $l^2+m^2-n^2=0$. Then the value of $\sin^4\alpha+\cos^4\alpha$ is :

(1) $\frac{3}{4}$ (2) $\frac{5}{8}$

(3) $\frac{1}{2}$ (4) $\frac{3}{8}$

Answer (2)

Sol. $l + m - n = 0 \Rightarrow l = n - m$ ...(i)

$l^2 + m^2 - n^2 = 0$ ...(ii)

Substitute l from (i) into (ii)

$\Rightarrow (n - m)^2 + m^2 - n^2 = 0$

$2m(m - n) = 0$

$m = 0$ or $m = n$

Case-I

$m = 0 \Rightarrow l = n$

$l^2 + m^2 + n^2 = 1 \Rightarrow l^2 = \frac{1}{2} \Rightarrow l_1, l_2 = \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$

$l = n \Rightarrow n_1, n_2 = \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$

DCs $\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$ or $\left(\frac{-1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}}\right)$ are DCs of same line $\rightarrow l_1$

Case-II

$m = n \Rightarrow l = 0 \Rightarrow l_1, l_2 = 0$

$l^2 + m^2 + n^2 = 1 \Rightarrow m^2 = \frac{1}{2} \Rightarrow m_1, m_2 = \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$

$m = n \Rightarrow n_1, n_2 = \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$

DCs $\left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ or $\left(0, \frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ are DCs of l_2

$\cos\alpha = l_1l_2 + m_1m_2 + n_1n_2 = 0 + 0 \pm \frac{1}{2} = \pm\frac{1}{2}$

$\cos^2\alpha = \frac{1}{4}, \sin^2\alpha = \frac{3}{4} \Rightarrow \sin^4\alpha + \cos^4\alpha = \frac{5}{8}$

2. If Rolle's theorem holds for the function $f(x) = x^3 - ax^2 + bx - 4$, $x \in [1, 2]$ with $f'\left(\frac{4}{3}\right) = 0$, then ordered pair (a, b) is equal to :

(1) (5, –8) (2) (5, 8)

(3) (–5, –8) (4) (–5, 8)

Answer (2)

Sol. $f(x) = x^3 - ax^2 + bx - 4$

$f(1) = f(2)$

$\Rightarrow$ $3a - b = 7$ ...(i)

$f'(x) = 3x^2 - 2ax + b$

$f'\left(\frac{4}{3}\right) = 0$

$\Rightarrow$ $8a - 3b = 16$ ...(ii)

(i) and (ii)

$\Rightarrow$ $a = 5, b = 8$

3. All possible values of $\theta \in [0, 2\pi]$ for which $\sin 2\theta + \tan 2\theta > 0$ lie in :

(1) $\left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$

(2) $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

(3) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$

(4) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

Answer (4)

Sol. $\sin2\theta + \tan2\theta > 0$ $\frac{\sin2\theta + \cos2\theta + \sin2\theta}{\cos2\theta} > 0$

$\tan2\theta(1 + \cos2\theta) > 0$

$\Rightarrow \tan2\theta > 0$ and $\cos2\theta \neq -1$

$\Rightarrow 2\theta \in \left(n\pi, n\pi+\frac{\pi}{2}\right)$ | $2\theta \neq (2n+1)\pi$

$\Rightarrow \theta \in \left(\frac{n\pi}{2}, (2n+1)\frac{\pi}{4}\right)$ ...(i) | $\theta \neq (2n+1)\frac{\pi}{2}$ ...(ii)

$\Rightarrow \theta \in [0, 2\pi]$

$\therefore \theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

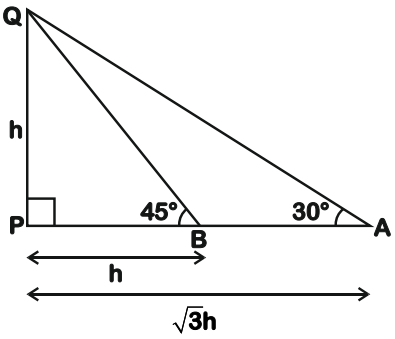
4. A man is observing, from the top of a tower, a boat speeding towards the tower from a certain point A, with uniform speed. At the point, angle of depression of the boat with the man's eye is 30° (Ignore man's height). After sailing for 20 seconds, towards the base of the tower (which is at the level of water), the boat has reached a point B, where the angle of depression is 45°. Then the time taken (in seconds) by the boat from B to reach the base of the tower is :

(1) $10\sqrt{3}$ (2) 10

(3) $10\left(\sqrt{3}+1\right)$ (4) $10\left(\sqrt{3}-1\right)$

Answer (3)

Sol.



Let PQ = h

PB = h cot45° = h

PA = h cot30 = $\sqrt{3}$h

AB = PA – PB

$= \left(\sqrt{3}-1\right)h$

$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$

$\frac{AB}{20} = \frac{PB}{t}$

$\frac{\left(\sqrt{3}-1\right)h}{20} = \frac{h}{t} \Rightarrow t = \frac{20}{\sqrt{3}-1} = 10\left(\sqrt{3}+1\right)$

5. The value of $\int_{-1}^{1} x^2 e^{[x^3]}dx$, where [t] denotes the greatest integer $\leq t$, is :

(1) $\frac{e+1}{3}$ (2) $\frac{1}{3e}$

(3) $\frac{e+1}{3e}$ (4) $\frac{e-1}{3e}$

Answer (3)

Sol. $\int_{-1}^{1} x^2 e^{[x^3]}dx = \int_{-1}^{0} x^2 e^{[x^3]}dx + \int_{0}^{1} x^2 e^{[x^3]}dx$

$= \int_{-1}^{0} x^2 \cdot e^{-1}dx + \int_{0}^{1} x^2 \cdot e^{0}dx$

$= \frac{1}{e}\int_{-1}^{0} x^2dx + \int_{0}^{1} x^2dx$

$= \frac{1}{e}\frac{x^3}{3}\Big|_{-1}^{0} + \frac{x^3}{3}\Big|_{0}^{1}$

$= \frac{1}{3e} + \frac{1}{3} = \frac{e+1}{3e}$

6. The statement $A \rightarrow (B \rightarrow A)$ is equivalent to :

(1) $A \rightarrow (A \leftrightarrow B)$ (2) $A \rightarrow (A \wedge B)$

(3) $A \rightarrow (A \rightarrow B)$ (4) $A \rightarrow (A \vee B)$

Answer (4)

Sol. $B \rightarrow A = \sim B \vee A$

Also $A \rightarrow (B \rightarrow A) = A \rightarrow (\sim B \vee A) = \sim A \vee (\sim B \vee A)$

$= \sim A \vee \sim B \vee A = \sim A \vee A \vee \sim B = t \vee \sim B = t$

$A \rightarrow (A \vee B)$

$= \sim A \vee (A \vee B)$

$= (\sim A \vee A) \vee B$

$= t \vee B = t$

7. Let f, g : $N \rightarrow N$ such that $f(n+1) = f(n) + f(1) \forall n \in N$ and g be any arbitrary function. Which of the following statements is NOT true ?

(1) If g is onto, then fog is one-one

(2) If f is onto, then $f(n) = n \forall n \in N$

(3) f is one-one

(4) If fog is one-one, then g is one-one

Answer (1)

Sol. Given f, g : N → N

& f(n + 1) = f(n) + 1

$$\left.\begin{array}{l}\Rightarrow f(2)=2f(1)\\ \Rightarrow f(3)=3f(1)\\ f(4)=4f(4)\\ \ldots\ldots\ldots\ldots\\ f(n)=nf(1)\end{array}\right\}\Rightarrow \text{f is one}-\text{one.}$$

Now if f is onto ⇒ f(1) = 1

$$\Rightarrow \boxed{f(n)=n}$$

Also it is clear if fog is one-one ⇒ g will be one-one.

So only option (1) is not correct.

8. The total number of positive integral solutions (x, y, z) such that xyz = 24 is :

(1) 36 (2) 30

(3) 45 (4) 24

Answer (2)

Sol. Given xyz = 24 = $2^3 \times 3$

So total number of positive integral solutions (x, y, z)

$= {}^{3+3-1}C_{3-1} \times {}^{1+3-1}C_{3-1}$

$= {}^5C_2 \times {}^3C_2$

$= 10 \times 3$

$= 30$

9. When a missile is fired a ship, the probability that it is intercepted is $\frac{1}{3}$ and the probability that the missile hits the target, given that it is not intercepted, is $\frac{3}{4}$. If three missiles are fired independently from the ship, then the probability that all three hit the target, is :

(1) $\frac{1}{27}$ (2) $\frac{3}{8}$

(3) $\frac{3}{4}$ (4) $\frac{1}{8}$

Answer (4)

Sol. Given P (when it is intercepted) $= \frac{1}{3}$

$\Rightarrow$ P (being not intercepted) $= 1-\frac{1}{3}=\frac{2}{3}$ & also when it is not intercepted, probability it hits the target $= \frac{3}{4}$

So when such 3 missiles launched then P (all 3 hitting the target)

$$=\left(\frac{2}{3}\times\frac{3}{4}\right)\times\left(\frac{2}{3}\times\frac{3}{4}\right)\times\left(\frac{2}{3}\times\frac{3}{4}\right)$$

$$=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$$

$$=\frac{1}{8}$$

10. Let the lines $(2-i)z=(2+i)\bar{z}$ and $(2+i)z+(i-2)\bar{z}-4i=0$, (here $i^2=-1$) be normal to a circle C. If the line $iz+\bar{z}+1+i=0$ is tangent to this circle C, then its radius is :

(1) $3\sqrt{2}$ (2) $\frac{3}{\sqrt{2}}$

(3) $\frac{3}{2\sqrt{2}}$ (4) $\frac{1}{2\sqrt{2}}$

Answer (3)

Sol. Given lines are

$(2 - i)\, z = (2 + i)\, \bar{z}$ ...(1)

and $(2 + i)z + (i - 2)\bar{z} - 4i = 0$

or $-i(2 + i)z - i\,(i - 2)\bar{z} - 4 = 0$

$\Rightarrow$ $(1 - 2i)z + (1 + 2i)\bar{z} - 4 = 0$ ...(2)

Let z = x + iy

So from (1) we get the line $y = \frac{x}{2}$ ...(3)

and from (2) $(1 - 2i)(x + iy) + (1 + 2i)(x - iy) - 4 = 0$

$\Rightarrow$ $x + 2y - 2 = 0$ ...(4)

On solving (3) and (4) we get x = 1, $y = \frac{1}{2}$

$\because$ These lines were normal to the circle.

So centre = $\left(1, \frac{1}{2}\right)$

Now the line $iz+\bar{z}$ + 1 + i = 0

or $i(1-i)z+(1-i)\bar{z}+(1+1)=0$

$\Rightarrow$ $(1+i)z+(1-i)\bar{z}+2=0$

$\Rightarrow$ $(z+\bar{z})+i\,(z-\bar{z})+2=0$ $\Rightarrow$ $2x - 2y + 2 = 0$

$x - y + 1 = 0$

$\because$ This line is tangent to circle

So, $r=\frac{\left|1-\frac{1}{2}+1\right|}{\sqrt{1+1}}=\frac{\left|\frac{3}{2}\right|}{\sqrt{2}}$

$$r=\frac{3}{2\sqrt{2}}$$

11. If the curves, $\frac{x^2}{a}+\frac{y^2}{b}=1$ and $\frac{x^2}{c}+\frac{y^2}{d}=1$ intersect each other at an angle of 90°, then which of the following relations is TRUE?

(1) a – c = b + d (2) a + b = c + d

(3) a – b = c – d (4) $ab=\frac{c+d}{a+b}$

Answer (3)

Sol. Given, Curves are $\frac{x^2}{a}+\frac{y^2}{b}=1$ [Ellipse]

and other curves can be written as

$\frac{x^2}{c}-\frac{y^2}{(-d)}=1$, Which is a hyperbola

Since these both are orthogonal

So, $\sqrt{a-b}=\sqrt{c-d}$

$\Rightarrow$ a – b = c – d

12. The image of the point (3, 5) in the line x – y + 1 = 0, lies on :

(1) $(x-4)^2+(y+2)^2=16$

(2) $(x-4)^2+(y-4)^2=8$

(3) $(x-2)^2+(y-2)^2=12$

(4) $(x-2)^2+(y-4)^2=4$

Answer (4)

Sol. Given the point (3, 5)

and the line x – y + 1 = 0

So, let the image is (x, y)

So, we have

$\frac{x-3}{1}=\frac{y-5}{-1}=-\frac{2(3-5+1)}{1+1}$

$\Rightarrow$ x = 4, y = 4

$\Rightarrow$ Point (4, 4)

Which will satisfy the curve

$(x-2)^2+(y-4)^2=4$

as $(4-2)^2+(4-4)^2$

= 4 + 0 = 4

13. The value of the integral

$\int \frac{\sin\theta.\sin2\theta\left(\sin^6\theta+\sin^4\theta+\sin^2\theta\right)}{\sqrt{2\sin^4\theta+3\sin^2\theta+6}}{1-\cos2\theta}d\theta$ is :

(where c is a constant of integration)

(1) $\frac{1}{18}\left[9-2\cos^6\theta-3\cos^4\theta-6\cos^2\theta\right]^{\frac{3}{2}}+c$

(2) $\frac{1}{18}\left[11-18\cos^2\theta+9\cos^4\theta-2\cos^6\theta\right]^{\frac{3}{2}}+c$

(3) $\frac{1}{18}\left[9-2\sin^6\theta-3\sin^4\theta-6\sin^2\theta\right]^{\frac{3}{2}}+c$

(4) $\frac{1}{18}\left[11-18\sin^2\theta+9\sin^4\theta-2\sin^6\theta\right]^{\frac{3}{2}}+c$

Answer (2)

Sol. $\int \frac{\sin\theta(2\sin\theta)\cos\theta\left(\sin^6\theta+\sin^4\theta+\sin^2\theta\right)}{1-\left(1-2\sin^2\theta\right)}\frac{\sqrt{2\sin^4\theta+3\sin^2\theta+6}}{}d\theta$

Put $\sin\theta = t$

$\Rightarrow \cos\theta\, d\theta = dt$

$\Rightarrow \int \frac{t^2\left(t^6+t^4+t^2\right)\sqrt{2t^4+3t^2+6}}{t^2}dt$

$\int\left(t^5+t^3+t\right)\sqrt{2t^6+3t^4+6t^2}\,dt$

Put $2t^6+3t^4+6t^2=k$

$\Rightarrow 12(t^5+t^3+t)\,dt = dk$

$\Rightarrow \frac{1}{12}\int\sqrt{k}\,dk$

$\Rightarrow \frac{2k^{\frac{3}{2}}}{12.3}$

$\Rightarrow \frac{1}{18}\left(2\sin^6\theta+3\sin^4+6\sin^2\right)^{\frac{3}{2}}+C$

$= \frac{1}{18}\left(11-18\cos^2\theta+9\cos^4\theta-2\sin^6\theta\right)^{\frac{3}{2}}+C$

14. If a curve passes through the origin and the slope of the tangent to it at any point (x, y) is $\frac{x^2-4x+y+8}{x-2}$, then this curve also passes through the point :

(1) (5, 5) (2) (4, 5)

(3) (4, 4) (4) (5, 4)

Answer (1)

Sol. $\frac{dy}{dx}=\frac{x^2-4x+y+8}{x-2}=\frac{(x-2)^2+(y+4)}{(x-2)}$

Put x – 2 = t

$\Rightarrow$ dx = dt

$\Rightarrow \frac{dy}{dt} = \frac{t^2+y+4}{t}$

$\Rightarrow \frac{dy}{dt} - \frac{y}{t} = t + \frac{4}{t}$

$I.F = e^{-\int \frac{1}{t} dt} = \frac{1}{t}$

$\Rightarrow \frac{y}{t} = t - \frac{4}{t} + C$

$y = (x-2)^2 - 4 + C(x-2)$

$\downarrow \quad (0, 0)$

$C = 0$

$y = (x-2)^2 - 4$ also passes through (5, 5)

15. If $0 < \theta, \phi < \frac{\pi}{2}, x = \sum_{n=0}^{\infty} \cos^{2n}\theta$, $y = \sum_{n=0}^{\infty} \sin^{2n}\phi$ and $z = \sum_{n=0}^{\infty} \cos^{2n}\theta \cdot \sin^{2n}\phi$ then :

(1) $xyz = 4$

(2) $xy - z = (x + y)z$

(3) $xy + yz + zx = z$

(4) $xy + z = (x + y)z$

Answer (4)

Sol. $x = \sum_{n=0}^{\infty} \cos^{2n}\theta = 1 + \cos^2\theta + \cos^4\theta + \ldots$

$= \frac{1}{1-\cos^2\theta} = \operatorname{cosec}^2\theta$

$y = \sum_{n=0}^{\infty} \sin^{2n}\phi = 1 + \sin^2\phi + \sin^4\phi + \ldots$

$= \frac{1}{1-\sin^2\phi} = \sec^2\phi$

$z = \sum_{n=0}^{\infty} \cos^{2n}\theta \sin^{2n}\phi$

$= 1 + \cos^2\theta\sin^2\phi + \left(\cos^2\theta\sin^2\phi\right)^2 + \ldots$

$= \frac{1}{1-\cos^2\theta\sin^2\phi}$

$\Rightarrow z = \frac{1}{1-\left(1-\frac{1}{x}\right)\left(1-\frac{1}{y}\right)}$

$\Rightarrow 1 - 1 + \frac{1}{x} + \frac{1}{y} - \frac{1}{xy} = \frac{1}{z}$

$\Rightarrow \frac{x+y}{xy} = \frac{z+xy}{xyz}$

$\Rightarrow (x + y)z = xy + z$

16. The equation of the line through the point (0, 1, 2) and perpendicular to the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{-2}$ is :

(1) $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{-3}$

(2) $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{3}$

(3) $\frac{x}{-3} = \frac{y-1}{4} = \frac{z-2}{3}$

(4) $\frac{x}{3} = \frac{y-1}{-4} = \frac{z-2}{3}$

Answer (3)

Sol. Let equation of line $\frac{x}{a} = \frac{y-1}{b} = \frac{z-2}{c}$

for being perpendicular to $\frac{x}{2} = \frac{y+1}{3} = \frac{z-1}{-2}$

we get

$2a + 3b - 2c = 0$

Hence satisfying this equation $a : b : c = -3 : 4 : 3$

Hence required line is $\frac{x-1}{-3} = \frac{y-1}{4} = \frac{z-2}{3}$

17. $\lim_{n\to\infty} \left(1 + \frac{1+\frac{1}{2}+\ldots\ldots+\frac{1}{n}}{n^2}\right)^n$ is equal to :

(1) $\frac{1}{2}$ (2) 0

(3) 1 (4) $\frac{1}{e}$

Answer (3)

Sol. $L = \lim_{n\to\infty} \left(\frac{1+\left(1+\frac{1}{2}+\frac{1}{3}+\ldots.+\frac{1}{n}\right)}{n^2}\right)^n$

if $n \to \infty \quad 1 + \frac{1}{2} + \frac{1}{3} + \ldots. + \frac{1}{n} < n$

hence $\lim_{n\to\infty} \frac{1+\frac{1}{2}+\ldots+\frac{1}{n}}{n^2} = 0$

L is of 1^∞ form

$L = e^{\lim_{n\to\infty} \left(\frac{1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}}{n^2}\right)\cdot n} = e^0 = 1$

18. The coefficients a, b and c of the quadratic equation, $ax^2 + bx + c = 0$ are obtained by throwing a dice three times. The probability that this equation has equal roots is :

(1) $\frac{1}{54}$ (2) $\frac{1}{36}$

(3) $\frac{5}{216}$ (4) $\frac{1}{72}$

Answer (3)

Sol. For equal roots $b^2 = 4ac$

$a, b, c \in \{1, 2, 3, 4, 5, 6\}$

Favourable case

$b = 2$ $a = c = 1$

$b = 4$ (a, c) = (1, 4), (4, 1) and (2, 2)

$b = 6$ (a, c) = (3, 3)

Total possible ordered triplets

$(a, b, c) = 6^3 = 216$

Favourable cases = 5

$\therefore$ Required probability $= \frac{5}{216}$

19. A tangent is drawn to the parabola $y^2 = 6x$ which is perpendicular to the line $2x + y = 1$. Which of the following points does NOT lie on it ?

(1) (0, 3) (2) (–6, 0)

(3) (4, 5) (4) (5, 4)

Answer (4)

Sol. Slope of line : $2x + y = 1$ is -2

Slope of line perpendicular to given line is $\frac{1}{2}$

$\therefore$ Equation of tangents to parabola $y^2 = 6x$ is

$$y = \frac{1}{2}x + \frac{\frac{6}{4}}{\frac{1}{2}}$$

$$y = \frac{1}{2}x + 3$$

$x - 2y + 6 = 0$

$\therefore$ (5, 4) does not lies on $x - 2y + 6 = 0$

20. The integer 'k', for which the inequality $x^2 - 2(3k - 1)x + 8k^2 - 7 > 0$ is valid for every x in R, is :

(1) 2 (2) 3

(3) 4 (4) 0

Answer (2)

Sol. $x^2 - 2(3x - 1)x + 8k^2 - 7 > 0, \ \forall x \in R$

Here $D < 0$

$$4(3k-1)^2 - 4 \cdot 1 \cdot (8k^2 - 7) < 0$$

$$9k^2 - 6k + 1 - 8k^2 + 7 < 0$$

$k^2 - 6k + 8 < 0$

$(k - 2)(k - 4) < 0$

$k \in (2, 4)$

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Let $A_1, A_2, A_3, \ldots$ be squares such that for each $n \geq 1$, the length of the side of A_n equals the length of diagonal of A_{n+1}. If the length of A_1 is 12 cm, then the smallest value of n for which area of A_n is less than one, is_____.

Answer (9)

Sol. $A_1 = 12$, Let side of square 2 be A_2

Given diagonal of A_{n+1} = Side of A_n

$\Rightarrow 2A_2^2 = A_1^2 \Rightarrow A_2 = A_{1/\sqrt{2}}$ (i.e., $A_{n+1} = \frac{A_n}{\sqrt{2}}$)

$$\Rightarrow A_2 = \frac{A_1}{\sqrt{2}}, A_3 = \frac{A_2}{\sqrt{2}} = \frac{A_1}{2} \ldots$$

$A_{n+1} = (\sqrt{2}.\sqrt{2} \ \ldots \ (n-1) \text{ times})^{-1} A_1$

$$\text{Area} = (A_{n+1})^2 = \frac{A_1^2}{2^{(n-1)}} < 1$$

$144 < 2^{n-1} \Rightarrow n - 1 \geq 8$

$n = 9$

2. The number of points, at which the function $f(x) = |2x+1| - 3|x+2| + |x^2 + x - 2|, x \in R$ is not differentiable is _____.

Answer (2)

Sol. $f(x) = |2x + 1| - 3|x + 2| + |x^2 + x - 2|$

$= |2x + 1| - 3|x + 2| + |(x + 2)(x - 1)$

$$\therefore \quad f(x)=\begin{cases} x^2+2x+3 & x<-2 \\ -x^2-6x-5 & -2\le x<-\frac{1}{2} \\ -x^2-2x-3 & \frac{-1}{2}\le x<1 \\ x^2-7 & 1\le x \end{cases}$$

at $x = -2$ f(x) is continuous,

LHD = –2 & RHD = –2 Hence differentiable

at $x=\frac{-1}{2}$ f(x) is continuous,

LHD = –5 & RHD = –1 Hence non-differentiable

at x = 1 f(x) is continuous,

LHD = –4 & RHD = 2 Hence non-differentiable

$\therefore$ f(x) is non differentiable at $x=\frac{-1}{2}$ and 1

3. The total number of numbers, lying between 100 and 1000 than can be formed with the digits 1, 2, 3, 4, 5, if the repetition of digits is not allowed and numbers are divisible by either 3 or 5, is _____.

Answer (32)

Sol. The numbers are lying between 100 and 1000 then each number is of three digits.

The possible combination of 3 digits numbers are

1, 2, 3; 1, 2, 4; 1, 2, 5; 1, 3, 4; 1, 3, 5; 1, 4, 5; 2, 3, 4; 2, 3, 5; 2, 4, 5; and 3, 4, 5.

The numbers which are divisible by 3 are 1, 2, 3; 3, 4, 5; 1, 3, 5 and 2, 3, 4.

$\therefore$ Total number of numbers = 4 × 3! = 24

The number divisible by 5 are 1, 2, 5; 2, 3, 5; 1, 4, 5 and 2, 4, 5.

$\therefore$ Number divisible by 5 = 4 × 2! = 8

$\therefore$ Total required number = 24 + 8 = 32

4. Let f(x) be a polynomial of degree 6 in x, in which the coefficient of x^6 is unity and it has extrema at x = –1, and x = 1. If $\lim\limits_{x\to 0}\frac{f(x)}{x^3}=1$, then $5\cdot f(2)$ is equal to _____.

Answer (144)

Sol. Let $f(x) = x^6 + ax^5 + bx^3 + cx^3 + dx^2 + ex + f$

$$\therefore \quad \lim_{x\to 0}\frac{f(x)}{x^3}=1 \Rightarrow d=e=f=0 \text{ and } c=1$$

So, $f(x) = x^6 + ax^5 + bx^4 + x^3$

$f'(x) = 6x^5 + 5ax^4 + 4bx^3 + 3x^2$

$\because$ f'(1) = 0 = f'(–1)

$\Rightarrow$ 5a + 4b = –9 and 5a – 4b = 3

$$\Rightarrow \quad a=-\frac{3}{5} \text{ and } b=-\frac{3}{2}$$

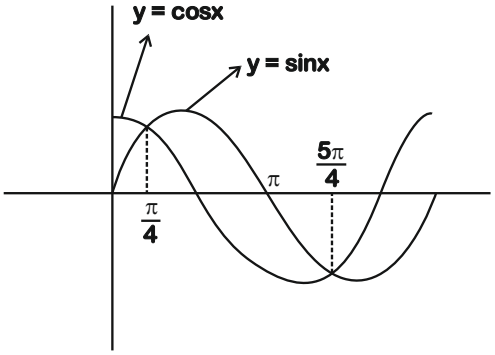
$$\text{Then } 5.f(2)=5\left[2^6-\frac{3}{5}.2^5-\frac{3}{2}.2^4+2^3\right]$$

= 144

5. The graphs of sine and cosine functions, intersect each other at a number of points and between two consecutive points of intersection, the two graphs enclose the same area A. Then A^4 is equal to _____.

Answer (64)

Sol.



$$A=\int_{\pi/4}^{5\pi/4}(\sin x-\cos x)dx=-\cos x-\sin x\Big|_{\pi/4}^{5\pi/4}$$

$$\Rightarrow \quad A=\sqrt{2}+\sqrt{2}=2\sqrt{2}$$

$\Rightarrow$ $A^4 = 64$

6. If $A=\begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$ and $(I_2, + A)(I_2 - A)^{-1}$

$=\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then $13(a^2 + b^2)$ is equal to _____.

Answer (13)

Sol. $$I_2+A=\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix} \quad \ldots (1)$$

$$I_2-A=\begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$$

$$\Rightarrow (I_2 - A)^{-1} = \frac{1}{\sec^2\frac{\theta}{2}}\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix} \quad \ldots (2)$$

$(I_2 + A)(I_2 - A)^{-1}$

$$= \frac{1}{\sec^2\frac{\theta}{2}}\begin{bmatrix} 1-\tan^2\frac{\theta}{2} & -2\tan\frac{\theta}{2} \\ 2\tan\frac{\theta}{2} & 1-\tan^2\frac{\theta}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Clearly $a = \cos\theta$ and $b = \sin\theta$, then $13(a^2 + b^2) = 13$

7. The locus of the point of intersection of the lines $\left(\sqrt{3}\right)kx + ky - 4\sqrt{3} = 0$ and $\sqrt{3}x - y - 4\left(\sqrt{3}\right)k = 0$ is a conic, whose eccentricity is _____.

Answer (2)

Sol. $L_1 : \sqrt{3x} + y = \frac{4\sqrt{3}}{k}$

and $L_2 : \sqrt{3x} - y = 4\sqrt{3}k$

So point of intersection will always satisfy

$$\left(\sqrt{3}x - y\right)\left(\sqrt{3}x - y\right) = 48$$

$$\Rightarrow \frac{x^2}{16} - \frac{y^2}{48} = 1$$

$$e = \sqrt{1 + \frac{48}{16}} = 2$$

8. If the system of equations

$kx + y + 2z = 1$

$3x - y - 2z = 2$

$-2x - 2y - 4z = 3$

has infinitely many solutions, then k is equal to _____.

Answer (21)

Sol. $kx + y + 2z = 1$ ...(1)

$-3x + y + 2z = -2$ ...(2)

$x + y + 2z = \frac{-3}{2}$ ...(3)

from (2) and (3) we get

$$x = \frac{1}{8} \text{ and } y + 2z = -\frac{13}{8}$$

Substituting these values in (1) we get

$k = 21$

9. Let $A = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$, where x, y and z are real numbers such that $x + y + z > 0$ and $xyz = 2$. If $A^2 = I_3$, then the value of $x^3 + y^3 + z^3$ is _____.

Answer (7*)

Sol. $\because$ $A^2 = I_3 \Rightarrow x^2 + y^2 + z^2 = 1$ and $xy + yz + zx = 0$

then $x + y + z = 1$

$\because$ $|A| = 3xyz - x^3 - y^3 - z^3$

$= -(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$\Rightarrow 6 - (x^3 + y^3 + z^3) = -1$

$\Rightarrow x^3 + y^3 + z^3 = 7$

*We will not get the real numbers x,y,z satisfying these conditions.

10. Let $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{b} = \hat{i} - \hat{j}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three given vectors, if $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$ and $\vec{r} \cdot \vec{b} = 0$, then $\vec{r} \cdot \vec{a}$ is equal to _____.

Answer (12)

Sol. $\because$ $\vec{a} \cdot \vec{b} = -1$, $\vec{b} \cdot \vec{c} = 2$, $\vec{c} \cdot \vec{a} = 0$

$\vec{r} \times \vec{a} = \vec{c} \times \vec{a} \Rightarrow (\vec{r} \times \vec{a}) \times \vec{b} = (\vec{c} \times \vec{a}) \times \vec{b}$

$\Rightarrow (\vec{r} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{b})\vec{r} = (\vec{b} \cdot \vec{c})\vec{a} - (\vec{a} \cdot \vec{b})\vec{c}$

$\Rightarrow \vec{r} = 2\vec{a} + \vec{c}$

then $\vec{r} \cdot \vec{a} = 2|\vec{a}|^2 + \vec{a} \cdot \vec{c}$

$= 12$

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