26/02/2021
Evening

Medical | IIT-JEE | Foundations

Regd. Office : Aakash Tower, 8, Pusa Road, New Delhi-110005. Ph.: 011-47623456

Time : 3 hrs.

Answers & Solutions

M.M. : 300

for

JEE (MAIN)-2021 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS :

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part has two sections.

(i) Section-I : This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) Section-II : This section contains 10 questions. In Section-II, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and there is no negative marking for wrong answer.

PART–A : PHYSICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

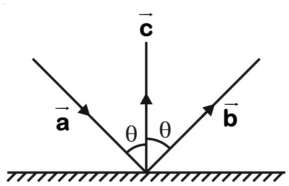
1. The incident ray, reflected ray and the outward drawn normal are denoted by the unit vectors $\vec{a}, \vec{b}$ and $\vec{c}$ respectively. Then choose the correct relation for these vectors.

(1) $\vec{b} = \vec{a} + 2\vec{c}$ (2) $\vec{b} = 2\vec{a} + \vec{c}$

(3) $\vec{b} = \vec{a} - 2(\vec{a}\cdot\vec{c})\vec{c}$ (4) $\vec{b} = \vec{a} - \vec{c}$

Answer (3)

Sol. $\vec{b} - \vec{a} = 2\cos\theta\vec{c}$



$\vec{b} = \vec{a} + 2\cos\theta\vec{c}$

$\vec{b} = \vec{a} - 2(\vec{a}\cdot\vec{c})\vec{c}$

2. A radioactive sample is undergoing α decay. At any time t_1, its activity is A and another time t_2, the activity is $\frac{A}{5}$. What is the average life time for the sample?

(1) $\frac{t_2 - t_1}{\ln 5}$ (2) $\frac{\ln(t_2 + t_1)}{2}$

(3) $\frac{\ln 5}{t_2 - t_1}$ (4) $\frac{t_1 - t_2}{\ln 5}$

Answer (1)

Sol. $\frac{A}{5} = Ae^{-\lambda(t_2 - t_1)}$

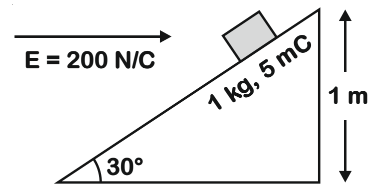
$\frac{1}{5} = e^{-\lambda(t_2 - t_1)}$

$\Rightarrow$ ln 5 = $\lambda(t_2 - t_1)$

$\Rightarrow \frac{1}{\lambda} = \frac{(t_2 - t_1)}{\ln 5} = t_{avg}$

3. An inclined plane making an angle of 30° with the horizontal is placed in a uniform horizontal electric field $200\,\frac{N}{C}$ as shown in the figure. A body of mass 1 kg and charge 5 mC is allowed to slide down from rest at a height of 1 m. If the coefficient of friction is 0.2, find the time taken by the body to reach the bottom.

$$\left[g = 9.8\text{ m/s}^2;\ \sin 30^\circ = \frac{1}{2};\ \cos 30^\circ = \frac{\sqrt{3}}{2}\right]$$

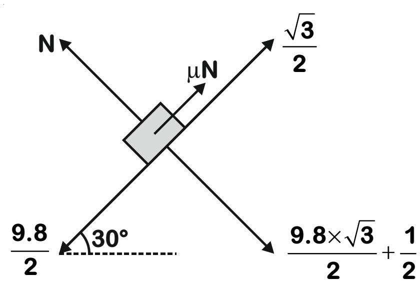


(1) 2.3 s (2) 1.3 s

(3) 0.92 s (4) 0.46 s

Answer (2)

Sol.



$$a = 4.9 - (0.2)\left\{\frac{9.8\sqrt{3}+1}{2}\right\} - \frac{\sqrt{3}}{2}$$

$= 2.24\text{ m/s}^2$

$$T = \sqrt{\frac{2\times 1}{(\sin 30)\times 2.24}} = 1.3\text{ s}$$

4. Given below are two statements :

Statement I : An electric dipole is placed at the centre of a hollow sphere. The flux of electric field through the sphere is zero but the electric field is not zero anywhere in the sphere.

Statement II : If R is the radius of a solid metallic sphere and Q be the total charge on it. The electric field at any point on the spherical surface of radius r (< R) is zero but the electric flux passing through this closed spherical surface of radius r is not zero.

In the light of the above statements, choose the correct answer from the options given below.

(1) Statement I is false but Statement II is true

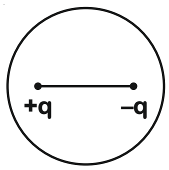
(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are true

Answer (3)

Sol. $\phi_{Tot} = \frac{q_{inc}}{\varepsilon_0}$

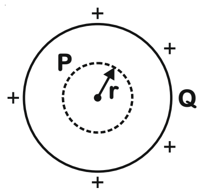


$q_{inc} = 0 \Rightarrow \phi_{Tot} = 0$

$\vec{E} \neq 0$

At P,

$E = 0$ and $\phi = 0$



5. A scooter accelerates from rest for time t_1 at constant rate a_1 and then retards at constant rate a_2 for time t_2 and comes to rest. The correct value of $\frac{t_1}{t_2}$ will be

(1) $\frac{a_1 + a_2}{a_1}$ (2) $\frac{a_1}{a_2}$

(3) $\frac{a_2}{a_1}$ (4) $\frac{a_1 + a_2}{a_2}$

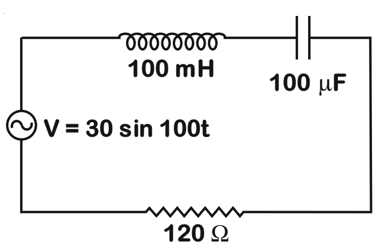
Answer (3)

Sol. We have,

$a_1 t_1 = a_2 t_2$

$\Rightarrow \frac{t_1}{t_2} = \frac{a_2}{a_1}$

6. Find the peak current and resonant frequency of the following circuit (as shown in figure).



(1) 0.2 A and 50 Hz (2) 2 A and 100 Hz

(3) 2 A and 50 Hz (4) 0.2 A and 100 Hz

Answer (1)

Sol. $I_{max} = \frac{V_{max}}{Z} = \frac{30}{\sqrt{R^2 + (X_L - X_C)^2}}$

$X_L = 100 \times 0.1 = 10\ \Omega$

$X_C = \frac{1}{100 \times 10^{-4}} = 100\ \Omega$

$I_{max} = \frac{30}{\sqrt{(120)^2 + (90)^2}} = 0.2\text{ A}$

$\omega = \frac{1}{\sqrt{LC}} \Rightarrow f = \frac{1}{2\pi\sqrt{LC}} \approx 50\text{ Hz}$

7. The length of metallic wire is l_1 when tension in it is T_1. It is l_2 when the tension is T_2. The original length of the wire will be

(1) $\frac{T_1 l_1 - T_2 l_2}{T_2 - T_1}$ (2) $\frac{l_1 + l_2}{2}$

(3) $\frac{T_2 l_1 + T_1 l_2}{T_1 + T_2}$ (4) $\frac{T_2 l_1 - T_1 l_2}{T_2 - T_1}$

Answer (4)

Sol. $T_1 = K(l_1 - l_0)$ ...(1)

$T_2 = K(l_2 - l_0)$ ...(2)

From (1) and (2),

$l_0 = \frac{T_2 l_1 - T_1 l_2}{(T_2 - T_1)}$

8. The trajectory of a projectile in a vertical plane is $y = \alpha x - \beta x^2$, where α and β are constants and x & y are respectively the horizontal and vertical distances of the projectile from the point of projection. The angle of projection θ and the maximum height attained H are respectively given by

(1) $\tan^{-1}\alpha, \frac{4\alpha^2}{\beta}$ (2) $\tan^{-1}\beta, \frac{\alpha^2}{2\beta}$

(3) $\tan^{-1}\left(\frac{\beta}{\alpha}\right), \frac{\alpha^2}{\beta}$ (4) $\tan^{-1}\alpha, \frac{\alpha^2}{4\beta}$

Answer (4)

Sol. $y = \alpha x - \beta x^2$

$\Rightarrow \tan\theta = \alpha \Rightarrow \theta = \tan^{-1}\alpha$

also, $\frac{dy}{dx} = \alpha - 2\beta x$

$\frac{dy}{dx} = 0 \Rightarrow x = \frac{\alpha}{2\beta}$

$y = \frac{\alpha^2}{2\beta} - \frac{\alpha^2}{4\beta} = \frac{\alpha^2}{4\beta}$ at $x = \frac{\alpha}{2\beta}$

9. An aeroplane, with its wings spread 10 m, is flying at a speed of 180 km/h in a horizontal direction. The total intensity of earth's field at that part is 2.5×10^{-4} Wb/m^2 and the angle of dip is 60°. The emf induced between the tips of the plane wings will be ________ .

(1) 108.25 mV (2) 88.37 mV

(3) 62.50 mV (4) 54.125 mV

Answer (1)

Sol. $\varepsilon_{ind} = (B_V) LV, \quad B_V = B_{Total} \sin 60$

$$= (2.5 \times 10^{-4})(\sin 60) \times 10 \times 180 \times \frac{5}{18}$$

= 108.25 mV

10. If 'C' and 'V' represent capacity and voltage respectively then what are the dimensions of λ where C/V = λ?

(1) $[M^{-1} L^{-3} I^{-2} T^{-7}]$
(2) $[M^{-2} L^{-4} I^{3} T^{7}]$
(3) $[M^{-2} L^{-3} I^{2} T^{6}]$
(4) $[M^{-3} L^{-4} I^{3} T^{7}]$

Answer (2)

Sol. $U = \frac{1}{2} CV^2$

$$\Rightarrow \frac{C}{V} = \frac{U}{V^3} = \frac{F \times L}{V^3}$$

$$V = \frac{F \times L}{IT}$$

$$\Rightarrow \frac{C}{V} = \frac{F \times L \times I^3 T^3}{F^3 \times L^3}$$

$$= [M^{-2} L^{-4} T^{7} I^{3}]$$

11. The recoil speed of a hydrogen atom after it emits a photon in going from n = 5 state to n = 1 state will be

(1) 4.34 m/s
(2) 2.19 m/s
(3) 4.17 m/s
(4) 3.25 m/s

Answer (3)

Sol. $P = \frac{E}{C}$

$$mv = \frac{E}{C}$$

$$\Rightarrow v = \frac{E}{mC} = \frac{(13.6) \times 24 \times 1.6 \times 10^{-19}}{25 \times 1.66 \times 10^{-27} \times 3 \times 10^{8}}$$

$v \simeq 4.17$ m/s

12. Given below are two statements: one is labeled as Assertion A and the other is labeled as Reason R.

Assertion A : For a simple microscope, the angular size of the object equals the angular size of the image.

Reason R : Magnification is achieved as the small object can be kept much closer to the eye than 25 cm and hence it subtends a large angle.

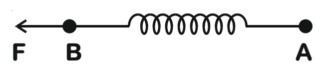
In the light of the above statements, choose the most appropriate answer from the options given below :

(1) A is false but R is true
(2) A is true but R is false
(3) Both A and R are true but R is NOT the correct explanation of A
(4) Both A and R are true and R is the correct explanation of A

Answer (4)

Sol. Though image size is bigger than object size, the angular size of the image is equal to the angular size of object

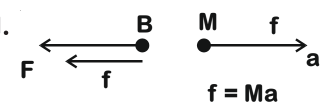
13. Two masses A and B, each of mass M are fixed together by a massless spring. A force acts on the mass B as shown in figure. If the mass A starts moving away from mass B with acceleration 'a', then the acceleration of mass B will be



(1) $\frac{Ma - F}{M}$
(2) $\frac{MF}{F + Ma}$
(3) $\frac{F - Ma}{M}$
(4) $\frac{F + Ma}{M}$

Answer (4)

Sol.



$F + f = Ma_1$

$$\Rightarrow a_1 = \left(\frac{F + f}{M}\right) = \left(\frac{F + Ma}{M}\right)$$

14. A tuning fork A of unknown frequency produces 5 beats/s with a fork of known frequency 340 Hz. When fork A is filed, the beat frequency decreases to 2 beats/s. What is the frequency of fork A?

(1) 335 Hz
(2) 338 Hz
(3) 345 Hz
(4) 342 Hz

Answer (1)

Sol. $f_A = 340 \pm 5$

If f_A increases, then beat frequency decreases $\Rightarrow f_A = 335$

15. A wire of 1 Ω has a length of 1 m. It is stretched till its length increases by 25%. The percentage change in resistance to the nearest integer is

(1) 76%
(2) 56%
(3) 12.5%
(4) 25%

Answer (2)

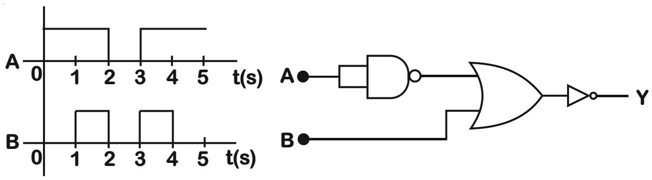
Sol. $R = \frac{\rho\ell}{A} = \frac{\rho\ell^2}{(\text{Vol.})}$

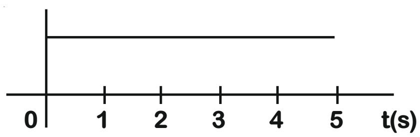
$R_1 = \left(\frac{\rho}{V}\right)(\ell)^2$

$R_2 = \left(\frac{\rho}{V}\right)\left(\frac{5\ell}{4}\right)^2$

$\frac{\Delta R}{R_1} \times 100 = \frac{9}{16} \times 100 \simeq 56\%$

16. Draw the output signal Y in the given combination of gates.



(1)


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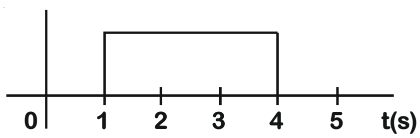

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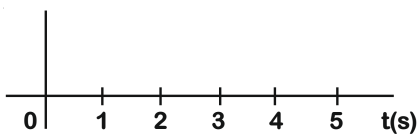
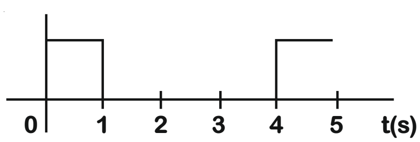


(4)


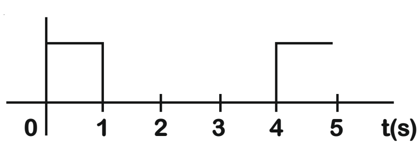
Answer (4)

Sol. A = 0 $\Rightarrow$ Y = 0 for B = 0 or B = 1

A = 1 $\Rightarrow$ Y = 0 for B = 1

A = 1 $\Rightarrow$ Y = 1 for B = 0

Hence output is 1 only when A = 1 and B = 0

$\Rightarrow$


17. A particle executes S.H.M., the graph of velocity as a function of displacement is

(1) a parabola (2) a helix

(3) a circle (4) an ellipse

Answer (4)

Sol. $v = \omega\sqrt{A^2 - x^2}$

$\Rightarrow \frac{v^2}{\omega^2} + x^2 = A^2$

$\Rightarrow \frac{x^2}{A^2} + \frac{v^2}{(A\omega)^2} = 1$ .... Ellipse equation

18. The internal energy (U), pressure (P) and volume (V) of an ideal gas are related as U = 3PV + 4. The gas is

(1) diatomic only

(2) either monoatomic or diatomic

(3) polyatomic only

(4) monoatomic only

Answer (3)

Sol. U = (3)nRT + 4

dU = 3nRdT $\Rightarrow$ f = 6

19. Given below are two statements :

Statement I : A second's pendulum has a time period of 1 second.

Statement II : It takes precisely one second to move between the two extreme positions.

In the light of the above Statement, choose the correct answer from the options given

(1) Both Statement I and Statement II are true

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are false

(4) Statement I is false but Statement II is true

Answer (4)

Sol. Time period of second pendulum is 2 sec

$\Delta t = \frac{T}{2} = 1$

20. A cord is wound round the circumference of wheel of radius r. The axis of the wheel is horizontal and the moment of inertia about it is I. A weight mg is attached to the cord at the end. The weight falls from rest. After falling through a distance 'h', the square of angular velocity of wheel will be

(1) 2gh (2) $\frac{2mgh}{I + 2mr^2}$

(3) $\frac{2mgh}{I + mr^2}$ (4) $\frac{2gh}{I + mr^2}$

Answer (3)

Sol. $\Delta PE = \Delta KE$

$mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}m(r\omega)^2$

$\Rightarrow \omega^2 = \frac{2mgh}{I + mr^2}$

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Two stream of photons, possessing energies equal to twice and ten times the work function of metal are incident on the metal surface successively. The value of ratio of maximum velocities of the photoelectrons emitted in the two respective cases is x : y. The value of x is ________.

Answer (01.00)

Sol. $\frac{1}{2}mv^2 = E - \phi$

$\Rightarrow \; V^2 = \frac{2}{m}(E - \phi)$

$\Rightarrow \; V_1^2 = \frac{2}{m}(\phi)$ ...(1)

$V_2^2 = \frac{2}{m}(9\phi)$ ...(2)

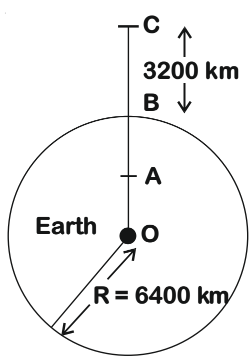
From (1) & (2),

$\frac{V_1}{V_2} = \frac{1}{3}$

$\Rightarrow \; \frac{x}{y} = \frac{1}{3}$

$\Rightarrow$ (x = 01.00)

2. In the reported figure of earth, the value of acceleration due to gravity is same at point A and C but it is smaller than that of its value at point B (surface of the earth). The value of OA : AB will be x : y. The value of x is _______.



Answer (04.00)

Sol. $g_C = \frac{GM}{\left(R + \frac{R}{2}\right)^2} = \frac{4}{9}g_0$

$g_A = g_C \Rightarrow \frac{4}{9}g_0 = g_0\left(1 - \frac{AB}{R}\right)$

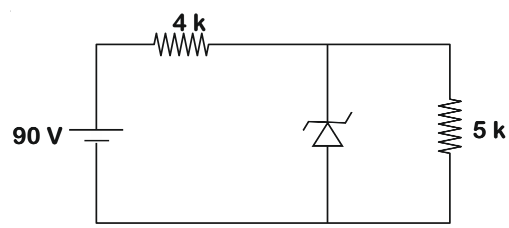
$\Rightarrow \; AB = \frac{5R}{9}$

$\Rightarrow \; OA = \frac{4R}{9}$

$\Rightarrow \; \frac{OA}{AB} = \frac{4}{5}$

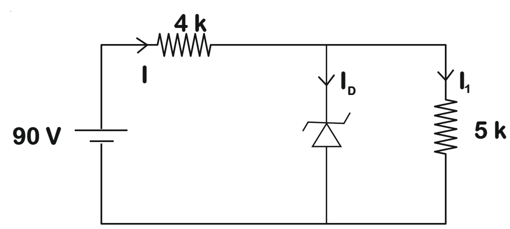
$\Rightarrow$ x = 04.00

3. The zener diode has a V_z = 30 V. The current passing through the diode for the following circuit is _______mA.



Answer (09.00)

Sol. $I_1 = \frac{30}{5{,}000} = 6 \text{ mA}$



also – 90 + 4000 I + 30 = 0

$\Rightarrow$ I = 15 mA

$\Rightarrow \; I_D = 15 - 6 = 9$ mA

4. Time period of a simple pendulum is T. The time taken to complete $\frac{5}{8}$ oscillations starting from mean position is $\frac{\alpha}{\beta}T$. The value of α is______.

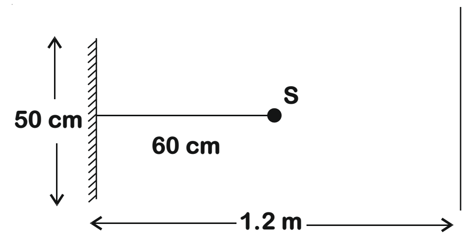
Answer (07.00)

Sol. $\frac{5}{8}$oscillation $= \frac{1}{2}$ oscillation $+ \frac{1}{8}$ oscillation

$\Delta t = \frac{T}{2} + \frac{T}{12} = \frac{7T}{12}$

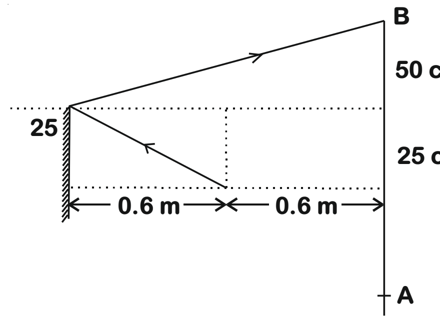
$\Rightarrow \alpha = 7$

5. A point source of light S, placed at a distance 60 cm in front of the centre of a plane mirror of width 50 cm, hangs vertically on a wall. A man walls in front of the mirror along a line parallel to the mirror at a distance 1.2 m from it (see in the figure). The distance between the extreme points where he can see the image of the light source in the mirror is _______ cm.



Answer (150.00)

Sol. AB = 2 × (50 + 25) cm = 150 cm



6. A particle executes S.H.M. with amplitude 'a' and time period 'T'. The displacement of the particle when its speed is half of maximum speed is $\frac{\sqrt{x}\,a}{2}$. The value of x is ______.

Answer (03.00)

Sol. $V = \omega\sqrt{a^2 - x^2}$

$\Rightarrow \frac{a\omega}{2} = \omega\sqrt{a^2 - x^2}$

$\Rightarrow \frac{a^2}{4} = a^2 - x^2$

$\Rightarrow x^2 = \frac{3a^2}{4}$

$\Rightarrow x = \frac{a\sqrt{3}}{2}$

7. 1 mole of rigid diatomic gas performs a work of $\frac{Q}{5}$ when heat Q is supplied to it. The molar heat capacity of the gas during this transformation is $\frac{xR}{8}$. The value of x is____.

[R = universal gas constant]

Answer (25.00)

Sol. $Q = \Delta U + \frac{Q}{5}$

$\Rightarrow \Delta U = \frac{4Q}{5} \Rightarrow \frac{5R}{2}\Delta T = \frac{4}{5}Q$ ... (i)

$\Rightarrow C_{process} = \left(\frac{Q}{\Delta T}\right)$ ... (ii)

From equation (i) and (ii),

$C_{process} = \frac{25}{8}R$

8. The volume V of a given mass of monoatomic gas changes with temperature T according to the relation $V = KT^{\frac{2}{3}}$. The work done when temperature changes by 90 K will be xR. The value of x is ____.

[R = universal gas constant]

Answer (60.00)

Sol. $W = \int pdV$

$p = \frac{nRT}{V}, V = KT^{2/3} \Rightarrow dV = \frac{2}{3}KT^{-1/3}dT$

$\Rightarrow W = \int \frac{nRT}{KT^{2/3}}\frac{2}{3}KT^{-1/3}dT$

$= \left(\frac{2}{3}\right)(n)R\ \Delta T$

$= \left(\frac{2}{3}\right)(n)R \times 90 = 60\,nR$ (assuming n = 1)

9. If the highest frequency modulating a carrier is 5 kHz, then the number of AM broadcast stations accommodated in a 90 kHz bandwidth are ____.

Answer (09.00)

Sol. Number of stations $= \frac{\text{Total B.W.}}{\text{B.W. for each channel}}$

$= \frac{90}{2 \times 5} = 9$

10. 27 similar drops of mercury are maintained at 10 V each. All these spherical drops combine into a single big drop. The potential energy of the bigger drop is ______ times that of a smaller drop.

Answer (243.00)

Sol. $U = \frac{(C)Q^2}{R}$

For smaller drop, $U_S = \frac{(C)(Q_0)^2}{r}$

For bigger drop, $U_B = \frac{(C)(27Q_0)^2}{3r}$

$\Rightarrow U_B = \frac{27 \times 27}{3}U_S = 243\,U_S$

PART–B : CHEMISTRY

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Which pair of oxides is acidic in nature?

(1) CaO, SiO_2

(2) B_2O_3, CaO

(3) B_2O_3, SiO_2

(4) N_2O, BaO

Answer (3)

Sol. CaO – Basic

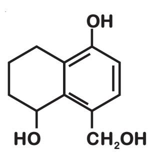
SiO_2 – Acidic

B_2O_3 – Acidic

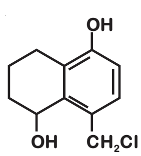
N_2O – Neutral

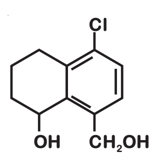
BaO – Basic

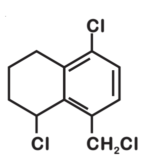
2. Identify A in the given reaction,

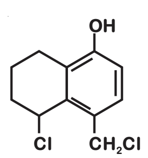


$\xrightarrow{SOCl_2}$ A (major product)

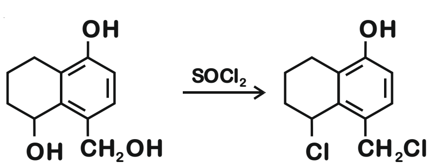
(1) 

(2) 

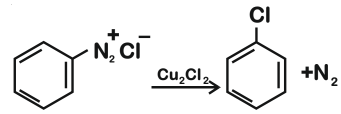
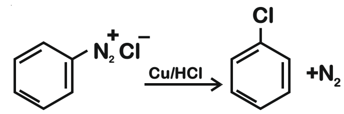
(3) 

(4) 

Answer (4)

Sol. 

3. Match List-I with List-II

List-I	List-II
(a) 	(i) Wurtz reaction
(b) 	(ii) Sandmeyer reaction
(c) $2CH_3CH_2Cl + 2Na \xrightarrow{Ether} C_2H_5 - C_2H_5 + 2NaCl$	(iii) Fittig reaction
(d) $2C_6H_5Cl + 2Na \xrightarrow{Ether} C_6H_5 - C_6H_5 + 2NaCl$	(iv) Gatterman reaction

Choose the correct answer from the options given below

(1) (a)-(iii); (b)-(iv); (c)-(i); (d)-(ii)

(2) (a)-(ii); (b)-(iv); (c)-(i); (d)-(iii)

(3) (a)-(iii); (b)-(i); (c)-(iv); (d)-(ii)

(4) (a)-(ii); (b)-(i); (c)-(iv); (d)-(iii)

Answer (2)

Sol. (a) – Sandmeyer reaction

(b) – Gatterman reaction

(c) – Wurtz reaction

(d) – Fittig reaction

(a)-(ii); (b)-(iv); (c)-(i); (d)-(iii)

4. Match list-I with list-II

List-I (Molecule)	List-II (Bond order)
(a) Ne_2	(i) 1
(b) N_2	(ii) 2
(c) F_2	(iii) 0
(d) O_2	(iv) 3

Choose the correct answer from the options given below

(1) (a)-(iv); (b)-(iii); (c)-(ii); (d)-(i)

(2) (a)-(ii); (b)-(i); (c)-(iv); (d)-(iii)

(3) (a)-(i); (b)-(ii); (c)-(iii); (d)-(iv)

(4) (a)-(iii); (b)-(iv); (c)-(i); (d)-(ii)

Answer (4)

Sol.

Molecule	Bond order
Ne_2	0
N_2	3
F_2	1
O_2	2

(a)-(iii); (b)-(iv); (c)-(i); (d)-(ii)

5. 2,4-DNP test can be used to identify
   (1) Aldehyde
   (2) Amine
   (3) Ether
   (4) Halogens

Answer (1)

Sol. 2,4 DNP test is used to identify $-\overset{\overset{O}{\|}}{C}-$ group. It gives addition reaction with carbonyl compounds. So, it can be used to identify aldehyde in the given option. It gives yellow/ orange PPt with carbonyl containing compounds.

6. Seliwanoff test and Xanthoproteic test are used for the identification of _______and _____ respectively.
   (1) Ketoses, aldoses (2) Proteins, ketoses
   (3) Ketoses, proteins (4) Aldoses, ketoses

Answer (3)

Sol. Seliwanoff test is used to distinguish ketoses from aldoses. On treatment with a concentrated acid, ketones are dehydrated more rapidly to give furfural derivative and on condensation with resorcinol give cherry red complex.

Positive Seliwanoff's test – Ketoses present

Positive Xanthoproteic test – Presence of aromatic amino acid

The Xanthoproteic reaction is a method that can be used to detect presence of protein soluble in a solution, using concentrated nitric acid.

7. The correct order of electron gain enthalpy is:
   (1) O > S > Se > Te
   (2) Te > Se > S > O
   (3) S > O > Se > Te
   (4) S > Se > Te > O

Answer (4)

Sol. Correct order of electron gain enthalpy is

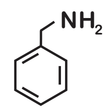
S > Se > Te > O

8. A. Phenyl methanamine
   B. N,N-Dimethylaniline
   C. N-Methyl aniline
   D. Benzenamine

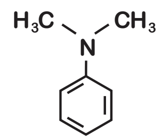
   Choose the correct order of basic nature of the above amines.
   (1) A > C > B > D (2) D > B > C > A
   (3) D > C > B > A (4) A > B > C > D

Answer (4)

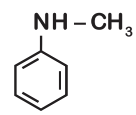
Sol.



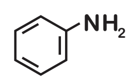
(A) Phenyl methanamine $pK_b = 4.7$



(B) N, N-Dimethylaniline $pK_b = 8.92$



(C) N-Methyl aniline $pK_b = 9.3$



(D) Benzenamine $pK_b = 9.38$

$$pK_b \propto \frac{1}{\text{Basicity}}$$

(A) > (B) > (C) > (D)

9. Match List-I with List-II.

List - I	List - II
(a) Sodium Carbonate	(i) Deacon
(b) Titanium	(ii) Castner-Kellner
(c) Chlorine	(iii) van-Arkel
(d) Sodium hydroxide	(iv) Solvay

Chose the correct answer from the options given below:
(1) (a) $\rightarrow$ (i), (b) $\rightarrow$ (iii), (c) $\rightarrow$ (iv), (d) $\rightarrow$ (ii)
(2) (a) $\rightarrow$ (iii), (b) $\rightarrow$ (ii), (c) $\rightarrow$ (i), (d) $\rightarrow$ (iv)
(3) (a) $\rightarrow$ (iv), (b) $\rightarrow$ (i), (c) $\rightarrow$ (ii), (d) $\rightarrow$ (iii)
(4) (a) $\rightarrow$ (iv), (b) $\rightarrow$ (iii), (c) $\rightarrow$ (i), (d) $\rightarrow$ (ii)

Answer (4)

Sol.

Compound	Method of preparation
Sodium Carbonate	Solvey
Titanium	van-Arkel
Chlorine	Deacon
Sodium hydroxide	Castner-Kellner

(a) $\rightarrow$ (iv), (b) $\rightarrow$ (iii), (c) $\rightarrow$ (i), (d) $\rightarrow$ (ii)

10. The nature of charge on resulting colloidal particles when $FeCl_3$ is added to excess of hot water is:

(1) Sometimes positive and sometimes negative

(2) Negative

(3) Neutral

(4) Positive

Answer (4)

Sol. Some $FeCl_3/Fe^{3+}$ will get hydrolyzed and form $Fe(OH)_3$. Over which some Fe^{3+} will get adsorbed. So the resulting charge on colloidal particle will be positive.

11. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : In TlI_3, isomorphous to CsI_3, the metal is present in +1 oxidation state.

Reason R : Tl metal has fourteen f electrons in its electronic configuration.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both A and R are correct but R is NOT the correct explanation of A

(2) Both A and R are correct and R is the correct explanation of A

(3) A is correct but R is not correct

(4) A is not correct but R is correct

Answer (1)

Sol.

A : Due to inert pair effect, Tl is more stable in +1 oxidation state

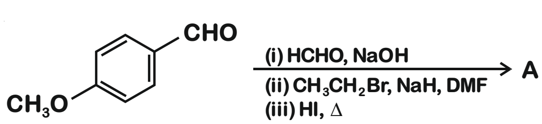
Hence TlI_3 and CSI_3 are isomorphous

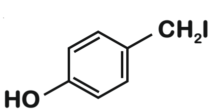
R : Electronic configuration of Tl (81) =

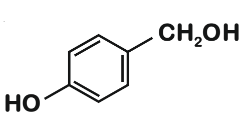
$Xe\ 4f^{14}\ 5d^{10}\ 6s^2\ 6p^1$

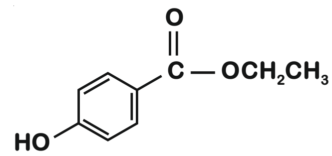
Both A and R are correct but R is not the correct explanation of A.

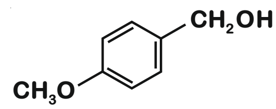
12. Identify A in the following chemical reaction.



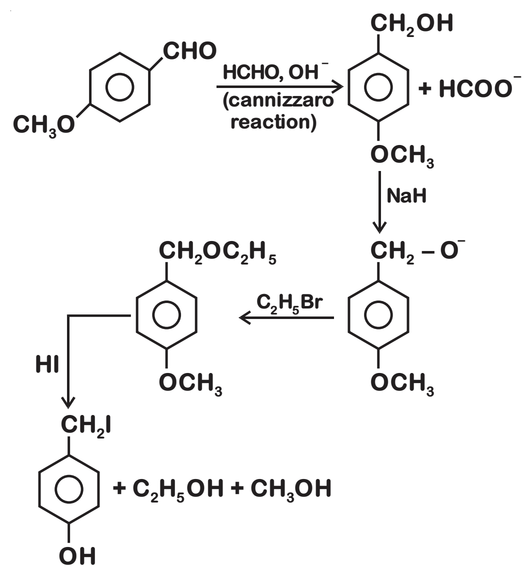
(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. 

13. Calgon is used for water treatment. Which of the following statement is NOT true about Calgon?

(1) It is polymeric compound and is water soluble

(2) Calgon contains the 2^{nd} most abundant element by weight in the Earth's crust

(3) It is also known as Graham's salt

(4) It doesnot remove Ca^{2+} ion by precipitation

Answer (2)

Sol. Calgon is sodium hexametaphosphate, a polymeric compound also called as Graham's salt.

Silicon is the 2^{nd} most abundant element which is absent in calgon.

14. Match List-I with List-II.

List-I	List-II
(a) Siderite	(i) Cu
(b) Calamine	(ii) Ca
(c) Malachite	(iii) Fe
(d) Cryolite	(iv) Al
	(v) Zn

Choose the **correct** answer from the options given below

(1) (a) (i), (b) (ii), (c) (iii), (d) (iv)

(2) (a) (i), (b) (ii), (c) (v), (d) (iii)

(3) (a) (iii), (b) (v), (c) (i), (d) (iv)

(4) (a) (iii), (b) (i), (c) (v), (d) (ii)

Answer (3)

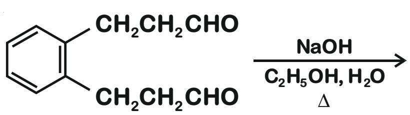
Sol. Siderite $FeCO_3$

Calamine $ZnCO_3$

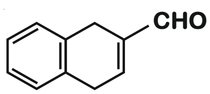
Malachite $CuCO_3 \cdot Cu(OH)_2$

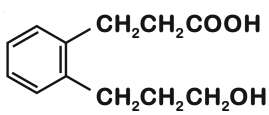
Cryolite Na_3AlF_6

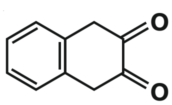
15. Identify A in the given chemical reaction.

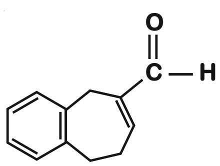


A (Major Product)

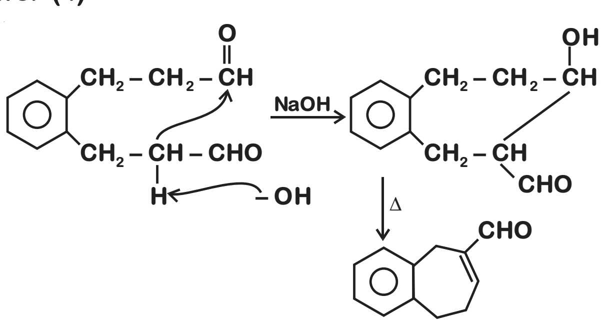
(1) 

(2) 

(3) 

(4) 

Answer (4)

Sol. 

16. Ceric ammonium nitrate and $CHCl_3$/alc. KOH are used for the identification of functional groups present in _____ and _____ respectively.

(1) Alcohol, phenol

(2) Amine, phenol

(3) Amine, alcohol

(4) Alcohol, amine

Answer (4)

Sol. Ceric ammonium nitrate is used for the identification of alcohol.

$$2R-OH + (NH_4)_2Ce(NO_3)_6 \longrightarrow \underset{\substack{\text{Alkoxy cerium ion (IV)} \\ \text{Compound} \\ \text{(Pink or Red colour)}}}{(ROH)_2Ce(NO_3)_4} + 2NH_4NO_3$$

$CHCl_3$/KOH is used for the identification of primary amines.

$$R-NH_2 + CHCl_3 + 3KOH(alc) \xrightarrow{\text{warm}} R-NC + 3KCl + 3H_2O$$

17. In $\overset{1}{C}H_2 = \overset{2}{C} = \overset{3}{C}H - \overset{4}{C}H_3$ molecule, the hybridization of carbon 1, 2, 3 and 4 respectively, are :

(1) sp^2, sp^2, sp^2, sp^3

(2) sp^2, sp, sp^2, sp^3

(3) sp^3, sp, sp^3, sp^3

(4) sp^2, sp^3, sp^2, sp^3

Answer (2)

Sol. $\underset{sp^2}{CH_2} = \underset{sp}{C} = \underset{sp^2}{CH} - \underset{sp^3}{CH_3}$

Hybridization of carbon 1, 2, 3 and 4 respectively are sp^2, sp, sp^2 and sp^3

18. Which of the following forms of hydrogen emits low energy β^- particles?

(1) Proton H^+

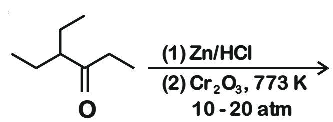
(2) Tritium ${}^3_1H$

(3) Protium ${}^1_1H$

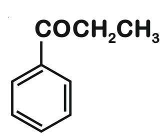
(4) Deuterium ${}^2_1H$

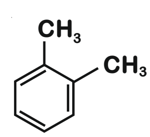
Answer (2)

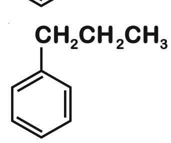
Sol. Out of isotopes of hydrogen, only tritium is radioactive and emits low energy β^- particles.

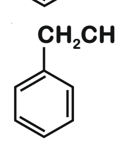
19. 

Considering the above reaction, the major product among the following is :

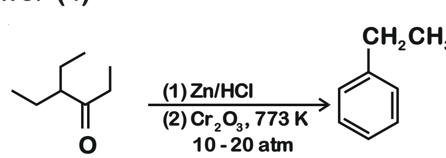
(1) 

(2) 

(3) 

(4) 

Answer (4)

Sol. 

20. Match List-I with List-II.

List-I	List-II
(a) Sucrose	(i) β-D-Galactose and β-D-Glucose
(b) Lactose	(ii) α-D-Glucose and β-D-Fructose
(c) Maltose	(iii) α-D-Glucose and α-D-Glucose

Choose the correct answer from the options given below :

(1) (a) → (i), (b) → (iii), (c) → (ii)

(2) (a) → (iii), (b) → (ii), (c) → (i)

(3) (a) → (ii), (b) → (i), (c) → (iii)

(4) (a) → (iii), (b) → (i), (c) → (ii)

Answer (3)

Sol.

Disaccharides	Monomer present
Sucrose	α-D-glucose and β-D-fructose
Lactose	β-D-Galactose and β-D-Glucose
Maltose	α-D-Glucose and α-D-Glucose

(a) → (ii), (b) → (i), (c) → (iii)

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. If the activation energy of a reaction is 80.9 kJ mol^{-1}, the fraction of molecules at 700 K, having enough energy to react to form products is e^{-x}. The value of x is _________. (Rounded off to the nearest integer)

[Use R = 8.31 J K^{-1} mol^{-1}]

Answer (14)

Energy of activation, E_a = 80.9 kJ mol^{-1}

Temperature of reaction, T = 700 K

Fraction of molecules having enough energy to react $= e^{-E_a/RT} = e^{-x}$

$$\therefore\ x = \frac{E_a}{RT} = \frac{80900}{8.31 \times 700} = 13.9 \simeq 14$$

2. The average S-F bond energy in kJ mol^{-1} of SF_6 is ____________. (Rounded off to the nearest integer)

[Given : The values of standard enthalpy of formation of $SF_{6(g)}$, $S_{(g)}$ and $F_{(g)}$ are - 1100, 275 and 80 kJ mol^{-1} respectively.]

Answer (309)

$$SF_6(g) \longrightarrow S(g) + 6F(g)$$

$$\Delta H^\circ = \Delta H_f^\circ(S) + 6\Delta H_f^\circ(F) - \Delta H_f^\circ(SF_6)$$

= 275 + 6 × 80 – (–1100)

= 1855 kJ mol^{-1}

Also, $\Delta H^\circ = 6\Delta H_{S-F}$

$$\therefore\ \Delta H_{S-F} = \frac{1855}{6} = 309.17 \simeq 309\ kJ\ mol^{-1}$$

3. The $NaNO_3$ weighed out to make 50 mL of an aqueous solution containing 70.0 mg Na^+ per mL is __________ g. (Rounded off to the nearest integer)

[Given : Atomic weight in g mol^{-1} - Na : 23; N : 14; O : 16]

Answer (13)

Mass of Na^+ in 50 mL = 70 × 50 mg

$$\text{Millimoles of } NaNO_3 = \frac{70 \times 50}{23}$$

$$\text{Mass of } NaNO_3 = \frac{70 \times 50 \times 85 \times 10^{-3}}{23}$$

$$= 12.9 \simeq 13\ g$$

4. When 12.2 g of benzoic acid is dissolved in 100 g of water, the freezing point of solution was found to be –0.93°C (K_f (H_2O) = 1.86 K kg mol^{-1}). The number (n) of benzoic acid molecules associated (assuming 100% association) is _________.

Answer (02.00)

$$\Delta T_f = iK_f\,m$$

$$0.93 = i \times 1.86 \times \frac{12.2 \times 1000}{122 \times 100}$$

i = 0.5

$$n(\text{Benzoic acid}) \longrightarrow (\text{Benzoic acid})_n$$

$$i = \frac{\text{Total number of particles after association}}{\text{Number of particles before association}}$$

$$0.5 = \frac{1}{n}$$

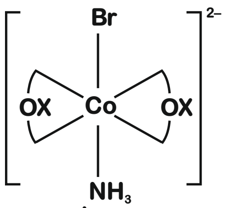
n = 2

5. The number of stereoisomers possible for $[Co(ox)_2(Br)(NH_3)]^{2-}$ is ________.

[ox = oxalate]

Answer (3)

Total number of stereoisomers possible for $[Co(OX)_2Br(NH_3)]^{2-}$ is 3.



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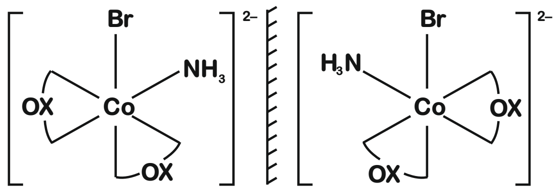


cis (+) (–)

6. The number of octahedral voids per lattice site in a lattice is _______. (Rounded off to the nearest integer)

Answer (1.00)

Sol. Number of octahedral voids present in a lattice (ccp or hcp) is equal to the number of close packed particles.

So the number of octahedral voids per particle = 1

7. The pH of ammonium phosphate solution, if pk_a of phosphoric acid and pk_b of ammonium hydroxide are 5.23 and 4.75 respectively, is ________.

Answer (7.00)

Sol. $(NH_4)_3PO_4$ is a salt of weak base and weak acid So pH in independent of concentration of salt. (Assuming no salt hydrolysis is occurring)

$$pH = \frac{1}{2}pK_w + \frac{1}{2}pK_a - \frac{1}{2}pK_b$$

$$= 7 + \frac{1}{2}(5.23) - \frac{1}{2}(4.75)$$

= 7.24

$\approx$ 7.00 (nearest integer)

8. A ball weighing 10 g is moving with a velocity of 90 ms^{-1}. If the uncertainty in its velocity is 5%, then the uncertainty in its position is __________ × 10^{-33} m. (Rounded off to the nearest integer)

[Given : h = 6.63 × 10^{-34} Js]

Answer (1)

Sol. According to Heisenberg uncertainty principle

$$\Delta x \Delta p \geq \frac{h}{4\pi}$$

$$\Delta x = \frac{h}{4\pi m \Delta V} (\Delta P = m\Delta V)$$

$$= \frac{6.63 \times 10^{-34} \text{ Js}}{4 \times 3.14 \times 10 \times 10^{-3} \text{ kg} \times 90 \text{ ms}^{-1} \times 0.05}$$

= 1.173 × 10^{-33} m

= 1 × 10^{-33} m

9. Emf of the following cell at 298 K in V is x × 10^{-2}.

$Zn|Zn^{2+}$ (0.1 M)||Ag^+ (0.01 M)|Ag

The value of x is ________. (Rounded off to the nearest integer)

[Given : $E^\theta_{Zn^{2+}/Zn} = -0.76\,V;$

$E^\theta_{Ag^+/Ag} = +0.80\,V; \frac{2.303RT}{F} = 0.059$]

Answer (147)

Sol. $Zn + 2Ag^+ \longrightarrow 2Ag + Zn^{2+}$

$$Q = \frac{[Zn^{2+}]}{[Ag^+]^2} = \frac{(0.1)}{(10^{-2})^2} = 10^3$$

$$\text{emf} = 0.80 + 0.76 - \frac{0.059}{2}\log 10^3$$

= 1.47 volt

emf = 147 × 10^{-2} volt

x = 147

10. In mildly alkaline medium, thiosulphate ion is oxidized by MnO_4^- to "A". The oxidation state of sulphur in "A" is _______.

Answer (6)

Sol. In neutral or faintly alkaline medium

$$8MnO_4^- + 3S_2O_3^{2-} + H_2O \longrightarrow 8MnO_2 + \underset{(A)}{6SO_4^{2-}} + 2OH^-$$

A is SO_4^{2-}. The oxidation state of sulphur in A is +6.

PART–C : MATHEMATICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. If the mirror image of the point (1, 3, 5) with respect to the plane 4x – 5y + 2z = 8 is (α, β, γ), then $5(\alpha + \beta + \gamma)$ equals :

(1) 43 (2) 47

(3) 41 (4) 39

Answer (2)

Sol. $\frac{\alpha-1}{4}=\frac{\beta-3}{-5}=\frac{\gamma-5}{2}=-2\frac{(4\times1-5\times3+2\times5-8)}{16+25+4}$

$\frac{\alpha-1}{4}=\frac{\beta-3}{-5}=\frac{\gamma-5}{2}=\frac{2}{5}$

$\alpha=\frac{8}{5}+1, \beta=\frac{-10}{5}+3, \gamma=\frac{4}{5}+5$

$5|\alpha+\beta+\gamma|=|5\alpha+5\beta+5\gamma|=47$

2. A natural number has prime factorization given by $n = 2^x3^y5^z$, where y and z are such that y + z = 5 and $y^{-1} + z^{-1} = \frac{5}{6}, y>z$. Then the number of odd divisors of n, including 1, is :

(1) 12 (2) 6x

(3) 11 (4) 6

Answer (1)

Sol. y + z = 5 ...(i)

$\frac{1}{y}+\frac{1}{z}=\frac{y+z}{yz}=\frac{5}{6} \Rightarrow yz=6$ ...(ii)

Equation with y and z as roots is

$x^2 - 5x + 6 = 0$

x = 2, 3, y = 3, z = 2 (y > z)

$n = 2^x \cdot 3^3 \cdot 5^2$

For odd divisors x = 1 only

No. of odd divisors = 1 × 4 × 3 = 12

3. Let A = {1, 2, 3, ..., 10} and f : A → A be defined as

$$f(k) = \begin{cases} k+1 & \text{if k is odd} \\ k & \text{if k is even} \end{cases}$$

Then the number of possible functions g : A → A such that gof = f is :

(1) 5^5 (2) 10^5

(3) 5! (4) $^{10}C_5$

Answer (2)

Sol. Not that f(1) = f(2) = 2

f(3) = f(4) = 4

f(5) = f(6) = 6

f(7) = f(8) = 8

f(9) = f(10) = 10

gof(1) = f(1) ⇒ g(2) = f(1) = 2

gof(2) = f(2) ⇒ g(2) = f(2) = 2

gof(3) = f(3) ⇒ g(4) = f(3) = 4

∴ In function g(x), 2, 4, 6, 8, 10 should be mapped to 2, 4, 6, 8, 10 respectively. Each of remaining elements can be mapped to any of 10 elements.

Number of possible g(x) is 10^5

4. Let slope of the tangent line to a curve at any point P(x, y) be given by $\frac{xy^2+y}{x}$. If the curve intersects the line x + 2y = 4 at x = –2, then the value of y, for which the point (3, y) lies on the curve, is :

(1) $\frac{18}{35}$ (2) $-\frac{4}{3}$

(3) $-\frac{18}{11}$ (4) $-\frac{18}{19}$

Answer (4)

Sol. $\frac{dy}{dx}=y^2+\frac{y}{x}$

$\frac{dy}{dx}-\frac{y}{x}=y^2$

$\frac{1}{y^2}\frac{dy}{dx}-\frac{1}{x}\times\frac{1}{y}=1$

Let $\frac{1}{y}=z$

$\frac{-1}{y^2}\frac{dy}{dx}=\frac{dz}{dx}$

$\frac{-dz}{dx}-\frac{1}{x}z=1$

$$\frac{dz}{dx}+\frac{1}{x}z=-1$$

$$\text{IF} = e^{\int \frac{1}{x}dx} = e^{\ell nx} = x$$

$$z\cdot x=\int -1\cdot x\,dx$$

$$z\cdot x=\frac{-x^2}{2}+c$$

$$\frac{x}{y}=\frac{-x^2}{2}+c \qquad \text{...(i)}$$

Putting x = –2 in x + 2y = 4, we get y = 3

Put (–2, 3) in (i)

$$\Rightarrow \quad c=\frac{4}{3}$$

$$\text{(i)} \Rightarrow \frac{x}{y}=\frac{-x^2}{2}+\frac{4}{3} \qquad \text{...(ii)}$$

Put x = 3 in (ii)

$$\frac{3}{y}=\frac{-9}{2}+\frac{4}{3}$$

$$y=\frac{-18}{19}$$

5. The triangle of maximum area that can be inscribed in a given circle of radius ‘r’ is :

(1) An isosceles triangle with base equal to 2r.

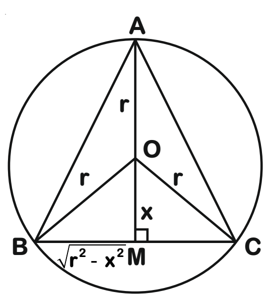
(2) An equilateral triangle of height $\frac{2r}{3}$.

(3) A right angle triangle having two of its sides of length 2r and r.

(4) An equilateral triangle having each of its side of length $\sqrt{3}$ r.

Answer (4)

Sol. Area of triangle ABC



$$A=\frac{1}{2}\times BC\times AM$$

$$=\frac{1}{2}\times 2\sqrt{r^2-x^2}\times(r+x)$$

$$A=(r+x)\sqrt{r^2-x^2}$$

$$\frac{dA}{dx}=\sqrt{r^2-x^2}-\frac{x}{\sqrt{r^2-x^2}}\times(r+x)=\frac{r^2-x^2-rx-x^2}{\sqrt{r^2-x^2}}$$

$$=\frac{r^2-rx-2x^2}{\sqrt{r^2-x^2}}=\frac{-(x+r)(2x-r)}{\sqrt{r^2-x^2}}$$

$$\frac{dA}{dx}=0 \;\Rightarrow\; x=\frac{r}{2}$$

Sign change of $\frac{dA}{dx}$ at $x=\frac{r}{2} \Rightarrow$ A has maximum at $x=\frac{r}{2}$ $BC=2\sqrt{r^2-x^2}=\sqrt{3}\,r$,

$$AM=\frac{3}{2}r$$

$$\Rightarrow \quad AB=AC=\sqrt{3}\,r$$

6. A seven digit number is formed using digits 3, 3, 4, 4, 4, 5, 5. The probability, that number so formed is divisible by 2, is :

(1) $\frac{1}{7}$

(2) $\frac{6}{7}$

(3) $\frac{4}{7}$

(4) $\frac{3}{7}$

Answer (4)

Sol. For even number, units place should be filled with 4 only.

$$P=\frac{\dfrac{6!}{2!2!2!}}{\dfrac{7!}{2!3!2!}}=\frac{6!}{2!}\times\frac{3!}{7!}=\frac{3}{7}$$

7. Let $F_1(A, B, C) = (A \wedge \sim B) \vee [\sim C \wedge (A \vee B)] \vee \sim A$ and $F_2(A, B) = (A \vee B) \vee (B \rightarrow \sim A)$ be two logical expressions. Then :

(1) F_1 and F_2 both are tautologies

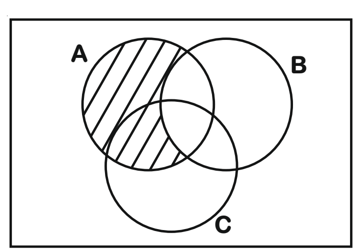
(2) Both F_1 and F_2 are not tautologies

(3) F_1 is a tautology but F_2 is not a tautology

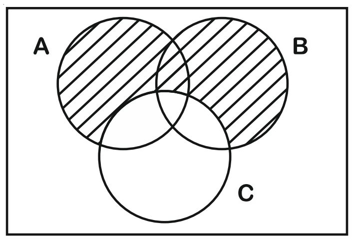
(4) F_1 is not a tautology but F_2 is a tautology

Answer (4)

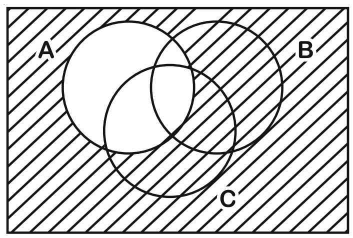
Sol.



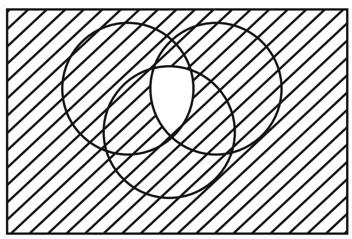
$A \wedge \sim B$



$\sim C \wedge (A \vee B)$



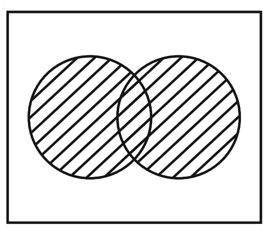
$\sim A$



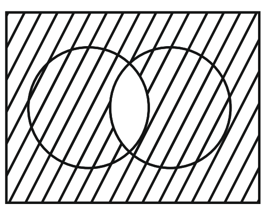
$\Rightarrow$ F1 is not a tautology

F1

$B \to \sim A = \sim B \vee \sim A$



$\vee$



$= F_2$

$A \vee B$ $\quad$ $\sim B \vee \sim A = \sim(A \cap B)$

$\Rightarrow$ F_2 is a tautology

8. Let $f : R \to R$ be defined as

$$f(x) = \begin{cases} 2\sin\left(-\frac{\pi x}{2}\right), \text{if } x < -1 \\ |ax^2 + x + b|, \text{if } -1 \le x \le 1 \\ \sin(\pi x), \quad \text{if } x > 1 \end{cases}$$

If f(x) is continuous on R, then a + b equals :

(1) –1 (2) –3

(3) 3 (4) 1

Answer (1)

Sol. $f(-1^-) = 2$

$f(-1^+) = |a + b - 1|$

$|a + b - 1| = 2$ ...(i)

$f(1^-) = |a + b + 1|$

$f(1^+) = 0$

$|a + b + 1| = 0 \Rightarrow a + b + 1 = 0$

$\Rightarrow$ a + b = –1 ...(ii)

9. Let L be a line obtained from the intersection of two planes x + 2y + z = 6 and y + 2z = 4. If point P(α, β, γ) is the foot of perpendicular from (3, 2, 1) on L, then the value of $21(\alpha + \beta + \gamma)$ equals :

(1) 68 (2) 102

(3) 142 (4) 136

Answer (2)

Sol. Direction of line L = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix} = 3\hat{i} - 2\hat{j} + \hat{k}$

d.r's = < 3, –2, 1>

A point on line (–2, 4, 0)

Line = $\frac{x+2}{3} = \frac{y-4}{-2} = \frac{z}{1}$

Foot of perpendicular from (3, 2, 1) be ($3\lambda - 2$, $-2\lambda + 4$, λ)

$(3\lambda - 5) . 3 + (-2\lambda + 2)(-2) + (\lambda - 1)1 = 0$

$9\lambda - 15 + 4\lambda - 4 + \lambda - 1 = 0$

$14\lambda - 20 = 0 \Rightarrow \lambda = \frac{10}{7}$

$(\alpha, \beta, \gamma) = \left(\frac{16}{7}, \frac{8}{7}, \frac{10}{7}\right)$

$\therefore 21(\alpha + \beta + \gamma) = (16 + 8 + 10)3 = 102$

10. Let f(x) be a differentiable function at x = a with $f'(a) = 2$ and $f(a) = 4$. Then $\lim_{x \to a} \frac{xf(a) - af(x)}{x - a}$ equals :

(1) 4 – 2a (2) a + 4

(3) 2a – 4 (4) 2a + 4

Answer (1)

Sol. $L = \lim_{x \to a} \frac{xf(a) - af(x)}{x - a} \quad \left[\frac{0}{0} \text{ form}\right]$

Using L' Hospital rule we get

$L = \lim_{x \to a} \frac{f(a) - af'(x)}{1}$

$f(a) - af'(a) = 4 - 2a$

11. Let A(1, 4) and B(1, –5) be two points. Let P be a point on the circle $(x-1)^2 + (y-1)^2 = 1$ such that $(PA)^2 + (PB)^2$ have maximum value, then the points, P, A and B lie on :

(1) an ellipse (2) a parabola

(3) a straight line (4) a hyperbola

Answer (3)

Sol. Let P be $(1 + \cos\theta, 1 + \sin\theta)$

$$(PA)^2 + (PB)^2 = (\cos\theta)^2 + (\sin\theta - 3)^2 + (\cos\theta)^2 + (\sin\theta + 6)^2$$

$$= 1 - 6\sin\theta + 9 + 1 + 12\sin\theta + 36$$

$$= 45 + 6\sin\theta \text{ maximum at } \theta = \frac{\pi}{2}$$

$\therefore$ P (1, 2)

$\therefore$ P, A and B are colinear

12. Let A_1 be the area of the region bounded by the curves y = sinx, y = cosx and y-axis in the first quadrant. Also, let A_2 be the area of the region bounded by the curves y = sinx, y = cosx, x-axis and $x = \frac{\pi}{2}$ in the first quadrant. Then,

(1) $A_1 : A_2 = 1 : \sqrt{2}$ and $A_1 + A_2 = 1$

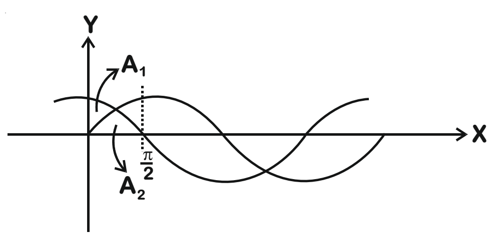
(2) $A_1 : A_2 = 1 : 2$ and $A_1 + A_2 = 1$

(3) $2A_1 = A_2$ and $A_1 + A_2 = 1 + \sqrt{2}$

(4) $A_1 = A_2$ and $A_1 + A_2 = \sqrt{2}$

Answer (1)

Sol.



$$A_1 = \int_0^{\frac{\pi}{2}} (\cos x - \sin x)\,dx = \sqrt{2} - 1$$

$$A_2 = \int_0^{\frac{\pi}{4}} \sin x\,dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x\,dx = \sqrt{2}(\sqrt{2} - 1)$$

$\therefore$ $A_1 : A_2 = 1 : \sqrt{2}$ & $A_1 + A_2 = 1$

13. Let $f(x) = \int_0^x e^t f(t)dt + e^x$ be a differentiable function for all $x \in R$. Then f(x) equals :

(1) $e^{(e^x - 1)}$

(2) $2e^{e^x} - 1$

(3) $2e^{(e^x - 1)} - 1$

(4) $e^{e^x} - 1$

Answer (3)

Sol. Apply Lebnitz' Rule we get

$t^1(x) = e^x + (y) + e^x$

$$\int \frac{dy}{y+1} = \int e^x dx$$

$\Rightarrow$ ln (y + 1) = e^x + c

$\downarrow$ (0, 1)

$$c = \ln\left(\frac{2}{e}\right)$$

$$y + 1 = e^{ex} \cdot \frac{2}{e} \Rightarrow y = \left(2 \cdot e^{e^x - 1}\right) - 1$$

14. If 0 < a, b < 1, and $\tan^{-1}a + \tan^{-1}b = \frac{\pi}{4}$, then the value of

$$(a + b) - \left(\frac{a^2 + b^2}{2}\right) + \left(\frac{a^3 + b^3}{3}\right) - \left(\frac{a^4 + b^4}{4}\right) + \ldots$$

is :

(1) $e^2 - 1$ (2) $\log_e\left(\frac{e}{2}\right)$

(3) e (4) $\log_e 2$

Answer (4)

Sol. $$\tan^{-1}a + \tan^{-1}b = \tan^{-1}\left(\frac{a+b}{a-ab}\right) = \frac{\pi}{4}$$

$\Rightarrow$ a + b + ab = 1

$\Rightarrow$ (1 + a)(1 + b) = 2

Given

$$\left(a - \frac{a^2}{2} + \frac{a^3}{3} + \ldots\right) + \left(b - \frac{b^2}{2} + \frac{b^3}{3} + \ldots\right)$$

ln(1 + a) + ln(1 + b)

$\Rightarrow$ ln(1 + a)(1 + b) = ln 2

15. The sum of the series $\sum_{n=1}^{\infty} \frac{n^2+6n+10}{(2n+1)!}$ is equal to :

(1) $\frac{41}{8}e+\frac{19}{8}e^{-1}-10$

(2) $\frac{41}{8}e-\frac{19}{8}e^{-1}-10$

(3) $-\frac{41}{8}e+\frac{19}{8}e^{-1}-10$

(4) $\frac{41}{8}e+\frac{19}{8}e^{-1}+10$

Answer (2)

Sol. $\sum_{n=1}^{\infty}\frac{n^2+6n+10}{(2n+1)!} = \frac{1}{2}\sum_{n=1}^{\infty}\frac{2n^2+12n+20}{(2n+1)!}$

$= \frac{1}{2}\sum_{n=1}^{\infty}\frac{n(2n+1)+11n+20}{(2n+1)!}$

$= \frac{1}{2}\sum_{n=1}^{\infty}\frac{n}{(2n)!}+\frac{1}{2}\sum_{n=1}^{\infty}\frac{11n+\frac{11}{2}+\frac{29}{2}}{(2n+1)!}$

$= \frac{1}{4}\sum_{n=1}^{\infty}\frac{2n}{(2n)!}+\frac{1}{2}\cdot\frac{11}{2}\sum_{n=1}^{\infty}\frac{2n+1}{(2n+1)!}+\frac{29}{4}\sum_{n=1}^{\infty}\frac{1}{(2n+1)!}$

$= \frac{1}{4}\sum_{n=1}^{\infty}\frac{2n}{(2n-1)!}+\frac{11}{4}\sum_{n=1}^{\infty}\frac{1}{2n!}+\frac{29}{4}\sum_{n=1}^{\infty}\frac{1}{(2n+1)!}$

$= \frac{1}{4}\left(\frac{e-e^{-1}}{2}\right)+\frac{11}{4}\left(\frac{e+e^{-1}}{2}-1\right)+\frac{29}{4}\left(\frac{e-e^{-1}}{2}-1\right)$

$= \frac{15}{2}\left(\frac{e-e^{-1}}{2}\right)+\frac{11}{4}\left(\frac{e+e^{-1}}{2}\right)-10$

$= \frac{41}{8}e-\frac{19}{8}e^{-1}-10$

16. For x > 0, if f(x) = $\int_1^x \frac{\log_e t}{(1+t)}dt$, then $f(e)+f\left(\frac{1}{e}\right)$ is equal to :

(1) 1 (2) $\frac{1}{2}$

(3) 0 (4) –1

Answer (2)

Sol. $f(x)=\int_1^x \frac{\ln t}{1+t}dt$

then $f\left(\frac{1}{x}\right)=\int_1^{1/x}\frac{\ln t}{1+t}dt$

Let $t=\frac{1}{u} \Rightarrow dt=-\frac{1}{u^2}du$

$\Rightarrow f\left(\frac{1}{x}\right)=\int_1^x \frac{\ln\frac{1}{u}}{1+\frac{1}{u}}\left(-\frac{1}{u^2}\right)dx$

$f\left(\frac{1}{x}\right)=\int_1^x \frac{\ln u}{u(1+u)}du=\int_1^x\frac{\ln t}{t(1+t)}dt$

$\therefore f(x)+f\left(\frac{1}{x}\right)=\int_1^x \ln t\left(\frac{1}{1+t}+\frac{1}{t(1+t)}\right)dt$

$=\int_1^x \ln t\left(\frac{1}{1+t}+\frac{1}{t}-\frac{1}{t+1}\right)dt$

$=\int_1^x \frac{\ln t}{t}dt=\frac{1}{2}(\ln x)^2$

$\therefore f(e)+f\left(\frac{1}{e}\right)=\frac{1}{2}(\ln e)^2=\frac{1}{2}$

17. If vectors $\vec{a}_1 = x\hat{i}-\hat{j}+\hat{k}$ and $\vec{a}_2=\hat{i}+y\hat{j}+z\hat{k}$ are collinear, then a possible unit vector parallel to the vector $x\hat{i}+y\hat{j}+z\hat{k}$ is :

(1) $\frac{1}{\sqrt{3}}\left(\hat{i}-\hat{j}+\hat{k}\right)$ (2) $\frac{1}{\sqrt{3}}\left(\hat{i}+\hat{j}-\hat{k}\right)$

(3) $\frac{1}{\sqrt{2}}\left(\hat{i}-\hat{j}\right)$ (4) $\frac{1}{\sqrt{2}}\left(-\hat{j}+\hat{k}\right)$

Answer (1)

Sol. $\vec{a_2}=\lambda\vec{a_1}$

$\hat{i}+y\hat{j}+z\hat{k}=\lambda\left(x\hat{i}-j+\hat{k}\right)$

$1=\lambda x, y=-\lambda, z=\lambda$

$x\hat{i}+y\hat{j}+z\hat{k}=\frac{1}{\lambda}\hat{i}-\lambda\hat{j}+\lambda\hat{k}$

Unit vector $=\frac{\frac{1}{\lambda}i-\lambda\hat{j}+\lambda\hat{k}}{\sqrt{\frac{1}{\lambda^2}+\lambda^2+\lambda^2}}$

$=\frac{\hat{i}-\lambda^2\hat{j}+\lambda^2\hat{k}}{\sqrt{1+2\lambda^4}}$

Let λ^2 = 1, possible unit vector $=\frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}$

18. If the locus of the mid-point of the line segment from the point (3, 2) to a point on the circle, $x^2 + y^2 = 1$ is a circle of radius r, then r is equal to :

(1) $\frac{1}{3}$ (2) 1

(3) $\frac{1}{4}$ (4) $\frac{1}{2}$

Answer (4)

Sol. Let P(h, k)

Required laws $\frac{3+\cos\theta}{2} = h$ and $\frac{2+\sin\theta}{2} = k$

$\cos\theta = 2h - 3$ and $\sin\theta = 2h - 2$

Squaring and adding we get

$(2h - 3)^2 + (2h - 2)^2 = 1$

$\Rightarrow 4x^2 - 12x + 9 + 4y^2 - 8y + 4 = 1$

$\Rightarrow 4x^2 + 4y^2 - 12x - 8y + 12 = 0$

$\Rightarrow x^2 + y^2 - 3x - 2y + 3 = 0$

Radius $= \sqrt{\frac{9}{4} + 1 - 3} = \frac{1}{2}$

19. Consider the following system of equations :

$x + 2y - 3z = a$

$2x + 6y - 11z = b$

$x - 2y + 7z = c$,

where a, b and c are real constants. Then the system of equations :

(1) has infinite number of solutions when $5a = 2b + c$

(2) has no solution for all a, b and c

(3) has a unique solution when $5a = 2b + c$

(4) has a unique solution for all a, b and c

Answer (1)

Sol. $0 = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 6 & -11 \\ 1 & -2 & 7 \end{vmatrix} = (20) - 2(25) - 3(-10) = 0$

$x + 2y - 3z = a$ ...(1)

$2x + 6y - 11z = b$ ...(2)

$x - 2y + 7z = c$ ...(3)

5eq (1) = 2eq (2) + eq (3)

it $5a = 2b + c \Rightarrow$ infinite solution

i.e., it will represent family of planes having a line (of intersection) as a solution

20. Let $f(x) = \sin^{-1} x$ and $g(x) = \frac{x^2 - x - 2}{2x^2 - x - 6}$. If $g(2) = \lim_{x \to 2} g(x)$, then the domain of the function fog is :

(1) $(-\infty, -2] \cup \left[-\frac{4}{3}, \infty\right)$

(2) $(-\infty, -2] \cup \left[-\frac{3}{2}, \infty\right)$

(3) $(-\infty, -1] \cup [2, \infty)$

(4) $(-\infty, -2] \cup [-1, \infty)$

Answer (1)

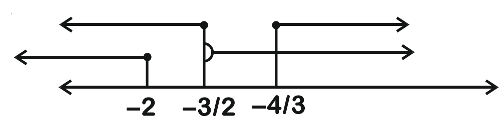
Sol. $g(2) = \lim_{x \to 2} g(x) = \frac{(x-2)(x+1)}{(2x+3)(x-2)} = \frac{3}{7}$

$\log(x) = \sin^{-1}\left(\frac{x+1}{2x+3}\right)$

for domain $-1 \le \frac{x+1}{2x+3} \le 1$

$\Rightarrow \frac{3x+4}{2x+3} \ge 0$ and $\frac{x+2}{2x+3} \ge 0$

$x \in (-\infty, -3/2) \cup [-4/3, \infty]$ and $x \in (-\infty, -2] \cup (-3/2, \infty)$



Hence $x \in (-\infty, -2]$ u $(-4/3, \infty]$

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Let α and β be two real numbers such that $\alpha + \beta = 1$ and $\alpha\beta = -1$. Let $p_n = (\alpha)^n + (\beta)^n$, $p_{n-1} = 11$ and $p_{n+1} = 29$ for some integer $n \ge 1$. Then, the value of p_n^2 is __________.

Answer (324)

Sol. $\because$ $\alpha + \beta = 1$ and $\alpha\beta = -1$

$\therefore$ Equation $x^2 - x = 0$ has two roots α and β.

$\therefore$ $\alpha^2 - \alpha = 1$ and $\beta^2 - \beta = 1$

$\Rightarrow$ $\alpha^{n+1} - \alpha^n = \alpha^{n-1}$ and $\beta^{n+n} - \beta^n = \beta^{n-1}$

$\Rightarrow$ $\alpha^{n+1} + \beta^{n+1} - \alpha^n - \beta^n = \alpha^{n-1} + \beta^{n-1}$

$\Rightarrow$ $P_{n+1} - P_n = P_{n-1}$

$\Rightarrow$ $P_n = 29 - 11$

$\Rightarrow$ $(P_n)^2 = 18^2 = 324$

2. The total number of 4-digit numbers whose greatest common divisor with 18 is 3, is _____.

Answer (1000)

Sol. Let A denotes a set of number divisible by 3.

B denotes a set of number divisible by 2.

and C denotes a set of number divisible by 9.

Required number of numbers

$= n(A) - n(A \cap B) - n(c) + n(A \cap B \cap C)$

$= 3000 - 1500 - 1000 + 500$

$= 1000$

3. Let L be a common tangent line to the curves $4x^2 + 9y^2 = 36$ and $(2x)^2 + (2y)^2 = 31$. Then the square of the slope of the line L is ________.

Answer (3)

Sol. Tangent to the curve $\frac{x^2}{9} + \frac{y^2}{14} = 1$ is

$y = mx + \sqrt{9m^2 + 4}$

and equation of tangent to the curve $x^2 + y^2 = \frac{31}{4}$ is

$y = mx + \sqrt{\frac{31}{4}\left(1 + m^2\right)}$

for common tangent $9m^2 + 4 = \frac{31}{4} + \frac{31}{4}m^2$

$\Rightarrow$ $\frac{5}{4}m^2 = \frac{15}{4}$

$\Rightarrow$ $m^2 = 3$

4. Let the normals at all the points on a given curve pass through a fixed point (a, b). If the curve passes through (3, –3) and (4, $-2\sqrt{2}$), and given that $a - 2\sqrt{2}\,b = 3$, then $(a^2 + b^2 + ab)$ is equal to _______.

Answer (9)

Sol. Clearly the curve is a circle with centre (a, b)

Centre lies on the line $x - 2\sqrt{2}y = 3$ ...(i)

$\because$ Circle passes through A(3, –3) and B(4, $-2\sqrt{2}$)

So centre lies on perpendicular bisector of AB, which is

$x + \left(3 - 2\sqrt{2}\right)y = 3$ ...(ii)

Clearly x = 3 and y = 0

a = 3 and b = 0

$\Rightarrow$ $a^2 + b^2 + ab = 9$

5. Let $X_1, X_2, \ldots, X_{18}$ be eighteen observations such that $\sum_{i=1}^{18}(X_i = \alpha) = 36$ and $\sum_{i=1}^{18}(X_i = \beta)^2 = 90$, where α and β are distinct real numbers. If the standard deviation of these observations is 1, then the value of $|\alpha - \beta|$ is ________.

Answer (4)

Sol. $\because$ $\sum_{i=1}^{18}(x_i - \beta)^2 = 90$

and $\sum_{i=1}^{18}(x_i - \beta) = \sum_{i=1}^{18}(x_i - \alpha) + 18(\alpha - \beta)$

$= 36 + 18(\alpha - \beta)$

So $\text{Var}(x_i) = \text{Var}(x_i - \beta) = \frac{\sum(x_i - \beta)^2}{18} - \left(\frac{\sum(x_i - \beta)}{18}\right)^2$

$\Rightarrow$ $1 = \frac{90}{18} - (2 + \alpha - \beta)^2$

$\Rightarrow$ $2 + \alpha - \beta = \pm 2$

$\Rightarrow$ $\alpha - \beta = 0, -4$

$\because$ α and β are distinct, so $|\alpha - \beta| = 4$

6. If the arithmetic mean and geometric mean of the p^{th} and q^{th} terms of the sequence –16, 8, –4, 2, ... satisfy the equation $4x^2 - 9x + 5 = 0$, then p + q is equal to _______.

Answer (10)

Sol. $T_p = -16\left(-\frac{1}{2}\right)^{p-1} = (-1)^p.2^{5-p}$

and $T_q = (-1)^q.2^{5-q}$

$\because$ A.M. of T_p and T_q is $\frac{5}{4}$ and G.M. is 1

$(-1)^{p+q}2^{10-p-q} = 1 \Rightarrow$ p + q = 10

7. If $I_{m,n} = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$, for m, n ≥ 1, and $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx = \alpha\, I_{m,n}$, $\alpha \in R$, then α equals ______.

Answer (1)

Sol. $\because \; I_{m,n} = \beta_{m,n}$

$$= \int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx \qquad \text{let } x = \tan^2\theta$$

$$= \int_0^{\pi/4} \frac{\tan^{2m-2}\theta + \tan^{2n-2}\theta}{\sec^{2(m+n)}\theta}.2\tan\theta\sec^2\theta\, d\theta$$

$$= 2\int_0^{\pi/4} \frac{\tan^{2m-1}\theta + \tan^{2n-1}\theta}{\sec^{2(m+n-1)}\theta} d\theta$$

$$= 2\int_0^{\pi/4} \left[\sin^{2m-1}\theta.\cos^{2n-1}\theta + \sin^{2n-1}\theta.\cos^{2m-1}\theta\right]d\theta$$

$$= 2\int_0^{\pi/2} \sin^{2m-1}\theta\cdot\cos^{2n-1}\theta\, d\theta$$

$= \beta_{m,\,n}$

Clearly α = 1

8. Let z be those complex numbers which satisfy $|z+5| \le 4$ and $z(1+i)+\bar{z}(1-i) \ge -10$, $i = \sqrt{-1}$.

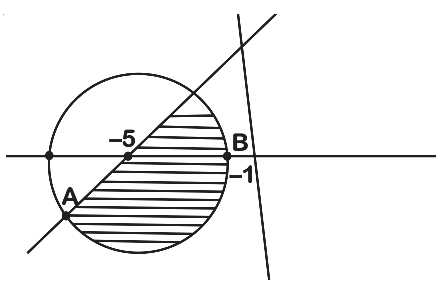
If the maximum value of $|z + 1|^2$ is $\alpha + \beta\sqrt{2}$, then the value of (α + β) is ______.

Answer (48)

Sol. $z(1+i)+\bar{z}(1+i) \ge -10 \Rightarrow x - y + 5 \ge 0$

and $|z+5| \le 4$ is interior of a circle with centre –5 and radius 4.

$\because$ $|z+1|$ represents the distance of z from –1.



$|z+1|$ is maximum is z is at A.

z is at A.

$AB^2 = |z+1|^2 = 4^2 + 4^2 - 2\cdot 4\cdot 4\cdot\cos 135° = 32 + 16\sqrt{2}$

$\Rightarrow$ α = 32 and β = 16

9. If the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$ satisfies the equation $A^{20} + \alpha A^{19} + \beta A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ for some real numbers α and β, then β – α is equal to ______.

Answer (4)

Sol. $\because \; A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}, A^4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \ldots\ldots$

So, $A^{20} + \alpha A^{19} + \beta A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{20} & 0 \\ 0 & 0 & 1 \end{bmatrix} + \alpha \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{19} & 0 \\ 3 & 0 & -1 \end{bmatrix} + \beta \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$

$$= \begin{bmatrix} 1+\alpha+\beta & 0 & 0 \\ 0 & 2^{20}+\alpha 2^{19}+2\beta & 0 \\ 3\alpha+3\beta & 0 & 1-\alpha-\beta \end{bmatrix}$$

Clearly $\alpha + \beta = 0$ and $2^{20} + \alpha\cdot 2^{19} + 2\beta = 4$

$\Rightarrow \alpha = -2$ and $\beta = 2$

10. Let a be an integer such that all the real roots of the polynomial $2x^5 + 5x^4 + 10x^3 + 10x^2 + 10x + 10$ lie in the interval (a, a + 1). Then, | a | is equal to ________.

Answer (2)

Sol. Let $f(x) = 2x^5 + 5x^4 + 10(x^3 + x^2 + x + 1)$

$\because$ f(–1) = 3

and f(–2) = –34

hence roots of f(x) lies in (–2, –1)

Clearly, $|a| = 2$

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