16/03/2021
Evening

Aakash
Medical | IIT-JEE | Foundations

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Time : 3 hrs. M.M. : 300

# Answers & Solutions

*for*

# JEE (MAIN)-2021 (Online) Phase-2

## (Physics, Chemistry and Mathematics)

**IMPORTANT INSTRUCTIONS :**

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part has two sections.

(i) Section-I : This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) Section-II : This section contains 10 questions. In Section-II, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and there is no negative marking for wrong answer.

# PART–A : PHYSICS

## SECTION - I

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer :**

1. Red light differs from blue light as they have:
   (1) Different frequencies and same wavelengths
   (2) Same frequencies and different wavelengths
   (3) Same frequencies and same wavelengths
   (4) Different frequencies and different wavelengths

**Answer (4)**

**Sol.** $\because \lambda\nu = C = \text{constant}$

$\therefore$ Red light and blue light have different frequencies and different wavelengths.

2. A charge Q is moving $\vec{dl}$ distance in the magnetic field $\vec{B}$. Find the value of work done by $\vec{B}$.
   (1) –1 (2) Zero
   (3) 1 (4) Infinite

**Answer (2)**

**Sol.** $\vec{F}_m = q\vec{V}\times\vec{B}$

$\because \vec{B} \perp \vec{V} \Rightarrow$ Work done is zero

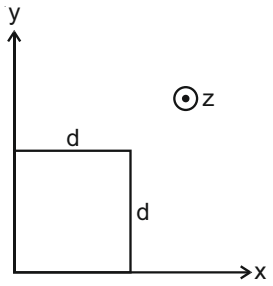
$\therefore$ W = 0

3. Find out the surface charge density at the intersection of point x = 3 m plane and x-axis, in the region of uniform line charge of 8 nC/m lying along the z-axis in free space.
   (1) 0.424 nC $m^{-2}$
   (2) 47.88 C/m
   (3) 4.0 nC $m^{-2}$
   (4) 0.07 nC $m^{-2}$

**Answer (*)**

**Sol.** *Wrong question

4. The magnetic field in a region is given by $\vec{B} = B_0\left(\frac{x}{a}\right)\hat{k}$. A square loop of side d is placed with its edges along the x and y axes. The loop is moved with a constant velocity $\vec{v} = v_0\hat{i}$. The emf induced in the loop is:

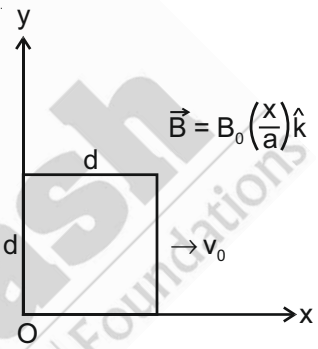



(1) $\frac{B_0 v_0 d^2}{2a}$ (2) $\frac{B_0 v_0 d^2}{a}$

(3) $\frac{B_0 v_0 d}{2a}$ (4) $\frac{B_0 v_0^2 d}{2a}$

**Answer (2)**

**Sol.**




$\therefore \varepsilon = B_0 \times \left(\frac{d}{a}\right) \times d \times v_0 - 0$

$= \frac{B_0 v_0 d^2}{a}$

5. The refractive index of a converging lens is 1.4. What will be the focal length of this lens if it is placed in a medium of same refractive index? Assume the radii of curvature of the faces of lens are $R_1$ and $R_2$ respectively.
   (1) $\frac{R_1R_2}{R_1 - R_2}$ (2) Zero
   (3) 1 (4) Infinite

**Answer (4)**

**Sol.** $\because \frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\Rightarrow \frac{1}{f} = \left(\frac{1.4}{1.4} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\Rightarrow$ f = $\infty$

6. A resistor develops 500 J of thermal energy in 20 s when a current of 1.5 A is passed through it. If the current is increased from 1.5 A to 3 A, what will be the energy developed in 20 s.

(1) 2000 J (2) 1000 J

(3) 1500 J (4) 500 J

**Answer (1)**

**Sol.** $H_1 = i_1^2 R\Delta t$

$H_2 = i_2^2 R\Delta t$

$\Rightarrow \frac{H_1}{H_2} = \frac{i_1^2}{i_2^2}$

$\Rightarrow \frac{500}{H_2} = \left(\frac{1}{2}\right)^2$

$\Rightarrow H_2 = 2000$ J

7. Amplitude of a mass-spring system, which is executing simple harmonic motion decreases with time. If mass = 500 g, Decay constant = 20 g/s then how much time is required for the amplitude of the system to drop to half of its initial value?

(ln2 = 0.693)

(1) 15.01 s (2) 0.034 s

(3) 34.65 s (4) 17.32 s

**Answer (3)**

**Sol.** $\because A = A_0 e^{-\frac{bt}{2m}}$

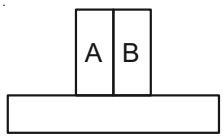
$\Rightarrow A = A_0 \times e^{-\frac{20\times10^{-3}}{2\times0.5}t}$

$\Rightarrow \frac{A_0}{2} = A_0 \times e^{-20\times10^{-3}t}$

$\Rightarrow 20 \times 10^{-3}\, t = \ln 2$

$\Rightarrow t = \frac{\ln 2}{20\times10^{-3}} = 34.65$ s

8. A bimetallic strip consists of metals A and B. It is mounted rigidly as shown. The metal A has higher coefficient of expansion compared to that of metal B. When the bimetallic strip is placed in a cold bath, it will :




(1) Bend towards the left

(2) Bend towards the right

(3) Not bend but shrink

(4) Neither bend nor shrink

**Answer (1)**

**Sol.** A will contract more than B, so it will bend towards left.

9. In order to determine the Young's Modulus of a wire of radius 0.2 cm (measured using a scale of least count = 0.001 cm) and length 1 m (measured using a scale of least count = 1 mm), a weight of mass 1 kg (measured using a scale of least count = 1 g) was hanged to get the elongation of 0.5 cm (measured using a scale of least count 0.001 cm). What will be the fractional error in the value of Young's Modulus determined by this experiment?

(1) 1.4% (2) 0.14%

(3) 9% (4) 0.9%

**Answer (1)**

**Sol.** $\because Y = \frac{F/A}{\Delta l/l}$

$= \frac{mg\times l}{(\pi r^2)\times\Delta l}$

$\therefore \frac{\Delta Y}{Y}\times100 = \left(\frac{1}{1000}+\frac{1}{1000}+2\times\frac{0.001}{0.2}+\frac{0.001}{0.5}\right)\times100$

$= 1.4\%$

10. Calculate the value of mean free path ($\lambda$) for oxygen molecules at temperature 27°C and pressure $1.01 \times 10^5$ Pa. Assume the molecular diameter 0.3 nm and the gas is ideal. ($k = 1.38 \times 10^{-23}$ JK$^{-1}$)

(1) 86 nm (2) 32 nm

(3) 58 nm (4) 102 nm

**Answer (4)**

**Sol.** $l_{mean} = \frac{RT}{\sqrt{2}\pi d^2 N_A P}$

$= \frac{1.38\times300\times10^{-23}}{\sqrt{2}\times3.14\times\left(0.3\times10^{-9}\right)^2\times1.01\times10^5}$

$= 102 \times 10^{-9}$ m

$= 102$ nm

11. What will be the nature of flow of water from a circular tap, when its flow rate increased from 0.18 L/min to 0.48 L/ min? The radius of the tap and viscosity of water are 0.5 cm and $10^{-3}$ Pa s, respectively.

(Density of water : $10^3$ kg/m$^3$)

(1) Steady flow to unsteady flow

(2) Remains turbulent flow

(3) Unsteady to steady flow

(4) Remains steady flow

**Answer (1)**

**Sol.** $R = \frac{\rho V D}{\eta}$

$R = \frac{4\rho Q}{\pi D \eta}$

$R_1 = \frac{4\times10^3\times0.18\times10^{-3}}{60\times\pi\times10^{-2}\times10^{-3}} = \frac{4\times10^5\times0.18}{60\pi}$

$= 0.0038 \times 10^5 = 380$

$R_2 = \frac{0.48}{0.18}\times380 = 1018$

12. Calculate the time interval between 33% decay and 67% decay if half-life of a substance is 20 minutes.

(1) 20 minutes (2) 40 minutes

(3) 60 minutes (4) 13 minutes

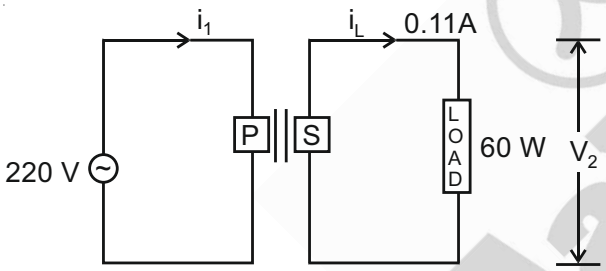
**Answer (1)**

**Sol.** $N_1 = N_0 e^{-\lambda t_1}$

$N_2 = N_0 e^{-\lambda t_2}$

$\frac{1}{2} = e^{-\lambda(t_2 - t_1)}$

13. For the given circuit, comment on the type of transformer used.




(1) Step-up transformer

(2) Auto transformer

(3) Step down transformer

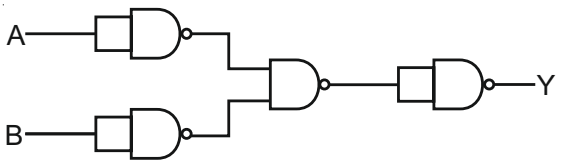
(4) Auxilliary transformer

**Answer (1)**

**Sol.** $V_2 I_2 = V_1 I_1$

Here $V_2 > V_1$

14. The following logic gate is equivalent to :




(1) OR Gate (2) NAND Gate

(3) NOR Gate (4) AND Gate

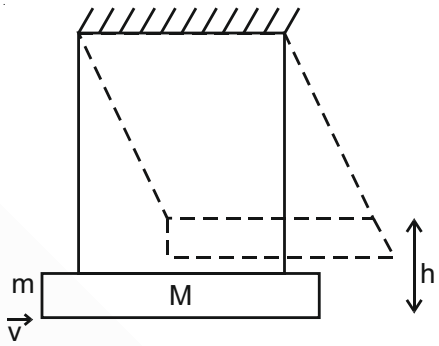
**Answer (3)**

**Sol.** $X = \overline{\overline{A}\,\overline{B}}$

$X = \overline{A + B}$

15. A large block of wood of mass M = 5.99 kg is hanging from two long massless cords. A bullet of mass m = 10 g is fired into the block and gets embedded in it. The (block + bullet) then swing upwards, their centre of mass rising a vertical distance h = 9.8 cm before the (block + bullet) pendulum comes momentarily to rest at the end of its arc. The speed of the bullet just before collision is :

(take g = 9.8 $ms^{-2}$)




(1) 821.4 m/s (2) 831.4 m/s

(3) 841.4 m/s (4) 811.4 m/s

**Answer (2)**

**Sol.** Let initial momentum be p.

$\frac{p^2}{2(m+M)} = (m+M)gH$

$p = (m+M)\sqrt{2gH}$

$= 6\times\sqrt{\frac{2\times9.8\times9.8}{100}}$

$p = 6\sqrt{2}\times0.98$

$6\sqrt{2}\times0.98 = 0.01\,V$

$V = 600\times0.98\sqrt{2}$

16. A mosquito is moving with a velocity $\vec{v} = 0.5t^2\hat{i} + 3t\hat{j} + 9\hat{k}\ m/s$ and accelerating in uniform conditions. What will be the direction of mosquito after 2 s?

(1) $\tan^{-1}\left(\frac{2}{3}\right)$ from y-axis

(2) $\tan^{-1}\left(\frac{5}{2}\right)$ from y-axis

(3) $\tan^{-1}\left(\frac{5}{2}\right)$ from x-axis

(4) $\tan^{-1}\left(\frac{2}{3}\right)$ from x-axis

**Answer (*)**

**Sol.** $\vec{v} = \left(0.5t^2\hat{i} + 3t\hat{j} + 9\hat{k}\right)$ m/s

At t = 2 s

$\vec{v} = \left(2\hat{i} + 6\hat{j} + 9\hat{k}\right)$

$$\cos\theta = \frac{\left(v.\hat{j}\right)}{\sqrt{9^2 + 6^2 + 2^2}} = \frac{6}{\sqrt{121}} = \frac{6}{11}$$

$$\sin\theta = \frac{\sqrt{85}}{11}$$

$$\tan\theta = \frac{\sqrt{85}}{6}$$

*None of the option matches

Question also seem incomplete

17. **Statement I :** A cyclist is moving on an unbanked road with a speed of 7 $kmh^{-1}$ and takes a sharp circular turn along a path of radius of 2 m without reducing the speed. The static friction coefficient is 0.2. The cyclist will not slip and pass the curve. ($g = 9.8\ m/s^2$)

**Statement II :** If the road is banked at an angle of 45°, cyclist can cross the curve of 2 m radius with the speed of 18.5 $kmh^{-1}$ without slipping.

In the light of the above statements, choose the correct answer from the options given below.

(1) Statement I is correct and statement II is incorrect

(2) Statement I is incorrect and statement II is correct

(3) Both statement I and statement II are true

(4) Both statement I and statement II are false

**Answer (3)**

**Sol.** For statement-I

$v \le \sqrt{\mu rg}$

$\le \sqrt{0.2\times 2\times 9.8}$

$v \le \sqrt{3.92} \Rightarrow$ statement-I is true

For statement-II

$$v_{allowable} = \sqrt{rg\frac{(\mu + \tan\theta)}{1-\mu}} = \sqrt{\frac{3}{2}rg}$$

So, both the statements are true.

18. Two identical antennas mounted on identical towers are separated from each other by a distance of 45 km. What should nearly be the minimum height of receiving antenna to receive the signals in line of sight?

(Assume radius of earth is 6400 km)

(1) 39.55 m (2) 158.2 m

(3) 79.1 m (4) 19.77 m

**Answer (1)**

**Sol.** $d = 2\sqrt{2Rh_R}$

$$45000 = 2\times\sqrt{2\times 6400\times 10^3\times h_R}$$

$h_R$ = 39.55 m

19. The half-life of $Au^{198}$ is 2.7 days. The activity of 1.50 mg of $Au^{198}$ if its atomic weight is 198 g $mol^{-1}$ is, ($N_A = 6 \times 10^{23}$/mol).

(1) 252 Ci (2) 535 Ci

(3) 357 Ci (4) 240 Ci

**Answer (3)**

**Sol.** $A = \lambda N$

$$N = \frac{0.693\times 1.5\times 10^{-3}\times 6\times 10^{23}}{2.7\times 86400\times 198} \text{ disinte./s}$$

$$N(\text{in Ci}) = \frac{0.693\times 1.5\times 10^{20}\times 6}{2.7\times 86400\times 198\times 3.7\times 10^{10}}$$

= 357 Ci

20. The de-Broglie wavelength associated with an electron and a proton were calculated by accelerating them through same potential of 100 V. What should nearly be the ratio of their wavelengths? ($m_p$ = 1.00727u $m_e$ = 0.00055u)

(1) 41.4 : 1 (2) $(1860)^2$ : 1

(3) 1860 : 1 (4) 43 : 1

**Answer (4)**

**Sol.** $\lambda = \frac{h}{p}$

$p = \sqrt{2mk}$

Kinetic energy of both are same.

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{m_p}{m_e}} = \sqrt{\frac{1.00727}{0.00055}}$$

## SECTION - II

**Numerical Value Type Questions:** This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. A deviation of 2° is produced in the yellow ray when prism of crown and flint glass are achromatically combined. Taking dispersive powers of crown and flint glass as 0.02 and 0.03 respectively and refractive index for yellow light for these glasses are 1.5 and 1.6 respectively. The refracting angles for crown glass prism will be _____° (in degree) (Round off to the Nearest Integer)

**Answer (12°)**

**Sol.** $\delta = \delta_1\left(1-\frac{\omega_1}{\omega_2}\right)$

$2 = \delta_1\left(1-\frac{.02}{.03}\right)$

$\Rightarrow \quad \delta_1 = 6 = A(1.5-1) \quad \Rightarrow \quad A = 12°$

2. The energy dissipated by a resistor is 10 mJ in 1 s when an electric current of 2 mA flows through it. The resistance is _____ Ω. (Round off to the Nearest Integer).

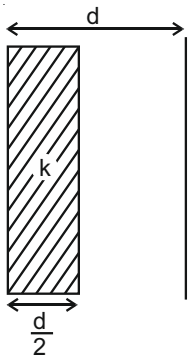
**Answer (2500)**

**Sol.** $U = i^2R\Delta t$

$10 \times 10^{-3} = (2 \times 10^{-3})^2 \times R \times 1$

$R = 2500\ \Omega$

3. In a parallel plate capacitor set up, the plate area of capacitor is 2 $m^2$ and the plates are separated by 1 m. If the space between the plates are filled with a dielectric material of thickness 0.5 m and area 2 $m^2$ (see fig) the capacitance of the set-up will be _____ $\varepsilon_0$. (Dielectric constant of the material = 3.2) (Round off to the Nearest Integer)

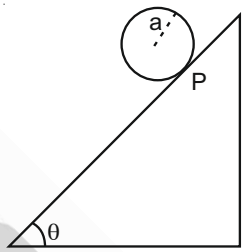



**Answer (4)**

**Sol.** $C = \frac{\varepsilon_0 A}{d-t+\frac{t}{k}} = \frac{\varepsilon_0 \times 2}{1-0.5+\frac{0.5}{3.2}} = 3.88\ \varepsilon_0$

4. A solid disc of radius 'a' and mass 'm' rolls down without slipping on an inclined plane making an angle θ with the horizontal. The acceleration of the disc will be $\frac{2}{b}g\sin\theta$ where b is _____. (Round off to the Nearest Integer)

(g = acceleration due to gravity, θ = angle as shown in figure.)




**Answer (3)**

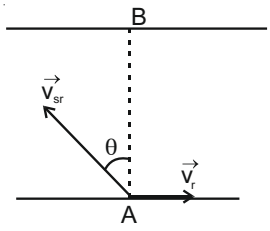
**Sol.** $a = \frac{g\sin\theta}{1+\frac{K^2}{R^2}} = \frac{2}{3}g\sin\theta$

5. A swimmer can swim with velocity of 12 km/h in still water. Water flowing in a river has velocity 6 km/h. The direction with respect to the direction of flow of river water he should swim in order to reach the point on the other bank just opposite to his starting point is ___°. (Round off to the Nearest Integer)

(Find the angle in degrees)

**Answer (120)**

**Sol.** To reach directly opposite point.




$v_{sr}\sin\theta = v_r$ $\qquad$ $\sin\theta = \frac{6}{12} = \frac{1}{2}$

$\theta = 30°$

Angle w.r.t. flow = 120°.

6. A force $\vec{F} = 4\hat{i} + 3\hat{j} + 4\hat{k}$ is applied on an intersection point of x = 2 plane and *x*-axis. The magnitude of torque of this force about a point (2, 3, 4) is _____. (Round off to the Nearest Integer)

**Answer (20)**

**Sol.** $\vec{\tau} = \vec{r} \times \vec{F}$

$= \left[(2-2)\hat{i} + (0-3)\hat{j} + (0-4)\hat{k}\right] \times \left(4\hat{i} + 3\hat{j} + 4\hat{k}\right)$

$= \left(-3\hat{j} - 4\hat{k}\right) \times \left(4\hat{i} + 3\hat{j} + 4\hat{k}\right)$

$= -16\hat{j} + 12\hat{k}$

$\left|\vec{\tau}\right| = 20$ units

7. For an ideal heat engine, the temperature of the source is 127°C. In order to have 60% efficiency the temperature of the sink should be _____ °C. (Round off to the Nearest Integer)

**Answer (–113)**

**Sol.** $\eta = 1 - \frac{T_L}{T_H}$

$0.6 = 1 - \frac{T_L}{400}$

$T_L = 160$ k

$= -113°C$

8. A body of mass 2 kg moves under a force of $\left(2\hat{i} + 3\hat{j} + 5\hat{k}\right)$N. It starts from rest and was at the origin initially. After 4 s, its new coordinates are (8, b, 20). The value of b is _____. (Round off to the Nearest Integer)

**Answer (12)**

**Sol.** $\vec{r} = \frac{1}{2}\vec{a}t^2$

$\frac{1}{2}\left(\hat{i} + \frac{3}{2}\hat{j} + \frac{5}{2}\hat{k}\right).16$

$= 8\hat{i} + 12\hat{j} + 20\hat{k}$

b = 12

9. If one wants to remove all the mass of the earth to infinity in order to break it up completely. The amount of energy that needs to be supplied will be $\frac{x}{5}\frac{GM^2}{R}$ where x is _____ (Round off to the Nearest Integer)

(M is the mass of earth, R is the radius of earth, G is the gravitational constant)

**Answer (3)**

**Sol.** Binding energy of uniform sphere = $\frac{3}{5}\frac{GM^2}{R}$

x = 3

10. A closed organ pipe of length L and an open organ pipe contain gases of densities $\rho_1$ and $\rho_2$ respectively. The compressibility of gases are equal in both the pipes. Both the pipes are vibrating in their first overtone with same frequency. The length of the open pipe is $\frac{x}{3}L\sqrt{\frac{\rho_1}{\rho_2}}$ where x is _____. (Round off to the Nearest Integer)

**Answer (4)**

**Sol.** $v = \sqrt{\frac{1}{k\rho}}$

$\frac{3v_1}{4L} = \frac{2v_2}{2L_2}$

$\Rightarrow L2 = \frac{4}{3}L\left(\frac{v_2}{v_1}\right)$

$= \frac{4}{3}L\sqrt{\frac{\rho_1}{\rho_2}}$

x = 4

# PART–B : CHEMISTRY

## SECTION - I

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer :**

1. The **INCORRECT** statement regarding the structure of $C_{60}$ is

   (1) It contains 12 six-membered rings and 24 five-membered rings

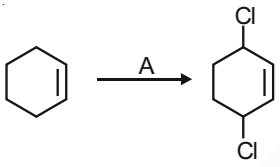
   (2) The six-membered rings are fused to both six and five-membered rings

   (3) Each carbon atom forms three sigma bonds

   (4) The five-membered rings are fused only to six-membered rings

**Answer (1)**

**Sol.** $C_{60}$ contains twenty six-membered rings and twelve five-membered rings.

2. 


   Identify the reagent(s) 'A' and condition(s) for the reaction

   (1) A = HCl ; Anhydrous $AlCl_3$

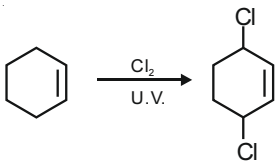
   (2) A = $Cl_2$ ; UV light

   (3) A = $Cl_2$ ; dark, Anhydrous $AlCl_3$

   (4) A = HCl, $ZnCl_2$

**Answer (2)**

**Sol.** In presence of U.V. light, free radical substitution reaction occurs, at allylic position.




3. The correct statements about $H_2O_2$ are

   (A) used in the treatment of effluents.

   (B) used as both oxidising and reducing agents.

   (C) the two hydroxyl groups lie in the same plane

   (D) miscible with water.

   Choose the **correct** answer from the options given below

   (1) (A), (B), (C) and (D)

   (2) (A), (B) and (D) only

   (3) (B), (C) and (D) only

   (4) (A), (C) and (D) only

**Answer (2)**

**Sol.**
- $H_2O_2$ is an important chemical used in pollution control treatment of domestic and industrial effluents.
- It has both oxidising and reducing properties.
- It has open-book like structure and both hydroxyl groups lie in different planes.
- It is miscible in water.

4. Which of the following polymer is used in the manufacture of wood laminates?

   (1) Urea formaldehyde resin

   (2) Phenol and formaldehyde resin

   (3) Melamine formaldehyde resin

   (4) cis-poly isoprene

**Answer (1)**

**Sol.** Urea-formaldehyde resin is used in wood laminates.

5. The characteristics of elements X, Y and Z with atomic numbers, respectively, 33, 53 and 83 are

   (1) X and Y are metalloids and Z is a metal

   (2) X is a metalloid, Y is a non-metal and Z is a metal

   (3) X and Z are non-metals and Y is a metalloid.

   (4) X, Y and Z are metals.

**Answer (2)**

**Sol.** X(Z = 33) = As (metalloid)

Y(Z = 53) = I (non-metal)

Z(Z = 83) = Bi (metal)

6. Identify the elements X and Y using the ionisation energy values given below :

| | **Ionization energy** | **(kJ/mol)** |
|---|---|---|
| | $1^{st}$ | $2^{nd}$ |
| X | 495 | 4563 |
| Y | 731 | 1450 |

   (1) X = Na ; Y = Mg (2) X = Mg ; Y = F

   (3) X = F ; Y = Mg (4) X = Mg ; Y = Na

**Answer (1)**

**Sol.** Due to $2p^6$, noble gas electronic configuration, the second ionisation enthalpy of Na is very high. That's why has large difference between $IE_1$, and $IE_2$

$Mg^+$ is $2p^6$, $3s^1$.

After the loss of one electron, $Mg^{2+}$ will be formed with noble gas electronic configuration. That's why has less difference between $I.E_1$ and $I.E_2$.

7. $Fex_2$ and $Fey_3$ are known when x and y are

(1) x = F, Cl, Br, I and y = F, Cl, Br

(2) x = Cl, Br, I and y = F, Cl, Br, I

(3) x = F, Cl, Br and y = F, Cl, Br, I

(4) x = F, Cl, Br, I and y = F, Cl, Br, I

**Answer (1)**

**Sol.** $FeI_3$ does not exist as $I^-$ reduces $Fe^{3+}$ to $Fe^{2+}$.

But $FeF_2$, $FeCl_2$, $FeBr_2$, $FeI_2$ all exist.

8. Which of the following reduction reaction **CANNOT** be carried out with coke?

(1) $Cu_2O \rightarrow Cu$ (2) $Fe_2O_3 \rightarrow Fe$

(3) $Al_2O_3 \rightarrow Al$ (4) $ZnO \rightarrow Zn$

**Answer (3)**

**Sol.** From the Ellingham diagram, the difference in the $\Delta G^\circ$ values is very much positive, that's why reduction of $Al_2O_3$ with coke is non-spontaneous.

9. Arrange the following metal complex/compounds in the increasing order of spin only magnetic moment. Presume all the three, high spin system.

(Atomic numbers Ce = 58, Gd = 64 and Eu = 63.)

(a) $(NH_4)_2[Ce(NO_3)_6]$ (b) $Gd(NO_3)_3$ and

(c) $Eu(NO_3)_3$

(1) (a) < (c) < (b) (2) (a) < (b) < (c)

(3) (c) < (a) < (b) (4) (b) < (a) < (c)

**Answer (1)**

**Sol.** (a) $(NH_4)_2$ $[Ce(NO_3)_6]$

$Ce^{+4} = 4f^0$ (diamagnetic)

(b) $Gd(NO_3)_3$

$Gd^{+3} = 4f^7$

$\mu = \sqrt{7(7+2)} = \sqrt{63}$ B.M.

(c) $Eu(NO_3)_3$

$Eu^{+3} = 4f^6$

$\mu = \sqrt{6(6+2)} = \sqrt{48}$ B.M.

$\therefore$ Order of spin only magnetic moment

= b > c > a

10. The **INCORRECT** statements below regarding colloidal solutions is

(1) The flocculating power of $Al^{3+}$ is more than that of $Na^+$.

(2) A colloidal solution shows Brownian motion of colloidal particles.

(3) An ordinary filter paper can stop the flow of colloidal particles.

(4) A colloidal solution shows colligative properties.

**Answer (3)**

**Sol.** Ordinary filter paper cannot stop the flow colloidal particles.

An ultra - filter paper is required.

11. Ammonolysis of Alkyl halides followed by the treatment with NaOH solution can be used to prepare primary, secondary and tertiary amines. The purpose of NaOH in the reaction is

(1) To remove basic impurities

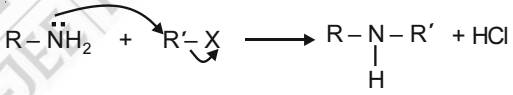
(2) To activate $NH_3$ used in the reaction

(3) To remove acidic impurities

(4) To increase the reactivity of alkyl halide

**Answer (3)**

**Sol.** With each substitution reaction, acidic concentration increases in reaction mixture



So to remove acidic impurities , NaOH is added.

12. Which of the following is least basic?

(1) $(CH_3CO)\ddot{N}HC_2H_5$

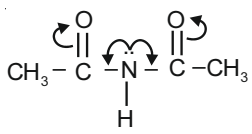
(2) $(CH_3CO)_2\ddot{N}H$

(3) $(C_2H_5)_2\ddot{N}H$

(4) $(C_2H_5)_3\ddot{N}$

**Answer (2)**

**Sol.** Basic strength $\propto$ availability of lone pair.



In this case lone pair of $\ddot{N}$ is highly participating in resonance.

13. **Statement I** : Sodium hydride can be used as an oxidising agent.

**Statement II** : The lone pair of electrons on nitrogen in pyridine makes it basic.

Choose the **CORRECT** answer from the options given below:

(1) Both statement I and statement II are true

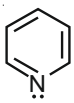
(2) Both statement I and statement II are false

(3) Statement I is false but statement II is true

(4) Statement I is true but statement II is false

**Answer (3)**

**Sol.**

- NaH is a strong $H^-$ (hydride) donor. Hence cannot be used as an oxidising agent.
-  Pyridine

In Pyridine, lone pair of 'N' is localised, makes it basic.

14. Match List-I with List-II :

| **List-I** **Test/Reagents/ Observation(s)** | **List-II** **Species detected** |
|---|---|
| (a) Lassaigne's Test | (i) Carbon |
| (b) Cu(II) oxide | (ii) Sulphur |
| (c) Silver nitrate | (iii) N, S, P, and halogen |
| (d) The sodium fusion extract gives black precipitate with acetic acid and lead acetate | (iv) Halogen Specifically |

The correct match is:

(1) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

(2) (a)-(i), (b)-(iv), (c)-(iii), (d)-(ii)

(3) (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)

(4) (a)-(i), (b)-(ii), (c)-(iv), (d)-(iii)

**Answer (1)**

**Sol.** Lassaigne's test is used in qualitative detection of N, S, P and halogens.

(a) $\rightarrow$ (iii)

Cu(II) oxide is used for the estimation of carbon.

(b) $\rightarrow$ (i)

Silver nitrate is used for detecting halogens in organic compounds (Carius Method)

(c) $\rightarrow$ (iv)

Sodium fusion extract when treated with acetic acid and lead acetate gives black precipitate if sulphur is present.

$$S^{2-} + Pb^{2+} \rightarrow \underset{(black)}{PbS}$$

(d) $\rightarrow$ (ii)

15. The exact volumes of 1 M NaOH solution required to neutralise 50 mL of 1 M $H_3PO_3$ solution and 100 mL of 2 M $H_3PO_2$ solution, respectively, are:

(1) 100 mL and 100 mL

(2) 50 mL and 50 mL

(3) 100 mL and 200 mL

(4) 100 mL and 50 mL

**Answer (3)**

**Sol.** $H_3PO_3$ – diprotic acid

$H_3PO_2$ – monoprotic acid

Using Law of equivalence:

m.equivalents of $H_3PO_3$ = m.equivalents of NaOH

$\Rightarrow 50 \times 1 \times 2 = V \times 1 \times 1$

$\Rightarrow V = 100$ mL

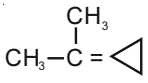
Similarly,

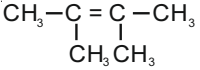
m.equivalents of $H_3PO_2$ = m.equivalents of NaOH

$\Rightarrow 100 \times 2 \times 1 = V \times 1 \times 1$

$\Rightarrow V = 200$ mL

16. An unsaturated hydrocarbon X on ozonolysis gives A. Compound A when warmed with ammonical silver nitrate forms a bright silver mirror along the sides of the test tube. The unsaturated hydrocarbon X is:

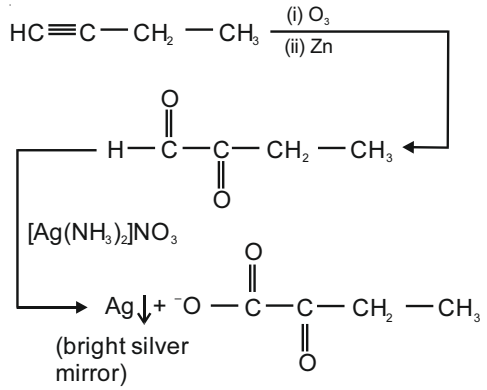
(1) 

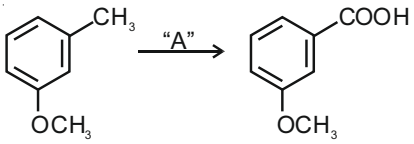
(2) 

(3) $HC \equiv C - CH_2 - CH_3$

(4) $CH_3 - C \equiv C - CH_3$

**Answer (3)**

**Sol.** 


17. 


In the above reaction, the reagent "A" is:

(1) HCl, Zn – Hg

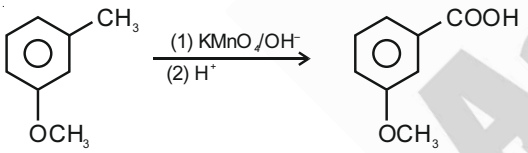
(2) Alkaline $KMnO_4$, $H^+$

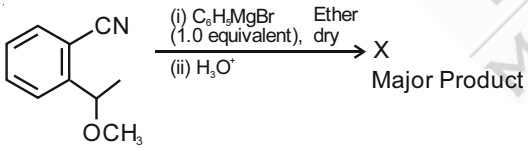
(3) $LiAlH_4$

(4) $NaBH_4$, $H_3O^+$

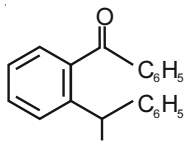
**Answer (2)**

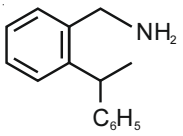
**Sol.**

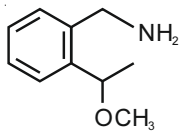


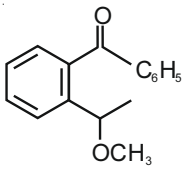

18. 


The structure of X is:

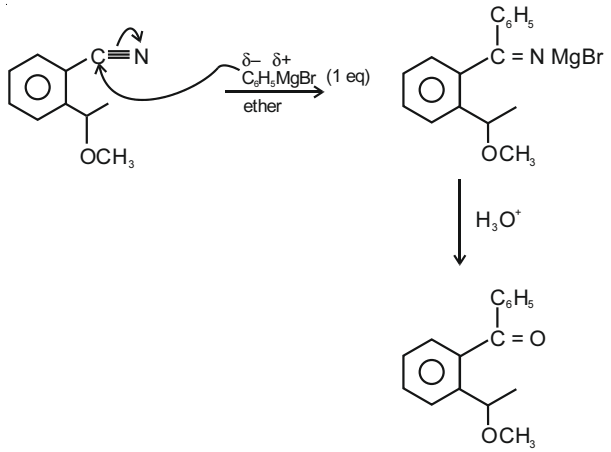
(1) 


(2) 


(3) 


(4) 


**Answer (4)**

**Sol.** 


19. The green house gas/es is (are):

(A) Carbon dioxide (B) Oxygen

(C) Water vapour (D) Methane

Choose the most appropriate answer from the options given below:

(1) (A), (C) and (D) only

(2) (A) and (B) only

(3) (A) and (C) only

(4) (A) only

**Answer (1)**

**Sol.** Carbon dioxide, water vapour, methane are greenhouse gases.

20. The secondary structure of protein is stabilised by:

(1) Hydrogen bonding (2) van der Waals forces

(3) Glycosidic bond (4) Peptide bond

**Answer (1)**

**Sol.** The secondary structure of protein is stabilised by hydrogen bonding.

## SECTION - II

**Numerical Value Type Questions:** This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. A and B decompose via first order kinetics with half-lives 54.0 min and 18.0 min respectively. Starting from an equimolar non reactive mixture of A and B, the time taken for the concentration of A to become 16 times that of B is _____min.

(Round off to the Nearest Integer).

**Answer (108)**

**Sol.** Initially : $[A_0] = [B_0] = a$

After time 't' min : $[A] = 16\,[B]$

$[A] = [A_0]\ e^{-k_A t}$

$[B] = [B_0]\ e^{-k_B t}$

$\Rightarrow a.e^{-k_A t} = 16ae^{-k_B t}$

$\Rightarrow e^{-(k_A - k_B)t} = 16$

$\Rightarrow (k_B - k_A)t = \ln 16$

$\Rightarrow \ln 2\left(\frac{1}{18} - \frac{1}{54}\right)t = 4\ln 2$

$\Rightarrow t = \frac{54 \times 18 \times 4}{36} = 108$ min

2. The number of orbitals with n = 5, $m_\ell = +2$ is _____.

(Round off to the Nearest Integer).

**Answer (03)**

**Sol.** n = 5

Possible values of $\ell$ = 4, 3, 2, 1, 0

$m_\ell = 2$ is possible for $\ell$ = 4, 3 & 2

as $m_\ell$ takes values from ($-\ell$ to $\ell$)

$\therefore$ Possible orbitals $(n, \ell, m_\ell)$ : (5, 4, 2) (5, 3, 2) (5, 2, 2)

3. In Duma's method of estimation of nitrogen, 0.1840 g of an organic compound gave 30 mL of nitrogen collected at 287 K and 758 mm of Hg pressure. The percentage composition of nitrogen in the compound is _____.

(Round off to the nearest Integer).

[Given : Aqueous tension at 287 K = 14 mm of Hg]

**Answer (19)**

**Sol.** Pressure due to nitrogen = 758 – 14 = 744 mm Hg

Using ideal gas equation : PV = nRT

$n_{N_2} = \frac{744 \times 30}{760 \times 1000 \times 0.082 \times 287}$

% of nitrogen = $\frac{n_{N_2} \times 28}{0.184} \times 100$

$= \frac{744 \times 30 \times 28 \times 100}{760 \times 1000 \times 0.082 \times 287 \times 0.184}$

= 18.99 %

$\approx$ 19%

4. $[Ti(H_2O)_6]^{3+}$ absorbs light of wavelength 498 nm during a d – d transition. The octahedral splitting, energy for the above complex is _____ $\times 10^{-19}$J.

(Round off to the nearest Integer).

h = $6.626 \times 10^{-34}$ Js; c = $3 \times 10^8$ $ms^{-1}$

**Answer (04)**

**Sol.** Octahedral splitting energy = $\frac{hc}{\lambda}$

$= \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{498 \times 10^{-9}}$

= $3.99 \times 10^{-19}$ J

$\approx 4 \times 10^{-19}$

5. When 35 mL of 0.15 M lead nitrate solution is mixed with 20 mL of 0.12 M chromic sulphate solution, _____ $\times 10^{-5}$ moles of lead sulphate precipitate out.

(Round off to the Nearest Integer).

**Answer (525)**

**Sol.** $3Pb(NO_3)_2 + Cr_2(SO_4)_3 \rightarrow 3PbSO_4 \downarrow + 2Cr(NO_3)_3$

m.moles of $Pb(NO_3)_2$ = 35 × 0.15 = 5.25 m.moles

m.moles of $Cr_2(SO_4)_3$ = 20 × 0.12 = 2.4 m.moles

$\therefore$ $Pb(NO_3)_2$ is limiting reagent.

m.moles of $PbSO_4$ formed = 5.25 m.moles

= $525 \times 10^{-5}$ moles

6. At 25°C, 50 g of iron reacts with HCl to form $FeCl_2$. The evolved hydrogen gas expands against a constant pressure of 1 bar. The work done by the gas during this expansion is _____ J.

(Round off to the Nearest Integer).

[Given : R = 8.314 J $mol^{-1}$ $K^{-1}$. Assume, hydrogen is an ideal gas]

[Atomic mass of Fe is 55.85 u]

**Answer (2218)**

**Sol.** $Fe + 2HCl \rightarrow FeCl_2 + H_2$

$\frac{50}{55.85}$ moles

No. of $H_2$ produced = $\frac{50}{55.85}$ moles

Work done = $-P_{ext} \cdot \Delta V$

$= -\Delta n_g RT$

$= \frac{50}{55.85} \times 8.314 \times 298$

$\approx$ 2218 J

7. Ga (atomic mass 70 u) crystallizes in a hexagonal close packed structure. The total number of voids in 0.581 g of Ga is _____ × $10^{21}$.

(Round off to the Nearest Integer).

[Given : $N_A = 6.023 \times 10^{23}$]

**Answer (15)**

**Sol.** Ga crystallizes in a hexagonal close packing.

No. of Ga atoms per unit cell

$$= \frac{1}{6}\times 12 + \frac{1}{2}\times 2 + 1\times 3 = 6$$

No. of tetrahedral voids = 2 × 6 = 12

No. of octahedral voids = 6

Total voids per unit cell = 18

Total voids per atom of Ga = $\frac{18}{6} = 3$

$\therefore$ Total no. of voids in given sample

$$= \frac{0.581}{70}\times 3\times 6.023\times 10^{23}$$

$$= 14.99 \times 10^{21}$$

$$\approx 15\times 10^{21}$$

8. A 5.0 m mol $dm^{-3}$ aqueous solution of KCl has a conductance of 0.55 mS when measured in a cell of cell constant 1.3 $cm^{-1}$. The molar conductivity of this solution is _____ $mSm^2\ mol^{-1}$.

(Round off to the Nearest Integer).

**Answer (14)**

**Sol.** Conductance = $\frac{\text{Conductivity}}{\text{Cell constant}}$

$\therefore$ Conductivity = $0.55 \times 10^{-3} \times 1.3$ S $cm^{-1}$

Molar conductivity

$$= \frac{\text{Conductivity (S cm}^{-1})\times 1000}{\text{Molarity (mol/L)}}$$

$$= \frac{0.55\times 10^{-3}\times 1.3\times 100}{5\times 10^{-3}}$$

$$= 143 \text{ S cm}^2 \text{ mol}^{-1}$$

$$= 14.3 \text{ mS m}^2 \text{ mol}^{-1}$$

$$\approx 14 \text{ mS m}^2 \text{ mol}^{-1}$$

9. At 363 K, the vapour pressure of A is 21 kPa and that of B is 18 kPa. One mole of A and 2 moles of B are mixed. Assuming that this solution is ideal, the vapour pressure of the mixture is _____ kPa.

(Round off to the Nearest Integer]

**Answer (19)**

**Sol.** An ideal solution is prepared by mixing 1 mol of A and 2 moles of B

Using Raoult's law

$$P_s = \chi_A P_A^o + \chi_B P_B^o$$

$$= \frac{1}{3}\times 21 + \frac{2}{3}\times 18$$

$$= 19 \text{ kPa}$$

10. Sulphurous acid ($H_2SO_3$) has $Ka_1 = 1.7 \times 10^{-2}$ and $Ka_2 = 6.4 \times 10^{-8}$. The pH of 0.588 M $H_2SO_3$ is _____.

(Round off to the Nearest Integer).

**Answer (01)**

**Sol.** $Ka_1$ of $H_2SO_3$ >> $Ka_2$ of $H_2SO_3$

$\therefore$ The contribution of $H^+$ from 2nd dissociation of $H_2SO_3$ can be neglected.

$$\underset{c(1-\alpha)}{H_2SO_3} \rightleftharpoons \underset{c\alpha}{H^+} + \underset{c\alpha}{HSO_3^-}$$

$$\Rightarrow \frac{c\alpha^2}{1-\alpha} = 1.7\times 10^{-2}$$

$$\Rightarrow \frac{0.588\alpha^2}{1-\alpha} = 1.7\times 10^{-2}$$

$$\Rightarrow 58.8\alpha^2 = 1.7 - 1.7\alpha$$

$$\Rightarrow 58.8\alpha^2 + 1.7\alpha - 1.7 = 0$$

$$\alpha = \frac{-1.7 + \sqrt{1.7^2 + 4\times 1.7\times 58.8}}{2\times 58.8} = 0.156$$

$[H^+] = c\alpha = 0.092$

$pH = -\log[H^+]$

$= 1.036$

$\approx 1$

# PART–C : MATHEMATICS

## SECTION - I

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer :**

1. Let C be the locus of the mirror image of a point on the parabola $y^2 = 4x$ with respect to the line y = x. Then the equation of tangent to C at P(2, 1) is :

(1) x + 2y = 4 (2) 2x + y = 5

(3) x – y = 1 (4) x + 3y = 5

**Answer (3)**

**Sol.** Equation of C $x^2 = 4y$

Tangent at (2, 1) is

2x = 2(y + 1)

x – y = 1

2. Let $\alpha \in R$ be such that the function

$$f(x)=\begin{cases}\dfrac{\cos^{-1}(1-\{x\}^2)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}, & x\neq 0\\ \alpha, & x=0\end{cases}$$ is

continuous at x = 0, where {x} = x – [x], [x] is the greatest integer less than or equal to x.

(1) No such $\alpha$ exists (2) $\alpha=\frac{\pi}{\sqrt{2}}$

(3) $\alpha$ = 0 (4) $\alpha=\frac{\pi}{4}$

**Answer (1)**

**Sol.** When $x \to 0^-$, {x} = 1 – h where $h \to 0$

$$LHL=\lim_{h\to 0}\frac{\cos^{-1}\left(1-(1-h)^2\right)\sin^{-1}(h)}{(1-h)\left(1-(1-h)^2\right)}$$

$$=\lim_{h\to 0}\frac{\cos^{-1}\left(1-(1-h)^2\right)\sin^{-1}h}{(1-h)\,h\,(2-h)}=\frac{\frac{\pi}{2}\times 1}{1\times 2}=\frac{\pi}{4}$$

When $x \to 0^+$, {x} = h where $h \to 0$

$$RHL=\lim_{h\to 0}\frac{\cos^{-1}\left(1-h^2\right)\sin^{-1}(1-h)}{h-h^3}$$

$$=\lim_{h\to 0}\frac{\sin^{-1}\sqrt{1-(1-h)^2}\,\sin^{-1}(1-h)}{h\left(1-h^2\right)}$$

$$=\lim_{h\to 0}\frac{\sin^{-1}\left(h\sqrt{2-h^2}\right)\sin^{-1}(1-h)}{h\left(1-h^2\right)}$$

$$=\lim_{h\to 0}\frac{\sin^{-1}\left(h\sqrt{2-h^2}\right)}{h\sqrt{2-h^2}}\times\frac{h\sqrt{2-h^2}}{h\left(1-h^2\right)}\times\sin^{-1}(1-h)$$

$$=1\times\frac{\sqrt{2}}{1}\times\frac{\pi}{2}=\frac{\pi}{\sqrt{2}}$$

LHL ≠ RHL

3. Let $P(x) = x^2 + bx + c$ be a quadratic polynomial with real coefficients such that $\int_0^1 P(x)dx = 1$ and P(x) leaves remainder 5 when it is divided by (x – 2). Then the value of 9(b + c) is equal to :

(1) 11 (2) 9

(3) 15 (4) 7

**Answer (4)**

**Sol.** $\int_0^1 x^2+bx+c=1$

$$\frac{x^3}{3}+\frac{bx^2}{2}+cx\Big]_0^1=1$$

$$\frac{1}{3}+\frac{b}{2}+c=1$$

3b + 6c = 4 ...(i)

P(2) = 5 $\Rightarrow$ 4 + 2b + c = 5

2b + c = 1 ...(ii)

(i) & (ii) $\Rightarrow b=\frac{2}{9}, c=\frac{5}{9}$

9(b + c) = 7

4. Let A = {2, 3, 4, 5, ..., 30} and '$\simeq$' be an equivalence relation on A × A, defined by $(a, b) \simeq (c, d)$, if and only if ad = bc. Then the number of ordered pairs which satisfy this equivalence relation with ordered pair (4, 3) is equal to :

(1) 7 (2) 8

(3) 5 (4) 6

**Answer (1)**

**Sol.** Let (4, 3) $\simeq$ (c, d)

$4d = 3c \Rightarrow \frac{c}{4} = \frac{d}{3} = k(\text{say})$

For c, d $\in$ A, k = 1, 2, 3, ...., 7

5. The maximum value of

$$f(x) = \begin{vmatrix} \sin^2 x & 1+\cos^2 x & \cos 2x \\ 1+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}, x \in R \text{ is :}$$

(1) $\frac{3}{4}$ (2) 5

(3) $\sqrt{7}$ (4) $\sqrt{5}$

**Answer (4)**

**Sol.** $C_1 \to C_1 + C_2$

$$f(x) = \begin{vmatrix} 2 & 1+\cos^2 x & \cos 2x \\ 2 & \cos^2 x & \cos 2x \\ 1 & \cos^2 x & \sin 2x \end{vmatrix}$$

$R_2 \to R_2 - R_1$, $R_1 \to R_1 - 2R_3$

$$f(x) = \begin{vmatrix} 0 & \sin^2 x & \cos 2x - 2\sin 2x \\ 0 & -1 & 0 \\ 1 & \cos^2 x & \sin 2x \end{vmatrix}$$

$f(x) = \cos 2x - 2\sin 2x$

$\text{Max} = \sqrt{5}$

6. Let f be a real valued function, defined on R – {–1, 1} and given by

$$f(x) = 3\log_e \left|\frac{x-1}{x+1}\right| - \frac{2}{x-1}.$$

Then in which of the following intervals, function f(x) is increasing?

(1) $(-\infty, \frac{1}{2}] - \{-1\}$

(2) $(-1, \frac{1}{2}]$

(3) $(-\infty, \infty) - \{-1, 1\}$

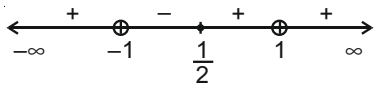
(4) $(-\infty, -1) \cup \left([\frac{1}{2}, \infty) - \{1\}\right)$

**Answer (4)**

**Sol.** $f' = \frac{3(x+1)}{x-1} \times \frac{(x+1)-(x-1)}{(x+1)^2} + \frac{2}{(x-1)^2} > 0$

$= \frac{6}{x^2-1} + \frac{2}{(x-1)^2} > 0$

$= \frac{2(3(x-1)+(x+1))}{(x-1)^2(x+1)} = \frac{4(2x-1)}{(x-1)^2(x+1)} > 0$




$\Rightarrow \quad x \in (-\infty, -1) \cup \left[\frac{1}{2}, \infty\right) - \{1\}$

7. If the points of intersections of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the circle $x^2 + y^2 = 4b$, $b > 4$ lie on the curve $y^2 = 3x^2$, then b is equal to :

(1) 5 (2) 6

(3) 12 (4) 10

**Answer (3)**

**Sol.** $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ ...(i)

$x^2 + y^2 = 4b$, $b > 4$ ...(ii)

$y^2 = 3x^2$ ...(iii)

Solving (ii) & (iii), $x^2 + 3x^2 = 4b$

$x^2 = b$ ...(iv)

(iii) & (i) $\Rightarrow$ $\frac{x^2}{16} + \frac{3x^2}{b^2} = 1$

$(48 + b^2)x^2 = 16b^2$

$(48 + b^2)b = 16b^2$ (using (iv))

$b^2 - 16b + 48 = 0$

$b = 12$ ($\because b > 4$)

8. Let A(–1, 1), B(3, 4) and C(2, 0) be given three points. A line y = mx, m > 0, intersects lines AC and BC at point P and Q respectively. Let $A_1$ and $A_2$ be the areas of $\Delta ABC$ and $\Delta PQC$ respectively, such that $A_1 = 3A_2$, then the value of m is equal to :

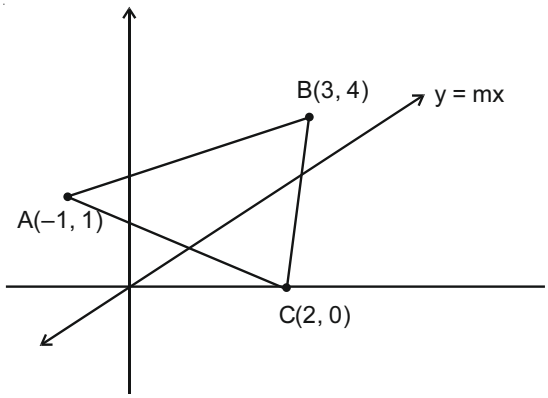
(1) 2 (2) 3

(3) $\frac{4}{15}$ (4) 1

**Answer (4)**

**Sol.** $y = mx$ ...(i)

Equation of AC




$x + 3y = 2$ ...(ii)

(i) and (ii)

$$\Rightarrow \quad P\left(\frac{2}{3m+1}, \frac{2m}{3m+1}\right)$$

Equation of BC is

$y = 4x - 8$ ...(iii)

(i) and (iii)

$$\Rightarrow \quad Q \equiv \left(\frac{8}{4-m}, \frac{8m}{4-m}\right)$$

$$A_1 = \frac{1}{2}\begin{vmatrix} -1 & 1 & 1 \\ 2 & 0 & 1 \\ 3 & 4 & 1 \end{vmatrix} = \frac{13}{2}$$

$$A_2 = \frac{1}{3}A_1 = \frac{13}{6}$$

$$\Rightarrow \quad \frac{1}{2}\begin{vmatrix} 2 & 0 & 1 \\ \frac{8}{4-m} & \frac{8m}{4-m} & 1 \\ \frac{2}{3m+1} & \frac{2m}{3m+1} & 1 \end{vmatrix} = \frac{13}{6}$$

[Taking points anticlockwise]

$15m^2 - 11m - 4 = 0$

$$m = 1, \frac{-4}{15} \quad \text{But} (m > 0)$$

$m = 1$

9. Consider the integral

$$I = \int_0^{10} \frac{[x]e^{[x]}}{e^{x-1}}dx,$$

where [x] denotes the greatest integer less than or equal to x. Then the value of I is equal to :

(1) 9(e – 1) (2) 45(e – 1)

(3) 9(e + 1) (4) 45(e + 1)

**Answer (2)**

**Sol.** $$I = \sum_{k=0}^{9}\int_k^{k+1} ke^{[x]}.e^{1-x}dx = \sum_{k=0}^{9}\int_k^{k+1} ke^{1-\{x\}}dx$$

$$= \sum_{k=0}^{9} k\int_0^1 e^{1-x}dx$$

[$\because$ {x} is periodic function with period 1]

$$= \sum_{k=0}^{9} k\left(-e^{(1-x)}\right)_0^1 = \sum_{k=0}^{9} k(e-1)$$

$$= (e-1)\sum_{k=0}^{9} k = 45(e-1)$$

10. If (x, y, z) be an arbitrary point lying on a plane P which passes through the point (42, 0, 0), (0, 42, 0) and (0, 0, 42), then the value of the expression

$$3 + \frac{x-11}{(y-19)^2(z-12)^2} + \frac{y-19}{(x-11)^2(z-12)^2} + \frac{z-12}{(x-11)^2(y-19)^2} - \frac{x+y+z}{14(x-11)(y-19)(z-12)}$$

is equal to :

(1) 39 (2) 3

(3) –45 (4) 0

**Answer (2)**

**Sol.** Equation of plane $x + y + z = 42$ ...(i)

Given expression is

$$E = 3 + \frac{(x-11)^3 + (y-19)^3 + (z-12)^3}{(x-11)^2(y-19)^2(z-12)^2} - \frac{42}{14(x-11)(y-19)(z-12)} \text{ (using (i))}$$

Now $(x - 11) + (y - 19) + (z - 12)$

$= x + y + z - 42 = 0$ (using (i))

$\therefore$ $(x - 11)^3 + (y - 19)^3 + (z - 12)^3$

$= 3(x - 11)(y - 19)(z - 12)$ ...(ii)

$$E = 3 + \frac{3(x-11)(y-19)(z-12)}{((x-11)(y-19)(z-12))^2} - \frac{3}{(x-11)(y-19)(z-12)}$$

$= 3$

11. The least value of $|z|$ where z is complex number which satisfies the inequality exp

$$\left(\frac{(|z|+3)(|z|-1)}{||z|+1|}\log_e 2\right) \geq \log_{\sqrt{2}}\left|5\sqrt{7}+9i\right|, i=\sqrt{-1},$$

is equal to :

(1) 2 (2) 8

(3) 3 (4) $\sqrt{5}$

**Answer (3)**

**Sol.** Let $|z| = t, t \geq 0$

$$e^{\frac{(t+3)(t-1)}{t+1}\log_e 2} \geq \log_{\sqrt{2}} 16 = 8 \quad (\because t+1>0)$$

$$2^{\frac{(t+3)(t-1)}{t+1}} \geq 2^3$$

$$\frac{(t+3)(t-1)}{t+1} \geq 3$$

$$t^2+2t-3 \geq 3t+3$$

$$t^2-t-6 \geq 0$$

$t \in (-\infty,-2] \cup [3,\infty)$ But $t \geq 0$

$\therefore t \in [3, \infty)$

12. Let $\vec{a} = \hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b} = 2\hat{i}-3\hat{j}+5\hat{k}$. If $\vec{r}\times\vec{a} = \vec{b}\times\vec{r}$, $\vec{r}\cdot\left(a\hat{i}+2\hat{j}+\hat{k}\right)=3$ and $\vec{r}\cdot\left(2\hat{i}+5\hat{j}-\alpha\hat{k}\right)=1$, $\alpha \in R$, then the value of $\alpha+|\vec{r}|^2$ is equal to :

(1) 15 (2) 13

(3) 9 (4) 11

**Answer (1)**

**Sol.** $\vec{r}\times\vec{a} = -\vec{r}\times\vec{b}$

$$\vec{r}\times(\vec{a}+\vec{b}) = 0$$

$$\vec{r} = \lambda(\vec{a}+\vec{b}) = \lambda(3\hat{i}-\hat{j}+2\hat{k})$$

$$\vec{r}\cdot(\alpha\hat{i}+2\hat{j}+\hat{k}) = 3 \Rightarrow \alpha\lambda = 1 \quad ...(i)$$

$$\vec{r}\cdot(2\hat{i}+5\hat{j}+\alpha\hat{k}) = -1 \Rightarrow \lambda - 2\alpha\lambda = -1$$

$\Rightarrow \lambda = 1$ and $\alpha = 1$ [using (i)]

$$\alpha + |\vec{r}|^2 = 1+(9+1+4) = 15$$

13. Let $f : S \to S$ where $S = (0, \infty)$ be a twice differentiable function such that $f(x+1) = xf(x)$. If $g : S \to R$ be defined as $g(x) = \log_e f(x)$, then the value of $|g''(5)-g''(1)|$ is equal to :

(1) $\frac{205}{144}$

(2) 1

(3) $\frac{187}{144}$

(4) $\frac{197}{144}$

**Answer (1)**

**Sol.** $f(x+1) = x\,f(x)$

$$\ln\left(f(x+1)\right) = \ln x + \ln f(x)$$

$$g(x+1) = \ln x + g(x)$$

$$g(x+1) - g(x) = \ln x \quad ...(i)$$

$$g'(x+1) - g'(x) = \frac{1}{x}$$

$$g''(x+1) - g''(x) = \frac{-1}{x^2}$$

$$g''(2) - g'(1) = \frac{-1}{1} \quad ...(ii)$$

$$g''(3) - g''(2) = \frac{-1}{4} \quad ...(iii)$$

$$g''(4) - g''(3) = \frac{-1}{9} \quad ...(iv)$$

$$g''(5) - g''(4) = \frac{-1}{16} \quad ...(v)$$

Adding (ii), (iii), (iv) & (v)

$$g''(5) - g''(1) = -\left(\frac{1}{1}+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}\right) = \frac{-205}{144}$$

$$|g''(5) - g''(1)| = \frac{205}{144}$$

14. Let A denote the event that a 6-digit integer formed by 0, 1, 2, 3, 4, 5, 6 without repetitions, be divisible by 3. Then probability of even A is equal to:

(1) $\frac{9}{56}$ (2) $\frac{11}{27}$

(3) $\frac{3}{7}$ (4) $\frac{4}{9}$

**Answer (4)**

**Sol.** Total number of numbers = 6 × (6 × 5 × 4 × 3 × 2)
= 6 × 6!

Required number of numbers

**Case (i)** 0 is not include → 6!

**Case (ii)** 0 is included → 5 × 5! × 2

Total 6! + 5 × 5! × 2 = 16 × 5!

Probability = $\frac{16\times5!}{6\times6!}=\frac{16}{36}=\frac{4}{9}$

15. Let $C_1$ be the curve obtained by the solution of differential equation $2xy\frac{dy}{dx}=y^2-x^2, x>0$. Let the curve $C_2$ be the solution of $\frac{2xy}{x^2-y^2}=\frac{dy}{dx}$. If both the curves pass through (1, 1), then the area enclosed by the curves $C_1$ and $C_2$ is equal to :

(1) $\pi-1$ (2) $\pi+1$

(3) $\frac{\pi}{4}+1$ (4) $\frac{\pi}{2}-1$

**Answer (4)**

**Sol.** $2\frac{dy}{dx}=\frac{y^2-x^2}{xy}=\frac{y}{x}-\frac{1}{\left(\frac{y}{x}\right)}$

Let $\frac{y}{x}=u$

$2\left(u+x\frac{du}{dx}\right)=u-\frac{1}{u}$

$\frac{2u}{u^2-1}du=\frac{-dx}{x}$

$\ln\left(u^2+1\right)+\ln_x=\ln C$

$x^2 + y^2 = Cx$

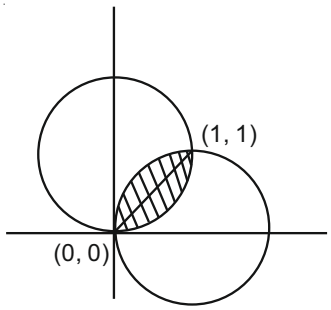
Curve passes through (1, 1) ⇒ C = 2

$x^2 + y^2 = 2x$ ...(i)

Similarly second curve can be obtained by interchanging x and y

$x^2 + y^2 = 2y$ ...(ii)

Required region is = $2\left(\frac{\pi}{4}-\frac{1}{2}\right)=\frac{\pi}{2}-1$




16. If y = y(x) is the solution of the differential equation $\frac{dy}{dx}+(\tan x)y=\sin x, 0\le x\le\frac{\pi}{3}$, with $y(0)=0$, then $y\left(\frac{\pi}{4}\right)$ equal to :

(1) $\log_e 2$ (2) $\left(\frac{1}{2\sqrt{2}}\right)\log_e 2$

(3) $\frac{1}{2}\log_e 2$ (4) $\frac{1}{4}\log_e 2$

**Answer (2)**

**Sol.** If = $e^{\int\tan x\,dx}=e^{\ln(\sec x)}=\sec x$

$y\sec x=\int(\sin x)\sec x\,dx=\ln(\sec x)+C$

y (0) = 0 ⇒ C = 0

$\therefore\quad y=\cos x\ln|\sec x|$

$y\left(\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}\ln\left(\sqrt{2}\right)=\frac{1}{2\sqrt{2}}\ln 2$

17. Given that the inverse trigonometric function take principal values only. Then, the number of real values of x which satisfy $\sin^{-1}\left(\frac{3x}{5}\right)+\sin^{-1}\left(\frac{4x}{5}\right)=\sin^{-1}x$ is equal to :

(1) 3 (2) 1

(3) 0 (4) 2

**Answer (1)**

**Sol.** $\sin^{-1}\left(\frac{3x}{5}\right)+\sin^{-1}\left(\frac{4x}{5}\right)=\sin^{-1}x$

$\sin^{-1}\left(\frac{3x}{5}\sqrt{1-\frac{16x^2}{25}}+\frac{4x}{5}\sqrt{1-\frac{9x^2}{25}}\right)=\sin^{-1}x$

$$\sin^{-1}\left(\frac{3x\sqrt{25-16x^2}+4x\sqrt{25-9x^2}}{25}\right)=\sin^{-1}x$$

$$3x\sqrt{25-16x^2}+4x\sqrt{25-9x^2}=25x$$

$x = 0$ or $3\sqrt{25-16x^2}+4\sqrt{25-9x^2}=25$

Squaring both sides

$$x^2=\frac{1}{2}$$

$$x=\pm\frac{1}{\sqrt{2}}$$

$$\therefore \quad x=0,\ \pm\frac{1}{\sqrt{2}}$$

18. Let the lengths of intercepts on x-axis and y-axis made by the circle $x^2 + y^2 + ax + 2ay + c = 0$, $(a < 0)$ be $2\sqrt{2}$ and $2\sqrt{5}$, respectively. Then the shortest distance from origin to a tangent to this circle which is perpendicular to the line $x + 2y = 0$, is equal to :

(1) $\sqrt{11}$ (2) $\sqrt{7}$

(3) $\sqrt{6}$ (4) $\sqrt{10}$

**Answer (3)**

**Sol.** $2\sqrt{\left(\frac{a}{2}\right)^2-c}=2\sqrt{2}\Rightarrow a^2-4c=8$ ...(i)

$$2\sqrt{a^2-c}=2\sqrt{5}\Rightarrow a^2-c=5 \quad ...(ii)$$

$\Rightarrow$ $a = -2$, $c = -1$

Equation of circle

$x^2 + y^2 - 2x - 4y - 1 = 0$

$$(x-1)^2+(y-2)^2=\left(\sqrt{6}\right)^2$$

$$x^2+y^2=\left(\sqrt{6}\right)^2$$

$m = 2$

Tangent $y=2\times+\sqrt{6}\sqrt{1+2^2}$

$$y-2=2(x-1)+\sqrt{30}$$

$$y=2x+\sqrt{30}\Rightarrow 2x-y+\sqrt{30}=0$$

Distance from (0, 0) $\frac{\sqrt{30}}{\sqrt{5}}=\sqrt{6}$

19. Consider a rectangle ABCD having 5, 7, 6, 9 points in the interior of the line segments AB, CD, BC, DA respectively. Let $\alpha$ be the number of triangles having these points from different sides as vertices and $\beta$ be the number of quadrilaterals having these points from different sides as vertices. Then $(\beta - \alpha)$ is equal to:

(1) 1890

(2) 717

(3) 795

(4) 1173

**Answer (2)**

**Sol.** Number of triangles = $5 \times 6 \times 7 + 6 \times 7 \times 9 + 7 \times 9 \times 5 + 9 \times 5 \times 6$

$= 210 + 378 + 315 + 270$

$\alpha = 1173$

$\beta = 5 \times 6 \times 7 \times 9 = 1890$

$\beta - \alpha = 717$

20. If the foot of the perpendicular from point (4, 3, 8) on the line $L_1:\frac{x-a}{l}=\frac{y-2}{3}=\frac{z-b}{4}$, $l \neq 0$ is (3, 5, 7), then the shortest distance between the line $L_1$ and line $L_2:\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$ is equal to :

(1) $\frac{1}{2}$ (2) $\sqrt{\frac{2}{3}}$

(3) $\sqrt{\frac{1}{\sqrt{6}}}$ (4) $\sqrt{\frac{1}{\sqrt{3}}}$

**Answer (3)**

**Sol.** Let A(4, 3, 8), B(3, 5, 7)

DRs of AB(1, –2, 1)

$AB \perp L_1 \Rightarrow l - 6 + 4 = 0 \Rightarrow l = 2$

Equation of $L_1$

$$\frac{x-3}{2}=\frac{y-5}{3}=\frac{z-7}{4}$$

$$L_2=\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$$

$$SD=\frac{\begin{vmatrix}1&1&2\\2&3&4\\3&4&5\end{vmatrix}}{\begin{vmatrix}i&j&k\\2&3&4\\3&4&5\end{vmatrix}}=\frac{|-1|}{\left|-\hat{i}+2\hat{j}-\hat{k}\right|}=\frac{1}{\sqrt{6}}$$

## SECTION - II

**Numerical Value Type Questions:** This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. For real numbers $\alpha$, $\beta$, $\gamma$ and $\delta$, if

$$\int \frac{(x^2-1)+\tan^{-1}\left(\frac{x^2+1}{x}\right)}{(x^4+3x^2+1)\tan^{-1}\left(\frac{x^2+1}{x}\right)}dx$$

$$=\alpha\log_e\left(\tan^{-1}\left(\frac{x^2+1}{x}\right)\right)+\beta\tan^{-1}\left(\frac{\gamma(x^2-1)}{x}\right)+\delta\tan^{-1}\left(\frac{x^2+1}{x}\right)+C$$

Where C is an arbitrary constant, then the value of $10(\alpha + \beta\gamma + \delta)$ is equal to _____.

**Answer (6)**

**Sol.** $$I=\int\frac{x^2-1}{(x^4+3x^2+1)\tan^{-1}\left(\frac{x^2+1}{x}\right)}dx+\int\frac{1}{x^4+3x^2+1}dx$$

$I = I_1 + I_2$ ...(i)

For $I_1$, Let $\tan^{-1}\frac{x^2+1}{x}=t$

$$I_1=\int\frac{1}{t}dt=\ln t=\ln\left|\tan^{-1}\left(\frac{x^2+1}{x}\right)\right|+C_1$$

$$I_2=\frac{1}{2}\int\frac{(x^2+1)-(x^2-1)}{x^4+3x^2+1}$$

$$=\frac{1}{2}\int\frac{x^2+1}{x^4+3x^2+1}-\frac{1}{2}\int\frac{x^2-1}{x^4+3x^2+1}$$

Divide Nr and Dr by $x^2$

$$=\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+5}-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)^2+1}$$

$$=\frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{5}x}\right)-\frac{1}{2}\tan^{-1}\left(\frac{x^2+1}{x}\right)$$

$$\alpha=1,\ \beta=\frac{1}{2\sqrt{5}},\ \gamma=\frac{1}{\sqrt{5}},\ \delta=\frac{-1}{2}$$

Required value $= 10\left(1+\frac{1}{10}-\frac{1}{2}\right)$

$= 6$

2. Let n be a positive integer. Let

$$A=\sum_{k=0}^{n}(-1)^k\ {}^nC_k\left[\left(\frac{1}{2}\right)^k+\left(\frac{3}{4}\right)^k+\left(\frac{7}{8}\right)^k+\left(\frac{15}{16}\right)^k+\left(\frac{31}{32}\right)^k\right]$$

If $63A = 1-\frac{1}{2^{30}}$, then n is equal to _____.

**Answer (6)**

**Sol.** $$\sum(-1)^k\ {}^nC_k\left(\frac{1}{2}\right)^k+\sum(-1)^k\ {}^nC_k\left(\frac{3}{4}\right)^k+\ldots$$

$$=\left(1-\frac{1}{2}\right)^n+\left(1-\frac{3}{4}\right)^n+\ldots+\left(1-\frac{31}{32}\right)^n$$

$$=\left(\frac{1}{2}\right)^n+\left(\frac{1}{2}\right)^{2n}+\left(\frac{1}{2}\right)^{3n}+\ldots+\left(\frac{1}{2}\right)^{5n}$$

$$=\left(\frac{1}{2}\right)^n\left(\frac{1-\left(\frac{1}{2}\right)^{5n}}{1-\left(\frac{1}{2}\right)^n}\right)=\frac{2^{5n}-1}{2^{5n}\left(2^n-1\right)}$$

Given, $A=\frac{1}{63}\left(1-\frac{1}{2^{30}}\right)$

$\Rightarrow$ n = 6

3. If the distance of the point (1, –2, 3) from the plane $x + 2y – 3z + 10 = 0$ measured parallel to the line, $\frac{x-1}{3}=\frac{2-y}{m}=\frac{z+3}{1}$ is $\sqrt{\frac{7}{2}}$, then the value of |m| is equal to _____.

**Answer (2)**

**Sol.** Line through (1, –2, 3) is

$$L_1:\frac{x-1}{3}=\frac{y+2}{-m}=\frac{z-3}{1}=r$$

Foot of $\perp$ Q(3r + 1, –mr –2, r + 3)

Q lies on $x + 2y –3z + 10 = 0$

$3r + 1 –2mr – 4 –3r –9 + 10 = 0$

$\Rightarrow$ mr = –1

$$PQ=\sqrt{\frac{7}{2}}\Rightarrow 10r^2+m^2r=\frac{7}{2}$$

$$r^2=\frac{1}{4}$$

$m^2 = 4$

$|m| = 2$

4. Let $A=\begin{bmatrix}a_1\\a_2\end{bmatrix}$ and $B=\begin{bmatrix}b_1\\b_2\end{bmatrix}$ be two 2 × 1 matrices with real entries such that A = XB, where $X=\frac{1}{\sqrt{3}}\begin{bmatrix}1 & -1\\1 & k\end{bmatrix}$, and $k\in R$. If $a_1^2+a_2^2=\frac{2}{3}\left(b_1^2+b_2^2\right)$ and $(k^2+1)b_2^2\neq -2b_1b_2$, then the value of k is _____.

**Answer (1)**

**Sol.** A = XB

$$\begin{bmatrix}a_1\\a_2\end{bmatrix}=\frac{1}{\sqrt{3}}\begin{bmatrix}1 & -1\\1 & k\end{bmatrix}\begin{bmatrix}b_1\\b_2\end{bmatrix}$$

$$\sqrt{3}a_1=b_1-b_2 \Rightarrow 3a_1^2=b_1^2+b_2^2-2b_1b_2 \quad ...(i)$$

$$\sqrt{3}a_2=b_1+kb_2 \Rightarrow 3a_2^2=b_1^2+k^2b_2^2+2kb_1b_2 \quad ...(ii)$$

(i) + (ii) $\Rightarrow$

$$3\left(a_1^2+a_2^2\right)=2b_1^2+\left(k^2+1\right)b_2^2+2(k-1)b_1b_2$$

$$\Rightarrow 2\left(b_1^2+b_2^2\right)=2b_1^2+\left(k^2+1\right)b_2^2+2(k-1)b_1b_2$$

$$\left(1-k^2\right)b_2^2=2(k-1)b_1b_2$$

$$(k-1)\left[(k+1)b_2^2+2b_1b_2\right]=0$$

$\Rightarrow k = 1$

5. In $\Delta ABC$, the lengths of sides AC and AB are 12 cm and 5 cm, respectively. If the area of $\Delta ABC$ is 30 $cm^2$ and R and r are respectively the radii of circumcircle and incircle of $\Delta ABC$, then the value of 2R + r (in cm) is equal to _____.

**Answer (15)**

**Sol.** $\Delta=\frac{1}{2}\times 12\times 5\sin\theta=30$

$\theta = 90°$

2R = hypotenuse = 13

$$r=\frac{\Delta}{s}=\frac{30}{\left(\frac{5+12+13}{2}\right)}=2$$

2R + r = 15

6. Let $\vec{c}$ be a vector perpendicular to the vectors $\vec{a}=\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$. If $\vec{c}\cdot\left(\hat{i}+\hat{j}+3\hat{k}\right)=8$ then the value of $\vec{c}\cdot(\vec{a}\times\vec{b})$ is equal to _____.

**Answer (28)**

**Sol.** $\vec{a}\times\vec{b}=3\hat{i}-2\hat{j}+\hat{k}$

$$C=\lambda\left(\vec{a}\times\vec{b}\right)=\lambda\left(3i-2j+\hat{k}\right)$$

$$C\cdot(i+j+3k)=8\Rightarrow\lambda=2$$

$$\vec{C}=2\left(\vec{a}\times\vec{b}\right)$$

$$\vec{C}\cdot\left(\vec{a}\times\vec{b}\right)=2\left(\vec{a}\times\vec{b}\right)\cdot\left(\vec{a}\times\vec{b}\right)$$

$$=2\left|\vec{a}\times\vec{b}\right|^2=2(9+4+1)=28$$

7. Let $f : R\to R$ and $g : R\to R$ be defined as

$$f(x)=\begin{cases}x+a, & x<0\\|x-1|, & x\geq 0\end{cases} \text{ and } g(x)=\begin{cases}x+1, & x<0\\(x-1)^2+b, & x\geq 0'\end{cases}$$

where a, b are non-negative real numbers. If $(gof)(x)$ is continuous for all $x\in R$, then a + b is equal to_____.

**Answer (1)**

**Sol.** f(x) should be continuous at x = 0

$\Rightarrow$ a = 1

g(x) should be continuous at x = 0

$\Rightarrow$ 1 = 1 + b $\Rightarrow$ b = 0

a + b = 1

8. Consider the statistics of two sets of observations as follows:

| | Size | Mean | Variance |
|---|---|---|---|
| Observation I | 10 | 2 | 2 |
| Observation II | n | 3 | 1 |

If the variance of the combined set of these two observations is $\frac{17}{9}$, then the value of n is equal to _____.

**Answer (5)**

**Sol.** $\overline{x_1}=2,\ \overline{x_2}=3,\ \overline{x}=\frac{3n+20}{n+10}$

$$d_1^2=\left(\overline{x}-\overline{x_1}\right)^2=\frac{n^2}{(n+10)^2},\ d_2^2=\left(\overline{x}-\overline{x_2}\right)^2=\frac{100}{(n+10)^2}$$

$$\sigma_1^2=2,\ \sigma_2^2=1,\ \sigma^2=\frac{17}{9}$$

$$(n_1+n_2)\sigma^2=n_1\left(\sigma_1^2+d_1^2\right)+n_2\left(\sigma_2^2+d_2^2\right)$$

$$(n+10)\times\frac{17}{9}=10\left(2+\frac{n^2}{(n+10)^2}\right)+n\left(1+\frac{100}{(n+10)^2}\right)$$

$$(n+10)17 = \left[20 + n + \frac{10n^2 + 100n}{(n+10)^2}\right] \times 9$$

$(8n - 10)(n+10)^2 = 90n^2 + 900n$

$(8n - 10)(n^2 + 20n + 100) = 90n^2 + 900n$

$(4n - 5)(n^2 + 20n + 100) = 45n^2 + 450n$

$2n^3 + 15n^2 - 75n - 250 = 0$

$(n - 5)(n + 10)(2n + 5) = 0$

$n = 5$

9. Let

$$S_n(x) = \log_{a^{1/2}} x + \log_{a^{1/3}} x + \log_{a^{1/6}} x + \log_{a^{1/11}} x + \log_{a^{1/18}} x + \log_{a^{1/27}} x + \ldots\ldots$$ up to n-terms, where $a > 1$. If $S_{24}(x) = 1093$ and $S_{12}(2x) = 265$, then value of a is equal to _____.

**Answer (16)**

**Sol.** $S_n = (2 + 3 + 6 + 11 + 18 + 27 \ldots\ldots)\log_a x$.

$T_n = 2, 3, 6, 11, 18, 27, \ldots\ldots$

$T_n' = 1, 3, 5, 7, 9, \ldots\ldots$A.P.

$T_n = An^2 + Bn + C$

$A + B + C = 2$

$4A + 2B + C = 3$

$9A + 3B + C = 6$

$A = 1, B = -2, C = 3$

$$S_n = \sum\left(n^2 - 2n + 3\right)\log_x$$

$$= \left(\frac{n(n+1)(2n+1)}{6} - 2\frac{n(n+1)}{2} + 3n\right)\log_a x$$

$$S_n(x) = \frac{n}{6}\left[2n^2 - 3n + 13\right]\log_a x$$

$$S_{24}(x) = 1093 \Rightarrow 4 \times 1093 \log_a x = 1093$$

$$\log_a x = \frac{1}{4} \Rightarrow x = a^{\frac{1}{4}} \Rightarrow a = x^4 \quad \ldots(i)$$

$$S_{12}(2x) = 265 \Rightarrow 2(265)\log_a 2x = 265$$

$$\Rightarrow 2x = a^{\frac{1}{2}} \Rightarrow a = 4x^2 \quad \ldots(ii)$$

(i) and (ii) $\Rightarrow$ $x^4 = 4x^2$

$\Rightarrow$ $x^2 = 4$

$a = x^4 = 16$

10. Let $\frac{1}{16}$, a and b be in G.P. and $\frac{1}{a}, \frac{1}{b}, 6$ be in A.P., where a, b > 0. Then 72(a+b) is equal to_____.

**Answer (14)**

**Sol.** $b \times \frac{1}{16} = a^2 \Rightarrow b = 16a^2$ ...(i)

$$\frac{1}{a} + 6 = \frac{2}{b}$$

$$\frac{6a+1}{a} = \frac{2}{16a^2}$$ [using (i)]

$48a^2 + 8a - 1 = 0$

$a = \frac{1}{12}$ $(a > 0)$

$b = \frac{1}{9}$

$a + b = \frac{7}{36}$

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