Aakash
Medical | IIT-JEE | Foundations

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Time : 3 hrs.

Answers & Solutions
for
JEE (MAIN)-2021 (Online) Phase-2

(Physics, Chemistry and Mathematics)

M.M. : 300

IMPORTANT INSTRUCTIONS :

(1)   The test is of **3 hours** duration.

(2)   The Test Booklet consists of 90 questions. The maximum marks are 300.

(3)   There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part has two sections.

(i) Section-I : This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) Section-II : This section contains 10 questions. In Section-II, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and there is no negative marking for wrong answer.

PART–A : PHYSICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. One main scale division of a vernier calipers is 'a' cm and n^{th} division of the vernier scale coincide with $(n – 1)^{th}$ division of the main scale. The least count of the calipers in mm is:

   (1) $\dfrac{10\,a}{(n-1)}$

   (2) $\dfrac{10\,na}{(n-1)}$

   (3) $\left(\dfrac{n-1}{10\,n}\right)a$

   (4) $\dfrac{10\,a}{n}$

Answer (4)

Sol. $1\,\text{VSD} = \dfrac{(n-1)\times(a\ \text{cm})}{n}$

$\therefore$ least count $= 1\,\text{MSD} - 1\,\text{VSD}$

$= a - \dfrac{(n-1)a}{n}$

$= \dfrac{a(n-n+1)}{n}$

$= \dfrac{a}{n}$ cm

$= \dfrac{a}{n}\times 10$ mm

2. For changing the capacitance of a given parallel plate capacitor, a dielectric material of dielectric constant K is used, which has the same area as the plates of the capacitor. The thickness of the dielectric slab is $\dfrac{3}{4}d$, where 'd' is the separation between the plates of parallel plate capacitor. The new capacitance (C′) in terms of original capacitance (C_0) is given by the following relation:

   (1) $C' = \dfrac{4+K}{3}C_0$

   (2) $C' = \dfrac{4}{3+K}C_0$

   (3) $C' = \dfrac{3+K}{4K}C_0$

   (4) $C' = \dfrac{4K}{K+3}C_0$

Answer (4)

Sol. $C = \dfrac{\varepsilon_0 A}{d - t + \dfrac{t}{K}}$

$= \dfrac{\varepsilon_0 A}{d - \dfrac{3d}{4} + \dfrac{3d}{4K}} = \dfrac{\varepsilon_0 A}{\dfrac{d}{4} + \dfrac{3d}{4K}}$

$= \dfrac{4K\times\varepsilon_0 A}{(K+3)d}$

$= \dfrac{4K}{K+3}C_0$

3. The maximum and minimum distances of a comet from the Sun are 1.6×10^{12} m and 8.0×10^{10} m respectively. If the speed of the comet at the nearest point is 6×10^4 ms^{-1}, the speed at the farthest point is:

   (1) 6.0×10^3 m/s

   (2) 3.0×10^3 m/s

   (3) 4.5×10^3 m/s

   (4) 1.5×10^3 m/s

Answer (2)

Sol. $\therefore\ mv_1 r_1 = mv_2 r_2$

$\Rightarrow\ 6 \times 10^4 \times 8 \times 10^{10} = v_2 \times 1.6 \times 10^{12}$

$\Rightarrow\ v_2 = \dfrac{6\times 8}{1.6}\times 10^2$

$= 3 \times 10^3$ m/s

4. A block of 200 g mass moves with a uniform speed in a horizontal circular groove, with vertical side walls of radius 20 cm. If the block takes 40 s to complete one round, the normal force by the side walls of the groove is:

   (1) 9.859×10^{-2} N

   (2) 9.859×10^{-4} N

   (3) 6.28×10^{-3} N
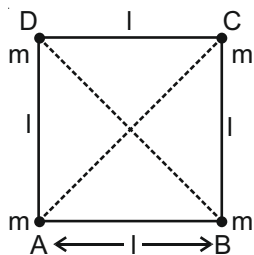
   (4) 0.0314 N

Answer (2)

Sol. $N = \dfrac{mv^2}{r}$

$= \dfrac{(200\times 10^{-3})\times\left(\dfrac{2\pi\times 0.2}{40}\right)^2}{0.2}$

$= 9.859 \times 10^{-4}$ N

5. Four equal masses, m each are placed at the corners of a square of length (l) as shown in the figure. The moment of inertia of the system about an axis passing through A and parallel to DB would be:
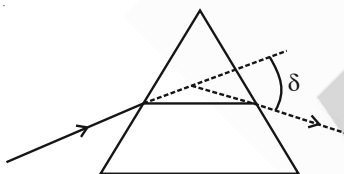

(1) $3 \, ml^2$

(2) $2 \, ml^2$

(3) $\sqrt{3} \, ml^2$

(4) ml^2

Answer (1)

Sol. $I_A = m \times \left(\dfrac{l}{\sqrt{2}}\right)^2 \times 2 + m \times \left(\sqrt{2} \, l\right)^2$

$= \dfrac{ml^2}{2} \times 2 + 2ml^2$

$= 3ml^2$

6. The angle of deviation through a prism is minimum when

(A) Incident ray and emergent ray are symmetric to the prism

(B) The refracted ray inside the prism becomes parallel to its base

(C) Angle of incidence is equal to that of the angle of emergence

(D) When angle of emergence is double the angle of incidence

Choose the correct answer from the options given below:

(1) Only statement (D) is true

(2) Statements (B) and (C) are true

(3) Only statements (A) and (B) are true

(4) Statements (A), (B) and (C) are true

Answer (4)

Sol.
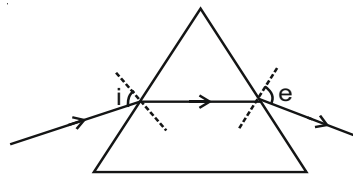
For minimum deviation,

i = e

and refracted ray is parallel to the base.

7. The volume V of an enclosure contains a mixture of three gases, 16 g of oxygen, 28 g of nitrogen and 44 g of carbon dioxide at absolute temperature T. Consider R as universal gas constant. The pressure of the mixture of gases is :

(1) $\dfrac{88RT}{V}$

(2) $\dfrac{4RT}{V}$

(3) $\dfrac{3RT}{V}$

(4) $\dfrac{5}{2}\dfrac{RT}{V}$

Answer (4)

Sol. $P \times V = \left(\dfrac{16}{32} + \dfrac{28}{28} + \dfrac{44}{44}\right) \times R \times T$

$\Rightarrow P \times V = \dfrac{5}{2} RT$

$\Rightarrow P = \dfrac{5RT}{2V}$

8. The stopping potential in the context of photoelectric effect depends on the following property of incident electromagnetic radiation:

(1) Intensity

(2) Amplitude

(3) Frequency

(4) Phase

Answer (3)

Sol. $\therefore eV_0 = h\nu - \phi$

$\therefore$ stopping potential is dependent on frequency.

9. A conducting wire of length 'l', area of cross-section A and electric resistivity ρ is connected between the terminals of a battery. A potential difference V is developed between its ends, causing an electric current.

If the length of the wire of the same material is doubled and the area of cross-section is halved, the resultant current would be :

(1) $4\dfrac{VA}{\rho l}$

(2) $\dfrac{3}{4}\dfrac{VA}{\rho l}$

(3) $\dfrac{1}{4}\dfrac{\rho l}{VA}$

(4) $\dfrac{1}{4}\dfrac{VA}{\rho l}$

Answer (4)

Sol. $R_1 = \dfrac{\rho l}{A}$

$R_2 = \dfrac{\rho \times (2l)}{\left(\dfrac{A}{2}\right)} = 4\dfrac{\rho l}{A}$

$\therefore \quad I = \dfrac{V}{R_2} = \dfrac{V}{\left(\dfrac{4\rho l}{A}\right)}$

$\Rightarrow \quad I = \dfrac{1}{4}\dfrac{VA}{\rho l}$

10. A 25 m long antenna is mounted on an antenna tower. The height of the antenna tower is 75 m. The wavelength (in meter) of the signal transmitted by this antenna would be :

    (1) 400   (2) 100

    (3) 300   (4) 200

Answer (2)

Sol. $l_{antenna} = \dfrac{\lambda}{4}$

$\Rightarrow \lambda = 4 \times (25) = 100$ m.

11. An RC circuit as shown in the figure is driven by a AC source generating a square wave. The output wave pattern monitored by CRO would look close to :
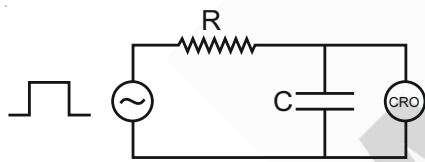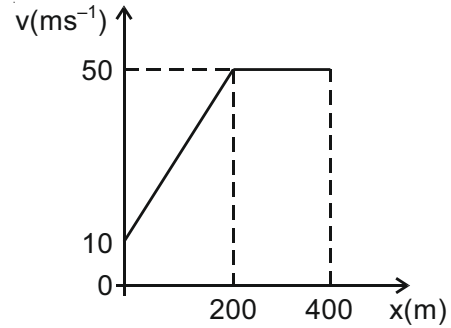

Answer (3)

Sol. For positive cycle capacitor will get charge and for zero input capacitor will discharge.

12. The velocity-displacement graph describing the motion of a bicycle is shown in the figure.

The acceleration-displacement graph of the bicycle's motion is best described by :

Answer (4)

Sol. $a = \dfrac{vdv}{dx}$

$v = 10 + \dfrac{x}{5}$

$a = \left(10 + \dfrac{x}{5}\right)\left(\dfrac{1}{5}\right) = 2 + \dfrac{x}{25}$

$a(x = 0) = 2$ m/s^2

$a(x = 200) = 10$ m/s^2

In graph [18 should be marked as 10 on y-axis in given options]

13. In thermodynamics, heat and work are :

(1) Point functions

(2) Extensive thermodynamic state variables

(3) Path functions

(4) Intensive thermodynamic state variables

Answer (3)

Sol. Heat and work are path functions.

14. Time period of a simple pendulum is T inside a lift when the lift is stationary. If the lift moves upwards with an acceleration g/2, the time period of pendulum will be :

(1) $\sqrt{\dfrac{2}{3}}\,T$

(2) $\sqrt{3}\,T$

(3) $\sqrt{\dfrac{3}{2}}\,T$

(4) $\dfrac{T}{\sqrt{3}}$

Answer (1)

Sol. $T = 2\pi\sqrt{\dfrac{l}{g_{eff}}}$

$T' = \sqrt{\dfrac{2}{3}}\,T$

15. A plane electromagnetic wave of frequency 500 MHz is travelling in vacuum along y-direction. At a particular point in space and time, $\vec{B} = 8.0 \times 10^{-8}\hat{z}\,T$. The value of electric field at this point is :

(speed of light = 3 × 10^8 ms^{-1})

$\hat{x}, \hat{y}, \hat{z}$ are unit vectors along x, y and z directions.

(1) $-24\,\hat{x}$ V/m

(2) $2.6\,\hat{x}$ V/m
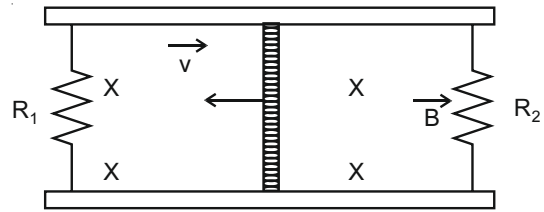
(3) $24\,\hat{x}$ V/m

(4) $-2.6\,\hat{y}$ V/m

Answer (1)

Sol. $\vec{E}.\vec{B} = 0$

$\vec{E} \times \vec{B}$ is along positive y-direction

$\dfrac{|\vec{E}|}{|\vec{B}|} = 3 \times 10^8$

16. A conducting bar of length L is free to slide on two parallel conducting rails as shown in the figure

Two resistors R_1 and R_2 are connected across the ends of the rails. There is a uniform magnetic field $\vec{B}$ pointing into the page. An external agent pulls the bar to the left at a constant speed v.
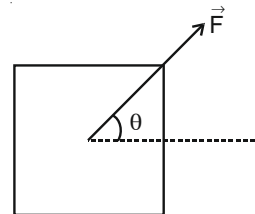
The correct statement about the directions of induced currents I_1 and I_2 flowing through R_1 and R_2 respectively is:

(1) I_1 is in clockwise direction and I_2 is in anticlockwise direction

(2) I_1 is in anticlockwise direction and I_2 is in clockwise direction

(3) Both I_1 and I_2 are in clockwise direction

(4) Both I_1 and I_2 are in anticlockwise direction

Answer (1)

Sol. It is based on Lenz Law. I_1 is in clockwise direction and I_2 is in anticlockwise direction.

17. A block of mass m slides along a floor while a force of magnitude F is applied to it at an angle θ as shown in figure. The coefficient of kinetic friction is μ_K. Then, the block's acceleration 'a' is given by: (g is acceleration due to gravity)

(1) $\dfrac{F}{m}\cos\theta + \mu_K\left(g - \dfrac{F}{m}\sin\theta\right)$

(2) $\dfrac{F}{m}\cos\theta - \mu_K\left(g - \dfrac{F}{m}\sin\theta\right)$

(3) $-\dfrac{F}{m}\cos\theta - \mu_K\left(g - \dfrac{F}{m}\sin\theta\right)$

(4) $\dfrac{F}{m}\cos\theta - \mu_K\left(g + \dfrac{F}{m}\sin\theta\right)$

Answer (2)

Sol. $F\cos\theta - \mu N = ma$

$N = (mg - F\sin\theta)$

$F\cos\theta - \mu mg + \mu F\sin\theta = ma$

18. For an electromagnetic wave travelling in free space, the relation between average energy densities due to electric (U_e) and magnetic (U_m) fields is:

    (1) $U_e > U_m$

    (2) $U_e = U_m$

    (3) $U_e \neq U_m$

    (4) $U_e < U_m$

Answer (2)

Sol. $U_e = U_m$

19. The pressure acting on a submarine is 3×10^5 Pa at a certain depth. If the depth is doubled, the percentage increase in the pressure acting on the submarine would be:

    (Assume that atmospheric pressure is 1×10^5 Pa, density of water is 10^3 kg m^{-3}, g = 10 ms^{-2})

    (1) $\dfrac{200}{5}\%$

    (2) $\dfrac{3}{200}\%$

    (3) $\dfrac{200}{3}\%$

    (4) $\dfrac{5}{200}\%$

Answer (3)

Sol. $P_1 = P_0 + \rho g h_1$

$h_1 = \dfrac{2P_0}{\rho g}$

$P_2 = P_0 + 4P_0 = 5P_0$

% increase $= \dfrac{P_2 - P_1}{P_1} \times 100 = \dfrac{200}{3}\%$

20. A bar magnet of length 14 cm is placed in the magnetic meridian with its north pole pointing towards the geographic north pole. A neutral point is obtained at a distance of 18 cm from the center of the magnet. If $B_H = 0.4$ G, the magnetic moment of the magnet is (1 G = 10^{-4} T)
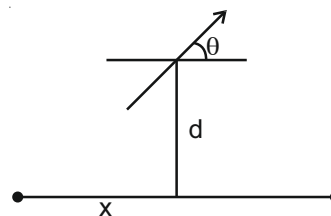
    (1) 2.880×10^2 J T^{-1}

    (2) 2.880 J T^{-1}

    (3) 2.880×10^3 J T^{-1}

    (4) 28.80 J T^{-1}

Answer (2)

Sol. Neutral point will lie on equatorial plane

$B_1 = \dfrac{\mu_0 m}{4\pi} \dfrac{1}{(d^2 + x^2)}$

$B_{net} = 2B_1 \cos\theta$

$B_{net} = \dfrac{2\mu_0 m}{4\pi} \times \dfrac{x}{(d^2 + x^2)^{\frac{3}{2}}}$

$B_{net} = 0.4$ G

Putting the values we get m × 2x = M = 2.881 JT^{-1}

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

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1. The value of power dissipated across the zener diode (V_z = 15 V) connected in the circuit as shown in the figure is x × 10^{-1} watt
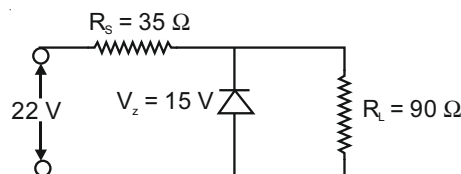
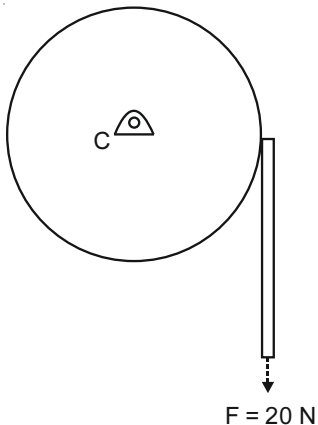   The value of x, to the nearest integer, is _____

Answer (5)

Sol. $P = V_z I_z$

$i = \dfrac{22 - 15}{35} = \dfrac{1}{5}$ A & $i' = \dfrac{15}{90} = \dfrac{1}{6}$ A

$i_z = \dfrac{1}{5} - \dfrac{1}{6} = \dfrac{1}{30}$ A

$P = 15 \times \dfrac{1}{30} = 5 \times 10^{-1}$ W

2. Consider a 20 kg uniform circular disk of radius 0.2 m. It is pin supported at its center and is at rest initially. The disk is acted upon by a constant force F = 20 N through a massless string wrapped around its periphery as shown in the figure.

F = 20 N

Suppose the disk makes n number of revolutions to attain an angular speed of 50 rad s^{-1}.

The value of n, to the nearest integer, is _____

[Given : In one complete revolution, the disk rotates by 6.28 rad]

Answer (20)

Sol. $\alpha = \dfrac{F.R}{I}$

$= \dfrac{20 \times 0.2}{\dfrac{1}{2} \times 20 \times 0.2^2}$

$= 10 \, \text{rad/s}^2$

$\omega^2 = \omega_0^2 + 2\alpha\theta$

$n = \dfrac{\theta}{2\pi} + \dfrac{\omega^2}{4\pi\alpha} = \dfrac{2500}{4 \times 3.14 \times 10} = 19.90$

3. The first three spectral lines of H-atom in the Balmer series are given λ_1, λ_2, λ_3 considering the Bohr atomic model, the wave lengths of first and third spectral lines $\left(\dfrac{\lambda_1}{\lambda_3}\right)$ are related by a factor of approximately x × 10^{-1}.
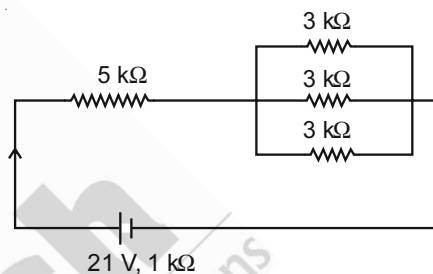
The value of x, to the nearest integer, is

Answer (15)

Sol. $\dfrac{1}{\lambda_1} = R\left(\dfrac{1}{4} - \dfrac{1}{9}\right)$

$\dfrac{1}{\lambda_2} = R\left(\dfrac{1}{4} - \dfrac{1}{16}\right)$

$\dfrac{1}{\lambda_3} = R\left(\dfrac{1}{4} - \dfrac{1}{25}\right)$

$\dfrac{\lambda_1}{\lambda_3} = \dfrac{21}{100} \times \dfrac{36}{5} = 1.512$

4. In the figure given, the electric current flowing through the 5 kΩ resistor is x mA.

21 V, 1 kΩ

The value of x to the nearest integer is _____

Answer (3)

Sol. $i = \dfrac{E}{R_{eq}} = \dfrac{21}{5 + 1 + 1} \text{mA} = 3\text{mA}$

5. A sinusoidal voltage of peak value 250 V is applied to a series LCR circuit , in which R = 8Ω, L = 24 mH and C = 60 μF. The value of power dissipated at resonant conditions is x kW.

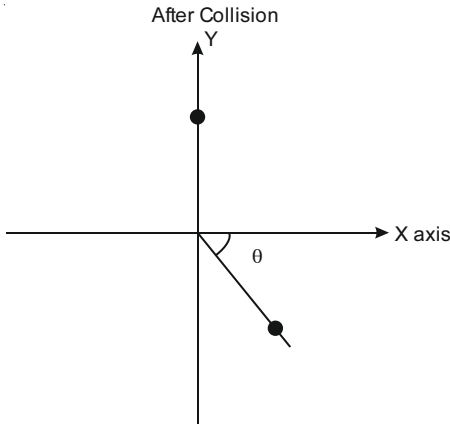The value of x to the nearest integer is _____

Answer (4)

Sol. Power dissipated at resonance

$= \dfrac{V_{rms}^2}{R} = \dfrac{(250)^2}{2 \times 8} = 3.906 \times 10^3 \, \text{W}$

6. A ball of mass 10 kg moving with a velocity $10\sqrt{3}$ m s^{-1} along X-axis, hits another ball of mass 20 kg which is at rest. After collision, the first ball comes to rest and the second one disintegrates into two equal pieces. One of the pieces starts moving along Y-axis at a speed of 10 m/s. The second piece starts moving at a speed of 20 m/s at an angle θ (degree) with respect to the X-axis.

The configuration of pieces after collision is shown in the figure.

The value of θ to the nearest integer is_____.

After Collision

Answer (30)

Sol. Using conservation of linear momentum

$-mv_1\sin\theta + mv_2 = 0$

$\sin\theta = \dfrac{v_2}{v_1} = \dfrac{10}{20}$

$\theta = 30°$

7. The resistance $R = \dfrac{V}{I}$, where $V = (50 \pm 2)V$ and $I = (20 \pm 0.2)A$. The percentage error in R is 'x' %. The value of 'x' to the nearest integer is_____.
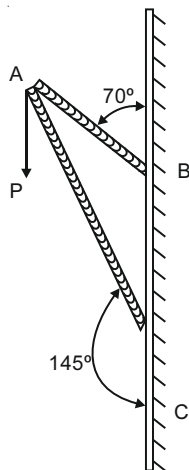
Answer (5)

Sol. $R = \dfrac{V}{I}$

$\Rightarrow \dfrac{\Delta R}{R} \times 100 = \dfrac{\Delta V}{V} \times 100 + \dfrac{\Delta I}{I} \times 100$

$= \dfrac{2}{50} \times 100 + \dfrac{0.2}{20} \times 100 = 5\%$

8. Consider a frame that is made up of two thin massless rods AB and AC as shown in the figure. A vertical force $\vec{P}$ of magnitude 100 N is applied at point A of the frame.

Suppose the force is $\vec{P}$ resolved parallel to the arms AB and AC of the frame. The magnitude of the resolved component along the arm AC is xN.

The value of x, to the nearest integer, is_____.

[Given : sin(35°) = 0.573,

cos(35°) = 0.819

sin(110°) = 0.939,

cos(110°) = −0.342]

Answer (164)

Sol. Let component be x, y

$x\cos35° + y\cos70° = P$

$x\sin35° + y\sin70° = 0$

$y = \dfrac{-x}{2\cos35°}$

$x\cos35° - \dfrac{x\cos70°}{2\cos35°} = P$

$x = 2P\cos35°$
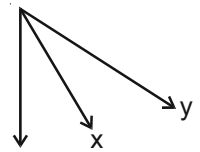
9. A fringe width of 6 mm was produced for two slits separated by 1 mm apart. The screen is placed 10 m away. The wavelength of light used is 'x' nm.

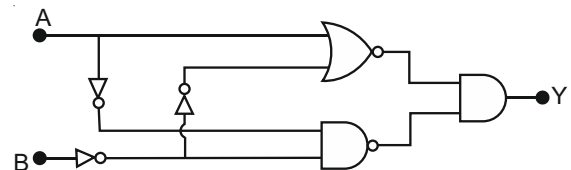The value of 'x' to the nearest integer is_____.

Answer (600)

Sol. $\beta = \dfrac{D\lambda}{d}$

$6 \times 10^{-3} = \dfrac{10\lambda}{10^{-3}}$
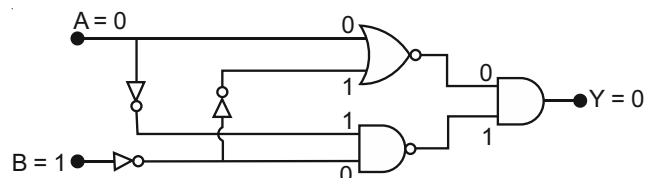
$\lambda = 600$ nm

10. In the logic circuit shown in the figure, if input A and B are 0 to 1 respectively, the output at Y would be 'x'.
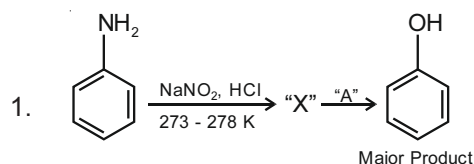
The value of x is_____.
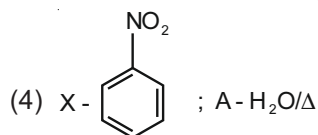

Answer (0)

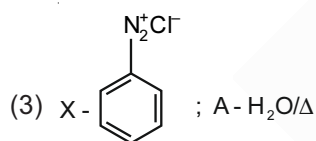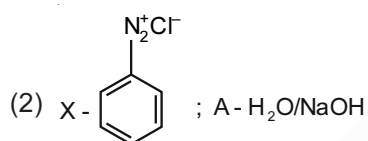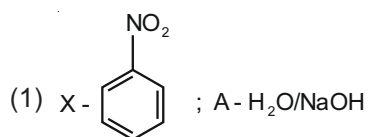**Sol.**

PART–B : CHEMISTRY

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.
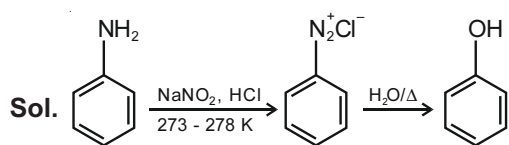
Choose the correct answer :

1.

   In the above chemical reaction, intermediate "X" and reagent/condition "A" are

   (1) ; A - H_2O/NaOH

   (2) ; A - H_2O/NaOH

   (3) ; A - H_2O/Δ

   (4) ; A - H_2O/Δ

Answer (3)

Sol.

2. In chromatography technique, the purification of compound is independent of

   (1) Solubility of the compound

   (2) Mobility or flow of solvent system

   (3) Length of the column or TLC plate

   (4) Physical state of the pure compound

Answer (4)

Sol. In chromatography technique, the purification of compound is independent of physical state of the pure compound.

3. Which among the following pairs of Vitamins is stored in our body relatively for longer duration?

   (1) Ascorbic acid and Vitamin D

   (2) Vitamin A and Vitamin D

   (3) Thiamine and Ascorbic acid

   (4) Thiamine and Vitamin A

Answer (2)

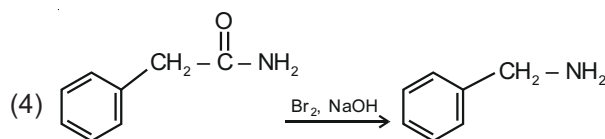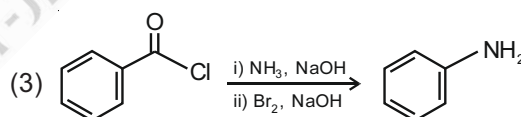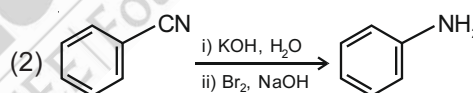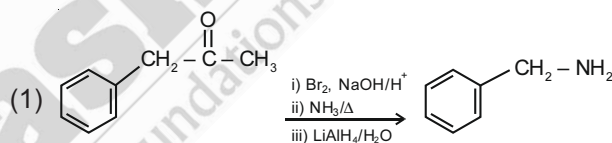**Sol.** Vitamins which are soluble in fat and oils but insoluble in water are fat soluble vitamins, which are stored in our body relatively for longer time.
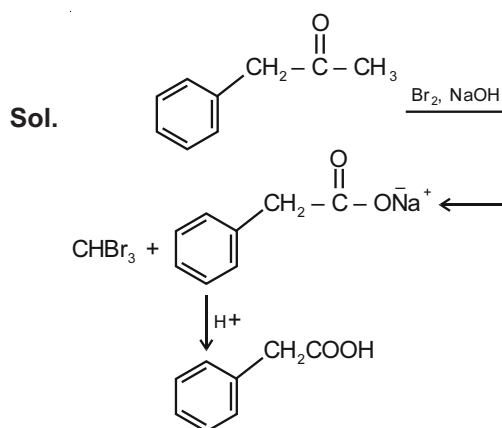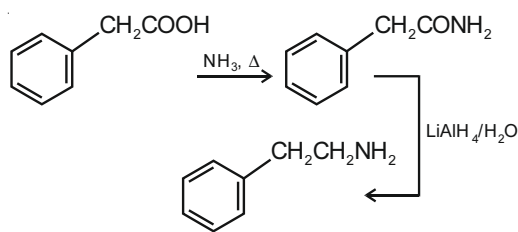
e.g. Vitamin A and Vitamin D

Thiamine (Vit B_1) and Ascorbic acid (Vit C) are water soluble.

4. Which of the following reaction DOES NOT involve Hoffmann bromamide degradation?

   (1)

   (2)

   (3)

   (4)

Answer (1)

Sol.

5. Match List-I with List-II :

List-I	List-II
Industrial process	**Application**
(a) Haber's process	(i) HNO_3 synthesis
(b) Ostwald's process	(ii) Aluminium extraction
(c) Contact process	(iii) NH_3 synthesis
(d) Hall-Heroult process	(iv) H_2SO_4 synthesis

Choose the **correct** answer from the options given below.

(1) (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)

(2) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

(3) (a)-(iv), (b)-(i), (c)-(ii), (d)-(iii)

(4) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

Answer (4)

Sol.

Process		Application
Haber's process	$\longrightarrow$	NH_3 synthesis
Ostwald's process	$\longrightarrow$	HNO_3 synthesis
Contact process	$\longrightarrow$	H_2SO_4 synthesis
Hall Heroults process	$\longrightarrow$	Aluminium extraction

(a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

6. The process that involves the removal of sulphur from the ores is

(1) Smelting

(2) Refining

(3) Roasting

(4) Leaching

Answer (3)

Sol. Removal of sulphur from the ore is done by Roasting.

7. Given below are two statement : one is labelled as **Assertion A** and the other is labelled as **Reason R** :

Assertion A : Size of Bk^{3+} ion is less than Np^{3+} ion.

Reason R : The above is a consequence of the lanthanoid contraction.

In the light of the above statements, choose the **correct** answer from the options given below.

(1) A is false but R is true

(2) A is true but R is false

(3) Both A and R are true but R is not the correct explanation of A

(4) Both A and R are true and R is the correct explanation of A

Answer (2)

Sol. Size of Bk^{3+} is 98 pm

Size of Np^{3+} is 101 pm

So size of Np^{3+} is more than Bk^{3+} ion.

there is a gradual decrease in the size of M^{3+} ions across the series. This may be referred to as the actinoid contraction.

8. The type of pollution that gets increased during the day time and in the presence of O_3 is

(1) Reducing smog

(2) Acid rain

(3) Global warming

(4) Oxidising smog

Answer (4)

Sol. Photochemical smog occurs in warm, dry and sunny climate. It is also called oxidising smog. The main components of photochemical smog are ozone, nitric oxide, acrolein, formaldehyde and PAN

9. Given below are two statements :

Statement I : H_2O_2 can act as both oxidising and reducing agent in basic medium.

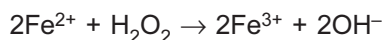**Statement II :** In the hydrogen economy, the energy is transmitted in the form of dihydrogen.

In the light of the above statements, choose the **correct** answer from the options given below:
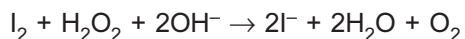
(1) Both statement I and statement II are false

(2) Statement I is true but statement II is false

(3) Both statement I and statement II are true

(4) Statement I is false but statement II is true

Answer (3)

Sol. Oxidising action in basic medium

$2Fe^{2+} + H_2O_2 \rightarrow 2Fe^{3+} + 2OH^-$

Reducing action in basic medium

$I_2 + H_2O_2 + 2OH^- \rightarrow 2I^- + 2H_2O + O_2$

Advantage of hydrogen economy is that energy is transmitted in the form of dihydrogen and not as electric power

10. Match List-I with List-II :

List-I		List-II	
Name of oxo acid		**Oxidation state of 'P'**	
(a) Hypophosphorous acid	(i)	+5	
(b) Orthophosphoric acid	(ii)	+4	
(c) Hypophosphoric acid	(iii)	+3	
(d) Orthophosphorous acid	(iv)	+2	
	(v)	+1	

Choose the correct answer from the options given below:

(1) (a)-(iv), (b)-(v), (c)-(ii), (d)-(iii)

(2) (a)-(v), (b)-(iv), (c)-(ii), (d)-(iii)

(3) (a)-(v), (b)-(i), (c)-(ii), (d)-(iii)

(4) (a)-(iv), (b)-(i), (c)-(ii), (d)-(iii)

Answer (3)

Sol.

| Hypophosphorous acid	H_3PO_2	+1
Orthophosphorous acid	H_3PO_3	+3
Hypophosphoric acid	$H_4P_2O_6$	+4
Orthophosphoric acid	H_3PO_4	+5

(a)-(v), (b)-(i), (c)-(ii), (d)-(iii)

11. Which of the following is Lindlar catalyst ?
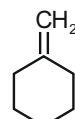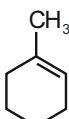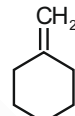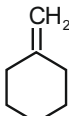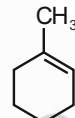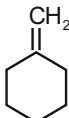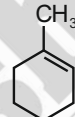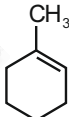
(1) Cold dilute solution of $KMnO_4$

(2) Partially deactivated palladised charcoal

(3) Zinc chloride and HCl

(4) Sodium and Liquid NH_3

Answer (2)

Sol. Partially deactivated palladised charcoal is called as Lindlar's catalyst.

12.

$\xrightarrow[358 \text{ K}]{20\% \ H_3PO_4}$ "A", (Major Product)

$\xrightarrow{(CH_3)_3CO^-K^+}$ "B" (Major Product)
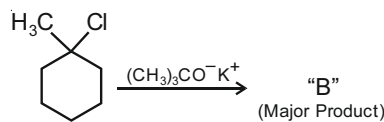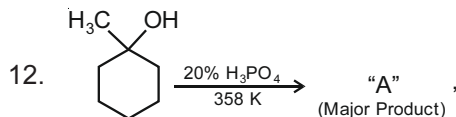
The products "A" and "B" formed in above reactions are

Answer (3)

Sol.
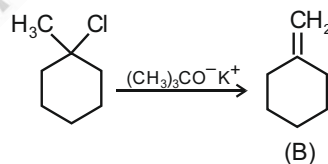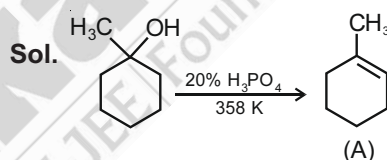
13. **Assertion A :** Enol form of acetone [CH_3COCH_3] exists in < 0.1% quantity. However, the enol form of acetyl acetone [$CH_3COCH_2OCCH_3$] exists in approximately 15% quantity.

Reason R : Enol form of acetyl acetone is stabilized by intramolecular hydrogen bonding, which is not possible in enol form of acetone.

Choose the correct statement :

(1) **A** is false but **R** is true

(2) Both **A** and **R** are true and **R** is the correct explanation of **A**

(3) **A** is true but **R** is false

(4) Both **A** and **R** are true but **R** is not the correct explanation of **A**

Answer (2)

Sol.
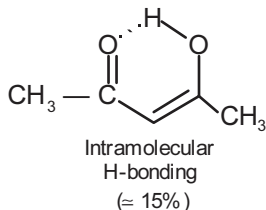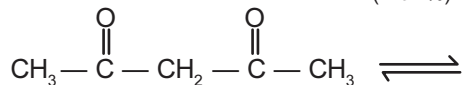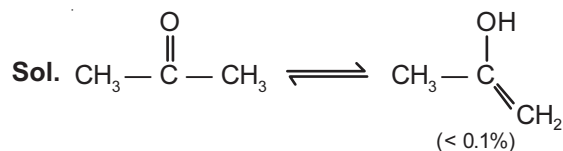
Acetyl acetone in enol form have intramolecular H-bonding, which is absent in acetone.

14. The functions of antihistamine are

   (1) Antiallergic and Analgesic

   (2) Analgesic and antacid

   (3) Antiallergic and antidepressant

   (4) Antacid and antiallergic

Answer (4)

Sol. The functions of antihistamine are antacid and antiallergic.

15. A group 15 element, which is a metal and forms a hydride with strongest reducing power among group 15 hydrides. The element is

   (1) As   (2) P

   (3) Bi   (4) Sb

Answer (3)

Sol. The stability of hydrides decreases from NH_3 to BiH_3 which can be observed from their bond dissociation enthalpy. Consequently, the reducing character of the hydrides increases.

Ammonia is only a mild reducing agent while BiH_3 is the strongest reducing agent amongst all the hydrides.

16. Given below are two statements :

   Statement I : The E° value for Ce^{4+}/Ce^{3+} is +1.74 V.

   Statement II : Ce is more stable in Ce^{4+} state than Ce^{3+} state.

   In the light of the above statements, choose the **most appropriate** answer from the options given below.

   (1) Both **statement I** and **statement II** are correct

   (2) **Statement I** is correct but **statement II** is incorrect

   (3) Both **statement I** and **statement II** are incorrect

   (4) **Statement I** is incorrect but **statement II** is correct

Answer (2)

Sol. $Ce^{4+} \xrightarrow{e^-} Ce^{3+}$ E° = +1.74 V

Positive SRP and higher SRP means greater oxidising power. So, Ce^{4+} wants to reduce to Ce^{3+}. Indicates Ce^{4+} is less stable than Ce^{3+}.

17. Given below are two statements :

   Statement I : Both $CaCl_2 \cdot 6H_2O$ and $MgCl_2 \cdot 8H_2O$ undergo dehydration on heating.

   Statement II : BeO is amphoteric whereas the oxides of other elements in the same group are acidic.

   In the light of the above statements, choose the **correct** answer from the options given below.

   (1) **Statement I** is false but **statement II** is true

   (2) Both **statement I** and **statement II** are true

   (3) Both **statement I** and **statement II** are false

   (4) **Statement I** is true but **statement II** is false

Answer (3)

Sol. $CaCl_2 \cdot 6H_2O$ undergoes dehydration on heating, but $MgCl_2 \cdot 8H_2O$ undergoes hydrolysis on heating.

BeO is amphoteric and other metal oxides of II A group are basic in nature.

18. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**.

   Assertion A : The H – O – H bond angle in water molecule is 104.5°.

   Reason R : The lone pair - lone pair repulsion of electrons is higher than the bond pair - bond pair repulsion.

   In the light of the above statements, choose the **correct** answer from the options given below.

   (1) **A** is false but **R** is true

   (2) **A** is true but **R** is false

   (3) Both **A** and **R** are true, and **R** is the correct explanation of **A**

   (4) Both **A** and **R** are true, but **R** is not the correct explanation of **A**

Answer (3)

Sol.
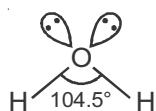
Repulsion between lone pair - lone pair electrons is higher than bond pair - bond pair electrons. Because bond pair electrons are stuck between two nuclei.

19. Among the following, the aromatic compounds are :

(A)

(B)

(C)

(D)
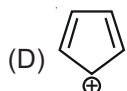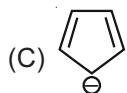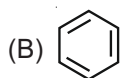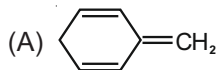
Choose the correct answer from the following options :

(1) (A), (B) and (C) only

(2) (B), (C) and (D) only

(3) (B) and (C) only

(4) (A) and (B) only

Answer (3)

Sol. Conditions for aromaticity are

(i) Planarity

(ii) Complete delocalisation of the π electrons in the ring

(iii) Presence of $(4n + 2)\pi$ electrons in the ring where n is an integer (n = 0, 1, 2....).

Compounds B and C are aromatic

Compound A is non-aromatic

Compound D is anti-aromatic

20.
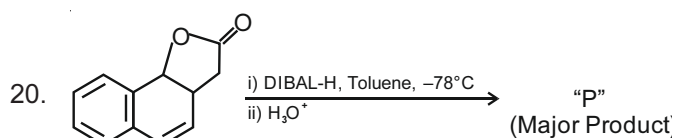
The product "P" in the above reaction is :

(1)

(2)

(3)

(4)
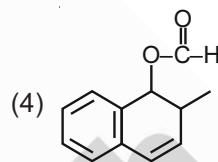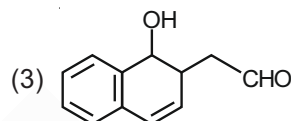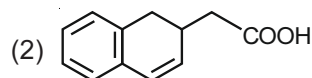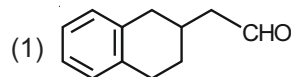
Answer (3)

Sol. DIBAL-H – diisobutylaluminium hydride selectively reduces nitriles and esters to aldehydes.
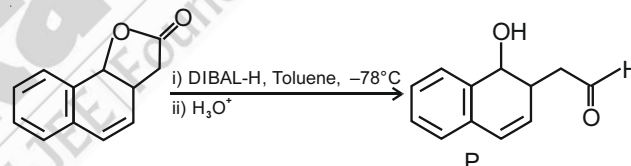
SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

---

1. When light of wavelength 248 nm falls on a metal of threshold energy 3.0 eV, the de-Broglie wavelength of emitted electrons is _____ Å. (Round off to the Nearest Integer).

[Use : $\sqrt{3} = 1.73$, $h = 6.63 \times 10^{-34}$ Js

$m_e = 9.1 \times 10^{-31}$ kg; $c = 3.0 \times 10^8$ ms^{-1} ; 1eV = 1.6 $\times 10^{-19}$J]

Answer (9)

Sol. Incident energy of = Work function of + K.E. of
    photon   metal   photoelectron

$h\nu = h\nu_0 + KE$

$$\frac{6.63 \times 10^{-34}\, Js \times 3 \times 10^8\, ms^{-1}}{248 \times 10^{-9}\, m \times 1.6 \times 10^{-19}\, J\, eV^{-1}} = 3.0 + K.E.$$

K. E. = 2.0 eV

$$\lambda = \frac{h}{\sqrt{2m\, K.E.}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 2 \times 10^{-19} \times 1.6}}$$

$= 8.68 \times 10^{-10}$ m ≈ 9 Å

2. A 6.50 molal solution of KOH (aq.) has a density of 1.89 g cm^{-3}. The molarity of the solution is ___ mol dm^{-3}. (Round off to the Nearest Integer).

[Atomic masses : K : 39.0 u; O : 16.0 u; H : 1.0 u]

Answer (9)

Sol. Molality = $\dfrac{\text{Moles of solute}}{\text{Mass of solvent (in kg)}}$

6.50 molal solution of KOH means

6.50 moles of KOH in 1000 g of water (solvent)

364 g of KOH in 1364 g of solution

Volume of solution = $\dfrac{\text{Mass of solution(g)}}{\text{density of solution (g mL}^{-1})}$

$= \dfrac{1364}{1.89}$

Molarity = $\dfrac{\text{Moles of solute}}{\text{Volume of solution (in L)}} = \dfrac{6.50 \times 1.89}{1364 \times 10^{-3}} = 9.00\,M$

3. The equivalents of ethylene diamine required to replace the neutral ligands from the coordination sphere of the trans-complex of $CoCl_3 \cdot 4NH_3$ is ___ (Round off to the Nearest Integer).

Answer (2)

Sol. $CoCl_3 \cdot 4NH_3 \Rightarrow$ trans-complex means octahedral with one Cl^- out of the coordination sphere i.e., $[Co(NH_3)_4Cl_2]Cl$. NH_3 are neutral ligands. Each ethylene diamine replaces two ammines ligands. So two ethylene diamine are required to replace all neutral monodentate (NH_3) ligands.

4. For the reaction A(g) $\rightleftharpoons$ B(g) at 495 K, $\Delta_r G^\circ = -9.478$ kJ mol^{-1}.

If we start the reaction in a closed container at 495 K with 22 millimoles of A, the amount of B in the equilibrium mixture is _____ millimoles. (Round off to the Nearest Integer).

[R = 8.314 J mol^{-1} K^{-1}; ln 10 = 2.303]

Answer (20)

Sol. $\Delta G^\circ = -2.303\, RT \log K = -RT \ln K$

$$\frac{+9.478 \times 10^3}{8.314 \times 495} = \ln K = 2.303$$

$\ln K = \ln 10$

$K = 10$

$$\begin{array}{ccc} A(g) & \rightleftharpoons & B(g) \\ 22 - x & & x \end{array}$$

$10 = \dfrac{x}{22 - x} \Rightarrow x = 20$

5. A certain element crystallises in a bcc lattice of unit cell edge length 27 Å. If the same element under the same conditions crystallises in the fcc lattice, the edge length of the unit cell in Å will be _____. (Round off to the Nearest Integer.)

[Assume each lattice point has a single atom]

[Assume $\sqrt{3} = 1.73, \sqrt{2} = 1.41$]

Answer (33)

Sol. In BCC the relationship between edge length and radius of an atom is given by.

$4r = \sqrt{3}a$

$4r = \sqrt{3} \times 27$

$4r = 1.73 \times 27\text{Å} = 46.71$ Å

In FCC the relationship between edge length and radius of atom is given by

$4r = \sqrt{2}\,a \Rightarrow \dfrac{46.71}{1.41} = a$ (edge length)

$\Rightarrow a \approx 33$ Å

6. The decomposition of formic acid on gold surface follows first order kinetics. If the rate constant at 300 K is 1.0×10^{-3} s^{-1} and the activation energy $E_a = 11.488$ kJ mol^{-1}, the rate constant at 200 K is _____ $\times 10^{-5}$ s^{-1}. (Round off to the Nearest Integer)

Answer (10)

Sol. $\log \dfrac{k_2}{k_1} = \dfrac{E_a}{2.303R}\left[\dfrac{1}{T_1} - \dfrac{1}{T_2}\right]$

k_1 (at 200 K) = ?

k_2 (at 300 K) = 1×10^{-3} s^{-1}

$\log \dfrac{1 \times 10^{-3}}{k_1} = \dfrac{11.488 \times 10^3}{2.303 \times 8.314}\left[\dfrac{1}{600}\right] = 1$

$\dfrac{1 \times 10^{-3}}{k_1} = 10$

$k_1 = 10 \times 10^{-5}$ s^{-1}

7. AB_2 is 10% dissociated in water to A^{2+} and B^-. The boiling point of a 10.0 molal aqueous solution of AB_2 is _____°C. (Round off to the Nearest Integer)

[Given : Molal elevation constant of water $K_b = 0.5$ K kg mol^{-1} boiling point of pure water = 100°C]

Answer (106)

Sol. $AB_2 \rightleftharpoons A^{2+} + 2B^-$

$1-\alpha \quad \alpha \quad 2\alpha$

$i = 1 + \alpha$

$i = 1.1$

$\Delta T_b = T_s - T° = i \times K_b \times$ molality

$T_s - 100 = 1.1 \times 0.5 \times 10$

$T_s = 105.5$

$\approx 106°C$

8. $2MnO_4^- + bC_2O_4^{2-} + cH^+ \rightarrow x\,Mn^{2+} + y\,CO_2 + z\,H_2O$

If the above equation is balanced with integer coefficients, the value of c is _____.

Anser (16)

Sol. $2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \rightarrow 2Mn^{2+} + 10CO_2 + 8H_2O$
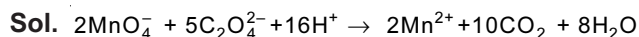
$b = 5$

$c = 16$

$x = 2$

$y = 10$

$z = 8$

9. Two salts A_2X and MX have the same value of solubility product of 4.0×10^{-12}. The ratio of their molar solubilities i.e. $\dfrac{S(A_2X)}{S(MX)} =$ _____. (Round off to the Nearest Integer)

Answer (50)

Sol. $A_2X \rightleftharpoons 2A^+ + X^{2-}$

$\qquad\qquad 2s_1 \quad s_1$

$K_{sp} = 4s_1^3 \Rightarrow s_1 = \sqrt[3]{\dfrac{K_{sp}}{4}} = 10^{-4}$

$MX \rightleftharpoons M^{2+} + X^{2-}$

$\qquad\qquad s_2 \quad s_2$

$K_{sp} = s_2^2 \Rightarrow s_2 = \sqrt{K_{sp}} = 2 \times 10^{-6}$

$s_1 = s(A_2x)$

$s_2 = s(MX)$

$\dfrac{s_1}{s_2} = \dfrac{10^{-4}}{2 \times 10^{-6}} = 50$

10. Complete combustion of 750 g of an organic compound provides 420 g of CO_2 and 210 g of H_2O. The percentage composition of carbon and hydrogen in organic compound is 15.3 and _____respectively. (Round off to the Nearest Integer).

Answer (3)

Sol. Weight of H = $\dfrac{210}{18} \times 2 = 23.333$ g (in 750 g compound)

% of H = $\dfrac{23.333}{750} \times 100$

= 3.111

≈ 3

PART–C : MATHEMATICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. If $y = y(x)$ is the solution of the different equation, $\frac{dy}{dx} + 2y\tan x = \sin x$, $y\left(\frac{\pi}{3}\right) = 0$, then the maximum value of the function $y(x)$ over **R** is equal to :

   (1) $-\frac{15}{4}$ (2) $\frac{1}{8}$

   (3) $\frac{1}{2}$ (4) 8

Answer (2)

Sol. $\frac{dy}{dx} + 2\tan x \cdot y = \sin x$

   I. F. $= e^{\int 2\tan x\, dx} = \sec^2 x$

   $\Rightarrow y \cdot \sec^2 x = \int \sin x \cdot \sec^2 x\, dx$

   $\Rightarrow y\sec^2 x = \sec x + c$

   $\because y\left(\frac{\pi}{3}\right) = 0$

   $\Rightarrow c = -2$

   $\Rightarrow y = \cos x - 2\cos^2 x = \frac{1}{8} - \left(\sqrt{2}\cos x - \frac{1}{2\sqrt{2}}\right)^2$

   Maximum value of $f(x)$ is $\frac{1}{8}$.

2. Let $S_k = \sum_{r=1}^{k} \tan^{-1}\left(\frac{6^r}{2^{2r+1} + 3^{2r+1}}\right)$. Then $\lim_{k\to\infty} S_k$ is equal to :

   (1) $\tan^{-1}(3)$

   (2) $\tan^{-1}\left(\frac{3}{2}\right)$

   (3) $\cot^{-1}\left(\frac{3}{2}\right)$

   (4) $\frac{\pi}{2}$

Answer (3)

Sol. Let $T_k = \tan^{-1}\left(\frac{6^r}{2^{2r+1} + 3^{2r+1}}\right) = \tan^{-1}\left(\frac{\frac{2^r}{3^{r+1}}}{1 + \left(\frac{2}{3}\right)^{2r+1}}\right)$

   $= \tan^{-1}\left(\frac{\left(\frac{2}{3}\right)^r - \left(\frac{2}{3}\right)^{r+1}}{1 + \left(\frac{2}{3}\right)^{2r+1}}\right) = \tan^{-1}\left(\frac{2}{3}\right)^r - \tan^{-1}\left(\frac{2}{3}\right)^{r+1}$

   then $S_k = \sum_{r=1}^{k} T_k = \tan^{-1}\left(\frac{2}{3}\right) - \tan^{-1}\left(\frac{2}{3}\right)^{k+1}$

   $\lim_{k\to\infty} S_k = \tan^{-1}\left(\frac{2}{3}\right)$.

3. If for $a > 0$, the feet of perpendiculars from the points $A(a, -2a, 3)$ and $B(0, 4, 5)$ on the plane $lx + my + nz = 0$ are points $C(0, -a, -1)$ and D respectively, then the length of line segment CD is equal to :

   (1) $\sqrt{41}$ (2) $\sqrt{66}$

   (3) $\sqrt{55}$ (4) $\sqrt{31}$

Answer (2)

Sol. DR's of AC $\propto a, -a, 4$

   So equation of the plane will be $ax - ay + 4z = 0$.

   $\because$ Point C lies on this plane, so $a^2 = 4 \Rightarrow a = 2$

   Equation of plane : $x - y + 2z = 0$.

   Projection of $B(0, 4, 5)$ on this plane is $D(-1, 5, 3)$.

   $CD = \sqrt{66}$

4. Consider three observations a, b and c such that $b = a + c$. If the standard deviation of $a + 2$, $b + 2$, $c + 2$ is d, then which of the following is true?

   (1) $b^2 = a^2 + c^2 + 3d^2$ (2) $b^2 = 3(a^2 + c^2) - 9d^2$

   (3) $b^2 = 3(a^2 + c^2) + 9d^2$ (4) $b^2 = 3(a^2 + c^2 + d^2)$

Answer (2)

Sol. $\because$ Standard deviation of a, b, c is d.

   $d^2 = \frac{a^2 + b^2 + c^2}{3} - \left(\frac{a+b+c}{3}\right)^2$

   $\Rightarrow 9d^2 = 3(a^2 + b^2 + c^2) - 4b^2$

   $\Rightarrow b^2 = 3(a^2 + c^2) - 9d^2$

5. Let P be a plane $lx + my + nz = 0$ containing the line, $\dfrac{1-x}{1} = \dfrac{y+4}{2} = \dfrac{z+2}{3}$. If plane P divides the line segment AB joining points A(−3, −6, 1) and B(2, 4, −3) in ratio k : 1 then the value of k is equal to:

(1) 2   (2) 4

(3) 1.5   (4) 3

Answer (1)

Sol. $L : \dfrac{x-1}{-1} = \dfrac{y+4}{2} = \dfrac{z+2}{3}$

∵ Plane P passes through origin and contains line L, then equation of plane P is

$$\begin{vmatrix} x & y & z \\ -1 & 2 & 3 \\ 1 & -4 & -2 \end{vmatrix} = 0$$

⇒ $8x + y + 2z = 0$

∵ Point $\left(\dfrac{2k-3}{k+1}, \dfrac{4k-6}{k+1}, \dfrac{-3k+1}{k+1}\right)$ lies on plane P,

then $8(2k − 3) + (4k − 6) + 2(−3k + 1) = 0$

⇒ $k = 2$

6. Let a complex number z, $|z| \neq 1$, satisfy $\log_{\frac{1}{\sqrt{2}}}\left(\dfrac{|z|+11}{(|z|-1)^2}\right) \leq 2$. Then, the largest value of $|z|$ is equal to _____.

(1) 8   (2) 7

(3) 6   (4) 5

Answer (2)

Sol. $\log_{\frac{1}{\sqrt{2}}}\left(\dfrac{|z|+11}{|z|^2 - 2|z|+1}\right) \leq 2$

⇒ $\dfrac{|z|+11}{|z|^2 - 2|z|+1} \geq \dfrac{1}{2}$

⇒ $|z|^2 - 2|z| + 1 \leq 2|z| + 22$

⇒ $(|z|-7)(|z|+3) \leq 0 \Rightarrow |z| \leq 7$

7. A pack of cards has one card missing. Two cards are drawn randomly and are found to be spades. The probability that the missing card is not a spade, is :

(1) $\dfrac{52}{867}$ (2) $\dfrac{22}{425}$

(3) $\dfrac{3}{4}$ (4) $\dfrac{39}{50}$

Answer (4)

Sol. Consider the events,

E_1 = missing card is spade

E_2 = missing card is not a spade

A = Two spade cards are drawn

$P(E_1) = \dfrac{1}{4}$ $P(E_2) = \dfrac{3}{4}$

$P\left(\dfrac{A}{E_1}\right) = \dfrac{{}^{12}C_2}{{}^{51}C_2}$ $P\left(\dfrac{A}{E_2}\right) = \dfrac{{}^{13}C_2}{{}^{51}C_2}$

Then $P\left(\dfrac{E_2}{A}\right) = \dfrac{P\left(\dfrac{A}{E_2}\right) \cdot P(E_2)}{P(E_1) \cdot P\left(\dfrac{A}{E_1}\right) + P(E_2) \cdot P\left(\dfrac{A}{E_2}\right)} =$

$\dfrac{\dfrac{3}{4} \cdot \dfrac{{}^{13}C_2}{{}^{51}C_2}}{\dfrac{1}{4} \cdot \dfrac{{}^{12}C_2}{{}^{51}C_2} + \dfrac{3}{4} \cdot \dfrac{{}^{13}C_2}{{}^{51}C_2}}$

$= \dfrac{3 \cdot {}^{13}C_2}{{}^{12}C_2 + 3 \cdot {}^{13}C_2} = \dfrac{3.78}{66 + 3.78} = \dfrac{39}{50}$

8. The number of elements in the set {x∈R : (| x | − 3) | x + 4 | = 6} is equal to :

(1) 4   (2) 2

(3) 1   (4) 3

Answer (2)

Sol. If $x \in (-\infty, -4)$

$(x + 3)(x + 4) = 6 \Rightarrow x^2 + 7x + 6 = 0 \Rightarrow x = -6$ only

If $x \in (-4, 0)$

$(x + 3)(x + 4) = -6 \Rightarrow x^2 + 7x + 18 = 0 \Rightarrow$ No solution.

If $x \in (0, \infty)$

$(x − 3)(x + 4) = 6 \Rightarrow x^2 + x - 18 = 0 \Rightarrow x = \dfrac{\sqrt{73}-1}{2}$

only.

So the given set contains only 2 elements.

9. Which of the following Boolean expression is a tautology?

(1) $(p \wedge q) \to (p \to q)$ (2) $(p \wedge q) \wedge (p \to q)$

(3) $(p \wedge q) \vee (p \vee q)$ (4) $(p \wedge q) \vee (p \to q)$

Answer (1)

Sol. ∵ $p \to q = {\sim} p \vee q$

so $(p \wedge q) \to (p \to q) = {\sim}(p \wedge q) \vee (p \to q)$

$= ({\sim} p \vee {\sim} q) \vee ({\sim} p \vee q)$

$= {\sim} p \vee q \vee {\sim} q$

is a tautology

10. If n is the number of irrational terms in the expansion

of $\left(3^{\frac{1}{4}} + 5^{\frac{1}{8}}\right)^{60}$, then (n – 1) is divisible by :

(1) 7

(2) 26

(3) 8

(4) 30

Answer (2)

Sol. $T_{r+1} = {}^{60}C_r \cdot \left(3^{\frac{1}{4}}\right)^{60-r} \cdot \left(5^{\frac{1}{8}}\right)^r$

$= {}^{60}C_r \cdot 3^{15-\frac{r}{4}} \cdot 5^{\frac{r}{8}}$

Term will be rational is r is divisible by 8.

r = 0, 8, 16, 24, 32, 40, 48, 56

Total number of irrational terms = n = 61 – 8 = 53 hence n – 1 is divisible by 26.

11. Let the function $f : \mathbf{R} \to \mathbf{R}$ and $g : \mathbf{R} \to \mathbf{R}$ be defined as:

$f(x) = \begin{cases} x+2, & x < 0 \\ x^2, & x \geq 0 \end{cases}$ and $g(x) = \begin{cases} x^3, & x < 1 \\ 3x - 2, & x \geq 1 \end{cases}$

Then, the number of points in **R** where (fog) (x) is NOT differentiable is equal to :

(1) 3

(2) 0

(3) 2

(4) 1

Answer (4)

Sol. $\because$ g(x) is always differentiable and f(x) is non-differentiable at x = 0

Clearly f(g(x)) is non-differentiable when g(x) = 0 (i.e. x = 0)

12. The range of $a \in \mathbf{R}$ for which the function

$f(x) = (4a - 3)(x + \log_e 5) + 2(a - 7) \cot\left(\frac{x}{2}\right)\sin^2\left(\frac{x}{2}\right)$, $x \neq 2n\pi$, $n \in \mathbf{N}$ has critical points, is :

(1) (–3, 1)

(2) [1, ∞]

(3) (–∞, –1)

(4) $\left[-\frac{4}{3}, 2\right]$

Answer (4)

Sol. $f(x) = (4a - 3)(x + \ln 5) + (a - 7)\sin x$

$\because$ $f(x)$ is always continuous and differentiable in its domain,

then $f(x)$ has critical points if

$f'(x) = 0$ has solutions

$(4a - 3) + (a - 7)\cos x = 0$

$\Rightarrow \cos x = \frac{3 - 4a}{a - 7}$ has solutions

$\because -1 \leq \frac{3 - 4a}{a - 7} \leq 1$

$\Rightarrow a \in \left[-\frac{4}{3}, 2\right]$

13. Let [x] denote greatest integer less than or equal to x. If for $n \in N$,

$\left(1 - x + x^3\right)^n = \sum_{j=0}^{3n} a_j x^j$, then

$\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4 \sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1}$ is equal to :

(1) 1

(2) 2^{n-1}

(3) n

(4) 2

Answer (1)

Sol. $f(x) = (1 - x + x^3)^n = \sum_{j=0}^{3n} a_j x^j$

$\because \sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} = \frac{1}{2}(f(1) + f(-1)) = \frac{1}{2}(1 + 1) = 1$

$\because \sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1} = \frac{1}{2}(f(1) - f(-1)) = 0$

Clearly $= \sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4 \sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1} = 1$

14. Let a vector $\alpha \hat{i} + \beta \hat{j}$ be obtained by rotating the vector $\sqrt{3}\hat{i} + \hat{j}$ by an angle 45° about the origin in counterclockwise direction in the first quadrant. Then the area of triangle having vertices (α, β), $(0, \beta)$ and $(0, 0)$ is equal to :

(1) $\frac{1}{2}$

(2) $2\sqrt{2}$

(3) 1

(4) $\frac{1}{\sqrt{2}}$

Answer (1)

Sol. $\because \ |\alpha\hat{i} + \beta\hat{j}| = |\sqrt{3}\,\hat{i} + \hat{j}| \Rightarrow \alpha^2 + \beta^2 = 4 \qquad \ldots(i)$

Also $\dfrac{\sqrt{3}\alpha + \beta}{2 \cdot 2} = \dfrac{1}{\sqrt{2}} \ \Rightarrow \sqrt{3}\alpha + \beta = 2\sqrt{2} \qquad \ldots(ii)$

$\because \ \alpha, \beta > 0$, then from (i) and (ii)

$\quad \alpha = \dfrac{\sqrt{3}-1}{\sqrt{2}}$ and $\beta = \dfrac{\sqrt{3}+1}{\sqrt{2}}$

Area of required triangle $= \dfrac{1}{2}\alpha\beta = \dfrac{1}{2}\left(\dfrac{\sqrt{3}-1}{\sqrt{2}}\right)\left(\dfrac{\sqrt{3}+1}{\sqrt{2}}\right)$

$\qquad\qquad = \dfrac{1}{2}$

15. Let the position vectors of two points P and Q be $3\hat{i} - \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} - 4\hat{k}$, respectively. Let R and S be two points such that the direction ratios of lines PR and QS are (4, −1, 2) and (−2, 1, −2), respectively. Let lines PR and QS intersect at T. If the vector $\overrightarrow{TA}$ is perpendicular to both $\overrightarrow{PR}$ and $\overrightarrow{QS}$ and the length of vector $\overrightarrow{TA}$ is $\sqrt{5}$ units, then the modulus of a position vector of A is :

(1) $\sqrt{482}$ (2) $\sqrt{227}$

(3) $\sqrt{5}$ (4) $\sqrt{171}$

Answer (4)

Sol. Equation of $PR : \dfrac{x-3}{4} = \dfrac{y+1}{-1} = \dfrac{z-2}{2}$

Equation of $QS : \dfrac{x-1}{-2} = \dfrac{y-2}{1} = \dfrac{z+4}{-2}$

Their point of intersection of PR and QS is T(11, −3, 6)

$\overrightarrow{PQ} \times \overrightarrow{QS} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 2 \\ -2 & 1 & -2 \end{vmatrix} = 2\hat{j} + \hat{k}$

Clearly $\overrightarrow{TA} = \pm(2\hat{j} + \hat{k})$

Position vector of $A = 11\hat{i} - 3\hat{j} + 6\hat{k} \pm (2\hat{j} + \hat{k})$

$11\hat{i} - \hat{j} + 7\hat{k}$ or $11\hat{i} - 5\hat{j} + 5\hat{k}$

Modulus of P. V. of $A = \sqrt{171}$

16. If the three normals drawn to the parabola, $y^2 = 2x$ pass through the point (a, 0) a ≠ 0, then 'a' must be greater than

(1) $-\dfrac{1}{2}$ (2) $\dfrac{1}{2}$

(3) 1   (4) −1

Answer (3)

Sol. Three normals can be drawn to the parabola $y^2 = 4bx$ from (a, 0) if a > 2b.

So, a > 1

17. If for $x \in \left(0, \dfrac{\pi}{2}\right), \log_{10}\sin x + \log_{10}\cos x = -1$ and $\log_{10}(\sin x + \cos x) = \dfrac{1}{2}(\log_{10} n - 1), n > 0$, then the value of n is equal to

(1) 9   (2) 16

(3) 12   (4) 20

Answer (3)

Sol. Given, $\log_{10}(\sin x \cos x) = -1$

$\Rightarrow \sin 2x = \dfrac{2}{10} \Rightarrow 1 + \sin 2x = \dfrac{6}{5}$

Also $\log_{10}(\sin x + \cos x) = \dfrac{1}{2}(\log_{10} n - 1)$

$\Rightarrow \dfrac{1}{2}\log_{10}(1 + \sin 2x) = \dfrac{1}{2}(\log_{10} n - \log_{10} 10)$

$\Rightarrow \dfrac{6}{5} = \dfrac{n}{10} \Rightarrow n = 12$

18. The number of roots of the equation, $(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$ in the interval [0, π] is equal to

(1) 4   (2) 2

(3) 8   (4) 3

Answer (1)

Sol. $81^{\sin^2 x} + 81^{1-\sin^2 x} = 30$

Let $81^{\sin^2 x} = t$

$\Rightarrow t + \dfrac{81}{t} = 30 \Rightarrow t^2 - 30t + 81 = 0$

$\qquad\qquad\qquad \Rightarrow t = 3 \text{ or } 27$

i.e. $81^{\sin^2 x} = 3$ or 3^3

$\Rightarrow 3^{4\sin^2 x} = 3^1$ or 3^3

$\Rightarrow \sin^2 x = \dfrac{1}{4}$ or $\dfrac{3}{4}$

$\Rightarrow \sin x = \pm\dfrac{1}{2}, \pm\dfrac{\sqrt{3}}{2}$

If $x \in (0, \pi)$ then $\sin x = \dfrac{1}{2}$ or $\dfrac{\sqrt{3}}{2}$ only

Hence 4 solutions.

19. Let $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}, i = \sqrt{-1}$. Then, the system of linear equations $A^8 \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$ has

(1) Exactly two solutions

(2) No solution

(3) A unique solution

(4) Infinitely many solutions

Answer (2)

Sol. $A^2 = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}\begin{bmatrix} i & -i \\ -i & i \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}$

$A^4 = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$

$A^8 = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}\begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} = \begin{bmatrix} 128 & -128 \\ -128 & 128 \end{bmatrix}$

Given $A^8 = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$

$\begin{bmatrix} 128 & -128 \\ -128 & 128 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$

$\Rightarrow$ $128(x - y) = 8$ and $-128(x - y) = 64$

$\Rightarrow$ $x - y = \dfrac{1}{16}$ and $x - y = -\dfrac{1}{2}$

Which cannot be equal on same time

Hence no solution.

20. The locus of the mid-points of the chord of the circle, $x^2 + y^2 = 25$ which is tangent to the hyperbola, $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$ is

(1) $(x^2 + y^2)^2 - 16x^2 + 9y^2 = 0$

(2) $(x^2 + y^2)^2 - 9x^2 + 144y^2 = 0$

(3) $(x^2 + y^2)^2 - 9x^2 - 16y^2 = 0$

(4) $(x^2 + y^2)^2 - 9x^2 + 16y^2 = 0$

Answer (4)

Sol. Let mid-point be (h, k)

$\therefore$ Chord of circle is $hx + ky = h^2 + k^2$

$\Rightarrow$ $y = \dfrac{-h}{k}x + \left(\dfrac{h^2 + k^2}{k}\right)$ ...(i)

$\Rightarrow$ Tangent to hyperbola (in slope form)

$y = mx \pm \sqrt{9m^2 - 16}$ ...(ii)

Comparing (i) and (ii) we get,

$\left(\dfrac{h^2 + k^2}{k}\right) = 9\left(\dfrac{h^2}{k^2}\right) - 16$

$\Rightarrow$ $(h^2 + k^2)^2 = 9h^2 - 16k^2$

$\Rightarrow$ $(x^2 + y^2)^2 - 9x^2 + 16y^2 = 0$

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. The total number of 3 × 3 matrices A having entries from the set {0, 1, 2, 3} such that the sum of all the diagonal entries of AA^T is 9, is equal to _____.

Answer (766)

Sol. Let matrix be $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

Then $AA^T = \begin{bmatrix} a^2+b^2+c^2 & - & - \\ - & d^2+e^2+f^2 & - \\ - & - & g^2+h^2+i^2 \end{bmatrix}$

$\therefore$ $a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 9$

Case-I 8 zeroes and one entry is 3 = 9 cases

Case-II Two 2's, one 1's and 6 zeroes

$= \dfrac{9!}{6!2!1!} = 252$ cases

Case-III One 2's, five is and three zero

$= \dfrac{9!}{5!3!} = 504$ cases

Case-IV Nine ones = 1 case

$\therefore$ Total cases = 9 + 252 + 504 + 1 = 766

2. Let $f : (0, 2) \rightarrow R$ be defined as $f(x) = \log_2\left(1 + \tan\left(\dfrac{\pi x}{4}\right)\right)$.

Then, $\lim\limits_{n\to\infty} \dfrac{2}{n}\left(f\left(\dfrac{1}{n}\right) + f\left(\dfrac{2}{n}\right) + ... + f(1)\right)$ is equal to _____.

Answer (01)

Sol. $\lim\limits_{n\to\infty}\dfrac{2}{n}\left(f\left(\dfrac{1}{n}\right)+f\left(\dfrac{2}{n}\right)+....+f\left(\dfrac{n}{n}\right)\right)$

$\Rightarrow \lim\limits_{n\to\infty}\dfrac{2}{n}\sum\limits_{r=1}^{n}\log_2\left(1+\tan\dfrac{\pi r}{4n}\right)$

$\Rightarrow I=2\int\limits_{0}^{1}\log_2\left(1+\tan\dfrac{\pi x}{4}\right)dx$

Using $\int\limits_{a}^{b}f(x)dx=\int\limits_{a}^{b}f(a+b-x)dx$ we get,

$I=2\int\limits_{0}^{1}\log_2\left(1+\tan\dfrac{\pi}{4}(1-x)\right)dx$

$\Rightarrow 2I=2\int\limits_{0}^{1}\log_2\left(\left(1+\tan\dfrac{\pi x}{4}\right)\left(1+\tan\dfrac{\pi(1-x)}{4}\right)\right)dx$

$\Rightarrow 2I=2\int\limits_{0}^{1}\log_2^2\,dx$

$\Rightarrow I=\int\limits_{0}^{1}dx=1$

3. Let ABCD be a square of side of unit length. Let a circle C_1 centered at A with unit radius is drawn. Another circle C_2 which touches C_1 and the lines AD and AB are tangent to it, is also drawn. Let a tangent line from the point C to the circle C_2 meet the side AB at E. If the length of EB is $\alpha+\sqrt{3}\beta$, where α, β are integers, then $\alpha+\beta$ is equal to _____.

Answer (1)

Sol. Let the centre of C_2 be O.
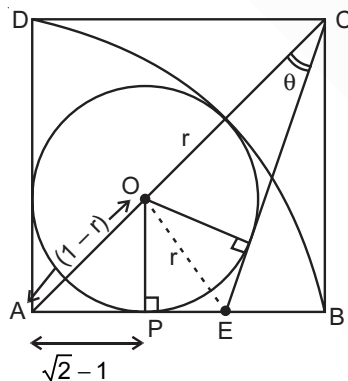
$\because \quad OC=\sqrt{2}-(1-r)$

where r is the radius of C_2.

$\because \quad 1-r=\sqrt{r^2+r^2}$

$\Rightarrow \quad r=\sqrt{2}-1$

hence $OC=\sqrt{2}-1+r=2r$

$\because \quad \sin\theta=\dfrac{r}{2r}=\dfrac{1}{2}\Rightarrow\theta=30°$

then $\angle BCE=15°$

So, $EB=\tan 15°=2-\sqrt{3}$

4. If the normal to the curve $y(x)=\int\limits_{0}^{x}(2t^2-15t+10)dt$ at a point (a, b) is parallel to the line $x+3y=-5$, $a>1$, then the value of $|a+6b|$ is equal to _____.

Answer (406)

Sol. $y(x)=\int\limits_{0}^{x}(2t^2-15t+10)dx$

by Leibnitz's rule

$y'(x)=2x^2-15x+10$ (given $m_N=3$)

$\Rightarrow 2x^2-15x+10=3 \Rightarrow x=\dfrac{1}{2}$ or 7 (but $a>1$)

$\therefore \quad a=7$

$b=y(7)=\int\limits_{0}^{7}(2t^2-15t+10)dt$

$=\left(\dfrac{2t^3}{3}-\dfrac{15t^2}{2}+10t\right)\Big|_{0}^{7}=\dfrac{686}{3}-\dfrac{735}{2}+70$

$=\dfrac{1372-2205+420}{6}$

$|a+6b|=|7-413|=406$

5. Consider an arithmetic series and a geometric series having four initial terms from the set {11, 8, 21, 16, 26, 32, 4}. If the last terms of these series are the maximum possible four digit numbers, then the number of common terms in these two series is equal to _____.

Answer (03)

Sol. G.P. from the set will be 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192 .....

and A.P. from the set will be 11, 16, 21, 26 .....

Common terms will be the terms of G.P. having unit digit 1 or 6
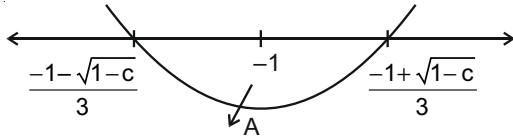
i.e. common terms 16, 256, 409

6. Let the curve $y=y(x)$ be the solution of the differential equation, $\dfrac{dy}{dx}=2(x+1)$. If the numerical value of area bounded by the curve $y=y(x)$ and x-axis is $\dfrac{4\sqrt{8}}{3}$, then the value of $y(1)$ is equal to _____.

Answer (02)

Sol. $\dfrac{dy}{dx} = 2x + 2 \Rightarrow y = x^2 + 2x + c$ ...(1)

Represents parabola with vertex at $x = -1$

Given area $\dfrac{4\sqrt{8}}{3} = A$

∴ Shifting origin to $(-1, 0)$ won't change the area

Hence equation (1) becomes $y = x^2 - 1 + c$

∴ $A = 2\displaystyle\sum_{0}^{\sqrt{1-c}}(x^2 + (c-1))dx = \dfrac{4\sqrt{8}}{3}$

$\Rightarrow \left.\dfrac{x^3}{3} + (c-1)x\right|_{0}^{\sqrt{1-c}} = \dfrac{2\sqrt{8}}{3}$

$\Rightarrow \sqrt{1-c}\,(1 - c + 3(-3)) = 2\sqrt{p}$

$\Rightarrow \sqrt{1-c}\,(c-1) = \sqrt{8} \Rightarrow c = -1$

Hence $y = x^2 + 2x - 1 \Rightarrow y(1) = 2$

7. Let $f : R \to R$ be a continuous function such that

$f(x) + f(x+1) = 2$, for $x \in R$. If $I_1 = \displaystyle\int_{0}^{8} f(x)dx$ and

$I_2 = \displaystyle\int_{-1}^{3} f(x)dx$, then the value of $I_1 + 2I_2$ is equal to

_____.

Answer (16)

Sol. $\Rightarrow$ $\begin{array}{c} f(x) + f(x+1) = 2 \\ \underline{f(x+1) + f(x+2) = 2} \\ f(x) - f(x+2) = 0 \end{array} \Rightarrow f(x)$ has fundamental

period = 2

∴ $f(x) = 2 - f(x+1)$

$\Rightarrow \displaystyle\int_{0}^{2} f(x)dx = \int_{0}^{2} 2dx - \int_{0}^{2} f(x+1)dx$

Now, $I = \displaystyle\int_{0}^{2} f(x+1)dx$

Put $x + 1 = t$

$dx = dt$

$\Rightarrow I = \displaystyle\int_{1}^{3} f(t)dt$

$\Rightarrow I = \displaystyle\int_{0}^{2} f(x)dx = \int_{0}^{2} f(x+1)dx = \int_{0}^{2} dx = 2 = I$

by periodicity $I_1 = 4I$ and $I_2 = 2I$

$\Rightarrow I_1 + 2I_2 = 8I = 16$

8. Let $P = \begin{bmatrix} -30 & 20 & 56 \\ 90 & 140 & 112 \\ 120 & 60 & 14 \end{bmatrix}$ and

$A = \begin{bmatrix} 2 & 7 & \omega^2 \\ -1 & -\omega & 1 \\ 0 & -\omega & -\omega+1 \end{bmatrix}$ where $\omega = \dfrac{-1+i\sqrt{3}}{3}$, and I_3

be the identity matrix of order 3. If the determinant of the matrix $(P^{-1}AP - I_3)^2$ is $\alpha\omega^2$, then the value α is equal to _____.

Answer (36)

Sol. ∵ $P^{-1}AP - I_3 = P^{-1}AP - P^{-1}P = P^{-1}(A-I)P$

$\Rightarrow |P^{-1}AP - I_3| = |P^{-1}||A-I||P| = |A-I|$

∵ $A - I = \begin{vmatrix} 1 & 7 & \omega^2 \\ -1 & \omega^2 & 1 \\ 0 & -\omega & -\omega \end{vmatrix}$

$\Rightarrow |A - I| = -6\omega$

$\Rightarrow |P^{-1}AP - I_3|^2 = (-6\omega)^2 = 36\omega^2$

9. Let z and w be two complex numbers such that

$w = z\bar{z} - 2z + 2, \left|\dfrac{z+i}{z-3i}\right| = 1$ and Re(w) has minimum

value. Then, the minimum value of $n \in N$ for which w^n is real, is equal to

Answer (04)

Sol. Let $z = x + iy$

$\Rightarrow w = x^2 + y^2 - 2x - 2iy + 2$

$\Rightarrow Re(w) = (x-1)^2 + y^2 + 1$ ...(i)

Also $|z + i| = |z - 3i|$

$(y+1)^2 = (y-3)^2$

$\Rightarrow 2y + 1 = -6y + 9$

$\Rightarrow y = 1$ ...(ii)

by (i) and (ii)

$Re(w)_{min} \Rightarrow x = 1$ and $y = 1$

$\Rightarrow w = 1 + i$

$(1 + i)^n = $ real $\Rightarrow n_{min} = 4$

10. If $\lim\limits_{x \to 0} \dfrac{ae^x - b\cos x + ce^{-x}}{x \sin x} = 2$, then a + b + c is equal to _____.

Answer (04)

Sol. Put x = 0 we get N_r = a – b + c = 0   ...(i)

(for indeterminancy to be present)

$\therefore \quad \lim\limits_{x \to 0} \dfrac{ae^x - b\cos x + ce^{-x}}{x^2} = 2 \quad \left(\lim\limits_{x \to 0} \dfrac{\sin x}{x} = 1 \right)$

using L' Hospital rule we get

$\lim\limits_{x \to 0} \dfrac{ae^x + b\sin x - cex^{-x}}{2x} = 2$

Put x = 0 $\Rightarrow$ a – c = 0   ...(ii)

Again by L' hospital rule we get

$\lim\limits_{x \to 0} \dfrac{ae^x + b\cos x + ce^{-x}}{2} = 2$

$\Rightarrow$ a + b + c = 4   ...(iii)

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