17/03/2021
Morning

Aakash

Medical | IIT-JEE | Foundations

Regd. Office : Aakash Tower, 8, Pusa Road, New Delhi-110005. Ph.: 011-47623456

Time : 3 hrs. M.M. : 300

Answers & Solutions

for

JEE (MAIN)-2021 (Online) Phase-2

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS :

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part has two sections.

(i) Section-I : This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) Section-II : This section contains 10 questions. In Section-II, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and there is no negative marking for wrong answer.

PART–A : PHYSICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Which level of the single ionized carbon has the same energy as the ground state energy of hydrogen atom?

(1) 4 (2) 8

(3) 1 (4) 6

Answer (4)

Sol. $\because \quad E = E_0 \times \frac{Z^2}{n^2}$

$\Rightarrow \quad E_0 = E_0 \times \frac{Z^2}{n^2}$

$\Rightarrow \quad n = z = 6$

2. The vernier scale used for measurement has a positive zero error of 0.2 mm. If while taking a measurement it was noted that '0' on the vernier scale lies between 8.5 cm and 8.6 cm, vernier coincidence is 6, then the correct value of measurement is _______ cm. (least count = 0.01 cm)

(1) 8.54 cm (2) 8.36 cm

(3) 8.56 cm (4) 8.58 cm

Answer (1)

Sol. e = +0.2 mm = + 0.02 cm

Measured value = 8.5 + 6 × LC

= 8.5 + 6 × 0.01

= 8.56 cm

$\therefore$ Correct measured value = 8.56 – 0.02

= 8.54 cm

3. Two ideal polyatomic gases at temperatures T_1 and T_2 are mixed so that there is no loss of energy. If F_1 and F_2, m_1 and m_2, n_1 and n_2 be the degrees of freedom, masses, number of molecules of the first and second gas respectively, the temperature of mixture of these two gases is:

(1) $\frac{n_1F_1T_1 + n_2F_2T_2}{F_1 + F_2}$ (2) $\frac{n_1T_1 + n_2T_2}{n_1 + n_2}$

(3) $\frac{n_1F_1T_1 + n_2F_2T_2}{n_1 + n_2}$ (4) $\frac{n_1F_1T_1 + n_2F_2T_2}{n_1F_1 + n_2F_2}$

Answer (4)

Sol. $\because \quad n_1F_1(T_1 - T) = n_2F_2(T - T_2)$

$\Rightarrow \quad T = \frac{n_1F_1T_1 + n_2F_2T_2}{n_1F_1 + n_2F_2}$

4. Two identical metal wires of thermal conductivities K_1 and K_2 respectively are connected in series. The effective thermal conductivity of the combination is:

(1) $\frac{2K_1K_2}{K_1 + K_2}$ (2) $\frac{K_1 + K_2}{2K_1K_2}$

(3) $\frac{K_1 + K_2}{K_1K_2}$ (4) $\frac{K_1K_2}{K_1 + K_2}$

Answer (1)

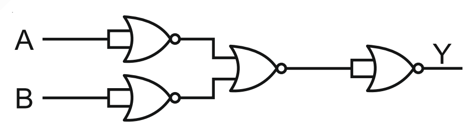
Sol. $\because \quad R_{eq} = R_1 + R_2$

$\Rightarrow \quad \frac{2l}{K_{eq} \times A} = \frac{l}{K_1A} + \frac{l}{K_2A}$

$\Rightarrow \quad \frac{2}{K_{eq}} = \frac{K_1 + K_2}{K_1K_2}$

$\Rightarrow \quad K_{eq} = \frac{2K_1K_2}{K_1 + K_2}$

5. The output of the given combination gates represents:



(1) AND Gate (2) NOR Gate

(3) NAND Gate (4) XOR Gate

Answer (3)

Sol. $\left(\overline{\overline{A} + \overline{B}}\right) = A \cdot B$

$\therefore \quad Y = \overline{A \cdot B} = \text{NAND gate}$

6. When two soap bubbles of radii a and b (b > a) coalesce, the radius of curvature of common surface is

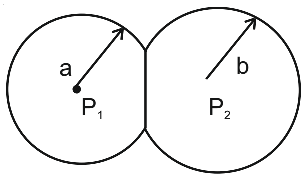
(1) $\frac{ab}{b - a}$ (2) $\frac{ab}{a + b}$

(3) $\frac{b - a}{ab}$ (4) $\frac{a + b}{ab}$

Answer (1)

Sol. $P_1 = P_0 + \frac{4T}{a}$

$P_2 = P_0 + \frac{4T}{b}$



$\therefore \quad P_1 - P_2 = \frac{4T}{r_c}$

$\Rightarrow \quad \frac{1}{a} - \frac{1}{b} = \frac{1}{r_c}$

$\Rightarrow \quad r_c = \frac{ab}{(b-a)}$

7. In an electron is moving in the n^{th} orbit of the hydrogen atom, then its velocity (v_n) for the n^{th} orbit is given as :

(1) $v_n \propto \frac{1}{n}$ (2) $v_n \propto n$

(3) $v_n \propto n^2$ (4) $v_n \propto \frac{1}{n^2}$

Answer (1)

Sol. $\because \quad v_n = v_0\left(\frac{Z}{n}\right)$

$\Rightarrow \quad v_n \propto \frac{1}{n}$

8. An AC current is given by $I = I_1 \sin\omega t + I_2 \cos\omega t$. A hot wire ammeter will give a reading

(1) $\sqrt{\frac{I_1^2 + I_2^2}{2}}$ (2) $\frac{I_1 + I_2}{\sqrt{2}}$

(3) $\frac{I_1 + I_2}{2\sqrt{2}}$ (4) $\sqrt{\frac{I_1^2 - I_2^2}{2}}$

Answer (1)

Sol. $I_{rms} = \sqrt{\frac{\int I^2\, dt}{T}}$

$$(I_{rms})^2 = \frac{\int_0^T [I_1^2 \sin^2 \omega t + I_2 \cos^2 \omega t + 2I_1I_2 \sin(\omega t) \times \cos(\omega t)]\, dt}{T}$$

$\Rightarrow \quad I_{rms} = \sqrt{\frac{I_1^2}{2} + \frac{I_2^2}{2} + 0} = \sqrt{\frac{I_1^2 + I_2^2}{2}}$

9. A current of 10 A exists in a wire of cross-sectional area of 5 mm^2 with a drift velocity of 2×10^{-3} ms^{-1}. The number of free electrons in each cubic meter of the wire is ________

(1) 2×10^{25} (2) 1×10^{23}

(3) 625×10^{25} (4) 2×10^{6}

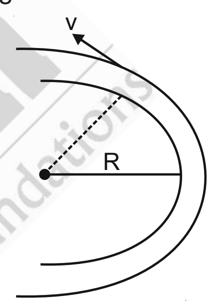
Answer (3)

Sol. $\because \quad V_d = \frac{I}{Ane}$

$\Rightarrow \quad n = \frac{I}{AeV_d}$

$= \frac{10}{5\times10^{-6} \times 1.6\times10^{-19} \times 2\times10^{-3}} = 625\times10^{25}$

10. A modern grand-prix racing car of mass m is travelling on a flat track in a circular arc of radius R with a speed v. If the coefficient of static friction between the tyres and the track is μ_s, then the magnitude of negative lift F_L acting downwards on the car is: (Assume forces on the four tyres are identical and g = acceleration due to gravity)

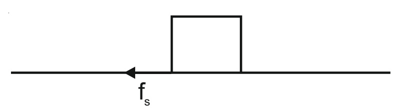


(1) $m\left(\frac{v^2}{\mu_s R} + g\right)$ (2) $m\left(\frac{v^2}{\mu_s R} - g\right)$

(3) $-m\left(g + \frac{v^2}{\mu_s R}\right)$ (4) $m\left(g - \frac{v^2}{\mu_s R}\right)$

Answer (2)

Sol.



$f_s = \frac{mv^2}{R}$

In limiting condition

$\mu_s N = \frac{mv^2}{R}$

$\Rightarrow \quad N = \frac{mv^2}{\mu_s R}$

$\therefore \quad F_L = mg - N = mg - \frac{mv^2}{\mu_s R}$

$-F_L = -m\left(g - \frac{v^2}{\mu_s R}\right)$

11. A Carnot's engine working between 400 K and 800 K has a work output of 1200 J per cycle. The amount of heat energy supplied to the engine from the source in each cycle is:

(1) 1800 J (2) 3200 J

(3) 1600 J (4) 2400 J

Answer (4)

Sol. $\frac{W}{Q}=\left(1-\frac{T_1}{T_2}\right)$

$Q = 2400$ J

12. An electron of mass m and a photon have same energy E. The ratio of wavelength of electron to that of photon is: (c being the velocity of light)

(1) $c(2mE)^{\frac{1}{2}}$ (2) $\left(\frac{E}{2m}\right)^{\frac{1}{2}}$

(3) $\frac{1}{c}\left(\frac{E}{2m}\right)^{\frac{1}{2}}$ (4) $\frac{1}{c}\left(\frac{2m}{E}\right)^{\frac{1}{2}}$

Answer (3)

Sol. $\lambda_1=\frac{h}{\sqrt{2mE}}$

$\lambda_2=\frac{hc}{E}$

$\frac{\lambda_1}{\lambda_2}=\frac{1}{c}\sqrt{\frac{E}{2m}}$

13. A polyatomic ideal gas has 24 vibrational modes. What is the value of γ?

(1) 1.30 (2) 10.3

(3) 1.37 (4) 1.03

Answer (4)

Sol. $\gamma=1+\frac{2}{f}$

$f = 2 \times 24 + 3 + 3 = 54$

$\gamma=1+\frac{2}{54}=1.03$

14. A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t seconds, the total distance travelled is:

(1) $\frac{2\alpha\beta}{(\alpha+\beta)}t^2$ (2) $\frac{\alpha\beta}{2(\alpha+\beta)}t^2$

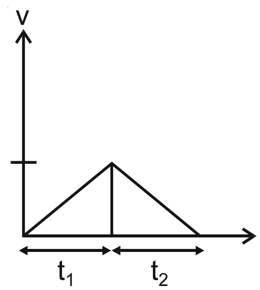
(3) $\frac{\alpha\beta}{4(\alpha+\beta)}t^2$ (4) $\frac{4\alpha\beta}{(\alpha+\beta)}t^2$

Answer (2)

Sol. $t_1 + t_2 = t$

$\alpha t_1 = \beta t_2$

S = Area under v-t curve



15. The thickness at the centre of a plano convex lens is 3 mm and the diameter is 6 cm. If the speed of light in the material of the lens is 2×10^8 ms^{-1}. The focal length of the lens is _____.

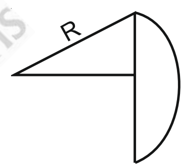
(1) 1.5 cm (2) 0.30 cm

(3) 15 cm (4) 30 cm

Answer (4)

Sol. $\mu = 1.5$

$R^2=(R-t)^2+\left(\frac{d}{2}\right)^2$



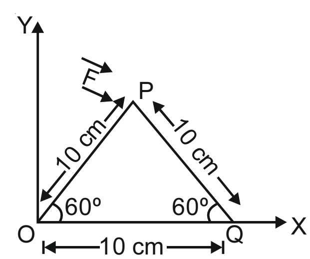
$2Rt=\frac{d^2}{4}$

$R=\frac{d^2}{8t}=\frac{36}{8\times0.3}=15$ cm

$\frac{1}{f}=\frac{(1.5-1)}{15}$

$f = 30$ cm

16. A triangular plate is shown. A force $\vec{F}=4\hat{i}-3\hat{j}$ is applied at point P. The torque at point P with respect to point 'O' and 'Q' are :



(1) $15+20\sqrt{3}, 15-20\sqrt{3}$

(2) $-15-20\sqrt{3}, 15-20\sqrt{3}$

(3) $-15+20\sqrt{3}, 15+20\sqrt{3}$

(4) $15-20\sqrt{3}, 15+20\sqrt{3}$

Answer (2)

Sol. $\vec{\tau} = \vec{r} \times \vec{f}$

$$\vec{r}_{OP} = \left(5\hat{i} + \frac{5\sqrt{3}}{2}\hat{j}\right)$$

$$\vec{\tau} = 5\left(\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right) \times \left(4\hat{i} - 3\hat{j}\right)$$

$$= 5\left(-3\hat{k} - 4\sqrt{3}\hat{k}\right)$$

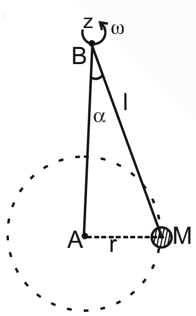
$$= \left(-15 - 20\sqrt{3}\right)\hat{k}$$

$$\overrightarrow{r_{QP}} = \left(-5\hat{i} + 5\sqrt{3}\hat{j}\right)$$

$$\vec{\tau} = \vec{r}_{QP} \times \vec{f}$$

$$= \left(15 - 20\sqrt{3}\right)\hat{k}$$

17. A mass M hangs on a massless rod of length l which rotates at a constant angular frequency. The mass M moves with steady speed in a circular path of constant radius. Assume that the system is in steady circular motion with constant angular velocity ω. The angular momentum of M about point A is L_A which lies in the positive z direction and the angular momentum of M about point B is L_B. The correct statement for this system is :



(1) L_A is constant, both in magnitude and direction

(2) L_B is constant in direction with varying magnitude

(3) L_A and L_B are both constant in magnitude and direction

(4) L_B is constant, both in magnitude and direction

Answer (1)

Sol. Net force on M is towards A, hence torque is zero about A.

$$\Rightarrow \vec{L}_A = \text{constant}$$

18. A solenoid of 1000 turns per metre has a core with relative permeability 500. Insulated windings of the solenoid carry an electric current of 5 A. The magnetic flux density produced by the solenoid is: (permeability of free space = $4\pi \times 10^{-7}$ H/m)

(1) $2 \times 10^{-3}\pi$ T

(2) $10^{-4}\pi$ T

(3) π T

(4) $\frac{\pi}{5}$T

Answer (3)

Sol. $B = (\mu_0 ni)\mu_r$

$= 4\pi \times 10^{-7} \times 10^3 \times 5 \times 500$

$= \pi$ T

19. A boy is rolling a 0.5 kg ball on the frictionless floor with the speed of 20 ms^{-1}. The ball gets deflected by an obstacle on the way. After deflection it moves with 5% of its initial kinetic energy. What is the speed of the ball now?

(1) 4.47 ms^{-1}

(2) 1.00 ms^{-1}

(3) 14.41 ms^{-1}

(4) 19.0 ms^{-1}

Answer (1)

Sol. $\frac{K_i}{K_f} = 20$

$$\frac{V_2}{V_0} = \frac{1}{\sqrt{20}}$$

$$V_2 = \sqrt{20}\ \text{m/s}$$

20. For what value of displacement the kinetic energy of a simple harmonic oscillation become equal?

(1) $x = \pm A$

(2) $x = 0$

(3) $x = \pm\frac{A}{\sqrt{2}}$

(4) $x = \frac{A}{2}$

Answer (3)

Sol. $\frac{1}{2}k\left(A^2 - x^2\right) = \frac{1}{2}kx^2$

$$\Rightarrow x = \pm\frac{A}{\sqrt{2}}$$

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Consider two identical springs each of spring constant k and negligible mass compared to the mass M as shown Fig. 1 shows one of them and Fig. 2 shows their series combination. The ratios of time period of oscillation of the two SHM is $T_b / T_a = \sqrt{x}$, where value of x is _____. (Round off to the Nearest Integer).

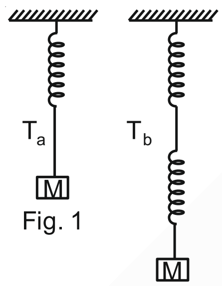


Fig. 1

Fig. 2

Answer (2)

Sol. $T_a = 2\pi\sqrt{\dfrac{m}{k}}$ $\qquad T_b = 2\pi\sqrt{\dfrac{m}{k/2}}$ $\qquad \dfrac{T_b}{T_a} = \sqrt{2}$

2. If 2.5 × 10^{-6} N average force is exerted by a light wave on a non-reflecting surface of 30 cm^2 area during 40 minutes of time span, the energy flux of light just before it falls on the surface is _____ W/cm^2. (Round off to the Nearest Integer)

(Assume complete adsorption and normal incidence conditions are there)

Answer (25)

Sol. $F = \dfrac{I}{c}A$

$$I = \frac{2.5\times10^{-6}\times3\times10^{8}}{30}\ \frac{\text{W}}{\text{cm}^2} = 25$$

3. The radius in kilometer to which the present radius of earth (R = 6400 km) to be compressed so that the escape velocity is increased 10 times is ______.

Answer (64)

Sol. $v_e = \sqrt{\dfrac{2GM}{R}}$ $\qquad 10v_e = \sqrt{\dfrac{2GM}{R'}}$

$$\Rightarrow R' = \frac{R}{100} = 64\ \text{km}$$

4. The following bodies,

(1) a ring

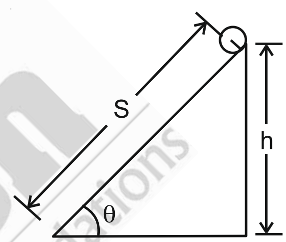
(2) a disc

(3) a solid cylinder

(4) a solid sphere,

of same mass 'm' and radius 'R' are allowed to roll down without slipping simultaneously from the top of the inclined plane. The body which will reach first at the bottom of the inclined plane is ______.

[Mark the body as per their respective numbering given in the question]



Answer (4)

Sol. The body having maximum acceleration will reach the bottom first.

$$a = \frac{g\sin\theta}{1+\frac{K^2}{R^2}}$$

$\dfrac{K^2}{R^2}$ is least for solid sphere.

5. The angular speed of truck wheel is increased from 900 rpm to 2460 rpm in 26 seconds. The number of revolutions by the truck engine during this time is ________.

(Assuming the acceleration to be uniform).

Answer (728)

Sol. $\omega = \omega_0 + \alpha t$

$$\alpha = 2\pi\ \text{rad/s}^2$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$2\pi n = 900\times\frac{2\pi}{60}\times26+\frac{1}{2}\times2\pi\times(26)^2$$

n = 728

6. The equivalent resistance of series combination of two resistors is 's'. When they are connected in parallel, the equivalent resistance is 'p'. If s = np, then the minimum value for n is ________. (Round off to the Nearest Integer)

Answer (4)

Sol. $s = R_1 + R_2$

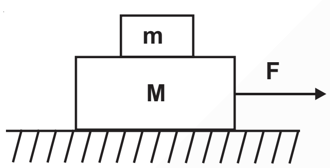
$$P = \frac{R_1R_2}{R_1+R_2}$$

$$\text{Given } (R_1 + R_2) = n\left(\frac{R_1R_2}{R_1+R_2}\right)$$

$$n = \frac{(R_1+R_2)^2}{R_1R_2} = \left(\frac{R_1}{R_2}+\frac{R_2}{R_1}+2\right)$$

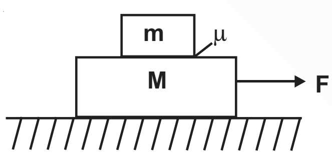
$$n \geq 4$$

7. Two block (m = 0.5 kg and M = 4.5 kg) are arranged on a horizontal frictionless table as shown in figure. The coefficient fo static friction between the two blocks is $\frac{3}{7}$. Then the maximum horizontal force that can be applied on the larger block so that the blocks move together is _________N. (Round off to the Nearest Integer) [Take g as 9.8 ms^{-2}]



Answer (21)

Sol.



$$a = \frac{F}{m+M}$$

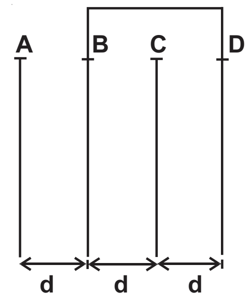
$$f = ma = m\frac{F}{m+M}$$

$$m\frac{F}{m+M} \leq \mu mg \quad \text{for no slipping}$$

$$F \leq \mu(m+M)g$$

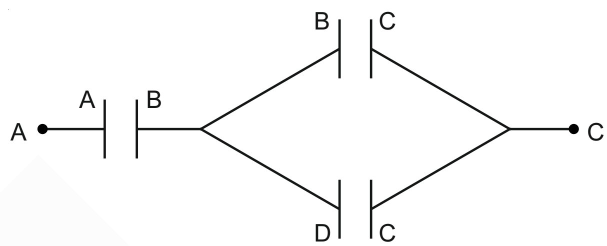
$$F_{max} = \frac{3}{7}(0.5+4.5)\ 9.8\ N = 21\ N$$

8. Four identical rectangular plates with length, l = 2 cm and breadth, b = $\frac{3}{2}$ cm are arranged as shown in figure. The equivalent capacitance between A and C is $\frac{x\varepsilon_0}{d}$. The value of x is _______. (Round off to the Nearest Integer)



Answer (2)

Sol.



$$C_{AB} = \frac{C.2C}{C+2C} = \frac{2}{3}.\frac{\varepsilon_0 lb}{d} = \frac{2\varepsilon_0}{d}$$

9. For VHF signal broadcasting, ________ km^2 of maximum service area will be covered by an antenna tower of height 30 m, if the receiving antenna is placed at ground. Let radius of the earth be 6400 km. (Round off to the Nearest Integer) (Take π as 3.14)

Answer (1206)

Sol. $A = \pi d^2$

$= 2\pi hR$

$= 2 \times 3.14 \times 30 \times 10^{-3} \times 6400$

$= 1205.76\ km^2$

10. A parallel plate capacitor whose capacitance C is 14 pF is charged by a battery to a potential difference V = 12 V between its plates. The charging battery is now disconnected and a porcelin plate with k = 7 is inserted between the plates, then the plate would oscillate back and forth between the plates with a constant mechanical energy of ________ pJ.

(Assume no friction)

Answer (864)

Sol. $U_i = \frac{1}{2}CV^2 = 1008\ pJ$

After releasing the slab it will have maximum K.E.(which is also equal to M.E.) while crossing mean position.

$$U_f = \frac{Q^2}{2C'} = \frac{C^2V^2}{2C'} = \frac{1}{2}\frac{CV^2}{k} = 144\ pJ$$

M.E. $= U_i - U_f$

$= 864\ pJ$

PART–B : CHEMISTRY

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. A central atom in a molecule has two lone pairs of electrons and forms three single bonds. The shape of this molecule is

   (1) trigonal pyramidal

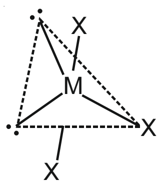
   (2) see-saw

   (3) T-shaped

   (4) planar triangular

Answer (3)

Sol. The shape of a molecule (MX_3) whose central atom (M) has two lone pairs of electrons and forms three single bonds is T-shaped.



2. Mesityl oxide is a common name of

   (1) 3-Methyl cyclohexane carbaldehyde

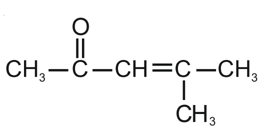
   (2) 2, 4-Dimethyl pentan-3-one

   (3) 2-Methyl cyclohexanone

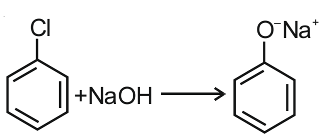
   (4) 4-Methyl pent-3-en-2-one

Answer (4)

Sol. Mesityl oxide is the common name of aldol condensation product of acetone. Its structure and IUPAC name are



4-methyl pent-3-en-2-one

3. 

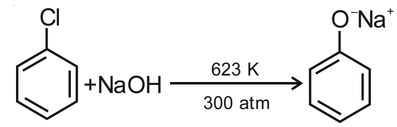
   The above reaction requires which of the following reaction conditions?

   (1) 573 K, 300 atm (2) 623 K, Cu, 300 atm

   (3) 573 K, Cu, 300 atm (4) 623 K, 300 atm

Answer (4)

Sol. Chlorobenzene is fused with NaOH at 623 K and 300 atmospheric pressure to get sodium phenoxide.



4. A colloidal system consisting of a gas dispersed in a solid is called a/an

   (1) aerosol (2) foam

   (3) solid sol (4) gel

Answer (3)

Sol. A colloidal system consisting of a gas dispersed in a solid is called a 'solid sol'.

5. What is the spin-only magnetic moment value (BM) of a divalent metal ion with atomic number 25, in it's aqueous solution?

   (1) 5.92 (2) 5.26

   (3) zero (4) 5.0

Answer (1)

Sol. The element having atomic number 25 is manganese. The electronic configuration of Mn^{2+} is

Mn^{2+} : $3d^5$

In aqueous solution it exists as $[Mn(H_2O)_6]^{2+}$. Since H_2O is a weak field ligand, it does not cause pairing of unpaired electrons. So, its spin only magnetic moment is

$$\mu = \sqrt{5\times7} = 5.92 \text{ BM}$$

6. The correct order of conductivity of ions in water is

   (1) $Na^+ > K^+ > Rb^+ > Cs^+$

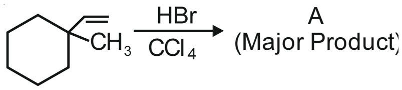
   (2) $Rb^+ > Na^+ > K^+ > Li^+$

   (3) $Cs^+ > Rb^+ > K^+ > Na^+$

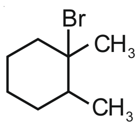
   (4) $K^+ > Na^+ > Cs^+ > Rb^+$

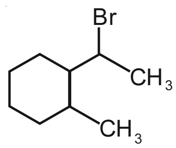
Answer (3)

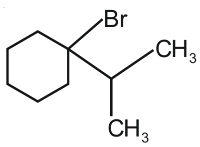
Sol. The alkali metal ions in aqueous solution get hydrated. The extent of hydration of an ion is directly proportional to its charge density. The size of hydrated metal ion decreases down the group and hence their mobility increases or their conductivity increases

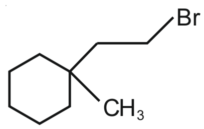
7. 

Product "A" in the above chemical reaction is

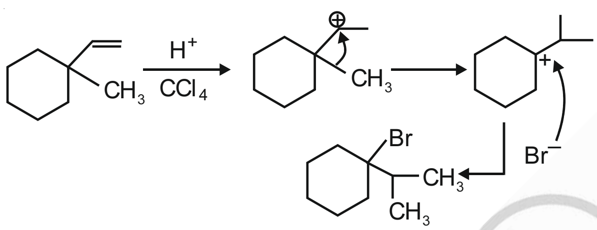
(1) 

(2) 

(3) 

(4) 

Answer (3)

Sol. 

(A)

The reaction involves the formation of 2° carbocation followed by methanide shift to give 3° carbocation. Br^- ion attacks the 3° carbocation to give the major product.

8. Given below are two statements :

Statements I : Potassium permanganate on heating at 573 K forms potassium manganate.

Statements II : Both potassium permanganate and potassium manganate are tetrahedral and paramagnetic in nature.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) Statement I is false but statement II is true

(2) Both statement I and statement II are false

(3) Both statement I and statement II are true

(4) Statement I is true but statement II is false

Answer (4)

Sol. $KMnO_4$ on heating dissociates as

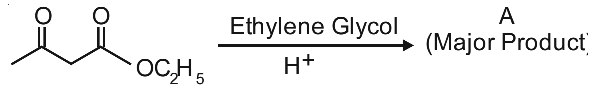
$$\underset{\text{Permanganate}}{\overset{+7}{2KMnO_4}} \xrightarrow{\Delta} \underset{\text{Manganate}}{\overset{+6}{K_2MnO_4}} + MnO_2 + O_2$$

Both permanganate and manganate are tetrahedral but only manganate is paramagnetic.

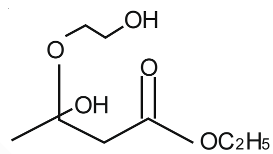
$\overset{+7}{Mn} : 3d^0 4s^0$ Diamagnetic

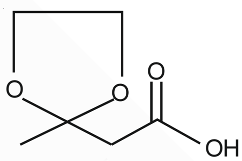
$\overset{+6}{Mn} : 3d^1 4s^0$ Paramagnetic

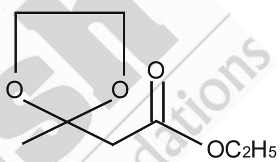
∴ Statement I is true but statement II is false.

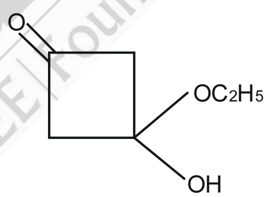
9. 

The product "A" in the above reaction is

(1) 

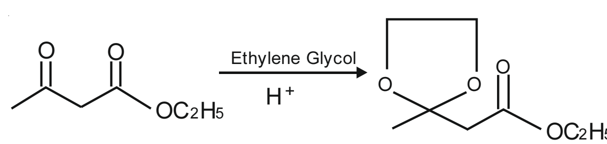
(2) 

(3) 

(4) 

Answer (3)

Sol. Ethylene glycol in presence of H^+ will convert ketone into cyclic ketal and the ester group remains intact.



10. Reducing smog is a mixture of

(1) Smoke, fog and $CH_2 = CH - CHO$

(2) Smoke, fog and SO_2

(3) Smoke, fog and N_2O_3

(4) Smoke, fog and O_3

Answer (2)

Sol. Classical smog is a mixture of smoke, fog and SO_2. Chemically it is a reducing mixture and so it is called as reducing smog.

11. Given below are two statements :

Statement I : Retardation factor (R_f) can be measured in meter/centimeter.

Statement II : R_f value of a compound remains constant in all solvents.

Choose the **most appropriate** answer from the options given below :

(1) Statement I is true but statement II is false

(2) Both statement I and statement II are true

(3) Both statement I and statement II are false

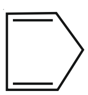
(4) Statement I is false but statement II is true

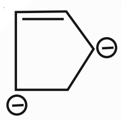
Answer (3)

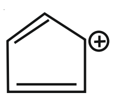
Sol. Retardation factor (R_f) is the ratio of distance moved by the substance from the base line to the distance moved by the solvent from the base line. So, it is dimensionless. The distance moved by the substance is due to adsorption of the substance on the stationary phase. It does not depend on the nature of solvent. But the distance moved by the solvent will change with the nature of solvent. Therefore, R_f will vary with the change in solvent.

So, both the statements are false.

12. Which of the following is an aromatic compound?

(1) 

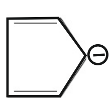
(2) 

(3) 

(4) 

Answer (4)

Sol. A compound which has $(4n + 2)\pi$ electrons completely delocalised over the cyclic ring is aromatic.



n = 1 (Aromatic)

13. The **INCORRECT** statement(s) about heavy water is (are)

(A) used as a moderator in nuclear reactor

(B) obtained as a by-product in fertilizer industry

(C) used for the study of reaction mechanism

(D) has a higher dielectric constant than water

Choose the correct answer from the options given below:

(1) (B) and (D) only

(2) (B) only

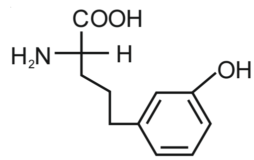
(3) (D) only

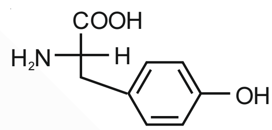
(4) (C) only

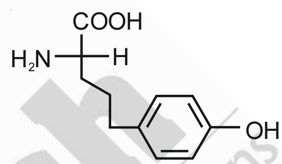
Answer (3)

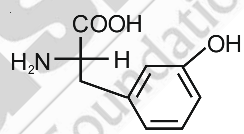
Sol. Heavy water (D_2O) is obtained as a by-product in fertilizer industry. It is used as a moderator in nuclear reactor and for the study of reaction mechanism. Its dielectric constant is lower than that of H_2O.

14. Which of the following is correct structure of tyrosine?

(1) 

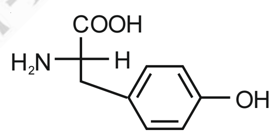
(2) 

(3) 

(4) 

Answer (2)

Sol. Tyrosine is p-hydroxyphenylalanine. Its structure is



15. Which of the following reaction is an example of ammonolysis?

(1) $C_6H_5CH_2Cl + NH_3 \longrightarrow C_6H_5CH_2NH_2$

(2) $C_6H_5NH_2 \xrightarrow{HCl} C_6H_5\overset{+}{N}H_3Cl^-$

(3) $C_6H_5COCl + C_6H_5NH_2 \longrightarrow C_6H_5CONHC_6H_5$

(4) $C_6H_5CH_2CN \xrightarrow{[H]} C_6H_5CH_2CH_2NH_2$

Answer (1)

Sol. Ammonolysis of alkyl halides is the reaction of alkyl halide with NH_3 which leads to the preparation of amines.

$$C_6H_5CH_2Cl + NH_3 \longrightarrow C_6H_5CH_2NH_2$$

16. With respect to drug-enzyme interaction, identify the wrong statement.

(1) Allosteric inhibitor competes with the enzyme's active site

(2) Allosteric inhibitor changes the enzyme's active site

(3) Non-Competitive inhibitor binds to the allosteric site

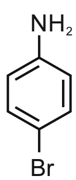
(4) Competitive inhibitor binds to the enzyme's active site

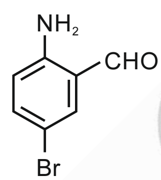
Answer (1)

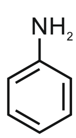
Sol. Allosteric inhibitor changes the enzyme's active site and they do not compete with the enzyme's active site. They bind to the allosteric site. Competitive inhibitor binds to the enzyme's active site.

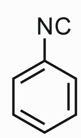
17. Hoffmann bromamide degradation of benzamide gives product A, which upon heating with $CHCl_3$ and NaOH gives product B.

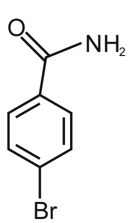
The structures of A and B are:

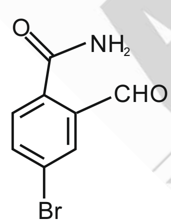
(1) A – 

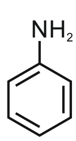
B – 

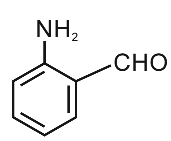
(2) A – 

B – 

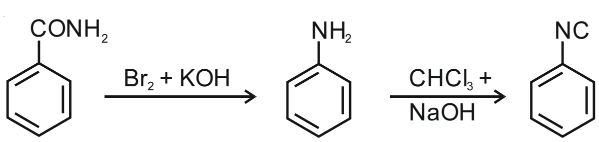
(3) A – 

B – 

(4) A – 

B – 

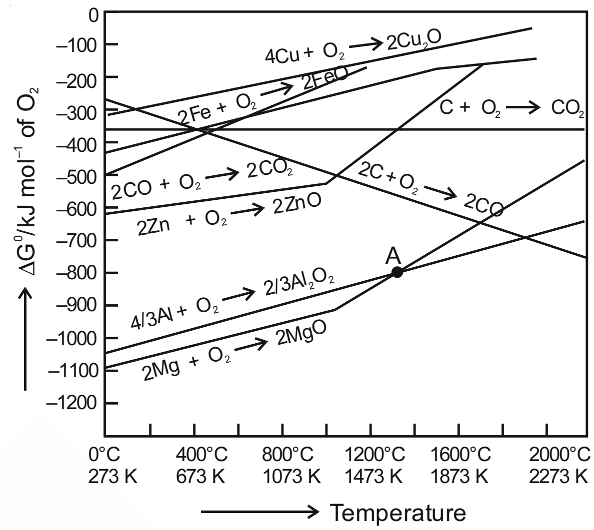
Answer (2)

Sol. 

(A) (B)

Hoffmann bromamide degradation of benzamide gives aniline (A) which upon heating with $CHCl_3$ and NaOH gives phenyl isocyanide (B).

18. The point of intersection and sudden increase in the slope, in the diagram given below, respectively, indicates:



(1) $\Delta G = 0$ and reduction of the metal oxide

(2) $\Delta G < 0$ and decomposition of the metal oxide

(3) $\Delta G = 0$ and melting or boiling point of the metal oxide

(4) $\Delta G > 0$ and decomposition of the metal oxide

Answer (3)

Sol. From the Ellingham diagram given, the point of intersection represents $\Delta G = 0$ and the temperature at which sudden increase in the slope occurs is indicated by melting or boiling.

19. Which of the following compound CANNOT act as a Lewis base?

(1) NF_3 (2) PCl_5

(3) ClF_3 (4) SF_4

Answer (2)

Sol. Lewis base should have at least one lone pair of electrons in the valence shell of the central atom which is available for donation. PCl_5 cannot function as a Lewis base as the central atom P does not have lone pair of electrons.

20. The absolute value of the electron gain enthalpy of halogens satisfies :

(1) Cl > Br > F > I (2) I > Br > Cl > F

(3) F > Cl > Br > I (4) Cl > F > Br > I

Answer (4)

Sol. The magnitude of electron gain enthalpy of halogen atoms down the group shows abnormal behaviour. The $|\Delta H_{eg}|$ of F is lower than that of Cl due to its smaller size. The incoming electron experiences higher repulsive force due to valence electrons of F than Cl. The correct order is Cl > F > Br > I

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. 0.01 moles of a weak acid HA (K_a = 2.0 × 10^{-6}) is dissolved in 1.0 L of 0.1 M HCl solution. The degree of dissociation of HA is _______ × 10^{-5} (Round off to the Nearest Integer).

   [Neglect volume change on adding HA.

   Assume degree of dissociation <<1]

Answer (2)

Sol.

$$\underset{}{HCl} \rightarrow \underset{0.1M}{H^+} + \underset{0.1M}{Cl^-}$$

$$\underset{0.01(1-\alpha)}{HA} \rightleftharpoons \underset{\substack{0.1+0.01\alpha \\ \simeq 0.1}}{H^+} + \underset{0.01\alpha}{A^-} \quad K_a = 2.0\times10^{-6}$$

$$K_a = \frac{[H^+][A^-]}{[HA]}$$

$$2\times10^{-6} = \frac{0.1\times0.01\alpha}{0.01(1-\alpha)} \simeq \frac{0.1\times0.01\alpha}{0.01}$$

α = 2.0 × 10^{-5}

2. The pressure exerted by a non-reactive gaseous mixture of 6.4 g of methane and 8.8 g of carbon dioxide in a 10 L vessel at 27°C is .... kPa.

   (Round off to the Nearest Integer).

   [Assume gases are ideal, R = 8.314 J mol^{-1} K^{-1}

   Atomic masses : C : 12.0 u, H : 1.0 u, O : 16.0 u]

Answer (150)

Sol. Number of moles of $CH_4 = \frac{6.4}{16} = 0.4$

Number of moles of $CO_2 = \frac{8.8}{44} = 0.2$

Total number of moles of the mixture = 0.6

Pressure of the mixture of gases in 10L (0.01 m^3)

Vessel at 300 K is given as

$$P = \frac{nRT}{V} = \frac{0.6\times8.314\times300}{0.01} = 149.65 \simeq 150\text{ kPa}$$

3. The mole fraction of a solute in a 100 molal aqueous solution is ____ × 10^{-2}.

   (Round off to the Nearest Integer).

   [Given : Atomic masses : H : 1.0 u, O : 16.0 u]

Answer (64)

Sol. Molality of an aqueous solution of a solute = 100 m

Number of moles of solvent = $\frac{1000}{18}$

Mole fraction of solute = $\frac{100}{100+\frac{1000}{18}} = \frac{100\times18}{2800}$

= 0.6428 = 64.28 × 10^{-2}

$\simeq 64$

4. The standard enthalpies of formation of Al_2O_3 and CaO are –1675 kJ mol^{-1} and –635 kJ mol^{-1} respectively.

   For the reaction

   3CaO + 2Al $\rightarrow$ 3Ca + Al_2O_3 the standard reaction enthalpy Δ_rH^0 = _____kJ.

   (Round off to the Nearest Integer).

Answer (230)

Sol. $\Delta H_f^\circ(Al_2O_3) = -1675\text{ kJ mol}^{-1}$

$\Delta H_f^\circ(CaO) = -635\text{ kJ mol}^{-1}$

3CaO + 2Al $\rightarrow$ 3Ca + Al_2O_3

$\Delta H_r^\circ = \Delta H_f^\circ(Al_2O_3) - 3\Delta H_f^\circ(CaO)$

= –1675 – 3(–635)

= 230 kJ mol^{-1}

5. The reaction of white phosphorus on boiling with alkali in inert atmosphere resulted in the formation of product 'A'. The reaction of 1 mol of 'A' with excess of $AgNO_3$ in aqueous medium gives _________ mol(s) of Ag.

   (Round off to the Nearest Integer).

Answer (6)

Sol. $P_4 + 3NaOH + 3H_2O \longrightarrow PH_3 + 3NaH_2PO_2$

$PH_3 + 6AgNO_3 \longrightarrow [Ag_3P.3AgNO_3] + 3HNO_3$

$Ag_3P.3AgNO_3 + 3H_2O \longrightarrow 6Ag + 3HNO_3 + H_3PO_3$

So, 1 mol of PH_3(A) on reaction with excess of aq. $AgNO_3$ gives 6 moles of Ag.

6. A certain orbital has n = 4 and m_L = –3. The number of radial nodes in this orbital is ________.

(Round off to the Nearest Integer).

Answer (0)

Sol. The orbital having n = 4 and m_L = –3 is 4*f*.

The number of radial nodes is an orbital is given by

Number of radial nodes = $n - m_L - 1$

$= 4 - 3 - 1$

$= 0$

7. For a certain first order reaction 32% of the reactant is left after 570 s. The rate constant of this reaction is ________ × 10^{-3} s^{-1}.

(Round off to the Nearest Integer).

[Given $\log_{10}2$ = 0.301, ln10 = 2.303]

Answer (2)

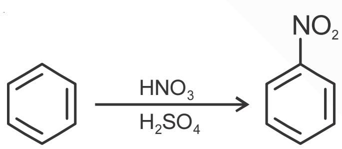
Sol. $A \longrightarrow$ Products

Rate constant of a first order is given as

$$k = \frac{2.303}{t}\log\frac{[A]_0}{[A]_t}$$

$$= \frac{2.303}{570}\log\frac{100}{32}$$

$$= 2 \times 10^{-3}\ s^{-1}$$

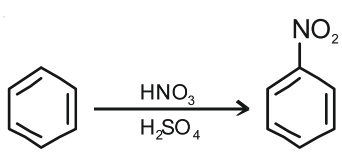
8. 

In the above reaction, 3.9 g of benzene on nitration gives 4.92 g of nitrobenzene. The percentage yield of nitrobenzene in the above reaction is ________%.

(Round off to the Nearest Integer)

(Given atomic mass : C : 12.0 u, H : 1.0 u, O : 16.0 u, N : 14.0 u)

Answer (80)

Sol. 

Number of moles of C_6H_6 = $\frac{3.9}{78}$ = 0.05

Theoretical moles of nitrobenzene = 0.05

Actual number of moles of nitrobenzene

$= \frac{4.92}{123} = 0.04$

Percentage yield of nitrobenzene

$= \frac{0.04}{0.05} \times 100$

$= 80\%$

9. 15 mL of aqueous solution of Fe^{2+} in acidic medium completely reacted with 20 mL of 0.03 M aqueous $Cr_2O_7^{2-}$. The molarity of the Fe^{2+} solution is ________ × 10^{-2} M.

(Round off to the Nearest Integer).

Answer (24)

Sol. $$\underset{n=1}{6Fe^{2+}} + \underset{n=6}{Cr_2O_7^{2-}} + 14H^+ \longrightarrow 2Cr^{3+} + 6Fe^{3+} + 7H_2O$$

milliequivalents of Fe^{2+} = milliequivalents of $Cr_2O_7^{2-}$

If M is the molarity of Fe^{2+} ion solution

$1 \times M \times 15 = 0.03 \times 6 \times 20$

$M = 0.24 = 24 \times 10^{-2}$

10. The oxygen dissolved in water exerts a partial pressure of 20 kPa in the vapour above water. The molar solubility of oxygen in water is ________ × 10^{-5} mol dm^{-3}.

(Round off to the Nearest Integer).

[Given : Henry's law constant = K_H = 8.0 × 10^4 kPa for O_2.

Density of water with dissolved oxygen = 1.0 kg dm^{-3}]

Answer (25)

Sol. P_{O_2} (over water) = 20 kPa

K_H for O_2 = 8.0 × 10^4 kPa

If X_{O_2} is the mole fraction of O_2 in soution, then according to Henry's law

$$P_{O_2} = K_H(X_{O_2})$$

$$X_{O_2} = \frac{20}{8.0 \times 10^4} = 2.5 \times 10^{-4}$$

Mass of 1 kg of water containing O_2 = 1 L

∴ Molarity of O_2 in solution = 25 × 10^{-5} M

PART–C : MATHEMATICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. In a triangle PQR, the co-ordinates of the points P and Q are (–2, 4) and (4, –2) respectively. If the equation of the perpendicular bisector of PR is 2x – y + 2 = 0, then the centre of the circumcircle of the ΔPQR is

(1) (–2, –2) (2) (0, 2)

(3) (1, 4) (4) (–1, 0)

Answer (1)

Sol. Mid point of $PQ \equiv \left(\dfrac{-2+4}{2}, \dfrac{4-2}{2}\right) \equiv (1, 1)$

Slope of $PQ = \dfrac{4+2}{-2-4} = -1$

Slope of perpendicular bisector of PQ = 1

Equation of perpendicular bisector of PQ

$y - 1 = 1(x - 1)$

$\Rightarrow$ y = x

Solving with perpendicular bisector of PR

Circumcentre is (–2, –2)

2. The area of the triangle with vertices A(z), B(iz) and C(z + iz) is

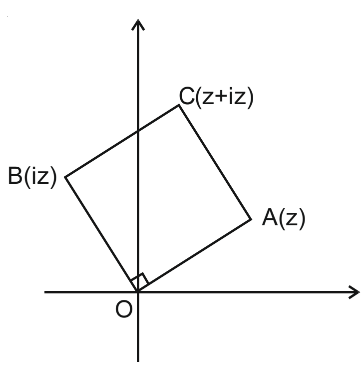
(1) $\dfrac{1}{2}|z|^2$ (2) $\dfrac{1}{2}|z+iz|^2$

(3) $\dfrac{1}{2}$ (4) 1

Answer (1)

Sol. Geometrically OABC form a square as shown

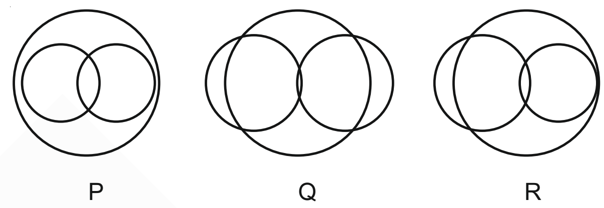
Each side length = |z|



Area of $\Delta ABC = \dfrac{1}{2}$(Area of square)

$= \dfrac{1}{2}.|z|^2$

3. In a school, there are three types of games to be played. Some of the students play two types of games, but none play all the three games. Which Venn diagrams can justify the above statement?



(1) Q and R (2) P and Q

(3) P and R (4) None of these

Answer (4)

Sol. As none play all three games the intersection of all three circles must be zero

Hence none of P, Q, R justify the given statement

4. If cot^{-1}(α) = cot^{-1}2 + cot^{-1}8 + cot^{-1}18 + cot^{-1}32 +... upto 100 terms, then α is :

(1) 1.01 (2) 1.02

(3) 1.03 (4) 1.00

Answer (1)

Sol. $\cot^{-1}(\alpha) = \cot^{-1}2 + \cot^{-1}8 + \cot^{-1}18 + \cot^{-1}32 + \ldots 100 \text{ terms}$

$= \tan^{-1}\dfrac{1}{2} + \tan^{-1}\dfrac{1}{8} + \tan^{-1}\dfrac{1}{18} + \tan^{-1}\dfrac{1}{32} + \ldots 100 \text{ term}$

$= \sum_{k=1}^{100} \tan^{-1}\dfrac{1}{2k^2}$

$= \sum_{k=1}^{100} \tan^{-1}\dfrac{2}{4k^2} = \sum_{k=1}^{n} \tan^{-1}\dfrac{(2k+1)-(2k-1)}{1+(2k-1)(2k+1)}$

$= \sum_{k=1}^{100}\left(\tan^{-1}(2k+1) - \tan^{-1}(2k-1)\right)$

= tan^{-1}201 – tan^{-1} 1

$= \tan^{-1}\dfrac{200}{202}$

= cot^{-1}(1.01)

Hence α = 1.01

5. The sum of possible values of x for $\tan^{-1}(x+1)+\cot^{-1}\left(\frac{1}{x-1}\right)=\tan^{-1}\left(\frac{8}{31}\right)$ is

(1) $-\frac{30}{4}$ (2) $-\frac{31}{4}$

(3) $-\frac{32}{4}$ (4) $-\frac{33}{4}$

Answer (3)

Sol. $\tan^{-1}(x+1)+\cot^{-1}\left(\frac{1}{x-1}\right)=\tan^{-1}\left(\frac{8}{31}\right)$

$\Rightarrow \tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31}$

$\Rightarrow \tan^{-1}\left(\frac{(x+1)+(x-1)}{1-(x+1)(x-1)}\right)=\tan^{-1}\frac{8}{31}$

$\Rightarrow \frac{x}{2-x^2}=\frac{4}{31}$

$\Rightarrow 4x^2 + 31x - 8 = 0 \Rightarrow x=\frac{1}{4}$ or $x=-8$

$x=\frac{1}{4}$ does not satisfy

Hence, sum of possible values of $x=-8=\frac{-32}{4}$

6. The line $2x - y + 1 = 0$ is a tangent to the circle at the point (2, 5) and the centre of the circle lies on $x - 2y = 4$. Then, the radius of the circle is

(1) $3\sqrt{5}$ (2) $5\sqrt{3}$

(3) $4\sqrt{5}$ (4) $5\sqrt{4}$

Answer (1)

Sol. Any line perpendicular to given tangent is

$x + 2y + \lambda = 0$

Passes through (2, 5) $\Rightarrow \lambda = -12$

Hence line in $x + 2y - 12 = 0$

Solving with $x - 2y - 4 = 0$ gives

Centre $\equiv$ (8, 2)

Radius = $\sqrt{(8-2)^2+(2-5)^2}$

$= 3\sqrt{5}$

7. The inverse of $y = 5^{\log x}$ is

(1) $x = y^{\log 5}$ (2) $x = 5^{\log y}$

(3) $x=y^{\frac{1}{\log 5}}$ (4) $x=5^{\frac{1}{\log y}}$

Answer (3)

Sol. $y=5^{\log x}$

$\Rightarrow \log y=\log x\cdot\log 5$

$\Rightarrow \log x=\frac{\log y}{\log 5}=\log_5 y$

$\Rightarrow x=e^{\log_5 y}$

$\Rightarrow x=y^{\log_5 e}$

$\Rightarrow x=y^{\frac{1}{\log 5}}$

8. Choose the incorrect statement about the two circles whose equations are given below :

$x^2 + y^2 - 10x - 10y + 41 = 0$ and

$x^2 + y^2 - 16x - 10y + 80 = 0$

(1) Distance between two centres is the average of radii of both the circles

(2) Circles have two intersection points

(3) Both circles pass through the centre of the each other

(4) Both circles' centres lie inside region of one another

Answer (4)

Sol. $S_1 \equiv x^2 + y^2 - 10x - 10y + 41 = 0$

Centre $C_1 \equiv (5, 5)$, radius $r_1 = 3$

$S_2 \equiv x^2 + y^2 - 16x - 10y + 80 = 0$

Centre $C_2 \equiv (8, 5)$, radius $r_2 = 3$

Distance between centres = 3

Hence both circles pass through the centre of each other, have two intersection point and distance between two centres in average of radii of both the circles.

9. If the fourth term in the expansion of $(x+x^{\log_2 x})^7$ is 4480, then the value of x where $x \in N$ is

(1) 2

(2) 1

(3) 3

(4) 4

Answer (1)

Sol. $T_4={}^7C_3\cdot(x^{\log_2 x})^3\cdot x^4=4480$

$\Rightarrow \left(x^{\log_2 x}\right)^3\cdot x^4=128$

$x = 2$ is the only solution for $x \in N$

10. The value of $\lim_{x\to 0^+}\frac{\cos^{-1}(x-[x]^2)\cdot\sin^{-1}(x-[x]^2)}{x-x^3}$, where [x] denote the greatest integer $\leq$ x is

(1) $\frac{\pi}{4}$ (2) 0

(3) $\frac{\pi}{2}$ (4) π

Answer (3)

Sol. $\lim_{x\to 0^+}\dfrac{\cos^{-1}\left(x-[x]^2\right)\cdot\sin^{-1}\left(x-[x]^2\right)}{x-x^3}$

$=\lim_{x\to 0^+}\dfrac{\cos^{-1}x}{1-x^2}\cdot\dfrac{\sin^{-1}x}{x}$

$=\cos^{-1}0=\dfrac{\pi}{2}$

11. The system of equations kx + y + z = 1, x + ky + z = k and x + y + zk = k^2 has no solution equal to :

(1) 0　　(2) 1

(3) –1　　(4) –2

Answer (4)

Sol. $\begin{vmatrix} k & 1 & 1\\ 1 & k & 1\\ 1 & 1 & k\end{vmatrix}=0$

$\Rightarrow$ $(k-1)^2(k+2)=0$

k = 1 makes the equation identical hence the system will have infinite solution

System will have no solution for k = –2.

12. Which of the following is true for y(x) that satisfies the differential equation $\dfrac{dy}{dx}=xy-1+x-y;\ y(0)-0$:

(1) $y(1)=e^{\frac{1}{2}}-e^{-\frac{1}{2}}$

(2) y(1) = 1

(3) $y(1)=e^{\frac{1}{2}}-1$

(4) $y(1)=e^{-\frac{1}{2}}-1$

Answer (4)

Sol. $\dfrac{dy}{dx}=xy-1+x-y$

$\Rightarrow \dfrac{dy}{dx}=(x-1)(y+1)$

$\Rightarrow \dfrac{dy}{y+1}=(x-1)dx$

$\Rightarrow \ln(y+1)=\dfrac{x^2}{2}-x+c$

y(0) = 0 $\Rightarrow$ c = 0

Hence $y(x)=e^{\left(\frac{x^2}{2}-x\right)}-1$

$y(1)=e^{\frac{-1}{2}}-1$

13. If $A-\begin{pmatrix}0 & \sin\alpha\\ \sin\alpha & 0\end{pmatrix}$ and $\det\left(A^2-\frac{1}{2}I\right)=0$, then possible value of α is :

(1) $\dfrac{\pi}{3}$　　(2) $\dfrac{\pi}{6}$

(3) $\dfrac{\pi}{2}$　　(4) $\dfrac{\pi}{4}$

Answer (4)

Sol. $A=\begin{bmatrix}0 & \sin\alpha\\ \sin\alpha & 0\end{bmatrix}$

$A^2=\begin{bmatrix}\sin^2\alpha & 0\\ 0 & \sin^2\alpha\end{bmatrix}$

$\det\left(A^2-\frac{1}{2}I\right)=\begin{vmatrix}\sin^2\alpha-\frac{1}{2} & 0\\ 0 & \sin^2\alpha-\frac{1}{2}\end{vmatrix}=0$

$\Rightarrow \left(\sin^2\alpha-\frac{1}{2}\right)^2=0$

$\sin\alpha=\pm\dfrac{1}{\sqrt{2}}$

$\alpha=\dfrac{\pi}{4}$ is one possibility

14. The equation of the plane which contains the y-axis and passes through the point (1, 2, 3) is :

(1) 3x + z = 6　　(2) x + 3x = 0

(3) 3x – z = 0　　(4) x + 3z = 10

Answer (3)

Sol. Any plane containing y-axis is of the form

$x+\lambda z=0$

It passes through (1, 2, 3)

$1+3\lambda=0,\ \Rightarrow \lambda=\dfrac{-1}{3}$

Required plane is

3x – z = 0

15. Let $\vec{a}=2\hat{i}-3\hat{j}+4\hat{k}$ and $\vec{b}=7\hat{i}+\hat{j}-6\hat{k}$.

If $\vec{r}\times\vec{a}=\vec{r}\times\vec{b},\ \vec{r}\cdot\left(\hat{i}+2\hat{j}+\hat{k}\right)=-3$, then $\vec{r}\cdot\left(2\hat{i}-3\hat{j}+\hat{k}\right)$ is equal to :

(1) 10　　(2) 13

(3) 8　　(4) 12

Answer (4)

Sol. $\bar{r}\times\bar{a}=\bar{r}\times\bar{b} \quad \Rightarrow \bar{r}\times(\bar{a}-\bar{b})=0$

$\Rightarrow \bar{r}=\lambda(\bar{a}-\bar{b}), \lambda\in R.$

$\Rightarrow \bar{r}=\lambda(-5\hat{i}-4\hat{j}+10\hat{k})$

$\because \quad \bar{r}\cdot(\hat{i}+2\hat{j}+\hat{k})=-3$

$\Rightarrow \lambda(-5-8+10)=-3$

$\Rightarrow \lambda=1$

Hence $\bar{r}=-5\hat{i}-4\hat{j}+10\hat{k}$

$\bar{r}\cdot(2\hat{i}-3\hat{j}+\hat{k})=12$

16. If the Boolean expression $(P\Rightarrow q)\Leftrightarrow(q*(\sim p))$ is a tautology, then the Boolean expression $p*(\sim q)$ is equivalent to :

(1) $p\Rightarrow q$ (2) $q\Rightarrow p$

(3) $\sim q\Rightarrow p$ (4) $p\Rightarrow \sim q$

Answer (2)

Sol. $p\Rightarrow q\Leftrightarrow q^{*}(\sim p)$ is a tautology

$\therefore \quad p\Rightarrow q$ and $q^{*}(\sim p)$ have same truth value

for all logical possibility

$\therefore \quad q*(\sim p)\equiv p\Rightarrow q$

And therefore, $p^{*}\sim q\equiv q\Rightarrow p$

17. The value of $4+\cfrac{1}{5+\cfrac{1}{4+\cfrac{1}{5+\cfrac{1}{4+.....\infty}}}}$ is :

(1) $2+\frac{4}{\sqrt{5}}\sqrt{30}$ (2) $4+\frac{4}{\sqrt{5}}\sqrt{30}$

(3) $2+\frac{2}{5}\sqrt{30}$ (4) $5+\frac{2}{5}\sqrt{30}$

Answer (3)

Sol. Let $k=4+\cfrac{1}{5+\cfrac{1}{4+\cfrac{1}{5+\cfrac{1}{4+....\infty}}}}$

$\Rightarrow k = 4+\cfrac{1}{5+\cfrac{1}{k}}$

$\Rightarrow 5k^2-20k-4=0$

$\Rightarrow k=2+\frac{2\sqrt{30}}{5}$ (taking positive value)

18. Two dices are rolled. If both dices have six faces numbered 1, 2, 3, 5, 7 and 11, then the probability that the sum of the numbers on the top faces is less than or equal to 8 is :

(1) $\frac{17}{36}$ (2) $\frac{1}{2}$

(3) $\frac{4}{9}$ (4) $\frac{5}{12}$

Answer (1)

Sol. Favourable outcomes are

(1,1), (1,2), (1,3), (1,5), (1,7)

(2,1), (2,2), (2,3), (2,5)

(3,1), (3,2), (3,3), (3,5)

(5,1), (5,2), (5,3)

(7,1)

i.e. total 17 favourable outcomes.

Required probability = $\frac{17}{36}$.

19. Team 'A' consists of 7 boys and n girls and Team 'B' has 4 boys and 6 girls. If a total of 52 single matches can be arranged between these two teams when a boy plays against a boy and a girl plays against a girl, then n is equal to

(1) 5 (2) 6

(3) 2 (4) 4

Answer (4)

Sol. Total matches of boys can be arranged in $7\times4=28$ ways

Total matches of girls can be arranged in $n\times6=6n$ ways

Given $28+6n=52$

$n=4$.

20. Which of the following statements is incorrect for the function $g(\alpha)$ for $\alpha\in R$ such that

$$g(\alpha)=\int_{\pi/6}^{\pi/3}\frac{\sin^{\alpha}x}{\cos^{\alpha}x+\sin^{\alpha}x}dx$$

(1) $g(\alpha)$ is a strictly increasing function

(2) $g(\alpha)$ has an inflection point at $\alpha=-\frac{1}{2}$

(3) $g(\alpha)$ is a strictly decreasing function

(4) $g(\alpha)$ is an even function

Answer (1, 2, 3)*

Sol. $g(\alpha)=\int_{\pi/6}^{\pi/3}\frac{\sin^{\alpha}x}{\cos^{\alpha}x+\sin^{\alpha}x}dx.$

$$g(\alpha)=\int_{\pi/6}^{\pi/3}\frac{\sin^{\alpha}\left(\frac{\pi}{2}-x\right)}{\cos^{\alpha}\left(\frac{\pi}{2}-x\right)x+\sin^{\alpha}\left(\frac{\pi}{2}-x\right)}dx$$

$$=\int_{\pi/6}^{\pi/3}\frac{\cos^{\alpha}x}{\sin^{\alpha}x+\cos^{\alpha}x}dx$$

$$2.g(\alpha)=\int_{\pi/6}^{\pi/3}\frac{\sin^{\alpha}x+\cos^{\alpha}x}{\sin^{\alpha}x+\cos^{\alpha}x}dx=\int_{\pi/6}^{\pi/3}dx=\frac{\pi}{3}-\frac{\pi}{6}=\frac{\pi}{6}.$$

$g(\alpha)=\frac{\pi}{12}$ i.e. a constant function hence an even function.

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. The minimum distance between any two points P_1 and P_2 while considering point P_1 on one circle and point P_2 on the other circle for the given circles' equations

   $x^2 + y^2 - 10x - 10y + 41 = 0$

   $x^2 + y^2 - 24x - 10y + 160 = 0$ is ________.

Answer (1)

Sol. $S_1 \equiv x^2 + y^2 - 10x - 10y + 41 = 0$

Centre $C_1 \equiv (5, 5)$ radius $r_1 = 3$

$S_2 = x^2 + y^2 - 24x - 10y + 160$

Centre $C_2 \equiv (12, 5)$ radius = 3

Distance between centres > Sum of radii

$\Rightarrow$ Circle are separated,

Required minimum possible distance = 7 – (3 + 3)

= 1

2. If $\vec{a}=\alpha\hat{i}+\beta\hat{j}+3\hat{k}$,

   $\vec{b}=-\beta\hat{i}-\alpha\hat{j}-\hat{k}$ and

   $\vec{c}=\hat{i}-2\hat{j}-\hat{k}$

   such that $\vec{a}.\vec{b}=1$ and $\vec{b}.\vec{c}=-3$, then $\frac{1}{3}\left(\left(\vec{a}\times\vec{b}\right).\vec{c}\right)$ is equal to ______.

Answer (2)

Sol. $\bar{a}.\bar{b}=1\Rightarrow-\alpha\beta-\alpha\beta-3=1$

$\Rightarrow \alpha\beta = -2$ ...(i)

$\bar{b}.\bar{c}=-3\Rightarrow-\beta+2\alpha+1=-3$

$2\alpha - \beta = -4$ ...(ii)

Solving (i) & (ii) $\alpha = -1$, $\beta = 2$,

$$\frac{1}{3}((\bar{a}\times\bar{b})\cdot\bar{c})=\frac{1}{3}\begin{vmatrix}-1 & 2 & 3\\ -2 & 1 & -1\\ 1 & -2 & -1\end{vmatrix}$$

= 2

3. If $f(x)=\sin\left(\cos^{-1}\left(\frac{1-2^{2x}}{1+2^{2x}}\right)\right)$ and its first derivative with respect to x is $-\frac{b}{a}\log_e 2$ when x = 1, where a and b are integers, then the minimum value of $|a^2 - b^2|$ is ________.

Answer (481)

Sol. $f(x)=\sin\cos^{-1}\left(\frac{1-(2^x)^2}{1+(2^x)^2}\right)$

$= \sin(2\tan^{-1}2^x)$

$$f'(x)=\cos(2\tan^{-1}2^x).2.\frac{1}{1+(2^x)^2}\times2^x.\log_e 2$$

$$f(1)=\cos(2\tan^{-1}2)\frac{2}{1+4}\times2\times lag_e 2$$

$$\Rightarrow f(1)=\cos\cos^{-1}\left(\frac{1-2^2}{1+2^2}\right).\frac{4}{5}\log_e 2$$

$$=-\frac{12}{25}\log_e 2$$

$\Rightarrow$ a = 25, b = 12

$|a^2 - b^2| = |625 - 144| = 481$

4. Let there be three independent events E_1, E_2 and E_3. The probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let 'p' denote the probability of none of events occurs that satisfies the equations $(\alpha - 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0, 1).

   Then, $\frac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3}$ is equal to ________.

Answer (6)

Sol. Let $p(E_1) = x$, $p(E_2) = y$ and $p(E_3) = z$

$$\alpha = p(E_1 \cap \bar{E}_2 \cap \bar{E}_3) = p(E_1).p(\bar{E}_2).p(\bar{E}_3)$$

$$\Rightarrow \alpha = x(1-y)(1-z) \quad \text{...(i)}$$

Similarly

$$\beta = (1-x).y(1-z) \quad \text{...(ii)}$$

$$\gamma = (1-x)(1-y).z \quad \text{...(iii)}$$

$$p = (1-x)(1-y)(1-z) \quad \text{...(iv)}$$

(i) and (iv) $\Rightarrow \dfrac{x}{1-x} = \dfrac{x}{p} \Rightarrow x = \dfrac{\alpha}{\alpha+p}$

(iii) and (iv) $\Rightarrow \dfrac{z}{1-z} = \dfrac{\gamma}{p} \Rightarrow z = \dfrac{\gamma}{\gamma+p}$

$$\frac{p(E_1)}{p(E_2)} = \frac{x}{z} = \frac{\dfrac{\alpha}{\alpha+p}}{\dfrac{\gamma}{\gamma+p}} = \frac{\dfrac{\gamma+p}{\gamma}}{\dfrac{\alpha+p}{\alpha}} = \frac{1+\dfrac{p}{\gamma}}{1+\dfrac{p}{\alpha}} \quad \text{...(v)}$$

Given that

$(\alpha - 2\beta)p = \alpha\beta \Rightarrow \alpha p = (\alpha + 2p)\beta$ ...(vi)

$(\beta - 3\gamma)p = 2\beta\gamma \Rightarrow 3\gamma p = (p - 2\gamma)\beta$ ...(vii)

(vi) and (vii) $\Rightarrow \dfrac{\alpha}{3\gamma} = \dfrac{\alpha+2p}{p-2\gamma}$

$\Rightarrow p\alpha - 6p\gamma = 5\gamma\alpha$

$$\frac{p}{\gamma} - \frac{6p}{\alpha} = 5.$$

$$\frac{p}{\gamma} + 1 = 6\left(\frac{p}{\alpha}+1\right) \quad \text{...(viii)}$$

(v) and (viii) $\Rightarrow \dfrac{p(E_1)}{p(E_3)} = 6$

5. If the function $f(x) = \dfrac{\cos(\sin x) - \cos x}{x^4}$ is continuous at each point in its domain and $f(0) = \dfrac{1}{k}$, then k is ______.

Answer (6)

Sol. $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \dfrac{\cos(\sin x) - \cos x}{x^4}$

$$= \lim_{x\to 0} \frac{2\sin\left(\frac{x+\sin x}{2}\right)\sin\left(\frac{x-\sin x}{2}\right)}{x^4}$$

$$= \lim_{x\to 0} \frac{2\sin\left(\frac{x+\sin x}{2}\right)}{\left(\frac{x+\sin x}{2}\right)} \times \frac{\sin\left(\frac{x-\sin x}{2}\right)}{\left(\frac{x-\sin x}{2}\right)} \times \frac{x^2 - \sin^2 x}{4x^4}$$

$$= \lim_{x\to 0} 2 \times 1 \times 1 \times \left(\frac{x+\sin x}{x}\right)\frac{(x-\sin x)}{x^3} \times \frac{1}{4}$$

$$= 2 \times 2 \times \frac{1}{6} \times \frac{1}{4} = \frac{1}{6}$$

For continuity at x = 0, $f(0) = \dfrac{1}{6} = \dfrac{1}{k} \Rightarrow k = 6$

6. If [·] represent the greatest integer function, then the value of $\left|\int_0^{\sqrt{\frac{\pi}{2}}} \left[[x^2] - \cos x\right] dx\right|$ is ______.

Answer (1)

Sol. $\left|\int_0^{\sqrt{\frac{\pi}{2}}} \left[[x^2] - \cos x\right] dx\right| = \left|\int_0^1 (-1)dx + \int_1^{\sqrt{\frac{\pi}{2}}} 0 \cdot dx\right|$

= 1

7. If $(2021)^{3762}$ is divided by 17, then the remainder is ______.

Answer (4)

Sol. $(2021)^{3762}$

$= (2023 - 2)^{3762} = m(17) + 2^{3762}$

$\{\because\ 2023 = 17 \times 119\}$

Where m(17) denotes "multiple of 17"

Required remainder = remainder on dividing 2^{3762} by 17.

Now $2^{3762} = 4.16^{940} = 4.(1 - 17)^{940} = m(17) + 4$

Here required remainder is 4.

8. If $A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$, then the value of $\det(A^4)$ + $\det\left(A^{10} - (\text{Adj}(2A))^{10}\right)$ is equal to______.

Answer (16)

Sol. $\because\ A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}, A^2 = \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix}, A^3 = \begin{bmatrix} 8 & 9 \\ 0 & -1 \end{bmatrix}, ...$

So by mathematical induction we can conclude that

$$A^n = \begin{bmatrix} 2^n & 2^n - (-1)^n \\ 0 & (-1)^n \end{bmatrix}$$

Also 2A · (adj (2A)) = |2A|I

$\Rightarrow$ A · adj(2A) = –4I

$$\text{Now, } |A^{10} - (\text{adj}2A)^{10}| = \frac{\left|A^{20} - A^{10}(\text{adj}(2A))^{10}\right|}{|A|^{10}}$$

$$= \frac{\left|A^{20} - 2^{20}I\right|}{\left|A^{10}\right|} \qquad \text{...(i)}$$

$$A^{20} - A^{20} \cdot I = \begin{bmatrix} 2^{20} & 2^{20}-1 \\ 0 & 1 \end{bmatrix} - 2^{20}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2^{20}-1 \\ 0 & 1-2^{20} \end{bmatrix}$$

$\Rightarrow$ $|A^{20} - 2^{20}I| = 0$

From (i) $|A^{10} - (\text{adj}(2A))^{10}| = 0$

Hence, $\det(A^4) + \det(A^{10} - (\text{adj}(2A)^{10})$

$= |A|^4 + 0$

$= (-2)^4 = 16$

9. If the equation of the plane passing through the line of intersection of the planes 2x – 7y + 4z – 3 = 0, 3x – 5y + 4z + 11 = 0 and the point (–2, 1, 3) is ax + by + cz – 7 = 0, then the value of 2a + b + c – 7 is _____.

Answer (4)

Sol. Let $p_1 \equiv 2x - 7y + 4z - 3 = 0$

and $p_2 = 3x - 5y + 4z + 11 = 0$

Any plane through line of intersection of p_1 and p_2 is

$(2x - 7y + 4z - 3) + \lambda(3x - 5y + 4z + 11) = 0$

If passes through (–2, 1, 3)

$$-2 + 12\lambda = 0 \Rightarrow \lambda = \frac{1}{6}$$

Required plane is

15x – 47y + 28z – 7 = 0

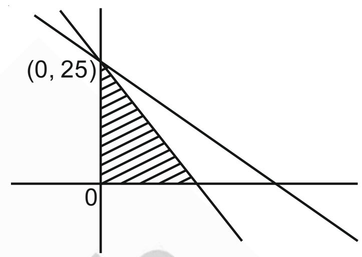
a = 15, b = –47, c = 28

2a + b + c – 7 = 4

10. The maximum value of z in the following equation $z = 6xy + y^2$, where $3x + 4y \le 100$ and $4x + 3y \le 75$ for $x \ge 0$ and $y \ge 0$ is _____.

Answer (904)

Sol.



$3x + 4y \le 100$

$4x + 3y \le 75$

$x \ge 0,\ y \ge 0$

Feasible region is shown in the graph

Let maximum value of $6xy + y^2 = c$

For a solution with feasible region,

$6xy + y^2 = c$ and $4x + 3y = 75$ must have atleast one positive solution.

$$y^2 + 6y\left(\frac{75-3y}{4}\right) - c = 0 \Rightarrow \frac{7}{2}y^2 - \frac{225}{2}y + c = 0$$

$$\Rightarrow \left(\frac{225}{2}\right)^2 \ge 4.\frac{7}{2}.c \Rightarrow c \le \frac{225^2}{56} \approx 904$$

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