

18/03/2021  
Evening



# Aakash

Medical | IIT-JEE | Foundations

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Time : 3 hrs.

## Answers & Solutions

M.M. : 300

*for*

## JEE (MAIN)-2021 (Online) Phase-2

(Physics, Chemistry and Mathematics)

### IMPORTANT INSTRUCTIONS :

- (1) The test is of **3 hours** duration.
- (2) The Test Booklet consists of 90 questions. The maximum marks are 300.
- (3) There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry and Mathematics** having 30 questions in each part of equal weightage. Each part has two sections.
  - (i) Section-I : This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **-1 mark** for wrong answer.
  - (ii) Section-II : This section contains 10 questions. In Section-II, attempt any **five questions out of 10**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and there is no negative marking for wrong answer.

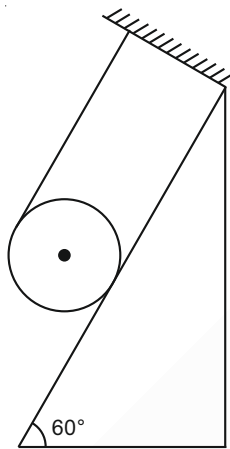
**PART-A : PHYSICS**

**SECTION - I**

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer :**

1. A solid cylinder of mass  $m$  is wrapped with an inextensible light string and, is placed on a rough inclined plane as shown in the figure. The frictional force acting between the cylinder and the inclined plane is:



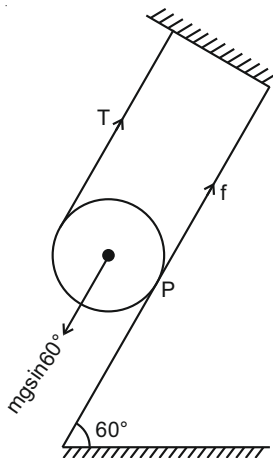
[The coefficient of static friction,  $\mu_s$ , is 0.4]

- (1) 0
- (2)  $5 mg$
- (3)  $\frac{7}{2} mg$
- (4)  $\frac{mg}{5}$

**Answer (4)**

**Sol.**  $T + f = mgsin60^\circ \dots(1)$

$\tau_p = 0$



$\Rightarrow T \times 2R = mgsin60^\circ \times R$

$\Rightarrow T = \frac{mg\sqrt{3}}{4} \dots(2)$

from (1) and (2)

$f = \frac{mg\sqrt{3}}{2} - \frac{mg\sqrt{3}}{4} = \frac{mg\sqrt{3}}{4}$

$f^{max} = (\mu) \frac{mg}{2} = \frac{mg}{5}$

$f > f_s^{max} \Rightarrow f = \frac{mg}{5}$

2. The correct relation between  $\alpha$  (ratio of collector current to emitter current) and  $\beta$  (ratio of collector current to base current) of a transistor is:

- (1)  $\alpha = \frac{\beta}{1+\beta}$
- (2)  $\alpha = \frac{\beta}{1-\alpha}$
- (3)  $\beta = \frac{1}{1-\alpha}$
- (4)  $\beta = \frac{\alpha}{1+\alpha}$

**Answer (1)**

**Sol.**  $\beta = \frac{\alpha}{1-\alpha}$  correct relation

$\Rightarrow \frac{1-\alpha}{\alpha} = \frac{1}{\beta}$

$\Rightarrow \frac{1}{\alpha} = 1 + \frac{1}{\beta} = \frac{\beta+1}{\beta}$

$\Rightarrow \alpha = \frac{\beta}{1+\beta}$

3. If the angular velocity of earth's spin is increased such that the bodies at the equator start floating, the duration of the day would be approximately:

[Take  $g = 10 \text{ ms}^{-2}$ , the radius of earth,  $R = 6400 \times 10^3 \text{ m}$ , Take  $\pi = 3.14$ ]

- (1) 60 minutes
- (2) Does not change
- (3) 84 minutes
- (4) 1200 minutes

**Answer (3)**

**Sol.**  $g = g_0 - R\omega^2$

$0 = g_0 - R\omega^2$

$\Rightarrow \omega = \sqrt{\frac{g_0}{R}}$

$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{R}{g_0}}$

$= 83.73 \text{ minutes}$

4. The time taken for the magnetic energy to reach 25% of its maximum value, when a solenoid of resistance  $R$ , inductance  $L$  is connected to a battery, is:

- (1)  $\frac{L}{R} \ln 5$  (2) Infinite  
 (3)  $\frac{L}{R} \ln 10$  (4)  $\frac{L}{R} \ln 2$

**Answer (4)**

**Sol.**  $U = \frac{1}{2} LI^2$

if  $U = \frac{U_{\max}}{4} \Rightarrow I = \frac{I_{\max}}{2}$

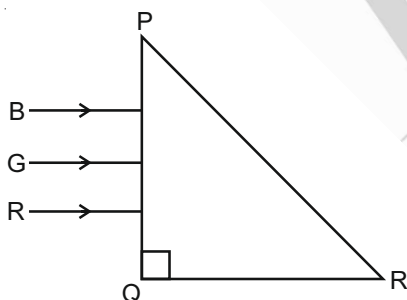
$I = I_0 \left(1 - e^{-\frac{Rt}{L}}\right)$

$\Rightarrow \frac{1}{2} = 1 - e^{-\frac{Rt}{L}}$

$\Rightarrow e^{-\frac{Rt}{L}} = \frac{1}{2} \Rightarrow (t) \frac{R}{L} = \ln 2$

$\Rightarrow t = \frac{L}{R} \ln 2$

5. Three rays of light, namely red (R), green (G) and blue (B) are incident on the face PQ of a right angled prism PQR as shown in the figure.



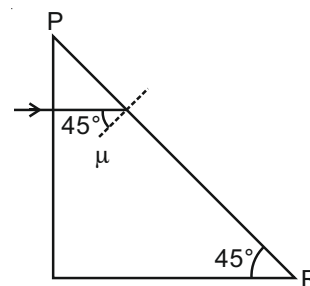
The refractive indices of the material of the prism for red, green and blue wavelength are 1.27, 1.42 and 1.49 respectively. The colour of the ray(s) emerging out of the face PR is:

- (1) Blue  
 (2) Green  
 (3) Red  
 (4) Blue and Green

**Answer (3)**

**Sol.** \* Data incomplete

\*Assuming angle at  $R = 45^\circ$



$\sin \theta_c = \frac{1}{\mu} \Rightarrow \mu = \sqrt{2}$

$\mu = 1.41$

$\mu_{\text{Red}} = 1.27 < 1.41$

$\Rightarrow$  Only red will emerge

6. A proton and an  $\alpha$ -particle, having kinetic energies  $K_p$  and  $K_\alpha$  respectively, enter into a magnetic field at right angles.

The ratio of the radii of trajectory of proton to that of  $\alpha$ -particle is 2 : 1. The ratio of  $K_p : K_\alpha$  is :

- (1) 1 : 8 (2) 8 : 1  
 (3) 1 : 4 (4) 4 : 1

**Answer (4)**

**Sol.**  $R = \frac{mV}{qB} = \frac{\sqrt{2mK}}{qB}$

$\frac{R_p}{R_\alpha} = \frac{\sqrt{m_p K_p} q_\alpha}{\sqrt{m_\alpha K_\alpha} q_p}$

$\Rightarrow \frac{K_p}{K_\alpha} = 4 : 1$

7. In a series LCR circuit, the inductive reactance ( $X_L$ ) is  $10\Omega$  and the capacitive reactance ( $X_C$ ) is  $4\Omega$ . The resistance ( $R$ ) in the circuit is  $6\Omega$ .

The power factor of the circuit is :

- (1)  $\frac{1}{\sqrt{2}}$  (2)  $\frac{1}{2}$   
 (3)  $\frac{1}{2\sqrt{2}}$  (4)  $\frac{\sqrt{3}}{2}$

**Answer (1)**

**Sol.**  $\cos \phi = \frac{R}{Z} = \frac{6}{\sqrt{(10-4)^2 + (6)^2}}$   
 $= \frac{6}{\sqrt{36 \times 2}} = \frac{1}{\sqrt{2}}$

8. Consider a uniform wire of mass  $M$  and length  $L$ . It is bent into a semicircle. Its moment of inertia about a line perpendicular to the plane of the wire passing through the centre is:

- (1)  $\frac{2 ML^2}{5 \pi^2}$  (2)  $\frac{1 ML^2}{4 \pi^2}$   
 (3)  $\frac{1 ML^2}{2 \pi^2}$  (4)  $\frac{ML^2}{\pi^2}$

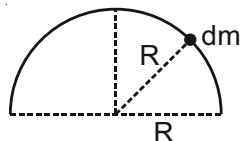
**Answer (4)**

**Sol.**  $I = \int (dm)r^2$

$I = (R^2) \int dm$

$= MR^2$

$= \frac{ML^2}{\pi^2}$



$\pi R = L$

$\Rightarrow R = \frac{L}{\pi}$

9. Consider a sample of oxygen behaving like an ideal gas. At 300 K, the ratio of root mean square (rms) velocity to the average velocity of gas molecule would be :

(Molecular weight of oxygen is 32 g/mol;  $R = 8.3 \text{ J K}^{-1} \text{ mol}^{-1}$ )

- (1)  $\sqrt{\frac{8\pi}{3}}$  (2)  $\sqrt{\frac{3}{8}}$   
 (3)  $\sqrt{\frac{8}{3}}$  (4)  $\sqrt{\frac{3\pi}{8}}$

**Answer (4)**

**Sol.**  $V_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

$V_{\text{avg}} = \sqrt{\frac{8RT}{\pi M}}$

$\Rightarrow \frac{V_{\text{rms}}}{V_{\text{avg}}} = \sqrt{\frac{3\pi}{8}}$

10. For an adiabatic expansion of an ideal gas, the fractional change in its pressure is equal to (where  $\gamma$  is the ratio of specific heats) :

- (1)  $-\gamma \frac{dV}{V}$  (2)  $-\gamma \frac{dV}{V}$   
 (3)  $-\frac{1}{\gamma} \frac{dV}{V}$  (4)  $\frac{dV}{V}$

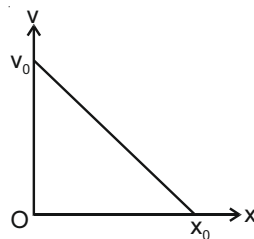
**Answer (2)**

**Sol.**  $PV^\gamma = C$

$\Rightarrow (P)(\gamma)V^{\gamma-1}dV + V^\gamma dP = 0$

$\Rightarrow dP = \frac{-\gamma P}{V} dV \Rightarrow \frac{dP}{P} = -\frac{\gamma dV}{V}$

11. The velocity-displacement graph of a particle is shown in the figure.



The acceleration-displacement graph of the same particle is represented by:

- (1) (2)   
 (3) (4)

**Answer (4)**

**Sol.**  $a = v \frac{dv}{dx}$

$\frac{dv}{dx} < 0$  and  $v$  is decreasing with  $x$

12. The decay of a proton to neutron is:

- (1) Always possible as it is associated only with  $\beta^+$  decay  
 (2) Not possible but neutron to proton conversion is possible  
 (3) Not possible as proton mass is less than the neutron mass  
 (4) Possible only inside the nucleus

**Answer (4)**

**Sol.**  $\beta^+$  decay is possible only inside the nucleus.

13. The speed of electrons in a scanning electron microscope is  $1 \times 10^7 \text{ ms}^{-1}$ . If the protons having the same speed are used instead of electrons, then the resolving power of scanning proton microscope will be changed by a factor of

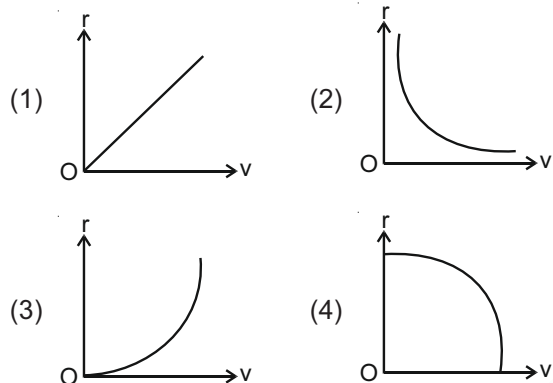
- (1)  $\frac{1}{\sqrt{1837}}$  (2)  $\frac{1}{1837}$   
 (3)  $\sqrt{1837}$  (4) 1837

**Answer (4)**

**Sol.**  $R.P \propto \frac{1}{\lambda}, \lambda \propto \frac{1}{P}$

14. A particle of mass  $m$  moves in a circular orbit under the central potential field,  $U(r) = -\frac{C}{r}$ , where  $C$  is a positive constant.

The correct radius – velocity graph of the particle's motion is



**Answer (2)**

**Sol.**  $F_r = -\frac{dU}{dr}$   
 $= -\frac{d}{dr}\left(-\frac{c}{r}\right) = -\frac{c}{r^2}$   
 $\Rightarrow \frac{mv^2}{r} = \frac{c}{r^2}$   
 $\Rightarrow v^2 = \frac{c}{mr}$

15. Which of the following statements are correct?
- (A) Electric monopoles do not exist whereas magnetic monopoles exist.
  - (B) Magnetic field lines due to a solenoid at its ends and outside cannot be completely straight and confined.
  - (C) Magnetic field lines are completely confined within a toroid.
  - (D) Magnetic field lines inside a bar magnet are not parallel.
  - (E)  $x = -1$  is the condition for a perfect diamagnetic material, where  $x$  is its magnetic susceptibility.

Choose the correct answer from the options given below:

- (1) (C) and (E) only      (2) (B) and (C) only
- (3) (B) and (D) only      (4) (A) and (B) only

**Answer (3)**

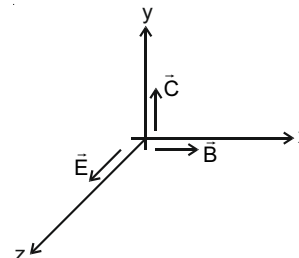
**Sol.** Theory based.

16. A plane electromagnetic wave propagating along  $y$ -direction can have the following pair of electric field ( $\vec{E}$ ) and magnetic field ( $\vec{B}$ ) components.

- (1)  $E_x, B_z$  or  $E_z, B_x$       (2)  $E_x, B_y$  or  $E_y, B_x$
- (3)  $E_y, B_y$  or  $E_z, B_z$       (4)  $E_y, B_x$  or  $E_x, B_y$

**Answer (1)**

**Sol.**  $\vec{E} \times \vec{B} = \vec{C}$



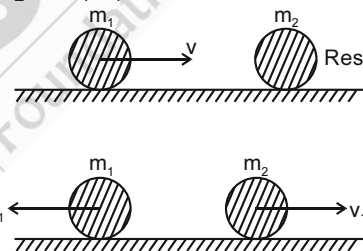
17. An object of mass  $m_1$  collides with another object of mass  $m_2$ , which is at rest. After the collision the objects move with equal speeds in opposite direction. The ratio of the masses  $m_2 : m_1$  is:

- (1) 2 : 1      (2) 3 : 1
- (3) 1 : 2      (4) 1 : 1

**Answer (2)**

**Sol.** Assuming  $e = 1$

$m_1 v = (m_2 - m_1)v_1$



$\Rightarrow v_1 = \frac{m_1 v}{(m_2 - m_1)} \dots(i)$

also  $2v_1 = v$

$\Rightarrow v_1 = \frac{v}{2}$

$\Rightarrow \frac{1}{2} = \frac{m_1}{m_2 - m_1}$

$\Rightarrow \frac{m_2}{m_1} = 3$

18. The angular momentum of a planet of mass  $M$  moving around the sun in an elliptical orbit is  $\vec{L}$ . The magnitude of the areal velocity of the planet is:

- (1)  $\frac{L}{2M}$       (2)  $\frac{L}{M}$
- (3)  $\frac{4L}{M}$       (4)  $\frac{2L}{M}$

**Answer (1)**

**Sol.**  $\frac{dA}{dt} = \frac{L}{2M}$  standard result

19. The function of time representing a simple harmonic motion with a period of  $\frac{\pi}{\omega}$  is:

- (1)  $3\cos\left(\frac{\pi}{4} - 2\omega t\right)$
- (2)  $\cos(\omega t) + \cos(2\omega t) + \cos(3\omega t)$
- (3)  $\sin^2(\omega t)$
- (4)  $\sin(\omega t) + \cos(\omega t)$

**Answer (1)**

**Sol.**  $T = \frac{2\pi}{\omega'} = \frac{\pi}{\omega}$   
 $\Rightarrow \omega' = 2\omega$   
 $\Rightarrow y = 3\cos\left(\frac{\pi}{4} - 2\omega t\right)$

20. An ideal gas in a cylinder is separated by a piston in such a way that the entropy of one part is  $S_1$  and that of the other part is  $S_2$ . Given that  $S_1 > S_2$ . If the piston is removed then the total entropy of the system will be:

- (1)  $S_1 - S_2$
- (2)  $S_1 \times S_2$
- (3)  $\frac{S_1}{S_2}$
- (4)  $S_1 + S_2$

**Answer (4)**

**Sol.** Entropy will increase.

**SECTION - II**

**Numerical Value Type Questions:** This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. The radius of a sphere is measured to be  $(7.50 \pm 0.85)$  cm. Suppose the percentage error in its volume is  $x$ .

The value of  $x$ , to the nearest  $x$ , is \_\_\_\_\_.

**Answer (34)**

**Sol.**  $v = \frac{4}{3}\pi R^3$   
 $\Rightarrow \frac{\Delta v}{v} \times 100 = (3) \frac{\Delta R}{R} \times 100$   
 $= (3) \times \frac{0.85}{7.5} \times 100$   
 $= 34$

2. An infinite number of point charges, each carrying  $1 \mu\text{C}$  charge, are placed along the  $y$ -axis at  $y = 1 \text{ m}, 2 \text{ m}, 4 \text{ m}, 8 \text{ m}$  \_\_\_\_\_.

The total force on a  $1 \text{ C}$  point charge, placed at the origin, is  $x \times 10^3 \text{ N}$ .

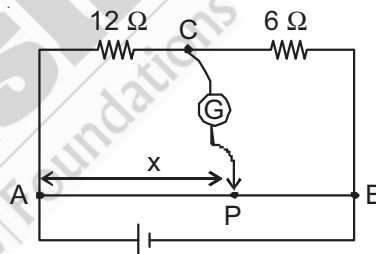
The value of  $x$ , to the nearest integer, is \_\_\_\_\_.

[Take  $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$ ]

**Answer (12)**

**Sol.**  $F = (k)(10^{-6}) \left[ \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{4^2} + \dots \right]$   
 $= \frac{(k)10^{-6}}{1 - \frac{1}{4}} = \frac{(9 \times 10^9) \times 4 \times 10^{-6} \text{ N}}{3}$   
 $= 12 \times 10^3 \text{ N}$

3. Consider a  $72 \text{ cm}$  long wire  $AB$  as shown in the figure. The galvanometer jockey is placed at  $P$  on  $AB$  at a distance  $x \text{ cm}$  from  $A$ . The galvanometer shows zero deflection.



The value of  $x$ , to the nearest integer, is \_\_\_\_\_.

**Answer (48)**

**Sol.**  $\frac{12}{6} = \frac{x}{(72 - x)}$   
 $\Rightarrow x = 48 \text{ cm}$

4. A galaxy is moving away from the earth at a speed of  $286 \text{ kms}^{-1}$ . The shift in the wavelength of a redline at  $630 \text{ nm}$  is  $x \times 10^{-10} \text{ m}$ .

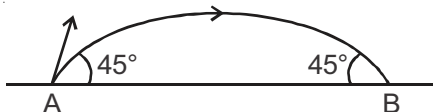
The value of  $x$ , to the nearest integer, is \_\_\_\_\_.

[Take the value of speed of light  $c$ , as  $3 \times 10^8 \text{ ms}^{-1}$ ]

**Answer (6)**

**Sol.**  $\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$   
 $\Rightarrow \Delta\lambda = \frac{v}{c} \times \lambda$   
 $= \frac{286 \times 10^3}{3 \times 10^8} \times 630 \times 10^{-9}$   
 $= 6.006 \times 10^{-10}$

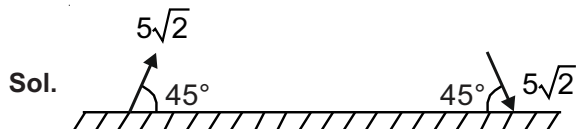
5. The projectile motion of a particle of mass 5 g is shown in the figure.



The initial velocity of the particle is  $5\sqrt{2}$  ms<sup>-1</sup> and the air resistance is assumed to be negligible. The magnitude of the change in momentum between the points A and B is  $x \times 10^{-2}$  kgms<sup>-1</sup>.

The value of x, to the nearest integer, is \_\_\_\_\_.

**Answer (5)**



Sol.  $|\Delta P| = 2 \times (5 \times 10^{-3}) (5)$   
 $= 5 \times 10^{-2}$  kg ms<sup>-1</sup>

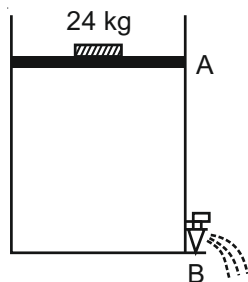
6. Two wires of same length and thickness having specific resistances  $6 \Omega \text{ cm}$  and  $3 \Omega \text{ cm}$  respectively are connected in parallel. The effective resistivity is  $\rho \Omega \text{ cm}$ . The value of  $\rho$ , to the nearest integer, is \_\_\_\_\_.

**Answer (4)**

Sol.  $R = \frac{R_1 R_2}{R_1 + R_2}$   
 $\rho(l) = \frac{\left(\frac{\rho_1 l}{A}\right) \left(\frac{\rho_2 l}{A}\right)}{\frac{\rho_1 l}{A} + \frac{\rho_2 l}{A}}$   
 $\frac{\rho}{2} = \frac{\rho_1 \rho_2}{\rho_1 + \rho_2} \Rightarrow \frac{\rho}{2} = \frac{6 \times 3}{6 + 3} = 2$   
 $\Rightarrow \rho = 4$

7. Consider a water tank as shown in the figure. It's cross-sectional area is  $0.4 \text{ m}^2$ . The tank has an opening B near the bottom whose cross-section area is  $1 \text{ cm}^2$ . A load of 24 kg is applied on the water at the top when the height of the water level is 40 cm above the bottom, the velocity of water coming out the opening B is  $v \text{ ms}^{-1}$ .

The value of v, to the nearest integer, is \_\_\_\_\_.  
 [Take value of g to be  $10 \text{ ms}^{-2}$ ]



**Answer (3)**

Sol.  $\frac{mg}{A} + \rho gH = \frac{1}{2} \rho v^2$   
 $\Rightarrow \frac{240}{0.4} + 1000 \times 10 \times 0.4 = \frac{1}{2} \times 1000 \times v^2$   
 $\Rightarrow v = 3 \text{ m/s}$

8. A ball of mass 4 kg, moving with a velocity of  $10 \text{ ms}^{-1}$ , collides with a spring of length 8 m and force constant  $100 \text{ Nm}^{-1}$ . The length of the compressed spring is x m. The value of x, to the nearest integer, is \_\_\_\_\_.

**Answer (6)**

Sol.  $\frac{1}{2} \times 4 \times (10)^2 = \frac{1}{2} \times 100 \times (\Delta x)^2$   
 $\Rightarrow \Delta x = 2 \text{ m}$   
 $\Rightarrow x = 8 - 2 = 6 \text{ m}$

9. A TV transmission tower antenna is at a height of 20 m. Suppose that the receiving antenna is at.

- (i) ground level
- (ii) a height of 5 m

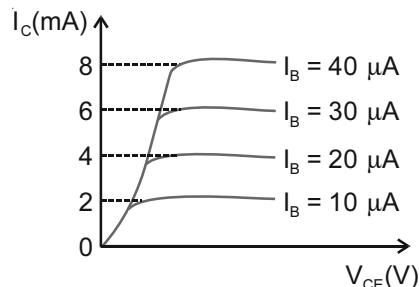
The increase in antenna range in case (ii) relative to case (i) is n%.

The value of n, to the nearest integer, is \_\_\_\_\_.

**Answer (50)**

Sol.  $d = \sqrt{2hR_e}$   
 $d_1 = \sqrt{2 \times 20 \times R_e}$   
 $d_2 = \sqrt{2 \times 20 \times R_e} + \sqrt{2 \times 5 \times R_e}$   
 $\Delta d = \frac{(d_2 - d_1)}{d_1} = \frac{\sqrt{10 R_e}}{\sqrt{40 R_e}} = \frac{1}{2}$   
 % increase = 50

10. The typical output characteristics curve for a transistor working in the common-emitter configuration is shown in the figure.



The estimated current gain from the figure is \_\_\_\_\_.

**Answer (200)**

Sol.  $\beta = \frac{\Delta I_C}{\Delta I_B}$   
 $= \frac{2 \times 10^{-3}}{10 \times 10^{-6}} = 200$

**PART-B : CHEMISTRY**

**SECTION - I**

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer :**

1. Given below are two statements :

**Statement I :** Non-biodegradable wastes are generated by the thermal power plants.

**Statement II :** Bio-degradable detergents leads to eutrophication.

In the light of the above statements, choose the most appropriate answer from the options given below :

- (1) Both statement I and statement II are false
- (2) Statement I is true but statement II is false
- (3) Statement I is false but statement II is true
- (4) Both statement I and statement II are true

**Answer (4)**

**Sol.** NCERT, Page No. – 415, 417 – Part-II, Class-XI

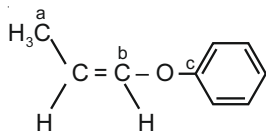
2. The oxide that shows magnetic property is :

- (1) MgO
- (2) Na<sub>2</sub>O
- (3) SiO<sub>2</sub>
- (4) Mn<sub>3</sub>O<sub>4</sub>

**Answer (4)**

**Sol.** Mn<sub>3</sub>O<sub>4</sub> is magnetic in nature.

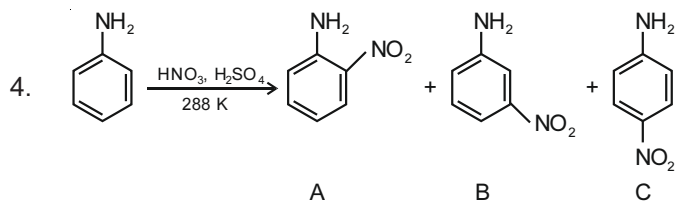
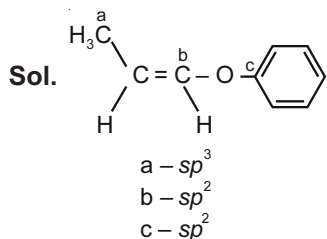
3. In the following molecule,



Hybridisation of Carbon a, b and c respectively are :

- (1) sp<sup>3</sup>, sp, sp
- (2) sp<sup>3</sup>, sp<sup>2</sup>, sp<sup>2</sup>
- (3) sp<sup>3</sup>, sp, sp<sup>2</sup>
- (4) sp<sup>3</sup>, sp<sup>2</sup>, sp

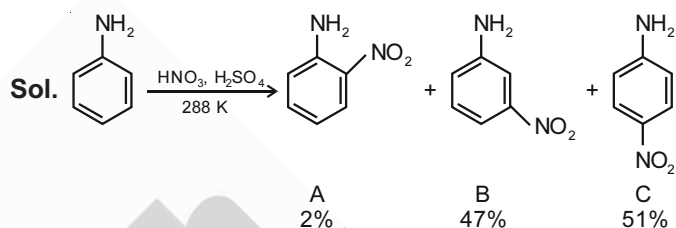
**Answer (2)**



Consider the given reaction, percentage yield of :

- (1) C > B > A
- (2) C > A > B
- (3) B > C > A
- (4) A > C > B

**Answer (1)**



In strongly acidic medium aniline is protonated to form anilinium ion making it m-directing but p-derivative is also formed in significant amount which is a major product.

5. Deficiency of vitamin K causes :

- (1) Cheilosis
- (2) Increase in blood clotting time
- (3) Increase in fragility of RBC's
- (4) Decrease in blood clotting time

**Answer (2)**

**Sol.** Deficiency of vitamin K causes increase in blood clotting time.

6. Match List-I with List-II :

List-I (Class of Chemicals)	List-II (Example)
(a) Antifertility drug	(i) Meprobamate
(b) Antibiotic	(ii) Alitame
(c) Tranquilizer	(iii) Norethindrone
(d) Artificial Sweetener	(iv) Salvarsan

Choose the most appropriate match :

- (1) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)
- (2) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)
- (3) (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)
- (4) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

**Answer (1)**



- Sol.** (a) Antifertility drug — Norethindrone  
 (b) Antibiotic — Salvarsan  
 (c) Tranquilizer — Meprobamate  
 (d) Artificial Sweetener — Alitame  
 (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

7. Given below are two statements :

**Statement I :**  $C_2H_5OH$  and  $AgCN$  both can generate nucleophile.

**Statement II :**  $KCN$  and  $AgCN$  both will generate nitrile nucleophile with all reaction conditions.

Choose the **most appropriate** option.

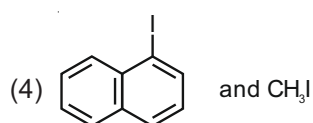
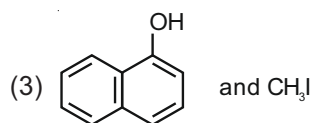
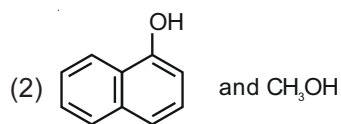
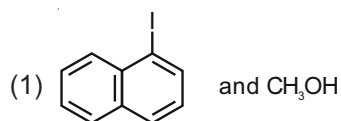
- (1) Statement I is true but statement II is false  
 (2) Statement I is false but statement II is true  
 (3) Both statement I and statement II are true  
 (4) Both statement I and statement II are false

**Answer (1)**

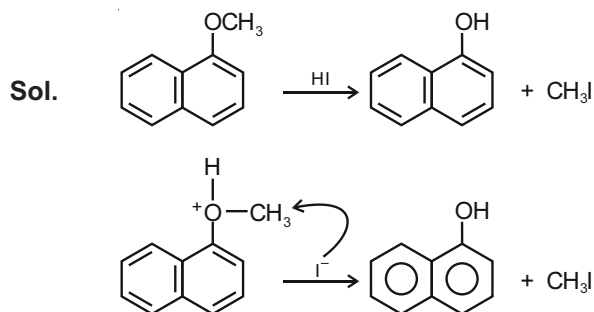
**Sol.** Both  $C_2H_5OH$  and  $AgCN$  can generate nucleophile.

$KCN$  generates nitriles on substitution reactions with haloalkanes where  $AgCN$  generates isonitriles on substitution reactions with haloalkanes. Because  $KCN$  is ionic and has 'C' nucleophilic centre whereas  $AgCN$  is covalent and has 'N' nucleophilic centre.

8. Main Products formed during a reaction of 1-methoxy naphthalene with hydroiodic acid are :



**Answer (3)**



9. The first ionization energy of magnesium is smaller as compared to that of elements X and Y, but higher than that of Z. The elements X, Y and Z, respectively, are

- (1) chlorine, lithium and sodium  
 (2) argon, lithium and sodium  
 (3) argon, chlorine and sodium  
 (4) neon, sodium and chlorine

**Answer (3)**

**Sol.** First ionisation energy of Mg is small than Argon and chlorine but higher than Na.

So  $X \rightarrow$  Argon

$Y \rightarrow$  Chlorine

$Z \rightarrow$  Sodium

10. The charges on the colloidal  $CdS$  sol and  $TiO_2$  sol are, respectively

- (1) positive and positive  
 (2) positive and negative  
 (3) negative and positive  
 (4) negative and negative

**Answer (3)**

**Sol.**  $CdS$  is negatively charged sol whereas  $TiO_2$  is positively charged sol.

11. In basic medium,  $H_2O_2$  exhibits which of the following reactions ?

(A)  $Mn^{2+} \rightarrow Mn^{4+}$

(B)  $I_2 \rightarrow I^-$

(C)  $PbS \rightarrow PbSO_4$

Choose the **most appropriate** answer from the options given below.

- (1) (A), (C) only                      (2) (A), (B) only  
 (3) (A) only                              (4) (B) only

**Answer (2)**

**Sol.** Possible reactions in basic medium are

$Mn^{2+} \rightarrow Mn^{4+}$

$I_2 \rightarrow I^-$

$PbS \rightarrow PbSO_4$  is possible in acidic medium

12. Given below are two statements :

**Statement I** : Bohr's theory accounts for the stability and line spectrum of  $\text{Li}^+$  ion.

**Statement II** : Bohr's theory was unable to explain the splitting of spectral lines in the presence of a magnetic field.

In the light of the above statements, choose the **most appropriate** answer from the options given below.

- (1) Both statement I and statement II are true
- (2) Statement I is false but statement II is true
- (3) Both statement I and statement II are false
- (4) Statement I is true but statement II is false

**Answer (2)**

**Sol.** Bohr's theory is applicable for unielectronic species only

$\text{Li}^+$  has two electrons

Bohr's theory could not explain the splitting of spectral lines in the presence of external magnetic field (Zeeman effect)

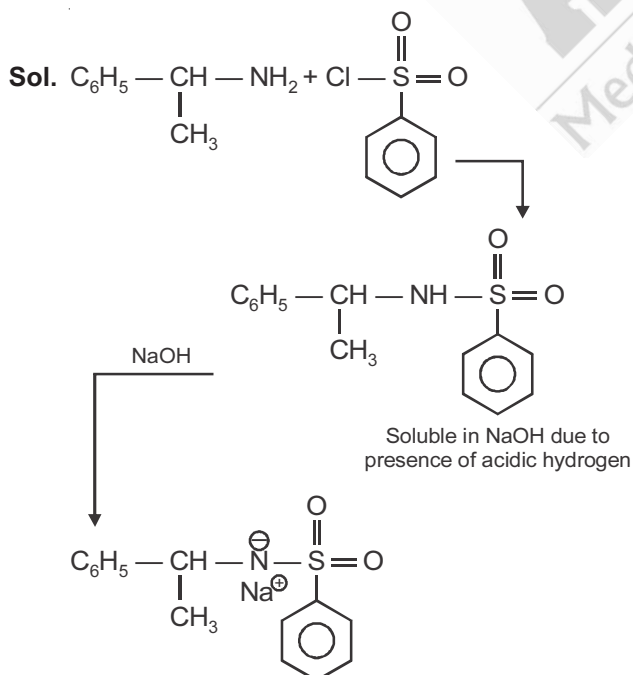
Statement I – false

Statement II – true

13. An organic compound "A" on treatment with benzene sulphonyl chloride gives compound B. B is soluble in dil. NaOH solution. Compound A is

- (1)  $\text{C}_6\text{H}_5 - \text{N} - (\text{CH}_3)_2$
- (2)  $\text{C}_6\text{H}_5 - \text{CH}_2\text{NHCH}_3$
- (3)  $\text{C}_6\text{H}_5 - \underset{\text{CH}_3}{\text{CH}} - \text{NH}_2$
- (4)  $\text{C}_6\text{H}_5 - \text{NHCH}_2\text{CH}_3$

**Answer (3)**



14. Match List-I with List-II :

List-I	List-II
(a) Mercury	(i) Vapour phase refining
(b) Copper	(ii) Distillation refining
(c) Silicon	(iii) Electrolytic refining
(d) Nickel	(iv) Zone refining

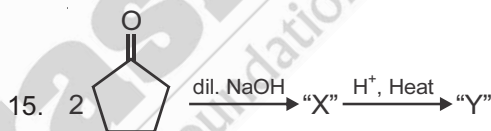
Choose the **most appropriate** answer from the option given below.

- (1) (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)
- (2) (a)-(ii), (b)-(iii), (c)-(i), (d)-(iv)
- (3) (a)-(i), (b)-(iv), (c)-(ii), (d)-(iii)
- (4) (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)

**Answer (4)**

Sol.	Element	Refining method
(a)	Mercury	Distillation refining
(b)	Copper	Electrolytic refining
(c)	Silicon	Zone refining
(d)	Nickel	Vapour phase refining

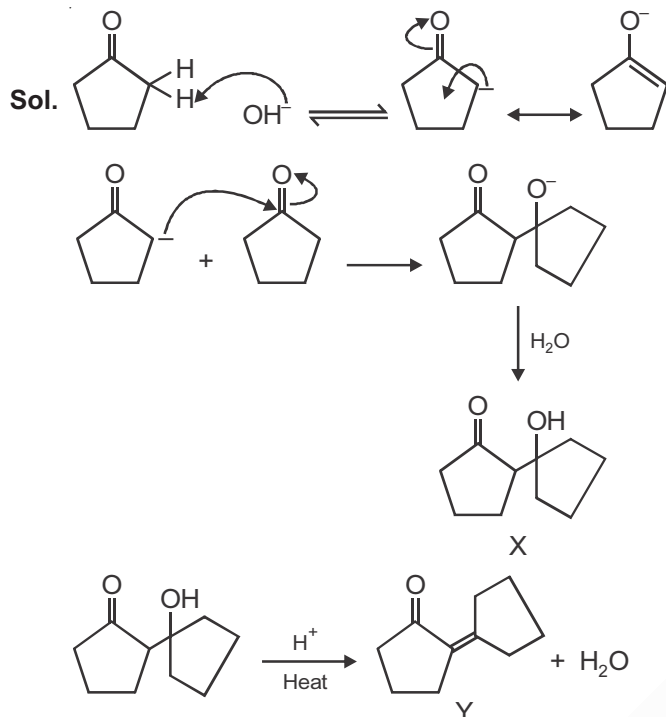
So, (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i).



Consider the above reaction, the product 'X' and 'Y' respectively are

- (1)
- (2)
- (3)
- (4)

**Answer (4)**



16. The secondary valency and the number of hydrogen bonded water molecule(s) in  $\text{CuSO}_4 \cdot 5\text{H}_2\text{O}$ , respectively, are

- (1) 5 and 1                      (2) 6 and 4  
(3) 6 and 5                      (4) 4 and 1

**Answer (4)**

**Sol.** Out of five  $\text{H}_2\text{O}$  molecules, four  $\text{H}_2\text{O}$  are the part of secondary valency and one  $\text{H}_2\text{O}$  is H-bonded.

17. Match **List-I** with **List-II** :

List-I	List-II
(a) Be	(i) treatment of cancer
(b) Mg	(ii) extraction of metals
(c) Ca	(iii) incendiary bombs and signals
(d) Ra	(iv) windows of X-ray tubes
	(v) bearings for motor engines

Choose the **most appropriate** answer from the option given below.

- (1) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(v)  
(2) (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii)  
(3) (a)-(iii), (b)-(iv), (c)-(v), (d)-(ii)  
(4) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)

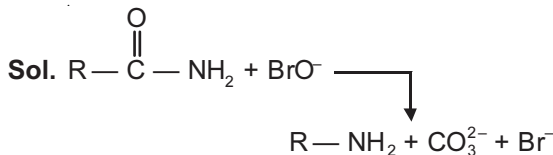
**Answer (4)**

**Sol.** Be – used in making windows of X-ray tube  
Mg – incendiary bombs and signals  
Ca – extraction of metals  
Ra – treatment of cancer

18. In the reaction of hypobromite with amide, the carbonyl carbon is lost as

- (1)  $\text{CO}_3^{2-}$                       (2)  $\text{HCO}_3^-$   
(3)  $\text{CO}_2$                       (4) CO

**Answer (1)**



Carbonyl carbon is lost as  $\text{CO}_3^{2-}$ .

19. A hard substance melts at high temperature and is an insulator in both solid and in molten state. This solid is most likely to be a/an:

- (1) Metallic solid                      (2) Ionic solid  
(3) Molecular solid                      (4) Covalent solid

**Answer (4)**

**Sol.** Covalent solid have high melting point due to network like structure and they are insulator e.g. diamond,  $\text{SiO}_2$

20. The oxidation states of nitrogen in NO,  $\text{NO}_2$ ,  $\text{N}_2\text{O}$  and  $\text{NO}_3^-$  are in the order of :

- (1)  $\text{NO}_2 > \text{NO}_3^- > \text{NO} > \text{N}_2\text{O}$   
(2)  $\text{NO}_3^- > \text{NO}_2 > \text{NO} > \text{N}_2\text{O}$   
(3)  $\text{N}_2\text{O} > \text{NO}_2 > \text{NO} > \text{NO}_3^-$   
(4)  $\text{NO} > \text{NO}_2 > \text{N}_2\text{O} > \text{NO}_3^-$

**Answer (2)**

**Sol.**

	O.S of N
$\text{NO}_3^-$	+5
$\text{NO}_2$	+4
NO	+2
$\text{N}_2\text{O}$	+1

## SECTION - II

**Numerical Value Type Questions:** This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. 10.0 mL of  $\text{Na}_2\text{CO}_3$  solution is titrated against 0.2 M HCl solution. The following titre values were obtained in 5 readings:

4.8 mL, 4.9 mL, 5.0 mL, 5.0 mL and 5.0 mL

Based on these readings and convention of titrimetric estimation the concentration of  $\text{Na}_2\text{CO}_3$  solution is \_\_\_\_\_ mM

(Round off the Nearest integer).

**Answer (50)**



equivalents of  $\text{Na}_2\text{CO}_3$  = equivalents of HCl

$$2 \times M \times 10 = 1 \times 0.2 \times 5$$

$$M = \frac{0.2 \times 5}{20} = 0.05 \text{ M}$$

$$= 5 \times 10^{-2} \text{ M}$$

$$= 50 \times 10^{-3} \text{ M} = 50 \text{ mM}$$

2. A reaction has a half life of 1 min. The time required for 99.9% completion of the reaction is \_\_\_\_\_ min. (Round off to the Nearest integer).

[Use :  $\ln 2 = 0.69$ ;  $\ln 10 = 2.3$ ]

**Answer (10\*\*)**

**Sol.** Order is not mentioned, question is incomplete. If we assume, the given reaction to be first order, then

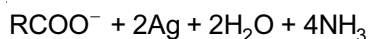
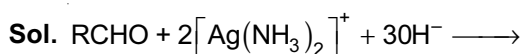
$$\ln(1000) = \frac{\ln 2}{t_{1/2}} t$$

$$t = \frac{1 \ln(1000)}{\ln 2}$$

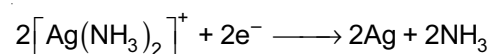
$$= 10 \text{ min}$$

3. In Tollen's test for aldehyde, the overall number of electron(s) transferred to the Tollen's reagent formula  $[\text{Ag}(\text{NH}_3)_2]^+$  per aldehyde group to form silver mirror is \_\_\_\_\_. (Round off to the Nearest Integer).

**Answer (2)**

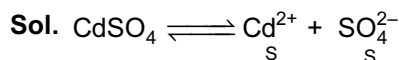


for 1 mol of R-CHO, 2 mol of  $[\text{Ag}(\text{NH}_3)_2]^+$  is reduced.



4. The solubility of  $\text{CdSO}_4$  in water is  $80 \times 10^{-4} \text{ mol L}^{-1}$ . Its solubility in 0.01 M  $\text{H}_2\text{SO}_4$  solution is \_\_\_\_\_  $\times 10^{-6} \text{ mol L}^{-1}$ . (Round off to the Nearest Integer). (Assume that solubility is much less than 0.01 M)

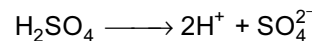
**Answer (64)**



$$k_{\text{sp}} = S^2$$

$$= (8 \times 10^{-4})^2$$

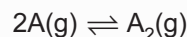
$$k_{\text{sp}} = 64 \times 10^{-8}$$



$$64 \times 10^{-8} = S' \times 0.01$$

$$S' = \frac{64 \times 10^{-8}}{0.01} = 64 \times 10^{-6} \text{ mol/L}$$

5. The gas phase reaction



at 400 K has  $\Delta G^\circ = +25.2 \text{ kJ mol}^{-1}$

The equilibrium constant  $K_C$  for this reaction is \_\_\_\_\_  $\times 10^{-2}$ . (Round off to the Nearest Integer).

[Use :  $R = 8.3 \text{ J mol}^{-1} \text{ K}^{-1}$ ,  $\ln 10 = 2.3 \log_{10}$ ,  $2 = 0.30$ ,  $1 \text{ atm} = 1 \text{ bar}$ ]

[antilog (- 0.3) = 0.501]

**Answer (166)**



$$\Delta G^\circ = 25.2 \text{ kJ mol}^{-1} \text{ at } 400 \text{ K}$$

$$\Delta G^\circ = -RT \ln K_p$$

$$25.2 \times 10^3 = -8.3 \times 400 \times 2.3 \log K_p$$

$$\log K_p = \frac{-25.2 \times 10^3}{8.3 \times 400 \times 2.3}$$

$$\log K_p = -3.3$$

$$K_p = 5 \times 10^{-4}$$

$$K_p = k_c (\text{RT})^{-1}$$

$$K_c = 5 \times 10^{-4} \times 8.3 \times 400 = 1.66$$

$$= 166 \times 10^{-2}$$

6. The molar conductivities at infinite dilution of barium chloride, sulphuric acid and hydrochloric acid are 280, 860 and 426  $\text{S cm}^2 \text{ mol}^{-1}$  respectively. The molar conductivity at infinite dilution of barium sulphate is \_\_\_\_\_  $\text{S cm}^2 \text{ mol}^{-1}$ . (Round off to the Nearest Integer).

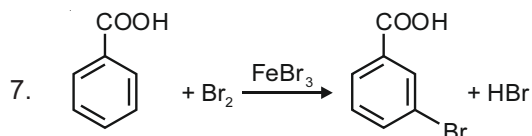
**Answer (288)**

**Sol.**  $\Lambda^\infty (\text{BaCl}_2) = 280 \text{ S cm}^2 \text{ mol}^{-1}$

$\Lambda^\infty (\text{H}_2\text{SO}_4) = 860 \text{ S cm}^2 \text{ mol}^{-1}$

$\Lambda^\infty (\text{HCl}) = 426 \text{ S cm}^2 \text{ mol}^{-1}$

$$\begin{aligned} \Lambda^\infty (\text{BaSO}_4) &= \Lambda^\infty (\text{BaCl}_2) + \Lambda^\infty (\text{H}_2\text{SO}_4) - 2\Lambda^\infty (\text{HCl}) \\ &= 280 + 860 - 2 \times 426 \\ &= 288 \text{ S cm}^2 \text{ mol}^{-1} \end{aligned}$$



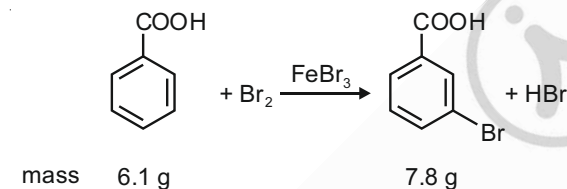
Consider the above reaction where 6.1 g of Benzoic acid is used to get 7.8 g of m-bromobenzoic acid. The percentage yield of the product is \_\_\_\_.

(Round off to Nearest Integer).

[Given : Atomic masses : C : 12.0 u, H : 1.0 u, O : 16.0 u, Br : 80.0 u]

**Answer (78)**

**Sol.**



moles  $\frac{6.1}{122} = 0.05$

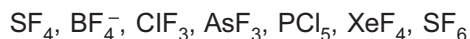
1 mol of benzoic acid give 1 mol of m-bromobenzoic acid. 0.05 mol of benzoic acid will give 0.05 mol of m-bromobenzoic acid.

So, percentage yield is

$$\% \text{ yield} = \frac{7.8 \times 100}{0.05 \times 201} = 77.61\%$$

$\approx 78\%$

8. The number of species below that have two lone pairs of electrons in their central atom is \_\_\_\_\_. (Round off to the Nearest Integer).



**Answer (2)**

Sol. Species	No. of lone pair of electron present on central atom
$\text{SF}_4$	1
$\text{BF}_4^-$	0
$\text{ClF}_3$	2

$\text{AsF}_3$	1
$\text{PCl}_5$	0
$\text{BrF}_5$	1
$\text{XeF}_4$	2
$\text{SF}_6$	0

9. A solute A dimerizes in water. The boiling point of a 2 molal solution of A is  $100.52^\circ\text{C}$ . The percentage association of A is \_\_\_\_\_. (Round off to the Nearest Integer).

[Use :  $K_b$  for water =  $0.52 \text{ K kg mol}^{-1}$

Boiling point of water =  $100^\circ\text{C}$ ]

**Answer (100)**



$$1 - \alpha \quad \frac{\alpha}{2}$$

$$i = 1 - \alpha + \frac{\alpha}{2}$$

Also

$$\Delta T_b = iK_b m$$

$$0.52 = i \times 0.52 \times 2$$

$$i = \frac{1}{2}$$

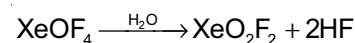
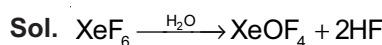
$$\frac{1}{2} = 1 - \alpha + \frac{\alpha}{2}$$

$$\Rightarrow 1 - \frac{\alpha}{2} = \frac{1}{2}$$

% of association of A is 100%

10. A xenon compound 'A' upon partial hydrolysis gives  $\text{XeO}_2\text{F}_2$ . The number of lone pair of electrons present in compound A is \_\_\_\_\_. (Round off to Nearest Integer).

**Answer (1\*\*)**



A can be both  $\text{XeF}_6$  and  $\text{XeOF}_4$ . Total number of lone pair in  $\text{XeF}_6$  is 19 and total number of lone pair in  $\text{XeOF}_4$  is 15.

Lone pair present on central atom is  $\text{XeF}_6$  and  $\text{XeOF}_4$  is 1.

**PART-C : MATHEMATICS**

**SECTION - I**

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer :**

1. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a function defined as

$$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin 2x}{2x}, & \text{if } x < 0 \\ b, & \text{if } x = 0 \\ \frac{\sqrt{x+bx^3} - \sqrt{x}}{bx^{5/2}}, & \text{if } x > 0 \end{cases}$$

If  $f$  is continuous at  $x = 0$ , then the value of  $a + b$  is equal to :

- (1)  $-\frac{5}{2}$                       (2)  $-\frac{3}{2}$   
(3)  $-3$                       (4)  $-2$

**Answer (1)**

**Sol.**  $\therefore f(x)$  is continuous at  $x = 0$

$$\therefore \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = f(0)$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{\sin(a+1)(-h) - \sin 2h}{-2h} = \lim_{h \rightarrow 0} \frac{\sqrt{h+bh^3} - \sqrt{h}}{bh^{5/2}} = b$$

$$\Rightarrow \frac{a+1}{2} + 1 = \lim_{h \rightarrow 0} \frac{bh^2}{bh^2(\sqrt{4+bh^2} + 1)} = b$$

$$\Rightarrow \frac{a+3}{2} = \frac{1}{2} = b$$

$$\Rightarrow a = -3, b = \frac{1}{2}$$

$$\therefore a+b = -3 + \frac{1}{2} = -\frac{5}{2}$$

2. If  $P$  and  $Q$  are two statements, then which of the following compound statement is a tautology?

- (1)  $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow (P \wedge Q)$   
(2)  $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow P$   
(3)  $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow \sim P$   
(4)  $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow Q$

**Answer (3)**

**Sol.**  $((P \rightarrow Q) \wedge \sim Q) \rightarrow \sim P$

$$= ((\sim P \vee Q) \wedge \sim Q) \rightarrow \sim P$$

$$= ((\sim P \wedge \sim Q) \vee C) \rightarrow \sim P$$

$$= (\sim P \wedge \sim Q) \rightarrow \sim P$$

$$= \sim(\sim P \wedge \sim Q) \vee \sim P$$

$$= (P \vee Q) \vee \sim P$$

$$= (P \vee \sim P) \vee Q$$

$$= t \vee Q$$

$$= t \text{ (tautology)}$$

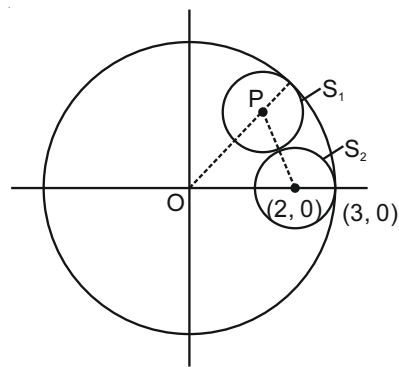
3. Let  $S_1 : x^2 + y^2 = 9$  and  $S_2 : (x - 2)^2 + y^2 = 1$ . Then the locus of center of a variable circle  $S$  which touches  $S_1$  internally and  $S_2$  externally always passes through the points:

(1)  $\left(\frac{1}{2}, \pm \frac{\sqrt{5}}{2}\right)$                       (2)  $(0, \pm\sqrt{3})$

(3)  $\left(2, \pm \frac{3}{2}\right)$                       (4)  $(1, \pm 2)$

**Answer (3)**

**Sol.** Let variable centre of required circle ( $S$ ) be  $(x_1, y_1)$  and its radius be  $r$  units.



$\therefore S$  touches  $S_1$  internally.

$$\therefore OP = 3 - r$$

$$\Rightarrow \sqrt{x_1^2 + y_1^2} = 3 - r \quad \dots(i)$$

and  $S$  touches  $S_2$  externally.

$$\therefore \sqrt{(x_1 - 2)^2 + y_1^2} = 1 + r \quad \dots(ii)$$

from eq. (i) and (ii), required locus of centre is

$$\sqrt{x^2 + y^2} + \sqrt{(x-2)^2 + y^2} = 4$$

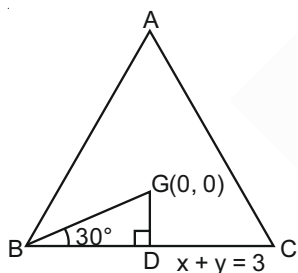
Clearly point  $\left(2, \pm \frac{3}{2}\right)$  lies on the locus.

4. Let the centroid of an equilateral triangle ABC be at the origin. Let one of the sides of the equilateral triangle be along the straight line  $x + y = 3$ . If R and r be the radius of circumcircle and incircle respectively of  $\Delta ABC$ , then  $(R + r)$  is equal to :

- (1)  $3\sqrt{2}$   
 (2)  $2\sqrt{2}$   
 (3)  $\frac{9}{\sqrt{2}}$   
 (4)  $7\sqrt{2}$

**Answer (3)**

**Sol.** Here  $GD = \frac{3}{\sqrt{2}}$



$$\therefore \frac{GD}{BD} = \tan 30^\circ$$

$$\therefore BD = \frac{3\sqrt{3}}{\sqrt{2}}$$

$$\therefore \text{side length} = a = 3\sqrt{6}$$

$$\text{Circumradius} = R = \frac{a}{2 \sin A} = \frac{3\sqrt{6}}{2 \cdot \frac{\sqrt{3}}{2}} = 3\sqrt{2}$$

$$\text{and inradius} = r = GD = \frac{3}{\sqrt{2}}$$

$$\therefore R + r = 3\sqrt{2} + \frac{3}{\sqrt{2}} = \frac{9}{\sqrt{2}}$$

5. Let  $y = y(x)$  be the solution of the differential equation  $\frac{dy}{dx} = (y+1)\left((y+1)e^{x^2/2} - x\right)$ ,  $0 < x < 2.1$ , with  $y(2) = 0$ . Then the value of  $\frac{dy}{dx}$  at  $x = 1$  is equal to :

- (1)  $\frac{e^{5/2}}{(1+e^2)^2}$       (2)  $\frac{-e^{3/2}}{(e^2+1)^2}$   
 (3)  $\frac{5e^{1/2}}{(e^2+1)^2}$       (4)  $-\frac{2e^2}{(1+e^2)^2}$

**Answer (2)**

**Sol.**  $\frac{dy}{dx} = (y+1)^2 e^{x^2/2} - x(y+1) \dots(A)$

$$\frac{dy}{dx} + x(y+1) = (y+1)^2 e^{x^2/2}$$

$$\frac{1}{(y+1)^2} \frac{dy}{dx} + x \times \frac{1}{y+1} = e^{x^2/2} \dots(i)$$

$$\text{Let } \frac{1}{y+1} = z \Rightarrow \frac{-1}{(y+1)^2} \frac{dy}{dx} = \frac{dz}{dx}$$

$$-\frac{dz}{dx} + xz = e^{x^2/2}$$

$$\frac{dz}{dx} - x \cdot z = -e^{x^2/2} \dots(ii)$$

$$\text{IF} = e^{\int -x dx} = e^{-x^2/2}$$

$$z \cdot e^{-x^2/2} = \int -e^{-x^2/2} \times e^{x^2/2} dx = \int -1 dx = -x + C$$

$$\frac{1}{(y+1)e^{x^2/2}} = -x + C \dots(iii)$$

$$y(2) = 0 \Rightarrow \frac{1}{1 \times e^2} = -2 + C$$

$$C = 2 + \frac{1}{e^2}$$

$$(iii) \Rightarrow \frac{1}{(y+1)e^{x^2/2}} = -x + 2 + \frac{1}{e^2}$$

at  $x = 1$ ,

$$\frac{1}{(y+1)e^2} = 1 + \frac{1}{e^2} = \frac{e^2 + 1}{e^2}$$

$$(y+1)e^2 = \frac{e^2}{e^2 + 1}$$

$$y+1 = \frac{e^{3/2}}{e^2 + 1}$$

$$(A) \Rightarrow \frac{dy}{dx} = \frac{e^{3/2}}{e^2 + 1} \left( \frac{e^{3/2}}{e^2 + 1} e^{1/2} - 1 \right)$$

$$= \frac{e^{3/2}}{e^2 + 1} \left( \frac{e^2}{e^2 + 1} - 1 \right)$$

$$= \frac{-e^{3/2}}{(e^2 + 1)^2}$$

6. Let in a series of  $2n$  observations, half of them are equal to  $a$  and remaining half are equal to  $-a$ . Also by adding a constant  $b$  in each of these observations, the mean and standard deviation of new set become 5 and 20, respectively. Then the value of  $a^2 + b^2$  is equal to :

- (1) 925                      (2) 650  
(3) 425                      (4) 250

**Answer (3)**

**Sol.** Old mean =  $\frac{\sum x_i}{n} = 0$

New mean =  $0 + b = 5$

$\Rightarrow b = 5$

Old S.D =  $\sqrt{\frac{\sum (x_i - \bar{x})^2}{n}} = \sqrt{\frac{\sum a^2}{n}} = a$

New S.D = old S.D =  $a = 20$

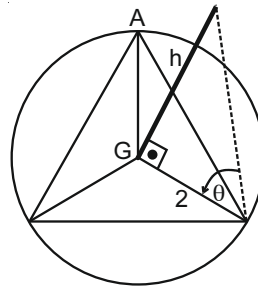
$a^2 + b^2 = 425$

7. A pole stands vertically inside a triangular park ABC. Let the angle of elevation of the top of the pole from each corner of the park be  $\frac{\pi}{3}$ . If the radius of the circumcircle of  $\triangle ABC$  is 2, then the height of the pole is equal to :

- (1)  $\frac{2\sqrt{3}}{3}$                       (2)  $\frac{1}{\sqrt{3}}$   
(3)  $2\sqrt{3}$                       (4)  $\sqrt{3}$

**Answer (3)**

**Sol.**



Triangle is equilateral

Pole is at circumcenter

$$h = 2 \tan \frac{\pi}{3}$$

$$h = 2\sqrt{3}$$

8. Let  $g(x) = \int_0^x f(t)dt$ , where  $f$  is continuous function

in  $[0, 3]$  such that  $\frac{1}{3} \leq f(t) \leq 1$  for all  $t \in [0, 1]$  and

$0 \leq f(t) \leq \frac{1}{2}$  for all  $t \in [1, 3]$ . The largest possible interval in which  $g(3)$  lies is :

- (1)  $[1, 3]$                       (2)  $\left[\frac{1}{3}, 2\right]$   
(3)  $\left[-1, -\frac{1}{2}\right]$                       (4)  $\left[-\frac{3}{2}, -1\right]$

**Answer (2)**

**Sol.**  $g(3) = \int_0^3 f(t)dt$

$$g(3) = \int_0^1 f(t)dt + \int_1^3 f(t)dt \quad \dots(i)$$

$$\int_0^1 \frac{1}{3} dt \leq \int_0^1 f(t)dt \leq \int_0^1 1 dt$$

$$\frac{1}{3} \leq \int_0^1 f(t)dt \leq 1 \quad \dots(ii)$$

$$\int_1^3 0 dt \leq \int_1^3 f(t)dt \leq \int_1^3 \frac{1}{2} dt$$

$$0 \leq \int_1^3 f(t)dt \leq \frac{1}{2} \times 2 = 1 \quad \dots(iii)$$

(i), (ii), (iii)  $\Rightarrow \frac{1}{3} \leq g(3) \leq 2$



9. Let  $f : \mathbf{R} - \{3\} \rightarrow \mathbf{R} - \{1\}$  be defined by

$$f(x) = \frac{x-2}{x-3}$$

Let  $g : \mathbf{R} \rightarrow \mathbf{R}$  be given as  $g(x) = 2x - 3$ . Then, the sum of all the values of  $x$  for which

$$f^{-1}(x) + g^{-1}(x) = \frac{13}{2} \text{ is equal to.}$$

- (1) 3 (2) 5  
(3) 7 (4) 2

**Answer (2)**

**Sol.** Finding inverse of  $f(x)$

$$y = \frac{x-2}{x-3} \Rightarrow xy - 3y = x - 2 \Rightarrow x(y-1) = 3y - 2$$

$$\therefore f^{-1}(x) = \frac{3x-2}{x-1}$$

Similarly for  $g^{-1}(x)$

$$y = 2x - 3 \Rightarrow x = \frac{y+3}{2} \Rightarrow g^{-1}(x) = \frac{x+3}{2}$$

$$\therefore \frac{3x-2}{x-1} + \frac{x+3}{2} = \frac{13}{2}$$

$$\Rightarrow 6x - 4 + x^2 + 2x - 3 = 13x - 13$$

$$\Rightarrow x^2 - 5x + 6 = 0$$

$$\Rightarrow (x-2)(x-3) = 0$$

$$\Rightarrow x = 2 \text{ or } 3$$

10. If  $15\sin^4\alpha + 10\cos^4\alpha = 6$ , for some  $\alpha \in \mathbf{R}$ , then the value of  $27\sec^6\alpha + 8\operatorname{cosec}^6\alpha$  is equal to :

- (1) 350 (2) 250  
(3) 400 (4) 500

**Answer (2)**

**Sol.**  $15\sin^2\alpha + 10(1 - \sin^2\alpha)^2 = 6$

$$\Rightarrow 25\sin^2\alpha - 20\sin^2\alpha + 4 = 0$$

$$\Rightarrow 25\sin^2\alpha - 10\sin^2\alpha - 10\sin^2\alpha + 4 = 0$$

$$\Rightarrow (5\sin^2\alpha - 2)^2 = 0 \Rightarrow \sin^2\alpha = \frac{2}{5}$$

$$\therefore \cos^2\alpha = \frac{3}{5}$$

$$\therefore 27\sec^6\alpha + 8\operatorname{cosec}^6\alpha = 27\left(\frac{5}{3}\right)^3 + 8\left(\frac{5}{2}\right)^3$$

$$= 125 + 125 = 250$$

11. Let a complex number be  $w = 1 - \sqrt{3}i$ . Let another complex number  $z$  be such that  $|zw| = 1$  and  $\arg(z) - \arg(w) = \frac{\pi}{2}$ . Then the area of the triangle with vertices origin,  $z$  and  $w$  is equal to :

- (1)  $\frac{1}{2}$  (2) 2  
(3) 4 (4)  $\frac{1}{4}$

**Answer (1)**

**Sol.**  $|wz| = 1 \Rightarrow |w||z| = 1$  and  $|w| = 2$

$$\Rightarrow |z| = \frac{1}{2}$$

$$\text{Also } \arg(z) - \arg(w) = \frac{\pi}{2}$$

$$\Rightarrow z = \frac{1}{2} \cdot \frac{(1-i\sqrt{3})}{2} \cdot i$$

$$\text{Area of triangle} = \frac{1}{2} \cdot 2 \times \frac{1}{2} = \frac{1}{2}$$

12. Let the system of linear equations

$$4x + \lambda y + 2z = 0$$

$$2x - y + z = 0$$

$$\mu x + 2y + 3z = 0, \lambda, \mu \in \mathbf{R}.$$

has a non-trivial solution. Then which of the following is true?

- (1)  $\mu = -6, \lambda \in \mathbf{R}$  (2)  $\lambda = 3, \mu \in \mathbf{R}$   
(3)  $\mu = 6, \lambda \in \mathbf{R}$  (4)  $\lambda = 2, \mu \in \mathbf{R}$

**Answer (3)**

**Sol.** 
$$\begin{vmatrix} 4 & \lambda & 2 \\ 2 & -1 & 1 \\ \mu & 2 & 3 \end{vmatrix} = 0$$

$$\Rightarrow \mu\lambda + 2\mu - 6\lambda - 12 = 0$$

$$\Rightarrow (\lambda + 2)(\mu - 6) = 0$$

$$\lambda = -2 \text{ or } \mu = 6$$

13. Define a relation  $R$  over a class of  $n \times n$  real matrices  $A$  and  $B$  as " $ARB$  iff there exists a non-singular matrix  $P$  such that  $PAP^{-1} = B$ ". Then which of the following is true?

- (1)  $R$  is reflexive, symmetric but not transitive  
(2)  $R$  is an equivalence relation  
(3)  $R$  is symmetric, transitive but not reflexive  
(4)  $R$  is reflexive, transitive but not symmetric

**Answer (2)**

**Sol.** For reflexive,

$$PAP^{-1} = A \text{ is true if } P = I$$

For symmetric,

If  $PAP^{-1} = B$  then  $PBP^{-1} = A$  must be true

$$\therefore PAP^{-1} = B \Rightarrow A = P^{-1}BP$$

and  $PBP^{-1} = P(PAP^{-1})P^{-1} = P^2A(P^{-1})^2$  is equal to  $A$  if  $P$  is involutory matrix (i.e.  $P^2 = I$ )

For transitive,

If  $PAP^{-1} = B$  and  $PBP^{-1} = C$  then  $PAP^{-1} = C$  must be true

$\therefore C = PBP^{-1} = P^2AP^{-1}$  will be equal to  $PAP^{-1}$  if  $P$  is idempotent matrix (i.e.  $P^2 = P$ )

Hence relation  $R$  is equivalence relation.

14. In a triangle  $ABC$ , if  $|\overline{BC}| = 8$ ,  $|\overline{CA}| = 7$ ,  $|\overline{AB}| = 10$ , then the projection of the vector  $\overline{AB}$  on  $\overline{AC}$  is equal to :

(1)  $\frac{115}{16}$                       (2)  $\frac{25}{4}$

(3)  $\frac{127}{20}$                       (4)  $\frac{85}{14}$

**Answer (4)**

**Sol.** Projection of  $\overline{AB}$  on  $\overline{AC} = \frac{(\overline{AB}) \cdot (\overline{AC})}{|\overline{AC}|} = p$  (say)

$$= \frac{|\overline{AB}| |\overline{AC}| \cos \theta}{|\overline{AC}|}$$

where  $\cos \theta = \frac{10^2 + 7^2 - 8^2}{2 \cdot 10 \cdot 7}$

$$\Rightarrow p = \frac{10 \cdot 85}{2 \cdot 10 \cdot 7} = \frac{85}{14}$$

15. Let  $S_1$  be the sum of first  $2n$  terms of an arithmetic progression. Let  $S_2$  be the sum of first  $4n$  terms of the same arithmetic progression. If  $(S_2 - S_1)$  is 1000, then the sum of the first  $6n$  terms of the arithmetic progression is equal to :

(1) 7000                      (2) 5000

(3) 3000                      (4) 1000

**Answer (3)**

**Sol.**  $S_1 = \frac{2n}{2} [2a + (2n-1)d]$       (where  $a = T_1$  and  $d$  is common difference)

$$S_2 = \frac{4n}{2} [2a + (4n-1)d]$$

$$S_2 - S_1 = 2n[2a + (4n-1)d] - n[2a + (2n-1)d] = 1000$$

$$= n[2a + d(8n-2-2n+1)] = 1000$$

$$= n[2a + (6n-1)d] = 1000$$

$$S_6 = \frac{6n}{2} [2a + (6n-1)d] = 3(S_2 - S_1) = 3000$$

16. The area bounded by the curve  $4y^2 = x^2(4-x)(x-2)$  is equal to :

(1)  $\frac{3\pi}{8}$                       (2)  $\frac{3\pi}{2}$

(3)  $\frac{\pi}{8}$                       (4)  $\frac{\pi}{16}$

**Answer (2)**

**Sol.** As RHS is always positive  $x \in [2, 4]$  only

$$\text{Area} = \int_2^4 x(6x-8-x^2)^{\frac{1}{2}} dx = A$$

$$= \int_2^4 x\sqrt{1-(x-3)^2} dx \quad \dots(i)$$

$$A = \int_2^4 (6-x)\sqrt{1-(x-3)^2} dx \quad \dots(ii)$$

$$\Rightarrow 2A = \int_2^4 6\sqrt{1-(x-3)^2} dx$$

$$\Rightarrow A = \left( x-3\sqrt{1-(x-3)^2} + \frac{1}{2}\sin^{-1}(x-3) \right) \Big|_2^4$$

$$= 3 \left( 3(0) + \frac{1}{2}\pi/2 - 0 - \frac{1}{2}(-\pi/2) \right)$$

$$= 3 \frac{\pi}{2} = \frac{3\pi}{2}$$

17. Let a tangent be drawn to the ellipse  $\frac{x^2}{27} + y^2 = 1$  at

$(3\sqrt{3}\cos\theta, \sin\theta)$  where  $\theta \in \left(0, \frac{\pi}{2}\right)$ . Then the value

of  $\theta$  such that the sum of intercepts on axes made by this tangent is minimum is equal to :

(1)  $\frac{\pi}{4}$                       (2)  $\frac{\pi}{8}$

(3)  $\frac{\pi}{6}$                       (4)  $\frac{\pi}{3}$

**Answer (3)**

**Sol.** Tangent =  $\frac{x}{3\sqrt{3}} \cos\theta + y \sin\theta = 1$

x-intercept =  $3\sqrt{3} \sec\theta$

y-intercept =  $\operatorname{cosec}\theta$

sum =  $3\sqrt{3} \sec\theta + \operatorname{cosec}\theta = f(\theta) \quad \theta \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow f'(\theta) = 3\sqrt{3} \sec\theta \tan\theta - \operatorname{cosec}\theta \cot\theta = 0$

$\Rightarrow \frac{3\sqrt{3} \sin\theta}{\cos^2\theta} = \frac{\cos\theta}{\sin\theta}$

$\Rightarrow \tan^3\theta = \left(\frac{1}{\sqrt{3}}\right)^3$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = \frac{\pi}{6}$

also  $f(\theta)$  changes sign – to + hence minimum.

18. Let in a Binomial distribution, consisting of 5 independent trials, probabilities of exactly 1 and 2 successes be 0.4096 and 0.2048 respectively. Then the probability of getting exactly 3 successes is equal to :

(1)  $\frac{80}{243}$

(2)  $\frac{32}{625}$

(3)  $\frac{40}{243}$

(4)  $\frac{128}{625}$

**Answer (2)**

**Sol.**  $n = 5$  (given)

also  $5c_1 p q^4 = 0.4096$

and  $5c_2 p^2 q^3 = 0.2048$

$\frac{5q}{10p} = \frac{0.4096}{0.2048}$

$\Rightarrow q = 4p$

also  $p + q = 1$

$\Rightarrow p = \frac{1}{5}$  and  $q = \frac{4}{5}$

$p(\text{exactly 3 successes}) = 5c_3 \cdot p^3 q^2$

$= 10 \frac{1}{5^3} \cdot \frac{16}{5^2} = \frac{32}{625}$

19. Let  $\vec{a}$  and  $\vec{b}$  be two non-zero vectors perpendicular to each other and  $|\vec{a}| = |\vec{b}|$ . If  $|\vec{a} \times \vec{b}| = |\vec{a}|$ , then the angle between the vectors  $(\vec{a} + \vec{b} + (\vec{a} \times \vec{b}))$  and  $\vec{a}$  is equal to :

(1)  $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(2)  $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(3)  $\sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$

(4)  $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

**Answer (4)**

**Sol.** Angle required is say  $\theta$

$\Rightarrow \cos\theta = \frac{\vec{a} \cdot (\vec{a} + \vec{b} + (\vec{a} \times \vec{b}))}{|\vec{a}| |\vec{a} + \vec{b} + (\vec{a} \times \vec{b})|}$

$= \frac{|\vec{a}|^2 + 0 + 0}{|\vec{a}| |\vec{a} + \vec{b} + (\vec{a} \times \vec{b})|}$

$= \frac{|\vec{a}|^2}{|\vec{a}| \sqrt{3} |\vec{a}|}$

(as  $\vec{a}, \vec{b}$  and  $\vec{a} \times \vec{b}$  are mutually perpendicular to each other)

$\cos\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

20. Consider a hyperbola  $H : x^2 - 2y^2 = 4$ . Let the tangent at a point  $P(4, \sqrt{6})$  meet the x-axis at Q and latus rectum at  $R(x_1, y_1)$ ,  $x_1 > 0$ . If F is a focus of H which is nearer to the point P, then the area of  $\Delta QFR$  is equal to.

(1)  $\sqrt{6} - 1$

(2)  $4\sqrt{6}$

(3)  $4\sqrt{6} - 1$

(4)  $\frac{7}{\sqrt{6}} - 2$

**Answer (4)**

**Sol.** Hyperbola  $x^2 - 2y^2 = 4 \Rightarrow \frac{x^2}{4} - \frac{y^2}{2} = 1$

$e = \sqrt{1 + \frac{2}{4}} = \frac{\sqrt{3}}{\sqrt{2}}$

$S \equiv (\pm\sqrt{6}, 0)$

Directrix  $\equiv x = \sqrt{6}$

Tangent at  $(4, \sqrt{6})$  is  $4x - 2\sqrt{6}y = 4$

$\therefore Q \equiv (1, 0), R \equiv \left(\sqrt{6}, \frac{4\sqrt{6} - 4}{2\sqrt{6}}\right)$

Area of  $\Delta QFR = \frac{1}{2}(\sqrt{6} - 1) \frac{4(\sqrt{6} - 1)}{2\sqrt{6}} = \frac{7}{\sqrt{6}} - 2$

**SECTION - II**

**Numerical Value Type Questions:** This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Let  ${}^n C_r$  denote the binomial coefficient of  $x^r$  in the expansion of  $(1+x)^n$ .

If  $\sum_{k=0}^{10} (2^2 + 3k) {}^n C_k = \alpha 3^{10} + \beta 2^{10}$ ,  $\alpha, \beta \in \mathbb{R}$ , then  $\alpha + \beta$  equal to \_\_\_\_\_.

**Answer (19\*)**

**Sol.** Take  $n = 10$

$$\begin{aligned} \sum_{k=0}^{10} (3k+4) {}^{10} C_k &= 30 \sum_{k=1}^{10} {}^9 C_{k-1} + 4 \sum_{k=0}^{10} {}^{10} C_k \\ &= 30 \cdot 2^9 + 4 \cdot 2^{10} \\ &= 19 \cdot 2^{10} \\ \Rightarrow \alpha &= 0 \text{ and } \beta = 19 \end{aligned}$$

2. The term independent of  $x$  in the expansion of

$$\left[ \frac{x+1}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1} - \frac{x-1}{x - x^{\frac{1}{2}}} \right]^{10}, \quad x \neq 1, \text{ is equal to}$$

**Answer (210)**

**Sol.**

$$\begin{aligned} &\left( \frac{x+1}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1} - \frac{x-1}{x - x^{\frac{1}{2}}} \right)^{10} \\ &= \left( \frac{\frac{1}{x^{\frac{2}{3}} + 1} - \frac{2}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1}}{\frac{2}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1} - \frac{1}{x^{\frac{1}{2}}(x^{\frac{1}{2}} - 1)}} - \frac{\frac{1}{x^{\frac{1}{2}} - 1} - \frac{1}{x^{\frac{1}{2}} + 1}}{\frac{1}{x^{\frac{1}{2}}(x^{\frac{1}{2}} - 1)}} \right)^{10} \\ &= \left( \frac{1}{x^{\frac{2}{3}} + 1} - (1 + x^{-\frac{1}{2}}) \right)^{10} \\ &= \left( \frac{1}{x^{\frac{2}{3}} - x^{-\frac{1}{2}}} \right)^{10} \\ T_{r+1} &= {}^{10} C_r \left( \frac{1}{x^{\frac{2}{3}}} \right)^{10-r} \left( x^{-\frac{1}{2}} \right)^r \end{aligned}$$

For being independent of  $x$  :  $\frac{10-r}{3} - \frac{r}{2} = 0 \Rightarrow r = 4$

Term independent of  $x = {}^{10} C_4 = 210$

3. Let the mirror image of the point  $(1, 3, a)$  with respect to the plane  $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) - b = 0$  be  $(-3, 5, 2)$ . Then, the value of  $|a + b|$  is equal to \_\_\_\_\_.

**Answer (1)**

**Sol.** Equation of plane :  $2x - y + z = b$

Midpoint of  $(1, 3, a)$  &  $(-3, 5, 2)$  lies on this plane

$$\Rightarrow 2 \left( \frac{1-3}{2} \right) - \left( \frac{3+5}{2} \right) + \left( \frac{2+a}{2} \right) = b$$

$$\Rightarrow a - 2b = 14 \quad \dots(i)$$

Also vector joining  $(1, 3, a)$  &  $(-3, 5, 2)$  is parallel to  $2\hat{i} - \hat{j} + \hat{k}$

$$\text{i.e., } 4\hat{i} - 2\hat{j} + (a-2)\hat{k} = \lambda(2\hat{i} - \hat{j} + \hat{k})$$

$$\Rightarrow \lambda = 2 \Rightarrow a - 2 = 2 \Rightarrow a = 4$$

From (i),  $b = -5$

Hence  $|a + b| = 1$

4. Let  $P$  be a plane containing the line  $\frac{x-1}{3} = \frac{y+6}{4} = \frac{z+5}{2}$  and parallel to the line  $\frac{x-3}{4} = \frac{y-3}{-3} = \frac{z+5}{7}$ . If the point  $(1, -1, \alpha)$  lies on the plane  $P$ , then the value of  $|5\alpha|$  is equal to \_\_\_\_\_.

**Answer (38)**

**Sol.** The plane is parallel to vectors

$$\vec{n}_1 = 3\hat{i} + 4\hat{j} + 2\hat{k} \quad \& \quad \vec{n}_2 = 4\hat{i} - 3\hat{j} + 7\hat{k}$$

Vector normal to plane =  $\vec{n} = \vec{n}_1 \times \vec{n}_2$

$$\Rightarrow \vec{n} = 34\hat{i} - 13\hat{j} - 25\hat{k}$$

Point  $(1, -6, -5)$  lies on the plane

Hence equation of plane is

$$\left( (x-1)\hat{i} + (y+6)\hat{j} + (z+5)\hat{k} \right) \cdot (34\hat{i} - 13\hat{j} - 25\hat{k}) = 0$$

Passes through  $(1, -1, \alpha)$

$$\Rightarrow \left( 0\hat{i} + 5\hat{j} + (\alpha+5)\hat{k} \right) \cdot (34\hat{i} - 13\hat{j} - 25\hat{k}) = 0$$

$$\Rightarrow 0 - 65 - 25\alpha - 125 = 0$$

$$\Rightarrow 5\alpha = 38$$

$$|5\alpha| = 38$$

5. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  satisfy the equation  $f(x + y) = f(x) \cdot f(y)$  for all  $x, y \in \mathbb{R}$  and  $f(x) \neq 0$  for any  $x \in \mathbb{R}$ . If the function  $f$  is differentiable at  $x = 0$  and  $f'(0) = 3$ , then

$$\lim_{h \rightarrow 0} \frac{1}{h} (f(h) - 1) \text{ is equal to } \underline{\hspace{2cm}}.$$

**Answer (3)**

**Sol.**  $\therefore f(x + y) = f(x)f(y) \quad \forall x, y \in \mathbb{R}$

$$x = y = 0 \Rightarrow f(0) = (f(0))^2 \Rightarrow f(0) = 0 \text{ or } f(0) = 1$$

$$f(x) \neq 0 \text{ for any } x \in \mathbb{R} \Rightarrow f(0) = 1$$

$$\text{Given } f'(0) = 3$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{f(0+h) + f(0)}{h} = 3$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{f(h) - 1}{h} = 3$$

6. Let  $I$  be an identity matrix of order  $2 \times 2$  and

$$P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}. \text{ Then the value of } n \in \mathbb{N} \text{ for which}$$

$$P_n = 5I - 8P \text{ is equal to } \underline{\hspace{2cm}}.$$

**Answer (6)**

**Sol.**  $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$

$$\begin{vmatrix} 2-\lambda & -1 \\ 5 & -3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^2 + \lambda - 1 = 0$$

$$\Rightarrow P^2 + P - I = 0$$

$$\Rightarrow P^2 = I - P$$

$$\Rightarrow P^4 = I + P^2 - 2P$$

$$\Rightarrow P^4 = 2I - 3P$$

$$\text{Now, } P^4 \cdot P^2 = (2I - 3P)(I - P) = 2I - 5P + 3P^2$$

$$\Rightarrow P^6 = 5I - 8P$$

so  $n = 6$ .

7. Let  $P(x)$  be a real polynomial of degree 3 which vanishes at  $x = -3$ . Let  $P(x)$  have local minima at

$$x=1, \text{ local maxima at } x = -1 \text{ and } \int_{-1}^1 P(x) dx = 18,$$

then the sum of all the coefficients of the polynomial  $P(x)$  is equal to  $\underline{\hspace{2cm}}$ .

**Answer (8)**

**Sol.**  $P'(x)$  is second degree polynomial and vanishes at

$$x = \pm 1$$

$$\text{let } P'(x) = k(x-1)(x+1) = kx^2 - k$$

$$\Rightarrow P(x) = \frac{k}{3} \cdot x^3 - kx + \lambda$$

$$\int_{-1}^1 p(x) dx = 18 \Rightarrow \int_{-1}^1 \left( \frac{k}{3} \cdot x^3 - kx + \lambda \right) dx = 2\lambda = 18$$

$$\Rightarrow \lambda = 9$$

$$\text{Also } P(-3) = 0$$

$$\Rightarrow \frac{k}{3}(-3)^3 - k(-3) + 9 = 0 \Rightarrow k = \frac{3}{2}$$

$$P(x) = \frac{1}{2}x^3 - \frac{3}{2}x + 9 \Rightarrow \text{sum of all the coefficients} = 8$$

8. Let  $y = y(x)$  be the solution of the differential

$$\text{equation } xdy - ydx = \sqrt{x^2 - y^2} dx, x \geq 1, \text{ with}$$

$y(1) = 0$ . If the area bounded by the line  $x = 1$ ,  $x = e^\pi$ ,  $y = 0$  and  $y = y(x)$  is  $\alpha e^{2\pi} + \beta$ , then the value of  $10(\alpha + \beta)$  is equal to  $\underline{\hspace{2cm}}$ .

**Answer (4)**

**Sol.**  $xdy - ydx = \sqrt{x^2 - y^2} dx$

$$\Rightarrow \frac{xdy - ydx}{x^2} = \frac{1}{x} \sqrt{1 - \left(\frac{y}{x}\right)^2} dx$$

$$\Rightarrow \frac{d\left(\frac{y}{x}\right)}{\sqrt{1 - \left(\frac{y}{x}\right)^2}} = \frac{dx}{x}$$

$$\Rightarrow \sin^{-1}\left(\frac{y}{x}\right) = \ln x + C$$

$$\Rightarrow y(1) = 0 \Rightarrow C = 0$$

$$\text{Here } y = x \cdot \sin(\ln x)$$

$A =$  The required area bounded is

$$= \int_1^{e^\pi} x \cdot \sin(\ln x) dx$$

$$\text{Put } x = e^t$$

$$\Rightarrow dx = e^t dt$$

$$A = \int_0^{\pi} e^t \cdot \sin t \cdot e^t dt$$

$$A = \int_0^{\pi} e^{2t} \cdot \sin t dt$$

$$= \frac{e^{2t}}{1^2 + 2^2} \cdot (2 \sin t - \cos t) \Big|_0^{\pi}$$

$$= \frac{1}{5} (e^{2\pi}(0+1) - 1(0-1))$$

$$= \frac{1}{5} \cdot e^{2\pi} + \frac{1}{5} = \alpha \cdot e^{2\pi} + \beta$$

$$\alpha = -\frac{1}{5}, \beta = \frac{1}{5}$$

$$10(\alpha + \beta) = 4$$

9. If  $f(x)$  and  $g(x)$  are two polynomials such that the polynomial  $P(x) = f(x^3) + x g(x^3)$  is divisible by  $x^2 + x + 1$ , then  $P(1)$  is equal to \_\_\_\_\_.

**Answer (Zero)**

**Sol.**  $x^2 + x + 1 = 0 \Rightarrow (x - \omega)(x - \omega^2) = 0$

where  $\omega$  is complex cube root of unity

$P(x)$  is divisible by  $x^2 + x + 1$

Here  $P(\omega) = 0$  and  $P(\omega^2) = 0$

$$\Rightarrow P(\omega) = f(\omega^3) + \omega g(\omega^3) = 0$$

$$0 = f(1) + \omega g(1) \quad \dots \text{eqn (i)}$$

Also,  $P(\omega^2) = f(\omega^6) + \omega^2 g(\omega^6) = 0$

$$0 = f(1) + \omega^2 g(1) \quad \dots \text{eqn (ii)}$$

from (i) and (ii),  $f(1) = g(1) = 0$

Here  $P(1) = f(1) + 1g(1) = 0$

10. If  $\sum_{r=1}^{10} r!(r^3 + 6r^2 + 2r + 5) = \alpha(11!)$ ,

then the value of  $\alpha$  is equal to \_\_\_\_\_.

**Answer (160)**

**Sol.**  $\sum_{r=1}^{10} r!(r^3 + 6r^2 + 2r + 5)$

$$= \sum_{r=1}^{10} ((r+3)! - (r+1)! - 8((r+1)! - r!))$$

$$= \sum_{r=1}^{10} ((r+3)! - (r+1)! - 8 \sum_{r=1}^{10} ((r+1)! - r!))$$

$$= 12! + 13! - 2! - 3! - 8(11! - 1!)$$

$$= 160 \cdot 11!$$

Hence  $\alpha = 160$

