

18/03/2021  
Morning



# Aakash

Medical | IIT-JEE | Foundations

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Time : 3 hrs.

## Answers & Solutions

M.M. : 300

*for*

## JEE (MAIN)-2021 (Online) Phase-2

(Physics, Chemistry and Mathematics)

### IMPORTANT INSTRUCTIONS :

- (1) The test is of **3 hours** duration.
- (2) The Test Booklet consists of 90 questions. The maximum marks are 300.
- (3) There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part has two sections.
  - (i) Section-I : This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **-1 mark** for wrong answer.
  - (ii) Section-II : This section contains 10 questions. In Section-II, attempt any **five questions out of 10**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and there is no negative marking for wrong answer.

**PART-A : PHYSICS**

**SECTION - I**

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer :**

- Your friend is having eye sight problem. She is not able to see clearly a distant uniform window mesh and it appears to her as non-uniform and distorted. The doctor diagnosed the problem as :
  - Myopia and hypermetropia
  - Presbyopia with Astigmatism
  - Myopia with Astigmatism
  - Astigmatism

**Answer (3)**

**Sol.** Myopia with Astigmatism causes distant objects to be blurry and distorted.

- A radioactive sample disintegrates via two independent decay processes having half lives  $T_{1/2}^{(1)}$  and  $T_{1/2}^{(2)}$  respectively. The effective half-life,  $T_{1/2}$  of the nuclei is

- $T_{1/2} = T_{1/2}^{(1)} + T_{1/2}^{(2)}$
- None of the above
- $T_{1/2} = \frac{T_{1/2}^{(1)} T_{1/2}^{(2)}}{T_{1/2}^{(1)} + T_{1/2}^{(2)}}$
- $T_{1/2} = \frac{T_{1/2}^{(1)} + T_{1/2}^{(2)}}{T_{1/2}^{(1)} - T_{1/2}^{(2)}}$

**Answer (3)**

**Sol.**  $-\frac{dN}{dt} = \lambda_A N + \lambda_B N$

$$\Rightarrow \lambda_{\text{eff}} = \lambda_A + \lambda_B$$

$$\text{Now } \lambda_{\text{eff}} = \frac{\ln 2}{T_{1/2 \text{ eff}}} \quad \lambda_A = \frac{\ln 2}{T_{1/2}^1} \quad \lambda_B = \frac{\ln 2}{T_{1/2}^2}$$

$$\Rightarrow \frac{1}{T_{1/2 \text{ eff}}} = \frac{1}{T_{1/2}^1} + \frac{1}{T_{1/2}^2}$$

$$\Rightarrow T_{1/2 \text{ eff}} = \frac{T_{1/2}^1 T_{1/2}^2}{T_{1/2}^1 + T_{1/2}^2}$$

- An AC source rated 220 V, 50 Hz is connected to a resistor. The time taken by the current to change from its maximum to the rms value is

- 2.5 s
- 2.5 ms
- 0.25 ms
- 25 ms

**Answer (2)**

**Sol.** For change in current from its maximum to rms value the current phasor will rotate by angle  $\frac{\pi}{4}$  or

the time required will be  $\frac{T}{8}$

$$\text{Time required} = \frac{T}{8} = \frac{1}{8 \times 50} = \frac{1}{400} \text{ s}$$

$$\text{Time required} = 2.5 \text{ ms}$$

- A constant power delivering machine has towed a box, which was initially at rest, along a horizontal straight line. The distance moved by the box in time 't' is proportional to:

- $t^{3/2}$
- $t^{2/3}$
- t
- $t^{1/2}$

**Answer (1)**

**Sol.**  $P = \text{constant} = k(\text{say})$

$$\frac{1}{2}mv^2 = kt$$

$$\Rightarrow v = \sqrt{\frac{2k}{m}} t^{1/2}$$

$$\Rightarrow \frac{ds}{dt} = \sqrt{\frac{2k}{m}} t^{1/2}$$

$$\Rightarrow \int_0^s ds = \sqrt{\frac{2k}{m}} \int_0^t t^{1/2} dt$$

$$s \propto t^{3/2}$$

- A thin circular ring of mass M and radius r is rotating about its axis with an angular speed  $\omega$ . Two particles having mass m each are now attached at diametrically opposite points. The angular speed of the ring will become:

$$(1) \omega \frac{M}{M+2m} \quad (2) \omega \frac{M+2m}{M}$$

$$(3) \omega \frac{M-2m}{M+2m} \quad (4) \omega \frac{M}{M+m}$$

**Answer (1)**

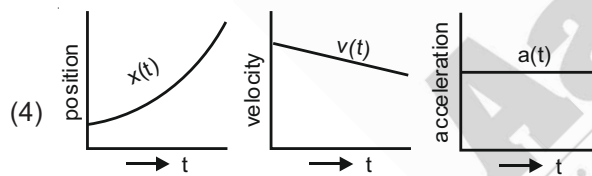
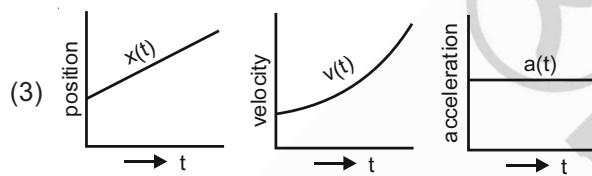
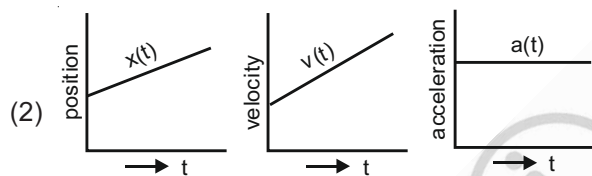
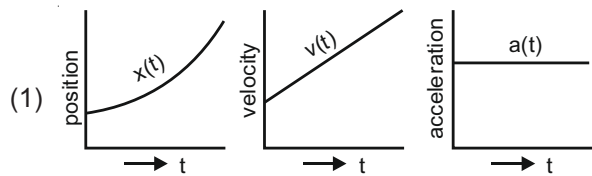
**Sol.** External torque being absent, net angular momentum should be conserved about the axis of ring

$$\Rightarrow I_{\text{ring}}\omega = (I_{\text{ring}} + I_{\text{particles}})\omega_f$$

$$\Rightarrow \omega_f = \frac{Mr^2\omega}{Mr^2 + 2mr^2}$$

$$\Rightarrow \omega_f = \frac{M}{M + 2m}\omega$$

6. The position, velocity and acceleration of a particle moving with a constant acceleration can be represented by:



**Answer (1)**

**Sol.** → Acceleration-time graph will be straight horizontal line.

→ Velocity-time graph will be straight inclined line.

→ Position-time graph will be parabolic graph opening upwards if both velocity and acceleration are positive.

7. In a series LCR resonance circuit, if we change the resistance only, from a lower to higher value:

(1) The quality factor and the resonance frequency will remain constant

(2) The resonance frequency will increase

(3) The bandwidth of resonance circuit will increase

(4) The quality factor will increase

**Answer (3)**

**Sol.**  $Q = \frac{X}{R}$

$$\text{Band width} = \frac{\text{Resonance Frequency}}{\text{Quality Factor}}$$

With increase in value of R the band width will increase.

8. Match List-I with List-II.

- | List-I                                 | List-II            |
|--|--------------------|
| (a) 10 km height over earth's surface  | (i) Thermosphere   |
| (b) 70 km height over earth's surface  | (ii) Mesosphere    |
| (c) 180 km height over earth's surface | (iii) Stratosphere |
| (d) 270 km height over earth's surface | (iv) Troposphere   |
- (1) (a)(iii), (b)(ii), (c)(i), (d)(iv)  
 (2) (a)(iv), (b)(iii), (c)(ii), (d)(i)  
 (3) (a)(i), (b)(iv), (c)(iii), (d)(ii)  
 (4) (a)(ii), (b)(i), (c)(iv), (d)(iii)

**Answer (2)**

**Sol.** Layer of atmosphere very close to surface of earth is troposphere and as we move upwards we encounter Stratosphere, Mesosphere & Thermosphere in that order.

9. A particle is travelling 4 times as fast as an electron. Assuming the ratio of de-Broglie wavelength of a particle to that of electron is 2 : 1, the mass of the particle is

- (1) 16 times the mass of  $e^-$   
 (2) 8 times the mass of  $e^-$   
 (3)  $\frac{1}{16}$  times the mass of  $e^-$   
 (4)  $\frac{1}{8}$  times the mass of  $e^-$

**Answer (3)**

**Sol.**  $v_{\text{particle}} = 4v_{\text{electron}}$

$$\lambda_{\text{particle}} = 2\lambda_{\text{electron}} \Rightarrow p_{\text{particle}} = \frac{1}{2}p_{\text{electron}}$$

$$\therefore m_{\text{particle}} v_{\text{particle}} = \frac{m_{\text{electron}} v_{\text{electron}}}{2}$$

$$\Rightarrow m_{\text{particle}} = \frac{m_{\text{electron}}}{8}$$

10. A loop of flexible wire of irregular shape carrying current is placed in an external magnetic field. Identify the effect of the field on the wire.

- (1) Wire gets stretched to become straight
- (2) Loop assumes circular shape with its plane normal to the field
- (3) Loop assumes circular shape with its plane parallel to the field
- (4) Shape of the loop remains unchanged

**Answer (2)**

**Sol.** For aligning magnetic moment with external magnetic field loop will align its plane normal to the field. This position of loop in magnetic field will cause the wire to stretch out and the loop will assume a circular shape.

11. What will be the average value of energy along one degree of freedom for an ideal gas in thermal equilibrium at a temperature  $T$ ? ( $k_B$  is Boltzmann constant)

- (1)  $\frac{3}{2} k_B T$
- (2)  $\frac{1}{2} k_B T$
- (3)  $k_B T$
- (4)  $\frac{2}{3} k_B T$

**Answer (2)**

**Sol.** Each degree of freedom as per law of equipartition of energy is associated with  $\frac{1}{2} k_B T$  energy per molecule.

12. An oil drop of radius 2 mm with a density  $3 \text{ g cm}^{-3}$  is held stationary under a constant electric field  $3.55 \times 10^5 \text{ V m}^{-1}$  in the Millikan's oil drop experiment. What is the number of excess electrons that the oil drop will possess?

Consider  $g = 9.81 \text{ m/s}^2$

- (1)  $1.73 \times 10^{12}$
- (2)  $1.73 \times 10^{10}$
- (3)  $48.8 \times 10^{11}$
- (4)  $17.3 \times 10^{10}$

**Answer (2)**

**Sol.** The electrostatic field will balance the weight of oil drop

$$\Rightarrow \frac{4}{3} \pi r^3 \times \rho \times g = neE$$

$$\Rightarrow n = \frac{\frac{4}{3} \pi r^3 \rho g}{eE} = \frac{\frac{4}{3} \times \pi \times (2 \times 10^{-3})^3 \times (3 \times 10^3) \times 9.81}{1.6 \times 10^{-19} \times 3.55 \times 10^5}$$

$$= 173.65 \times 10^8$$

$$= 1.73 \times 10^{10}$$

13. The time period of a satellite in a circular orbit of radius  $R$  is  $T$ . The period of another satellite in a circular orbit of radius  $9R$  is :

- (1) 12 T
- (2) 27 T
- (3) 9 T
- (4) 3 T

**Answer (2)**

**Sol.**  $T^2 \propto R^3$

$$\Rightarrow \frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$$

$$\Rightarrow \frac{T^2}{T_2^2} = \frac{R^3}{729R^3}$$

$$\Rightarrow T_2 = 27 T$$

14. Imagine that the electron in a hydrogen atom is replaced by a muon ( $\mu$ ). The mass of muon particle is 207 times that of an electron and charge is equal to the charge of an electron. The ionization potential of this hydrogen atom will be :

- (1) 331.2 eV
- (2) 2815.2 eV
- (3) 13.6 eV
- (4) 27.2 eV

**Answer (2)**

**Sol.** Ionization potential is directly proportional to mass of revolving particle

$$\Rightarrow \frac{IP_{\mu\text{on}}}{IP_{\text{electron}}} = \frac{m_{\mu\text{on}}}{m_{\text{electron}}}$$

$$\Rightarrow IP_{\mu\text{on}} = 207 \times IP_{\text{electron}}$$

$$= 207 \times 13.6 \text{ eV}$$

$$= 2815.2 \text{ eV}$$

15. A plane electromagnetic wave of frequency 100 MHz is travelling in vacuum along the x-direction. At a particular point in space and time,  $\vec{B} = 2.0 \times 10^{-8} \hat{k} \text{ T}$ . (where,  $\hat{k}$  is unit vector along z-direction) What is  $\vec{E}$  at this point?

(speed of light  $c = 3 \times 10^8 \text{ m/s}$ )

- (1)  $6.0 \hat{j} \text{ V/m}$
- (2)  $0.6 \hat{j} \text{ V/m}$
- (3)  $0.6 \hat{k} \text{ V/m}$
- (4)  $6.0 \hat{k} \text{ V/m}$

**Answer (1)**

**Sol.**  $|E_0| = |B_0| c$

$$\Rightarrow |E_0| = 2 \times 10^{-8} \times 3 \times 10^8 \text{ V/m}$$

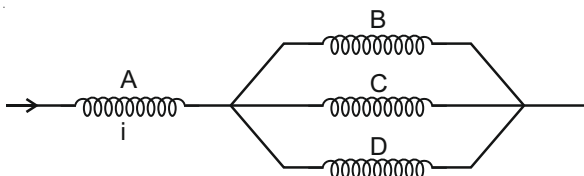
$$\Rightarrow |E_0| = 6 \text{ V/m}$$

$$\vec{S} = \vec{E} \times \vec{B}$$

$\Rightarrow$  Electric field vector at given point of time should point toward positive y-axis.

16. Four identical long solenoids A, B, C and D are connected to each other as shown in the figure. If the magnetic field at the center of A is 3 T, the field at the centre of C would be:

(Assume that the magnetic field is confined within the volume of respective solenoid).



- (1) 12 T                      (2) 9 T  
(3) 1 T                        (4) 6 T

**Answer (3)**

**Sol.** As  $I_B + I_C + I_D = I_A$

By symmetry  $I_B = I_C = I_D$

$$\Rightarrow 3LI_C = LI_A$$

$$\Rightarrow 3\phi_C = \phi_A$$

$$\Rightarrow B_C = \frac{B_A}{3} = 1 \text{ T}$$

17. The time period of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

The measured value of the length of pendulum is 10 cm known to a 1 mm accuracy. The time for 200 oscillations of the pendulum is found to be 100 second using a clock of 1 s resolution. The percentage accuracy in the determination of 'g' using the pendulum is 'x'. The value of 'x' to the nearest integer is,

- (1) 3%                              (2) 5%  
(3) 2%                              (4) 4%

**Answer (1)**

**Sol.**  $T = 2\pi\sqrt{\frac{l}{g}}$

$$\text{or } g = \frac{l(2\pi)^2}{T^2}$$

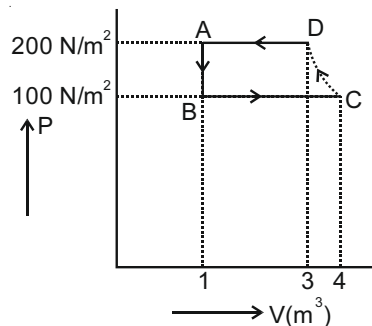
$$\Rightarrow \frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T}$$

$$\Rightarrow x = \frac{10^{-3}}{10 \times 10^{-2}} \times 100 + 2 \times \frac{1}{100} \times 100$$

$$x = (1 + 2)\%$$

$$x = 3\%$$

18. The P-V diagram of a diatomic ideal gas system going under cyclic process as shown in figure. The work done during an adiabatic process CD is (Use  $\gamma = 1.4$ ):



- (1) - 500 J                      (2) 200 J  
(3) - 400 J                      (4) 400 J

**Answer (1)**

**Sol.** (C → D) work done =  $\frac{P_C V_C - P_D V_D}{\gamma - 1}$

$$= \frac{400 - 600}{\frac{2}{5}} \text{ J} = -500 \text{ J}$$

19. In Young's double slit arrangement, slits are separated by a gap of 0.5 mm, and the screen is placed at a distance of 0.5 m from them. The distance between the first and the third bright fringe formed when the slits are illuminated by a monochromatic light of 5890 Å is:

- (1)  $1178 \times 10^{-9} \text{ m}$                       (2)  $1178 \times 10^{-6} \text{ m}$   
(3)  $1178 \times 10^{-12} \text{ m}$                       (4)  $5890 \times 10^{-7} \text{ m}$

**Answer (2)**

**Sol.**  $d = 0.5 \text{ mm}$

$$D = 0.5 \text{ m}$$

$$\text{Required distance} = 2\beta = \frac{2\lambda D}{d}$$

$$= \frac{2 \times 5890 \times 10^{-10} \times 0.5}{0.5 \times 10^{-3}}$$

$$= 1178 \times 10^{-6} \text{ m}$$

20. In the experiment of Ohm's law, a potential difference of 5.0 V is applied across the end of a conductor of length 10.0 cm and diameter of 5.00 mm. The measured current in the conductor is 2.00 A. The maximum permissible percentage error in the resistivity of the conductor is:

- (1) 3.0                                      (2) 3.9  
(3) 7.5                                      (4) 8.4

**Answer (2)**

**Sol.**  $V = I \times \rho \frac{\ell}{A}$

$$\Rightarrow \rho = \frac{VA}{I\ell} = \frac{\pi V d^2}{4 I \ell}$$

$$\frac{\Delta \rho}{\rho} = \frac{2\Delta d}{d} + \frac{\Delta V}{V} + \frac{\Delta I}{I} + \frac{\Delta \ell}{\ell}$$

$$= 2\left(\frac{0.01}{5}\right) + \frac{0.1}{5} + \frac{0.01}{2} + \frac{0.1}{10}$$

$$\frac{\Delta \rho}{\rho} = 0.039 = 3.9\%$$

**SECTION - II**

**Numerical Value Type Questions:** This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. A parallel plate capacitor has plate area  $100 \text{ m}^2$  and plate separation of  $10 \text{ m}$ . The space between the plates is filled up to a thickness  $5 \text{ m}$  with a material of dielectric constant of  $10$ . The resultant capacitance of the system is ' $x$ ' pF.

The value of  $\epsilon_0 = 8.85 \times 10^{-12} \text{ F.m}^{-1}$

The value of ' $x$ ' to the nearest integer is \_\_\_\_\_.

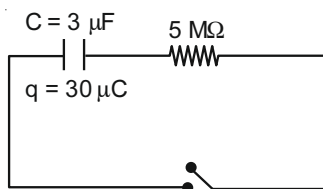
**Answer (161)**

**Sol.**  $C = \frac{\epsilon_0 A}{d - t + t/k}$

$$= \frac{8.85 \times 10^{-12} \times 100}{10 - 5 + 5/10} = 1.609 \times 10^{-10}$$

$$= 160.9 \times 10^{12} = 161$$

2. The circuit shown in the figure consists of a charged capacitor of capacity  $3 \mu\text{F}$  and a charge of  $30 \mu\text{C}$ . At time  $t = 0$ , when the key is closed, the value of current flowing through the  $5 \text{ M}\Omega$  resistor is ' $x$ '  $\mu\text{A}$ .



The value of ' $x$ ' to the nearest integer is \_\_\_\_\_.

**Answer (2)**

**Sol.** Potential difference across resistance at  $t = 0$

$$= \frac{Q}{C} = 10 \text{ V}$$

$$\Rightarrow I = \frac{10}{R} = \frac{10}{5 \times 10^6} = 2 \times 10^{-6} \text{ A} = 2 \mu\text{A}$$

3. Two separate wires A and B are stretched by  $2 \text{ mm}$  and  $4 \text{ mm}$  respectively, when they are subjected to a force of  $2 \text{ N}$ . Assume that both the wires are made up of same material and the radius of wire B is 4 times that of the radius of wire A. The length of the wires A and B are in the ratio of  $a : b$ . Then

$\frac{a}{b}$  can be expressed as  $\frac{1}{x}$  where  $x$  is \_\_\_\_\_.

**Answer (32)**

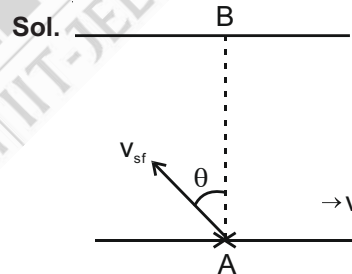
**Sol.**  $\Delta \ell = \frac{F \ell}{A y}$

$$\Rightarrow \frac{\Delta \ell_A}{\Delta \ell_B} = \frac{\ell_A}{\ell_B} \times \frac{r_B^2}{r_A^2} \Rightarrow \frac{2}{4} = \frac{\ell_A}{\ell_B} \times 16$$

$$\Rightarrow \frac{\ell_A}{\ell_B} = \frac{1}{32} \Rightarrow x = 32$$

4. A person is swimming with a speed of  $10 \text{ m/s}$  at an angle of  $120^\circ$  with the flow and reaches to a point directly opposite on the other side of the river. The speed of the flow is ' $x$ '  $\text{m/s}$ . The value of ' $x$ ' to the nearest integer is \_\_\_\_\_.

**Answer (5)**



$$\sin \theta = \frac{v_f}{v_{sf}} \quad \sin 30^\circ = \frac{x}{10}$$

$$x = 5$$

5. An npn transistor operates as a common emitter amplifier with a power gain of  $10^6$ . The input circuit resistance is  $100 \Omega$  and the output load resistance is  $10 \text{ k}\Omega$ . The common emitter current gain ' $\beta$ ' will be \_\_\_\_\_. (Round off to the Nearest Integer)

**Answer (100)**

**Sol.** Power gain =  $\frac{R_{out}}{R_{in}}$  (Current gain)<sup>2</sup>

$$\Rightarrow 10^6 = \frac{10 \times 10^3}{100} \times \beta^2 \Rightarrow \beta = 100$$



6. A bullet of mass 0.1 kg is fired on a wooden block to pierce through it, but it stops after moving a distance of 50 cm into it. If the velocity of bullet before hitting the wood is 10 m/s and it slows down with uniform deceleration, then the magnitude of effective retarding force on the bullet is 'x' N.

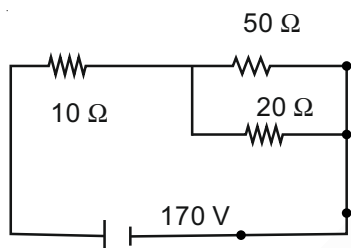
The value of 'x' to the nearest integer is \_\_\_\_\_.

**Answer (10)**

$$\text{Sol. } F_{\text{avg}} = \frac{\Delta \text{K.E.}}{\text{distance}} = \frac{\frac{1}{2}mv^2}{s}$$

$$= \frac{\frac{1}{2} \times 0.1 \times 100}{0.5} = 10 \text{ N}$$

7. The voltage across the 10 Ω resistor in the given circuit is x volt.



The value of 'x' to the nearest integer is \_\_\_\_\_.

**Answer (70)**

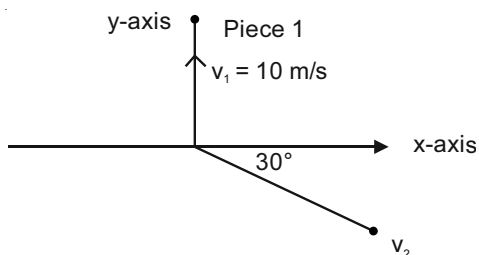
$$\text{Sol. } V_{10} = \frac{170 \times 10}{10 + \frac{1000}{70}} = 70 \text{ V}$$

8. A ball of mass 10 kg moving with a velocity  $10\sqrt{3}$  m/s along the x-axis, hits another ball of mass 20 kg which is at rest. After the collision, first ball comes to rest while the second ball disintegrates into two equal pieces. One piece starts moving along y-axis with a speed of 10 m/s. The second piece starts moving at an angle of 30° with respect to the x-axis.

The velocity of the ball moving at 30° with x-axis is x m/s.

The configuration of pieces after collision is shown in the figure below.

The value of x to the nearest integer is \_\_\_\_\_.



**Answer (20)**

$$\text{Sol. Velocity of 10 kg ball} = v_{10} = 10\sqrt{3}\hat{i}$$

$$\text{initial total momentum of system} = 10 \times 10\sqrt{3}\hat{i}$$

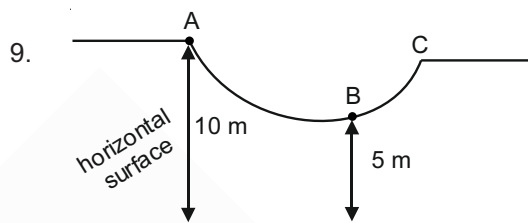
Final total momentum of system

$$= 10 \times 10\hat{j} + 10 \times x(\cos 30^\circ \hat{i} - \sin 30^\circ \hat{j})$$

Now by conservation of momentum

$$10 \times 10\sqrt{3}\hat{i} = 10 \times 10\hat{j} + 10 \times x \left( \frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j} \right)$$

$$\Rightarrow x = 20$$



As shown in the figure, a particle of mass 10 kg is placed at a point A. When the particle is slightly displaced to its right, it starts moving and reaches the point B. The speed of the particle at B is x m/s. (Take  $g = 10 \text{ m/s}^2$ )

The value of 'x' to the nearest integer is \_\_\_\_\_.

**Answer (10)**

**Sol.** By Energy Conservation

$$T.E_A = T.E_B$$

$$mg(10) + 0 = mg(5) + \frac{1}{2}mv^2$$

$$\Rightarrow v = \sqrt{2 \times g \times 5} = 10 \text{ m/s}$$

$$\Rightarrow x = 10$$

10. A particle performs simple harmonic motion with a period of 2 second. The time taken by the particle to cover a displacement equal to half of its amplitude

from the mean position is  $\frac{1}{a}$  s.

The value of 'a' to the nearest integer is \_\_\_\_\_.

**Answer (6)**

$$\text{Sol. } \frac{A}{2} = A \sin \left( \frac{2\pi t}{T} \right)$$

$$\Rightarrow \frac{2\pi t}{T} = \frac{\pi}{6}$$

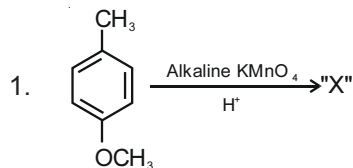
$$\Rightarrow t = \frac{T}{12} = \frac{1}{6} \text{ seconds}$$

$$x = 6$$

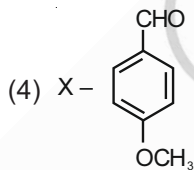
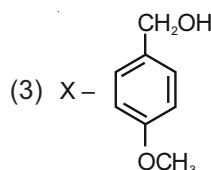
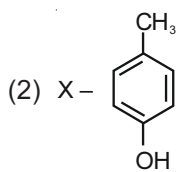
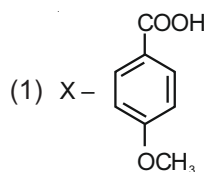
**PART-B : CHEMISTRY**

**SECTION - I**

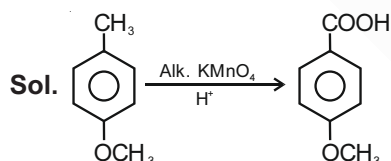
**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.



Considering the above chemical reaction, identify the product "X" :



**Answer (1)**



2. Match **List-I** with **List-II** :

<b>List-I</b>	<b>List-II</b>
(a) $\text{Ca}(\text{OCl})_2$	(i) Antacid
(b) $\text{CaSO}_4 \cdot \frac{1}{2}\text{H}_2\text{O}$	(ii) Cement
(c) $\text{CaO}$	(iii) Bleach
(d) $\text{CaCO}_3$	(iv) Plaster of Paris

Choose the **most appropriate** answer from the options given below :

- (1) (a)-(iii), (b)-(ii), (c)-(i), (d)-(iv)  
 (2) (a)-(iii), (b)-(ii), (c)-(iv), (d)-(i)  
 (3) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)  
 (4) (a)-(i), (b)-(iv), (c)-(iii), (d)-(ii)

**Answer (3)**

**Sol.**  $\text{CaSO}_4 \cdot \frac{1}{2}\text{H}_2\text{O}$  - Plaster of Paris

$\text{Ca}(\text{OCl})_2$  - Bleaching agent

$\text{CaO}$  is used in cement

$\text{CaCO}_3$  is used as an Antacid

3. Match **List-I** with **list-II** :

<b>List-I</b>	<b>List-II</b>
(a) Chlorophyll	(i) Ruthenium
(b) Vitamin - $\text{B}_{12}$	(ii) Platinum
(c) Anticancer drug	(iii) Cobalt
(d) Grubbs catalyst	(iv) Magnesium

Choose the **most appropriate** answer from the options given below :

- (1) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)  
 (2) (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i)  
 (3) (a)-(iii), (b)-(ii), (c)-(iv), (d)-(i)  
 (4) (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii)

**Answer (1)**

**Sol.** Chlorophyll - Mg

Vitamin -  $\text{B}_{12}$  - Co

Anticancer drug - Pt

4. Match **List-I** with **list-II** :

<b>List-I</b>	<b>List-II</b>
<b>(Class of Drug)</b>	<b>(Example)</b>
(a) Antacid	(i) Novestrol
(b) Artificial Sweetener	(ii) Cimetidine
(c) Antifertility	(iii) Valium
(d) Tranquilizers	(iv) Alitame

Choose the **most appropriate** match :

- (1) (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii)  
 (2) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)  
 (3) (a)-(iv), (b)-(i), (c)-(ii), (d)-(iii)  
 (4) (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)

**Answer (2)**

**Sol.** Antacid - Cimetidine

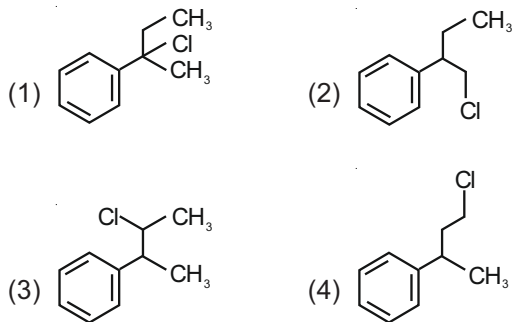
Artificial Sweetener - Alitame

Antifertility - Novestrol

Tranquilizers - Valium

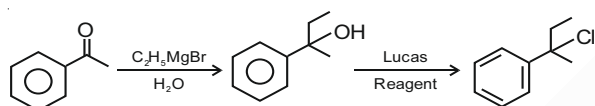


5. Reaction of Grignard reagent,  $C_2H_5MgBr$  with  $C_8H_8O$  followed by hydrolysis gives compound "A" which reacts instantly with Lucas reagent to give compound B,  $C_{10}H_{13}Cl$ .



**Answer (1)**

**Sol.** Reaction occur through formation of carbocation.



6. Match **List-I** and with **List-II**.

<b>List-I</b> <b>(Process)</b>	<b>List-II</b> <b>(Catalyst)</b>
(a) Deacon's process	(i) ZSM-5
(b) Contact process	(ii) $CuCl_2$
(c) Cracking of hydrocarbons	(iii) Particles 'Ni'
(d) Hydrogenation of vegetable oils	(iv) $V_2O_5$

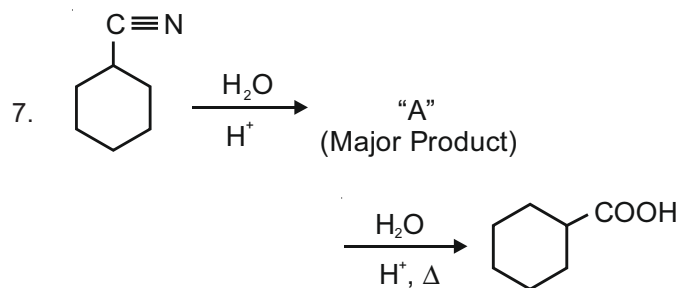
Choose the **most appropriate** answer from the options given below :

- (1) (a)-(iv), (b)-(ii), (c)-(i), (d)-(iii)  
 (2) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)  
 (3) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)  
 (4) (a)-(i), (b)-(iii), (c)-(ii), (d)-(iv)

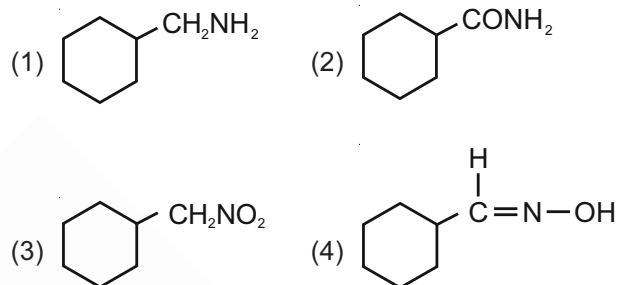
**Answer (2)**

**Sol.**

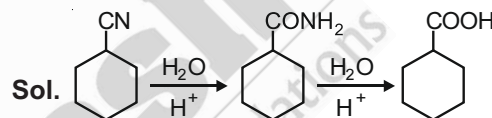
(a) Deacon's process	-	$CuCl_2$
(b) Contact process	-	$V_2O_5$
(c) Cracking of hydrocarbons	-	ZSM-5
(d) Hydrogenation of vegetable oils	-	Particles 'Ni'



Consider the above chemical reaction and identify product "A"



**Answer (2)**



8. A certain orbital has no angular nodes and two radial nodes. The orbital is

- (1)  $2p$  (2)  $3p$   
 (3)  $3s$  (4)  $2s$

**Answer (3)**

**Sol.**  $3s$  has no angular node two radial nodes.

9. In a binary compound, atoms of element A form a hcp structure and those of element M occupy  $2/3$  of the tetrahedral voids of the hcp structure. The formula of the binary compound is

- (1)  $M_2A_3$  (2)  $M_4A_3$   
 (3)  $M_4A$  (4)  $MA_3$

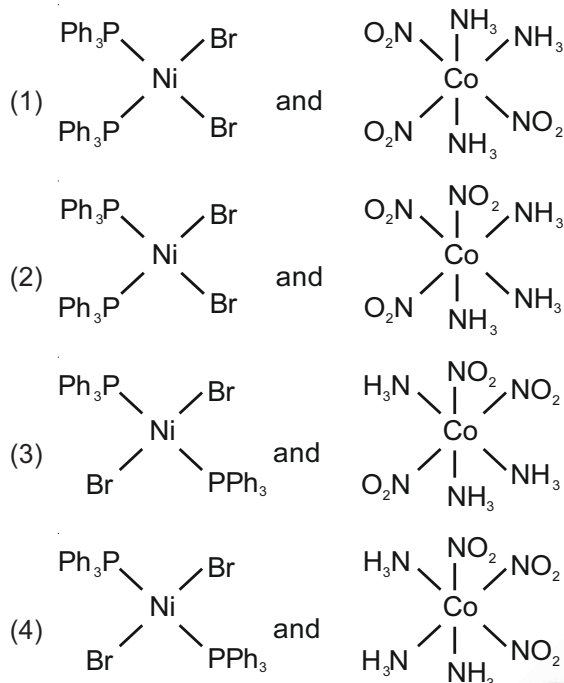
**Answer (2)**

**Sol.**  $A = 6$

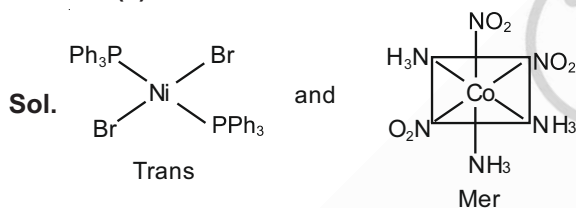
$$M = \frac{2}{3} \times 12 = 8$$



10. The correct structures of trans-[NiBr<sub>2</sub>(PPh<sub>3</sub>)<sub>2</sub>] and meridional-[Co(NH<sub>3</sub>)<sub>3</sub>(NO<sub>2</sub>)<sub>3</sub>], respectively



**Answer (3)**

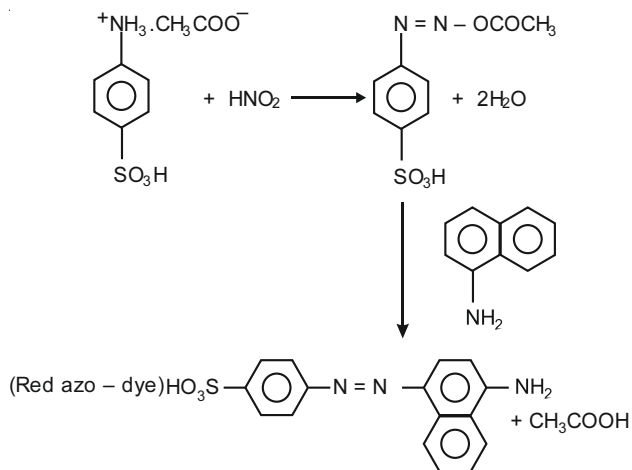
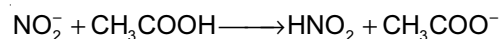


11. Reagent, 1-naphthylamine and sulphanilic acid in acetic acid is used for the detection of

- (1) NO<sub>3</sub><sup>-</sup> (2) N<sub>2</sub>O  
(3) NO<sub>2</sub><sup>-</sup> (4) NO

**Answer (3)**

**Sol.** 1-naphthyl amine and sulphanilic acid in acetic acid is used for the detection of NO<sub>2</sub><sup>-</sup>

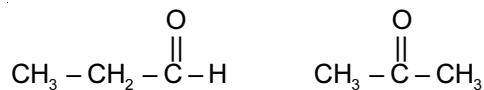


12. Compound with molecular formula C<sub>3</sub>H<sub>6</sub>O can show

- (1) Positional isomerism  
(2) Both positional isomerism and metamerism  
(3) Metamerism  
(4) Functional group isomerism

**Answer (4)**

**Sol.** C<sub>3</sub>H<sub>6</sub>O (degree of unsaturation = 1)



Functional group isomerism.

13. The chemical that is added to reduce the melting point of the reaction mixture during the extraction of aluminium is

- (1) Cryolite  
(2) Calamine  
(3) Kaolite  
(4) Bauxite

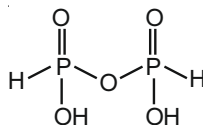
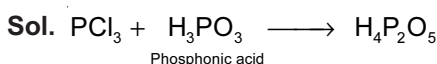
**Answer (1)**

**Sol.** Cryolite is used to lower M.P. of Alumina.

14. The number of ionisable hydrogens present in the product obtained from a reaction of phosphorus trichloride and phosphonic acid is

- (1) 0  
(2) 3  
(3) 2  
(4) 1

**Answer (3)**



There are two ionisable H's.

15. Given below are two Statements : One is labelled as **Assertion A** and the other is labelled as **Reason R** :

**Assertion A** : During the boiling of water having temporary hardness,  $\text{Mg}(\text{HCO}_3)_2$  is converted to  $\text{MgCO}_3$ .

**Reason R** : The solubility product of  $\text{Mg}(\text{OH})_2$  is greater than that of  $\text{MgCO}_3$ .

In the light of the above statements, choose the **most appropriate** answer from the options given below:

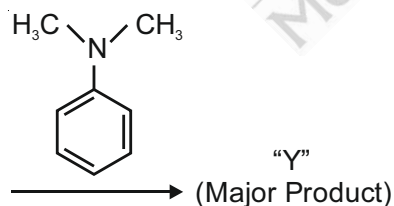
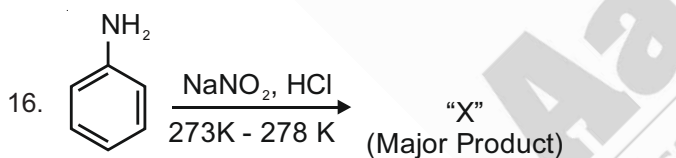
- (1) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**
- (2) **A** is true but **R** is false
- (3) Both **A** and **R** are true and **R** is the correct explanation of **A**
- (4) **A** is false but **R** is true

**Answer (4)**

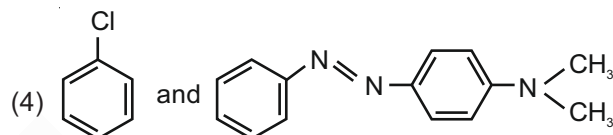
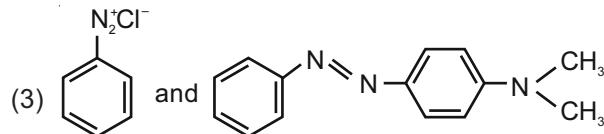
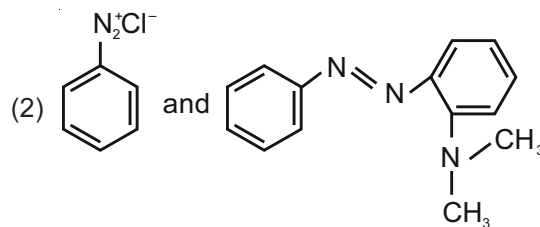
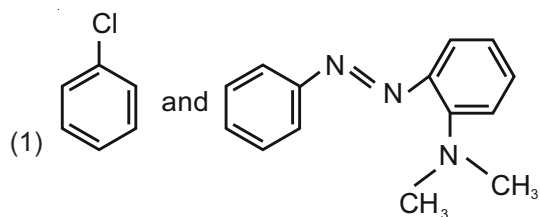
**Sol.** A : During the boiling of water having temporary hardness  $\text{Mg}(\text{OH})_2$  is formed.

Given Assertion is false.

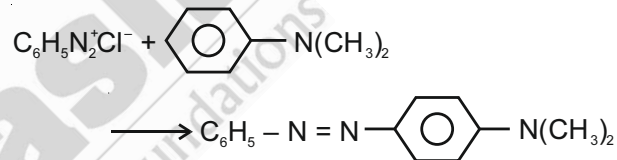
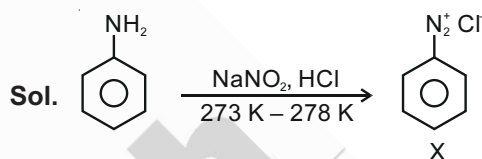
R :  $\text{Mg}(\text{OH})_2$  has greater solubility product than that of  $\text{MgCO}_3$ .



Considering the above reaction, X and Y respectively are



**Answer (3)**



17. The Statements that are TRUE :

- (A) Methane leads to both global warming and photochemical smog
- (B) Methane is generated from paddy fields
- (C) Methane is a stronger global warming gas than  $\text{CO}_2$
- (D) Methane is a part of reducing smog

Choose the **most appropriate** answer from the options given below

- (1) (A), (B), (C) only
- (2) (A), (B), (D) only
- (3) (A) and (B) only
- (4) (B), (C), (D) only

**Answer (1)**

**Sol.** •  $\text{CH}_4$  causes global warming and also a constituent of photochemical smog.

It is a part of oxidising smog.

- $\text{CH}_4$  is generated from paddy fields.
- Due to high heat capacity,  $\text{CH}_4$  is a stronger global warming causing gas than  $\text{CO}_2$ .

18. Match List-I with List-II

List-I (Chemicals)	List-II (Use/Preparation/ Constituent)
(a) Alcoholic potassium hydroxide	(i) electrodes in batteries
(b) Pd/BaSO <sub>4</sub>	(ii) obtained by addition reaction
(c) BHC (Benzene hexachloride)	(iii) used for β-elimination reaction
(d) Polyacetylene	(iv) Lindlar's Catalyst

Choose the **most appropriate** match

- (1) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)  
 (2) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)  
 (3) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)  
 (4) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

**Answer (1)**

- Sol.** • Alc. KOH causes elimination  
 • Pd / BaSO<sub>4</sub> – Lindlar's catalyst  
 • BHC is obtained by the addition reaction of Cl<sub>2</sub> with benzene in presence of U.V.  
 • Thin film of polyacetylene can be used as electrode in batteries.
19. A non-reducing sugar "A" hydrolyses to give two reducing mono saccharides. Sugar A is :
- (1) Galactose                      (2) Sucrose  
 (3) Fructose                      (4) Glucose

**Answer (2)**

- Sol.** Sucrose  $\xrightarrow{\text{Hydrolyses}}$  glucose + fructose
- Glucose and fructose both are monosaccharides. Sucrose is non-reducing sugar.
20. The ionic radius of Na<sup>+</sup> ion is 1.02 Å. The ionic radii (in Å) of Mg<sup>2+</sup> and Al<sup>3+</sup>, respectively, are
- (1) 0.72 and 0.54                  (2) 1.05 and 0.99  
 (3) 0.68 and 0.72                  (4) 0.85 and 0.99

**Answer (1)**

**Sol.** Order of ionic size Na<sup>+</sup> > Mg<sup>2+</sup> > Al<sup>3+</sup>

**SECTION - II**

**Numerical Value Type Questions:** This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

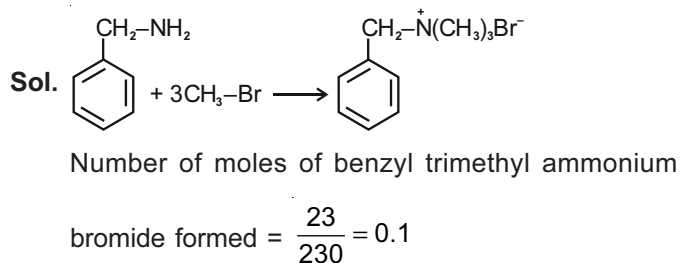
1. 2 molal solution of a weak acid HA has a freezing point of 3.885°C. The degree of dissociation of this acid is \_\_\_\_\_ × 10<sup>-3</sup>.  
 (Round off to the Nearest Integer).  
 [Given : Molal depression constant of water = 1.85 K kg mol<sup>-1</sup>  
 Freezing point of pure water = 0°C]

**Answer (50)**

- Sol.** HA  $\rightleftharpoons$  H<sup>+</sup> + A<sup>-</sup>  
 $\frac{1-\alpha}{1-\alpha} \quad \frac{\alpha}{\alpha} \quad \frac{\alpha}{\alpha}$   
 (α – degree of dissociation)  
 van't Hoff factor (i) = 1 + α  
 Assuming given freezing point is -3.885°C  
 $\Delta T_f = i k_f \cdot m$   
 $\Rightarrow 3.885 = (1 + \alpha) \times 1.85 \times 2$   
 $\Rightarrow (1 + \alpha) = 1.05$   
 $\alpha = 0.05$   
 $= 50 \times 10^{-3}$
2. A reaction of 0.1 mole of Benzylamine with bromomethane gave 23 g of Benzyl trimethyl ammonium bromide. The number of moles of bromomethane consumed in this reaction are n × 10<sup>-1</sup>, when n = \_\_\_\_\_  
 (Round off to the Nearest Integer).

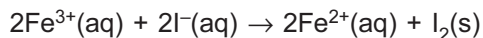
[Given : Atomic masses : C : 12.0 u, H : 1.0 u, N : 14.0 u, Br : 80.0 u]

**Answer (03)**



∴ No. of moles of bromomethane consumed  
= 3 × 0.1  
= 3 × 10<sup>-1</sup>

3. For the reaction



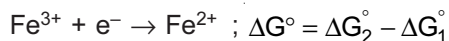
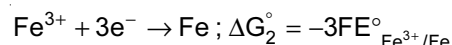
the magnitude of the standard molar free energy change,  $\Delta_r G_m^\circ = -$  \_\_\_\_\_ kJ

(Round off to the Nearest Integer).

$$\left[ \begin{array}{l} E^\circ_{\text{Fe}^{2+}/\text{Fe}(\text{s})} = -0.440 \text{ V}; E^\circ_{\text{Fe}^{3+}/\text{Fe}(\text{s})} = -0.036 \text{ V} \\ E^\circ_{\text{I}_2/2\text{I}^{-}} = 0.539 \text{ V}; F = 96500 \text{ C} \end{array} \right]$$

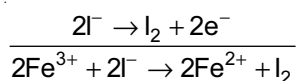
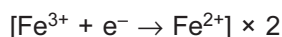
**Answer (45)**

**Sol.**  $\text{Fe}^{2+} + 2\text{e}^{-} \rightarrow \text{Fe}; \Delta G_1^\circ = -2FE^\circ_{\text{Fe}^{2+}/\text{Fe}}$



$$\Rightarrow FE^\circ_{\text{Fe}^{3+}/\text{Fe}^{2+}} = -3FE^\circ_{\text{Fe}^{3+}/\text{Fe}} + 2FE^\circ_{\text{Fe}^{2+}/\text{Fe}}$$

$$\begin{aligned} \Rightarrow E^\circ_{\text{Fe}^{3+}/\text{Fe}^{2+}} &= 3E^\circ_{\text{Fe}^{3+}/\text{Fe}} - 2E^\circ_{\text{Fe}^{2+}/\text{Fe}} \\ &= -3 \times 0.036 + 2 \times 0.440 \\ &= 0.772 \text{ V} \end{aligned}$$



$$\begin{aligned} \Delta_r G_m^\circ &= -2F \times 0.772 - 2F(-0.539) \\ &= -2F(0.772 - 0.539) \\ &= 44969 \text{ J} \\ &= 44.969 \text{ kJ} \\ &\approx 45 \text{ kJ} \end{aligned}$$

4. In order to prepare a buffer solution of pH 5.74, sodium acetate is added to acetic acid. If the concentration of acetic acid in the buffer is 1.0 M, the concentration of sodium acetate in the buffer is \_\_\_\_\_ M.

(Round off to the Nearest Integer).

[Given :  $\text{p}K_a$  (acetic acid) = 4.74]

**Answer (10)**

**Sol.** For an acidic buffer of  $\text{CH}_3\text{COOH}$  and  $\text{CH}_3\text{COO}^{-}$

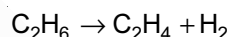
$$\text{pH} = \text{p}K_a + \log \frac{[\text{CH}_3\text{COO}^{-}]}{[\text{CH}_3\text{COOH}]}$$

$$\Rightarrow 5.74 = 4.74 + \log \frac{x}{1}$$

$$\Rightarrow \frac{x}{1} = 10$$

$$x = 10 \text{ M}$$

5. For the reaction



the reaction enthalpy  $\Delta_r H =$  \_\_\_\_\_ kJ mol<sup>-1</sup>.

(Round off to the Nearest Integer).

[Given : Bond enthalpies in kJ mol<sup>-1</sup> :

$$\text{C} - \text{C} : 347, \text{C} = \text{C} : 611;$$

$$\text{C} - \text{H} : 414, \text{H} - \text{H} : 436]$$

**Answer (128)**

**Sol.**  $\text{C}_2\text{H}_6 \rightarrow \text{C}_2\text{H}_4 + \text{H}_2$

$$\Delta_r H = (\text{B.E})_{\text{C}-\text{C}} + 6 \times (\text{B.E})_{\text{C}-\text{H}}$$

$$- [(\text{B.E})_{\text{C}=\text{C}} + 4 \times (\text{B.E})_{\text{C}-\text{H}} + (\text{B.E})_{\text{H}-\text{H}}]$$

$$= 347 + 6 \times 414 - (611 + 4 \times 414 + 436)$$

$$= 128 \text{ kJ mol}^{-1}$$

6. Complete combustion of 3 g of ethane gives  $x \times 10^{22}$  molecules of water. The value of x is \_\_\_\_\_.

(Round off to the Nearest Integer).

[Use :  $N_A = 6.023 \times 10^{23}$ ;

Atomic masses in u : C : 12.0 ; O : 16.0 ; H : 1.0]

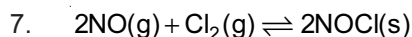
**Answer (18)**

**Sol.**  $\text{C}_2\text{H}_6 + \frac{7}{2}\text{O}_2 \rightarrow 2\text{CO}_2 + 3\text{H}_2\text{O}$

$$\text{Number of moles of ethane} = \frac{3}{30} = 0.1$$

$$\text{Number of moles of water} = 3 \times 0.1 = 0.3$$

$$\begin{aligned} \therefore \text{Number of molecules of water} \\ &= 6.023 \times 10^{23} \times 0.3 \\ &= 18.069 \times 10^{22} \\ &\approx 18 \times 10^{22} \end{aligned}$$



This reaction was studied at  $-10^\circ\text{C}$  and the following data was obtained

run	$[\text{NO}]_0$	$[\text{Cl}_2]_0$	$r_0$
1	0.10	0.10	0.18
2	0.10	0.20	0.35
3	0.20	0.20	1.40

$[\text{NO}]_0$  and  $[\text{Cl}_2]_0$  are the initial concentrations and  $r_0$  is the initial reaction rate.

The overall order of the reaction is \_\_\_\_\_.

(Round off to the Nearest Integer).

**Answer (03)**

**Sol.** Rate  $(r) = k[\text{NO}]^x [\text{Cl}_2]^y$

From run (2) and (3)

$$\frac{(r_0)_3}{(r_0)_2} = \frac{(0.20)^x (0.20)^y}{(0.10)^x (0.20)^y}$$

$$\Rightarrow 2^x = \frac{1.40}{0.35} = 2^2$$

$$\therefore x = 2$$

From run (1) and (2)

$$\frac{(r_0)_2}{(r_0)_1} = \frac{(0.10)^x (0.20)^y}{(0.10)^x (0.10)^y}$$

$$\frac{0.35}{0.18} = 2^y$$

$$\Rightarrow y \approx 1$$

$$\therefore r = k[\text{NO}]^2 [\text{Cl}_2]^1$$

$$\therefore \text{Overall order of reaction} = 2 + 1 = 3$$

8. The total number of unpaired electrons present in the complex  $\text{K}_3[\text{Cr}(\text{oxalate})_3]$  is \_\_\_\_\_.

**Answer (03)**

**Sol.**  $\text{K}_3[\text{Cr}(\text{oxalate})_3]$

Oxidation state of Cr = +3

Electronic configuration of  $\text{Cr}^{+3} = [\text{Ar}] 3d^3$

$\therefore$  Number of unpaired electrons = 3

9. AX is a covalent diatomic molecule where A and X are second row elements of periodic table. Based on Molecular orbital theory, the bond order of AX is 2.5. The total number of electrons in AX is \_\_\_\_\_.

(Round off to the Nearest Integer).

**Answer (15)**

**Sol.** Bond order

$$= \frac{\text{No. of } e^- \text{ s in BMO} - \text{No. of } e^- \text{ s in ABMO}}{2}$$

$$\Rightarrow \text{No. of } e^- \text{ s in BMO} - \text{No. of } e^- \text{ s in ABMO} = 5$$

As AX is diatomic molecule (neutral)

The only possible case is CO

Total number of electrons = 15

**Note :** Total number of electrons equal to 13 will also have the 2.5 bond order. But in this case neutral diatomic molecule will not be possible.

10. \_\_\_\_\_ grams of 3-Hydroxy propanal (MW = 74) must be dehydrated to produce 7.8 g of acrolein (MW = 56) ( $\text{C}_3\text{H}_4\text{O}$ ) if the percentage yield is 64.

(Round off to the Nearest Integer).

[Given : Atomic masses : C : 12.0 u, H : 1.0 u, O : 16.0 u]

**Answer (16)**



Let x g 7.8

$$\text{Moles of acrolein produced} = \frac{7.8}{56} \text{ moles}$$

$$\therefore \frac{56}{x} \times 100 = 64$$

$$x \approx 16$$



**PART-C : MATHEMATICS**

**SECTION - I**

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer :**

1. The differential equation satisfied by the system of parabolas  $y^2 = 4a(x + a)$  is

(1)  $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) + y = 0$

(2)  $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) - y = 0$

(3)  $y\left(\frac{dy}{dx}\right) + 2x\left(\frac{dy}{dx}\right) - y = 0$

(4)  $y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$

**Answer (4)**

**Sol.**  $y^2 = 4ax + 4a^2$  ... (i)

Differentiate both sides we get

$2yy' = 4a \Rightarrow a = \frac{yy'}{2}$  ... (ii)

By (i) and (ii) we get

$y^2 = \frac{4 \cdot yy'}{2} \cdot x + \frac{4y^2(y')^2}{4}$

$\Rightarrow y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$

2.  $\frac{1}{3^2-1} + \frac{1}{5^2-1} + \frac{1}{7^2-1} + \dots + \frac{1}{(201)^2-1}$  is equal to

(1)  $\frac{101}{404}$  (2)  $\frac{99}{400}$

(3)  $\frac{25}{101}$  (4)  $\frac{101}{408}$

**Answer (3)**

**Sol.**  $T_n = \frac{1}{(2n+1)^2-1} = \frac{1}{2n(2n+2)} = \frac{1}{4(n(n+1))}$

$\Rightarrow T_n = \frac{1}{4} \left( \frac{n+1-n}{n(n+1)} \right) = \frac{1}{4} \left( \frac{1}{n} - \frac{1}{n+1} \right)$

$$S_n = \sum_{n=1}^{100} T_n = \frac{1}{4} \left( 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{100} - \frac{1}{101} \right)$$

$$= \frac{1}{4} \left( 1 - \frac{1}{101} \right) = \frac{1}{4} \cdot \frac{100}{101} = \frac{25}{101}$$

3. The value of  $3 + \frac{1}{4 + \frac{1}{3 + \frac{1}{4 + \frac{1}{3 + \dots \infty}}}}$  is equal to

(1)  $2 + \sqrt{3}$

(2)  $3 + 2\sqrt{3}$

(3)  $4 + \sqrt{3}$

(4)  $1.5 + \sqrt{3}$

**Answer (4)**

**Sol.** Let  $y = 3 + \frac{1}{4 + \frac{1}{3 + \frac{1}{4 + \dots}}}$

$\Rightarrow y = 3 + \frac{1}{4 + \frac{1}{y}}$

$\Rightarrow (y - 3)(4y + 1) = y$

$\Rightarrow 4y^2 - 11y - 3 = y$

$\Rightarrow 4y^2 - 12y - 3 = 0$

$4\left(y - \frac{3}{2}\right)^2 = 12$

$\Rightarrow y = \sqrt{3} + \frac{3}{2}$

4. A vector  $\vec{a}$  has components  $3p$  and  $1$  with respect to a rectangular cartesian system. This system is rotated through a certain angle about the origin in the counter clockwise sense. If, with respect to new system,  $\vec{a}$  has components  $p + 1$  and  $\sqrt{10}$ , then a value of  $p$  is equal to

- (1)  $-1$  (2)  $1$   
(3)  $\frac{4}{5}$  (4)  $-\frac{5}{4}$

**Answer (1)**

**Sol.** Magnitude of vector remains same hence

$$\begin{aligned} 9p^2 + 1 &= (p + 1)^2 + 10 \\ \Rightarrow 8p^2 - 2p - 10 &= 0 \\ \Rightarrow 4p^2 - p - 5 &= 0 \\ \Rightarrow 4p^2 - 5p + 4p - 5 &= 0 \\ \Rightarrow (p + 1)(4p - 5) &= 0 \Rightarrow p = -1 \text{ or } \frac{5}{4} \end{aligned}$$

5. Let  $(1 + x + 2x^2)^{20} = a_0 + a_1x + a_2x^2 + \dots + a_{40}x^{40}$ . Then,  $a_1 + a_3 + a_5 + \dots + a_{37}$  is equal to

- (1)  $2^{19}(2^{20} - 21)$  (2)  $2^{19}(2^{20} + 21)$   
(3)  $2^{20}(2^{20} - 21)$  (4)  $2^{20}(2^{20} + 21)$

**Answer (1)**

**Sol.**  $(1 + x + 2x^2)^{20} = a_0 + a_1x + a_2x^2 + \dots + a_{40}x^{40}$

Put  $x = 1$   
 $\Rightarrow 4^{20} = a_0 + a_1 + \dots + a_{40}$  ... (i)

Put  $x = -1$   
 $\Rightarrow 2^{20} = a_0 - a_1 + \dots - a_{39} + a_{40}$  ... (ii)

by (i) - (ii) we get,

$$4^{20} - 2^{20} = 2(a_1 + a_3 + \dots + a_{37} + a_{39})$$

$$\Rightarrow a_1 + a_3 + \dots + a_{37} = \frac{4^{20} - 2^{20}}{2} = 2^{39} - 2^{19} - a_{39}$$
 ... (iii)

$a_{39} = \text{coeff. } x^{39} \text{ in } (1 + x + 2x^2)^{20}$

$$= \frac{20!}{0! 1! 19!} (1)^0 (1)^1 (2)^{19}$$

$$= 20 \cdot 2^{19}$$

$$\therefore a_1 + a_3 + \dots + a_{37} = 2^{39} - 2^{19} \cdot 21$$

$$\Rightarrow 2^{19}(2^{20} - 21)$$

6. The equation of one of the straight lines which passes through the point  $(1, 3)$  and makes an angle  $\tan^{-1}(\sqrt{2})$  with the straight line,  $y + 1 = 3\sqrt{2}x$  is :

- (1)  $4\sqrt{2}x - 5y - (5 + 4\sqrt{2}) = 0$   
(2)  $4\sqrt{2}x + 5y - 4\sqrt{2} = 0$   
(3)  $4\sqrt{2}x + 5y - (15 + 4\sqrt{2}) = 0$   
(4)  $5\sqrt{2}x + 4y - (15 + 4\sqrt{2}) = 0$

**Answer (3)**

**Sol.** Let slope of line be  $m$

$$\therefore \left| \frac{m - 3\sqrt{2}}{1 + 3\sqrt{2}m} \right| = \sqrt{2}$$

$$\Rightarrow m - 3\sqrt{2} = \pm\sqrt{2} \pm 6m$$

$$\Rightarrow m \mp 6m = \pm\sqrt{2} + 3\sqrt{2}$$

$$\Rightarrow m = -\frac{4\sqrt{2}}{5} \text{ or } \frac{2\sqrt{2}}{7}$$

Hence line can be

$$y - 3 = \frac{-4\sqrt{2}}{5}(x - 1)$$

$$\Rightarrow 5y - 15 = -4\sqrt{2}x + 4\sqrt{2}$$

$$\Rightarrow 4\sqrt{2}x + 5y - (15 + 4\sqrt{2}) = 0$$

7. Let  $A + 2B = \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix}$  and

$2A - B = \begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix}$ . If  $\text{Tr}(A)$  denotes the sum of all diagonal elements of the matrix  $A$ , then  $\text{Tr}(A) - \text{Tr}(B)$  has value equal to:

- (1)  $1$  (2)  $3$   
(3)  $0$  (4)  $2$

**Answer (4)**

**Sol.**  $A = \frac{1}{5}((A + 2B) + 2(2A - B))$

$$= \frac{1}{5} \left( \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix} + \begin{bmatrix} 4 & -2 & 10 \\ 4 & -2 & 12 \\ 0 & 2 & 4 \end{bmatrix} \right)$$

$$= \frac{1}{5} \begin{bmatrix} 5 & 0 & 10 \\ 10 & -5 & 15 \\ -5 & 5 & 5 \end{bmatrix} \Rightarrow \text{tr}(A) = 1$$

Similarly,

$$B = \frac{1}{5}(2(A + 2B) - (2A - B))$$

$$= \frac{1}{5} \left( \begin{bmatrix} 2 & 4 & 0 \\ 12 & -6 & 6 \\ -10 & 6 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix} \right)$$

$$= \frac{1}{5} \begin{bmatrix} 0 & 6 & -5 \\ 10 & -5 & 0 \\ -10 & 5 & 0 \end{bmatrix} \Rightarrow \text{tr}(B) = -1$$

$$\text{Tr}(A) - \text{Tr}(B) = 1 - (-1) = 2$$

8. The real valued function  $f(x) = \frac{\operatorname{cosec}^{-1}x}{\sqrt{x - [x]}}$ , where  $[x]$

denotes the greatest integer less than or equal to  $x$ , is defined for all  $x$  belonging to :

- (1) all non-integers except the interval  $[-1, 1]$
- (2) all integers except  $0, -1, 1$
- (3) all reals except integers
- (4) all reals except the interval  $[-1, 1]$

**Answer (1)**

**Sol.**  $\operatorname{cosec}^{-1}x$  defined for  $x \in (-\infty, -1] \cup [1, \infty)$

also  $\sqrt{\{x\}} > 0 \Rightarrow x \neq Z$

$\therefore f(x)$  is defined for all non-integer except interval  $[-1, 1]$

9. The solutions of the equation

$$\begin{vmatrix} 1 + \sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1 + \cos^2 x & \cos^2 x \\ 4 \sin 2x & 4 \sin 2x & 1 + 4 \sin 2x \end{vmatrix} = 0, (0 < x < \pi), \text{ are :}$$

- (1)  $\frac{\pi}{6}, \frac{5\pi}{6}$
- (2)  $\frac{\pi}{12}, \frac{\pi}{6}$
- (3)  $\frac{5\pi}{12}, \frac{7\pi}{12}$
- (4)  $\frac{7\pi}{12}, \frac{11\pi}{12}$

**Answer (4)**

**Sol.** By using  $C_1 \rightarrow C_1 - C_2$  and  $C_3 \rightarrow C_3 - C_2$  we get

$$\begin{vmatrix} 1 & \sin^2 x & 0 \\ -1 & 1 + \cos^2 x & -1 \\ 0 & 4 \sin 2x & 1 \end{vmatrix} = 0$$

Expanding by  $R_1$  we get

$$1(1 + \cos^2 x + 4 \sin 2x) - \sin^2 x(-1) = 0$$

$$\Rightarrow 2 + 4 \sin 2x = 0$$

$$\Rightarrow \sin 2x = \frac{-1}{2}$$

$$\Rightarrow 2x = n\pi + (-1)^n \left( \frac{-\pi}{6} \right), n \in Z$$

$$\therefore 2x = \frac{7\pi}{6}, \frac{11\pi}{6} \Rightarrow x = \frac{7\pi}{12}, \frac{11\pi}{12}$$

10. If  $\alpha, \beta$ , are natural numbers such that  $100^\alpha - 199^\beta = (100)(100) + (99)(101) + (98)(102) + \dots + (1)(199)$ , then the slope of the line passing through  $(\alpha, \beta)$  and origin is:

- (1) 550
- (2) 530
- (3) 540
- (4) 510

**Answer (1)**

**Sol.**  $100^\alpha - 199^\beta = 100^2 + (100 + 1)(100 - 1) + (100 - 2)(100 + 2) + \dots + (100 - 99)(100 + 99)$   
 $= 100^2 + 100^2 - 1^2 + 100^2 - 2^2 + \dots + 100^2 - 99^2$   
 $= 100 \cdot 100^2 - (1^2 + 2^2 + \dots + 99^2)$   
 $= 100^3 - \frac{99 \cdot 100 \cdot 199}{6}$

$$= 100^3 - 1650 \cdot 199$$

$$\therefore (\alpha, \beta) = (3, 1650)$$

$$m = \frac{\beta - 0}{\alpha - 0} = \frac{1650}{3} = 550$$

11. The integral  $\int \frac{(2x-1)\cos\sqrt{(2x-1)^2+5}}{\sqrt{4x^2-4x+6}} dx$  is equal

to :

(where  $c$  is a constant of integration)

- (1)  $\frac{1}{2} \sin \sqrt{(2x+1)^2+5} + c$
- (2)  $\frac{1}{2} \sin \sqrt{(2x-1)^2+5} + c$
- (3)  $\frac{1}{2} \cos \sqrt{(2x+1)^2+5} + c$
- (4)  $\frac{1}{2} \cos \sqrt{(2x-1)^2+5} + c$

**Answer (2)**

**Sol.** Put  $(2x - 1)^2 + 5 = t^2$

$$4(2x - 1)dx = 2t dt$$

$$\Rightarrow \int \frac{t \cos t}{2t} \cdot dt$$

$$\Rightarrow \frac{1}{2} \sin t + c$$

$$\Rightarrow \frac{1}{2} \sin(\sqrt{(2x-1)^2+5}) + c$$

12. For the four circles M, N, O and P, following four equations are given :

Circle M :  $x^2 + y^2 = 1$

Circle N :  $x^2 + y^2 - 2x = 0$

Circle O :  $x^2 + y^2 - 2x - 2y + 1 = 0$

Circle P :  $x^2 + y^2 - 2y = 0$

If the centre of circle M is joined with centre of the circle N, further centre of circle N is joined with centre of the circle O, centre of circle O is joined with the centre of circle P and lastly, centre of circle P is joined with centre of circle M, then these lines form the sides of a :

- (1) Rectangle
- (2) Parallelogram
- (3) Square
- (4) Rhombus

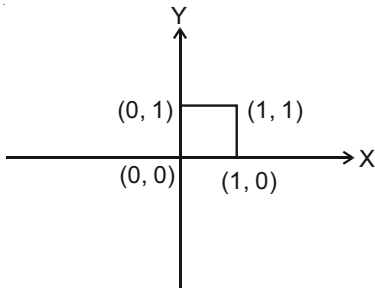
**Answer (3)**

**Sol.** Centre of M = (0, 0)

Centre of N = (1, 0)

Centre of O = (1, 1)

Centre of P = (0, 1)



Clearly these points form a square

(\*But every square is also rectangle and parallelogram)

13. If  $\lim_{x \rightarrow 0} \frac{\sin^{-1} x - \tan^{-1} x}{3x^3}$  is equal to L, then the value of  $(6L + 1)$  is :

(1) 6 (2) 2

(3)  $\frac{1}{2}$  (4)  $\frac{1}{6}$

**Answer (2)**

**Sol.**  $\lim_{x \rightarrow 0} \frac{\sin^{-1} x - \tan^{-1} x}{3x^3}$

$$\lim_{x \rightarrow 0} \frac{\tan^{-1} \left( \frac{x}{\sqrt{1-x^2}} \right) - \tan^{-1} x}{3x^3}$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\tan^{-1} \left( \frac{x - x\sqrt{1-x^2}}{\sqrt{1-x^2} + x^2} \right)}{3x^3}$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{x(1 - \sqrt{1-x^2})}{(x^2 + \sqrt{1-x^2}) \cdot 3x^3}$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{(1 - \sqrt{1-x^2})(1 + \sqrt{1-x^2})}{3x^2(1 + \sqrt{1-x^2})}$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{x^2}{3x^2(1 + \sqrt{1-x^2})} = \frac{1}{6} = L$$

$$\therefore 6L + 1 = 2$$

14. Let  $\alpha, \beta, \gamma$  be real roots of the equation,  $x^3 + ax^2 + bx + c = 0$ , ( $a, b, c \in \mathbb{R}$  and  $a, b \neq 0$ ). If the system of equations (in  $u, v, w$ ) given by  $\alpha u + \beta v + \gamma w = 0$ ;  $\beta u + \gamma v + \alpha w = 0$ ;  $\gamma u + \alpha v + \beta w = 0$  has non-trivial solution, then the value of  $\frac{a^2}{b}$  is :

(1) 5 (2) 1

(3) 3 (4) 0

**Answer (3)**

**Sol.** For non-trivial solutions of given system we have

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = 0$$

$$\therefore -(\alpha + \beta + \gamma)(\alpha^2 + \beta^2 + \gamma^2 - \alpha\beta - \beta\gamma - \gamma\alpha) = 0$$

$$\Rightarrow -(-a)(a^2 - 3b) = 0$$

$$\Rightarrow \frac{a^2}{b} = 3 \quad (\text{as } a \neq 0)$$

15. The number of integral values of  $m$  so that the abscissa of point of intersection of lines  $3x + 4y = 9$  and  $y = mx + 1$  is also an integer, is :

(1) 0 (2) 3

(3) 1 (4) 2

**Answer (4)**

$$3x + 4y = 9 \quad \times 1$$

$$mx - y = -1 \quad \times 4$$

**Sol.**  $(3 + 4m)x = 5$

$$\Rightarrow x = \frac{5}{3 + 4m}$$

$m = -1$  and  $-2$  only

gives x-coordinate as integer

16. Choose the correct statement about two circles whose equations are given below:

$$x^2 + y^2 - 10x - 10y + 41 = 0$$

$$x^2 + y^2 - 22x - 10y + 137 = 0$$

(1) circles have only one meeting point

(2) circles have two meeting points

(3) circles have no meeting point

(4) circles have same centre

**Answer (1)**

**Sol.**  $C_1 \equiv x^2 + y^2 - 10x - 10y + 41 = 0$

$$\Rightarrow C = (5, 5) \quad R = 3$$

$$C_2 \equiv x^2 + y^2 - 22x - 10y + 137 = 0$$

$$\Rightarrow C = (11, 5) \quad R = 3$$

$$d_{C_1 C_2} = \sqrt{(11-5)^2 + (5-5)^2} = 6$$

$$d_{C_1 C_2} = r_1 + r_2$$

i.e., circles touch each other externally

17. If the equation  $a|z|^2 + \overline{\alpha z} + \alpha \overline{z} + d = 0$  represents a circle where  $a, d$  are real constants, then which of the following condition is correct?

- (1)  $|\alpha|^2 - ad > 0$  and  $a \in \mathbf{R} - \{0\}$
- (2)  $|\alpha|^2 - ad \neq 0$
- (3)  $\alpha = 0, a, d \in \mathbf{R}^+$
- (4)  $|\alpha|^2 - ad \geq 0$  and  $a \in \mathbf{R}$

**Answer (1)**

**Sol.**  $az\bar{z} + \alpha\bar{z} + \overline{\alpha z} + d = 0$

for equation of circle radius  $> 0$

$$\Rightarrow z\bar{z} + \frac{\alpha}{a}\bar{z} + \frac{\overline{\alpha}}{a}z + \frac{d}{a} = 0$$

$$\text{Radius} = \sqrt{\frac{\alpha \overline{\alpha}}{a} - \frac{d}{a}}$$

$$\Rightarrow \frac{\alpha \overline{\alpha}}{a^2} > \frac{d}{a}$$

$$\Rightarrow |\alpha|^2 - ad > 0 \text{ and } a \neq 0$$

18. If the functions are defined as  $f(x) = \sqrt{x}$  and  $g(x) = \sqrt{1-x}$ , then what is the common domain of the following functions :

$f + g, f - g, f/g, g/f, g - f$

where  $(f \pm g)(x) = f(x) \pm g(x), (f/g)(x) = \frac{f(x)}{g(x)}$

- (1)  $0 < x < 1$
- (2)  $0 < x \leq 1$
- (3)  $0 \leq x \leq 1$
- (4)  $0 \leq x < 1$

**Answer (1)**

**Sol.** For common domain  $\equiv (\text{domain of } f) \cap (\text{domain of } g) - \{\text{Points where either or both of } f, g \text{ vanishes}\}$

$$\Rightarrow x > 0 \text{ and } 1 - x > 0$$

$$\Rightarrow x \in (0, 1)$$

19. If  $f(x) = \begin{cases} \frac{1}{|x|} & ; |x| \geq 1 \\ ax^2 + b & ; |x| < 1 \end{cases}$  is differentiable at

every point of the domain, then the values of  $a$  and  $b$  are respectively :

- (1)  $-\frac{1}{2}, \frac{3}{2}$
- (2)  $\frac{5}{2}, -\frac{3}{2}$
- (3)  $\frac{1}{2}, -\frac{3}{2}$
- (4)  $\frac{1}{2}, \frac{1}{2}$

**Answer (1)**

**Sol.**  $f(x)$  must be continuous at  $x = \pm 1$

$$\Rightarrow 1 = a + b \quad \dots(i)$$

For differentiable at  $x = 1$

$$f(x) = \begin{cases} 1/x & x \geq 1 \\ ax^2 + b & x < 1 \end{cases}$$

$$\text{LHD} = \lim_{h \rightarrow 0} \frac{a(1-h)^2 + b - 1}{-h} = \frac{-2ah + h^2}{-h} = 2a$$

(For existence  $a + b = 1$ )

$$\text{RHD} = \lim_{h \rightarrow 0} \frac{1}{1+h} - 1 = -1$$

$$\Rightarrow a = \frac{-1}{2}, b = \frac{3}{2}$$

(Similar can be done for  $x = -1$ )

20. The sum of all the 4-digit distinct numbers that can be formed with the digits 1, 2, 2 and 3 is :

- (1) 122234
- (2) 122664
- (3) 22264
- (4) 26664

**Answer (4)**

**Sol.** Digits to be used 1, 2, 2, 3

Total contribution of 3  $\rightarrow$

$$(3 + 30 + 300 + 3000) = 9999$$

Similarly total contribution of 1  $\rightarrow$

$$(1 + 10 + 100 + 1000)3 = 3333$$

And Total contribution of 2  $\rightarrow$

$$(2 + 20 + 200 + 2000)6 = 13332$$

$$\therefore \text{Sum of number} = 26664$$

**SECTION - II**

**Numerical Value Type Questions:** This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. If  $f(x) = \int \frac{5x^8 + 7x^6}{(x^2 + 1 + 2x^7)^2} dx, (x \geq 0), f(0) = 0$  and  $f(1) = \frac{1}{K}$ , then the value of  $K$  is \_\_\_\_\_.

**Answer (04)**

**Sol.**  $\int \frac{5x^8 + 7x^6}{x^{14} \left( \frac{1}{x^5} + \frac{1}{x^7} + 2 \right)} dx$

$$\Rightarrow \int \frac{5x^{-6} + 7x^{-8}}{\left(2 + \frac{1}{x^5} + \frac{1}{x^7}\right)^2} dx$$

Put  $2 + \frac{1}{x^5} + \frac{1}{x^7} = t$

$$(-5x^{-6} - 7x^{-8})dx = dt$$

$$\Rightarrow \int \frac{-dt}{t^2} = \frac{1}{t} + c$$

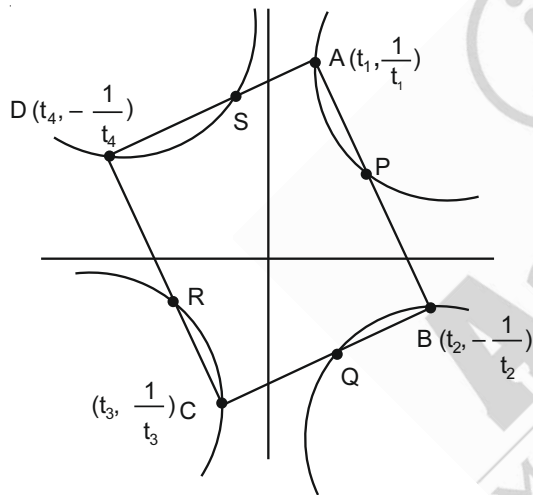
$$f(x) = \frac{x^7}{2x^7 + x^2 + 1} + c$$

$$f(0) = 0 \Rightarrow c = 0 \Rightarrow f(1) = \frac{1}{4} \Rightarrow k = 4$$

2. A square ABCD has all its vertices on the curve  $x^2y^2 = 1$ . The midpoints of its sides also lie on the same curve. Then, the square of area of ABCD is \_\_\_\_\_.

**Answer (80)**

**Sol.** Refer to diagram



$$(t_1 + t_2) \left( \frac{1}{t_1} - \frac{1}{t_2} \right) = 4$$

$$\Rightarrow \frac{t_2}{t_1} - \frac{t_1}{t_2} - 4 = 0$$

$$\frac{t_2}{t_1} = \sqrt{5} + 2$$

Similarly,  $\frac{t_4}{t_1} = -(\sqrt{5} + 2)$

$\therefore AB \perp AD$ , then

$$\text{Slope of } AB = -\left(\frac{\sqrt{5}+1}{\sqrt{5}+3}\right)t_1t_2$$

$$\text{Slope of } AD = -\left(\frac{3+\sqrt{5}}{\sqrt{5}+1}\right)t_1t_4$$

$$\Rightarrow t_1^2t_2t_4 = -1 \Rightarrow t_1^2 = \sqrt{5} - 2,$$

then,  $t_2^2 = \sqrt{5} + 2$

area of require triangle

$$= (t_1 - t_2)^2 + \left(\frac{1}{t_1} + \frac{1}{t_2}\right)^2 = 4\sqrt{5}$$

$$\Rightarrow \Delta^2 = 80$$

3. The number of times the digit 3 will be written when listing the integers from 1 to 1000 is \_\_\_\_\_.

**Answer (300)**

**Sol.** In single digit numbers = 1

In double digit numbers =  $10 + 9 = 19$

In triple digit numbers =  $100 + 90 + 90 = 280$

Total = 300 times

4. Let  $z_1, z_2$  be the roots of the equation  $z^2 + az + 12 = 0$  and  $z_1, z_2$  form an equilateral triangle with origin. Then, the value of  $|a|$  is \_\_\_\_\_.

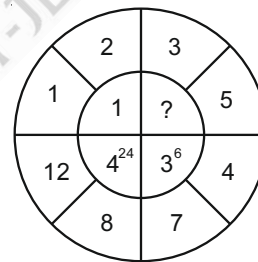
**Answer (06)**

**Sol.**  $z_1^2 + z_2^2 = z_1z_2$  (Condition for equilateral triangle)


$$a^2 - 2(12) = 12$$

$$\Rightarrow |a| = 6$$

5. The missing value in the following figure is \_\_\_\_\_.



**Answer (04)**

**Sol.** In every 

Where  $c = |a - b|^{[a][b]}$

Where [ ] is g. i. f.

Hence unknown is  $2^2 = 4$

6. The mean age of 25 teachers in a school is 40 years. A teacher retires at the age of 60 years and a new teacher is appointed in his place. If the mean age of the teachers in this school now is 39 years, then the age (in years) of the newly appointed teacher is \_\_\_\_\_.

**Answer (35)**



**Sol.**  $x_1 + x_2 + \dots + x_{25} = 25 \times 40 \dots (i)$

Let age of new teacher is A

then  $(x_1 + x_2 + \dots + x_{25}) - 60 + A = 25 \times 39$

$\Rightarrow A = 975 + 60 - 1000 = 35$  years

7. Let the plane  $ax + by + cz + d = 0$  bisect the line joining the points  $(4, -3, 1)$  and  $(2, 3, -5)$  at the right angles. If  $a, b, c, d$  are integers, then the minimum value of  $(a^2 + b^2 + c^2 + d^2)$  is \_\_\_\_\_.

**Answer (28)**

**Sol.** Slope of normal to the plane  $\langle 2, -6, 6 \rangle$

Hence plane is  $\pi = 2x - 6y + 6z + k = 0$

(where  $k = \lambda d$ )

Also mid point of  $(4, -3, 1)$  and  $(2, 3, 5)$

i.e.,  $(3, 0, -2)$

satisfies  $\pi$

$\Rightarrow 6 - 0 - 12 + k = 0$

$\Rightarrow k = 6$

Hence,  $\pi = 2x - 6y + 6z + 6 = 0$

Minimum  $(a^2 + b^2 + c^2 + d^2) = (1^2 + 3^2 + 3^2 + 3^2) = 28$

8. Let  $f(x)$  and  $g(x)$  be two functions satisfying  $f(x^2) + g(4 - x) = 4x^3$  and  $g(4 - x) + g(x) = 0$ , then the value of  $\int_{-4}^4 f(x^2) dx$  is \_\_\_\_\_.

**Answer (512)**

**Sol.**  $I = \int_{-4}^4 f(x)^2 dx = 2 \cdot \int_0^4 f(x)^2 dx$

$\therefore f(x^2) = 4x^3 - g(4 - x)$

$I = 2 \cdot \int_0^4 (4x^3 - g(4 - x)) dx = 2 \cdot 4 \cdot \frac{x^4}{4} \Big|_0^4 - 2 \cdot \int_0^4 g(4 - x) dx$

$I = 512 - 2 \cdot I_1$

$I_1 = \int_0^4 g(4 - x) dx =$

$= \int_0^4 g(4 - (0 + 4 - x)) dx = \int_0^4 g(x) dx$

$I_1 = - \int_0^4 g(4 - x) dx$

$\Rightarrow I_1 = 0$

Hence  $I = 512$

9. The equation of the planes parallel to the plane  $x - 2y + 2z - 3 = 0$  which are at unit distance from the point  $(1, 2, 3)$  is  $ax + by + cz + d = 0$ . If  $(b - d) = K(c - a)$ , then the positive value of  $K$  is \_\_\_\_\_.

**Answer (04)**

**Sol.** Required plane is of form

$x - 2y + 2z + d = 0$

Also it is at unit distance from  $(1, 2, 3)$

$\Rightarrow \left| \frac{1 - 4 + 6 + d}{3} \right| = 1$

$\Rightarrow |d + 3| = 3 \Rightarrow d = 0$  or  $-6$

$\therefore$  Plane is  $x - 2y + 2z = 0$  or  $x - 2y + 2z - 6 = 0$

$(-2 - 0) = K(2 - 1)$  or  $(-2 + 6) = K(2 - 1)$

$K = -2$  or  $K = 4$

10. The number of solutions of the equation

$|\cot x| = \cot x + \frac{1}{\sin x}$  in the interval  $[0, 2\pi]$  is \_\_\_\_\_.

**Answer (01)**

**Sol.** If  $x \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, 3\frac{\pi}{2}\right)$

$\Rightarrow \cot x = \cot x + \frac{1}{\sin x}$

$\Rightarrow$  No solution

if  $x \in \left(\frac{\pi}{2}, \pi\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)$

$\Rightarrow \frac{2\cos x + 1}{\sin x} = 0$

$\Rightarrow \cos x = \frac{-1}{2}$

$\Rightarrow x = \frac{2\pi}{3}$

$\Rightarrow$  One solution

