20/07/2021
Evening

Corporate Office: Aakash Tower, 8, Pusa Road, New Delhi-110005 Ph. 011-47623456

Time : 3 hrs. **M.M. : 300**

Answers & Solutions

for

JEE (MAIN)-2021 (Online) Phase-3

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS :

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part has two sections.

(i) Section-I : This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) Section-II : This section contains 10 questions. In Section-II, attempt any **five questions out of 10.** There will be **no negative marking for Section-II**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and there is no negative marking for wrong answer.

PART–A : PHYSICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Two vectors $\vec{P}$ and $\vec{Q}$ have equal magnitudes. If the magnitude of $\vec{P}+\vec{Q}$ is n times the magnitude of $\vec{P}-\vec{Q}$, then angle between $\vec{P}$ and $\vec{Q}$ is

(1) $\cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$ (2) $\sin^{-1}\left(\frac{n-1}{n+1}\right)$

(3) $\sin^{-1}\left(\frac{n^2-1}{n^2+1}\right)$ (4) $\cos^{-1}\left(\frac{n-1}{n+1}\right)$

Answer (1)

Sol. $|\vec{P}+\vec{Q}|^2 = n^2|\vec{P}-\vec{Q}|^2$

$\Rightarrow P^2 + Q^2 + 2PQ\cos\theta = n^2(P^2 + Q^2 - 2PQ\cos\theta)$

$\Rightarrow 1 + 1 + 2\cos\theta = n^2(1 + 1 - 2\cos\theta)$

$\Rightarrow 1 + \cos\theta = n^2(1 - \cos\theta)$

$\Rightarrow \frac{1+\cos\theta}{1-\cos\theta} = \frac{n^2}{1}$

$\Rightarrow \frac{1}{\cos\theta} = \frac{n^2+1}{n^2-1}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$

2. Two small drops of mercury each of radius R coalesce to form a single large drop. The ratio of total surface energy before and after the change is

(1) 2 : 1 (2) 1 : 2

(3) $2^{\frac{1}{3}}:1$ (4) $1:2^{\frac{1}{3}}$

Answer (3)

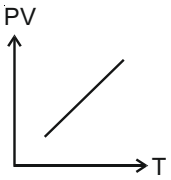
Sol. $\frac{4}{3}\pi R^3 \times 2 = \frac{4}{3}\pi R_2^3$

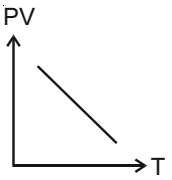
$\Rightarrow R_2 = R \times 2^{\frac{1}{3}}$

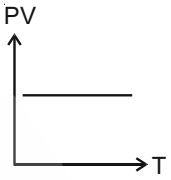
$\therefore \frac{E_1}{E_2} = \frac{T \times 4\pi R^2 \times 2}{T \times 4\pi R^2 \times 2^{\frac{2}{3}}}$

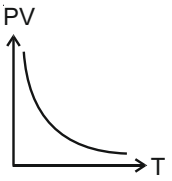
$= \frac{2}{2^{2/3}} = 2^{\frac{1}{3}}$

3. Which of the following graphs represent the behavior of an ideal gas? Symbols have their usual meaning.

(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. PV = nRT

$\Rightarrow PV \propto T$

$\Rightarrow$ straight line with positive slope.

4. The correct relation between the degrees of freedom f and the ratio of specific heat γ is

(1) $f = \frac{1}{\gamma+1}$ (2) $f = \frac{2}{\gamma-1}$

(3) $f = \frac{2}{\gamma+1}$ (4) $f = \frac{\gamma+1}{2}$

Answer (2)

Sol. $\because \gamma = 1 + \frac{2}{f}$

$\Rightarrow \frac{2}{f} = \gamma - 1$

$\Rightarrow f = \frac{2}{\gamma-1}$

5. The magnetic susceptibility of a material of a rod is 499. Permeability in vacuum is $4\pi \times 10^{-7}$ H/m. Absolute permeability of the material of the rod is:

(1) $\pi \times 10^{-4}$ H/m (2) $4\pi \times 10^{-4}$ H/m

(3) $2\pi \times 10^{-4}$ H/m (4) $3\pi \times 10^{-4}$ H/m

Answer (3)

Sol. $\chi = 499$

$\therefore \mu = (1 + \chi) \times \mu_0$

$= 500 \times 4\pi \times 10^{-7}$

$= 2\pi \times 10^{-4}$ H/m

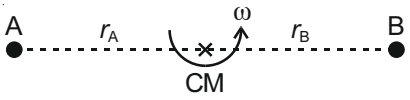
6. Consider a binary star system of star A and star B with masses m_A and m_B revolving in a circular orbit of radii r_A and r_B, respectively. If T_A and T_B are the time period of star A and star B, respectively, then:

(1) $T_A = T_B$ (2) $\frac{T_A}{T_B} = \left(\frac{r_A}{r_B}\right)^{\frac{3}{2}}$

(3) $T_A > T_B$ (if $r_A > r_B$) (4) $T_A > T_B$ (if $m_A > m_B$)

Answer (1)

Sol. $T_A = \frac{2\pi}{\omega}$



$T_B = \frac{2\pi}{\omega}$

$\Rightarrow T_A = T_B$

7. A body rolls down an inclined plane without slipping. The kinetic energy of rotation is 50% of its translational kinetic energy. The body is:

(1) Solid cylinder (2) Hollow cylinder

(3) Ring (4) Solid sphere

Answer (1)

Sol. $\frac{k_T}{k_R} = \frac{k_T}{\frac{1}{2}k_T} = 2$

$\frac{MR^2}{I_{CM}} = 2$

$\Rightarrow I_{CM} = \frac{MR^2}{2}$

8. At an angle of 30° to the magnetic meridian, the apparent dip is 45°. Find the true dip:

(1) $\tan^{-1}\frac{2}{\sqrt{3}}$ (2) $\tan^{-1}\sqrt{3}$

(3) $\tan^{-1}\frac{1}{\sqrt{3}}$ (4) $\tan^{-1}\frac{\sqrt{3}}{2}$

Answer (4)

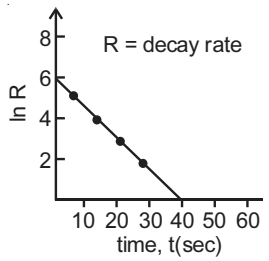
Sol. $\tan\theta = \frac{B_V}{B_H}$

$\tan 45 = \frac{B_V}{B_H \cos 30} = \frac{2B_V}{B_H\sqrt{3}}$

$\Rightarrow B_V = \frac{\sqrt{3}}{2}B_H$

$\Rightarrow \theta = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

9. For a certain radioactive process the graph between ln R and t(sec) is obtained as shown in the figure. Then the value of half life for the unknown radioactive material is approximately:



(1) 4.62 sec (2) 6.93 sec

(3) 9.15 sec (4) 2.62 sec

Answer (1)

Sol. $R = R_0 e^{-\lambda t}$

$\Rightarrow \ln R = \ln R_0 - \lambda t$

at $t = 0$, $\ln R = 6$

$\Rightarrow 6 = \ln R_0$ ...(1)

at $t = 40$ s, $\ln R = 0$

$\Rightarrow \lambda = \frac{\ln R_0}{t} = \frac{6}{40}$

$\Rightarrow t_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \frac{0.693\times 40}{6} = 4.62$ s

10. The length of a metal wire is l_1, when the tension in it is T_1 and is l_2 When the tension is T_2. The natural length of the wire is:

(1) $\sqrt{l_1 l_2}$ (2) $\frac{l_1T_2 - l_2T_1}{T_2 - T_1}$

(3) $\frac{l_1 + l_2}{2}$ (4) $\frac{l_1T_2 + l_2T_1}{T_2 + T_1}$

Answer (2)

Sol. $T_1 = k(l_1 - l_0)$ ...(i)

$T_2 = k(l_2 - l_0)$ ...(ii)

From (i) and (ii)

$l_0 = \frac{l_1T_2 - l_2T_1}{T_2 - T_1}$

11. In an electromagnetic wave the electric field vector and magnetic field vector are given as $\vec{E} = E_0\hat{i}$ and $\vec{B} = B_0\hat{k}$ respectively. The direction of propagation of electromagnetic wave is along:

(1) $(-\hat{k})$ (2) $(-\hat{j})$

(3) $\hat{j}$ (4) $(\hat{k})$

Answer (2)

Sol. Direction of propagation of wave = Direction of $\vec{E}\times\vec{B}$

= Direction of $E_0\hat{i}\times B_0\hat{k}$

= Direction of $E_0B_0(-\hat{j})$

$=-\hat{j}$

12. If the Kinetic energy of a moving body becomes four times its initial Kinetic energy, then the percentage change in its momentum will be:

(1) 200% (2) 100%

(3) 400% (4) 300%

Answer (2)

Sol. Momentum, $P=\sqrt{2km}$

$$\% \text{ change in } P=\frac{P_2-P_1}{P_1}\times 100\%$$

$$=\frac{\sqrt{2k_2m}-\sqrt{2k_1m}}{\sqrt{2k_1m}}\times 100\%$$

$$=\frac{\sqrt{k_2}-\sqrt{k_1}}{\sqrt{k_1}}\times 100\%$$

$$=\left(\sqrt{\frac{k_2}{k_1}}-1\right)\times 100\%$$

$$=\left(\sqrt{\frac{4k_1}{k_1}}-1\right)\times 100\%$$

= 100%

13. With what speed should a galaxy move outward with respect to earth so that the sodium-D line at wavelength 5890 Å is observed at 5896 Å?

(1) 322 km/sec (2) 306 km/sec

(3) 336 km/sec (4) 296 km/sec

Answer (2)

Sol. $\lambda_{obs}=\lambda_{actual}\sqrt{\frac{1+\frac{V}{C}}{1-\frac{V}{C}}}$

$$5896 = 5890\left(1+\frac{V}{C}\right) \text{ [for V << C]}$$

$$V=\frac{C\times 6}{5890}=306 \text{ km/s}$$

14. A body at rest is moved along a horizontal straight line by a machine delivering a constant power. The distance moved by the body in time '*t*' is proportional to

(1) $t^{\frac{1}{2}}$ (2) $t^{\frac{1}{4}}$

(3) $t^{\frac{3}{2}}$ (4) $t^{\frac{3}{4}}$

Answer (3)

Sol. P = C

So. K.E. = P*t*

$$\frac{1}{2}mv^2=Pt$$

$$\Rightarrow v=\sqrt{\frac{2P}{m}}\sqrt{t}$$

$$\Rightarrow v=C\sqrt{t}$$

$$\frac{ds}{dt}=C\sqrt{t}$$

$$\int ds=\int C\sqrt{t}\,dt$$

Distance = C $t^{3/2}$

15. A satellite is launched into a circular orbit of radius R around earth, while a second satellite is launched into a circular orbit of radius 1.02R. The percentage difference in the time periods of the two satellites is

(1) 1.5 (2) 2.0

(3) 0.7 (4) 3.0

Answer (4)

Sol. $T^2 = CR^3$

$$2\frac{dT}{T}=3\frac{dR}{R}$$

$$\frac{dT}{T}=\frac{3}{2}\times\frac{0.02R}{R}$$

= 0.03

So % difference in the time period

= 0.03 × 100= 3%

16. For a series LCR circuit with R = 100 Ω, L = 0.5 mH and C = 0.1 pF connected across 220 V - 50 Hz AC supply, the phase angle between current and supplied voltage and the nature of the circuit is

(1) ≈ 90°, predominantly capacitive circuit

(2) 0°, resonance circuit

(3) 0°, resistive circuit

(4) ≈ 90°, predominantly inductive circuit

Answer (1)

Sol. $X_L = 2 \times \pi \times 50 \times 0.5 \times 10^{-3}$

$= 0.05\ \Omega$

$$X_C = \frac{1}{\omega C} = \frac{10^{12}}{2\pi \times 50 \times 0.1}$$

$$= \frac{10^{11}}{\pi}\Omega$$

$X_C - X_L >> R$

17. An electron having de-Broglie wavelength λ is incident on a target in a X-ray tube. Cut-off wavelength of emitted X-ray is:

(1) 0 (2) $\frac{hc}{mc}$

(3) $\frac{2m^2c^2\lambda^2}{h^2}$ (4) $\frac{2mc\lambda^2}{h}$

Answer (4)

Sol. $p = \frac{h}{\lambda}$

$$\frac{p^2}{2m} = \frac{hc}{\lambda_0}$$

$$\frac{h^2}{2m\lambda^2} = \frac{hc}{\lambda_0}$$

$$\lambda_0 = \frac{2mc\lambda^2}{h}$$

18. If time (t), velocity (v), and angular momentum (l) are taken as the fundamental units. Then the dimension of mass (m) in terms of t, v, and l is:

(1) $[t^{-1}v^1l^{-2}]$ (2) $[t^{-1}v^{-2}l^1]$

(3) $[t^1v^2l^{-1}]$ (4) $[t^{-2}v^{-1}l^1]$

Answer (2)

Sol. $l = [M^1L^2T^{-1}]$

$M = l^\alpha t^\beta v^\gamma$

$M = [M^1L^2T^{-1}]^\alpha\ [T]^\beta\ [L^1T^{-1}]^\gamma$

$\alpha = 1$

$-\alpha + \beta - \gamma = 0$

$2\alpha + \gamma = 0$

$\gamma = -2$

$\beta = -1$

19. A particle is making simple harmonic motion along the X-axis. If at a distances x_1 and x_2 from the mean position the velocities of the particle are v_1 and v_2 respectively. The time period of its oscillation is given as:

(1) $T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 + v_2^2}}$

(2) $T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$

(3) $T = 2\pi\sqrt{\frac{x_2^2 + x_1^2}{v_1^2 - v_2^2}}$

(4) $T = 2\pi\sqrt{\frac{x_2^2 + x_1^2}{v_1^2 + v_2^2}}$

Answer (2)

Sol. $v_1^2 = \omega^2(A^2 - x_1^2)$

$v_2^2 = \omega^2(A^2 - x_2^2)$

$v_1^2 - v_2^2 = \omega^2(x_2^2 - x_1^2)$

$$v_1^2 - v_2^2 = \left[\frac{2\pi}{T}\right]^2 (x_2^2 - x_1^2)$$

$$T = 2\pi\sqrt{\frac{(x_2^2 - x_1^2)}{(v_1^2 - v_2^2)}}$$

20. A boy reaches the airport and finds that the escalator is not working. He walks up the stationary escalator in time t_1. If he remains stationary on a moving escalator then the escalator takes him up in time t_2. The time taken by him to walk up on the moving escalator will be:

(1) $t_2 - t_1$ (2) $\frac{t_1 t_2}{t_2 - t_1}$

(3) $\frac{t_1 + t_2}{2}$ (4) $\frac{t_1 t_2}{t_2 + t_1}$

Answer (4)

Sol. Let l be the length of escalator, v_1 be the velocity of man w.r.t. escalator, v_2 be the velocity of escalator.

$$t_1 = \frac{l}{v_1}$$

$$t_2 = \frac{l}{v_2}$$

$$t = \frac{l}{v_1 + v_2} = \frac{l}{\frac{l}{t_1} + \frac{l}{t_2}}$$

$$t = \frac{t_1 t_2}{t_1 + t_2}$$

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. A certain metallic surface is illuminated by monochromatic radiation of wavelength λ. The stopping potential for photoelectric current for this radiation is $3V_0$. If the same surface is illuminated with a radiation of wavelength 2λ, the stopping potential is V_0. The threshold wavelength of this surface for photoelectric effect is _____ λ.

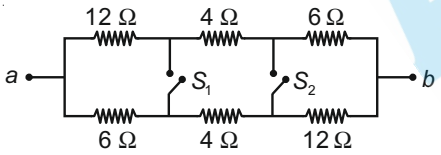
Answer (4)

Sol. $e\,3V_0 = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$ ...(i)

$eV_0 = \frac{hc}{2\lambda} - \frac{hc}{\lambda_0}$ ...(ii)

$\Rightarrow \lambda_0 = 4\lambda$

2. In the given figure switches S_1 and S_2 are in open condition. The resistance across *ab* when the switches S_1 and S_2 are closed is _____ Ω.



Answer (10)

Sol. $R_{ab} = \frac{12\times6}{12+6} + \frac{4\times4}{4+4} + \frac{6\times12}{6+12}$

$= 10\ \Omega$

3. Two bodies, a ring and a solid cylinder of same material are rolling down without slipping an inclined plane. The radii of the bodies are same. The ratio of velocity of the centre of mass at the bottom of the inclined plane of the ring to that of the cylinder is $\frac{\sqrt{x}}{2}$. Then, the value of *x* is _____.

Answer (3)

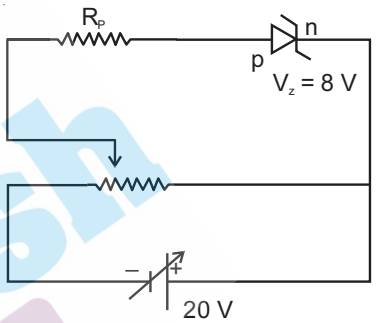
Sol. $v = \sqrt{\frac{2gh}{1+\frac{K^2}{R^2}}}$

$v_{ring} = \sqrt{gh}$

$v_{cylinder} = \sqrt{\frac{4}{3}gh}$

$\frac{v_{ring}}{v_{cylinder}} = \sqrt{\frac{3}{4}}$

4. A zener diode having zener voltage 8 V and power dissipation rating of 0.5 W is connected across a potential divider arranged with maximum potential drop across zener diode is as shown in the diagram. The value of protective resistance R_P is _____ Ω.



Answer (192)

Sol. $P = V_z I_z \Rightarrow I_z = \frac{1}{16}A$

$I_z = \frac{V - V_z}{R_P}$

$R_P = (20-8)\times16 = 192\ \Omega$

5. A body of mass '*m*' is launched up on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of friction between the body and plane is $\frac{\sqrt{x}}{5}$ if the time of ascent is half of the time of descent. The value of *x* is _____

Answer (3)

Sol. $t_A = \sqrt{\frac{2\ell}{g(\sin\theta+\mu\cos\theta)}}$

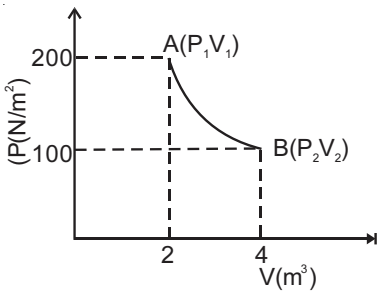
$t_D = \sqrt{\frac{2\ell}{g(\sin\theta-\mu\cos\theta)}}$

$t_A = \frac{1}{2}t_D$

$\Rightarrow \mu = \frac{3}{5}\tan\theta = \frac{\sqrt{3}}{5}$

6. One mole of an ideal gas at 27°C is taken from A to B as shown in the given PV indicator diagram. The work done by the system will be _____ × 10^{-1} J.

[Given : R = 8.3 J / mole K, ln2 = 0.6931] (Round off to the nearest integer)



Answer (17258)

Sol. Assuming process to be isothermal

$$W = nRT\ln\left(\frac{V_f}{V_i}\right)$$

$= 8.3 \times 300 \times \ln(2)$

$= 1725.819$ J

$= 17258 \times 10^{-1}$ J

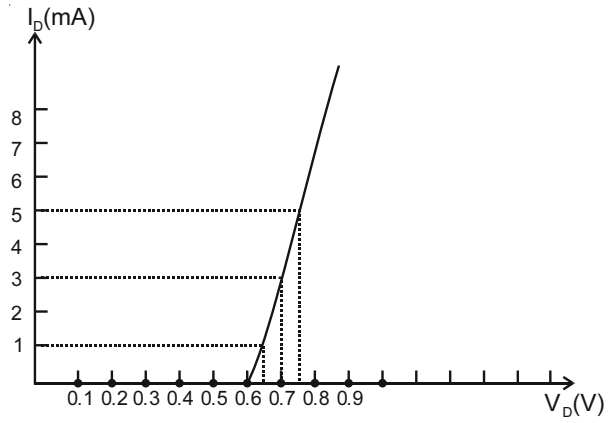
7. A series LCR circuit of R = 5 Ω, L = 20 mH and C = 0.5 μF is connected across an AC supply of 250 V, having variable frequency. The power dissipated at resonance condition is ____ × 10^2 W.

Answer (125)

Sol. $P_r = \frac{V_{rms}^2}{R}$ as at resonance Z = R

$$= \frac{250\times250}{50} = 125\times10^2\,W$$

8. For the forward biased diode characteristics shown in the figure, the dynamic resistance at I_D = 3 mA will be _____ Ω.



Answer (25)

Sol. $R_D = \frac{\Delta V}{\Delta I} = \frac{0.1}{4\times10^{-3}} = 25\ \Omega$

9. A radioactive substance decays to $\left(\frac{1}{16}\right)^{th}$ of its initial activity in 80 days. The half life of the radioactive substance expressed in days is _____.

Answer (20)

Sol. $\frac{x}{16} = x\left(\frac{1}{2}\right)^n$

$n = 4$

$\Rightarrow$ 4 half lives = 80 days

1 half life = 20 days

10. A body rotating with an angular speed of 600 rpm is uniformly accelerated to 1800 rpm in 10 sec. The number of rotations made in the process is _____.

Answer (200)

Sol. $4\pi^2\frac{(18^2-6^2)10^4}{60^2} = \frac{2\times1200}{10}\times\frac{\pi2\times\theta}{60}$

$$\theta = \frac{\pi\times24\times12\times10^4}{120\times60} = \frac{\pi24\times10^3}{60}$$

$$\theta = \frac{(12000)(2\pi)}{60}$$

$= (200)2\pi$

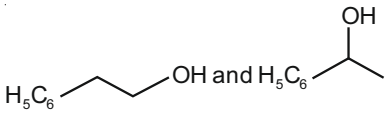
PART–B : CHEMISTRY

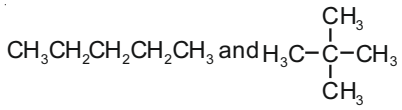
SECTION - I

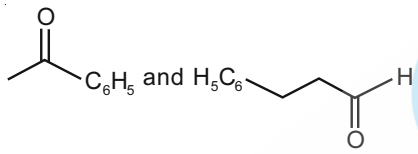
Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

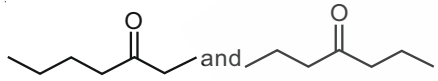
Choose the correct answer :

1. Which one of the following pairs of isomers is an example of metamerism?

(1) 

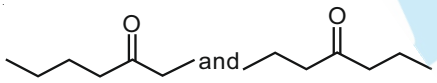
(2) 

(3) 

(4) 

Answer (4)

Sol. Metamers have a common functional group and differ in the type of alkyl groups attached to the functional group

 are metamers

2. Outermost electronic configuration of a group 13 element, E, is $4s^2, 4p^1$. The electronic configuration of an element of p-block period-five placed diagonally to element, E is:

(1) $[Xe]5d^{10}6s^26p^2$ (2) $[Kr]3d^{10}4s^24p^2$

(3) $[Ar]3d^{10}4s^24p^2$ (4) $[Kr]4d^{10}5s^25p^2$

Answer (4)

Sol. The element E belongs to group-13 and period-4. The element belonging to period-5 and placed diagonally to E has the electronic configuration $[Kr]4d^{10}5s^25p^2$

3. Which one of the following gases is reported to retard photosynthesis?

(1) NO_2 (2) CFCs

(3) CO (4) CO_2

Answer (1)

Sol. The gas responsible to retard the rate of photosynthesis is NO_2. It is also known to damage plant leaves.

4. Cu^{2+} salt reacts with potassium iodide to give:

(1) Cu_2I_2 (2) Cu_2I_3

(3) $Cu(I_3)_2$ (4) CuI

Answer (1) or (4)

Sol. Cu^{2+} salt reacts with potassium iodide to form Cu_2I_2 and KI_3

$$2Cu^{2+} + 5KI \longrightarrow Cu_2I_2 \downarrow + KI_3 + 4K^+$$

Cu_2I_2 is sometimes also written as CuI.

5. Metallic sodium does not react normally with:

(1) Ethyne

(2) Gaseous ammonia

(3) But-2-yne

(4) Tert-butyl alcohol

Answer (3)

Sol. Sodium metal is a very strong base. It can remove proton from all compounds having acidic hydrogen like water, alcohol, ammonia and terminal alkynes. But-2-yne does not have acidic hydrogen. Therefore, it will not react with sodium metal.

6. Which one of the following statements is not true about enzymes ?

(1) The action of enzymes is temperature and pH specific

(2) Enzymes are non-specific for a reaction and substrate

(3) Enzymes work as catalysts by lowering the activation energy of a biochemical reaction

(4) Almost all enzymes are proteins

Answer (2)

Sol. Enzymes are mostly proteins. They function as catalysts in biochemical reactions by lowering the energy of activation. They are highly specific w.r.t. temperature and pH in their action.

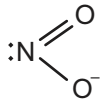
7. The hybridisations of the atomic orbitals of nitrogen in NO_2^-, NO_2^+ and NH_4^+ respectively are :

(1) sp^3, sp^2 and sp (2) sp, sp^2 and sp^3

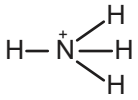
(3) sp^2, sp and sp^3 (4) sp^3, sp and sp^2

Answer (3)

Sol. The type of hybridisation of atomic orbitals of nitrogen in the given species is

NO_2^- : sp^2 

NO_2^+ : sp $O=\overset{+}{N}=O$

NH_4^+ : sp^3 

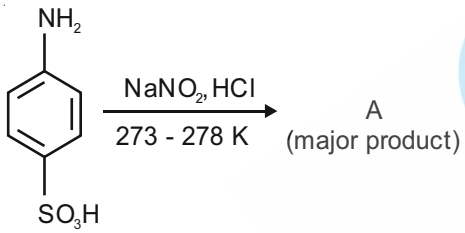
8. Bakelite is a cross-linked polymer of formaldehyde and :

(1) Buna-S (2) Novolac

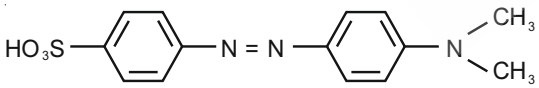
(3) Dacron (4) PHBV

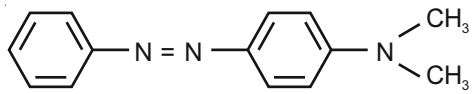
Answer (2)

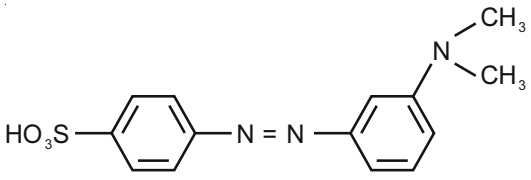
Sol. Novolac is a linear condensation polymer of phenol and formaldehyde. But, bakelite is a cross-linked polymer of formaldehyde and Novolac.

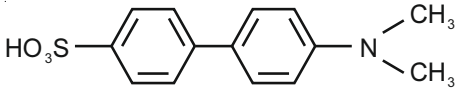
9. 

Consider the above reaction, compound B is :

(1) 

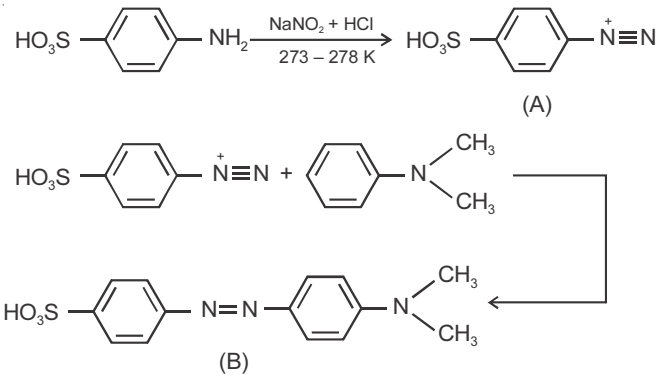
(2) 

(3) 

(4) 

Answer (1)

Sol.



10. Benzene on nitration gives nitrobenzene in presence of HNO_3 and H_2SO_4 mixture, where :

(1) HNO_3 acts as a base and H_2SO_4 acts as an acid

(2) Both H_2SO_4 and HNO_3 act as an acids

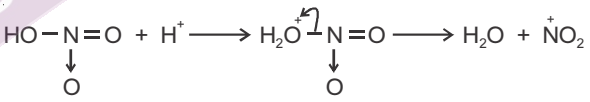
(3) HNO_3 acts as an acid and H_2SO_4 acts as a base

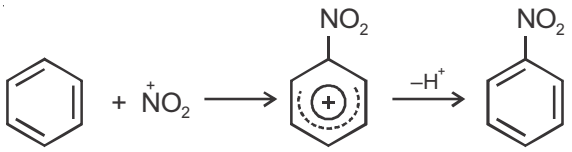
(4) Both H_2SO_4 and HNO_3 act as a bases

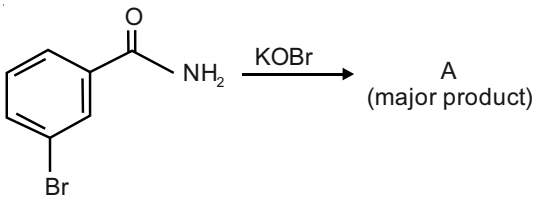
Answer (1)

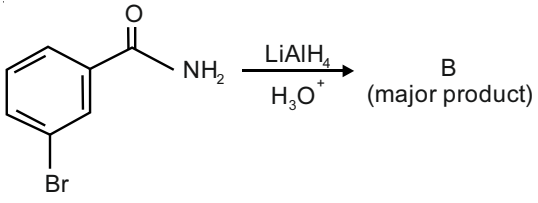
Sol. In the nitration of benzene using nitrating mixture, HNO_3 acts as a base and H_2SO_4 acts as an acid to generate NO_2^+ ion.

$$H_2SO_4 \longrightarrow H^+ + HSO_4^-$$

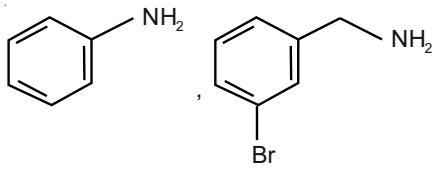


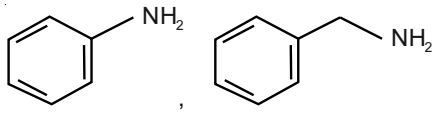


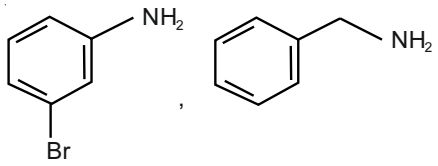
11. 

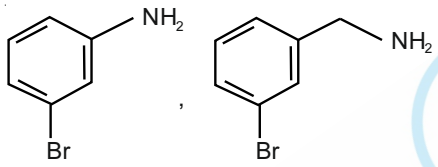


In the above reactions, product A and product B respectively are

(1) 

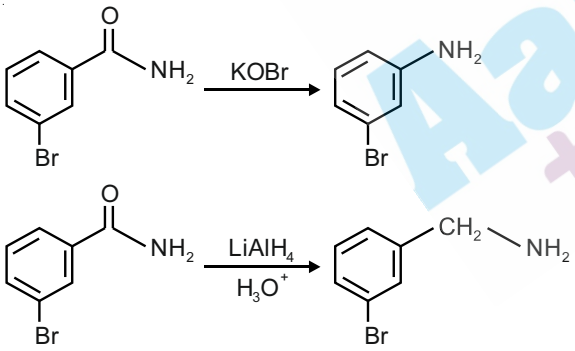
(2) 

(3) 

(4) 

Answer (4)

Sol.



12. Spin only magnetic moment of an octahedral complex of Fe^{2+} in the presence of a strong field ligand in BM is

(1) 3.46

(2) 2.82

(3) 0

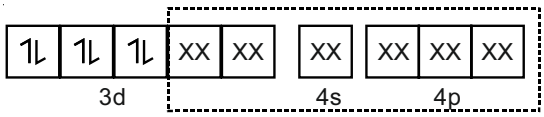
(4) 4.89

Answer (3)

Sol. The electronic configuration of Fe^{2+} is $3d^6 4s^0$. In presence of strong field ligand (L^{-1}) resulting in the formation of inner orbital octahedral complex the electronic configuration of Fe^{2+} would be

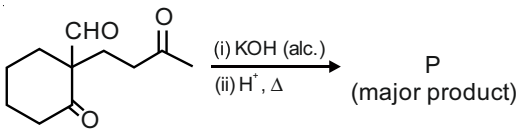
$[FeL_6]^{4-}$: $t_{2g}^6 e_g^o$

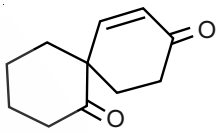
Magnetic moment, $\mu = 0$

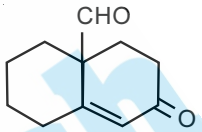


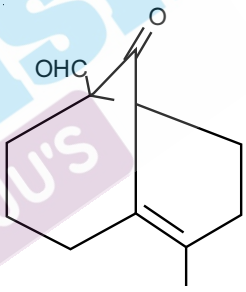
d^2sp^3 hybridisation

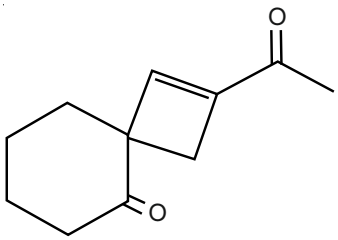
13. The major product (P) in the following reaction is



(1) 

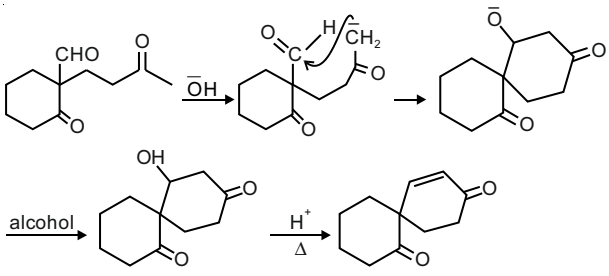
(2) 

(3) 

(4) 

Answer (1)

Sol. This problem is based on intramolecular aldol condensation reaction.



P(major product)

14. Which one of the following species **doesn't** have a magnetic moment of 1.73 BM (spin only value)?

(1) $[Cu(NH_3)_4]Cl_2$ (2) CuI

(3) O_2^- (4) O_2^+

Answer (2)

Sol. $[Cu(NH_3)_4]Cl_2$: Cu^{2+} : 1 unpaired electron; μ = 1.73 BM

CuI : Cu^+: No unpaired electron ; $\mu = 0$

O_2^-: 1 unpaired electron; μ = 1.73 BM

O_2^+: 1 unpaired electron; μ = 1.73 BM

15. The single largest industrial application of dihydrogen is

(1) In the synthesis of nitric acid

(2) Rocket fuel in space research

(3) In the synthesis of ammonia

(4) Manufacture of metal hydrides

Answer (3)

Sol. The single largest industrial application of dihydrogen is in the synthesis of ammonia which is mainly used in the manufacture of fertiliser.

16. Consider two chemical reactions (A) and (B) that take place during metallurgical process :

(A) $ZnCO_{3(s)} \xrightarrow{\Delta} ZnO_{(s)} + CO_{2(g)}$

(B) $2ZnS_{(s)} + 3O_{2(g)} \xrightarrow{\Delta} 2ZnO_{(s)} + 2SO_{2(g)}$

The correct option of names given to them respectively is

(1) Both (A) and (B) are producing same product so both are calcination

(2) (A) is calcination and (B) is roasting

(3) Both (A) and (B) are producing same product so both are roasting

(4) (A) is roasting and (B) is calcination

Answer (2)

Sol. Heating of carbonate and hydroxide ore in absence or limited supply of air is calcination.

So, A is calcination

B is roasting

17. In Carius method, halogen containing organic compound is heated with fuming nitric acid in the presence of

(1) HNO_3

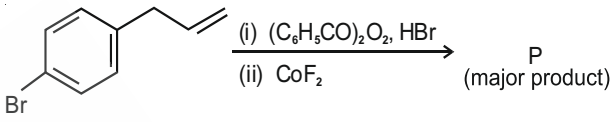
(2) $CuSO_4$

(3) $BaSO_4$

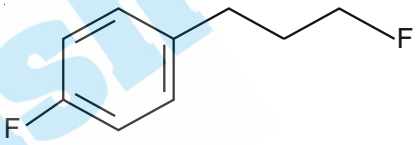
(4) $AgNO_3$

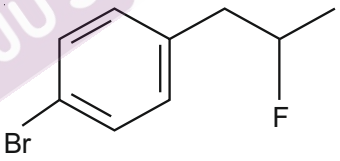
Answer (4)

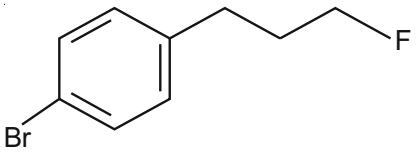
Sol. Halide ion reacts with silver nitrate to give silver halide precipitate.

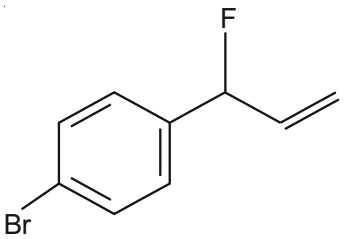
18. 

Major product P of above reaction, is

(1) 

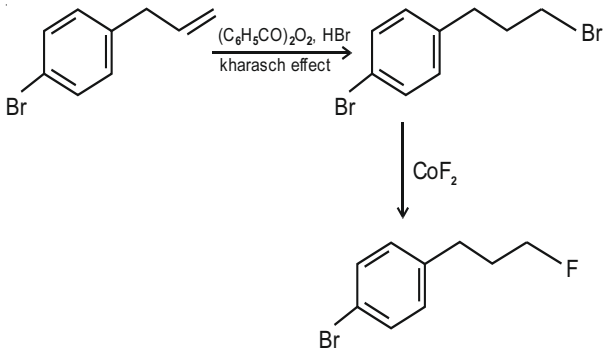
(2) 

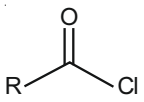
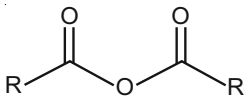
(3) 

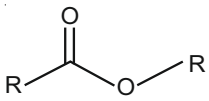
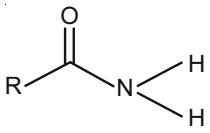
(4) 

Answer (3)

Sol.



19. (A)  (B) 

(C)  (D) 

The **correct** order of their reactivity towards hydrolysis at room temperature is

(1) (A) > (B) > (C) > (D)

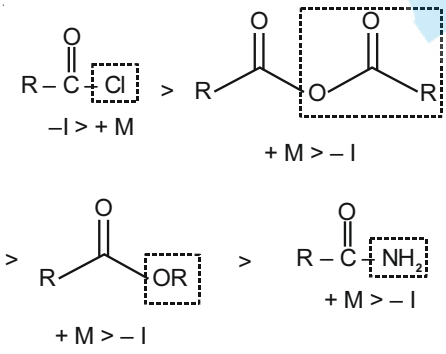
(2) (A) > (C) > (B) > (D)

(3) (D) > (A) > (B) > (C)

(4) (D) > (B) > (A) > (C)

Answer (1)

Sol. Order of hydrolysis



$-NH_2$ has greater denoting power than –OR group making –C = O less electron deficient.

20. A solution is 0.1 M in Cl^- and 0.001 M in CrO_4^{2-}.

Solid $AgNO_3$ is gradually added to it. Assuming that the addition does not change in volume and $K_{sp}(AgCl) = 1.7 \times 10^{-10}\ M^2$ and $K_{sp}(Ag_2CrO_4) = 1.9 \times 10^{-12}\ M^3$.

Select **correct** statement from the following

(1) AgCl precipitates first because its K_{sp} is high.

(2) Ag_2CrO_4 precipitates first as its K_{sp} is low.

(3) Ag_2CrO_4 precipitates first because the amount of Ag^+ needed is low.

(4) AgCl will precipitate first as the amount of Ag^+ needed to precipitate is low.

Answer (4)

Sol. Conc. of Cl^- = 0.1 M = 10^{-1} M

Conc. of CrO_4^{2-} = 0.001 M = 10^{-3} M

$K_{sp}(AgCl) = [Ag^+][Cl^-]$

$$[Ag^+]_{AgCl} = \frac{1.7\times10^{-10}}{10^{-1}} = 1.7\times10^{-9}$$

$$K_{sp}(Ag_2CrO_4) = \left[Ag^+\right]^2\left[CrO_4^{2-}\right]$$

$$[Ag^+] = \sqrt{\frac{1.9\times10^{-12}}{10^{-3}}} = \sqrt{19}\times10^{-4}$$

$\therefore$ AgCl will be precipitated first

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. 4 g equimolar mixture of NaOH and Na_2CO_3 contains x g of NaOH and y g of Na_2CO_3. The value of x is _____ g.

(Nearest integer)

Answer (1)

Sol. Mass of NaOH = x

Moles of NaOH = $\frac{x}{40}$

Mass of Na_2CO_3 = y

Moles of $Na_2CO_3 = \frac{y}{106}$

$\frac{x}{40} = \frac{y}{106}$

x + y = 4

x = 1.1, y = 2.9

x = 1.1 ≈ 1 (nearest integer)

2. The vapour pressures of A and B at 25°C are 90 mm Hg and 15 mm Hg respectively. If A and B are mixed such that the mole fraction of A in the mixture is 0.6, then the mole fraction of B in the vapour phase is x × 10^{-1}. The value of *x* is _____. (Nearest integer)

Answer (1)

Sol. $x_A = 0.6$

$P_T = x_A P_A^\circ + x_B P_B^\circ$

= 0.6 × 90 + 0.4 × 15

= 54 + 6 = 60

$x_A P_A^\circ = y_A P_T$

$0.6 \times 90 = y_A(60)$

$\Rightarrow y_A = 0.9$

$y_B = 0.1 = 1 \times 10^{-1}$

$\therefore x = 1$

3. Potassium chlorate is prepared by electrolysis of KCl in basic solution as shown by following equation.

$$6OH^- + Cl^- \longrightarrow ClO_3^- + 3H_2O + 6e^-$$

A current of xA has to be passed for 10 h to produce 10.0 g of potassium chlorate. The value of x is _____. (Nearest integer)

(Molar mass of $KClO_3$ = 122.6 g mol^{-1}, F = 96500 C)

Answer (1)

Sol. Mass of $KClO_3$ = 10 g

$\Rightarrow \frac{10}{122.6} = \frac{x \times 10 \times 3600}{6 \times 96500}$

$\Rightarrow$ x = 1.311 ≈ 1 (nearest integer)

4. An aqueous solution of $NiCl_2$ was heated with excess sodium cyanide in presence of strong oxidizing agent to form $[Ni(CN)_6]^{2-}$. The total change in number of unpaired electrons on metal centre is _____.

Answer (2)

Sol. $NiCl_2(aq) \rightarrow [Ni(H_2O)_6]Cl_2$

H_2O : weak field ligand

$$[Ni(H_2O)_6]Cl_2 \xrightarrow[(excess)]{NaCN} [Ni(CN)_6]^{2-}$$

$Ni^{2+} \rightarrow sp^3d^2$ unpaired electrons = 2

$Ni^{4+} \rightarrow d^2sp^3$ d^6 number of unpaired electrons = 0

$\therefore$ Total change in number of unpaired electrons = 2

5. For a given chemical reaction A → B at 300 K the free energy change is –49.4 kJ mol^{-1} and the enthalpy of reaction is 51.4 kJ mol^{-1}. The entropy change of the reaction is _____ J K^{-1} mol^{-1}.

Answer (336)

Sol. ΔG = –49.4 kJ/mol

ΔH = 51.4 kJ/mol

$\Delta G = \Delta H - T\Delta S$

$-49400 = 51400 - 300\Delta S$

$\Delta S = \frac{+100800}{300} = 336\,JK^{-1}mol^{-1}$

6. When 0.15 g of an organic compound was analyzed using Carius method for estimation of bromine, 0.2397 g of AgBr was obtained. The percentage of bromine in the organic compound is _____. (Nearest integer)

[Atomic mass :
Silver = 108
Bromine = 80]

Answer (68)

Sol. % of Br = $\frac{\text{Atomic mass of Br} \times m_1}{\text{molecular mass of AgBr} \times m} \times 100$

m = mass of organic compound taken

m_1 = mass of AgBr obtained.

$\therefore$ % of Br = $\frac{80 \times 0.2397}{188 \times 0.15} \times 100$

= 68

7. The wavelength of electrons accelerated from rest through a potential difference of 40 kV is $x \times 10^{-2}$ m. The value of x is _____. (Nearest integer)

Given: Mass of electron = 9.1×10^{-31} kg

Charge on an electron = 1.6×10^{-19} C

Planck's constant = 6.63×10^{-34} Js

Answer (6)

Sol. Wavelength of electron is given by

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

Here q = charge on electron, V = potential difference

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 40 \times 10^3}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{1164.8 \times 10^{-47}}} = 6.144 \times 10^{-12} \approx 6 \times 10^{-12}$$

x = 6

8. Diamond has a three dimensional structure of C atoms formed by covalent bonds. The structure of diamond has face centred cubic lattice where 50% of the tetrahedral voids are also occupied by carbon atoms. The number of carbon atoms present per unit cell of diamond is _____.

Answer (8)

Sol. Effective atoms of carbon from FCC lattice =

$$8 \times \frac{1}{8} + 6 \times \frac{1}{2} = 4$$

Number of atoms occupied in 1 unit cell (TV)

= 4 × 1 = 4

$\therefore$ Total atoms = 4 + 4 = 8

9. $PCl_5(g) \rightarrow PCl_3(g) + Cl_2(g)$

In the above first order reaction the concentration of PCl_5 reduces from initial concentration 50 mol L^{-1} to 10 mol L^{-1} in 120 minutes at 300 K.The rate constant for the reaction at 300 K is $x \times 10^{-2}$ min^{-1}. The value of x is _____.

Given: log5 = 0.6989

Answer (1)

Sol. a_0 = 50 mol L^{-1}

a_t = 10 mol L^{-1}

$$K = \frac{1}{120} \times 2.303 \log \frac{50}{10}$$

= 0.01341

= 1.34×10^{-2} min^{-1}

x = 1.34 $\approx$ 1 (nearest integer)

10. 100 ml of 0.0018% (w/v) solution of Cl^- ion was the minimum concentration of Cl^- required to precipitate a negative sol in one h. The coagulating value of Cl^- ion is _____.

(Nearest integer)

Answer (1) Bonus*

Assuming coagulating of positive sol

Sol. 0.0018 g of Cl^- in 100 ml solution

mmoles in 100 ml = $\frac{0.0018}{35.5} \times 1000$

= 0.0507

$\therefore$ Coagulating value = $\frac{0.0507}{0.1}$

= 0.507 = 0.51

$\approx$ 1 (nearest integer)

PART–C : MATHEMATICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Let r_1 and r_2 be the radii of the largest and smallest circles, respectively, which pass through the point (–4, 1) and having their centres on the circumference of the circle $x^2 + y^2 + 2x + 4y - 4 = 0$. If $\frac{r_1}{r_2} = a + b\sqrt{2}$, then a + b is equal to

(1) 3 (2) 7

(3) 11 (4) 5

Answer (4)

Sol. $C \equiv (x + 1)^2 + (y + 2)^2 = 9$

Distance between (–1, –2) and (–4, 1)

$\sqrt{3^2 + 3^2} = \sqrt{18}$

Maximum radius of required circle $= \sqrt{18} + 3$

Minimum radius of required circle $= \sqrt{18} - 3$

$$\frac{r_1}{r_2} = \frac{3\sqrt{2}+3}{3\sqrt{2}-1} = \frac{\sqrt{2}+1}{\sqrt{2}-1} = \frac{\left(\sqrt{2}+1\right)^2}{1} = 3 + 2\sqrt{2}$$

2. If the mean and variance of six observations 7, 10, 11, 15, a, b are 10 and $\frac{20}{3}$, respectively, then the value of |a – b| is equal to

(1) 7 (2) 1

(3) 11 (4) 9

Answer (2)

Sol. Given $\frac{7+10+11+15+a+b}{6} = 10$

$\Rightarrow$ a + b = 17 ...(i)

& $\frac{7^2+10^2+11^2+15^2+a^2+b^2}{6} - 10^2 = \frac{20}{3}$

$\frac{4095 + a^2 + b^2}{6} = \frac{320}{3}$

$\Rightarrow$ $a^2 + b^2 = 145$ ...(ii)

$\therefore$ $a^2 + b^2 + 2ab = 289$

$\Rightarrow$ $2ab = 144$

$(a - b)^2 = 145 - 144$

$\therefore$ $(a - b) = 1$

3. The value of $\tan\left(2\tan^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right)\right)$ is equal to

(1) $\frac{220}{21}$ (2) $\frac{151}{63}$

(3) $\frac{-181}{69}$ (4) $\frac{-291}{76}$

Answer (1)

Sol. $2\tan^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{6/5}{1 - 9/25}\right) = \tan^{-1}\left(\frac{6/5}{16/25}\right) = \tan^{-1}\frac{15}{8}$

$\therefore$ $2\tan^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right) = \tan^{-1}\left(\frac{15}{8}\right) + \tan^{-1}\left(\frac{5}{12}\right)$

$= \tan^{-1}\left(\frac{\frac{15}{8} + \frac{5}{12}}{1 - \frac{15}{8}, \frac{5}{12}}\right)$

$= \tan^{-1}\left(\frac{180 + 40}{21}\right) = \tan^{-1}\left(\frac{220}{21}\right)$

4. For the natural numbers m, n, if $(1 - y)^m (1 + y)^n = 1 + a_1 y + a_2 y^2 + ... + a_{m+n} y^{m+n}$ and $a_1 = a_2 = 10$, then the value of (m + n) is equal to

(1) 64 (2) 80

(3) 88 (4) 100

Answer (2)

Sol. $(1 - y)^n (1 + y)^n = 1 + a_1 y + a_2 y^2 + ... + a_{m+n} y^{m+n}$

Given $(a_1 = a_2 = 10)$

$(1 - my + {}^mC_2 y^2 + ...)(1 + ny + {}^nC_2 y^2 + ...)$

$= 1 + a_1 y + a_2 y^2 + ...$

$\Rightarrow$ n – m = 10 ...(i)

$\Rightarrow$ ${}^mC_2 + {}^nC_2 - mn = 10$ ...(ii)

$\frac{m(m-1)}{2} + \frac{n(n-1)}{2} - mn = 10$

$\Rightarrow$ $\frac{m^2 - m}{2} + \frac{(10+m)(9+m)}{2} - m(10+m) = 10$

$\Rightarrow$ $m^2 - m + m^2 + 19m + 90 - 2(m^2 + 10m) = 20$

$\Rightarrow$ $18m + 90 - 20m = 20$

$\Rightarrow$ $2m = 70$

$\Rightarrow$ m = 35 & n = 45

m + n = 80

5. Let $y = y(x)$ satisfies the equation $\frac{dy}{dx} - |A| = 0$, for all $x > 0$, where $A = \begin{bmatrix} y & \sin x & 1 \\ 0 & -1 & 1 \\ 2 & 0 & \frac{1}{x} \end{bmatrix}$. If $y(\pi) = \pi + 2$, then the value of $y\left(\frac{\pi}{2}\right)$ is

(1) $\frac{\pi}{2} - \frac{4}{\pi}$ (2) $\frac{\pi}{2} + \frac{4}{\pi}$

(3) $\frac{3\pi}{2} - \frac{1}{\pi}$ (4) $\frac{\pi}{2} - \frac{1}{\pi}$

Answer (2)

Sol. $|A| = \frac{-y}{x} - \sin x(-2) + 1(2)$

$= 2 + 2\sin x - \frac{y}{x}$

$\frac{dy}{dx} + \frac{y}{x} = 2 + 2\sin x$

I.f $= e^{\ln x} = x$

$\int d(xy) = \int 2x(1 + \sin x)\, dx$

$\Rightarrow xy = x^2 - 2x\cos x + \int 2\cos x\, dx$

$\Rightarrow xy = x^2 - 2x\cos x + 2\sin x + c$

$\therefore y(\pi) = \pi + 2$

$\Rightarrow \pi(\pi + 2) = \pi^2 - 2\pi(-1) + 0 + c$

$\Rightarrow c = 0$

For $y\left(\frac{\pi}{2}\right)$

$\frac{\pi}{2}y = \frac{\pi^2}{4} - \frac{2\pi}{2}(0) + 2$

$\Rightarrow y\left(\frac{\pi}{2}\right) = \frac{\pi}{2} + \frac{4}{\pi}$

6. If sum of the first 21 terms of the series $\log_{9^{1/2}} x + \log_{9^{1/3}} x + \log_{9^{1/4}} x + \ldots$, where $x > 0$ is 504, then x is equal to

(1) 81 (2) 243

(3) 9 (4) 7

Answer (1)

Sol. $\log_{9^{1/2}} x + \log_{9^{1/3}} x + \log_{9^{1/4}} x + \ldots$

$\Rightarrow \log_9 x^2 + \log_9 x^3 + \log_9 x^4 + \ldots$

$\Rightarrow \log_9\left(x^{2+3+\ldots\ldots 21-\text{terms}}\right) = 504$

$\Rightarrow 252 \log_9 x = 504$

$\Rightarrow x = 9^2 = 81$

7. Let in a right angled triangle, the smallest angle be θ. If a triangle formed by taking the reciprocal of its sides is also a right angled triangle, then $\sin\theta$ is equal to :

(1) $\frac{\sqrt{5}+1}{4}$ (2) $\frac{\sqrt{2}-1}{2}$

(3) $\frac{\sqrt{5}-1}{2}$ (4) $\frac{\sqrt{5}-1}{4}$

Answer (3)

Sol. Let a ΔABC having $C = 90°$ and $A = \theta$

$\frac{\sin\theta}{a} = \frac{\cos\theta}{b} = \frac{1}{c}$ ...(i)

Also for triangle of reciprocals

$\cos A = \frac{\left(\frac{1}{c}\right)^2 + \left(\frac{1}{b}\right)^2 - \left(\frac{1}{a}\right)^2}{2\left(\frac{1}{c}\right)\left(\frac{1}{b}\right)}$

$\frac{1}{c^2} + \frac{1}{(c\cos\theta)^2} = \frac{1}{(c\sin\theta)^2}$

$\Rightarrow 1 + \sec^2\theta = \text{cosec}^2\theta$

$\Rightarrow \frac{1}{4} = \frac{\cos^2\theta}{4\sin^2\theta\cos^2\theta}$

$\Rightarrow \frac{1}{4} = \frac{\cos^2\theta}{\sin^2 2\theta}$

$\Rightarrow 1 - \cos^2 2\theta = 4\cos 2\theta$

$\cos^2 2\theta + 4\cos 2\theta - 1 = 0$

$\cos 2\theta = \frac{-4 \pm \sqrt{16+4}}{2}$

$\cos 2\theta = -2 \pm \sqrt{5}$

$\cos 2\theta = \sqrt{5} - 2 = 1 - 2\sin^2\theta$

$\Rightarrow 2\sin^2\theta = 3 - \sqrt{5}$

$\Rightarrow \sin^2\theta = \frac{3-\sqrt{5}}{2}$

$\Rightarrow \sin\theta = \frac{\sqrt{5}-1}{2}$

8. Let $g(t) = \int_{-\pi/2}^{\pi/2} \cos\left(\frac{\pi}{4}t + f(x)\right) dx$, where $f(x) = \log_e\left(x + \sqrt{x^2+1}\right)$, $x \in \mathbf{R}$. Then which one of the following is correct?

(1) $g(1) + g(0) = 0$ (2) $g(1) = \sqrt{2}\, g(0)$

(3) $\sqrt{2}\, g(1) = g(0)$ (4) $g(1) = g(0)$

Answer (3)

Sol. $\because \quad f(x)=\ln\left(x+\sqrt{x^2+1}\right)$

$\therefore \quad f(x)+f(-x)=\ln\left(\sqrt{x^2+1}+x\right)+\ln\left(\sqrt{x^2+1}-x\right)$

$\therefore \quad f(x)+f(-x)=0 \qquad ...(i)$

$\because \quad g(t)=\int_{-\pi/2}^{\pi/2}\cos\left(\frac{\pi}{4}t+f(x)\right)dx.$

$=\int_0^{\pi/2}\left\{\cos\left(\frac{\pi}{4}t+f(x)\right)+\cos\left(\frac{\pi}{4}t+f(-x)\right)\right\}dx.$

$=\int_0^{\pi/2}\left\{\cos\left(\frac{\pi t}{4}+f(x)\right)+\cos\left(\frac{\pi t}{4}-f(x)\right)\right\}dx.$

$g(t)=2\int_0^{\pi/2}\cos\frac{\pi t}{4}\cdot\cos(f(x))\,dx.$

$\therefore \quad g(1)=\sqrt{2}\int_0^{\pi/2}\cos(f(x))\,dx$

and $g(0)=2\int_0^{\pi/2}\cos(f(x))\,dx.$

$\therefore \quad \sqrt{2}\,g(1)=g(0)$

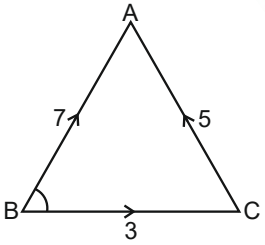
9. In a triangle ABC, if $|\overrightarrow{BC}|=3, |\overrightarrow{CA}|=5$ and $|\overrightarrow{BA}|=7$, then the projection of the vector $\overrightarrow{BA}$ on $\overrightarrow{BC}$ is equal to :

(1) $\frac{13}{2}$ (2) $\frac{19}{2}$

(3) $\frac{15}{2}$ (4) $\frac{11}{2}$

Answer (4)

Sol. Projection of $\overrightarrow{BA}$ on $\overrightarrow{BC}$



$=\left|\frac{\overrightarrow{BA}\cdot\overrightarrow{BC}}{|\overrightarrow{BC}|}\right|$

$=\frac{|\overrightarrow{BA}|\cdot|\overrightarrow{BC}|\cos B}{|\overrightarrow{BC}|}$

$=7\cdot\left(\frac{7^2+3^2-5^2}{2\times7\times3}\right)$

$=\frac{11}{2}$ units

10. If $[x]$ denotes the greatest integer less than or equal to x, then the value of the integral $\int_{-\pi/2}^{\pi/2}[[x]-\sin x]dx$ is equal to :

(1) $-\pi$ (2) 0

(3) π (4) 1

Answer (1)

Sol. $I=\int_{-\pi/2}^{\pi/2}[[x]-\sin x]dx$

$=\int_{-\pi/2}^{\pi/2}([x]+[-\sin x])dx$

$=\int_0^{\pi/2}([x]+[-\sin x]+[-x]+[\sin x])dx$

$=\int_0^{\pi/2}(-2)dx$

$=-\pi$

11. Let P be a variable point on the parabola $y = 4x^2 + 1$. Then, the locus of the mid-point of the point P and the foot of the perpendicular drawn from the point P to the line $y = x$ is :

(1) $(3x - y)^2 + 2(x - 3y) + 2 = 0$

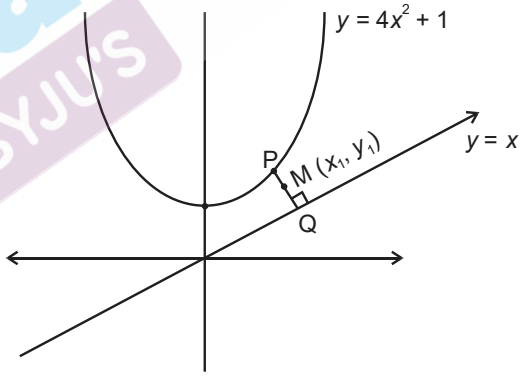
(2) $2(3x - y)^2 + (x - 3y) + 2 = 0$

(3) $2(x - 3y)^2 + (3x - y) + 2 = 0$

(4) $(3x - y)^2 + (x - 3y) + 2 = 0$

Answer (2)

Sol.



Let coordinate of mid-point M of PQ be (x_1, y_1)

Let coordinate of Q be (α, β).

$\therefore \quad \frac{\alpha-x_1}{1}=\frac{\beta-y_1}{-1}=\frac{-(x_1-y_1)}{2}$

$\therefore \quad Q=(\alpha,\ \beta)=\left(\frac{x_1+y_1}{2},\ \frac{x_1+y_1}{2}\right)$

and coordinate of $P=\left(\frac{3x_1-y_1}{2},\ \frac{3y_1-x_1}{2}\right)$

$\therefore$ P lies on parabola

$\therefore \quad \frac{3y_1-x_1}{2}=4\left(\frac{3x_1-y_1}{2}\right)^2+1$

$\therefore$ Required locus is $2(3x - y)^2 + (x - 3y) + 2 = 0$

12. If the real part of the complex number $(1 - \cos\theta + 2i\sin\theta)^{-1}$ is $\frac{1}{5}$ for $\theta \in (0, \pi)$, then the value of the integral $\int_0^{\theta} \sin x\,dx$ is equal to :

(1) 1 (2) 2

(3) 0 (4) –1

Answer (1)

Sol. $Z = \frac{1}{1-\cos\theta+2i\sin\theta} = \frac{(1-\cos\theta)-2i\sin\theta}{(1-\cos\theta)^2+4\sin^2\theta}$

$\therefore \quad \text{Re}(Z) = \frac{1-\cos\theta}{2-2\cos\theta+3\sin^2\theta} = \frac{1}{5}$

$\therefore \quad 5 - 5\cos\theta = 2 - 2\cos\theta + 3\sin^2\theta$

$3\cos\theta(1 - \cos\theta) = 0$

$\therefore \quad \theta = \frac{\pi}{2}$, when $\theta \in (0, \pi)$

$\therefore \quad \int_0^{\theta} \sin x\,dx = \int_0^{\pi/2} \sin x\,dx$

$= 1$

13. The lines $x = ay - 1 = z - 2$ and $x = 3y - 2 = bz - 2$, $(ab \neq 0)$ are coplanar, if

(1) a = 2, b = 2 (2) a = 2, b = 3

(3) a = 1, b $\in$ **R** – {0} (4) b = 1, a $\in$ **R** – {0}

Answer (4)

Sol. Lines are $x = ay - 1 = z - 2$

$\therefore \quad \frac{x}{1} = \frac{y - 1/a}{1/a} = \frac{z-2}{1}$ ... (i)

and $x = 3y - 2 = bz - 2$

$\therefore \quad \frac{x}{1} = \frac{y - 2/3}{1/3} = \frac{z - 2/b}{1/b}$ ...(ii)

$\therefore$ lines are co-planar

$\therefore \quad \begin{vmatrix} 0 & -\frac{1}{a}+\frac{2}{3} & -2+\frac{2}{b} \\ 1 & \frac{1}{a} & 1 \\ 1 & \frac{1}{3} & \frac{1}{b} \end{vmatrix} = 0$

$\therefore \quad \begin{vmatrix} 0 & \frac{2}{3}-\frac{1}{a} & \frac{2}{b}-2 \\ 0 & \frac{1}{a}-\frac{1}{3} & 1-\frac{1}{b} \\ 1 & \frac{1}{3} & \frac{1}{b} \end{vmatrix} = 0$

$\therefore \quad \frac{1}{a} - \frac{1}{ab} = 0$

$\Rightarrow$ b = 1 and a $\in$ **R** – {0}

14. Consider the following three statements

(A) If 3 + 3 = 7 then 4 + 3 = 8.

(B) If 5 + 3 = 8 then earth is flat.

(C) If both (A) and (B) are true then 5 + 6 = 17.

Then, which of the following statements is correct?

(1) (A) and (B) are false while (C) is true

(2) (A) is false, but (B) and (C) are true

(3) (A) and (C) are true while (B) is false

(4) (A) is true while (B) and (C) are false

Answer (3)

Sol. $\because$ 3 + 3 = 7 is false and 4 + 3 = 8 is false

then statement (A) is true

For (B) 5 + 3 = 8 is true and earth is flat is false.

Then statement (B) is false

For (C) if A and B are true then 5 + 6 = 17 is false, then (C) is true.

$\therefore$ (A) and (C) are true and (B) is false.

15. Let $f : \mathbf{R} - \left\{\frac{\alpha}{6}\right\} \to \mathbf{R}$ be defined by $f(x) = \frac{5x+3}{6x-\alpha}$. Then the value of α for which $(fof)(x) = x$, for all $x \in \mathbf{R} - \left\{\frac{\alpha}{6}\right\}$, is

(1) 5 (2) 8

(3) No such α exists (4) 6

Answer (1)

Sol. For $f(f(x)) = x$

$f(x) = f^{-1}(x)$

finding $f^{-1}(x)$

$y = \frac{3x+3}{6x-\alpha}$

$\Rightarrow \quad f^{-1}(x) = \frac{3+\alpha x}{6x-5}$

$\therefore \quad f(x) = f^{-1}(x)$ gives

$\frac{3+\alpha x}{6x-5} = \frac{5x+3}{6x-\alpha}$

$\Rightarrow \quad (30 - 6\alpha)x^2 + (\alpha^2 - 25)x + (3\alpha - 15) = 0$

$\therefore \quad \alpha = 5$

16. If $f : \mathbf{R} \to \mathbf{R}$ is given by $f(x) = x + 1$, then the value of $\lim\limits_{n\to\infty} \frac{1}{n}\left[f(0)+f\left(\frac{5}{n}\right)+f\left(\frac{10}{n}\right)+...+f\left(\frac{5(n-1)}{n}\right)\right]$, is

(1) $\frac{7}{2}$ (2) $\frac{3}{2}$

(3) $\frac{5}{2}$ (4) $\frac{1}{2}$

Answer (1)

Sol. $f(0)+f\left(\frac{5}{n}\right)+f\left(\frac{10}{n}\right)+....+f\left(\frac{5(n-1)}{n}\right)$

$\Rightarrow 1 + 1+\frac{5}{n}+1+\frac{10}{n}+...+1+\frac{5(n-1)}{n}$

$\Rightarrow n+\frac{5}{n}\frac{(n-1)n}{2}=\frac{2n+5n-5}{2}=\frac{7n-5}{2}$

$\lim\limits_{n\to\infty}\frac{1}{n}\left(\frac{7n-5}{2}\right)=\frac{7}{2}$

17. Consider the line L given by the equation $\frac{x-3}{2}=\frac{y-1}{1}=\frac{z-2}{1}$. Let Q be the mirror image of the point (2, 3, –1) with respect to L. Let a plane P be such that it passes through Q, and the line L is perpendicular to P. Then which of the following points is on the plane P?

(1) (–1, 1, 2) (2) (1, 2, 2)

(3) (1, 1, 1) (4) (1, 1, 2)

Answer (2)

Sol. L is normal to P and plane P will pass through (2, 3, –1)

Equation of P is $2x + y + z = \lambda = 6$ which is satisfied by (1, 2, 2)

18. The sum of all the local minimum values of the twice differentiable function $f : R \to R$ defined by $f(x) = x^3 - 3x^2 - \frac{3f''(2)}{2}x + f''(1)$ is

(1) –22 (2) 5

(3) –27 (4) 0

Answer (3)

Sol. $f''(x) = 6x - 6$

$f''(2) = 6, f''(1) = 0$

$f(x) = x^3 - 3x^2 - 9x$

$f'(x) = 3x^2 - 6x - 9 = 3(x + 1)(x - 3)$

Local min at $x = 3$

Local min value = $f(3) = -27$

19. Let A, B and C be three events such that the probability that exactly one of A and B occurs is (1 – k), the probability that exactly one of B and C occurs is (1 – 2k), the probability that exactly one of C and A occurs is (1 – k) and the probability of all A, B and C occur simultaneously is k^2, where $0 < k < 1$. Then the probability that at least one of A, B and C occur is

(1) Greater than $\frac{1}{2}$

(2) Exactly equal to $\frac{1}{2}$

(3) Greater than $\frac{1}{8}$ but less than $\frac{1}{4}$

(4) Greater than $\frac{1}{4}$ but less than $\frac{1}{2}$

Answer (1)

Sol. $P(A) + P(B) - 2P(A \cap B) = 1 - k$ ...(i)

$P(B) + P(C) - 2P(B \cap C) = 1 - 2k$ ...(ii)

$P(C) + P(A) - 2P(C \cap A) = 1 - k$ ...(iii)

(i) + (ii) + (iii)

$\Rightarrow \sum P(A)-\sum P(A\cap B)=\frac{3-4k}{2}$

$P(A\cup B\cup C)=\sum P(A)-\sum P(A\cap B)+P(A\cap B\cap C)$

$=\frac{3-4k}{2}+k^2$

$=(k-1)^2+\frac{1}{2}>\frac{1}{2}$

20. The value of $k \in R$, for which the following system of linear equations

$3x - y + 4z = 3,$

$x + 2y - 3z = -2,$

$6x + 5y + kz = -3,$

has infinitely many solutions, is

(1) –3 (2) –5

(3) 5 (4) 3

Answer (2)

Sol. $\Delta=\begin{vmatrix}3 & -1 & 4\\ 1 & 2 & -3\\ 6 & 5 & k\end{vmatrix}=0 \Rightarrow k=-5$

For $k = -5$, $\Delta_1 = \Delta_2 = \Delta_3 = 0$

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Let a curve $y = y(x)$ be given by the solution of the differential equation

$$\cos\left(\frac{1}{2}\cos^{-1}\left(e^{-x}\right)\right)dx = \sqrt{e^{2x}-1}\,dy$$

If it intersects y-axis at $y = -1$, and the intersection point of the curve with x-axis is $(\alpha, 0)$, then e^{α} is equal to _____.

Answer (2)

Sol. $$\int dy = \int \frac{\cos\frac{1}{2}\cos^{-1}(e^{-x})}{\sqrt{e^{2x}-1}}dx$$

Let $\frac{1}{2}\cos^{-1}(e^{-x}) = \theta$

$e^{-x} = \cos 2\theta$

$x = \ln\sec 2\theta$

$dx = 2\tan 2\theta\, d\theta$

$$y = \int 2\cos\theta\, d\theta = 2\sin\theta + C = \sqrt{2}\sqrt{1-\cos 2\theta} + C$$

$$= \sqrt{2}\sqrt{1-e^{-x}} + C$$

$y(0) = -1 \Rightarrow C = -1$

$y = \sqrt{2(1-e^{-x})} - 1$

$y = 0 \Rightarrow e^{\alpha} = 2$

2. Let a function $g : [0, 4] \to \mathbf{R}$ be defined as

$$g(x) = \begin{cases} \max\ \{t^3 - 6t^2 + 9t - 3\}, & 0 \le x \le 3 \\ 0 \le t \le x & \\ 4 - x, & 3 < x \le 4 \end{cases}$$,then

the number of points in the interval $(0, 4)$ where $g(x)$ is NOT differentiable, is_____.

Answer (1)

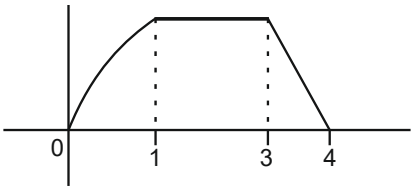
Sol. $f(t) = t^3 - 6t^2 + 9t - 3$

$f'(t) = 3(t - 1)(t - 3)$

Local max at $x = 1$, $f(1) = 1$

$$g(x) = \begin{cases} f(x) = x^3 - 6x^2 + 9x - 3, & t \in [0, 1] \\ 1 & t \in (1, 3] \\ 4 - x & 3 < x \le 4 \end{cases}$$

$y = g(x)$ =



Not diff. at $x = 3$

3. If the point on the curve $y^2 = 6x$, nearest to the point $\left(3, \frac{3}{2}\right)$ is (α, β), then $2(\alpha + \beta)$ is equal to_____.

Answer (09.00)

Sol. Let a point on $y^2 = 6x$ is $P\left(\frac{3}{2}t^2, 3t\right)$

The distance between P and $\left(3, \frac{3}{2}\right)$ is D.

$$\therefore\quad D^2 = \left(\frac{3t^2}{2} - 3\right)^2 + \left(3t - \frac{3}{2}\right)^2$$

$$= 9\left\{\frac{t^4}{4} - t^2 + 1 + t^2 - t + \frac{1}{4}\right\}$$

$$= \frac{9}{4}\left(t^4 - 4t + 5\right)$$

$$\therefore\quad 2D.\frac{dD}{dt} = \frac{9}{4}(4t^3 - 4) = 9(t-1)(t^2 + t + 1)$$

$\therefore$ For $t = 1$, D^2 will be minimum.

$$\therefore\quad P = \left(\frac{3}{2}, 3\right) = (\alpha, \beta)$$

$\therefore\quad 2(\alpha + \beta) = 9$

4. If $\lim\limits_{x\to 0}\dfrac{\alpha x e^{x}-\beta\log_e(1+x)+\gamma x^2e^{-x}}{x\sin^2 x}=10$, $\alpha, \beta, \gamma \in \mathbf{R}$, then the value of $\alpha+\beta+\gamma$ is_____.

_____.

Answer (3)

Sol. $$\alpha x\left(1+\frac{x}{\lfloor 1}+\frac{x^2}{\lfloor 2}+\ldots\right)-\beta\left(x-\frac{x^2}{2}+\frac{x^3}{3}+\ldots\right)+\gamma x^2\left(1-\frac{x}{\lfloor 1}+\frac{x^2}{\lfloor 2}+\ldots\right)$$

$$\lim_{x\to 0}\frac{}{x^3\left(\frac{\sin x}{x}\right)^2}=10$$

$$\Rightarrow \lim_{x\to 0}\frac{x(\alpha-\beta)+x^2\left(\alpha+\frac{\beta}{2}+\gamma\right)+x^3\left(\frac{\alpha}{2}-\frac{\beta}{3}-\gamma\right)+\ldots}{x^3}=10$$

$$\Rightarrow \alpha-\beta=0,\ \alpha+\frac{\beta}{2}+\gamma=0,\ \frac{\alpha}{2}-\frac{\beta}{3}-\gamma=10$$

$\Rightarrow \alpha = 6, \beta = 6, \gamma = -9$

5. For $k \in N$, let $\dfrac{1}{\alpha(\alpha+1)(\alpha+2).....(\alpha+20)}=\sum\limits_{k=0}^{20}\dfrac{A_k}{\alpha+k}$, where $\alpha > 0$. Then the value of $100\left(\dfrac{A_{14}+A_{15}}{A_{13}}\right)^2$ is equal to_____.

Answer (9)

Sol. $$\alpha(\alpha+1)(\alpha+2)...(\alpha+20)\sum_{k=0}^{20}\frac{A_k}{\alpha+k}=1$$

Put $\alpha = -13$, $-A_{13}\cdot\lfloor 13\lfloor 7=1\Rightarrow A_{13}=\dfrac{1}{\lfloor 7\,\lfloor 13}$

Put $\alpha = -14$, $-A_{14}\cdot\lfloor 14\lfloor 6=1\Rightarrow A_{14}=\dfrac{-1}{\lfloor 14\,\lfloor 6}$

Put $\alpha = 15$, $-A_{15}\cdot\lfloor 15\lfloor 5=1\Rightarrow A_{15}=\dfrac{-1}{\lfloor 15\,\lfloor 5}$

$$100\left(\frac{A_{14}+A_{15}}{A_{13}}\right)^2=\frac{100\left(\frac{1}{\lfloor 14\lfloor 6}-\frac{1}{\lfloor 15\lfloor 5}\right)^2}{\left(\frac{1}{\lfloor 7\lfloor 13}\right)^2}$$

$$=100\left(\frac{\frac{9}{\lfloor 15\lfloor 6}}{\frac{1}{\lfloor 7\lfloor 13}}\right)^2=9$$

6. For $p > 0$, a vector $\vec{v}_2=2\hat{i}+(p+1)\hat{j}$ is obtained by rotating the vector $\vec{v}_1=\sqrt{3}p\hat{i}+\hat{j}$ by an angle θ about origin in counter clockwise direction. If $\tan\theta=\dfrac{(\alpha\sqrt{3}-2)}{(4\sqrt{3}+3)}$, then the value of α is equal to _____.

Answer (6)

Sol. $\because \cos\theta=\dfrac{\vec{v}_1\cdot\vec{v}_2}{|\vec{v}_1|\cdot|\vec{v}_2|}$ and $|\vec{v}_1|=|\vec{v}_2|$

$$\Rightarrow \cos\theta=\frac{2\sqrt{3}p+p+1}{|\vec{v}_1|^2}$$ and $4 + (p + 1)^2 = 3p^2 + 1$

$\Rightarrow p = 2$

$$\Rightarrow \cos\theta=\frac{4\sqrt{3}+3}{13}\Rightarrow\tan\theta=\frac{6\sqrt{3}-2}{4\sqrt{3}+3}$$

7. The number of solutions of the equation $\log_{(x+1)}(2x^2 + 7x + 5) + \log_{(2x+5)}(x + 1)^2 - 4 = 0$, $x > 0$, is _____.

Answer (1)

Sol. $\log_{(x+1)}(x + 1)(2x + 5) + \log_{(2x+5)}(x + 1)^2 = 4$

$\Rightarrow 1 + \log_{(x+1)}(2x + 5) + 2\log_{(2x+5)}(x + 1) = 4$

Let $\log_{(x+1)}(2x + 5) = t$

then $t+\dfrac{2}{t}=3\Rightarrow t = 1, 2$

$\Rightarrow 2x + 5 = x + 1$ or $2x + 5 = (x + 1)^2$

$\Rightarrow x = -4, +2, -2$ out of which only $x = 2$ is acceptable.

8. Consider a triangle having vertices A(–2, 3), B(1, 9) and C(3, 8). If a line L passing through the circum-center of triangle ABC, bisects line BC, and intersects y-axis at point $\left(0,\dfrac{\alpha}{2}\right)$, then the value of real number α is _____.

Answer (9)

Sol. Line L is perpendicular bisector of BC, which is

L : $4x - 2y + 9 = 0$

L cuts the *y*-axis at $\left(0,\dfrac{9}{2}\right)$

Clearly $\alpha = 9$

9. Let $A = \{a_{ij}\}$ be a 3×3 matrix, where

$$a_{ij} = \begin{cases} (-1)^{j-i} & \text{if } i < j, \\ 2 & \text{if } i = j, \\ (-1)^{i+j} & \text{if } i > j, \end{cases}$$

then $\det(3\,\text{Adj}(2A^{-1}))$ is equal to _____.

Answer (108)

Sol. $\text{adj}(2A^{-1}) = |2A^{-1}|(2A^{-1})^{-1} = \frac{8}{|A|}\cdot\frac{1}{2}A = \frac{4A}{|A|}$

So, $|3\text{adj}(2A^{-1})| = \left|12\frac{A}{|A|}\right| = \left(\frac{12}{|A|}\right)^3 \cdot |A| = \frac{12^3}{|A|^2}$

$\because \quad A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \Rightarrow |A| = 4$

Hence, $|3\text{adj}(2A^{-1})| = \frac{12^3}{4^2} = 108$

10. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence such that $a_1 = 1$, $a_2 = 1$ and $a_{n+2} = 2a_{n+1} + a_n$ for all $n \geq 1$. Then the value of $47\sum_{n=1}^{\infty}\frac{a_n}{2^{3n}}$ is equal to _____.

Answer (7)

Sol. $a_{n+2} = 2a_{n+1} + a_n$ has its characteristic equation as

$x^2 = 2x + 1 \Rightarrow x = 1 \pm \sqrt{2}$

So $a_n = a(1+\sqrt{2})^{n-1} + b(1-\sqrt{2})^{n-1}$

$\because \quad a_1 = 1 \Rightarrow a + b = 1$

and $a_2 = 1 \Rightarrow (a+b) + \sqrt{2}(a-b) = 1$

$\Rightarrow \quad a = \frac{1}{2}$ and $b = \frac{1}{2}$

So, $a_n = \frac{(1+\sqrt{2})^{n-1} + (1-\sqrt{2})^{n-1}}{2}$

$$\sum_{n=1}^{\infty}\frac{a_n}{2^{3n}} = \frac{1}{16}\left[\sum_{n=1}^{\infty}\left(\frac{1+\sqrt{2}}{8}\right)^{n-1} + \sum_{n=1}^{\infty}\left(\frac{1-\sqrt{2}}{8}\right)^{n-1}\right]$$

$$= \frac{1}{16}\left[\frac{8}{7-\sqrt{2}} + \frac{8}{7+\sqrt{2}}\right]$$

$$= \frac{7}{47}$$

❑ ❑ ❑