22/07/2021
Evening

Corporate Office: Aakash Tower, 8, Pusa Road, New Delhi-110005 Ph. 011-47623456

Time : 3 hrs.

M.M. : 300

Answers & Solutions

for

JEE (MAIN)-2021 (Online) Phase-3

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS :

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part has two sections.

(i) Section-I : This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) Section-II : This section contains 10 questions. In Section-II, attempt any **five questions out of 10.** There will be **no negative marking for Section-II**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and there is no negative marking for wrong answer.

PART–A : PHYSICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Choose the correct option

(1) True dip is always equal to apparent dip.

(2) True dip is not mathematically related to apparent dip.

(3) True dip is less than the apparent dip.

(4) True dip is always greater than the apparent dip.

Answer (3)

Sol. $\tan(\theta_a) = \dfrac{\tan(\theta_T)}{\cos\phi}$

$\Rightarrow \theta_a \geq \theta_T$

$\therefore$ True dip is less than apparent dip.

2. What will be the projection of vector $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ on vector $\vec{B} = \hat{i} + \hat{j}$?

(1) $\sqrt{2}(\hat{i} + \hat{j} + \hat{k})$

(2) $(\hat{i} + \hat{j})$

(3) $\sqrt{2}(\hat{i} + \hat{j})$

(4) $2(\hat{i} + \hat{j} + \hat{k})$

Answer (2)

Sol. Projection $= \dfrac{\vec{A}\cdot\vec{B}}{|\vec{B}|}(\hat{B})$

$$= \frac{(\hat{i}+\hat{j}+\hat{k})\cdot(\hat{i}+\hat{j})}{\sqrt{2}}\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$$

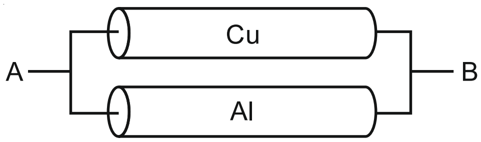
$$= \frac{2}{\sqrt{2}}\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$$

$$= (\hat{i}+\hat{j})$$

3. A Copper (Cu) rod of length 25 cm and cross-sectional area 3 mm^2 is joined with a similar Aluminium (Al) rod as shown in figure. Find the resistance of the combination between the ends A and B.

(Take Resistivity of Copper = 1.7 × 10^{-8} Ωm

Resistivity of Aluminium = 2.6 × 10^{-8} Ωm)



(1) 0.0858 mΩ

(2) 1.420 mΩ

(3) 0.858 mΩ

(4) 2.170 mΩ

Answer (3)

Sol. $R_{Cu} = \dfrac{\rho_{Cu}\times\ell}{A}$

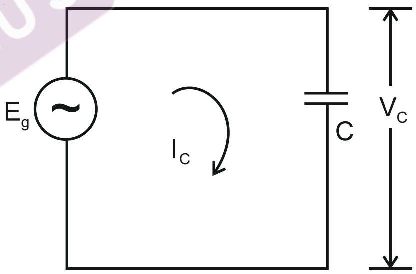
$R_{Al} = \dfrac{\rho_{Al}\times\ell}{A}$

$R_{Eq} = \dfrac{\rho_{Cu}\times\rho_{Al}}{\rho_{Cu}+\rho_{Al}}\times\left(\dfrac{\ell}{A}\right)$

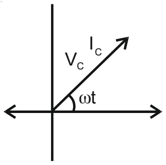
$$= \frac{1.7\times10^{-8}\times2.6\times10^{-8}}{(1.7+2.6)\times10^{-8}}\times\frac{0.25}{3\times10^{-6}}$$

= 0.856 mΩ

4. In a circuit consisting of a capacitance and a generator with alternating emf $E_g = E_{g_0}\sin\omega t$, V_C and I_C are the voltage and current. Correct phasor diagram for such circuit is



(1)



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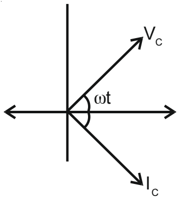

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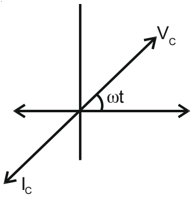

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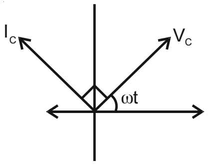
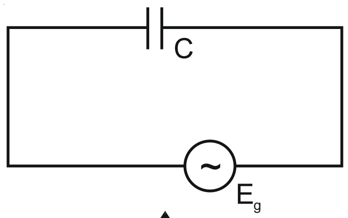
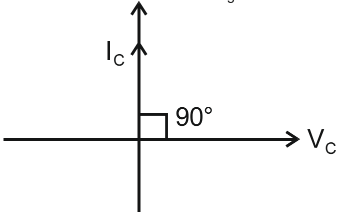


Answer (4)

Sol.





and phasors rotate by ωt.

5. A porter lifts a heavy suitcase of mass 80 kg and at the destination lowers it down by a distance of 80 cm with a constant velocity. Calculate the workdone by the porter in lowering the suitcase.

   (take g = 9.8 ms^{-2})

   (1) +627.2 J (2) –62720.0 J

   (3) –627.2 J (4) 784.0 J

Answer (3)

Sol. $W = -N \times \Delta x$

$$= -80 \times 9.8 \times \frac{80}{100}$$

$$= -627.2 \text{ J}$$

6. Consider a situation in which a ring, a solid cylinder and a solid sphere roll down on the same inclined plane without slipping. Assume that they start rolling from rest and having identical diameter.

   The **correct** statement for this situation is

   (1) All of them will have same velocity.

   (2) The ring has greatest and the cylinder has the least velocity of the centre of mass at the bottom of the inclined plane.

   (3) The sphere has the greatest and the ring has the least velocity of the centre of mass at the bottom of the inclined plane.

   (4) The cylinder has the greatest and the sphere has the least velocity of the centre of mass at the bottom of the inclined plane.

Answer (3)

Sol. $\frac{K_T}{K_R} = \frac{MR^2}{I_{CM}}$

I_{CM} is maximum for ring.

$\Rightarrow$ v is least for ring.

7. Intensity of sunlight is observed as 0.092 Wm^{-2} at a point in free space. What will be the peak value of magnetic field at the point?

   ($\varepsilon_0 = 8.85 \times 10^{-12}\ C^2N^{-1}m^{-2}$)

   (1) 2.77×10^{-8} T (2) 1.96×10^{-8} T

   (3) 8.31 T (4) 5.88 T

Answer (1)

Sol. $\frac{I}{C} = \frac{1}{2}\varepsilon_0.E_0^2$

$$\Rightarrow \quad E_0 = \sqrt{\frac{2I}{C\varepsilon_0}}$$

$$\frac{E_0}{B_0} = C \Rightarrow B_0 = \frac{E_0}{C}$$

$$\Rightarrow \quad B_0 = \sqrt{\frac{2I}{\varepsilon_0 C^3}} = \sqrt{\frac{2 \times 0.092}{8.85 \times 10^{-12} \times 27 \times 10^{+24}}}$$

$$= 2.77 \times 10^{-8} \text{ T}$$

8. A body is projected vertically upwards from the surface of earth with a velocity sufficient enough to carry it to infinity. The time taken by it to reach height h is ________s.

   (1) $\sqrt{\frac{2R_e}{g}}\left[\left(1+\frac{h}{R_e}\right)^{\frac{3}{2}} - 1\right]$

   (2) $\frac{1}{3}\sqrt{\frac{R_e}{2g}}\left[\left(1+\frac{h}{R_e}\right)^{\frac{3}{2}} - 1\right]$

   (3) $\sqrt{\frac{R_e}{2g}}\left[\left(1+\frac{h}{R_e}\right)^{\frac{3}{2}} - 1\right]$

   (4) $\frac{1}{3}\sqrt{\frac{2R_e}{g}}\left[\left(1+\frac{h}{R_e}\right)^{\frac{3}{2}} - 1\right]$

Answer (4)

Sol. $\frac{1}{2}mv^2 - \frac{GMm}{r} = 0 \Rightarrow v = \sqrt{\frac{2GM}{r}}$

$$\frac{dr}{dt} = \sqrt{\frac{2GM}{r}}$$

$$\Rightarrow \int_{R_e}^{(R_e+h)} \sqrt{r}\, dr = \int_0^t \sqrt{2GM}\, dt$$

$$\Rightarrow \frac{2}{3}\left[(R_e+h)^{3/2} - R_e^{3/2}\right] = (t)\sqrt{2GM}$$

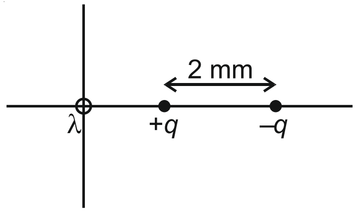
$$\Rightarrow t = \frac{1}{3}\sqrt{\frac{2R_e}{g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2} - 1\right]$$

9. An electric dipole is placed on x-axis in proximity to a line charge of linear charge density 3.0×10^{-6} C/m. Line charge is placed on z-axis and positive and negative charge of dipole is at a distance of 10 mm and 12 mm from the origin respectively. If total force of 4 N is exerted on the dipole, find out the amount of positive or negative charge of the dipole.

(1) 0.485 mC
(2) 815.1 nC
(3) 8.8 μC
(4) 4.44 μC

Answer (4)

Sol.



$|F| = q(E_1 - E_2)$

$$= (q)2k\lambda\left[\frac{2}{10\times12\times10^{-3}}\right]$$

$$4 = (q)\times2\times9\times10^{9}\times(3\times10^{-6})\left[\frac{2}{120\times10^{-3}}\right]$$

$\Rightarrow$ $q = 4.44\ \mu$C

10. A nucleus with mass number 184 initially at rest emits an α-particle. If the Q value of the reaction is 5.5 MeV, calculate the kinetic energy of the α-particle.

(1) 5.5 MeV
(2) 5.0 MeV
(3) 5.38 MeV
(4) 0.12 MeV

Answer (3)

Sol. $k_\alpha + k_N = 5.5$

$$k = \frac{p^2}{2\ \text{m}}$$

$$\Rightarrow \frac{k_\alpha}{k_N} = \frac{180}{4} = 45$$

$$\Rightarrow k_\alpha = \frac{45}{46}\times5.5\ \text{MeV}$$

$= 5.38$ MeV

11. A bullet of '4 g' mass is fired from a gun of mass 4 kg. If the bullet moves with the muzzle speed of 50 ms^{-1}, the impulse imparted to the gun and velocity of recoil of gun are:

(1) 0.2 kg ms^{-1}, 0.1 ms^{-1}
(2) 0.4 kg ms^{-1}, 0.05 ms^{-1}
(3) 0.2 kg ms^{-1}, 0.05 ms^{-1}
(4) 0.4 kg ms^{-1}, 0.1 ms^{-1}

Answer (3)

Sol. $m_{\text{Bullet}} = 4$ g, $M_{\text{Gun}} = 4$ kg

$v_{\text{Bullet}} \simeq 50$ m/s

Now $P_B = P_g$

$P_g = m \times v_{\text{Bullet}}$

$$= \frac{4}{1000}\times50$$

$= 0.2$ kg m/s

So impulse = 0.2 kg m/s

$$v_G = \frac{0.2}{M_{\text{Gun}}} = \frac{0.2}{4} = 0.05\ \text{m/s}$$

12. **Statement I:** The ferromagnetic property depends on temperature. At high temperature, ferromagnet becomes paramagnet.

Statement II: At high temperature, the domain wall area of a ferromagnetic substance increases.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) **Statement I** is false but **statement II** is true
(2) **Statement I** is true but **statement II** is false
(3) Both **Statement I** and **statement II** are False
(4) Both **Statement I** and **statement II** are true

Answer (2)

Sol. With increase in temperature domain volume decreases.

Statement 1 true

Statement 2 false

13. Match List-I with List-II

List-I	**List-II**
(a) $\omega L > \frac{1}{\omega C}$	(i) Current is in phase with emf
(b) $\omega L = \frac{1}{\omega C}$	(ii) Current lags behind the applied emf
(c) $\omega L < \frac{1}{\omega C}$	(iii) Maximum current occurs
(d) Resonant frequency	(iv) Current leads the emf

Choose the **correct** answer from the options given below

(1) a(ii), b(i), c(iv), d(iii)
(2) a(ii), b(i), c(iii), d(iv)
(3) a(iii), b(i), c(iv), d(ii)
(4) a(iv), b(iii), c(ii), d(i)

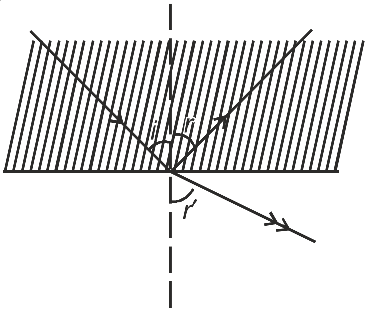
Answer (1)

Sol. $\omega L = \frac{1}{\omega C}, X_L = X_C$

So current in phase with EMF

At resonance, current have maximum value.

14. A ray of light passes from a denser medium to a rarer medium at an angle of incidence *i*. The reflected and refracted rays make an angle of 90° with each other. The angle of reflection and refraction are respectively *r* and *r*′. The critical angle is given by

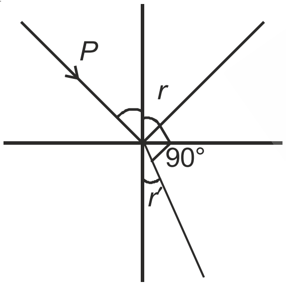


(1) $\sin^{-1}(\tan r)$ (2) $\sin^{-1}(\cot r)$

(3) $\sin^{-1}(\tan r')$ (4) $\tan^{-1}(\sin i)$

Answer (1)

Sol.



$n \sin\theta_c = 1$

$\sin\theta_c = \frac{1}{n}$

$i = r$

$r' + r + 90° = 180°$

$r' = 90° - r = 90° - i$

$\cos r' = \cos(90° - i)$

$\cos r' = \sin i$ ...(i)

$n \sin i = \sin r'$ ...(ii)

$n = \tan r'$

$\frac{1}{\sin\theta_c} = \tan r'$

$\sin\theta_c = \cot r'$

$\sin\theta_c = \tan r$

$\theta_c = \sin^{-1}(\tan r)$

15. An electron of mass m_e and a proton of mass m_p are accelerated through the same potential difference. The ratio of the de-Broglie wavelength associated with the electron to that with the proton is

(1) $\frac{m_e}{m_p}$

(2) 1

(3) $\frac{m_p}{m_e}$

(4) $\sqrt{\frac{m_p}{m_e}}$

Answer (4)

Sol. $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2km}}$

$\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{k_p m_p}{k_e m_e}} = \sqrt{\frac{m_p}{m_e}}$

16. What will be the average value of energy for a monoatomic gas in thermal equilibrium at temperature T?

(1) $\frac{3}{2}k_B T$ (2) $k_B T$

(3) $\frac{2}{3}k_B T$ (4) $\frac{1}{2}k_B T$

Answer (1)

Sol. $E = \frac{3}{2}k_B T$

17. T_0 is the time period of a simple pendulum at a place. If the length of the pendulum is reduced to $\frac{1}{16}$ times of its initial value, the modified time period is :

(1) $4\ T_0$ (2) $\frac{1}{4}T_0$

(3) T_0 (4) $8\pi\ T_0$

Answer (2)

Sol. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$T' = \frac{T_0}{4}$

18. What should be the height of transmitting antenna and the population covered if the television telecast is to cover a radius of 150 km? The average population density around the tower is 2000/km^2 and the value of $R_e = 6.5 \times 10^6$ m.

(1) Height = 1241 m

Population Covered = 7×10^5

(2) Height = 1731 m

Population Covered = 1413×10^5

(3) Height = 1800 m

Population Covered = 1413×10^8

(4) Height = 1600 m

Population Covered = 2×10^5

Answer (2)

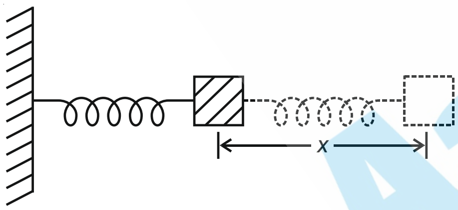
Sol. $d = \sqrt{2R_e h}$

Area covered = πd^2

$$h = \frac{d^2}{2R_e} = 1731\text{ m}$$

Population covered = $2000 \times \pi(d^2)$

19. The motion of a mass on a spring, with spring constant K is as shown in figure.



The equation of motion is given by $x(t) = A\sin\omega t + B\cos\omega t$ with $\omega = \sqrt{\frac{K}{m}}$

Suppose that at time $t = 0$, the position of mass is $x(0)$ and velocity $v(0)$, then its displacement can also be represented as $x(t) = C\cos(\omega t - \phi)$, where C and ϕ are:

(1) $C = \sqrt{\frac{2v(0)^2}{\omega^2} + x(0)^2}, \phi = \tan^{-1}\left(\frac{x(0)\omega}{2v(0)}\right)$

(2) $C = \sqrt{\frac{v(0)^2}{\omega^2} + x(0)^2}, \phi = \tan^{-1}\left(\frac{x(0)\omega}{v(0)}\right)$

(3) $C = \sqrt{\frac{v(0)^2}{\omega^2} + x(0)^2}, \phi = \tan^{-1}\left(\frac{v(0)}{x(0)\omega}\right)$

(4) $C = \sqrt{\frac{2v(0)^2}{\omega^2} + x(0)^2}, \phi = \tan^{-1}\left(\frac{v(0)}{x(0)\omega}\right)$

Answer (3)

Sol. $C\cos\phi = x(0)$

$tC\omega\sin\phi = v(0)$

$$\left[\frac{v(0)}{\omega}\right]^2 + [x(0)]^2 = C^2$$

$$\tan\phi = \frac{v(0)}{x(0)\omega}$$

20. Consider a situation in which reverse biased current of a particular P-N junction increases when it is exposed to a light of wavelength ≤ 621 nm. During this process, enhancement in carrier concentration takes place due to generation of hole-electron pairs. The value of band gap is nearly.

(1) 1 eV

(2) 4 eV

(3) 0.5 eV

(4) 2 eV

Answer (4)

Sol. Band gap = $\frac{hc}{\lambda} = 2$ eV

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

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1. A ray of light passing through a prism $(\mu = \sqrt{3})$ suffers minimum deviation. It is found that the angle of incidence is double the angle of refraction within the prism. Then, the angle of prism is ________ (in degrees).

Answer (60)

Sol. For minimum deviation $r_1 = r_2 = A/2$

given $i = 2r$

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin 2r}{\sin r}$$

$$\Rightarrow \cos r = \frac{\mu}{2}$$

$\Rightarrow r = 30°$

$\Rightarrow A = 60°$

2. Three students S_1, S_2 and S_3 perform an experiment for determining the acceleration due to gravity (g) using a simple pendulum. They use different lengths of pendulum and record time for different number of oscillations. The observations are as shown in the table.

Student No.	Length of Pendulum (cm)	No. of oscillations (n)	Total time for n oscillations	Time period (s)
1	64.0	8	128.0	16.0
2	64.0	4	64.0	16.0
3	20.0	4	36.0	9.0

(Least count of length = 0.1 cm

Least count for time = 0.1 s)

If E_1, E_2 and E_3 are the percentage errors in '*g*' for students 1, 2 and 3 respectively, then the minimum percentage error is obtained by student no. ______.

Answer (1)

Sol. $$T = \frac{t}{n} = 2\pi\sqrt{\frac{l}{g}}$$

$$\Rightarrow \quad g = \frac{4\pi^2 l}{T^2}$$

$$\Rightarrow \quad \frac{\Delta g}{g}\times 100 = \frac{\Delta l}{l}\times 100 + 2\frac{\Delta T}{T}\times 100$$

$$= \left(\frac{\Delta l}{l} + \frac{2\Delta t}{nT}\right)100\%$$

$$E_1 = \frac{20}{64}\%$$

$$E_2 = \frac{30}{64}\%$$

$$E_3 = \frac{19}{18}\%$$

3. Three particles P, Q and R are moving along the vectors $\vec{A} = \hat{i} + \hat{j}, \vec{B} = \hat{j} + \hat{k}$ and $\vec{C} = -\hat{i} + \hat{j}$ respectively. They strike on a point and start to move in different directions. Now particle P is moving normal to the plane which contains vector $\vec{A}$ and $\vec{B}$. Similarly particle Q is moving normal to the plane which contains vector $\vec{A}$ and $\vec{C}$. The angle between the direction of motion of P and Q is $\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)$. Then the value of *x* is _____.

Answer (3)

Sol. $$\hat{n}_1 = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|} = \frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$$

$$\hat{n}_2 = \frac{\vec{A}\times\vec{C}}{|\vec{A}\times\vec{C}|} = \hat{k}$$

$$\cos\theta = \hat{n}_1 \cdot \hat{n}_2 = \frac{1}{\sqrt{3}}$$

4. In 5 minutes, a body cools from 75°C to 65°C at room temperature of 25°C. The temperature of body at the end of next 5 minutes is ___ °C.

Answer (57)

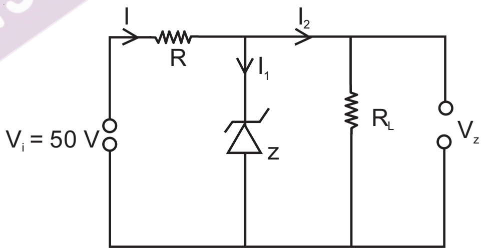
Sol. $$\frac{75-65}{5} = k\left(\frac{75+65}{2} - 25\right)$$

$$\Rightarrow \quad k = \frac{2}{45}$$

$$\frac{65-T}{5} = k\left(\frac{65+T}{2} - 25\right)$$

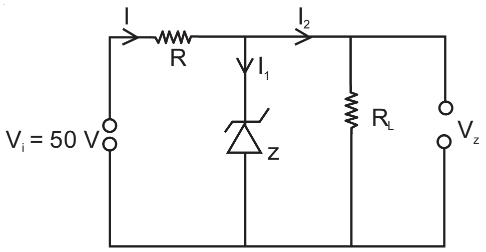
$$\Rightarrow \quad T = 57°C$$

5. In a given circuit diagram, a 5 V zener diode along with a series resistance is connected across a 50 V power supply. The minimum value of the resistance required, if the maximum zener current is 90 mA will be _____ Ω.



Answer (500)

Sol. $$I = \frac{50 - V_z}{R} = \frac{5}{R_L} + 90\times 10^{-3}$$



For $R_L \to \infty$

$R = 500\ \Omega$

6. The total charge enclosed in an incremental volume of 2×10^{-9} m^3 located at the origin is _____ nC, if electric flux density of its field is found as

$$D = e^{-x}\sin y\,\hat{i} - e^{-x}\cos y\,\hat{j} + 2z\,\hat{k}\ \text{C/m}^2$$

Answer (4)

Sol. $\bar{D} = \varepsilon_0 \bar{E}$

$$\text{Div. } \bar{E} = \frac{\rho}{\varepsilon_0}$$

$$\Rightarrow \text{div.} \bar{D} = \rho$$

$$\Rightarrow \frac{\partial}{\partial x}\left(e^{-x}\sin y\right) + \frac{\partial}{\partial y}\left(-e^{-x}\cos y\right) + \frac{\partial}{\partial z}(2z) = \rho$$

$$\Rightarrow \rho = 2 \text{ (a constant)}$$

$$V = 2 \times 10^{-9}\,\text{m}^3$$

$$q = 2 \times 2 \times 10^{-9} = 4\text{nC}$$

7. The position of the centre of mass of a uniform semi-circular wire of radius 'R' placed in x-y plane with its centre at the origin and the line joining its ends as x-axis is given by $\left(0, \frac{xR}{\pi}\right)$. Then, the value of $|x|$ is _______.

Answer (2)

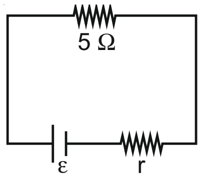
Sol. Centre of mass of half ring is located at a distance $\frac{2R}{\pi}$ from centre of the ring on its axis of symmetry so position of centre of mass in the given question will be $\left(0, \frac{2R}{\pi}\right)$

$$\Rightarrow |x| = 2$$

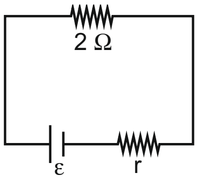
8. In an electric circuit, a cell of certain emf provides a potential difference of 1.25 V across a load resistance of 5 Ω. However, it provides a potential difference of 1 V across a load resistance of 2 Ω. The emf of the cell is given by $\frac{x}{10}$ V. Then the value of x is ______.

Answer (15)

Sol.



(a)



(b)

In case (a) $\varepsilon = \frac{1.25}{5}(5 + r)$

$\Rightarrow 4\varepsilon = 5 + r$ ...(1)

In case (b), $\varepsilon = \frac{1}{2}(2 + r)$

$\Rightarrow 2\varepsilon = 2 + r$ ...(2)

From equation (1) & (2)

$2\varepsilon = 3 \Rightarrow \varepsilon = 1.5$

or $x = 15$

9. The area of cross-section of a railway track is 0.01 m^2. The temperature variation is 10°C. Coefficient of linear expansion of material of track is 10^{-5}/°C. The energy stored per meter in the track is ______ J/m.

(Young's modulus of material of track is 10^{11} Nm^{-2})

Answer (05)

Sol. As the tracks won't be allowed to expand linearly, the rise in temperature would lead to developing thermal stress in track.

$$\frac{\text{(Stress)}}{y} = \alpha\Delta T \ \text{ or } \ \sigma = Y\alpha\Delta T$$

$$\text{Energy stored per unit volume } = \frac{1}{2}\frac{\sigma^2}{Y}$$

$$\Rightarrow \text{Energy stored per unit length } = \frac{A\sigma^2}{2Y}$$

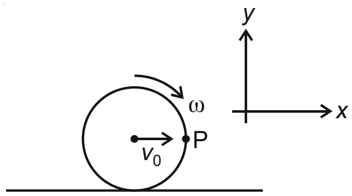
$$= \frac{A}{2} \times Y\alpha^2\Delta T^2$$

$$= \frac{10^{-2} \times 10^{11} \times 10^{-10} \times 100}{2} = 5 \text{ J/m}$$

10. The centre of a wheel rolling on a plane surface moves with a speed v_0. A particle on the rim of the wheel at the same level as the centre will be moving at a speed $\sqrt{x}v_0$. Then the value of x is ______.

Answer (02)

Sol.



$$|\omega| = \frac{v_0}{R}$$

$$\vec{v}_P = v_0\hat{i} + \omega R(-\hat{j}) = v_0\hat{i} - v_0\hat{j}$$

$$|\vec{v}_P| = \sqrt{2}v_0$$

$x = 02$

PART–B : CHEMISTRY

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Sulphide ion is soft base and its ores are common for metals.

(a) Pb (b) Al

(c) Ag (d) Mg

Choose the **correct** answer from the options given below :

(1) (a) and (d) only (2) (c) and (d) only

(3) (a) and (b) only (4) (a) and (c) only

Answer (4)

Sol. ∵ Sulphide ion is a soft base, it has more tendency to make salt with soft acids.

Pb^{2+} and Ag^{+} are soft acids, Al^{3+} and Mg^{2+} are hard acids.

2. Thiamine and pyridoxine are also known respectively as:

(1) Vitamin E and Vitamin B_2

(2) Vitamin B_2 and Vitamin E

(3) Vitamin B_1 and Vitamin B_6

(4) Vitamin B_6 and Vitamin B_2

Answer (3)

Sol. Vitamin B_1 — Thiamine

Vitamin B_6 — Pyridoxine

3. Match **List-I** and **List-II**:

List-I	**List-II**
(Elements)	**(Properties)**
(a) Ba	(i) Organic solvent soluble compounds
(b) Ca	(ii) Outer electronic configuration $6s^2$
(c) Li	(iii) Oxalate insoluble in water
(d) Na	(iv) Formation of very strong monoacidic base

Choose the **correct** answer from the options given below:

(1) (a)-(iii), (b)-(ii), (c)-(iv) and (d)-(i)

(2) (a)-(ii), (b)-(iii), (c)-(i) and (d)-(iv)

(3) (a)-(i), (b)-(iv), (c)-(ii) and (d)-(iii)

(4) (a)-(iv), (b)-(i), (c)-(ii) and (d)-(iii)

Answer (2)

Sol. Ba — $[Xe]6s^2$

Ca — Calcium oxalate is insoluble (sparingly soluble) in water

Li — LiCl is soluble in organic solvents like pyridine

Na — NaOH is a very strong monoacidic base

4. Given below are the statements about diborane.

(a) Diborane is prepared by the oxidation of $NaBH_4$ with I_2.

(b) Each boron atom is in sp^2 hybridized state.

(c) Diborane has one bridged 3 centre-2-electron bond.

(d) Diborane is a planar molecule.

The option with **correct** statement(s) is:

(1) (c) only (2) (a) and (b) only

(3) (c) and (d) only (4) (a) only

Answer (4)

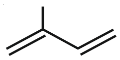
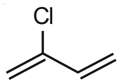
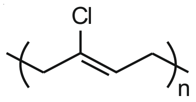
Sol. $2NaBH_4 + I_2 \longrightarrow B_2H_6 + 2NaI + H_2$

In B_2H_6, B atoms are sp^3 hybridised

B_2H_6 has two bridged 3c-2e bonds

B_2H_6 is non planar

5. Match **List-I** with **List-II**

List-I	**List-II**
(a) Chloroprene	(i) 
(b) Neoprene	(ii) 
(c) Acrylonitrile	(iii) 
(d) Isoprene	(iv) $CH_2 = CH — CN$

Choose the **correct** answer from the options given below:

(1) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

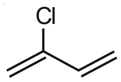
(2) (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)

(3) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

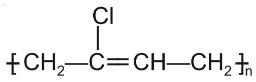
(4) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

Answer (2)

Sol. Chloroprene



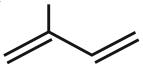
Neoprene



$$-\!\!\left[CH_2-\overset{\overset{\displaystyle Cl}{|}}{C}=CH-CH_2\right]\!\!-_n$$

Acrylonitrile $CH_2=CH-CN$

Isoprene



6. Which purification technique is used for high boiling organic liquid compound (decomposes near its boiling point)?

(1) Reduced pressure distillation

(2) Simple distillation

(3) Steam distillation

(4) Fractional distillation

Answer (1)

Sol. Reduced pressure distillation technique is used to purify the liquid which decomposes near its boiling point.

7. The water having more dissolved O_2 is

(1) Boiling water

(2) Water at 4°C

(3) Polluted water

(4) Water at 80°C

Answer (2)

Sol. Solubility of gas (O_2) in liquid decreases with the increase in temperature.

8. Match List-I with List-II

	List-I		**List-I**
	(Species)		**(Hybrid Orbitals)**
(a)	SF_4	(i)	sp^3d^2
(b)	IF_5	(ii)	d^2sp^3
(c)	NO_2^+	(iii)	sp^3d
(d)	NH_4^+	(iv)	sp^3
		(v)	sp

Choose the correct answer from the options given below :

(1) (a)-(ii), (b)-(i), (c)-(iv) and (d)-(v)

(2) (a)-(iv), (b)-(iii), (c)-(ii) and (d)-(v)

(3) (a)-(i), (b)-(ii), (c)-(v) and (d)-(iii)

(4) (a)-(iii), (b)-(i), (c)-(v) and (d)-(iv)

Answer (4)

Sol. $SF_4 - sp^3d$ hybridised

$IF_5 - sp^3d^2$ hybridised

$\overset{+}{N}O_2 - sp$ hybridised

$\overset{+}{N}H_4 - sp^3$ hybridised

9. Which one of the following statements for D.I. Mendeleev, is incorrect?

(1) He authored the textbook-Principles of Chemistry

(2) He invented accurate barometer

(3) At the time, he proposed Periodic Table of elements structure of atom was known

(4) Element with atomic number 101 is named after him

Answer (3)

Sol. At the time of D.I. Mendeleev, structure of atom was not known.

Element with atomic number 101 is known as Mendelevium.

10. Isotope(s) of hydrogen which emits low energy β^- particles with $t\frac{1}{2}$ value > 12 years is/are

(1) Deuterium

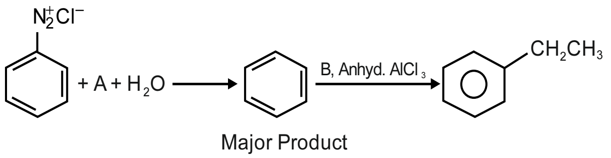
(2) Protium

(3) Tritium

(4) Deuterium and Tritium

Answer (3)

Sol. Only Tritium is the radioactive isotope of Hydrogen which emits low energy β^- particle with $t_{1/2}$ 12.33 years.

11. 

In the chemical reactions given above A and B respectively are

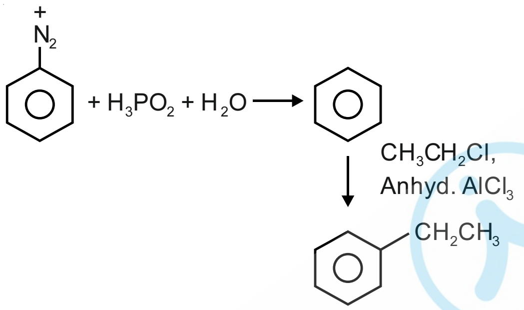
(1) CH_3CH_2Cl and H_3PO_2

(2) H_3PO_2 and CH_3CH_2OH

(3) H_3PO_2 and CH_3CH_2Cl

(4) CH_3CH_2OH and H_3PO_2

Answer (3)

Sol. 

12. The set having ions which are coloured and paramagnetic both is

(1) Cu^{2+}, Cr^{3+}, Sc^{+}

(2) Cu^{+}, Zn^{2+}, Mn^{4+}

(3) Sc^{3+}, V^{5+}, Ti^{4+}

(4) Ni^{2+}, Mn^{7+}, Hg^{2+}

Answer (1)

Sol. To show colour and paramagnetic behaviour, the ion must have unpaired electron(s)

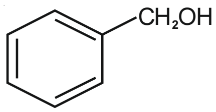
Cu^{2+} - $3d^9$ (one unpaired e^-)

Cr^{3+} - $3d^3$ (three unpaired e^-)

Sc^+ - $3d^2$ (two unpaired e^-)

13. Which one of the following compounds does not exhibit resonance?

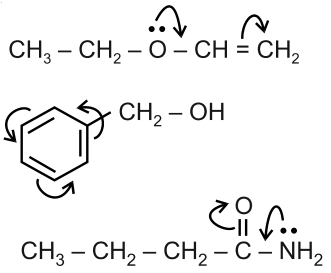
(1) $CH_3CH_2OCH = CH_2$

(2) 

(3) $CH_3CH_2CH_2CONH_2$

(4) $CH_3CH_2CH = CHCH_2NH_2$

Answer (4)

Sol. 

$CH_3 - CH_2 - CH = CH - CH_2 - NH_2$

(Resonance is not possible in this compound)

14. Which one of the following molecules does not show stereo isomerism?

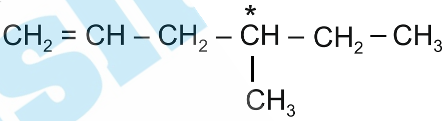
(1) 4-Methylhex-1-ene

(2) 3-Ethylhex-3-ene

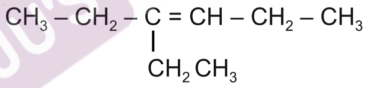
(3) 3,4-Dimethylhex-3-ene

(4) 3-Methylhex-1-ene

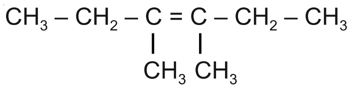
Answer (2)

Sol. (i) 
$$CH_2 = CH - CH_2 - \underset{CH_3}{\underset{|}{\overset{*}{C}H}} - CH_2 - CH_3$$

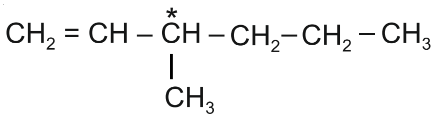
4-methylhex-1-ene has chiral centre

(ii) 
$$CH_3 - CH_2 - \underset{CH_2CH_3}{\underset{|}{C}} = CH - CH_2 - CH_3$$

3-ethylhex-3-ene cannot show geometerical isomerism

(iii) 
$$CH_3 - CH_2 - \underset{CH_3}{\underset{|}{C}} = \underset{CH_3}{\underset{|}{C}} - CH_2 - CH_3$$

3, 4–dimethylhex –3-ene can show geometerical isomerism

(iv) 
$$CH_2 = CH - \underset{CH_3}{\underset{|}{\overset{*}{C}H}} - CH_2 - CH_2 - CH_3$$

3-methylhex-1-ene has Chiral centre

15. Which one of the following 0.06 M aqueous solutions has lowest freezing point?

(1) KI

(2) $Al_2(SO_4)_3$

(3) $C_6H_{12}O_6$

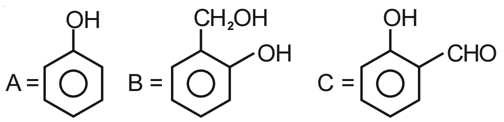
(4) K_2SO_4

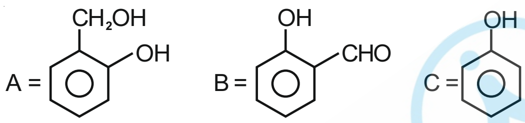
Answer (2)

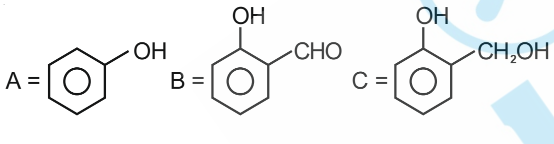
Sol. $\Delta T_F \propto i$ (for equimolar solutions)

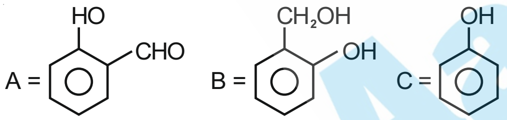
Solute	i
KI	2
$Al_2(SO_4)_3$	5
$C_6H_{12}O_6$	1
K_2SO_4	3

16. An organic compound A (C_6H_6O) gives dark green colouration with ferric chloride. On treatment with $CHCl_3$ and KOH, followed by acidification gives compound B. Compound B can also be obtained from compound C on reaction with pyridinium chlorochromate (PCC). Identify A, B and C

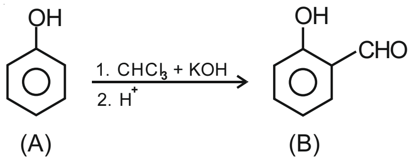
(1) 

(2) 

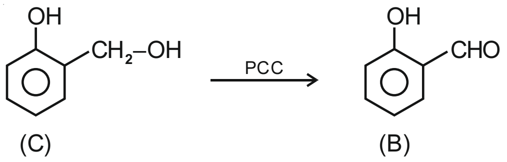
(3) 

(4) 

Answer (3)

Sol. 

(Reimer-Tiemann Reaction)

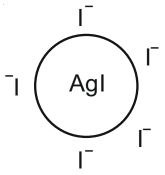


17. When silver nitrate solution is added to potassium iodide solution then the sol produced is :

(1) AgI/Ag^+
(2) $AgNO_3/NO_3^-$
(3) KI/NO_3^-
(4) AgI/I^-

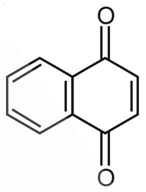
Answer (4)

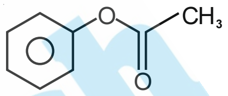
Sol. $AgNO_3 + \underset{(excess)}{KI} \rightarrow \underset{colloid}{AgI}$

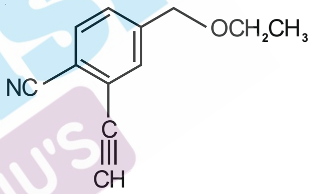


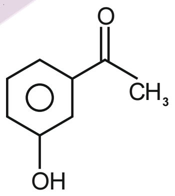
Negative colloid will be formed AgI/I^-

18. Which one of the following compounds will provide a tertiary alcohol on reaction with excess of CH_3MgBr followed by hydrolysis?

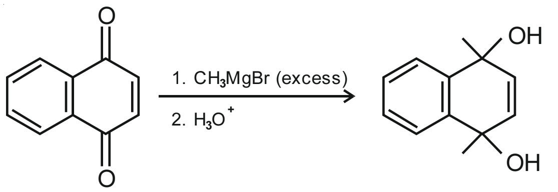
(1) 

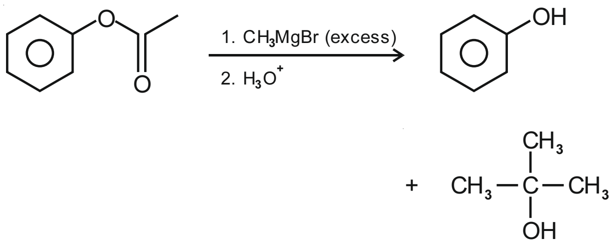
(2) 

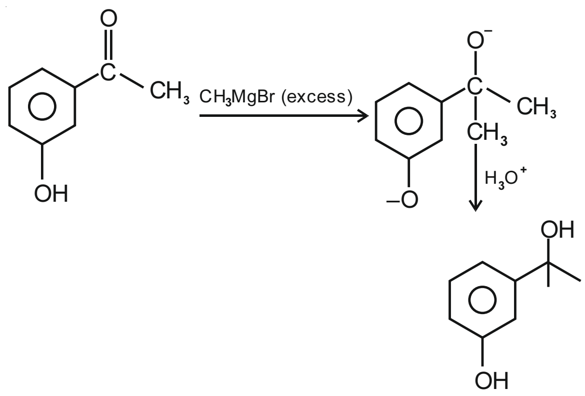
(3) 

(4) 

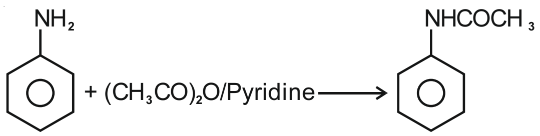
Answer (Bonus)

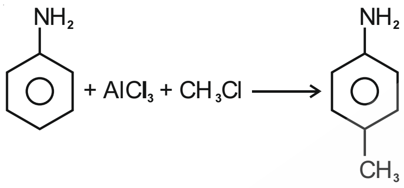
Sol. 

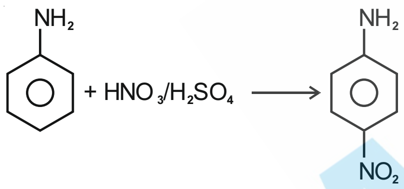


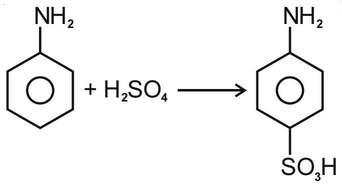


19. Which one of the following reactions does not occur?

(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. Aniline does not give Friedel craft reactions

20. Which one of the following group-15 hydride is the strongest reducing agent?

(1) AsH_3

(2) PH_3

(3) SbH_3

(4) BiH_3

Answer (4)

Sol. BiH_3 is most reducing among the group-15 hydrides.

Reducing property of the hydrides increases down the group

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

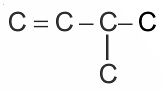
1. The number of acyclic structural isomers (including geometrical isomers) for pentene are _______.

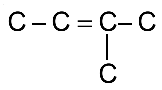
Answer (6)

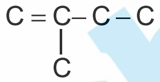
Sol. C_5H_{10}

$C = C - C - C - C$

$C - C = C - C - C$ + geometrical isomer


$C=C-\underset{\underset{C}{|}}{C}-C$


$C-C=\underset{\underset{C}{|}}{C}-C$


$C=\underset{\underset{C}{|}}{C}-C-C$

Total 6 isomers are possible

2. Number of electrons that Vanadium (Z = 23) has in p-orbitals is equal to ________.

Answer (12)

Sol. Vanadium (Z = 23)

$V = 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^3\ 4s^2$

Number of electrons in p-orbital are 12

3. Value of K_p for the equilibrium reaction $N_2O_{4(g)} \rightleftharpoons 2NO_{2(g)}$ at 288 K is 47.9. The K_C for this reaction at same temperature is _______. (Nearest integer) (R = 0.083 L bar K^{-1} mol^{-1})

Answer (2)

Sol. $K_P = K_C(RT)\Delta n_g$

$$\underset{(g)}{N_2O_4} \rightleftharpoons \underset{(g)}{2NO_2}$$

$\Delta n_g = 1$

$$K_C = \frac{Kp}{(RT)} = \frac{47.9}{(0.083\times 288)}$$

≈ 2

4. The total number of unpaired electrons present in $[Co(NH_3)_6]Cl_2$ and $[Co(NH_3)_6]Cl_3$ is ______.

Answer (1)

Sol. $[Co(NH_3)_6]\ Cl_2$

Co^{2+} • NH_3 is a strong field ligand

• Pairing will occur

$Co^{2+} = 3d^7 \; t_{2g}^{6} \quad eg^{1} \quad 5s$

e^- from the eg orbital will get excited to 5s orbital hence the hybridisation will be d^2sp^3 with one unpaired e^-

$[Co(NH_3)_6]\ Cl_3$ – d^2sp^3 hybridised with no unpaired e^-

5. $N_2O_{5(g)} \longrightarrow 2NO_{2(g)} + \frac{1}{2}O_{2(g)}$

In the above first order reaction the initial concentration of N_2O_5 is 2.40×10^{-2} mol L^{-1} at 318 K. The concentration of N_2O_5 after 1 hour was 1.60×10^{-2} mol L^{-1}. The rate constant of the reaction at 318 K is ______ × 10^{-3} min^{-1}. (Nearest integer)

[Given : log 3 = 0.477, log 5 = 0.699]

Answer (7)

Sol. For the first order reaction

$$Kt = \ln\frac{[R]_0}{[R]}$$

$$K \times 60 = \ln\frac{(2.4 \times 10^{-2})}{(1.6 \times 10^{-2})}$$

= 2.303 × (log3 – log2)

= 2.303 × (0.477 – 0.301)

$K = 6.7 \times 10^{-3}\ min^{-1}$

6. A copper complex crystallising in a CCP lattice with a cell edge of 0.4518 nm has been revealed by employing X-ray diffraction studies. The density of a copper complex is found to be 7.62 g cm^{-3}. The molar mass of copper complex is ______ g mol^{-1}. (Nearest integer) [Given : $N_A = 6.022 \times 10^{23}$ mol^{-1}]

Answer (106)

Sol. $d = \frac{Z \times M}{a \times N_A}$

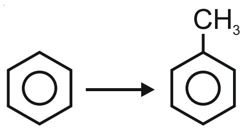
$$7.62 = \frac{4 \times M}{(0.4518 \times 10^{-7})^3 \times 6.022 \times 10^{23}}$$

M = 105.8 g/mol

7. Methylation of 10 g of benzene gave 9.2 g of toluene. Calculate the percentage yield of toluene ________. (Nearest integer)

Answer (78)

Sol.



10 g of $C_6H_6 = \frac{10}{78}$ moles

moles of methylbenzene should be obtained

$= \frac{10}{78}$ mole

$= \frac{10}{78} \times 92$ g

$$\%\ \text{yield} = \frac{9.2}{10 \times 92} \times 78 \times 100 = 78\%$$

8. If the concentration of glucose ($C_6H_{12}O_6$) in blood is 0.72 g L^{-1}, the molarity of glucose in blood is ________ × 10^{-3} M. (Nearest integer)

(Given : Atomic mass of C = 12, H = 1, O = 16 u)

Answer (4)

Sol. Concentration of glucose in blood = 0.72 g/L

$$= \frac{0.72}{180} = 4 \times 10^{-3} \text{ molar}$$

9. Assume a cell with the following reaction

$Cu_{(s)} + 2Ag^+ (1 \times 10^{-3}\ M) \rightarrow Cu^{2+} (0.250\ M) + 2\ Ag_{(s)}$

$E^{\ominus}_{cell} = 2.97$ V

E_{cell} for the above reaction is ______ V. (Nearest integer)

[Given : log 2.5 = 0.3979, T = 298 K]

Answer (3)

Sol. $Cu + 2Ag^+ \longrightarrow Cu^{2+} \quad + 2Ag$

10^{-3} M $\qquad$ 0.250 M

$$E_{cell} = E^{o}_{cell} - \frac{0.059}{2} \log\frac{[Cu^{2+}]}{[Ag^+]^2}$$

$$= 2.97 - \frac{0.059}{2} \log\frac{(0.25)}{(10^{-3})^2}$$

= 2.81 V

10. If the standard molar enthalpy change for combustion of graphite powder is -2.48×10^2 kJ mol^{-1}, the amount of heat generated on combustion of 1 g of graphite powder is _________ kJ. (Nearest integer)

Answer (21)

Sol. Heat of combustion per mol (for 12 g)

$= -2.48 \times 10^2$ KJ

for 1 g of graphite $= \frac{-2.48 \times 10^2}{12}$

= –20.66 KJ

PART–C : MATHEMATICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Let a vector $\vec{a}$ be coplanar with vectors $\vec{b}=2\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}+\hat{k}$. If $\vec{a}$ is perpendicular to $\vec{d}=3\hat{i}+2\hat{j}+6\hat{k}$, and $|\vec{a}|=\sqrt{10}$. Then a possible value of $\left[\vec{a}\vec{b}\vec{c}\right]+\left[\vec{a}\vec{b}\vec{d}\right]+\left[\vec{a}\vec{c}\vec{d}\right]$ is equal to

(1) –42 (2) –40

(3) –38 (4) –29

Answer (1)

Sol. $[\vec{b}\ \vec{c}\ \vec{d}]=\begin{vmatrix}2 & 1 & 1\\ 1 & -1 & 1\\ 3 & 2 & 6\end{vmatrix}=2(-6-2)-1(3)+1(5)$

$=-16-3+5=-14$

Let $\vec{a}=\lambda\vec{b}+\mu\vec{c}$

$\therefore\ [\vec{a}\ \vec{b}\ \vec{c}]+[\vec{a}\ \vec{b}\ \vec{d}]+[\vec{a}\ \vec{c}\ \vec{d}]$

$=-\mu[\vec{b}\ \vec{c}\ \vec{d}]+\lambda[\vec{b}\ \vec{c}\ \vec{d}]$

$=(\lambda-\mu)[\vec{b}\ \vec{c}\ \vec{d}]$

$\therefore\ \vec{a}=(2\lambda+\mu)\hat{i}+(\lambda-\mu)\hat{j}+(\lambda+\mu)\hat{k}$

$(2\lambda+\mu)^2+(\lambda-\mu)^2+(\lambda+\mu)^2=10$ (as $|\vec{a}|=\sqrt{10}$)

$\Rightarrow\ 6\lambda^2+3\mu^2+4\lambda\mu=10$ ...(i)

& $\hat{a}\cdot\hat{b}=0\Rightarrow 3(2\lambda+\mu)+2(\lambda-\mu)+6(\lambda+\mu)=0$

$14\lambda+7\mu=0\Rightarrow\mu=-2\lambda$ ...(ii)

by (i) & (ii), $6\lambda^2+12\lambda^2-8\lambda^2=10$

$\Rightarrow\ \lambda=\pm1\ \Rightarrow\ \mu=\mp2$

$(\lambda-\mu)=3$ or -3

$\therefore$ Required quantity = –42

2. Let L be the line of intersection of planes $\vec{r}\cdot(\hat{i}-\hat{j}+2\hat{k})=2$ and $\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k})=2$. If P(α, β, γ) is the foot of perpendicular on L from the point (1, 2, 0), then the value of 35(α + β + γ) is equal to

(1) 143 (2) 101

(3) 134 (4) 119

Answer (4)

Sol. $P_1\equiv x-y+2z=2$ & $P_2\equiv 2x+y-z=2$

Line of intersection

$$L\equiv\frac{x-\frac{4}{3}}{-1}=\frac{y-\frac{2}{3}}{5}=\frac{z-0}{3}=\lambda$$

general point on $L\equiv\left(-\lambda+\frac{4}{3},5\lambda+\frac{2}{3},3\lambda\right)$

for it being foot of perpendicular from (1, 2, 0)

$$\left(-\lambda+\frac{1}{3}\right)(-1)+\left(5\lambda-\frac{4}{3}\right)5+(3\lambda)3=0$$

$$\lambda-\frac{1}{3}+25\lambda-\frac{20}{3}+9\lambda=0$$

$$\Rightarrow\ 35\lambda=7\Rightarrow\lambda=\frac{7}{35}$$

$35(\alpha+\beta+\gamma)=35(7\lambda+2)\Rightarrow 70+49=119$

3. Let $E_1:\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ a>b$. Let E_2 be another ellipse such that it touches the end points of major axis of E_1 and the foci of E_2 are the end points of minor axis of E_1. If E_1 and E_2 have same eccentricities, then its value is

(1) $\frac{-1+\sqrt{6}}{2}$ (2) $\frac{-1+\sqrt{8}}{2}$

(3) $\frac{-1+\sqrt{3}}{2}$ (4) $\frac{-1+\sqrt{5}}{2}$

Answer (4)

Sol. Let $E_2=\frac{x^2}{A^2}+\frac{y^2}{B^2}=1\ (B>A)$

focus (O, Be)

$\therefore$ Be = b and A = a

$$e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{A^2}{B^2}}$$

$$\therefore\ 1-\frac{a^2e^2}{b^2}=1-\frac{b^2}{a^2}\Rightarrow e^2=\frac{b^4}{a^4}$$

$$\therefore\ e=\sqrt{1-e}\Rightarrow e^2+e-1=0$$

$$e=\frac{-1\pm\sqrt{5}}{2}$$

$$e=\frac{\sqrt{5}-1}{2}$$

4. Let n denote the number of solutions of the equation $z^2 + 3\bar{z} = 0$, where z is a complex number. Then the value of $\sum_{k=0}^{\infty} \frac{1}{n^k}$ is equal to

(1) 1

(2) 2

(3) $\frac{4}{3}$

(4) $\frac{3}{2}$

Answer (3)

Sol. $z^2 + 3\bar{z} = 0$

$x^2 - y^2 + 2ixy + 3x - 3iy = 0$

$x^2 - y^2 + 3x = 0$ & $(2x - 3)\, y = 0$

i.e. if $y = 0 \quad \Rightarrow \; x = 0$ or -3

$$\text{if } x = \frac{3}{2} \Rightarrow y^2 = \frac{9}{4} + \frac{9}{2} = \frac{27}{4} \Rightarrow y = \pm\frac{3\sqrt{3}}{2}$$

Number of solutions 4.

$$\therefore \sum_{k=0}^{\infty} \frac{1}{n^k} = 1 + \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \ldots\ldots$$

$$= \frac{1}{1 - \frac{1}{4}} = \frac{4}{3}$$

5. If the domain of the function

$$f(x) = \frac{\cos^{-1}\sqrt{x^2 - x + 1}}{\sqrt{\sin^{-1}\left(\frac{2x-1}{2}\right)}}$$ is the interval $(\alpha, \beta]$, then $\alpha + \beta$ is equal to

(1) 1

(2) $\frac{3}{2}$

(3) $\frac{1}{2}$

(4) 2

Answer (2)

Sol. $$f(x) = \frac{\cos^{-1}\left(\sqrt{x^2 - x + 1}\right)}{\sqrt{\sin^{-1}\left(\frac{2x-1}{2}\right)}}$$

$$0 < \frac{2x-1}{2} \le 1 \qquad \ldots\text{(i)}$$

$\Rightarrow \; 0 < 2x - 1 \le 2$

$\Rightarrow \; 1 < 2x \le 3$

$\Rightarrow \; \frac{1}{2} < x \le \frac{3}{2}$

and $0 \le x^2 - x + 1 \le 1 \qquad \ldots$(ii)

$x^2 - x \le 0$

$x(x - 1) \le 0$

$0 \le x \le 1$

$$\therefore \text{ domain } x \in \left(\frac{1}{2}, 1\right] = (\alpha, \beta] \Rightarrow \alpha + \beta = \frac{3}{2}$$

6. The number of solutions of $\sin^7 x + \cos^7 x = 1$, $x \in [0, 4\pi]$ is equal to :

(1) 5 (2) 7

(3) 11 (4) 9

Answer (1)

Sol. $\sin^7 x + \cos^7 x = 1$

As $\sin^7 x + \cos^7 x \le \sin^2 x + \cos^2 x$

≤ 1

The equation gives solution only when one of $\sin x$, $\cos x$ is unity and other vanishes

i.e., $x = 0, \frac{\pi}{2}, 2\pi, \frac{5\pi}{2}, 4\pi$

7. Let the circle S : $36x^2 + 36y^2 - 108x + 120y + c = 0$ be such that it neither intersects nor touches the co-ordinate axes. If the point of intersection of the lines, $x - 2y = 4$ and $2x - y = 5$ lies inside the circle S, then :

(1) $\frac{25}{9} < c < \frac{13}{3}$ (2) $81 < c < 156$

(3) $100 < c < 156$ (4) $100 < c < 165$

Answer (3)

Sol. Intersection point of $x - 2y = 4$ and $2x - y = 5$ is (2, –1)

$\therefore \; 36(4) + 36(1) - 108(2) + 120(-1) + C < 0 \qquad \ldots$(i)

$$\text{and } \left(\frac{108}{72}\right)^2 + \left(\frac{-120}{72}\right)^2 - \frac{C}{36} < \frac{3}{2} \qquad \ldots\text{(ii)}$$

(Neither touches any axis)

$\therefore$ by (i) $\; C < 156$

$$\text{and by (ii) } \frac{9}{4} + \frac{25}{9} - \frac{C}{36} < \frac{9}{4}$$

$\Rightarrow \; 100 < C$

8. If $\int_0^{100\pi} \frac{\sin^2 x}{e^{\left(\frac{x}{\pi}-\left[\frac{x}{\pi}\right]\right)}} dx = \frac{\alpha\pi^3}{1+4\pi^2}, \alpha \in \mathbf{R},$

where [*x*] is the greatest integer less than or equal to *x*, then the value of α is :

(1) 50(e – 1) (2) 100(1 – e)

(3) $150(e^{-1} - 1)$ (4) $200(1 - e^{-1})$

Answer (4)

Sol. $I = \int_0^{100\pi} \frac{\sin^2 x}{e^{\left(\frac{x}{\pi}-\left[\frac{x}{\pi}\right]\right)}} dx$

$\because$ Integrand is periodic with period 1

$\therefore \quad I = 100 \int_0^{\pi} \frac{\sin^2 x}{e^{\left\{\frac{x}{\pi}\right\}}} dx$

Let $\frac{x}{\pi} = t \Rightarrow dx = \pi dt$

$= 100\pi \int_0^1 \frac{\sin^2(\pi t)\,dt}{e^t}$

$= 50\pi \int_0^1 e^{-t}(1-\cos 2\pi t)\,dt$

$= 50\pi \int_0^1 e^{-t} dt - 50\pi \int_0^1 e^{-t}\cos(2\pi t)\,dt$

$= -\,50\pi \left[e^{-t}\right]_0^1$

$-50\pi\left[\frac{e^{-t}}{1+4\pi^2}(-\cos 2\pi t + 2\pi \sin 2\pi t)\right]_0^1$

$= -\,50\pi\,(e^{-1} - 1) - \frac{50\pi}{1+4\pi^2}(e^{-1}(-1+0) - (-1+0))$

$= -\,50\pi\,(e^{-1} - 1) - \frac{50\pi}{1+4\pi^2}\left(1-e^{-1}\right)$

$= 50\pi\,(1 - e^{-1}) - \frac{50\pi\left(1-e^{-1}\right)}{1+4\pi^2}$

$= \frac{200\pi^3\left(1-e^{-1}\right)}{1+4\pi^2} = \frac{\alpha\pi^3}{1+4\pi^3}$ (Given)

$\therefore \quad \alpha = 200\,(1 - e^{-1})$

9. Let a line L : 2*x* + *y* = k, k > 0 be a tangent to the hyperbola $x^2 - y^2 = 3$. If L is also a tangent to the parabola $y^2 = \alpha x$, the α is equal to :

(1) –24 (2) 24

(3) 12 (4) –12

Answer (1)

Sol. $\because$ Line L: 2x + y = k, k > 0

$\Rightarrow$ L: y = –2x + k is tangent to $\frac{x^2}{3} - \frac{y^2}{3} = 1$

$\therefore \quad k^2 = 3 \cdot 4 - 3 = 9$

$\because$ L = 0 is also tangent to $y^2 = \alpha x$

$\therefore \quad k = \frac{\alpha/4}{-2}$

$\therefore \quad \alpha = -8k$

α = –24

10. Let *f* : **R** → **R** be defined as

$$f(x) = \begin{cases} -\frac{4}{3}x^3 + 2x^2 + 3x, & x > 0 \\ 3xe^x, & x \le 0 \end{cases}$$

Then *f* is increasing function in the interval.

(1) (–3, –1) (2) (0, 2)

(3) $\left(-1, \frac{3}{2}\right)$ (4) $\left(-\frac{1}{2}, 2\right)$

Answer (3)

Sol. $f(x) = \begin{cases} -\frac{4}{3}x^3 + 2x^2 + 3x, & x > 0 \\ 3xe^x, & x \le 0 \end{cases}$

$$f'(x) = \begin{cases} -4x^2 + 4x + 3, & x > 0 \\ 3e^x(x+1), & x \le 0 \end{cases}$$

Here f(x) is differentiable at x = 0

$\therefore \quad f'(x) = \begin{cases} 4 - (2x-1)^2, & x > 0 \\ 3e^x(x+1), & x \le 0 \end{cases}$

Here f′(x) > 0 when $x \in \left(-1, \frac{3}{2}\right)$

$\therefore$ f(x) is increasing in $\left(-1, \frac{3}{2}\right)$

11. Let *f* : **R** → **R** be defined as

$$f(x) = \begin{cases} \frac{x^3}{(1-\cos 2x)^2}\log_e\left(\frac{1+2xe^{-2x}}{\left(1-xe^{-x}\right)^2}\right), & x \ne 0 \\ \alpha, & x = 0 \end{cases}$$

If *f* is continuous at *x* = 0, then α is equal to

(1) 0 (2) 1

(3) 2 (4) 3

Answer (2)

Sol. ∴ f(x) is continuous at x = 0

$$\therefore \quad \alpha = \lim_{x\to 0}\frac{x^3}{(1-\cos 2x)^2}\log_e\left(\frac{1+2xe^{-2x}}{(1-xe^{-x})^2}\right)$$

$$\alpha = \lim_{x\to 0}\frac{x^4}{4\sin^4 x}\cdot\frac{1}{x}\log_e\left(\frac{e^{2x}+2x}{x^2-2xe^x+e^{2x}}\right)$$

$$=\frac{1}{4}\lim_{x\to 0}\left\{\frac{\ln(e^{2x}+2x)}{x}-\frac{\ln(x^2-2xe^x+e^{2x})}{x}\right\}$$

$$=\frac{1}{4}\lim_{x\to 0}\left\{\frac{2e^{2x}+2}{e^{2x}+2x}-\frac{2x-2e^x(x+1)+2e^{2x}}{x^2-2xe^x+e^{2x}}\right\}$$

$$=\frac{1}{4}(4-0)$$

= 1

12. Let [*x*] denote the greatest integer less than or equal to *x*. Then, the values of $x \in \mathbf{R}$ satisfying the equation $[e^x]^2 + [e^x + 1] - 3 = 0$ lie in the interval.

(1) [0, 1/e)

(2) [1, e)

(3) $[0, \log_e 2)$

(4) $[\log_e 2, \log_e 3)$

Answer (3)

Sol. ∵ $[e^x]^2 + [e^x+1] - 3 = 0$

⇒ $[e^x]^2 + [e^x] - 2 = 0$

⇒ $([e^x] + 2)([e^x] - 1) = 0$

$[e^x] = -2$ not possible

and $[e^x] = 1$

∴ $e^x \in [1, 2)$

∴ $x \in [0, \ln 2)$

13. Let three vectors $\vec a$, $\vec b$ and $\vec c$ be such that $\vec a \times \vec b = \vec c$, $\vec b \times \vec c = \vec a$ and $|\vec a| = 2$. Then which one of the following is **not** true?

(1) $\vec a \times \left(\left(\vec b + \vec c\right)\times\left(\vec b - \vec c\right)\right) = \vec 0$

(2) Projection of $\vec a$ on $\left(\vec b \times \vec c\right)$ is 2

(3) $\left[\vec a\ \vec b\ \vec c\right] + \left[\vec c\ \vec a\ \vec b\right] = 8$

(4) $\left|3\vec a + \vec b - 2\vec c\right|^2 = 51$

Answer (4)

Sol. ∵ $\vec a \times \vec b = \vec c \Rightarrow \left(\vec a \times \vec b\right)\cdot\vec c = \vec c \cdot \vec c$

∴ $[\vec a\ \vec b\ \vec c] = |\vec c|^2$ ...(i)

and $\left(\vec b \times \vec c\right) = \vec a \Rightarrow \vec a \cdot (\vec b \times \vec c) = \vec a \cdot \vec a$

∴ $[\vec a\ \vec b\ \vec c] = |\vec a|^2 = 4$ ...(ii)

∴ $|\vec a| = |\vec c| = 2$

Option: (1)

$$\vec a \times \left(\left(\vec b + \vec c\right)\times\left(\vec b - \vec c\right)\right) = \vec a \times \left(-\vec b \times \vec c + \vec c \times \vec b\right)$$

$$= 2\vec a \times (\vec c \times \vec b) = 2\vec a \times (\vec a) = 0$$

Option: (2)

$$\text{Projection of } \vec a \text{ on } (\vec b \times \vec c) = \frac{\vec a \cdot (\vec b \times \vec c)}{\left|\vec b \times \vec c\right|} = \frac{|\vec a|^2}{|\vec a|} = 2$$

Option: (3)

$$[\vec a\ \vec b\ \vec c] + [\vec c\ \vec a\ \vec b] = 2[\vec a\ \vec b\ \vec c] = 8$$

Option: (4)

$$\left|3\vec a + \vec b - 2\vec c\right|^2$$

$$= 9\vec a^2 + \vec b^2 + 4\vec c^2 + 6\vec a\cdot\vec b - 4\vec b\cdot\vec c - 12\vec a\cdot\vec c$$

$$= 9.2^2 + 1^2 + 4.2^2 + 0$$

= 53

$$\because\ \vec a\cdot\vec b = \vec b\cdot\vec c = \vec c\cdot\vec a = 0$$

$$\text{and } [\vec a\ \vec b\ \vec c]^2 = \begin{vmatrix} \vec a\cdot\vec a & \vec a\cdot\vec b & \vec a\cdot\vec c \\ \vec b\cdot\vec a & \vec b\cdot\vec b & \vec b\cdot\vec c \\ \vec c\cdot\vec a & \vec c\cdot\vec b & \vec c\cdot\vec c \end{vmatrix}$$

$$\therefore\ 16 = |\vec a|^2|\vec b|^2|\vec c|^2$$

$$\therefore\ |\vec b| = 1$$

14. Four dice are thrown simultaneously and the numbers shown on these dice are recorded in 2 × 2 matrices. The probability that such formed matrices have all different entries and are non-singular, is

(1) $\frac{43}{162}$

(2) $\frac{45}{162}$

(3) $\frac{22}{81}$

(4) $\frac{23}{81}$

Answer (1)

Sol. Total matrices = 6^4

Number of matrices with distinct entries

= 6 × 5 × 4 × 3 = 360

Number of singular matrices with distinct entries

(i.e. 1, 2, 3, 6 or 2, 3, 4, 6) = 8 + 8 = 16

Favourable cases = 360 – 16 = 344

Required probability $=\dfrac{344}{6^4}=\dfrac{43}{162}$

15. Let S_n denote the sum of first n-terms of an arithmetic progression. If S_{10} = 530, S_5 = 140, then $S_{20} - S_6$ is equal to

(1) 1842 (2) 1852

(3) 1862 (4) 1872

Answer (3)

Sol. Let first term of A.P. be a and common difference is d.

$\therefore \quad S_{10}=\dfrac{10}{2}\{2a+9d\}=530$

$\therefore$ 2a + 9d = 106 ...(i)

$S_5=\dfrac{5}{2}\{2a+4d\}=140$

a + 2d = 28 ...(ii)

from equation (i) and (ii), a = 8, d = 10

$\therefore \quad S_{20}-S_6=\dfrac{20}{2}\{2\times8+19\times10\}-\dfrac{6}{2}\{2\times8+5\times10\}$

= 2060 – 198

= 1862

16. If the shortest distance between the straight lines 3(*x* – 1) = 6(*y* – 2) = 2(*z* – 1) and 4(*x* – 2) = 2(*y* – λ) = (*z* – 3), λ∈R is $\dfrac{1}{\sqrt{38}}$, then the integral value of λ is equal to

(1) 2 (2) 5

(3) 3 (4) –1

Answer (3)

Sol. The given equation lines are

3(x – 1) = 6(y –2) = 2(z – 1)

$\Rightarrow \quad \dfrac{x-1}{2}=\dfrac{y-2}{1}=\dfrac{z-1}{3}$

$\therefore \quad \vec{y}=(\hat{i}+2\hat{j}+\hat{k})+t(2\hat{i}+\hat{j}+3\hat{k}) \quad ...(i)$

and 4(x – 2) = 2(y – λ) = (z – 3)

$\Rightarrow \quad \dfrac{x-2}{1}=\dfrac{y-\lambda}{2}=\dfrac{z-3}{4}$

$\therefore \quad \vec{y}=(2\hat{i}+\lambda\hat{j}+3\hat{k})+s(\hat{i}+2\hat{j}+4\hat{k}) \ ...(ii)$

$\therefore \quad \vec{a}_1=\hat{i}+2\hat{j}+\hat{k},\ \vec{a}_2=2\hat{i}+\lambda\hat{j}+3\hat{k}$

$\therefore \quad \vec{a}_1-\vec{a}_2=-\hat{i}+(2-\lambda)\hat{j}+2\hat{k}$

$\vec{b}_1\times\vec{b}_2=(2\hat{i}+\hat{j}+3\hat{k})\times(\hat{i}+2\hat{j}+4\hat{k})$

$=-2\hat{i}-5\hat{j}+3\hat{k}$

$\therefore \quad \text{S.D.}=\left|\dfrac{(\vec{a}_1-\vec{a}_2)\cdot\vec{b}_1\times\vec{b}_2}{|\vec{b}_1\times\vec{b}_2|}\right|=\dfrac{1}{\sqrt{38}}$

$\therefore \quad |5\lambda-14|=1$

$\therefore \quad \lambda=3$

17. Let A = $[a_{ij}]$ be a real matrix of order 3 × 3, such that $a_{i1}+a_{i2}+a_{i3}=1$, for *i* = 1, 2, 3. Then, the sum of all the entries of the matrix A^3 is equal to

(1) 1 (2) 3

(3) 2 (4) 9

Answer (2)

Sol. Let a matrix B = $\begin{bmatrix}1\\1\\1\end{bmatrix}$

$\because$ A · B = B

$\because$ Sum of all entries of A^3 is equal to the only element of $B^T \cdot A^3 \cdot B$

$\because$ $B^T\cdot A^3\cdot B = B^T\cdot A^2\cdot(AB) = B^T\cdot A^2\cdot B = B^T\cdot B$

$= B^T\cdot B = [3]_{1\times1}$

18. Which of the following Boolean expressions is **not** a tautology?

(1) (~ p ⇒ q) ∨ (~ q ⇒ p)

(2) (p ⇒ q) ∨ (~ q ⇒ p)

(3) (q ⇒ P) ∨ (~ q ⇒ p)

(4) (p ⇒ ~ q) ∨ (~ q ⇒ p)

Answer (1)

Sol. 1. $(\sim p\rightarrow q)\vee(\sim q\rightarrow p)\equiv(p\vee q)\vee(q\vee p)$

$\equiv p\vee q \not\equiv T$

2. $(\sim p\vee q)\vee(q\vee p)\equiv\sim(p\wedge\sim q)\vee(p\vee q)\equiv T$

3. $(\sim q\vee p)\vee(q\vee p)\equiv\sim(\sim p\wedge q)\vee(p\vee q)\equiv T$

4. $(\sim p\vee\sim q)\vee(q\vee p)\equiv\sim(p\wedge q)\vee(p\vee q)\equiv T$

19. Let $y = y(x)$ be the solution of the differential equation $\text{cosec}^2 x\, dy + 2dx = (1 + y\cos 2x)\,\text{cosec}^2 x\, dx$, with $y\left(\frac{\pi}{4}\right) = 0$. Then, the value of $(y(0) + 1)^2$ is equal to

(1) e

(2) $e^{1/2}$

(3) e^{-1}

(4) $e^{-1/2}$

Answer (3)

Sol. $\text{cosec}^2x\, dy = (\text{cosec}^2x - 2)\, dx + (\cos 2x\, \text{cosec}^2x)y\, dx$

$$\frac{dy}{dx} = \left(1 - 2\sin^2 x\right) + \cos 2x \cdot y$$

$$\frac{dy}{dx} - \cos 2x \cdot y = \cos 2x$$

If $e^{-\int \cos 2x\, dx} = e^{\frac{-\sin 2x}{2}}$

$$y \cdot e^{\frac{-\sin 2x}{2}} = \int \cos 2x \cdot e^{\frac{-\sin 2x}{2}} dx$$

$$\Rightarrow \quad y \cdot e^{\frac{-\sin 2x}{2}} = -e^{\frac{-\sin 2x}{2}} + c \Rightarrow y = -1 + ce^{\frac{\sin 2x}{2}}$$

$$y\left(\frac{\pi}{4}\right) = 0 \Rightarrow c = e^{\frac{-1}{2}}$$

$$\Rightarrow \quad y = -1 + e^{\frac{-1}{2}(1 - \sin 2x)}$$

$$\Rightarrow \quad y(0) = -1 + e^{\frac{-1}{2}}$$

$\Rightarrow$ $(y(0) + 1)^2 = e^{-1}$

20. The values of λ and μ such that the system of equations $x + y + z = 6$, $3x + 5y + 5z = 26$, $x + 2y + \lambda z = \mu$ has no solution, are

(1) $\lambda \neq 2$, $\mu = 10$

(2) $\lambda = 3$, $\mu \neq 10$

(3) $\lambda = 3$, $\mu = 5$

(4) $\lambda = 2$, $\mu \neq 10$

Answer (4)

Sol. $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 5 & 5 \\ 1 & 2 & \lambda \end{vmatrix} = 0$

$\Rightarrow \lambda = 2$

For $\mu = 10$, Δ_1, Δ_2, Δ_3 = 0 which corresponds to the case of infinite solutions

$\therefore \mu \neq 10$

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

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1. Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Then the number of 3 × 3 matrices B with entries from the set {1, 2, 3, 4, 5} and satisfying AB = BA is ________.

Answer (3125)

Sol. Let $B = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{bmatrix}$

$$AB = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{bmatrix} = \begin{bmatrix} \beta_1 & \beta_2 & \beta_3 \\ \alpha_1 & \alpha_2 & \alpha_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{bmatrix}$$

$$BA = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \alpha_2 & \alpha_1 & \alpha_3 \\ \beta_2 & \beta_1 & \beta_3 \\ \gamma_2 & \gamma_1 & \gamma_3 \end{bmatrix}$$

AB = BA $\Rightarrow \beta_1 = \alpha_2$, $\beta_2 = \alpha_1$, $\beta_3 = \alpha_3$, $\gamma_1 = \gamma_2$

5 places can be filled independently in 5^5 = 3125 ways = 3125 matrices

2. The sum of all the elements in the set $\{n \in \{1, 2, \ldots\ldots, 100\} \mid \text{H.C.F. of } n \text{ and } 2040 \text{ is } 1\}$ is equal to ________.

Answer (1251)

Sol. $\because$ 2040 = 2^3.3.5.17

Let A = Sum of all numbers which are divisible by 2 upto 100

B = Sum of all numbers which are divisible by 3 upto 100

C = Sum of all numbers which are divisible by 5 upto 100

D = Sum of all numbers which are divisible by 17 upto 100

$$A\cup B\cup C\cup D = (A + B + C + D) - (A\cap B + A\cap C + A\cap D + B\cap C + B\cap D + C\cap D) + (A\cap B\cap C + A\cap B\cap D + A\cap C\cap D + B\cap C\cap D) - (A\cap B\cap C\cap D)$$

$$= (50 \times 51 + 33 \times 51 + 1050 + 51 \times 5) - (51 \times 16 + 550 + 102 + 315 + 51 + 85) + (180 + 0 + 0 + 0) - 0 = 3799$$

Required sum = 5050 – 3799 = 1251

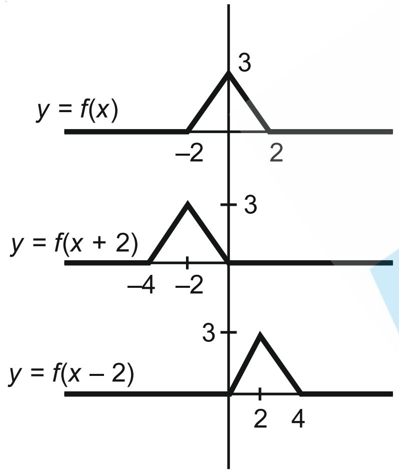
3. Let $f : \mathbf{R} \to \mathbf{R}$ be a function defined as

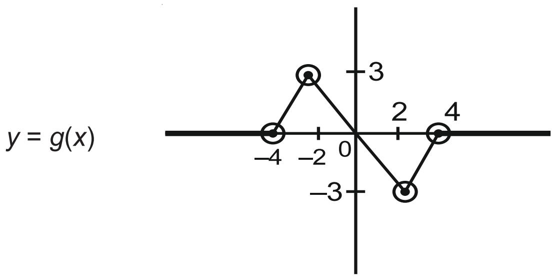
$$f(x) = \begin{cases} 3\left(1-\frac{|x|}{2}\right) & \text{if } |x| \le 2 \\ 0 & \text{if } |x| > 2 \end{cases}$$

Let $g : \mathbf{R} \to \mathbf{R}$ be given by $g(x) = f(x + 2) - f(x - 2)$. If n and m denote the number of points in **R** where g is not continuous and not differentiable, respectively, then n + m is equal to ________.

Answer (4)

Sol.





Clearly $n = 0$ and $m = 4$.

4. If the digits are not allowed to repeat in any number formed by using the digits 0, 2, 4, 6, 8, then the number of all numbers greater than 10,000 is equal to ________.

Answer (96)

Sol. $\underset{\overset{\uparrow}{2,4,6,8}}{\underline{4}}\ \underbrace{\underline{4}\ \underline{3}\ \underline{2}\ \underline{1}}_{\text{No restriction}}$

Total number of numbers = 4 × 4 × 3 × 2 × 1 = 96

5. The number of elements in the set $\{n \in \{1, 2, 3, ..., 100\} \mid (11)^n > (10)^n + (9)^n\}$ is ________.

Answer (96)

Sol. $11^n - 9^n > 10^n$

$$\Rightarrow \left(1+\frac{1}{10}\right)^n - \left(1-\frac{1}{10}\right)^n > 1$$

$$\Rightarrow {}^nC_1.\frac{1}{10} + \underbrace{{}^nC_3.\frac{1}{10^3} + {}^nC_5.\frac{1}{10^5} + ...}_{\text{Neglecting these terms}} > \frac{1}{2}$$

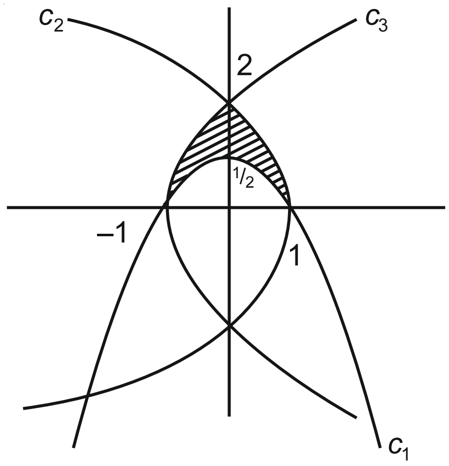
$$\Rightarrow n \ge 5$$

Possible values of n = 5, 6, 7, 8, …, 100

6. The area (in sq. units) of the region bounded by the curves $x^2 + 2y - 1 = 0$, $y^2 + 4x - 4 = 0$ and $y^2 - 4x - 4 = 0$, in the upper half plane is ________.

Answer (2)

Sol. $c_1 : y = \frac{-x^2+1}{2}, c_2 : y^2 = -4(x-1), c_3 : y^2 = 4(x+1)$



$$\text{Required area } = 2\int_0^1 \left(2\sqrt{1-x} - \frac{1-x^2}{2}\right)dx$$

$$= \left.\frac{4(1-x)^{\frac{3}{2}}}{\frac{3}{2}} - x + \frac{x^3}{3}\right|_0^1 = \left(-\frac{2}{3}\right) - \left(\frac{8}{3}\right) = 2$$

7. Let $y = y(x)$ be the solution of the differential equation $\left((x+2)e^{\left(\frac{y+1}{x+2}\right)}+(y+1)\right)dx=(x+2)dy$, $y(1) = 1$. If the domain of $y = y(x)$ is an open interval (α, β), then $|\alpha + \beta|$ is equal to ________.

Answer (4)

Sol. Let $y + 1 = Y$ and $x + 2 = X$

$dy = dY \qquad dx = dX$

$$\left(Xe^{\frac{X}{X}}+Y\right)dX = XdY$$

$$\Rightarrow \frac{XdY-YdX}{X^2}=\frac{e^{\frac{Y}{X}}}{X}dX$$

$$\Rightarrow e^{-\frac{Y}{X}}d\left(\frac{Y}{X}\right)=\frac{dX}{X}$$

$$\Rightarrow -e^{-\frac{Y}{X}}=\ln|X|+c$$

$$\Rightarrow -e^{-\left(\frac{y+1}{x+2}\right)}=\ln|x+2|+c$$

∵ (1, 1) satisfy this equation

So, $c=-e^{-\frac{2}{3}}-\ln 3$

Now $y = -1 - (x + 2)\ \ln\left(\ln\left(\left|\frac{3}{x+2}\right|\right)+e^{-\frac{2}{3}}\right)$

Domain :

$$\ln\left|\frac{3}{x+2}\right|>e^{-e^{-\frac{2}{3}}}$$

$$\Rightarrow \frac{3}{|x+2|}>\bar{e}^{e^{-\frac{2}{3}}}$$

$$\Rightarrow |x+2|<3e^{e^{-\frac{2}{3}}}$$

$$\Rightarrow -3e^{e^{-\frac{2}{3}}}-2<x<3e^{e^{-\frac{2}{3}}}-2$$

So $\alpha + \beta = -4$

$\Rightarrow |\alpha + \beta| = 4$

8. Consider the following frequency distribution :

Class :	0 – 6	6 – 12	12 – 18	18 – 24	24 – 30
Frequency :	a	b	12	9	5

If mean $=\frac{309}{22}$ and median = 14, then the value $(a - b)^2$ is equal to ________.

Answer (4)

Sol.

Class Interval	x_i	f_i	x_if_i	C.F.
0 – 6	3	a	3a	a
6 – 12	9	b	9b	a + b
12 – 18	15	12	180	$\boxed{12 + a + b}$ → Median Class
18 – 24	21	9	189	21 + a + b
24 – 30	27	5	135	26 + a + b
		a+b+26	3a+9b+504	

$$\bar{x}=\frac{\Sigma x_if_i}{\Sigma f_i}=\frac{3a+9b+504}{a+b+26}=\frac{309}{22}\Rightarrow 81a+37b=1018 \qquad \text{...(1)}$$

$$\text{Median }=12+\frac{13+\frac{a+b}{2}-(a+b)}{12}\times 6=14\Rightarrow a+b=18 \qquad \text{...(2)}$$

From (1) and (2), $a = 8$ and $b = 10$

9. If the constant term, in binomial expansion of $\left(2x^r+\frac{1}{x^2}\right)^{10}$ is 180, then r is equal to ________.

Answer (8)

Sol. $T_{k+1}={}^{10}C_k.(2x^r)^{10-k}.\left(\frac{1}{x^2}\right)^k={}^{10}C_k.2^{10-k}.x^{(10-k)r-2k}$

∵ $(10 - k)r - 2k = 0$ and ${}^{10}C_k.2^{10-k} = 180$

$\Rightarrow$ ${}^{10}C_k.2^{10-k} = 45.2^2$

$\Rightarrow$ $k = 8$

and $r = 8$

10. Let A = {0, 1, 2, 3, 4, 5, 6, 7}. Then the number of bijective functions $f : \mathrm{A} \rightarrow \mathrm{A}$ such that $f(1) + f(2) = 3 - f(3)$ is equal to ________.

Answer (720)

Sol. Clearly $f(1)$, $f(2)$ and $f(3)$ are the permutations of 0, 1, 2; and $f(0)$, $f(4)$, $f(5)$, $f(6)$ and $f(7)$ are the permutations of 3, 4, 5, 6 and 7.

Total number of bijective functions $=\underline{|5}\,.\,\underline{|3}=720$

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