

25/07/2021
Morning



Corporate Office: Aakash Tower, 8, Pusa Road, New Delhi-110005 Ph. 011-47623456

Time : 3 hrs.

Answers & Solutions

M.M. : 300

for

JEE (MAIN)-2021 (Online) Phase-3

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS :

- (1) The test is of **3 hours** duration.
- (2) The Test Booklet consists of 90 questions. The maximum marks are 300.
- (3) There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry and Mathematics** having 30 questions in each part of equal weightage. Each part has two sections.
 - (i) Section-I : This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **-1 mark** for wrong answer.
 - (ii) Section-II : This section contains 10 questions. In Section-II, attempt any **five questions out of 10**. There will be **no negative marking for Section-II**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and there is no negative marking for wrong answer.

PART-A : PHYSICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Match **List I** with **List II**.

List I	List II
(a) $\vec{C} - \vec{A} - \vec{B} = 0$	(i)
(b) $\vec{A} - \vec{C} - \vec{B} = 0$	(ii)
(c) $\vec{B} - \vec{A} - \vec{C} = 0$	(iii)
(d) $\vec{A} + \vec{B} = -\vec{C}$	(iv)

Choose the correct answer from the options given below:

- (1) (a) → (iv), (b) → (i), (c) → (iii), (d) → (ii)
- (2) (a) → (i), (b) → (iv), (c) → (ii), (d) → (iii)
- (3) (a) → (iii), (b) → (ii), (c) → (iv), (d) → (i)
- (4) (a) → (iv), (b) → (iii), (c) → (i), (d) → (ii)

Answer (4)

- Sol.** (a) $\vec{C} = \vec{A} + \vec{B} \Rightarrow$ (iv)
 (b) $\vec{A} = \vec{B} + \vec{C} \Rightarrow$ (iii)
 (c) $\vec{B} = \vec{A} + \vec{C} \Rightarrow$ (i)
 (d) $-\vec{C} = \vec{A} + \vec{B} \Rightarrow$ (ii)

2. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A: Moment of inertia of a circular disc of mass 'M' and radius 'R' about X, Y axes (passing through its plane) and Z-axis which is perpendicular to its plane were found to be I_x , I_y & I_z respectively. The respective radii of gyration about all the three axes will be the same.

Reason R: A rigid body making rotational motion has fixed mass and shape.

In the light of the above statements, choose the most appropriate answer from the options given below:

- (1) Both **A** and **R** are correct but **R** is NOT the correct explanation of **A**.
- (2) **A** is correct but **R** is not correct.
- (3) **A** is not correct but **R** is correct.
- (4) Both **A** and **R** are correct and **R** is the correct explanation of **A**.

Answer (3)

Sol. I_x, I_y, I_z are not equal.

So, radius of gyration can not be equal.

3. A parallel plate capacitor with plate area 'A' and distance of separation 'd' is filled with a dielectric. What is the capacity of the capacitor when permittivity of the dielectric varies as

$$\epsilon(x) = \epsilon_0 + kx, \text{ for } \left(0 < x \leq \frac{d}{2}\right)$$

$$\epsilon(x) = \epsilon_0 + k(d - x), \text{ for } \left(\frac{d}{2} \leq x \leq d\right)$$

(1) $\left(\epsilon_0 + \frac{kd}{2}\right)^{2/kA}$

(2) $\frac{kA}{2} \ln\left(\frac{2\epsilon_0}{2\epsilon_0 - kd}\right)$

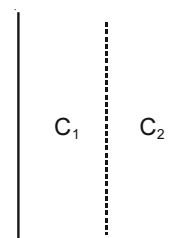
(3) $\frac{kA}{2 \ln\left(\frac{2\epsilon_0 + kd}{2\epsilon_0}\right)}$

(4) 0

Answer (3)

Sol. $\epsilon(x) = \epsilon_0 + kx, \text{ for } 0 \leq x \leq \frac{d}{2}$

and $\epsilon(x) = \epsilon_0 + k(d - x) \text{ for } \frac{d}{2} \leq x \leq d$



$$C_1 = C_2$$

$$d\left(\frac{1}{C_1}\right) = \int_0^{d/2} \frac{dx}{(\epsilon_0 + kx)A}$$

$$\Rightarrow \frac{1}{C_1} = \frac{1}{Ak} \ln\left(\frac{\epsilon_0 + k\frac{d}{2}}{\epsilon_0}\right) \Rightarrow C_1 = \frac{Ak}{\ln\left(1 + \frac{kd}{2\epsilon_0}\right)}$$

$$\therefore C = \frac{C_1}{2} = \frac{Ak}{2 \ln\left(\frac{2\epsilon_0 + kd}{2\epsilon_0}\right)}$$

4. Two wires of same length and radius are joined end to end and loaded. The Young's moduli of the materials of the two wires are Y_1 and Y_2 . The combination behaves as a single wire then its Young's modulus is

(1) $Y = \frac{2Y_1Y_2}{3(Y_1 + Y_2)}$ (2) $Y = \frac{Y_1Y_2}{2(Y_1 + Y_2)}$

(3) $Y = \frac{2Y_1Y_2}{Y_1 + Y_2}$ (4) $Y = \frac{Y_1Y_2}{Y_1 + Y_2}$

Answer (3)

Sol. $\Delta L = \Delta L_1 + \Delta L_2$

$$\Rightarrow \frac{F}{\left(\frac{YA}{2L}\right)} = \frac{F}{\left(\frac{Y_1A}{L}\right)} + \frac{F}{\left(\frac{Y_2A}{L}\right)}$$

$$\Rightarrow \frac{2L}{YA} = \frac{L}{Y_1A} + \frac{L}{Y_2A}$$

$$\Rightarrow \frac{2}{Y} = \frac{1}{Y_1} + \frac{1}{Y_2}$$

$$\Rightarrow Y = \frac{2Y_1Y_2}{Y_1 + Y_2}$$

5. In Amplitude Modulation, the message signal

$$V_m(t) = 10 \sin(2\pi \times 10^5 t) \text{ volts and}$$

Carrier signal

$$V_c(t) = 20 \sin(2\pi \times 10^7 t) \text{ volts}$$

The modulated signal now contains the message signal with lower side band and upper side band frequency, therefore the bandwidth of modulated signal is α kHz.

The value of α is

- (1) 50 kHz (2) Zero
(3) 200 kHz (4) 100 kHz

Answer (3)

Sol. $f_m = \frac{2\pi \times 10^5}{2\pi} = 10^5 \text{ Hz}$

$$f_c = 10^7 \text{ Hz}$$

$$\therefore \text{Bandwidth} = 2 \times f_m = 2 \times 10^5 \text{ Hz} = 200 \text{ kHz}$$

6. Water droplets are coming from an open tap at a particular rate. The spacing between a droplet observed at 4th second after its fall to the next droplet is 34.3 m. At what rate the droplets are coming from the tap? (Take $g = 9.8 \text{ m/s}^2$)

- (1) 1 drop / 7 seconds
(2) 3 drops / 2 seconds
(3) 2 drops / second
(4) 1 drop / second

Answer (4)

Sol. Let N drop/sec

$$V_1 = (g) \frac{1}{N}$$

$$H_0 = \frac{1}{2}(g) \left(\frac{1}{N}\right)^2$$

At $t = 4 \text{ s}$,

$$9.8 \times \left(4 - \frac{1}{N}\right) + (4.9) \frac{1}{N^2} = 34.3 \Rightarrow N = 1$$

$$N = 1$$

7. Some nuclei of a radioactive material are undergoing radioactive decay. The time gap between the instances when a quarter of the nuclei have decayed and when half of the nuclei have decayed is given as (where λ is the decay constant)

(1) $\frac{\ln 2}{\lambda}$ (2) $\frac{1}{2} \frac{\ln 2}{\lambda}$

(3) $\frac{\ln 3}{2\lambda}$ (4) $\frac{2 \ln 2}{\lambda}$

Answer (3)

Sol. $N = N_0 \cdot e^{-\lambda t}$

$$\frac{3}{4} N_0 = N_0 \cdot e^{-\lambda t_1}$$

$$\Rightarrow t_1 = \frac{1}{\lambda} \ln \frac{4}{3}$$

$$t_2 = \frac{\ln 2}{\lambda}$$

$$\Rightarrow |t_1 - t_2| = \left(\frac{1}{\lambda}\right) \left| \left[2 \ln 2 - \ln 3 - \ln 2\right] \right|$$

$$= \left(\frac{1}{\lambda}\right) \ln \frac{3}{2}$$

8. What should be the order of arrangement of de-Broglie wavelength of electron (λ_e), an α -particle (λ_α) and proton (λ_p) given that all have the same kinetic energy?

- (1) $\lambda_e = \lambda_p > \lambda_\alpha$ (2) $\lambda_e = \lambda_p = \lambda_\alpha$
 (3) $\lambda_e > \lambda_p > \lambda_\alpha$ (4) $\lambda_e < \lambda_p < \lambda_\alpha$

Answer (3)

Sol. $\lambda = \frac{h}{p}$

$$\lambda = \frac{h}{\sqrt{2mk}}$$

$$\Rightarrow \lambda \propto \frac{1}{\sqrt{m}}$$

$$\Rightarrow \lambda_e > \lambda_p > \lambda_\alpha$$

9. The minimum and maximum distances of a planet revolving around the Sun are x_1 and x_2 . If the minimum speed of the planet on its trajectory is v_0 then its maximum speed will be :

- (1) $\frac{v_0 x_2^2}{x_1^2}$ (2) $\frac{v_0 x_1}{x_2}$
 (3) $\frac{v_0 x_1^2}{x_2^2}$ (4) $\frac{v_0 x_2}{x_1}$

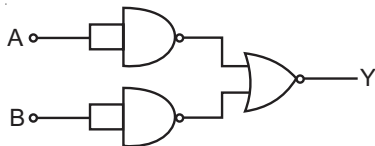
Answer (4)

Sol. $\vec{L}_i = \vec{L}_f$

$$v_0 x_2 = v x_1$$

$$\Rightarrow v = \frac{v_0 x_2}{x_1}$$

10. Identify the logic operation carried out.



- (1) NAND (2) NOR
 (3) OR (4) AND

Answer (4)

Sol.

A	B	Y
0	0	0
1	0	0
0	1	0
1	1	1

\Rightarrow AND gate

11. The half-life of ^{198}Au is 3 days. If atomic weight of ^{198}Au is 198 g/mol then the activity of 2 mg of ^{198}Au is [in disintegration/second]:

- (1) 2.67×10^{12} (2) 16.18×10^{12}
 (3) 32.36×10^{12} (4) 6.06×10^{18}

Answer (2)

Sol. $\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{3 \times 24 \times 60 \times 60}$

$$N = \frac{2 \times 10^{-3}}{198} \times 6.022 \times 10^{23}$$

Activity = λN

$$= \frac{\ln 2}{3 \times 24 \times 3600} \times \frac{2 \times 10^{-3} \times 6.022 \times 10^{23}}{198}$$

$$\approx 16.18 \times 10^{12} \text{ dps}$$

12. A particle of mass $4M$ at rest disintegrates into two particles of mass M and $3M$ respectively having non zero velocities. The ratio of de-Broglie wavelength of particle of mass M to that of mass $3M$ will be:

- (1) 1 : 3 (2) 1 : $\sqrt{3}$
 (3) 1 : 1 (4) 3 : 1

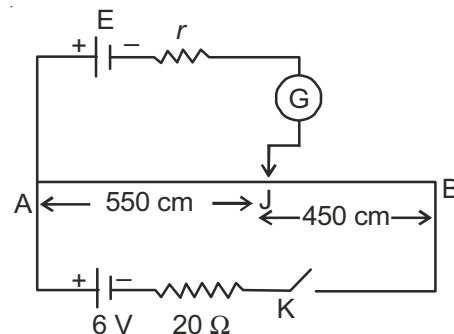
Answer (3)

Sol. $P_1 = P_2$

$$\lambda_1 = \frac{h}{P_1}, \lambda_2 = \frac{h}{P_2}$$

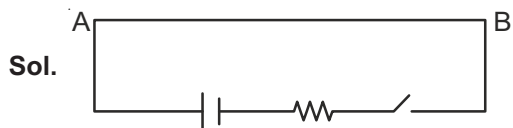
So $\lambda_1 : \lambda_2 = 1 : 1$

13. In the given figure, there is a circuit of potentiometer of length $AB = 10 \text{ m}$. The resistance per unit length is 0.1Ω per cm. Across AB , a battery of emf E and internal resistance ' r ' is connected. The maximum value of emf measured by this potentiometer is



- (1) 5 V (2) 2.25 V
 (3) 2.75 V (4) 6 V

Answer (1)



Resistance of AB wire = $10 \times 100 \times 0.1 = 100 \Omega$

$$\text{current through AB} = \frac{6}{20+100} = \frac{6}{120}$$

$$\text{P.D. across AB} = \frac{6}{120} \times 100 = 5 \text{ V}$$

14. A monoatomic ideal gas, initially at temperature T_1 is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand adiabatically to a temperature T_2 by releasing the piston suddenly. If l_1 and l_2 are the lengths of the gas column, before and after the expansion respectively, then the value of

$\frac{T_1}{T_2}$ will be

- (1) $\frac{l_2}{l_1}$ (2) $\frac{l_1}{l_2}$
 (3) $\left(\frac{l_2}{l_1}\right)^{\frac{2}{3}}$ (4) $\left(\frac{l_1}{l_2}\right)^{\frac{2}{3}}$

Answer (3)

Sol. For Adiabatic process

$$TV^{\gamma-1} = C$$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{l_2}{l_1}\right)^{\frac{5}{3}-1}$$

$$\frac{T_1}{T_2} = \left(\frac{l_2}{l_1}\right)^{\frac{2}{3}}$$

15. For a gas $C_p - C_v = R$ in a state P and $C_p - C_v = 1.10 R$ in a state Q, T_P and T_Q are the temperatures in two different states P and Q respectively, Then

- (1) $T_P < T_Q$ (2) $T_P > T_Q$
 (3) $T_P = T_Q$ (4) $T_P = 0.9 T_Q$

Answer (2)

Sol. State P $\Rightarrow C_p - C_v = R$ Ideal gas

State Q $\Rightarrow C_p - C_v = 1.10 R$ Real gas

As gas behaves like ideal gas at high temperature

So, $T_P > T_Q$

16. A ray of laser of a wavelength 630 nm is incident at an angle of 30° at the diamond-air interface. It is going from diamond to air. The refractive index of diamond is 2.42 and that of air is 1. Choose the correct option.

- (1) refraction is not possible
 (2) angle of refraction is 30°
 (3) angle of refraction is 24.41°
 (4) angle of refraction is 53.4°

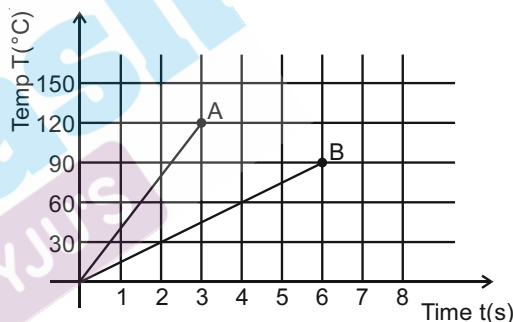
Answer (1)

Sol. $\theta_C = \sin^{-1}\left(\frac{1}{2.42}\right)$

$$\theta > \theta_C$$

hence, refraction is not possible

17. Two different metal bodies A and B of equal mass are heated at a uniform rate under similar conditions. The variation of temperature of the bodies is graphically represented as shown in the figure. The ratio of specific heat capacities is:



- (1) $\frac{3}{8}$ (2) $\frac{4}{3}$
 (3) $\frac{3}{4}$ (4) $\frac{8}{3}$

Answer (1)

Sol. $Q = ms \frac{dT}{dt}$

$$\frac{S_A}{S_B} \times \frac{120}{3} \times \frac{6}{90} = 1$$

$$\frac{S_A}{S_B} \times \frac{8}{3} = 1$$

$$\Rightarrow \frac{S_A}{S_B} = \frac{3}{8}$$

18. A linearly polarized electromagnetic wave in vacuum is

$$E = 3.1 \cos[(1.8)z - (5.4 \times 10^6)t] \hat{i} \text{ N/C}$$

is incident normally on a perfectly reflecting wall at $z = a$. Choose the correct option

- (1) The frequency of electromagnetic wave is 54×10^4 Hz.
- (2) The reflected wave will be $3.1 \cos[(1.8)z + (5.4 \times 10^6)t] \hat{i} \text{ N/C}$
- (3) The transmitted wave will be $3.1 \cos[(1.8)z - (5.4 \times 10^6)t] \hat{i} \text{ N/C}$
- (4) The wavelength is 5.4 m

Answer (Bonu)

Sol. $E = 3.1 \cos[5.4 \times 10^6 t - 1.8z] \hat{i} \text{ N/C}$

Since it is reflected by $z = a$, so z will be replaced by $(2a - z)$ also there would be phase change of π .

$$E_r = 3.1 \cos[5.4 \times 10^6 t - 1.8(2a - z)\pi] \hat{i}$$

$$E_r = -3.1 \cos[5.4 \times 10^6 t + 1.8z - 3.6a] \hat{i}$$

Answer is dependent on a so none of option matches.

19. In the Young's double slit experiment, the distance between the slits varies in time as $d(t) = d_0 + a_0 \sin \omega t$; where d_0 , ω and a_0 are constants. The difference between the largest fringe width and the smallest fringe width obtained over time is given as:

- (1) $\frac{2\lambda D(d_0)}{(d_0^2 - a_0^2)}$
- (2) $\frac{\lambda D}{d_0 + a_0}$
- (3) $\frac{2\lambda D a_0}{(d_0^2 - a_0^2)}$
- (4) $\frac{\lambda D}{d_0^2} a_0$

Answer (3)

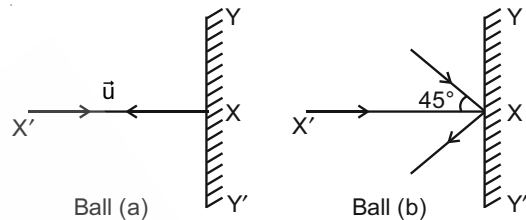
Sol. $\beta_{\max} = \frac{\lambda D}{d_0 + a_0}$

$$\beta_{\min} = \frac{\lambda D}{d_0 - a_0}$$

$$\beta_{\max} - \beta_{\min} = \lambda D \left[\frac{1}{d_0 - a_0} - \frac{1}{(d_0 + a_0)} \right]$$

$$= \frac{2\lambda D a_0}{(d_0^2 - a_0^2)}$$

20. Two billiard balls of equal mass 30 g strike a rigid wall with same speed of 108 kmph (as shown) but at different angles. If the balls get reflected with the same speed then the ratio of the magnitude of impulses imparted to ball 'a' and ball 'b' by the wall along 'X' direction is:



- (1) $\sqrt{2} : 1$
- (2) $1 : \sqrt{2}$
- (3) $2 : 1$
- (4) $1 : 1$

Answer (1)

Sol. $I_a = 2mv$

$$I_b = 2mv \cos(45^\circ)$$

$$\text{ratio} = \sqrt{2}$$

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. An inductor of 10 mH is connected to a 20 V battery through a resistor of 10 kΩ and a switch. After a long time, when maximum current is set up in the circuit, the current is switched off. The current in the circuit after $1 \mu\text{s}$ is $\frac{x}{100}$ mA. Then x is equal to _____. (Take $e^{-1} = 0.37$)

Answer (74)

Sol. $i = i_0 e^{-t/\tau}$

$$= \frac{20}{10 \times 10^3} \cdot e^{-\frac{10^{-6}}{10^{-6}}}$$

$$= 2 \times 10^{-3} e^{-1}$$

$$= 0.74 \text{ mA}$$

2. A circular conducting coil of radius 1 m is being heated by the change of magnetic field \vec{B} passing perpendicular to the plane in which the coil is laid. The resistance of the coil is $2 \mu\Omega$. The magnetic field is slowly switched off such that its magnitude changes in time as

$$B = \frac{4}{\pi} \times 10^{-3} T \left(1 - \frac{t}{100} \right)$$

The energy dissipated by the coil before the magnetic field is switched off completely is $E = \underline{\hspace{2cm}}$ mJ.

Answer (80)

Sol. $\phi = B \cdot A = \frac{4 \times 10^{-3}}{\pi} \cdot \left(1 - \frac{t}{100} \right) \cdot \pi \cdot 1^2$

$$= 4 \times 10^{-3} \left(1 - \frac{t}{100} \right)$$

$$\varepsilon = \left| \frac{d\phi}{dt} \right| = 40 \times 10^{-6}$$

$$i = \frac{\varepsilon}{R} = \frac{40 \times 10^{-6}}{2 \times 10^{-6}} = 20 \text{ A}$$

$$E = i^2 R \cdot t = 20^2 \times 2 \times 10^{-6} \times 100$$

$$= 80 \times 10^{-3} \text{ J}$$

3. A particle of mass 'm' is moving in time 't' on a trajectory given by

$$\vec{r} = 10\alpha t^2 \hat{i} + 5\beta(t-5) \hat{j}$$

Where α and β are dimensional constants.

The angular momentum of the particle becomes the same as it was for $t = 0$ at time $t = \underline{\hspace{2cm}}$ seconds.

Answer (10)

Sol. $\vec{L} = \vec{r} \times m\vec{v}$

$$= (10\alpha t^2 \hat{i} + 5\beta(t-5) \hat{j}) \times m(20\alpha t \hat{i} + 5\beta \hat{j})$$

$$= 50\alpha\beta m t(10-t) \hat{k}$$

$$\vec{L}_{t=0} = 0$$

$$\vec{L}_t = 0 \Rightarrow t = 10 \text{ sec.}$$

4. The value of aluminium susceptibility is 2.2×10^{-5} . The percentage increase in the magnetic field if space within a current carrying toroid is filled with

Aluminium is $\frac{x}{10^4}$. Then the value of x is $\underline{\hspace{2cm}}$

Answer (22)

Sol. % increase in $\vec{B} = (\mu_r - 1) \times 100$

$$= x \times 100$$

$$= 2.2 \times 10^{-3}$$

$$= \frac{22}{10^4}$$

5. A body of mass 2 kg moving with a speed of 4 m/s makes an elastic collision with another body at rest and continues to move in the original direction but with one fourth of its initial speed. The

speed of the two body centre of mass is $\frac{x}{10}$ m/s.

Then the value of x is $\underline{\hspace{2cm}}$.

Answer (25)

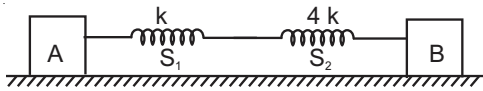
Sol. $m_1 \frac{v_0}{4} + m_2 v_2 = m_1 v_0 \dots (1)$

$$v_2 - \frac{v_0}{4} = v_0 \dots (2)$$

$$\Rightarrow v_2 = 5 \text{ m/s and } m_2 = \frac{6}{5} \text{ kg}$$

$$v_c = \frac{m_1 v_0}{m_1 + m_2} = 2.5 \text{ m/s}$$

6. In the reported figure, two bodies A and B of masses 200 g and 800 g are attached with the system of springs. Springs are kept in a stretched position with some extension when the system is released. The horizontal surface is assumed to be frictionless. The angular frequency will be ____ rad/s when $k = 20 \text{ N/m}$.



Answer (10)

Sol. $\mu = 160 \text{ g} = 0.16 \text{ kg}$

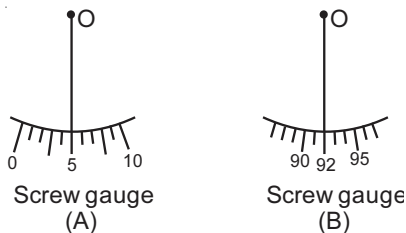
$$k_{\text{eff}} = 16$$

$$\omega = \sqrt{\frac{k_{\text{eff}}}{\mu}} = 100 = 10$$

7. Student A and Student B used two screw gauges of equal pitch and 100 equal circular divisions to measure the radius of a given wire. The actual value of the radius of the wire is 0.322 cm. The absolute value of the difference between the final circular scale readings observed by the students A and B is _____.

[Figure shows position of reference 'O' when jaws of screw gauge are closed]

Given pitch = 0.1 cm.



Answer (13)

Sol. Difference in CSR = $(100 - 92) + 5$
 $= 8 + 5$
 $= 13$

8. A pendulum bob has a speed of 3 m/s at its lowest position. The pendulum is 50 cm long. The speed of bob, when the length makes an angle of 60° to the vertical will be ($g = 10 \text{ m/s}^2$) _____ m/s.

Answer (2)

Sol. $mg(1 - \cos\theta) + \frac{1}{2}mv^2 = \frac{1}{2}mu^2$

$$\frac{mg(0.5)}{2} + \frac{1}{2}mv^2 = \frac{1}{2}m(9)$$

$$5 + v^2 = 9$$

$$v = 2$$

9. A particle of mass 1 mg and charge q is lying at the mid-point of two stationary particles kept at a distance '2 m' when each is carrying same charge 'q'. If the free charged particle is displaced from its equilibrium position through distance 'x' ($x \ll 1 \text{ m}$). The particle executes SHM. Its angular frequency of oscillation will be _____ $\times 10^5 \text{ rad/s}$ if $q^2 = 10 \text{ C}^2$.

Answer (6000)

Sol. $F_{\text{res}} = kq^2 \left(\frac{1}{(1-x)^2} - \frac{1}{(1+x)^2} \right)$

$$= \frac{kq^2 \times 4x}{(1-x^2)^2} \approx 4kq^2 x$$

$$a = \frac{4 \times 10^9 \times 9 \times 10}{10^{-6}} x$$

$$\Rightarrow \omega = 6 \times 10^8 \text{ rad/s}$$

10. An electric bulb rated as 200 W at 100 V is used in a circuit having 200 V supply. The resistance 'R' that must be put in series with the bulb so that the bulb delivers the same power is _____ Ω .

Answer (50)

Sol. $200 = \frac{(100)^2}{R}$

$$R = 50 \Omega$$

Same amount of resistance must be put in series to have potential drop half of the total potential difference.

$$R_{\text{req}} = 50 \Omega$$

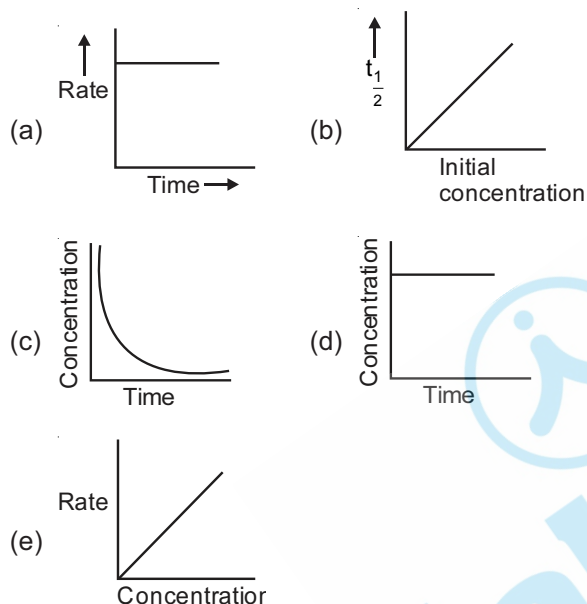
PART-B : CHEMISTRY

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. For the following graphs,



Choose from the options given below, the **correct** one regarding order of reaction is

- (1) (a) and (b) Zero order
(c) and (e) First order
- (2) (b) and (d) Zero order
(e) First order
- (3) (a) and (b) Zero order
(e) First order
- (4) (b) Zero order
(c) and (e) First order

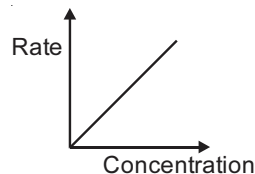
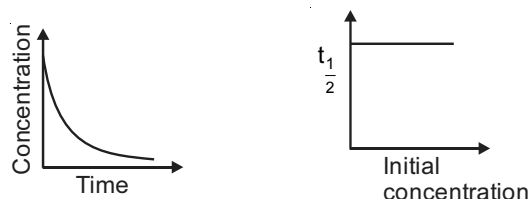
Answer (3)

Sol. For 1st order reaction

$$\text{Rate} = k [\text{Reactant}]$$

$$[A] = [A]_0 e^{-kt}$$

$$t_{1/2} = \frac{0.693}{k}$$

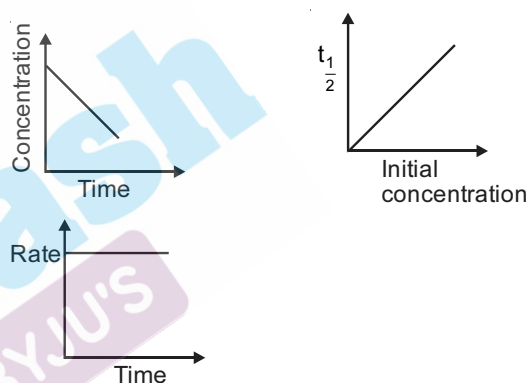


For zero order reaction

$$\text{Rate} = k$$

$$[A] = [A]_0 - kt$$

$$t_{1/2} = \frac{[A]_0}{2k}$$



2. Given below are two statements, one is labelled as **Assertion (A)** and other is labelled as **Reason (R)**.

Assertion (A): Gabriel phthalimide synthesis cannot be used to prepare aromatic primary amines.

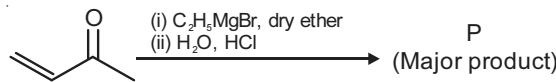
Reason (R): Aryl halides do not undergo nucleophilic substitution reaction.

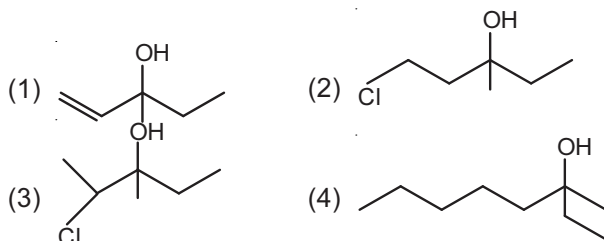
In the light of the above statements, choose the correct answer from the options given below :

- (1) **(A)** is false but **(R)** is true
- (2) Both **(A)** and **(R)** are true and **(R)** is correct explanation of **(A)**
- (3) **(A)** is true but **(R)** is false
- (4) Both **(A)** and **(R)** are true but **(R)** is not the correct explanation of **(A)**

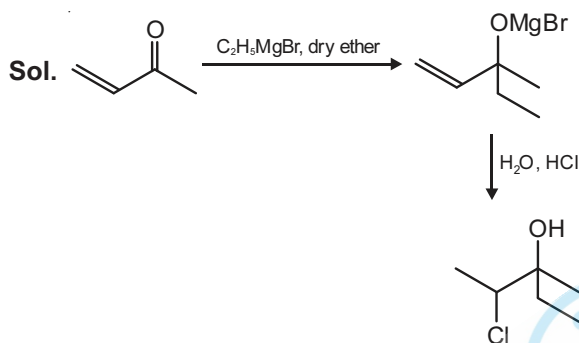
Answer (2)

Sol. Gabriel phthalimide synthesis cannot be used to prepare aromatic primary amines because aryl halides do not undergo nucleophilic substitution with the anion formed by phthalimide.

3.  Consider the above reaction, the major product 'P' is



Answer (3)



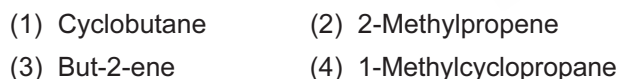
4. Which one of the following compounds of Groups-14 elements is not known?



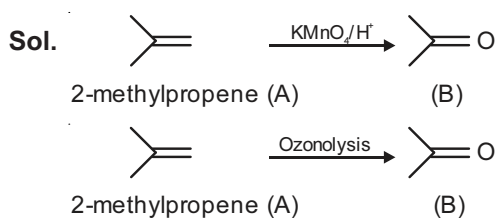
Answer (4)

Sol. $[\text{SiCl}_6]^{2-}$ is not known. The main reasons are (i) six large chloride ion cannot be accommodated around Si^{4+} due to limitation of its size and (ii) interaction between lone pair of chloride ion and Si^{4+} is not very strong.

5. An Organic compounds 'A' C_4H_8 on treatment with KMnO_4/H^+ yields compound 'B' $\text{C}_3\text{H}_6\text{O}$. Compound 'A' also yields compound 'B' an ozonolysis. Compound 'A' is



Answer (2)



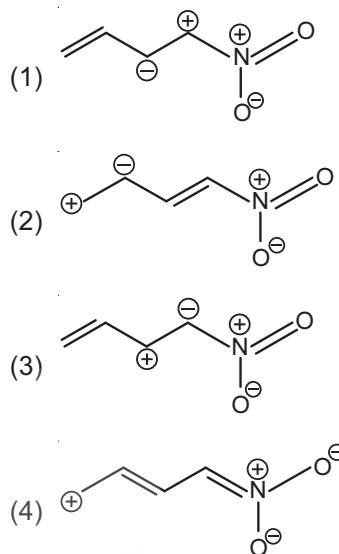
6. In the leaching of alumina from bauxite, the ore expected to leach out in the process by reacting with NaOH is



Answer (3)

Sol. Along with Al_2O_3 , SiO_2 too dissolves forming Na_2SiO_3 .

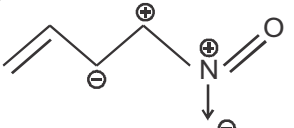
7. Which one among the following resonating structures is **not** correct?



Answer (1)

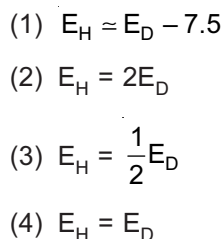


C - C double bond π electrons move towards $-\text{NO}_2$

group, so the structure  is

incorrect.

8. At 298.2 K the relationship between enthalpy of bond dissociation (in kJ mol^{-1}) for hydrogen (E_{H}) and its isotope, deuterium (E_{D}), is best described by

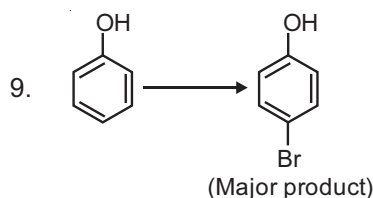


Answer (1)

Sol. Enthalpy of bond dissociation (kJ mol^{-1}) of hydrogen = 435.88

Enthalpy of bond dissociation (kJ mol^{-1}) of deuterium = 443.35

$$E_{\text{H}} = E_{\text{D}} - 7.47$$



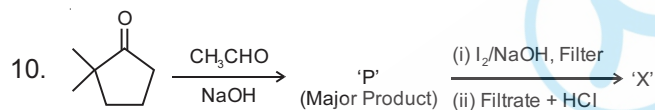
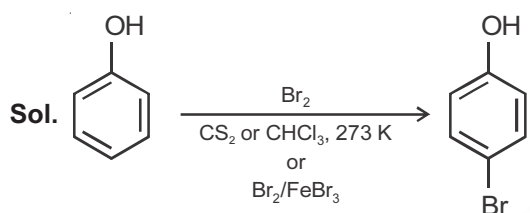
The given reaction can occur in the presence of

- (a) Bromine water (b) Br₂ in CS₂, 273 K
(c) Br₂/FeBr₃ (d) Br₂ in CHCl₃, 273 K

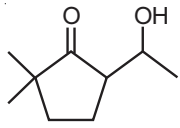
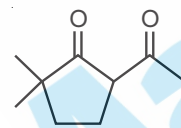
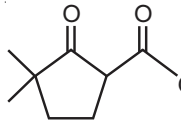
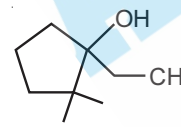
Choose the **correct** answer from the options given below

- (1) (a) and (c) only (2) (a), (b) and (d) only
(3) (b) and (d) only (4) (b), (c) and (d) only

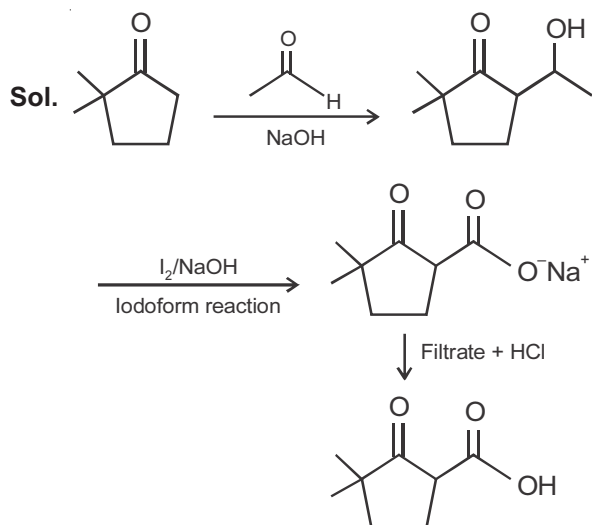
Answer (4)



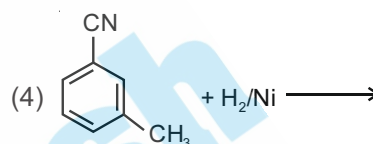
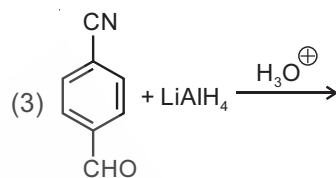
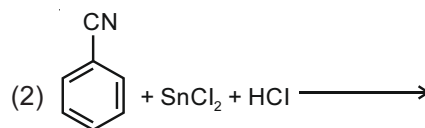
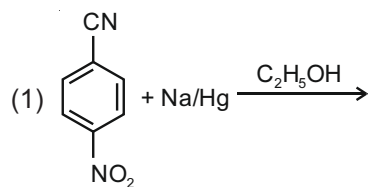
Consider the given reaction, the product 'X' is

- (1)  (2) 
(3)  (4) 

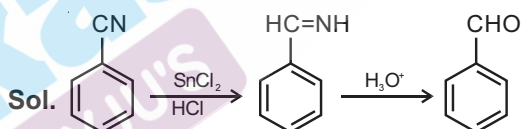
Answer (3)



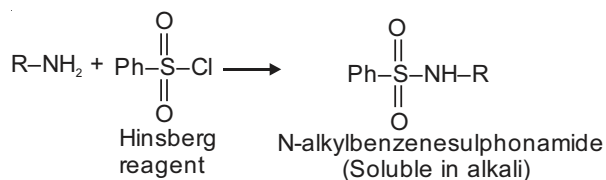
11. Which one of the products of the following reactions **does not** react with Hinsberg reagent to form sulphonamide?



Answer (2)



Rest of the reactions give amine group containing product. So, they can react with Hinsberg reagent



12. Which one of the following chemical agent is **not** being used for dry-cleaning of clothes?

- (1) H₂O₂
(2) Liquid CO₂
(3) CCl₄
(4) Cl₂C = CCl₂

Answer (3)

Sol. H₂O₂ is used for the purpose of bleaching clothes in the process of laundry.

13. Sodium stearate $\text{CH}_3(\text{CH}_2)_{16}\text{COO}^-\text{Na}^+$ is an anionic surfactant which forms micelles in oil. Choose the **correct** statement for it from the following

- (1) It forms non-spherical micelles with $\text{CH}_3(\text{CH}_2)_{16}$ – group pointing towards the centre
- (2) It forms spherical micelles with $\text{CH}_3(\text{CH}_2)_{16}$ – group pointing outwards on the surface of sphere
- (3) It forms non-spherical micelles with $-\text{COO}^\ominus$ group pointing outwards on the surface
- (4) It forms spherical micelles with $\text{CH}_3(\text{CH}_2)_{16}$ – group pointing towards the centre of sphere

Answer (4)

Sol. Surfactant form micelles by pointing the hydrophobic part towards the centre of sphere.

14. The correct order of following 3d metal oxides, according to their oxidation number is

- (a) CrO_3
 - (b) Fe_2O_3
 - (c) MnO_2
 - (d) V_2O_5
 - (e) Cu_2O
- (1) (a) > (d) > (c) > (b) > (e)
 - (2) (d) > (a) > (b) > (c) > (e)
 - (3) (a) > (c) > (d) > (b) > (e)
 - (4) (c) > (a) > (d) > (e) > (b)

Answer (1)

Sol. Metal oxide | Oxidation number

CrO_3	+6
Fe_2O_3	+3
MnO_2	+4
V_2O_5	+5
Cu_2O	+1

$$a > d > c > b > e$$

15. The ionic radii of K^+ , Na^+ , Al^{3+} and Mg^{2+} are in the order

- (1) $\text{Na}^+ < \text{K}^+ < \text{Mg}^{2+} < \text{Al}^{3+}$
- (2) $\text{Al}^{3+} < \text{Mg}^{2+} < \text{K}^+ < \text{Na}^+$
- (3) $\text{Al}^{3+} < \text{Mg}^{2+} < \text{Na}^+ < \text{K}^+$
- (4) $\text{K}^+ < \text{Al}^{3+} < \text{Mg}^{2+} < \text{Na}^+$

Answer (3)

Sol.	Ionic radii (in pm)
Na^+	102
K^+	138
Mg^{2+}	72
Al^{3+}	53.5

Generally higher the charge on cation smaller will be its ionic radius.

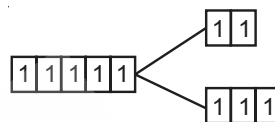
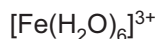
$$\text{Al}^{3+} < \text{Mg}^{2+} < \text{Na}^+ < \text{K}^+$$

16. Which one of the following species responds to an external magnetic field?

- (1) $[\text{Ni}(\text{CN})_4]^{2-}$
- (2) $[\text{Fe}(\text{H}_2\text{O})_6]^{3+}$
- (3) $[\text{Co}(\text{CN})_6]^{3-}$
- (4) $[\text{Ni}(\text{CO})_4]$

Answer (2)

Sol. CN^- and CO are strong field ligands and causes pairing of electrons. Whereas H_2O is weak field ligand so do not cause pairing, generally.



So, $[\text{Fe}(\text{H}_2\text{O})_6]^{3+}$ is paramagnetic. In the rest of the complexes no unpaired electron is present hence they are diamagnetic. Paramagnetic substances get weakly attracted by external magnetic field whereas diamagnetic gets weakly repelled.

17. is a repeating unit for

- (1) Acrilan
- (2) Novolac
- (3) Buna-N
- (4) Neoprene

Answer (2)

Sol. Given polymer is novolac

18. Which one of the following compounds will liberate CO_2 , when treated with NaHCO_3 ?

- (1) CH_3NH_2
- (2) $(\text{CH}_3)_3\text{N}^+\text{HCl}^-$
- (3) $\text{CH}_3-\overset{\text{O}}{\parallel}{\text{C}}-\text{NH}_2$
- (4) $(\text{CH}_3)_4\text{N}^+\text{OH}^-$

Answer (2)

Sol. $(\text{CH}_3)_3\text{N}^+\text{HCl}^-$ } contains strong conjugate acid

19. The water soluble protein is

- (1) Albumin
- (2) Collagen
- (3) Myosin
- (4) Fibrin

Answer (1)

Sol. Globular proteins are usually soluble in water. Insulin and albumin are the common examples of globular proteins.

20. Given below are two statements :

Statement I : None of the alkaline earth metal hydroxides dissolve in alkali.

Statement II : Solubility of alkaline earth metal hydroxides in water increases down the group.

In the light of the above statements, choose the **most appropriate** answer from the options given below

- (1) **Statement I** and **Statement II** both are correct
- (2) **Statement I** and **Statement II** both are incorrect
- (3) **Statement I** is incorrect but **Statement II** is correct
- (4) **Statement I** is correct but **Statement II** is incorrect

Answer (1)

- Sol.**
- $\text{Be}(\text{OH})_2$ is amphoteric in nature hence it dissolves in both acid and alkali
 - Solubility of alkaline earth metal hydroxides in water increases down the group.
 - Beryllium is not alkaline earth metal.
- *(NCERT Pg. No. 306, Part-2, XI)

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. At 298 K, the enthalpy of fusion of a solid (X) is 2.8 kJ mol^{-1} and the enthalpy of vaporisation of the liquid (X) is 98.2 kJ mol^{-1} . The enthalpy of sublimation of the substance (X) in kJ mol^{-1} is _____. (in nearest integer)

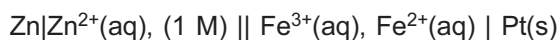
Answer (101)

Sol. $\Delta_{\text{sub}}H = \Delta_{\text{fus}}H + \Delta_{\text{vap}}H$ (These values should be at a given temperature)

$$\Delta_{\text{sub}}H = 98.2 + 2.8$$

$$\Delta_{\text{sub}}H = 101 \text{ kJ mol}^{-1}$$

2. Consider the cell at 25°C

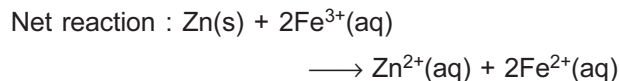


The fraction of total iron present as Fe^{3+} ion at the cell potential of 1.500 V is $x \times 10^{-2}$. The value of x is _____. (Nearest integer)

(Given : $E_{\text{Fe}^{3+}/\text{Fe}^{2+}}^0 = 0.77 \text{ V}$, $E_{\text{Zn}^{2+}/\text{Zn}}^0 = -0.76 \text{ V}$)

Answer (24)

Sol. $\text{Zn}(\text{s})|\text{Zn}^{2+}(\text{aq}, 1 \text{ M})||\text{Fe}^{3+}(\text{aq}), \text{Fe}^{2+}(\text{aq})|\text{Pt}(\text{s})$



$$Q = \frac{[\text{Zn}^{2+}][\text{Fe}^{2+}]^2}{[\text{Fe}^{3+}]^2}$$

$$E_{\text{cell}} = E_{\text{cell}}^0 - \frac{0.0591}{n} \log Q$$

$$1.500 = 1.53 - \frac{0.0591}{2} \log \left(\frac{[\text{Fe}^{2+}]^2}{[\text{Fe}^{3+}]^2} \right)$$

$$\frac{[\text{Fe}^{2+}]}{[\text{Fe}^{3+}]} = 3.218$$

$$\text{Fraction of Fe}^{3+} = \frac{1}{4.218} = 0.237 = 23.7 \times 10^{-2} \approx 24 \times 10^{-2}$$

3. Consider the complete combustion of butane, the amount of butane utilized to produce 72.0 g of water is _____ $\times 10^{-1} \text{ g}$. (in nearest integer)

Answer (464)

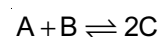
Sol. $2\text{C}_4\text{H}_{10}(\text{g}) + 13\text{O}_2(\text{g}) \longrightarrow 8\text{CO}_2(\text{g}) + 10\text{H}_2\text{O}(\text{l})$

$$116 \text{ g} \qquad \qquad \qquad 180 \text{ g}$$

$$46.4 \text{ g} \qquad \qquad \qquad 72.0 \text{ g}$$

So, the amount of butane required is $464 \times 10^{-1} \text{ g}$ for the production of 72.0 g of H_2O .

4. For the reaction



The value of equilibrium constant is 100 at 298 K . If the initial concentration of all the three species is 1 M each, then the equilibrium concentration of C is $x \times 10^{-1} \text{ M}$. The value of x is _____. (Nearest integer)

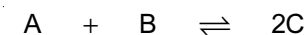
Answer (25)

Sol. $\text{A} + \text{B} \rightleftharpoons 2\text{C} \qquad K = 100$

$$1 \quad 1 \quad 1$$

$$Q = 1$$

$Q < K$ so reaction moves forward



$$1 \quad 1 \quad 1$$

$$1-x \quad 1-x \quad 1+2x$$

$$100 = \frac{(1+2x)^2}{(1-x)^2}$$

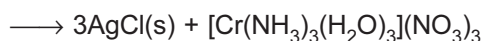
$$\Rightarrow 2x = \frac{9}{6}$$

$$[C] = 1 + 2x = \frac{15}{6} = 25 \times 10^{-1} M$$

5. Three moles of AgCl get precipitated when one mole of an octahedral co-ordination compound with empirical formula $CrCl_3 \cdot 3NH_3 \cdot 3H_2O$ reacts with excess of silver nitrate. The number of chloride ions satisfying the secondary valency of the metal ion is _____.

Answer (0)

Sol. $[Cr(NH_3)_3(H_2O)_3]Cl_3 + AgNO_3$



None of the chloride ion is directly bonded to metal ion. Hence number of chloride ions satisfying the secondary valence of the metal ion is zero.

6. A source of monochromatic radiation of wavelength 400 nm provides 1000 J of energy in 10 seconds. When this radiation falls on the surface of sodium, $x \times 10^{20}$ electrons are ejected per second. Assume that wavelength 400 nm is sufficient for ejection of electron from the surface of sodium metal. The value of x is _____. (Nearest integer)

$$(h = 6.626 \times 10^{-34} \text{ Js})$$

Answer (2)

Sol. $E = nh\nu$

$$1000 = \frac{n \times 6.626 \times 10^{-34} \times 3 \times 10^8}{400 \times 10^{-9}}$$

$n = 20.122 \times 10^{20}$ photons incidented on metal surface in 10 seconds

$n = 2.0122 \times 10^{20}$ photon incidented on metal surface in 1 second

Number of electrons ejected equal to number of photon incidented.

7. When 10 mL of an aqueous solution of Fe^{2+} ions was titrated in the presence of dil H_2SO_4 using diphenylamine indicator, 15 mL of 0.02 M solution of $K_2Cr_2O_7$ was required to get the end point. The molarity of the solution containing Fe^{2+} ions is $x \times 10^{-2}$ M. The value of x is _____. (Nearest integer)

Answer (18)

Sol. $6Fe^{2+} + 14H^+ + Cr_2O_7^{2-} \longrightarrow 2Cr^{3+} + 6Fe^{3+} + 7H_2O$

At equivalence point

$$(\text{Number of gram equivalents})_{OA} = (\text{Number of gram equivalents})_{RA}$$

$$(15 \times 0.02 \times 6)_{K_2Cr_2O_7} = (10 \times M \times 1)_{Fe^{2+}}$$

$$M = 18 \times 10^{-2} M$$

8. CO_2 gas is bubbled through water during a soft drink manufacturing process at 298 K. If CO_2 exerts a partial pressure of 0.835 bar then x m mol of CO_2 would dissolve in 0.9 L of water. The value of x is _____. (Nearest integer)

(Henry's law constant for CO_2 at 298 K is 1.67×10^3 bar)

Answer (25)

Sol. According to Henry's law

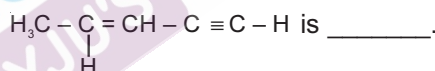
$$P_{\text{gas}} = X_{\text{gas}} \times K_H$$

$$X_{\text{gas}} = \frac{0.835}{1.67 \times 10^3} = 5 \times 10^{-4}$$

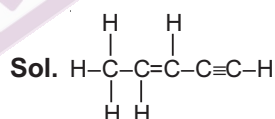
$$X_{\text{gas}} = \frac{n_{\text{gas}}}{n_{\text{gas}} + n_{H_2O}} \approx \frac{n_{\text{gas}}}{n_{H_2O}}$$

$$n_{\text{gas}} = 5 \times 10^{-4} \times \frac{900}{18} = 0.025 \text{ mole or } 25 \text{ mmol}$$

9. The number of sigma bonds in



Answer (10)



\rightarrow 10 sigma bonds and 3 pi bonds.

10. A home owner uses $4.00 \times 10^3 \text{ m}^3$ of methane (CH_4) gas, (assume CH_4 is an ideal gas) in a year to heat his home. Under the pressure of 1.0 atm and 300 K, mass of gas used is $x \times 10^5$ g. The value of x is _____. (Nearest integer)

(Given $R = 0.083 \text{ L atm K}^{-1} \text{ mol}^{-1}$)

Answer (26)

Sol. $PV = nRT$

$$\text{Mass} = \frac{1 \times 4 \times 10^6}{0.083 \times 300} \times 16$$

$$= 25.7 \times 10^5 \text{ g}$$

$$\approx 26 \times 10^5 \text{ g}$$

Sol. $S_{3n} = \frac{3n}{2} [2a + (3n-1)d]$

$S_{2n} = n[2a + (2n-1)d]$

(where a is first term & d is common difference of A.P.)

if $S_{3n} = 3S_{2n}$

$\Rightarrow \frac{3}{2} [2a + (3n-1)d] = 3 [2a + (2n-1)d]$

$\Rightarrow 2a + (3n-1)d = 4a + 2(2n-1)d$

$\Rightarrow 2a = (3n-1-4n+2)d$

$\Rightarrow \frac{a}{d} = \frac{1-n}{2} \dots(i)$

$\Rightarrow \frac{S_{4n}}{S_{2n}} = \frac{\frac{4n}{2} [2a + (4n-1)d]}{\frac{2n}{2} [2a + (2n-1)d]}$

$= \frac{2 \left[2 \left(\frac{1-n}{2} \right) + (4n-1) \right]}{\left[2 \left(\frac{1-n}{2} \right) + (2n-1) \right]}$

$= \frac{2(1-n+4n-1)}{1-n+2n-1} = \frac{2(3n)}{n} = 6$

5. The value of the definite integral $\int_{\pi/24}^{5\pi/24} \frac{dx}{1+\sqrt[3]{\tan 2x}}$

is

(1) $\frac{\pi}{18}$

(2) $\frac{\pi}{6}$

(3) $\frac{\pi}{3}$

(4) $\frac{\pi}{12}$

Answer (4)

Sol. $I = \int_{\pi/24}^{5\pi/24} \frac{dx}{1+(\tan 2x)^{1/3}}$

applying $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$, we get

$I = \int_{\pi/24}^{5\pi/24} \frac{dx}{1+\left(\tan\left(\frac{\pi}{2}-2x\right)\right)^{1/3}}$

$2I = \int_{\pi/24}^{5\pi/24} \frac{dx}{1+(\tan 2x)^{2/3}} + \int_{\pi/24}^{5\pi/24} \frac{(\tan 2x)^{1/3}}{1+(\tan 2x)^{1/3}} dx$

$= \int_{\pi/24}^{5\pi/24} dx$

$2I = \frac{5\pi}{24} - \frac{\pi}{24} \Rightarrow I = \frac{\pi}{12}$

6. Let the vectors

$(2+a+b)\hat{i} + (a+2b+c)\hat{j} - (b+c)\hat{k}$,

$(1+b)\hat{i} + 2b\hat{j} - b\hat{k}$ and

$(2+b)\hat{i} + 2b\hat{j} + (1-b)\hat{k}, a, b, c \in \mathbb{R}$

be co-planar. Then which of the following is true?

(1) $3c = a + b$

(2) $2b = a + c$

(3) $2a = b + c$

(4) $a = b + 2c$

Answer (2)

Sol. For coplanarity S.T.P. of given vectors shall vanish

i.e. $\begin{vmatrix} 2+a+b & a+2b+c & -(b+c) \\ 1+b & 2b & -b \\ 2+b & 2b & (1-b) \end{vmatrix} = 0$

$R_3 \rightarrow R_3 - R_2$ and $R_1 \rightarrow R_1 - R_2$

$\begin{vmatrix} 1+a & a+c & -c \\ 1+b & 2b & -b \\ 1 & 0 & 1 \end{vmatrix} = 0$

Expanding

$\therefore 1(-b(a+c) + 2bc) + 1(2b(1+a) - (b+1)(a+c)) = 0$

$\therefore a + c = 2b$

7. If b is very small as compared to the value of a, so

that the cube and other higher powers of $\frac{b}{a}$ can be neglected in the identity

$\frac{1}{a-b} + \frac{1}{a-2b} + \frac{1}{a-3b} + \dots + \frac{1}{a-nb} = \alpha n + \beta n^2 + \gamma n^3$

then the value of γ is

(1) $\frac{a+b^2}{3a^3}$

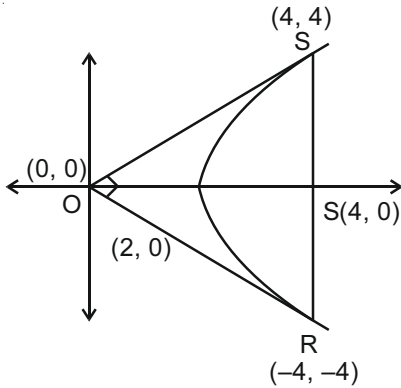
(2) $\frac{a+b}{3a^2}$

(3) $\frac{a^2+b}{3a^3}$

(4) $\frac{b^2}{3a^3}$

Sol. From given condition y axis is directrix and equation parabola is

$$y^2 = 8(x - 2)$$



Let $y = mx$ be tangent

$$\therefore m^2x^2 - 8x + 16 = 0 \Rightarrow m = \pm 1$$

So that lines $y = x$ and $y = -x$ are tangents.

\therefore Coordinate of S and R be (4, 4) and (-4, 4)

$$\therefore \text{area of } \Delta \text{SOR} = \left(\frac{1}{2} \times 4 \times 4\right) \times 2 = 16$$

12. Let $g : \mathbb{N} \rightarrow \mathbb{N}$ be defined as

$$g(3n + 1) = 3n + 2,$$

$$g(3n + 2) = 3n + 3,$$

$$g(3n + 3) = 3n + 1, \text{ for all } n \geq 0.$$

Then which of the following statements is true?

- (1) $g \circ g \circ g = g$
- (2) There exists an onto function $f : \mathbb{N} \rightarrow \mathbb{N}$ such that $f \circ g = f$
- (3) There exists a one-one function $f : \mathbb{N} \rightarrow \mathbb{N}$ such that $f \circ g = f$
- (4) There exists a function $f : \mathbb{N} \rightarrow \mathbb{N}$ such that $g \circ f = f$

Answer (2)

Sol. $\therefore g(3n + 1) = 3n + 2, g(3n + 2) = 3n + 3$ and $g(3n + 3) = 3n + 1$

$$\therefore g \circ g \circ g(3n + 1) = g(g(g(3n + 1))) = g(g(3n + 2)) = g(3n + 3) = 3n + 1$$

Similarly we can see that $g \circ g \circ g = x$ (identity)

For $f \circ g = f$ to hold

'f' must be an onto function

13. Let $f(x) = 3\sin^4x + 10\sin^3x + 6\sin^2x - 3$, $x \in \left[-\frac{\pi}{6}, \frac{\pi}{2}\right]$. Then, f is

- (1) Decreasing in $\left(-\frac{\pi}{6}, 0\right)$
- (2) Decreasing in $\left(0, \frac{\pi}{2}\right)$
- (3) Increasing in $\left(-\frac{\pi}{6}, 0\right)$
- (4) Increasing in $\left(-\frac{\pi}{6}, \frac{\pi}{2}\right)$

Answer (1)

Sol. $f(x) = 3\sin^4x + 10\sin^3x + 6\sin^2x - 3$

$$\begin{aligned} f'(x) &= 12\sin^3x \cdot \cos x + 30\sin^2x \cos x + 12\sin x \cdot \cos x \\ &= 3\sin 2x(2\sin^2x + 5\sin x + 2) \\ &= 3\sin 2x \cdot (\sin x + 2)(2\sin x + 1) \end{aligned}$$

\therefore Changing points are $-\frac{\pi}{6}, 0$ and $\frac{\pi}{2}$



$\therefore f(x)$ is decreasing in $\left(-\frac{\pi}{6}, 0\right)$

and increasing in $\left(0, \frac{\pi}{2}\right)$

14. Let $y = y(x)$ be the solution of the differential

$$\text{equation } \frac{dy}{dx} = 1 + xe^{y-x}, -\sqrt{2} < x < \sqrt{2}, y(0) = 0$$

then, the minimum value of $y(x), x \in (-\sqrt{2}, \sqrt{2})$ is equal to

- (1) $(2 - \sqrt{3}) - \log_e 2$
- (2) $(1 - \sqrt{3}) - \log_e(\sqrt{3} - 1)$
- (3) $(1 + \sqrt{3}) - \log_e(\sqrt{3} - 1)$
- (4) $(2 + \sqrt{3}) + \log_e 2$

Answer (2)

Sol. $\frac{dy}{dx} = 1 + xe^{y-x}$

$$\Rightarrow e^{-y} \frac{dy}{dx} = e^{-y} + xe^{-x}$$

Let $e^{-y} = t \Rightarrow -e^{-y} \frac{dy}{dx} = \frac{dt}{dx}$

$$-\frac{dt}{dx} = t + xe^{-x}$$

$$\frac{dt}{dx} + t = -xe^{-x}$$

\therefore Integrating function $= e^{\int 1 dx} = e^x$

\therefore Solution is : $t \cdot e^x = \int -xe^{-x} \cdot e^x dx$

$$t \cdot e^x = -\frac{x^2}{2} + c$$

$$e^{x-y} = -\frac{x^2}{2} + c$$

$\therefore y(0) = 0 \Rightarrow c = 1$

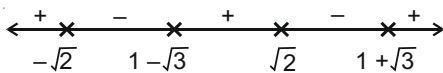
$\therefore e^{x-y} = 1 - \frac{x^2}{2} \Rightarrow y(x) = x - \ln\left(1 - \frac{x^2}{2}\right)$

Now $g'(x) = 1 - \frac{1}{1 - \frac{x^2}{2}}(0 - x)$

$$= 1 + \frac{x}{1 - \frac{x^2}{2}} = \frac{2 - x^2 + 2x}{2 - x^2}$$

$$= \frac{(x-1)^2 - 3}{x^2 - 2}$$

\therefore Critical points are $-\sqrt{2}, \sqrt{2}, 1 - \sqrt{3}, 1 + \sqrt{3}$



$\therefore x \in (-\sqrt{2}, \sqrt{2})$ then $x = 1 - \sqrt{3}$ is point of local minima

\therefore Minimum value of $y(x) = y(1 - \sqrt{3})$

$$= (1 - \sqrt{3}) - \log_e \left(1 - \frac{(1 - \sqrt{3})^2}{2} \right)$$

$$= (1 - \sqrt{3}) - \log_e (\sqrt{3} - 1)$$

15. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as

$$f(x) = \begin{cases} \frac{\lambda |x^2 - 5x + 6|}{\mu(5x - x^2 - 6)}, & x < 2 \\ \frac{\tan(x-2)}{e^{x-(x)}} & , x > 2 \\ \mu & , x = 2 \end{cases}$$

where $[x]$ is the greatest integer less than or equal to x . If f is continuous at $x = 2$, then $\lambda + \mu$ is equal to

(1) 1 (2) $e(e - 2)$

(3) $e(-e + 1)$ (4) $2e - 1$

Answer (3)

Sol. For continuity $\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x) = f(2)$

$$\therefore \lim_{x \rightarrow 2^-} \frac{\lambda |(x-2)(x-3)|}{\mu(x-2)(3-x)} = \lim_{x \rightarrow 2^+} e^{\frac{\tan(x-2)}{x-2}} = \mu$$

$$\Rightarrow \frac{\lambda(-1)}{\mu(1)} = e = \mu$$

$$\Rightarrow \mu = e \text{ and } \lambda = e^2$$

$$\lambda + \mu = e - e^2 = e(1 - e)$$

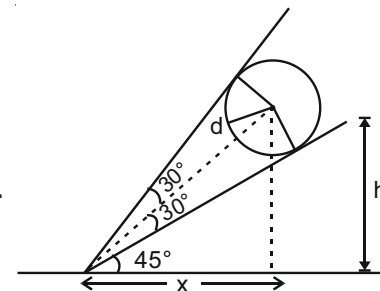
16. A spherical gas balloon of radius 16 meter subtends an angle 60° at the eye of the observer A while the angle of elevation of its center from the eye of A is 75° . Then the height (in meter) of the top most point of the balloon from the level of the observer's eye is

(1) $8(2 + 2\sqrt{3} + \sqrt{2})$ (2) $8(\sqrt{6} - \sqrt{2} + 2)$

(3) $8(\sqrt{2} + 2 + \sqrt{3})$ (4) $8(\sqrt{6} + \sqrt{2} + 2)$

Answer (4)

Sol.



$$\tan 45^\circ = \frac{h}{x}$$

Sol. Let $e^y = y$, $\frac{dy}{dx} - 2y \sin x = -\sin x \cdot \cos^2 x$

$$\text{I.F.} = e^{2\cos x}$$

$$\Rightarrow y \cdot e^{2\cos x} = -\int e^{2\cos x} (\sin x \cdot \cos^2 x) dx + C$$

$$\Rightarrow e^{2\cos x} \cdot e^y = \frac{1}{4} e^{2\cos x} (2\cos^2 x - 2\cos x + 1) + C$$

$$\therefore y\left(\frac{\pi}{2}\right) = 0 \Rightarrow C = \frac{3}{4}$$

$$\text{Now, } y(0) = \ln\left(\frac{1}{4} + \frac{3}{4}e^{-2}\right)$$

$$\Rightarrow \alpha = \frac{1}{4} \text{ and } \beta = \frac{3}{4}$$

4. Let $S = \left\{ n \in \mathbb{N} \mid \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}^n \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \forall a, b, c, d \in \mathbb{R} \right\}$,

where $i = \sqrt{-1}$. Then the number of 2-digit numbers in the set S is _____.

Answer (11)

Sol. Let $B = \begin{bmatrix} 0 & i \\ 1 & 0 \end{bmatrix}$

$$\Rightarrow B^2 = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$$

$$\Rightarrow B^4 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow B^8 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

hence n must be a multiple of 8.

So n = 16, 24, 32,, 96

No. of values of n = 11.

5. If α, β are roots of the equation $x^2 + 5(\sqrt{2})x + 10 = 0$, $\alpha > \beta$ and $P_n = \alpha^n - \beta^n$ for each positive integer n, then the value of

$$\left(\frac{P_{17}P_{20} + 5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19} + 5\sqrt{2}P_{18}^2} \right) \text{ is equal to } \underline{\hspace{2cm}}.$$

Answer (1)

Sol. $\therefore P_n + 5\sqrt{2} P_{n-1} = -10P_{n-2}$

$$\frac{P_{17}(P_{20} + 5\sqrt{2} P_{19})}{P_{18}(P_{19} + 5\sqrt{2} P_{18})} = \frac{P_{17} \cdot (-10 P_{18})}{P_{18} \cdot (-10 P_{17})} = 1$$

6. Consider the following frequency distribution :

Class :	10-20	20-30	30-40	40-50	50-60
Frequency :	α	110	54	30	β

If the sum of all frequencies is 584 and median is 45, then $|\alpha - \beta|$ is equal to _____.

Answer (164)

Sol.

C.I.	x_i	f_i	$x_i \cdot f_i$	C.F.
10-20	15	α	15α	α
20-30	25	110	2750	$110 + \alpha$
30-40	35	54	1890	$164 + \alpha$
40-50	45	30	1350	$194 + \alpha$
50-60	55	β	55β	$194 + \alpha + \beta$
		$194 + \alpha + \beta$	$5990 + 15\alpha + 55\beta$	median class

$$\therefore \alpha + \beta = 584 - 194$$

$$\Rightarrow \alpha + \beta = 390 \quad \dots(1)$$

$$\text{and median} = 40 + \left(\frac{194 + \alpha - 292}{30} \right) 10 = 45$$

$$\Rightarrow \alpha = 113 \quad \dots(2)$$

$$\text{So } \beta = 277$$

7. Let $M = \left\{ A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \{\pm 3, \pm 2, \pm 1, 0\} \right\}$.

Define $f : M \rightarrow \mathbb{Z}$, as $f(A) = \det(A)$, for all $A \in M$, where \mathbb{Z} is set of all integers. Then the number of $A \in M$ such that $f(A) = 15$ is equal to _____.

Answer (16)

Sol. $f(A) = 15 \Rightarrow ad - bc = 15$

$$(ad, bc) = (9, -6) \text{ or } (6, -9)$$

(i) Number of ways to select (a, d) = 2

Number of ways to select (b, c) = 4

(ii) Number of ways to select (a, d) = 4

Number of ways to select (b, c) = 2

Total number of possible matrix A = $2 \times 4 + 2 \times 4$

$$= 16$$

8. The ratio of the coefficient of the middle term in the expansion of $(1+x)^{20}$ and the sum of the coefficients of two middle terms in expansion of $(1+x)^{19}$ is _____.

Answer (1)

Sol. Coefficient of middle term in $(1+x)^{20} = {}^{20}C_{10}$

Sum of coefficient of two middle terms in

$$(1+x)^{19} = {}^{19}C_9 + {}^{19}C_{10} = {}^{20}C_{10}$$

9. If the value of

$$\left(1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \dots \text{upto } \infty\right)^{\log_{(0.25)}\left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots \text{upto } \infty\right)}$$

is l , then l^2 is equal to _____.

Answer (3)

Sol. $S_{\infty} = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \dots \infty$

$$\frac{1}{-3} S_{\infty} = \frac{1}{-3} + \frac{2}{-3^2} + \frac{6}{-3^3} + \dots \infty$$

$$\frac{2}{3} S_{\infty} = 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \dots \infty$$

$$\frac{2}{3} S_{\infty} = \frac{4}{3} \left(1 + \frac{1}{3} + \frac{1}{3^2} + \dots \infty\right) = \frac{4}{3} \times \frac{1}{1 - \frac{1}{3}}$$

$$\therefore S_{\infty} = 3.$$

$$\begin{aligned} \therefore \text{Given expression} &= 3^{\log_{\left(\frac{1}{4}\right)}\left(\frac{1}{2}\right)} \\ &= \sqrt{3} = l \\ &= l^2 = 3 \end{aligned}$$

10. The term independent of 'x' in the expansion of

$$\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}\right)^{10}, \text{ where } x \neq 0, 1 \text{ is}$$

equal to _____.

Answer (210)

Sol. $\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}\right)^{10} = \left(\left(x^{1/3} + 1\right) - \frac{\sqrt{x} + 1}{\sqrt{x}}\right)^{10}$

$$= \left(x^{1/3} - x^{-1/2}\right)^{10}$$

$$T_{r+1} = {}^{10}C_r \cdot x^{\frac{10-r}{3}} \cdot x^{-\frac{r}{2}}$$

$$\therefore \frac{10-r}{3} = \frac{r}{2} \Rightarrow r = 4$$

put $r = 4, T_5 = {}^{10}C_4 = 210$

