27/07/2021
Evening

Aakash + BYJU'S

**Corporate Office:** Aakash Tower, 8, Pusa Road, New Delhi-110005 Ph. 011-47623456

Time : 3 hrs.

M.M. : 300

# Answers & Solutions

*for*

# JEE (MAIN)-2021 (Online) Phase-3

## (Physics, Chemistry and Mathematics)

**IMPORTANT INSTRUCTIONS :**

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part has two sections.

(i) Section-I : This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) Section-II : This section contains 10 questions. In Section-II, attempt any **five questions out of 10.** There will be **no negative marking for Section-II**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and there is no negative marking for wrong answer.

# PART–A : PHYSICS

## SECTION - I

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer :**

1. An electron and proton are separated by a large distance. The electron starts approaching the proton with energy 3 eV. The proton captures the electron and forms a hydrogen atom in second excited state. The resulting photon is incident on a photosensitive metal of threshold wavelength 4000 Å. What is the maximum kinetic energy of the emitted photoelectron?

(1) 3.3 eV

(2) No photoelectron would be emitted

(3) 1.41 eV

(4) 7.61 eV

**Answer (3)**

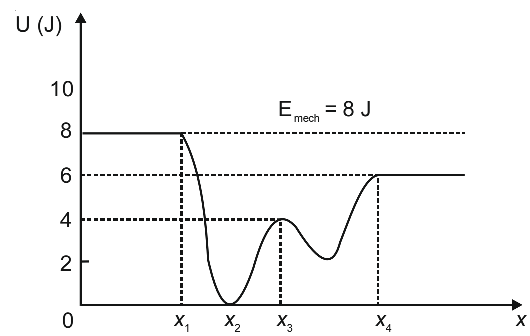
**Sol.** $\phi_0 = 3.1\,\text{eV}$

$$E_1 = \frac{-13.6}{9}\ \text{eV}$$

$$\therefore\ E_{photo} = \left(3.0 + \frac{13.6}{9}\right)\ \text{eV} = 4.511$$

$\therefore$ KE of photoelectron = 4.511 – 3.1 = 1.41 eV

2. Given below is the plot of a potential energy function U($x$) for a system, in which a particle is in one dimensional motion, while a conservative force F($x$) acts on it. Suppose that $E_{mech}$ = 8 J, the incorrect statement for this system is :




[where K.E. = kinetic energy]

(1) At $x < x_1$, K.E. is smallest and the particle is moving at the slowest speed.

(2) At $x > x_4$, K.E. is constant throughout the region.

(3) At $x = x_2$, K.E. is greatest and the particle is moving at the fastest speed.

(4) At $x = x_3$, K.E. = 4 J.

**Answer (1)**

**Sol.** At $x < x_1$, KE = 0

$\Rightarrow$ Particle is at rest

3. A particle of mass M originally at rest is subjected to a force whose direction is constant but magnitude varies with time according to the relation

$$F = F_0\left[1 - \left(\frac{t - T}{T}\right)^2\right]$$

Where $F_0$ and T are constants. The force acts only for the time interval 2T. The velocity $v$ of the particle after time 2T is :

(1) $\frac{F_0T}{3M}$

(2) $\frac{F_0T}{2M}$

(3) $\frac{2F_0T}{M}$

(4) $\frac{4F_0T}{3M}$

**Answer (4)**

**Sol.** $F = F_0\left[1 - \left(\frac{t-T}{T}\right)^2\right]$

$$a = \frac{F_0}{M}\left[1 - \left(\frac{t-T}{T}\right)^2\right]$$

$$\Rightarrow \int_0^v dv = \int \frac{F_0}{M\times T^2}\left[T^2 - \left(t^2 + T^2 - 2tT\right)\right]dt$$

$$\Rightarrow V = \frac{F_0}{MT^2}\times\left[2T\times\frac{t^2}{2} - \frac{t^3}{3}\right]_0^{2T}$$

$$= \frac{F_0}{MT^2}\times\left[T\times 4T^2 - \frac{1}{3}\times 8T^3\right]$$

$$= \frac{F_0}{MT^2}\times\frac{4T^3}{3} = \frac{4F_0T}{3M}$$

4. One mole of an ideal gas is taken through an adiabatic process where the temperature rises from 27°C to 37°C. If the ideal gas is composed of polyatomic molecule that has 4 vibrational modes, which of the following is true?

[R = 8.314 J mol$^{-1}$ K$^{-1}$]

(1) Work done by the gas is close to 332 J

(2) Work done on the gas is close to 332 J

(3) Work done by the gas is close to 582 J

(4) Work done on the gas is close to 582 J

**Answer (4)**

**Sol.** $\Delta T = 37 - 27 = 10°C$

$f = 3 + 3 + 4 \times 2 = 14$

$\therefore \quad \gamma = 1 + \frac{2}{f} = 1 + \frac{2}{14} = \frac{8}{7}$

$\therefore \quad W = \frac{-nR\Delta T}{(\gamma - 1)} = \frac{-1 \times 8.314 \times 10}{\left(\frac{1}{7}\right)} = -582 \text{ J}$

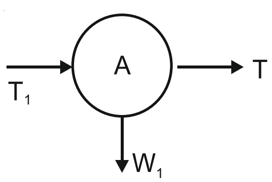
5. Two Carnot engines A and B operate in series such that engine A absorbs heat at $T_1$ and rejects heat to a sink at temperature T. Engine B absorbs half of the heat rejected by Engine A and rejects heat to the sink at $T_3$. When workdone in both the cases is equal, the value of T is

(1) $\frac{2}{3}T_1 + \frac{1}{3}T_3$ (2) $\frac{1}{3}T_1 + \frac{2}{3}T_3$

(3) $\frac{3}{2}T_1 + \frac{1}{3}T_3$ (4) $\frac{2}{3}T_1 + \frac{3}{2}T_3$

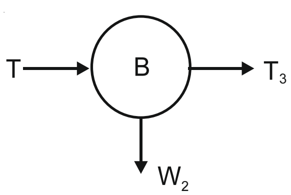
**Answer (1)**

**Sol.**




$W_1 = Q_1 - Q \Rightarrow \frac{W_1}{Q} = \frac{Q_1}{Q} - 1$

$= \frac{T_1}{T} - 1$ ... (i)




$W_2 = \frac{Q}{2} - Q_3 \Rightarrow \frac{W_2}{\frac{Q}{2}} = 1 - \frac{Q_3}{\frac{Q}{2}}$

$\Rightarrow \frac{2W_2}{Q} = 1 - \frac{T_3}{T}$ ...(ii)

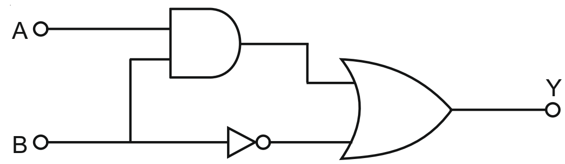
From (i) and (ii)

$2\left(\frac{T_1}{T} - 1\right) = 1 - \frac{T_3}{T}$

$\Rightarrow \frac{2T_1}{T} + \frac{T_3}{T} = 3$

$\Rightarrow T = \frac{2T_1}{3} + \frac{T_3}{3}$

6. Find the truth table for the function Y of A and B represented in the following figure




(1)

| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

(2)

| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

(3)

| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

(4)

| A | B | Y |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

**Answer (4)**

**Sol.**

| A | B | Y |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

7. Match **List I** with **List II**.

| **List I** | **List II** |
|---|---|
| (a) Capacitance, C | (i) $M^1 L^1 T^{-3} A^{-1}$ |
| (b) Permittivity of free space, $\varepsilon_0$ | (ii) $M^{-1} L^{-3} T^4 A^2$ |
| (c) Permeability of free space, $\mu_0$ | (iii) $M^{-1} L^{-2} T^4 A^2$ |
| (d) Electric field, E | (iv) $M^1 L^1 T^{-2} A^{-2}$ |

Choose the correct answer from the options given below

(1) (a) → (iii), (b) → (iv), (c) → (ii), (d) → (i)
(2) (a) → (iii), (b) → (ii), (c) → (iv), (d) → (i)
(3) (a) → (iv), (b) → (ii), (c) → (iii), (d) → (i)
(4) (a) → (iv), (b) → (iii), (c) → (ii), (d) → (i)

**Answer (2)**

**Sol.** (a) $U = \frac{Q^2}{2C}$

$\Rightarrow [C] = \frac{I^2t^2}{U} = \frac{A^2T^2}{[ML^2T^{-2}]} = [M^{-1}L^{-2}T^4A^2]$

(b) $[\varepsilon_0] \to [M^{-1}L^{-3}T^4A^2]$

(c) $[\mu_0] \to [MLT^{-2}A^{-2}]$

(d) $W = qE \times d \Rightarrow E = \frac{[ML^2T^{-2}]}{[AT][L]}$

8. A raindrop with radius R = 0.2 mm falls from a cloud at a height $h$ = 2000 m above the ground. Assume that the drop is spherical throughout its fall and the force of buoyance may be neglected, then the terminal speed attained by the raindrop is

[Density of water $f_w$ = 1000 kg m$^{-3}$ and

Density of air $f_a$ = 1.2 kg m$^{-3}$,

g = 10 m/s$^2$

Coefficient of viscosity of air = 1.8 × 10$^{-5}$ Nsm$^{-2}$]

(1) 250.6 ms$^{-1}$

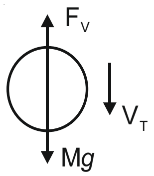
(2) 14.4 ms$^{-1}$

(3) 43.56 ms$^{-1}$

(4) 4.94 ms$^{-1}$

**Answer (4)**

**Sol.**




$$F_V = Mg$$

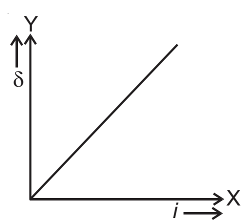
$$6\pi\eta R V_T = \left(\frac{4}{3}\pi R^3\right)\rho_w \times g$$

$$V_T = \frac{2R^2\rho_w g}{9\times\eta} = \frac{2\times(2\times10^{-4})^2\times1000\times10}{9\times1.8\times10^{-5}}$$

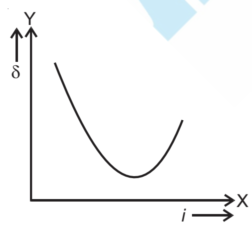
$\simeq 4.94$ m/s

9. The expected graphical representation of the variation of angle of deviation 'δ' with angle of incidence '$i$' in a prism is :

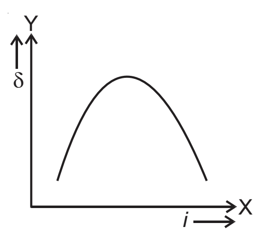
(1)



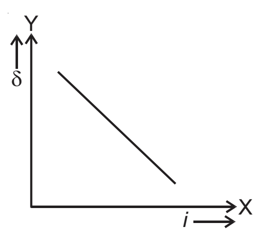

(2)




(3)

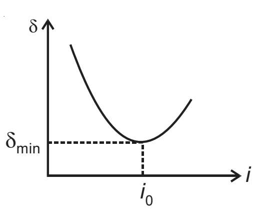



(4)

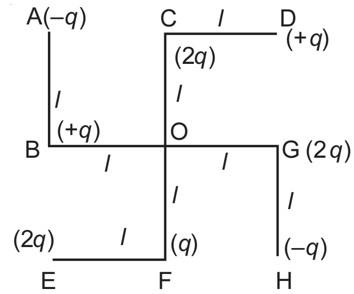



**Answer (2)**

**Sol.**




10. What will be the magnitude of electric field at point O as shown in figure? Each side of the figure is $l$ and perpendicular to each other?




(1) $\frac{q}{4\pi\varepsilon_0(2l)^2}$

(2) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{2l^2}(\sqrt{2})$

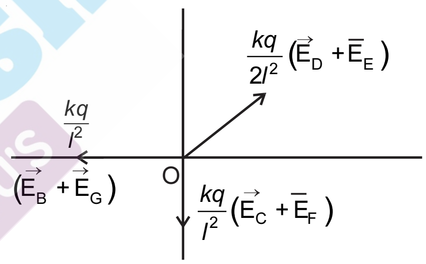
(3) $\frac{1}{4\pi\varepsilon_0}\frac{q}{l^2}$

(4) $\frac{1}{4\pi\varepsilon_0}\frac{q}{(2l^2)}(2\sqrt{2}-1)$

**Answer (4)**

**Sol.** $\bar{E}_0 = \bar{E}_A + \bar{E}_B + \bar{E}_C + \bar{E}_D + \bar{E}_E + \bar{E}_F + \bar{E}_G + \bar{E}_H$

$\bar{E}_A + \bar{E}_H = 0$




$$\Rightarrow |\bar{E}_0| = \left|\frac{kq}{2l^2} - \frac{kq}{l^2}\sqrt{2}\right|$$

$$= \frac{kq}{l^2}\left|\left(\frac{1}{2} - \sqrt{2}\right)\right|$$

$$= \frac{kq}{2l^2}\left(2\sqrt{2} - 1\right)$$

11. Figure A and B show two long straight wires of circular cross-section (a and b with a < b), carrying current I which is uniformly distributed across the cross-section. The magnitude of magnetic field B varies with radius r and can be represented as :

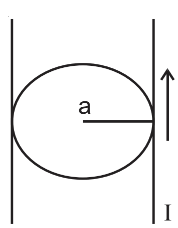



Fig. A

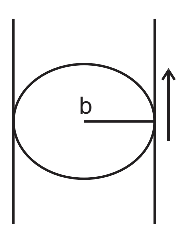
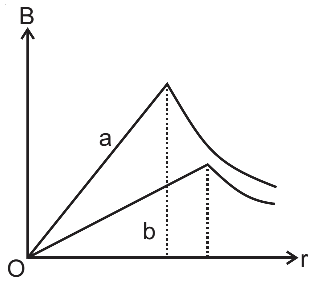
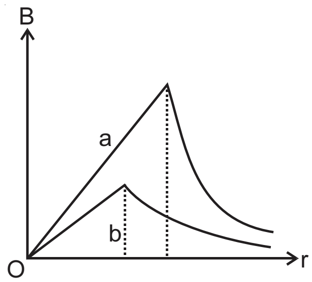



Fig. B

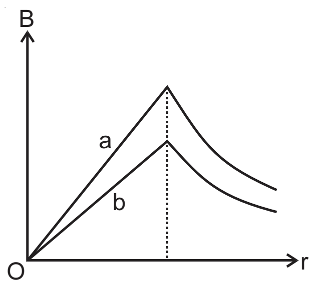
(1)



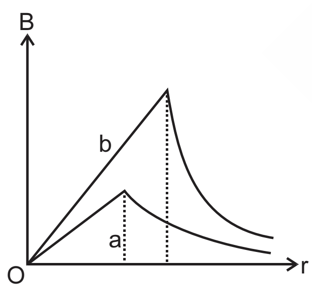

(2)




(3)

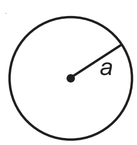



(4)




**Answer (1)**

**Sol.** For inside point




$$B \cdot 2\pi r = \frac{\mu_0 I \cdot \pi r^2}{\pi a^2}$$

$$B = \frac{\mu_0 I r}{2\pi a^2}$$

For outside point

$B \cdot 2\pi r = \mu_0 I$

$$B = \frac{\mu_0 I}{2\pi r}$$

For fig. A B (at $r = a$) $= \dfrac{\mu_0 I}{2\pi a}$

For fig. B B (at $r = b$) $= \dfrac{\mu_0 I}{2\pi b}$

So option (1) is correct.

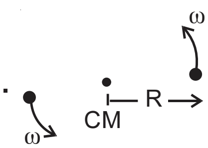
12. Two identical particles of mass 1 kg each go round a circle of radius R, under the action of their mutual gravitational attraction. The angular speed of each particle is:

(1) $\sqrt{\dfrac{G}{2R^3}}$ (2) $\sqrt{\dfrac{2G}{R^3}}$

(3) $\dfrac{1}{2R}\sqrt{\dfrac{1}{G}}$ (4) $\dfrac{1}{2}\sqrt{\dfrac{G}{R^3}}$

**Answer (4)**

**Sol.**




$$\frac{GM^2}{(2R)^2} = M\omega^2 R$$

$$\frac{GM}{4R^3} = \omega^2$$

$$\Rightarrow \quad \omega = \frac{1}{2}\sqrt{\frac{GM}{R^3}}$$

$$= \frac{1}{2}\sqrt{\frac{G}{R^3}}$$

Option 4 is correct.

13. Consider the following statements

A. Atoms of each element emit characteristics spectrum.

B. According to Bohr's Postulate, an electron in a hydrogen atom, revolves in a certain stationary orbit.

C. The density of nuclear matter depends on the size of the nucleus.

D. A free neutron is stable but a free proton decay is possible.

E. Radioactivity is an indication of the instability of nuclei.

Choose the correct answer from the options given below

(1) B and D only

(2) A, B and E only

(3) A, B, C, D and E

(4) A, C and E only

**Answer (2)**

**Sol.** Density of nuclear matter is independent of size. Free neutron can decay

$n \rightarrow P + e$

As $m_n > m_p + m_e$

So, A, B and E are correct

option 2 is correct

14. An automobile of mass '*m*' accelerates starting from origin and initially at rest, while the engine supplies constant power P. The position is given as a function of time by

(1) $\left(\frac{9P}{8m}\right)^{\frac{1}{2}} t^{\frac{3}{2}}$

(2) $\left(\frac{8P}{9m}\right)^{\frac{1}{2}} t^{\frac{3}{2}}$

(3) $\left(\frac{8P}{9m}\right)^{\frac{1}{2}} t^{\frac{2}{3}}$

(4) $\left(\frac{9m}{8P}\right)^{\frac{1}{2}} t^{\frac{3}{2}}$

**Answer (2)**

**Sol.** Power = P

So, K.E = P*t*

$$\frac{1}{2}mv^2 = Pt$$

$$v^2 = \frac{2P}{m}t$$

$$v = \sqrt{\frac{2P}{m}}t^{\frac{1}{2}}$$

$$dx = \sqrt{\frac{2P}{m}}t^{\frac{1}{2}}dt$$

$$\int dx = \sqrt{\frac{2P}{m}}\int_0^t t^{\frac{1}{2}}dt$$

$$x = \sqrt{\frac{2P}{m}}\frac{t^{3/2}}{3/2}$$

$$x = \left(\frac{8P}{9m}\right)^{\frac{1}{2}} t^{3/2}$$

15. An object of mass 0.5 kg is executing simple harmonic motion. Its amplitude is 5 cm and time period (T) is 0.2 s. What will be the potential energy of the object at an instant $t = \frac{T}{4}$ s starting from mean position. Assume that the initial phase of the oscillation is zero

(1) $6.2 \times 10^{-3}$ J

(2) $6.2 \times 10^{3}$ J

(3) 0.62 J

(4) $1.2 \times 10^{3}$ J

**Answer (3)**

**Sol.** $m = \frac{1}{2}$kg  A = 5 cm, T = 0.2 s

$x = A\sin\omega t$

$$P.E = \frac{1}{2}m\omega^2 A^2 \sin\omega t$$

$$= \frac{1}{2}m\omega^2 A^2 \sin\frac{2\pi}{T}\times\frac{T}{4}$$

$$= \frac{1}{2}m\omega^2 A^2$$

$$= \frac{1}{2}\times\frac{1}{2}\times\frac{4\pi^2}{(0.2)^2}\times\left(\frac{5}{100}\right)^2$$

$$= \frac{\pi^2}{16}$$

$\approx$ 0.62 J

16. A physical quantity '*y*' is represented by the formula $y = m^2 r^{-4} g^x l^{-\frac{3}{2}}$. If the percentage errors found in *y*, *m*, *r*, *l* and *g* are 18, 1, 0.5, 4 and *p* respectively, then find the value of *x* and *p*.

(1) $\frac{16}{3}$ and $\pm\frac{3}{2}$  (2) 8 and ± 2

(3) 4 and ± 3  (4) 5 and ± 2

**Answer (1)**

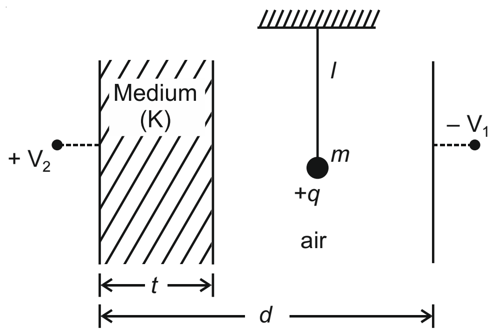
**Sol.** $\frac{\Delta y}{y} = \frac{2\Delta m}{m} + \frac{4\Delta r}{r} + \frac{x\Delta g}{g} + \frac{3}{2}\frac{\Delta l}{l}$

$$18 = 2\times1 + 4\times0.5 + \frac{3}{2}\times4 + px$$

8 = *px*

As per given option, option 1 is correct match

17. A simple pendulum of mass '*m*', length '*l*' and charge '+q' suspended in the electric field produced by two conducting parallel plates as shown. The value of deflection of pendulum in equilibrium position will be




(1) $\tan^{-1}\left[\frac{q}{mg}\times\frac{C_2(V_2-V_1)}{(C_1+C_2)(d-t)}\right]$

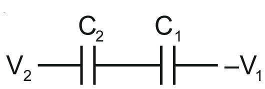
(2) $\tan^{-1}\left[\frac{q}{mg}\times\frac{C_1(V_2-V_1)}{(C_1+C_2)(d-t)}\right]$

(3) $\tan^{-1}\left[\frac{q}{mg}\times\frac{C_1(V_1+V_2)}{(C_1+C_2)(d-t)}\right]$

(4) $\tan^{-1}\left[\frac{q}{mg}\times\frac{C_2(V_1+V_2)}{(C_1+C_2)(d-t)}\right]$

**Answer (Bonus)**

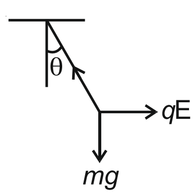
**Sol.** Potential across capacitor, $C_1 = \frac{C_2(V_1+V_2)}{(C_1+C_2)}$




Field in the region of $C_1 = \frac{C_2(V_1+V_2)}{(C_1+C_2)(d-t)}$

$T\cos\theta = mg$

$T\sin\theta = qE$




$$\tan\theta = \frac{qE}{mg}$$

$$\tan\theta = \frac{qC_2(V_1+V_2)}{(C_1+C_2)(d-t)mg}$$

In the given problem $C_1$ and $C_2$ is not mentioned, so option (3) and (4) both can be correct.

18. The planet Mars has two moons, if one of them has a period 7 hours, 30 minutes and an orbital radius of $9.0 \times 10^3$ km. Find the mass of Mars.

$$\left\{\text{Given}\frac{4\pi^2}{G} = 6\times10^{11}\ \text{N}^{-1}\text{m}^{-2}\ \text{kg}^2\right\}$$

(1) $3.25 \times 10^{21}$ kg  (2) $5.96 \times 10^{19}$ kg

(3) $6.00 \times 10^{23}$ kg  (4) $7.02 \times 10^{25}$ kg

**Answer (3)**

**Sol.** $m\omega^2 r = \frac{GMm}{r^2}$

$$\omega^2 = \frac{GM}{r^3}$$

$$\frac{4\pi^2}{T^2} = \frac{GM}{r^3}$$

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

$$M = \frac{4\pi^2 r^3}{GT^2} = \frac{6\times10^{11}\times729\times10^{18}}{T^2}$$

$M = 6.0 \times 10^{23}$ kg

19. The resistance of a conductor at 15°C is 16 Ω and at 100°C is 20 Ω. What will be the temperature coefficient of resistance of the conductor?

(1) $0.010°C^{-1}$

(2) $0.003°C^{-1}$

(3) $0.033°C^{-1}$

(4) $0.042°C^{-1}$

**Answer (2)**

**Sol.** $R_t = R_0[1 + \alpha(\Delta T)]$

$20 = 16[1 + \alpha(85)]$

$$\alpha = \frac{4}{16\times85}$$

$= 0.003°C^{-1}$

20. A 100 Ω resistance, a 0.1 μF capacitor and an inductor are connected in series across a 250 V supply at variable frequency. Calculate the value of inductance of inductor at which resonance will occur. Given that the resonant frequency is 60 HZ.

(1) 70.3 mH  (2) 70.3 H

(3) $7.03 \times 10^{-5}$ H  (4) 0.70 H

**Answer (2)**

**Sol.** $\omega = \frac{1}{\sqrt{LC}}$

$$L = \frac{1}{\omega^2 C} = \frac{1}{[120\pi]^2 10^{-7}}$$

$$L = \frac{10^7}{120^2\pi^2}\text{H}$$

$= 70.3$ H

## SECTION - II

**Numerical Value Type Questions:** This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. The maximum amplitude for an amplitude modulated wave is found to be 12 V while the minimum amplitude is found to be 3 V. The modulation index is 0.6*x* where x is __________.

**Answer (1)**

**Sol.** M + A = 12

A – M = 3

$$\Rightarrow \quad A = \frac{15}{2} \text{ and } M = \frac{9}{2}$$

$$m = \frac{M}{A} = \frac{9}{15} = 0.6$$

2. The difference in the number of waves when yellow light propagates through air and vacuum columns of the same thickness is one. The thickness of the air column is _______ mm.

[Refractive index of air = 1.0003, wavelength of yellow light in vacuum = 6000Å]

**Answer (2)**

**Sol.** $\Delta n = \frac{d}{\lambda_1} - \frac{d}{\lambda_2}$

$$1 = \frac{(\mu - 1)d}{\lambda_0}$$

$\Rightarrow \quad d = 6 \times 10^{-7}/.0003$

$= 2 \times 10^{-3}$ m

3. The $K_\alpha$ X-ray of molybdenum has wavelength 0.071 nm. If the energy of a molybdenum atom with a K electron knocked out is 27.5 keV, the energy of this atom when an L electron is knocked out will be _______ keV. (Round off to the nearest integer)

[$h = 4.14 \times 10^{-15}$ eVs, $c = 3 \times 10^8$ ms$^{-1}$]

**Answer (10)**

**Sol.** $\lambda_{K_\alpha} = \frac{hc}{E_K - E_L}$

$$E_L = E_K - \frac{hc}{\lambda_{K_\alpha}}$$

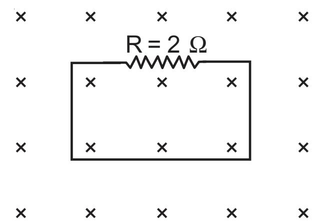
$$= 27.5 \times 10^3 - \frac{4.14 \times 10^{-15} \times 3 \times 10^8}{0.071 \times 10^{-9}} \text{ eV}$$

= 27.5 – 17.49 KeV

= 10 KeV

4. In the given figure the magnetic flux through the loop increases according to the relation $\phi_\beta(t) = 10t^2 + 20t$, where $\phi_\beta$ is in milliwebers and *t* is in seconds.

The magnitude of current through R = 2 Ω resistor at *t* = 5 s is _____ mA.




**Answer (60)**

**Sol.** $\varepsilon = \left|\frac{d\phi}{dt}\right| = (20t + 20) \times 10^{-3}$

$$i = \frac{\varepsilon}{R} = \frac{(20 \times 5 + 20)}{2} \times 10^{-3} = 60 \text{ mA}$$

5. A particle executes simple harmonic motion represented by displacement function as

$x(t) = A \sin(\omega t + \phi)$

If the position and velocity of the particle at *t* = 0 s are 2 cm and 2ω cm s$^{-1}$ respectively, then its amplitude is $x\sqrt{2}$ cm where the value of *x* is _____.

**Answer (2)**

**Sol.** | A sinϕ | = 2

| ωA cosϕ | = 2ω

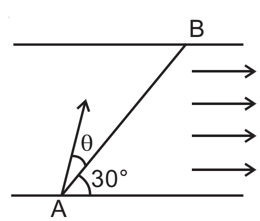
$$\Rightarrow \quad |\tan\phi| = 1 \quad \Rightarrow \quad \phi = \frac{\pi}{4}$$

$$x(0) = A\sin\left(\frac{\pi}{4}\right) = 2 \text{ cm}$$

$$\Rightarrow \quad A = 2\sqrt{2} \text{ cm}$$

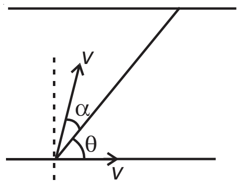
$\Rightarrow \quad x = 2$

6. A swimmer wants to cross a river from point A to point B. Line AB makes an angle of 30° with the flow of river. Magnitude of velocity of the swimmer is same as that of the river. The angle θ with the line AB should be _____°, so that the swimmer reaches point B.




**Answer (30)**

**Sol.**




For equal magnitude $v_s$ and $v_R$

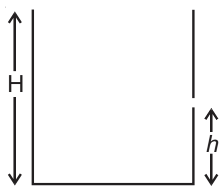
$\alpha = \theta$

$\Rightarrow \quad \alpha = 30°$

7. The water is filled upto height of 12 m in a tank having vertical sidewalls. A hole is made in one of the walls at a depth '*h*' below the water level. The value of '*h*' for which the emerging stream of water strikes the ground at the maximum range is _____ m.

**Answer (6)**

**Sol.**




For maximum range

$h = H - h$

$h = \dfrac{H}{2}$

$= \dfrac{12}{2}\text{m} = 6\text{ m}$

8. For the circuit shown, the value of current at time $t$ = 3.2 s will be _______ A.

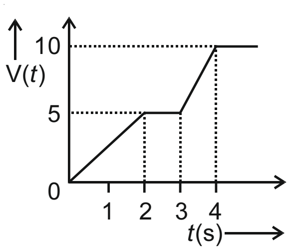



Figure 1

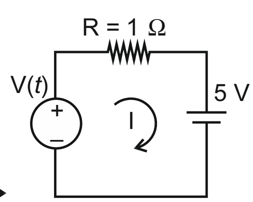



Figure 2

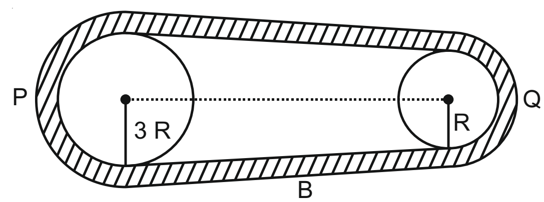
[Voltage distribution V($t$) is shown by Fig. (1) and the circuit is shown in Fig. (2)]

**Answer (1)**

**Sol.** V(3.2) = 6V

$$I = \frac{6-5}{1}\text{ A} = 1\text{ A}$$

9. In the given figure, two wheels P and Q are connected by a belt B. The radius of P is three times as that of Q. In case of same rotational kinetic energy, the ratio of rotational inertias $\left(\dfrac{I_1}{I_2}\right)$ will be $x$ : 1. The value of $x$ will be _______.




**Answer (9)**

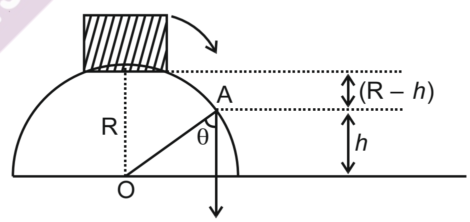
**Sol.** $\dfrac{1}{2}I_1\omega_1^2 = \dfrac{1}{2}I_2\omega_2^2$

$$\frac{1}{2}I_1\left(\frac{v}{R_1}\right)^2 = \frac{1}{2}I_2\left(\frac{v}{R_2}\right)^2$$

$$\frac{I_1}{I_2} = \left(\frac{R_2}{R_1}\right)^2 = 9$$

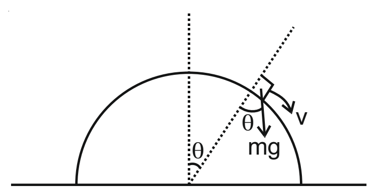
10. A small block slides down from the top of hemisphere of radius R = 3 m as shown in the figure. The height '*h*' at which the block will lose contact with the surface of the sphere is _____ m.

(Assume there is no friction between the block and the hemisphere)




**Answer (2)**

**Sol.**




$$mg\cos\theta = \frac{mv^2}{R}$$

$$\frac{1}{2}mv^2 = mgR(1-\cos\theta)$$

On solving $\cos\theta = \dfrac{2}{3}$

$$h = \frac{2R}{3} = 2\text{ m}$$

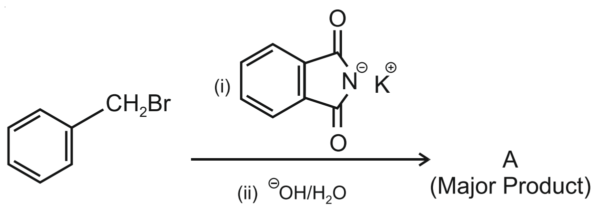
# PART–B : CHEMISTRY

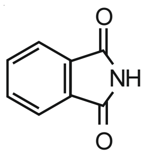
## SECTION - I

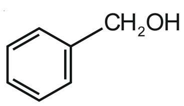
**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

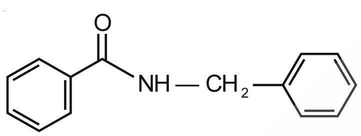
**Choose the correct answer :**

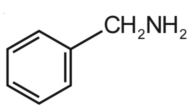
1. What is A in the following reaction ?



(1) 

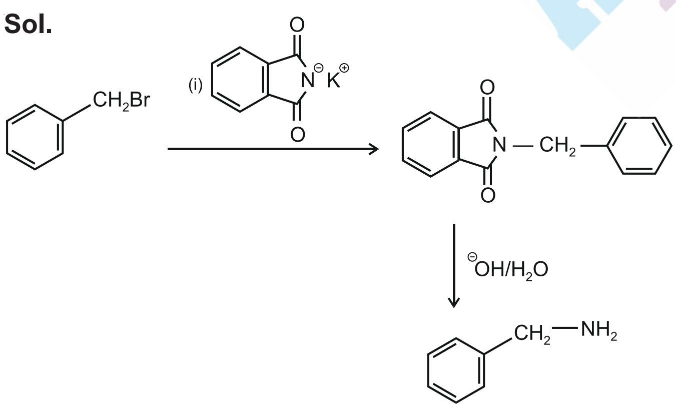
(2) 

(3) 

(4) 

**Answer (4)**

**Sol.**



(Major product)

2. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**.

**Assertion A :** $SO_2(g)$ is adsorbed to a larger extent than $H_2(g)$ on activated charcoal.

**Reason R :** $SO_2(g)$ has a higher critical temperature than $H_2(g)$.

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) **A** is correct but **R** is not correct

(2) Both **A** and **R** are correct but **R** is not the correct explanation of **A**

(3) **A** is not correct but **R** is correct

(4) Both **A** and **R** are correct and **R** is the correct explanation of **A**

**Answer (4)**

**Sol.** $SO_2$ has higher mass and larger surface area. It has higher critical temperature than $H_2$ and that's why it adsorbed to a larger extent.

3. If the Thompson model of the atom was correct, then the result of Rutherford's gold foil experiment would have been :

(1) All $\alpha$-particles get bounced back by 180°

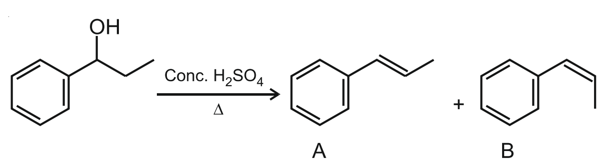
(2) $\alpha$-particles pass through the gold foil deflected by small angles and with reduced speed

(3) $\alpha$-particles are deflected over a wide range of angles

(4) All of the $\alpha$-particles pass through the gold foil without decrease in speed

**Answer (2)**

**Sol.** According to thompson model of atom, the mass of each gold atom is uniformly distributed. And as the $\alpha$-particles had enough energy to pass directly through such mass, it slowed down with small changes in its directions.

4. 

Consider the above reaction, and choose the correct statement :

(1) Both compounds **A** and **B** are formed equally

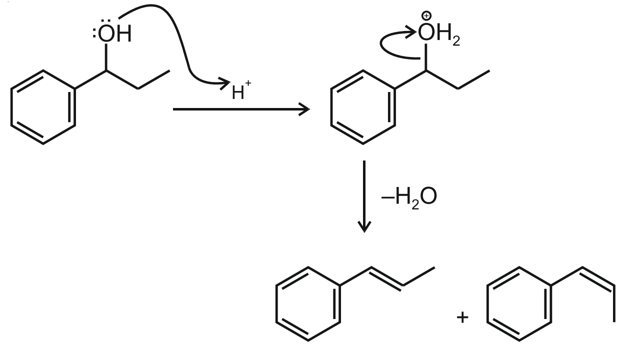
(2) Compound **A** will be the major product

(3) Compound **B** will be the major product

(4) The reaction is not possible in acidic medium

**Answer (2)**

**Sol.**



A (Major) (Greater stability of alkene) + B (Minor)

5. To an aqueous solution containing ions such as $Al^{3+}$, $Zn^{2+}$, $Ca^{2+}$, $Fe^{3+}$, $Ni^{2+}$, $Ba^{2+}$ and $Cu^{2+}$ was added conc. HCl, followed by $H_2S$.

The total number of cations precipitated during this reaction is/are :

(1) 2 (2) 1

(3) 3 (4) 4

**Answer (2)**

**Sol.** Only group I and group II cations will get precipitated.

∴ Only $Cu^{2+}$ gets precipitated here.

6. The CORRECT order of first ionisation enthalpy is:

(1) Al < Mg < S < P (2) Mg < Al < P < S

(3) Mg < S < Al < P (4) Mg < Al < S < P

**Answer (1)**

**Sol.** All elements belong to 3rd period in periodic table.

Al < Mg < S < P

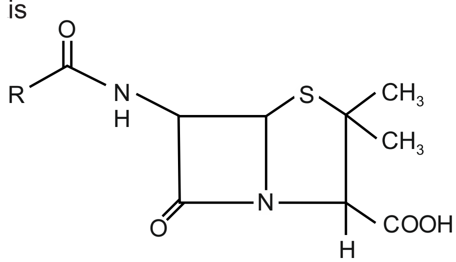
Mg: Electron is removing from *s* subshell

P: $3s^2 3p^3$ Half filled configuration

7. Given below are two statements :

**Statement I :** Penicillin is a bacteriostatic type antibiotic.

**Statement II :** The general structure of Penicillin is



Choose the correct option :

(1) Statement I is incorrect but Statement II is true

(2) Statement I is correct but Statement II is false

(3) Both Statement I and Statement II are true

(4) Both Statement I and Statement II are false

**Answer (1)**

**Sol.** Penicillin is a bactericidal antibiotics.

8. The addition of silica during the extraction of copper from its sulphide ore

(1) Converts iron oxide into iron silicate

(2) Converts copper sulphide into copper silicate

(3) Reduces copper sulphide into metallic copper

(4) Reduces the melting point of the reaction mixture

**Answer (1)**

**Sol.** Silica converts iron oxide into iron silicate.

$$FeO + SiO_2 \longrightarrow FeSiO_3$$

9. Given below are two statements :

**Statement I :** $[Mn(CN)_6]^{3-}$, $[Fe(CN)_6]^{3-}$ and $[Co(C_2O_4)_3]^{3-}$ are $d^2sp^3$ hybridised.

**Statement II :** $[MnCl_6]^{3-}$ and $[FeF_6]^{3-}$ are paramagnetic and have 4 and 5 unpaired electrons, respectively.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both statement I and statement II are true

(2) Statement I is correct but statement II is false

(3) Both statement I and statement II are false

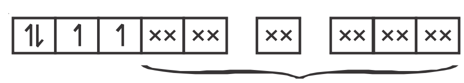
(4) Statement I is incorrect but statement II is true

**Answer (1)**

**Sol.** $[Mn(CN)_6]^{3-}$

$Mn^{3+} \longrightarrow 3d^4 4s^0$

$CN^-$ is a strong field ligand.




$[Co(C_2O_4)_3]^{3-} \longrightarrow d^2sp^3$ (0 unpaired electron)

$[Fe(CN)_6]^{3-} \longrightarrow d^2sp^3$ (1 unpaired electron)

$Cl^-$ and $F^-$ are weak field.

So, $[MnCl_6]^{3-} \longrightarrow sp^3d^2$ (4 unpaired electron)

$[FeF_6]^{3-} \longrightarrow sp^3d^2$ (5 unpaired electron)

10. Given below are two statements :

Statement I : Hyperconjugation is a permanent effect.

Statement II : Hyperconjugation in ethyl cation $\left(CH_3 - \overset{+}{C}H_2\right)$ involves the overlapping of $C_{sp^2} - H_{1s}$ bond with empty 2p orbital of other carbon.

Choose the correct option:

(1) Both statement I and statement II are true

(2) Statement I is incorrect but statement II is true

(3) Statement I is correct but statement II is false

(4) Both statement I and statement II are false

**Answer (3)**

**Sol.** Hyperconjugation, inductive and mesomeric effect are permanent electronic effect.

In $CH_3 - \overset{\oplus}{C}H_2$

Overlapping of $C_{sp^3} - H_{1s}$ bond with empty $2p$ orbital of other carbon takes place.

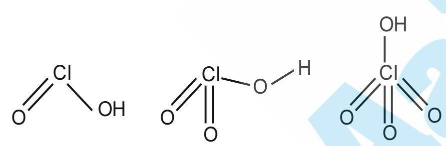
11. Number of Cl = O bonds in chlorous acid, chloric acid and perchloric acid respectively are

(1) 1, 2 and 3 (2) 4, 1 and 0

(3) 1, 1 and 3 (4) 3, 1 and 1

**Answer (1) (Bonus*)**

**Sol.**




| Number of Cl = O bonds. | Chlorous acid | Chloric acid | Perchloric acid |
|---|---|---|---|
| | 1 | 2 | 3 |

12. Match List-I with List-II :

| List-I | List-II |
|---|---|
| (a) Li | (i) Photoelectric cell |
| (b) Na | (ii) Absorbent of $CO_2$ |
| (c) K | (iii) Coolant in fast breeder nuclear reactor |
| (d) Cs | (iv) Treatment of cancer |
| | (v) Bearings for motor engines |

Choose the correct answer from the options given below :

(1) (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii)

(2) (a)-(v), (b)-(iii), (c)-(ii), (d)-(i)

(3) (a)-(v), (b)-(i), (c)-(ii), (d)-(iv)

(4) (a)-(v), (b)-(ii), (c)-(iv), (d)-(i)

**Answer (2)**

**Sol.** Li – (v) bearing for motor engines

Na – (iii) coolant in fast breeder reactor

K – (ii) absorbent of $CO_2$

Cs – (i) Photoelectric cell

13. $R - CN \xrightarrow[\text{(ii) } H_2O]{\text{(i) DIBAL-H}} R - Y$

Consider the above reaction and identify "Y".

(1) $-CH_2NH_2$

(2) –CHO

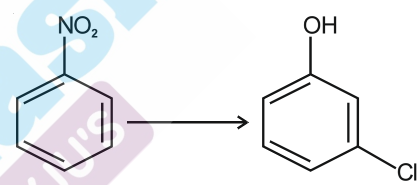
(3) –COOH

(4) $-CONH_2$

**Answer (2)**

**Sol.** $R - CN \xrightarrow[\text{ii) } H_2O]{\text{i) DIBAL} - H} R - CHO$

14. The correct sequence of correct reagents for the following transformation is




(1) (i) Fe, HCl

(ii) $NaNO_2$, HCl, 0°C

(iii) $H_2O/H^+$

(iv) $Cl_2$, $FeCl_3$

(2) (i) Fe, HCl

(ii) $Cl_2$, HCl

(iii) $NaNO_2$, HCl, 0°C

(iv) $H_2O/H^+$

(3) (i) $Cl_2$, $FeCl_3$

(ii) $NaNO_2$, HCl, 0°C

(iii) Fe, HCl

(iv) $H_2O/H^+$

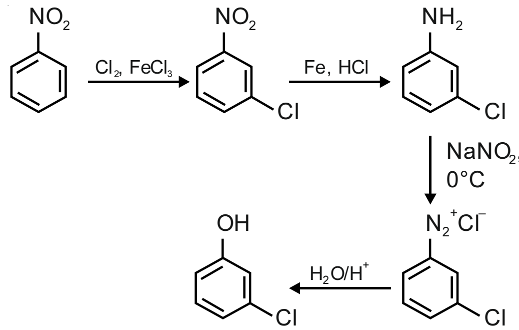
(4) (i) $Cl_2$, $FeCl_3$

(ii) Fe, HCl

(iii) $NaNO_2$, HCl, 0°C

(iv) $H_2O/H^+$

**Answer (4)**

**Sol.** 

15. Select the correct statements

    (A) Crystalline solids have long range order.

    (B) Crystalline solids are isotropic.

    (C) Amorphous solids are sometimes called pseudo solids.

    (D) Amorphous solids soften over a range of temperatures

    (E) Amorphous solids have a definite heat of fusion.

    Choose the most appropriate answer from the options given below

    (1) (A), (C), (D) only  (2) (C), (D) only

    (3) (B), (D) only  (4) (A), (B), (E) only

**Answer (1)**

**Sol.** Crystalline solids are anisotropic and having long range order.

Amorphous solids

- Pseudo solids.
- Softer over a range of temperature.
- do not have definite heat of fusion.

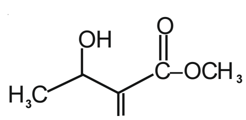
16. Compound A gives D-Galactose and D-Glucose on hydrolysis. The compound A is

    (1) Amylose  (2) Lactose

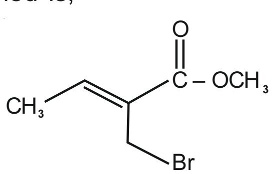
    (3) Maltose  (4) Sucrose

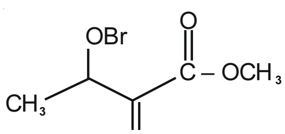
**Answer (2)**

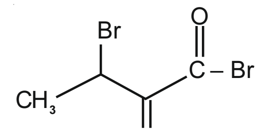
**Sol.** $\underset{\text{(Reducing sugar)}}{\text{Lactose}} \xrightarrow{\text{hydrolysis}} \beta-D-\text{galactose} + \beta-D-\text{glucose}$

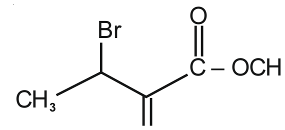
17.  $\xrightarrow{\text{Conc. HBr}}$ "P" (Major Product)

    Consider the above reaction, the major product "P" formed is,

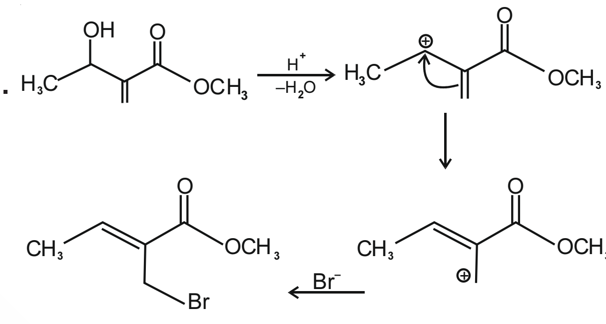
    (1) 

    (2) 

    (3) 

    (4) 

**Answer (1)**

**Sol.** 

(Major)

18. Match List-I with List-II

| List-I (compound) | List-II (effect/affected species) |
|---|---|
| (a) Carbon monoxide | (i) Carcinogenic |
| (b) Sulphur dioxide | (ii) Metabolized by pyrus plants |
| (c) Polychlorinated biphenyls | (iii) Haemoglobin |
| (d) Oxides of nitrogen | (iv) Stiffness of flower buds |

Choose the **correct** answer from the options given below :

(1) (a) - (i), (b) - (ii), (c) - (iii), (d) - (iv)

(2) (a) - (iii), (b) - (iv), (c) - (i), (d) - (ii)

(3) (a) - (iv), (b) - (i), (c) - (iii), (d) - (ii)

(4) (a) - (iii), (b) - (iv), (c) - (ii), (d) - (i)

**Answer (2)**

**Sol.** (a) CO ......... (iii) Haemoglobin

(b) $SO_2$ ......... (iv) Stiffness to flower buds

(c) Polychlorinated biphenyls ...(i) Carcinogenic

(d) Oxides of nitrogen ...(ii) Metabolized by pyrus plants

19. Which one of the following set of elements can be detected using sodium fusion extract?

    (1) Phosphorous, Oxygen, Nitrogen, Halogens

    (2) Nitrogen, Phosphorous, Carbon, Sulfur

    (3) Sulfur, Nitrogen, Phosphorous, Halogens

    (4) Halogens, Nitrogen, Oxygen, Sulfur

**Answer (3)**

**Sol.** Lassaigne's test – Sodium fusion extract

↓

Used for detection of N, S, X, P

X = halogen

20. The number of neutrons and electrons, respectively, present in the radioactive isotope of hydrogen is

(1) 1 and 1 (2) 2 and 1

(3) 2 and 2 (4) 3 and 1

**Answer (2)**

**Sol.** Radioactive isotope of hydrogen is Tritium

Tritium → 1 proton

1 electron

2 neutron

## SECTION - II

**Numerical Value Type Questions:** This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. The dihedral angle in staggered form of Newman projection of 1, 1, 1-Trichloro ethane is _____ degree. (Round off to the Nearest Integer).

**Answer (60)**

**Sol.** dihedral angle is 60°

2. The equilibrium constant for the reaction

$$A(s) \rightleftharpoons M(s) + \frac{1}{2}O_2(g)$$

is $K_P$ = 4. At equilibrium, the partial pressure of $O_2$ is _______ atm. (Round off to the Nearest Integer).

**Answer (16)**

**Sol.** $A(s) \rightleftharpoons M(s) + \frac{1}{2}O_2(g)$

$$K_P = (P_{O_2})^{1/2} = 4$$

$$P_{O_2} = 16$$

3. In a solvent 50% of an acid HA dimerizes and the rest dissociates. The van't Hoff factor of the acid is _______ × $10^{-2}$. (Round off to the Nearest Integer).

**Answer (125)**

**Sol.** $i = \frac{\text{total number of particle after dissociation / association}}{\text{total number of particle before dissociation / association}}$

let 'a' is total moles of HA

$$\underset{0.5a}{HA} \longrightarrow \underset{0.5a}{H^+} + \underset{0.5a}{A^-}$$

$$\underset{0.5a}{HA} \longrightarrow \underset{\frac{0.5a}{2}}{(HA)_2}$$

$$i = \frac{\left(0.5 + 0.5 + \frac{0.5}{2}\right)a}{a} = 1.25 = 125 \times 10^{-2}$$

4. When 400 mL of 0.2 M $H_2SO_4$ solution is mixed with 600 mL of 0.1 M NaOH solution, the increase in temperature of the final solution is _________ × $10^{-2}$ K. (Round off to the Nearest Integer).

[Use : $H^+$ (aq) + $OH^-$ (aq) → $H_2O$ :

$\Delta_\gamma H$ = –57.1 kJ $mol^{-1}$

Specific heat of $H_2O$ = 4.18 J $K^{-1}$ $g^{-1}$

Density of $H_2O$ = 1.0 g $cm^{-3}$

Assume no change in volume of solution on mixing.]

**Answer (82)**

**Sol.** millimoles of $H_2SO_4$ = 400 × 0.2 = 80

meq of NaOH = 600 × 0.1 = 60

| | $H_2SO_4$ | + 2NaOH | ⟶ $Na_2SO_4$ | + $2H_2O$ |
|---|---|---|---|---|
| t = 0 | 80 | 60 | – | – |
| | 50 | – | 30 | |

30 mmoles of product is formed.

$H^+ + OH^- \longrightarrow H_2O$ $\Delta H$ = –57.1 kJ/mol.

Moles of $H^+$ & $OH^-$ neutralised = $\frac{60}{1000}$

$$\therefore \quad \Delta H = \frac{60}{1000} \times (-57.1) = 3426 \text{ J/mol}$$

Total volume = 1 L, Mass = 1000 g

1000 × 4.18 × ΔT = 3426

ΔT = 0.8196

= 81.9 × $10^{-2}$ K ≈ 82 × $10^{-2}$ K

5. For the cell Cu(s) | $Cu^{2+}$ (aq) (0.1 M) || $Ag^+$ (aq) (0.01 M) | Ag(s) the cell potential $E_1$ = 0.3095 V

For the cell Cu(s) | $Cu^{2+}$ (aq) (0.01 M) || $Ag^+$ (aq) (0.001 M) Ag(s) the cell potential = _____ × $10^{-2}$ V. (Round off to the Nearest Integer).

[Use : $\frac{2.303\,RT}{F} = 0.059$ ]

**Answer (28)**

**Sol.** $Cu \longrightarrow Cu^{2+} + 2e^-$

$2e^- + 2Ag^+ \longrightarrow 2Ag$

$2Ag^+ + Cu \longrightarrow Cu^{2+} + 2Ag$

$$E_1 = 0.3095 = E^\circ - \frac{RT}{nF}\ln\left(\frac{[Cu^{2+}]}{[Ag^+]^2}\right)$$

$$0.3095 = E^\circ - \frac{0.059}{2}\log\left(\frac{0.1}{(0.01)^2}\right)$$

$$E^\circ = 0.3095 + \frac{0.059}{2}\log(10^3)$$

$$= 0.3095 + \frac{0.059}{2}\times 3 = 0.398 \text{ V}$$

For second cell,

$$E = 0.398 - \frac{0.059}{2}\log\left[\frac{0.01}{(0.001)^2}\right]$$

$= 0.28 \text{ V} = 28 \times 10^{-2} \text{ V}$

6. The total number of electrons in all bonding molecular orbitals of $O_2^{2-}$ is ________.

(Round off to the Nearest Integer).

**Answer (10)**

**Sol.** $O_2^{2-}$

$$\sigma 1s^2\ \sigma^*1s^2\ \sigma 2s^2\ \sigma^*2s^2\ \sigma 2p_z^2\ \begin{matrix}\pi 2p\pi_x^2 & \pi^*2p_x^2\\ \pi 2p_y^2 & \pi^*2p_y^2\end{matrix}\ \sigma^*2p_z^0$$

Total number of electrons in bonding molecular orbitals = 10

7. 10.0 mL of 0.05 M $KMnO_4$ solution was consumed in a titration with 10.0 mL of given oxalic acid dihydrate solution. The strength of given oxalic acid solution is ________ $\times 10^{-2}$ g/L.

(Round off to the Nearest Integer).

**Answer (1575)**

**Sol. At equivalence point**

(Number of gram equivalence)$_{OA}$

= (Number of gram equivalence)$_{RA}$

$(10 \times 0.05 \times 5)\ KMnO_4 = (10 \times M \times 2)\ H_2C_2O_4 \cdot 2H_2O$

M = 0.125 Molar

Strength of solution = molarity × molar mass ($gL^{-1}$)

= 0.125 × 126

= $1575 \times 10^{-2}\ gL^{-1}$

8. 3 moles of metal complex with formula $Co(en)_2Cl_3$ gives 3 moles of silver chloride on treatment with excess of silver nitrate. The secondary valency of Co in the complex is ________.

(Round off to the Nearest Integer).

**Answer (6)**

**Sol.** Each mole of complex gives one mole of AgCl. Which indicates two chloride ions present in coordination sphere. So, the complex is $[Co(en)_2Cl_2]Cl$ having a coordination number of 6. Secondary valency is equal to the coordination number

9. For the first order reaction A $\rightarrow$ 2B, 1 mole of reactant A gives 0.2 moles of B after 100 minutes. The half life of the reaction is ________ min. (Round off to the Nearest Integer).

[Use : ln 2 = 0.69, ln 10 = 2.3

Properties of logarithms : $\ln x^y = y \ln x$;

$\ln\left(\frac{x}{y}\right) = \ln x - \ln y$]

**Answer (658)**

**Sol.**

| | A | $\rightarrow$ | 2B |
|---|---|---|---|
| t = 0 | 1 | | 0 |
| t = 100 min | 1 – x | | 2x |

$2x = 0.2 \Rightarrow x = 0.1$

$$k = \frac{1}{t}\ln\frac{[A]_0}{[A]}$$

$$k = \frac{\ln 2}{t_{\frac{1}{2}}}$$

$$\frac{\ln 2}{t_{\frac{1}{2}}} = \frac{1}{100}\ln\frac{1}{0.9}$$

$$t_{\frac{1}{2}} = \frac{\ln 2 \times 100}{\ln 10 - \ln 9} \approx (600 - 700)\text{ min*}$$

(depending on value of log 3)

10. $2SO_2(g) + O_2(g) \rightarrow 2SO_3(g)$

The above reaction is carried out in a vessel starting with partial pressures $P_{SO_2}$ = 250 m bar, $P_{O_2} = 750$ m bar and $P_{SO_3} = 0$ bar. When the reaction is complete, the total pressure in the reaction vessel is ________ m bar. (Round off to the nearest Integer).

**Answer (875)**

**Sol.**

| | $2SO_2(g)$ + | $O_2(g)$ $\rightarrow$ | $2SO_3(g)$ |
|---|---|---|---|
| Initial pressures : (in m bar) | 250 | 750 | 0 |
| Completion: (in m bar) | 0 | 625 | 250 |

$P_T$ = 875 m bar

# PART–C : MATHEMATICS

## SECTION - I

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer :**

1. A student appeared in an examination consisting of 8 true - false type questions. The student guesses the answers with equal probability. The smallest value of n, so that the probability of guessing at least 'n' correct answers is less than $\frac{1}{2}$, is

(1) 4 (2) 3

(3) 5 (4) 6

**Answer (3)**

**Sol.** $P(x=n) = {}^8C_n\left(\frac{1}{2}\right)^8$

$P(x=n+1) = {}^8C_{n+1}\left(\frac{1}{2}\right)^8$

$P(x=8) = {}^8C_8\left(\frac{1}{2}\right)^8$

$\left(\frac{1}{2}\right)^8\left({}^8C_n + {}^8C_{n+1} + ..... + {}^8C_8\right) < \frac{1}{2}$

$\Rightarrow \quad 2^8 - \left({}^8C_0 + {}^8C_1 + .... + {}^8C_{n-1}\right) < 2^7$

$\Rightarrow \quad {}^8C_0 + {}^8C_1 + .... + {}^8C_{n-1} > 2^7$

Minimum value of n – 1 = 4

n = 5

2. Let N be the set of natural numbers and a relation R on N be defined by

$R = \{(x, y) \in N \times N : x^3 - 3x^2y - xy^2 + 3y^3 = 0\}.$

Then the relation R is

(1) An equivalence relation

(2) Reflexive and symmetric, but not transitive

(3) Reflexive but neither symmetric nor transitive

(4) Symmetric but neither reflexive nor transitive

**Answer (3)**

**Sol.** $x^2(x - 3y) - y^2(x - 3y) = 0$

$(x - y)(x + y)(x - 3y) = 0$ ...(i)

$\therefore$ (i) holds for all $(x, x)$ $\therefore$ R is reflexive

if $(x, y)$ holds then $(y, x)$ may or may not holds for factors $(x + y)$, $(x - 3y)$ $\therefore$ R is NOT symmetric

Similarly $(x - 3y)$ factor doesn't hold for transitive

3. A possible value of '$x$', for which the ninth term in the expansion of $\left\{3^{\log_3\sqrt{25^{x-1}+7}} + 3^{\left(-\frac{1}{8}\right)\log_3\left(5^{x-1}+1\right)}\right\}^{10}$ in the increasing powers of $3^{\left(-\frac{1}{8}\right)\log_3\left(5^{x-1}+1\right)}$ is equal to 180, is

(1) –1 (2) 0

(3) 1 (4) 2

**Answer (3)**

**Sol.** Given expression reduces to

$\left[\left(5^{2(x-1)}+7\right)^{\frac{1}{2}} + \left(5^{x-1}+1\right)^{-\frac{1}{8}}\right]^{10}$

${}^{10}C_8\left(5^{2(x-1)}+7\right)\left(5^{x-1}+1\right)^{-1} = 180$

Let $5^{x-1} = t$

$(t^2 + 7)(t + 1)^{-1} = 4$

$t^2 + 7 = 4t + 4$

$t^2 - 4t + 3 = 0$

$(t - 3)(t - 1) = 0$

$5^{x-1} = 1$ or $3$

$x = 1$ or $x = 1 + \log_5 3$

4. Let $\mathbb{C}$ be the set of all complex numbers. Let

$S_1 = \{z \in \mathbb{C} : |z - 2| \le 1\}$ and

$S_2 = \{z \in \mathbb{C} : z(1+i) + \bar{z}(1-i) \ge 4\}.$

Then, the maximum value of $\left|z - \frac{5}{2}\right|^2$ for $z \in S_1 \cap S_2$ is equal to

(1) $\frac{5+2\sqrt{2}}{2}$ (2) $\frac{5+2\sqrt{2}}{4}$

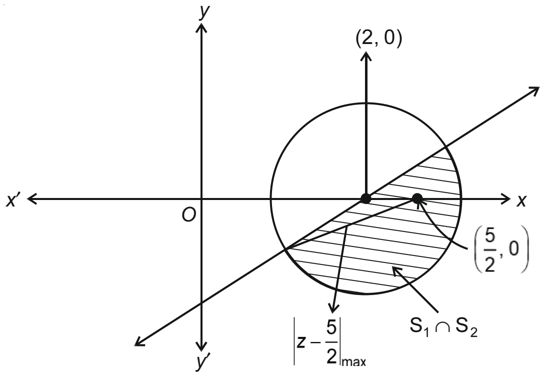
(3) $\frac{3+2\sqrt{2}}{4}$ (4) $\frac{3+2\sqrt{2}}{2}$

**Answer (2)**

**Sol.** $S_1 \equiv |z-2| \le 1 \Rightarrow (x-2)^2 + y^2 \le 1$

$S_2 \equiv x - y \ge 2$

$S_1 \cap S_2$




Solving equation from (i) & (ii), we get

$y^2 = \frac{1}{2} \Rightarrow y = -\frac{1}{2} \quad x = 2 - \frac{1}{\sqrt{2}}$

$\left|z - \frac{5}{2}\right|^2 = \left(x - \frac{5}{2}\right)^2 + y^2 = \left(\frac{1}{2} + \frac{1}{\sqrt{2}}\right)^2 + \frac{1}{2}$

$= \frac{3+2\sqrt{2}}{4} + \frac{2}{4} = \frac{5+2\sqrt{2}}{4}$

5. Let $f : (a, b) \to R$ be twice differentiable function such that $f(x) = \int_a^x g(t)dt$ for a differentiable function $g(x)$. If $f(x) = 0$ has exactly five distinct roots in (a, b), then $g(x)g'(x) = 0$ has at least

(1) Twelve roots in (a, b)

(2) Three roots in (a, b)

(3) Five roots in (a, b)

(4) Seven roots in (a, b)

**Answer (4)**

**Sol.** $f'(x) = g(x)$

As $f(x)$ has 5 roots $f'(x) = 0$, 4 times for $x \in (a, b)$

$\therefore$ $g(x)$ has 4 roots in $x \in (a, b)$

$\therefore$ $g'(x)$ has 3 roots in $x \in (a, b)$

$\therefore$ $g(x)\, g'(x)$ has 7 roots in $x \in (a, b)$

6. For real numbers $\alpha$ and $\beta \ne 0$, if the point of intersection of the straight lines $\frac{x-\alpha}{1} = \frac{y-1}{2} = \frac{z-1}{3}$ and $\frac{x-4}{\beta} = \frac{y-6}{3} = \frac{z-7}{3}$, lies on the plane $x + 2y - z = 8$, then $\alpha - \beta$ is equal to

(1) 9 (2) 5

(3) 3 (4) 7

**Answer (4)**

**Sol.** Let point on line $L_1$ be $(\lambda + \alpha, 2\lambda + 1, 3\lambda + 1)$ and a point on line $L_2$ be $(\mu\beta + 4, 3\mu + 6, 3\mu + 7)$

$\therefore$ $\lambda + \alpha = \mu\beta + 4,\ 2\lambda + 1 = 3\mu + 6\ \&\ 3\lambda + 1 = 3\mu + 7$

$\lambda = 1$ and $\mu = 1$

$\Rightarrow$ $1 + \alpha = -\beta + 4 \Rightarrow \alpha + \beta = 3$

$\therefore$ Point of intersection $(1 + \alpha, 3, 4)$

$1 + \alpha + 6 - 4 = 8 \quad \Rightarrow \quad \alpha = 5, \beta = -2$

$\alpha - \beta = 7$

7. The value of $\lim_{x\to 0}\left(\frac{x}{\sqrt[8]{1-\sin x} - \sqrt[8]{1+\sin x}}\right)$ is equal to

(1) 4 (2) –4

(3) –1 (4) 0

**Answer (2)**

**Sol.** $\lim_{x\to 0} \frac{x}{\left((1-\sin x)^{\frac{1}{8}} - (1+\sin x)^{\frac{1}{8}}\right)} \times \left(\frac{(1-\sin x)^{\frac{1}{8}} + (1+\sin x)^{\frac{1}{8}}}{(1-\sin x)^{\frac{1}{8}} + (1+\sin x)^{\frac{1}{8}}}\right)$

$\Rightarrow \lim_{x\to 0} \frac{x\left((1-\sin x)^{\frac{1}{8}} + (1+\sin x)^{\frac{1}{8}}\right)}{(1-\sin x)^{\frac{1}{4}} - (1+\sin x)^{\frac{1}{4}}} \times \frac{(1-\sin x)^{\frac{1}{4}} + (1+\sin x)^{\frac{1}{4}}}{(1-\sin x)^{\frac{1}{4}} + (1+\sin x)^{\frac{1}{4}}}$

$\Rightarrow \lim_{x\to 0} \frac{x\, 2\cdot 2}{(1-\sin x)^{\frac{1}{2}} - (1+\sin x)^{\frac{1}{2}}} \times \frac{(1-\sin x)^{\frac{1}{2}} + (1+\sin x)^{\frac{1}{2}}}{(1-\sin x)^{\frac{1}{2}} + (1+\sin x)^{\frac{1}{2}}}$

$\Rightarrow \lim_{x\to 0} \frac{8x}{1-\sin x - 1 - \sin x} = -4$

8. Let $\alpha = \max_{x\in\mathbb{R}}\{8^{2\sin 3x} \cdot 4^{4\cos 3x}\}$ and $\beta = \min_{x\in\mathbb{R}}\{8^{2\sin 3x} \cdot 4^{4\cos 3x}\}$.

If $8x^2 + bx + c = 0$ is a quadratic equation whose roots are $\alpha^{\frac{1}{5}}$ and $\beta^{\frac{1}{5}}$, then the value of c – b is equal to:

(1) 43 (2) 42

(3) 50 (4) 47

**Answer (2)**

**Sol.** $\alpha = \max\{2^{6\sin 3x + 8\cos 3x}\} = 2^{10}$

$\beta = \min\{2^{6\sin 3x + 8\cos 3x}\} = 2^{-10}$

$\alpha^{\frac{1}{5}} = 4$ and $\beta^{\frac{1}{5}} = \frac{1}{4}$

Sum of roots $= \frac{17}{4}$ & Product of roots = 1

$\frac{-b}{8} = \frac{17}{4} \Rightarrow b = -34$ & $\frac{c}{8} = 1 \Rightarrow c = 8$

c – b = 8 + 34 = 42

9. Two sides of a parallelogram are along the lines $4x + 5y = 0$ and $7x + 2y = 0$. If the equation of one of the diagonals of the parallelogram is $11x + 7y = 9$, then other diagonal passes through the point

(1) (2, 2)

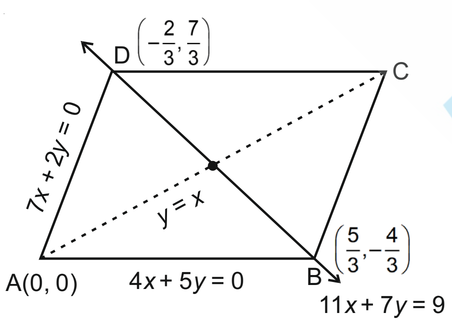
(2) (2, 1)

(3) (1, 3)

(4) (1, 2)

**Answer (1)**

**Sol.** On solving equation $4x + 5y = 0$




and $11x + 7y = 9$ we get

$B = \left(\frac{5}{3}, -\frac{4}{3}\right)$

and on solving equation

$7x + 2y = 0$ and $11x + 7y = 9$, we get

Coordinate of $D = \left(-\frac{2}{3}, \frac{7}{3}\right)$

$\therefore$ Mid point of $BD = M = \left(\frac{1}{2}, \frac{1}{2}\right)$

$\therefore$ Equation of other diagonal is $y = x$

$\therefore$ Point (2, 2) lies on other diagonal.

10. Let $f : \mathbb{R} \to \mathbb{R}$ be defined as

$f(x+y) + f(x-y) = 2f(x)f(y), f\left(\frac{1}{2}\right) = -1$. Then,

the value of $\sum_{k=1}^{20} \frac{1}{\sin(k)\sin(k+f(k))}$ is equal to

(1) $\text{cosec}^2(21)\cos(20)\cos(2)$

(2) $\sec^2(21)\sin(20)\sin(2)$

(3) $\text{cosec}^2(1)\,\text{cosec}(21)\sin(20)$

(4) $\sec^2(1)\sec(21)\cos(20)$

**Answer (3)**

**Sol.** $\because$ $f(x + y) + f(x - y) = 2f(x).f(y)$

$\therefore$ $f(x) = \cos(\lambda x)$

$\therefore$ $f\left(\frac{1}{2}\right) = -1$, then $\lambda = 2n\pi$, $n \in I$

$\therefore$ $f(x) = \cos(2\pi x) \Rightarrow f(k) = 1$, $k \in I$

$$\sum_{k=1}^{20} \frac{1}{\sin k \sin(k+f(k))} = \sum_{k=1}^{20} \frac{1}{\sin k.\sin(k+1)}$$

$$= \sum_{k=1}^{20} \frac{1}{\sin 1} \frac{\sin\{(k+1)-k\}}{\sin k.\sin(k+1)}$$

$$= \frac{1}{\sin 1} \sum_{k=1}^{20} (\cot k - \cot(k+1))$$

$$= \frac{1}{\sin 1}(\cot 1 - \cot 21)$$

$$= \frac{1}{\sin 1} . \frac{\sin(21-1)}{\sin 1.\sin 21} = \text{cosec}^2(1).\text{cosec}(21).\sin(20)$$

11. The point P (a, b) undergoes the following three transformations successively :

(a) Reflection about the line $y = x$.

(b) Translation through 2 units along the positive direction of $x$-axis.

(c) Rotation through angle $\frac{\pi}{4}$ about the origin in the anti-clockwise direction.

It the co-ordinates of the final position of the point P are $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$, then the value of 2a + b is equal to

(1) 7

(2) 9

(3) 5

(4) 13

**Answer (2)**

**Sol.** Reflection of P(a, b) about line $y = x$ is P′ = (b, a). After translation of 2 units the new coordinate in P″ = (b + 2, a)

On rotation of $\frac{\pi}{4}$ the new coordinate be $(x_1, y_1)$.

$$\therefore \quad \frac{(x_1 + iy_1) - 0}{(b + 2 + ai) - 0} = e^{i\frac{\pi}{4}}$$

$$x_1 + iy_1 = ((b+2) + ai)\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)$$

$$= \frac{1}{\sqrt{2}}(b + 2 + (b+2)i + ai - a)$$

$$= \frac{1}{\sqrt{2}}((a + b + 2)i + (b - a + 2))$$

$\therefore$ b – a + 2 = –1, a + b + 2 = 7

$\therefore$ a = 4, b = 1

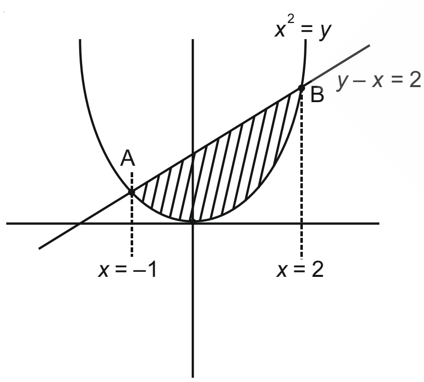
12. The area of the region bounded by $y - x = 2$ and $x^2 = y$ is equal to

(1) $\frac{2}{3}$ (2) $\frac{4}{3}$

(3) $\frac{16}{3}$ (4) $\frac{9}{2}$

**Answer (4)**

**Sol.**




On solving equations $x^2 = y$

and $y - x = 2$ we get

$x^2 = 2 + x$

$x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$

$$\therefore \quad \text{Required area} = \int_{-1}^{2}(x + 2 - x^2)\,dx$$

$$= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2}$$

$$= \left(2 + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right).$$

$$= \frac{9}{2} \text{ square units.}$$

13. Let the mean and variance of the frequency distribution

| | | | | |
|---|---|---|---|---|
| $x$ : | $x_1 = 2$ | $x_2 = 6$ | $x_3 = 8$ | $x_4 = 9$ |
| $f$ : | 4 | 4 | $\alpha$ | $\beta$ |

be 6 and 6.8 respectively. If $x_3$ is changed from 8 to 7, then the mean for the new data will be :

(1) 5 (2) 4

(3) $\frac{17}{3}$ (4) $\frac{16}{3}$

**Answer (3)**

**Sol.** $\because$

$$\bar{x} = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{8 + 24 + 8\alpha + 9\beta}{8 + \alpha + \beta} = 6 \Rightarrow 2\alpha + 3\beta = 16 \quad \text{...(i)}$$

$$\sigma^2 = \frac{\Sigma x_i^2 f_i}{\Sigma f_i} - (\bar{x})^2 \Rightarrow \frac{16 + 144 + 64\alpha + 81\beta}{8 + \alpha + \beta} = 42.8$$

$\Rightarrow$ $106\alpha + 191\beta = 912$ ...(ii)

from (i) and (ii), $\alpha = 5$ and $\beta = 2$

Now, correct mean = $\frac{8 + 24 + 35 + 18}{15} = \frac{17}{3}$

14. Let A and B be two 3 × 3 real matrices such that $(A^2 - B^2)$ is invertible matrix. If $A^5 = B^5$ and $A^3B^2 = A^2B^3$, then the value of the determinant of the matrix $A^3 + B^3$ is equal to

(1) 1 (2) 2

(3) 4 (4) 0

**Answer (4)**

**Sol.** $A^5 = B^5$ ...(i)

$A^3B^2 = A^2B^3$ ...(ii)

$A^5 - A^3B^2 = B^5 - A^2B^3$

$A^3(A^2 - B^2) = B^3(B^2 - A^2) = -B^3(A^2 - B^2)$

$A^3(A^2 - B^2) + B^3(A^2 - B^2) = 0$

$(A^3 + B^3)(A^2 - B^2) = 0$

$$|(A^3 + B^3)(A^2 - B^2)| = 0$$

$$|A^3 + B^3| \times |A^2 - B^2| = 0$$

$$\Rightarrow \quad |A^3 + B^3| = 0 \quad (\because |A^2 - B^2| \neq 0)$$

15. Let $y = y(x)$ be the solution of the differential equation $(x - x^3)dy = (y + yx^2 - 3x^4)dx$, $x > 2$. If $y(3) = 3$, then $y(4)$ is equal to

(1) 12 (2) 8

(3) 4 (4) 16

**Answer (1)**

**Sol.** $(x - x^3)dy = y(1 + x^2)dx - 3x^4dx$

$$\therefore \quad \frac{dy}{dx} + y\frac{1+x^2}{x(x^2-1)} = \frac{3x^3}{x^2-1}$$

$$\therefore \quad \text{I.F.} = e^{\int \frac{1+x^2}{x(x^2-1)}dx} = e^{\int\left(\frac{1}{x-1}+\frac{1}{x+1}-\frac{1}{x}\right)dx}$$

$$= e^{\ln\left(\frac{x^2-1}{x}\right)} = \frac{x^2-1}{x}$$

$$\therefore \quad \text{Solution is } y\cdot\left(\frac{x^2-1}{x}\right) = \int \frac{3x^3}{x^2-1}\cdot\frac{x^2-1}{x}dx$$

$$y\left(\frac{x^2-1}{x}\right) = x^3 + c$$

$\because$ $y(3) = 3$ then $c = -1$

$$\therefore \quad y(x) = \frac{(x^3-19)\cdot x}{x^2-1}$$

$$\therefore \quad y(4) = \frac{45\times 4}{15} = 12$$

16. Let $f : [0, \infty) \to [0, 3]$ be a function defined by

$$f(x) = \begin{cases} \max\{\sin t : 0 \le t \le x\}, & 0 \le x \le \pi \\ 2 + \cos x, & x > \pi \end{cases}$$

Then which of the following is true?

(1) $f$ is continuous everywhere but not differentiable exactly at two points in $(0, \infty)$

(2) $f$ is continuous everywhere but not differentiable exactly at one point in $(0, \infty)$

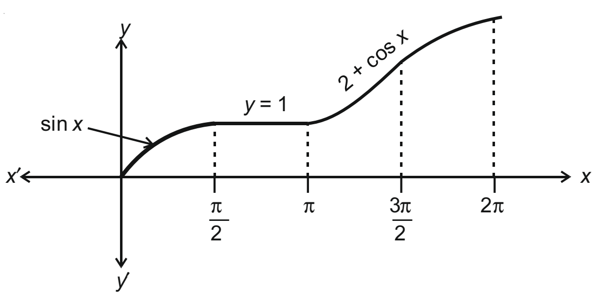
(3) $f$ is differentiable everywhere in $(0, \infty)$

(4) $f$ is not continuous exactly at two points in $(0, \infty)$

**Answer (3)**

**Sol.** $f : [0, \infty) \to [0, 3]$

and $f(x) = \begin{cases} \max\{\sin t : 0 \le t \le x\}, & 0 \le x \le \pi \\ 2 + \cos x, & x > \pi \end{cases}$




Clearly $f(x)$ is continuous everywhere

and $f(x)$ is differentiable at $x = \frac{\pi}{2}$ and $x = \pi$

$\therefore$ $f(x)$ is differentiable everywhere

17. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a} = \vec{b} \times (\vec{b} \times \vec{c})$. If magnitudes of the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are $\sqrt{2}$, 1 and 2 respectively and the angle between $\vec{b}$ and $\vec{c}$ is $\theta\left(0 < \theta < \frac{\pi}{2}\right)$, then the value of $1 + \tan\theta$ is equal to

(1) 2

(2) $\frac{\sqrt{3}+1}{\sqrt{3}}$

(3) 1

(4) $\sqrt{3}+1$

**Answer (1)**

**Sol.** $\vec{a} = \vec{b} \times (\vec{b} \times \vec{c}) = (\vec{b}\cdot\vec{c})\vec{b} - |\vec{b}|^2\vec{c}$

$$= (\vec{b}\cdot\vec{c})\vec{b} - \vec{c} \quad (\because |\vec{b}| = 1)$$

$$|\vec{a}|^2 = (\vec{b}\cdot\vec{c})^2|\vec{b}|^2 + |\vec{c}|^2 - 2(\vec{b}\cdot\vec{c})(\vec{b}\cdot\vec{c})$$

$$2 = |\vec{c}|^2 - (\vec{b}\cdot\vec{c})^2$$

$2 = 4 - (2\cos\theta)^2$

$(2\cos\theta)^2 = 2$

$$\cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \tan\theta = 1$$

18. Consider a circle C which touches the $y$-axis at (0, 6) and cuts off an intercept $6\sqrt{5}$ on the $x$-axis. Then the radius of the circle C is equal to

(1) $\sqrt{53}$

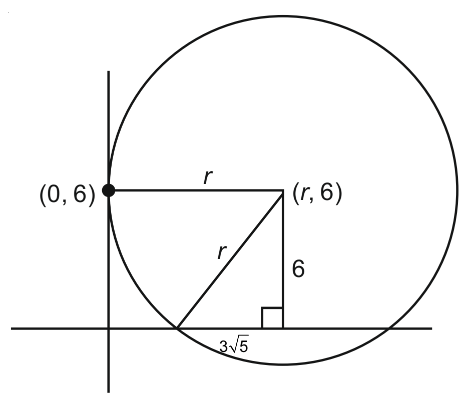
(2) 9

(3) 8

(4) $\sqrt{82}$

**Answer (2)**

**Sol.**




$$r^2 = 6^2 + (3\sqrt{5})^2 = 81$$

$r = 9$

19. Which of the following is the negation of the statement "for all M > 0, there exists $x \in S$ such that $x \geq M$"?

(1) There exists M > 0, there exists $x \in S$ such that $x < M$

(2) There exists M > 0, there exists $x \in S$ such that $x \geq M$

(3) There exists M > 0, such that $x < M$ for all $x \in S$

(4) There exists M > 0, such that $x \geq M$ for all $x \in S$

**Answer (3)**

**Sol.** Statement : For all M > 0, there exists $x \in S$ such that $x \geq M$.

Negation : There exist M > 0, such that $x \not\geq M$ for all $x \in S$.

20. If $\tan\left(\frac{\pi}{9}\right), x, \tan\left(\frac{7\pi}{18}\right)$ are in arithmetic progression and $\tan\left(\frac{\pi}{9}\right), y, \tan\left(\frac{5\pi}{18}\right)$ are also in arithmetic progression, then $|x - 2y|$ is equal to

(1) 0 (2) 1

(3) 3 (4) 4

**Answer (1)**

**Sol.** $x - 2y = \frac{\tan 20^\circ + \tan 70^\circ}{2} - (\tan 20^\circ + \tan 50^\circ)$

$\frac{1}{2}(\tan 70^\circ - \tan 20^\circ - 2\tan 50^\circ)$

$= \frac{1}{2}[(\tan 70^\circ - \tan 50^\circ) - (\tan 20^\circ + \tan 50^\circ)]$

$= \frac{1}{2}\left[\frac{\sin 20^\circ}{\cos 70^\circ \cos 50^\circ} - \frac{\sin 70^\circ}{\cos 20^\circ \cos 50^\circ}\right]$

$= \frac{1}{2}\left[\frac{1}{\cos 50^\circ} - \frac{1}{\cos 50^\circ}\right] = 0$

## SECTION - II

**Numerical Value Type Questions:** This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and $M = A + A^2 + A^3 + ... + A^{20}$, then the sum of all the elements of the matrix M is equal to ________.

**Answer (2020)**

**Sol.** $A^n = \begin{bmatrix} 1 & n & \frac{n(n+1)}{2} \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$

$M = A + A^2 + .......... + A^{20}$

$= \begin{bmatrix} \sum 1 & \sum n & \sum \frac{n(n+1)}{2} \\ 0 & \sum 1 & \sum n \\ 0 & 0 & \sum 1 \end{bmatrix} = \begin{bmatrix} 20 & 210 & 1540 \\ 0 & 20 & 210 \\ 0 & 0 & 20 \end{bmatrix}$

Because $\sum 1 = 20,\ \sum_{n=1}^{20} n = \frac{20 \times 21}{2} = 210$

$\frac{1}{2}\sum_{n=1}^{20} n(n+1) = \frac{1}{2} \times \frac{20 \times 21 \times 22}{3} = 1540$

Sum = 20 + 20 + 20 + 210 + 210 + 1540 = 2020

2. The number of real roots of the equation $e^{4x} - e^{3x} - 4e^{2x} - e^x + 1 = 0$ is equal to ____.

**Answer (2)**

**Sol.** Let $e^x = t$, $(t > 0)$

$t^4 - t^3 - 4t^2 - t = 1 = 0$

$\left(t^2 + \frac{1}{t^2}\right) - \left(t^3 + t\right) - 4 = 0$

$\left(t + \frac{1}{t}\right)^2 - \left(t + \frac{1}{t}\right) - 6 = 0$

Let $t + \frac{1}{t} = u \quad (u > 2)$

$u^2 - u - 6 = 0$

$(u - 3)(u + 2) = 0$

$u = 3, -2$ (rejected)

$u = 3$

$t + \frac{1}{t} = 3 \quad \Rightarrow t^2 - 3t + 1 = 0$

$t = \frac{3 \pm \sqrt{5}}{2} = e^x$

$x = \ln\frac{3+\sqrt{5}}{2},\ \ln\frac{3-\sqrt{5}}{2}$

3. Let n be a non-negative integer. Then the number of divisors of the form "4n + 1" of the number $(10)^{10}\cdot(11)^{11}\cdot(13)^{13}$ is equal to _______.

**Answer (924)**

**Sol.** $N = 2^{10}\cdot 5^{10}\cdot 11^{11}\cdot 13^{13}$

$$\underbrace{(2^0+2^1+\ldots+2^{10})}_{\text{only } 2^0 \text{ is allowed to be selected } (2^0 \text{ is of the type } 4\lambda+1)}\ \underbrace{(5^0+5^1+\ldots+5^{10})}_{\text{All terms are of the type } 4\lambda+1}$$

$$\underbrace{(11^0+11^1+\ldots+11^{11})}_{11^{\text{even}} \text{ are of the type } 4\lambda+1}\ \underbrace{(13^0+13^1+\ldots+13^{13})}_{\text{All terms are of the type } 4\lambda+1}$$

Number of required divisors = 1 × 11 × 6 × 14

= 924

4. The distance of the point P(3, 4, 4) from the point of intersection of the line joining the points Q(3, –4, –5) and R(2, –3, 1) and the plane $2x + y + z = 7$, is equal to _______.

**Answer (7)**

**Sol.** $QR: \dfrac{x-2}{-1} = \dfrac{y+3}{1} = \dfrac{z-1}{6}$

Let point of intersection be $S(-\lambda + 2, \lambda - 3, 6\lambda + 1)$

$2(-\lambda + 2) + \lambda - 3 + 6\lambda + 1 = 7 \Rightarrow \lambda = 1$

So S(1, –2, 7)

$PS = \sqrt{2^2 + 6^2 + 3^2} = 7$

5. Let $A = \{n \in N \mid n^2 \le n + 10{,}000\}$, $B = \{3k + 1 \mid k \in N\}$ and $C = \{2k \mid k \in N\}$, then the sum of all the elements of the set $A \cap (B - C)$ is equal to _______.

**Answer (832)**

**Sol.** A = {1, 2, 3, ...., 100}

and $B - C = \{3k + 1 \mid k \in \text{even}\}$

$\Rightarrow$ B – C = {7, 13, 19, ....., 97}

Sum of all elements $= \dfrac{16}{2}[7 + 97] = 832$

6. Let E be an ellipse whose axes are parallel to the co-ordinates axes, having its center at (3, –4), one focus at (4, –4) and one vertex at (5, –4). If $mx - y = 4$, $m > 0$ is a tangent to the ellipse E, then the value of $5m^2$ is equal to _______.

**Answer (3)**

**Sol.** $\because$ ae = 1 and a = 2 so $b = \sqrt{3}$

$E: \dfrac{(x-3)^2}{4} + \dfrac{(y+4)^2}{3} = 1$

Equation of tangent

$y + 4 = m(x-3) \pm \sqrt{4m^2 + 3}$

$\Rightarrow\ y = mx - 3m - 4 \pm \sqrt{4m^2 + 3}$

Comparing with $y = mx - 4$

we get $-3 \pm \sqrt{4m^2 + 3} = 0$

$\Rightarrow\ 9m^2 = 4m^2 + 3$

$\Rightarrow\ 5m^2 = 3$

7. If $\int_0^{\pi} (\sin^3 x)\, e^{-\sin^2 x}\, dx = \alpha - \dfrac{\beta}{e}\int_0^1 \sqrt{t}\, e^t dt$, then $\alpha + \beta$ is equal to _______.

**Answer (5)**

**Sol.** $\int_0^{\pi}\left(\sin^3 x\right)\cdot e^{\sin^{-2x}}dx = \dfrac{1}{e}\int_0^{\pi} \sin^2 x \cdot e^{\cos^2 x}\cdot \sin x dx$

Let cosx = t, sindx = –dt

$= \dfrac{1}{e}\int_1^{-1}\left(t^2 - 1\right)e^{t^2}dt = \dfrac{2}{e}\int_0^1\left(1 - t^2\right)e^{t^2}dt$

Let $t^2 = z$, $dt = \dfrac{dz}{2\sqrt{z}}$

$= \dfrac{1}{e}\int_0^1\left(\dfrac{1}{\sqrt{z}} - \sqrt{z}\right)e^z dz$

$= \dfrac{1}{e}\left[e^z\cdot 2\sqrt{z}\Big|_0^1 - \int_0^1 2e^z\cdot\sqrt{z}dz - \int_0^1\sqrt{z}e^z dz\right]$

$= \dfrac{1}{e}\left[2e - 3\int_0^1 e^t\cdot\sqrt{t}\,dt\right]$

Clearly $\alpha = 2$ and $\beta = 3$

8. If the real part of the complex number $z = \dfrac{3 + 2i\cos\theta}{1 - 3i\cos\theta}$, $\theta \in \left(0, \dfrac{\pi}{2}\right)$ is zero, then the value of $\sin^2 3\theta + \cos^2\theta$ is equal to _______.

**Answer (1)**

**Sol.** $z = \dfrac{(3 + 2i\cos\theta)(1 + 3\cos\theta)}{1 + 9\cos^2\theta}$

$\because\ \text{Re}(z) = 0 = \dfrac{3 - 6\cos^2\theta}{1 + 9\cos^2\theta} = 0$

$\Rightarrow\ \cos^2\theta = \dfrac{1}{2}$

$\Rightarrow\ \theta = \dfrac{\pi}{4}$

$\sin^2 3\theta + \cos^2\theta = \dfrac{1}{2} + \dfrac{1}{2} = 1$

9. Let $\vec{a} = \hat{i} - \alpha\hat{j} + \beta\hat{k}, \vec{b} = 3\hat{i} + \beta\hat{j} - \alpha\hat{k}$ and $\vec{c} = -\alpha\hat{i} - 2\hat{j} + \hat{k}$, where $\alpha$ and $\beta$ are integers. If $\vec{a}\cdot\vec{b} = -1$ and $\vec{b}\cdot\vec{c} = 10$, $(\vec{a}\times\vec{b})\cdot\vec{c}$ is equal to ________.

**Answer (9)**

**Sol.** $\because$ $\vec{a}\cdot\vec{b} = -1 = 3 - 2\alpha\beta \Rightarrow \alpha\beta = 2$

$\vec{b}\cdot\vec{c} = 10 = -3\alpha - 2\beta - \alpha \Rightarrow 2\alpha + \beta = -5$

Clearly $(\alpha,\beta) = (-2, -1)$

$$[\vec{a}\ \ \vec{b}\ \ \vec{c}] = \begin{vmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 2 & -2 & 1 \end{vmatrix} = 9$$

10. Let $y = y(x)$ be the solution of the differential equation $dy = e^{\alpha x + y}dx$; $\alpha \in N$. If $y(\log_e 2) = \log_e 2$ and $y(0) = \log_e\left(\frac{1}{2}\right)$, then the value of $\alpha$ is equal to ________.

**Answer (2)**

**Sol.** $e^{-y}\,dy = e^{\alpha x}\,dx$

$$\Rightarrow\ -e^{-y} = \frac{1}{\alpha}e^{\alpha x} + c$$

Put $x = y = \ln 2$ and $x = 0$, $y = -\ln 2$

$$-\frac{1}{2} = \frac{2^{\alpha}}{\alpha} + c \qquad -2 = \frac{1}{\alpha} + c$$

$$\Rightarrow\ \alpha = 2 \text{ and } c = -\frac{5}{2}$$

❑ ❑ ❑