27/07/2021
Morning

Corporate Office: Aakash Tower, 8, Pusa Road, New Delhi-110005 Ph. 011-47623456

Time : 3 hrs. **M.M. : 300**

Answers & Solutions

for

JEE (MAIN)-2021 (Online) Phase-3

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS :

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part has two sections.

(i) Section-I : This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

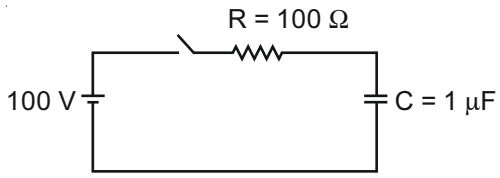
(ii) Section-II : This section contains 10 questions. In Section-II, attempt any **five questions out of 10.** There will be **no negative marking for Section-II**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and there is no negative marking for wrong answer.

PART–A : PHYSICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. 

A capacitor of capacitance C = 1 μF is suddenly connected to a battery of 100 volt through a resistance R = 100 Ω. The time taken for the capacitor to be charged to get 50 V is :

[Take ln 2 = 0.69]

(1) 0.30×10^{-4} s (2) 1.44×10^{-4} s

(3) 0.69×10^{-4} s (4) 3.33×10^{-4} s

Answer (3)

Sol. $50 = 100\left(1 - e^{-\frac{t}{RC}}\right)$

$\Rightarrow t = RC \ln 2$

$= 100 \times 10^{-6} \times (0.69)$

$= 0.69 \times 10^{-4}$ s

2. The number of molecules in one litre of an ideal gas at 300 K and 2 atmospheric pressure with mean kinetic energy 2×10^{-9} J per molecule is

(1) 0.75×10^{11} (2) 6×10^{11}

(3) 1.5×10^{11} (4) 3×10^{11}

Answer (None)

Sol. $2 \times 10^5 \times 10^{-3} = (n) \times 8.31 \times 300$

$\Rightarrow n = 0.08$ moles

$N = nx\ N_A$

$= 4.8 \times 10^{22}$ molecules

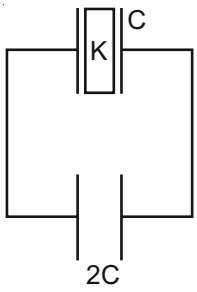
3. Two capacitors of capacities 2C and C are joined in parallel and charged up to potential V. The battery is removed and the capacitor of capacity C is filled completely with a medium of dielectric constant K. The potential difference across the capacitors will now be

(1) $\frac{3V}{K+2}$ (2) $\frac{3V}{K}$

(3) $\frac{V}{K}$ (4) $\frac{V}{K+2}$

Answer (1)

Sol. $Q_{total} = 3C \times V$



$\therefore V_f = \frac{3CV}{(2C+KC)}$

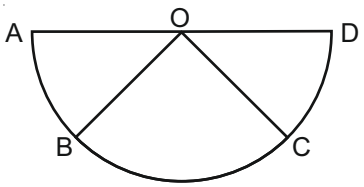
$= \frac{3}{K+2} \times V$

4. **Assertion A :** If A, B, C, D are four points on a semi-circular arc with centre at 'O' such that $|\overrightarrow{AB}| = |\overrightarrow{BC}| = |\overrightarrow{CD}|$, then

$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} = 4\overrightarrow{AO} + \overrightarrow{OB} + \overrightarrow{OC}$

Reason R : Polygon law of vector addition yields

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{AD} = 2\overrightarrow{AO}$



In the light of the above statements, choose the **most appropriate** answer from the options given below

(1) **A** is not correct but **R** is correct.

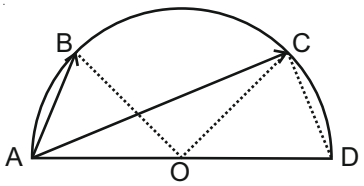
(2) **A** is correct but **R** is not correct.

(3) Both **A** and **R** are correct and **R** is the correct explanation of **A**.

(4) Both **A** and **R** are correct but **R** is not the correct explanation of **A**.

Answer (4)

Sol. $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD}$



$= (\overrightarrow{AO} + \overrightarrow{OB}) + (\overrightarrow{AO} + \overrightarrow{OC}) + 2\overrightarrow{AO}$

$= 4\overrightarrow{AO} + \overrightarrow{OB} + \overrightarrow{OC}$

5. A 0.07 H inductor and a 12 Ω resistor are connected in series to a 220 V, 50 Hz ac source. The approximate current in the circuit and the phase angle between current and source voltage are respectively $\left[\text{Take } \pi \text{ as } \frac{22}{7}\right]$

(1) 8.8 A and $\tan^{-1}\left(\frac{11}{6}\right)$

(2) 8.8 A and $\tan^{-1}\left(\frac{6}{11}\right)$

(3) 88 A and $\tan^{-1}\left(\frac{11}{6}\right)$

(4) 0.88 A and $\tan^{-1}\left(\frac{11}{6}\right)$

Answer (1)

Sol. $Z = \sqrt{12^2 + (100\pi \times 0.07)^2} \approx 25\ \Omega$

$\therefore \quad I = \frac{220}{25} = 8.8\ A$

$$\tan\phi = \frac{X_L}{R} = \frac{100 \times \frac{22}{7} \times \frac{7}{100}}{12} = \frac{11}{6}$$

6. In Young's double slit experiment, if the source of light changes from orange to blue then

(1) The distance between consecutive fringes will increase

(2) The central bright fringe will become a dark fringe

(3) The distance between consecutive fringes will decrease

(4) The intensity of the minima will increase

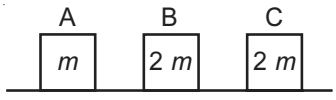
Answer (3)

Sol. $\lambda_O > \lambda_B$

$$\text{Fringe Width, } \beta = \frac{\lambda D}{d}$$

$\Rightarrow \beta_O > \beta_{Blue}$

7. Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. The masses of A, B and C are *m*, 2 *m* and 2 *m* respectively. A moves towards B with a speed of 9 m/s and makes an elastic collision with it. Thereafter B makes a completely inelastic collision with C. All motions occur along same straight line. The final speed of C is



(1) 6 m/s

(2) 4 m/s

(3) 9 m/s

(4) 3 m/s

Answer (4)

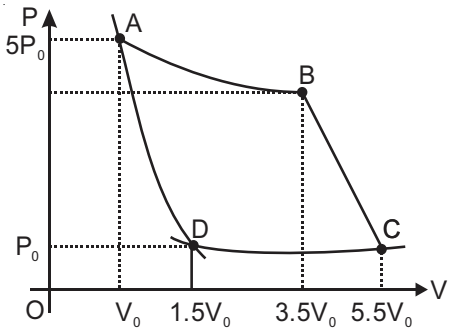
Sol. After 1st collision

$$V_B = \frac{2V_0}{3}$$

After inelastic collision

$$V_C = \frac{V_B}{2}$$

8. In the reported figure, there is a cyclic process ABCDA on a sample of 1 mol of a diatomic gas. The temperature of the gas during the process A → B and C → D are T_1 and T_2 ($T_1 > T_2$) respectively



Choose the correct option out of the following for work done if processes BC and DA are adiabatic

(1) $W_{AD} = W_{BC}$

(2) $W_{AB} < W_{CD}$

(3) $W_{BC} + W_{DA} > 0$

(4) $W_{AB} = W_{DC}$

Answer (1)

Sol. $W_{A\to D} = \frac{5P_0V_0 - (P_0)(1.5V_0)}{\gamma - 1}$

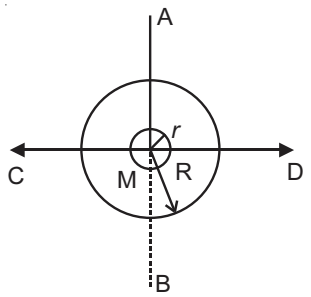
$$W_{BC} = \frac{(P_B)(3.5V_0) - P_C(5.5V_0)}{\gamma - 1}$$

$$P_B = \frac{5P_0}{3.5} = \frac{10}{7}P_0,\ P_C = \frac{1.5P_0}{5.5} = \frac{3}{11}P_0$$

$$\Rightarrow W_{BC} = \frac{5P_0V_0 - 1.5P_0V_0}{\gamma - 1}$$

$\Rightarrow W_{AD} = W_{BC}$

9. The figure shows two solid discs with radius R and *r* respectively. If mass per unit area is same for both, what is the ratio of MI of bigger disc around axis AB (Which is ⊥ to the plane of the disc and passing through its centre) to MI of smaller disc around one of its diameters lying on its plane? Given 'M' is the mass of the larger disc. (MI stands for moment of inertia)



(1) $2r^4 : R^4$ (2) $R^2 : r^2$

(3) $2R^2 : r^2$ (4) $2R^4 : r^4$

Answer (4)

Sol. $I_1 = \frac{MR^2}{2}$

$I_2 = \frac{mr^2}{4}$

$m = (\rho)\pi r^2 = \frac{m}{\pi R^2}\pi r^2 = \frac{Mr^2}{R^2}$

$\Rightarrow \frac{I_1}{I_2} = \frac{2R^4}{r^4}$

10.

List-I	List-II
(a) MI of the rod (length L, Mass M, about and axis ⊥ to the rod passing through the midpoint)	(i) $8\ ML^2/3$
(b) MI of the rod (length L, Mass 2M, about an axis ⊥ to the rod passing through one of its end)	(ii) $ML^2/3$
(c) MI of the rod (length 2L, Mass M, about an axis ⊥ to the rod passing through its midpoint)	(iii) $ML^2/12$
(d) MI of the rod (length 2L, Mass 2M, about an axis ⊥ to the rod passing through one of its end)	(iv) $2\ ML^2/3$

Choose the **correct** answer from the options given below :

(1) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

(2) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

(3) (a)-(ii), (b)-(iii), (c)-(i), (d)-(iv)

(4) (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)

Answer (1)

Sol. (a) $I_1 = \frac{ML^2}{12}$

(b) $I_2 = \frac{(2M)L^2}{3} = \frac{2ML^2}{3}$

(c) $I_3 = \frac{(M)(2L)^2}{12} = \frac{ML^2}{3}$

(d) $I_4 = \frac{(2M)(2L)^2}{3} = \frac{8}{3}ML^2$

11. A ball is thrown up with a certain velocity so that it reaches a height '*h*'. Find the ratio of the two different times of the ball reaching $\frac{h}{3}$ in both the directions.

(1) $\frac{1}{3}$

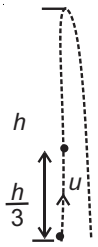
(2) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

(3) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

(4) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$

Answer (2)

Sol. $u = \sqrt{2gh}$



Now using, $s = ut + \frac{1}{2}at^2$

$\frac{h}{3} = ut - \frac{1}{2}gt^2$

$\frac{1}{2}gt^2 - ut + \frac{4}{3} = 0$

$t = \frac{u \pm \sqrt{u^2 - 4\cdot\frac{1}{2}g\frac{h}{3}}}{g}$

$\frac{t_1}{t_2} = \frac{\sqrt{2}-\sqrt{\frac{4}{3}}}{\sqrt{2}+\sqrt{\frac{4}{3}}} = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

12. **Assertion A :** If in five complete rotations of the circular scale, the distance travelled on main scale of the screw gauge is 5 mm and there are 50 total divisions on circular scale, then least count is 0.001 cm.

Reason R : $\text{Least Count} = \dfrac{\text{Pitch}}{\text{Total divisions on circular scale}}$

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both A and R are correct and R is the correct explanation of A.

(2) A is not correct but R is correct.

(3) Both A and R are correct and R is not the correct explanation of A

(4) A is correct but R is not correct

Answer (2)

Sol. $\text{Pitch} = \dfrac{5\,\text{mm}}{5} = 1\,\text{mm}$

$$\text{So least count} = \frac{\text{Pitch}}{\text{Total division on circular scale}}$$

$$= \frac{1\,\text{mm}}{50}$$

$$= 0.02\ \text{mm}$$

$$= 0.002\ \text{cm}$$

So A is not correct but R is correct.

13. The relative permittivity of distilled water is 81. The velocity of light in it will be

(Given $\mu_r = 1$)

(1) 2.33×10^7 m/s (2) 3.33×10^7 m/s

(3) 5.33×10^7 m/s (4) 4.33×10^7 m/s

Answer (2)

Sol. $v = \dfrac{c}{\sqrt{\mu_r \varepsilon_r}} = \dfrac{3\times10^8\,\text{m/s}}{\sqrt{1\times81}}$

$$= \frac{1}{3}\times10^8\ \text{m/s}$$

$$= 3.33 \times 10^7\ \text{m/s}$$

14. If '*f*' denotes the ratio of the number of nuclei decayed (N_d) to the number of nuclei at $t = 0$ (N_0) then for a collection of radioactive nuclei, the rate of change of '*f*' with respect to time is given as

[λ is the radioactive decay constant]

(1) $\lambda e^{-\lambda t}$ (2) $-\lambda(1 - e^{-\lambda t})$

(3) $-\lambda e^{-\lambda t}$ (4) $\lambda(1 - e^{-\lambda t})$

Answer (1)

Sol. $N = N_0\, e^{-\lambda t}$

$$N_d = N_0 - N$$

$$N_d = N_0(1 - e^{-\lambda t})$$

$$f = \frac{N_d}{N_0} = 1 - e^{-\lambda t}$$

$$\frac{df}{dt} = 0 - (-\lambda)e^{-\lambda t} = \lambda e^{-\lambda t}$$

15. Two identical tennis balls each having mass '*m*' and charge '*q*' are suspended from a fixed point by threads of length '*l*'. What is the equilibrium separation when each thread makes a small angle 'θ' with the vertical?

(1) $x = \left(\dfrac{q^2l^2}{2\pi\varepsilon_0 m^2 g}\right)^{1/3}$ (2) $x = \left(\dfrac{q^2l^2}{2\pi\varepsilon_0 m^2 g^2}\right)^{1/3}$

(3) $x = \left(\dfrac{q^2 l}{2\pi\varepsilon_0 mg}\right)^{1/2}$ (4) $x = \left(\dfrac{q^2 l}{2\pi\varepsilon_0 mg}\right)^{1/3}$

Answer (4)

Sol. $T\cos\theta = mg$

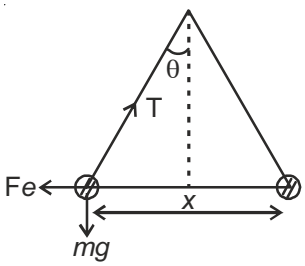
$T\sin\theta = F_e$

$$\tan\theta = \frac{F_e}{mg}$$

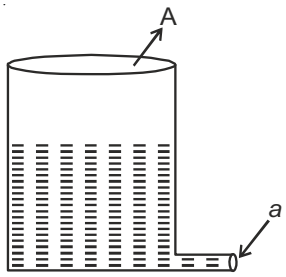
$$\frac{x}{2l} = \frac{1q^2}{4\pi\varepsilon_0 x^2 mg}$$

$$x^3 = \frac{q^2 l}{2\pi\varepsilon_0 mg}$$

$$x = \left(\frac{q^2 l}{2\pi\varepsilon_0 mg}\right)^{\frac{1}{3}}$$



16. A light cylindrical vessel is kept on a horizontal surface. Area of base is A. A hole of cross-sectional area '*a*' is made just at its bottom side. The minimum coefficient of friction necessary to prevent sliding the vessel due to the impact force of the emerging liquid is ($a << A$)



(1) $\dfrac{A}{2a}$ (2) None of these

(3) $\dfrac{2a}{A}$ (4) $\dfrac{a}{A}$

Answer (3)

Sol. $V_e = \sqrt{2gh}$

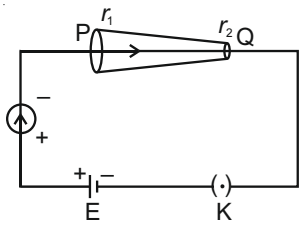
Thrust force = friction

$\rho av^2 = \mu(\rho Ah)g$

$a(2gh) = \mu Agh$

$$\mu = \frac{2a}{A}$$

17. In the given figure, a battery of emf E is connected across a conductor PQ of length 'l' and different area of cross-sections having radii r_1 and $r_2(r_2 < r_1)$.



Choose the correct option as one moves from P to Q

(1) All of these

(2) Electron current decreases

(3) Electric field decreases

(4) Drift velocity of electron increases

Answer (4)

Sol. On moving from P to Q

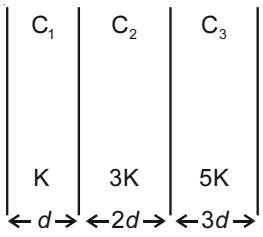
Current density increase

$J = \sigma E$

Electric field increase

Hence, Drift velocity increases.

18. In the reported figure, a capacitor is formed by placing a compound dielectric between the plates of parallel plate capacitor. The expression for the capacity of the said capacitor will be : (Given area of plate = A)



(1) $\frac{15}{6}\frac{K\varepsilon_0 A}{d}$ (2) $\frac{15}{34}\frac{K\varepsilon_0 A}{d}$

(3) $\frac{25}{6}\frac{K\varepsilon_0 A}{d}$ (4) $\frac{9}{6}\frac{K\varepsilon_0 A}{d}$

Answer (2)

Sol. $C_1 = \frac{K\varepsilon_0 A}{d}$

$$C_2 = \frac{3K\varepsilon_0 A}{2d}$$

$$C_3 = \frac{5K\varepsilon_0 A}{3d}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C_{eq}} = \frac{d}{K\varepsilon_0 A} + \frac{2d}{3K\varepsilon_0 A} + \frac{3d}{5K\varepsilon_0 A}$$

$$\frac{1}{C_{eq}} = \frac{d}{K\varepsilon_0 A}\left[1 + \frac{2}{3} + \frac{3}{5}\right]$$

$$= \frac{d}{15K\varepsilon_0 A}[15 + 10 + 9]$$

$$C_{eq} = \frac{15K\varepsilon_0 A}{34d}$$

19. A particle starts executing simple harmonic motion (SHM) of amplitude 'a' and total energy E. At any instant, its kinetic energy is $\frac{3E}{4}$ then its displacement 'y' is given by

(1) $y = \frac{a}{\sqrt{2}}$ (2) $y = a$

(3) $y = \frac{a}{2}$ (4) $y = \frac{a\sqrt{3}}{2}$

Answer (3)

Sol. $E = \frac{1}{2}m\omega^2 a^2$

$$K.E. = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2[a^2 - y^2]$$

$$a^2 - y^2 = \frac{3}{4}a^2$$

$$y = \frac{a}{2}$$

20. A body takes 4 min. to cool from 61°C to 59°C. If the temperature of the surroundings is 30°C, the time taken by the body to cool from 51°C to 49°C is

(1) 8 min. (2) 3 min.

(3) 6 min. (4) 4 min.

Answer (3)

Sol. $-\frac{dT}{dt} = k(T - T_0)$

$$\frac{(60-30)}{(50-30)} = \frac{t}{4}$$

t = 6 min.

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. A radioactive sample has an average life of 30 ms and is decaying. A capacitor of capacitance 200 μF is first charged and later connected with resistor 'R'. If the ratio of charge on capacitor to the activity of radioactive sample is fixed with respect to time then the value of 'R' should be _____ Ω.

Answer (150)

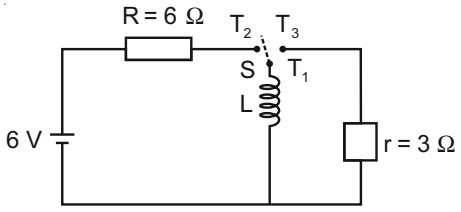
Sol. $\frac{q}{A}=\frac{Q}{A_0}$

$\Rightarrow \frac{1}{RC}=\lambda=\frac{1}{T_{avg.}}$

$\Rightarrow R=\frac{T_{avg.}}{C}=\frac{30\times10^{-3}}{200\times10^{-6}}$

$= 150\ \Omega$

2. Consider an electrical circuit containing a two way switch 'S'. Initially S is open and then T_1 is connected to T_2. As the current in R = 6 Ω attains a maximum value of steady state level, T_1 is disconnected from T_2 and immediately connected to T_3. Potential drop across r = 3 Ω resistor immediately after T_1 is connected to T_3 is _____ V. (Round off to the Nearest Integer)



Answer (3)

Sol. $i_0=\frac{\varepsilon}{R}=\frac{6}{6}=1\,A$

As current through inductor will not change instantly

$V_r = i_0 r = 1 \times 3 = 3\ V$

3. The amplitude of upper and lower side bands of A.M. wave where a carrier signal with frequency 11.21 MHz, peak voltage 15 V is amplitude modulated by a 7.7 kHz sine wave of 5V amplitude are $\frac{a}{10}$ V and $\frac{b}{10}$ V respectively. Then the value of $\frac{a}{b}$ is _____.

Answer (1)

Sol. Amplitudes of side band for both left and right side band will be equal.

Ratio of both amplitudes = 1 (as both will be equal)

4. In Bohr's atomic model, the electron is assumed to revolve in a circular orbit of radius 0.5 Å. If the speed of electron is 2.2×10^6 m/s, then the current associated with the electron will be ___ $\times 10^{-2}$ mA.

[Take π as $\frac{22}{7}$]

Answer (112)

Sol. $i=\frac{qv}{2\pi r}$

$=\frac{1.6\times10^{-19}\times2.2\times10^{-6}}{2\times\pi\times0.5\times10^{-10}}$

$= 112 \times 10^{-5}$ A

5. In a uniform magnetic field, the magnetic needle has a magnetic moment 9.85×10^{-2} A/m^2 and moment of inertia 5×10^{-6} kg m^2. If it performs 10 complete oscillations in 5 seconds then the magnitude of the magnetic field is ____ mT. [Take π^2 as 9.85]

Answer (8)

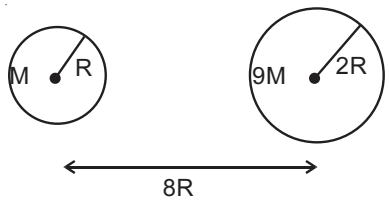
Sol. $T=2\pi\sqrt{\frac{I}{MB}}$

$\frac{5}{10}=2\pi\sqrt{\frac{5\times10^{-6}}{9.85\times10^{-2}\times B}}$

$B = 8 \times 10^{-3}$ T

6. Suppose two planets (spherical in shape) of radii R and 2R, but mass M and 9 M respectively have a centre to centre separation 8 R as shown in the figure. A satellite of mass '*m*' is projected from the surface of the planet of mass 'M' directly towards the centre of the second planet. The minimum speed '*v*' required for the satellite to reach the surface of the second planet is $\sqrt{\frac{a}{7}\frac{GM}{R}}$ then the value of 'a' is _____.

[Given : The two planets are fixed in their position]



Answer (4)

Sol. $-\frac{GMm}{2R}-\frac{9GMm}{6R}=\frac{1}{2}mv^2-\frac{GMm}{R}-\frac{9GMm}{7R}$

$\frac{1}{2}mv^2=\frac{16\,GM}{7R}-\frac{2GM}{R}$

$\frac{1}{2}mv^2=\frac{2\,GM}{7R}$

$v=\sqrt{\frac{4GM}{7R}}$

7. A transistor is connected in common emitter circuit configuration, the collector supply voltage is 10 V and the voltage drop across a resistor of 1000 Ω in the collector circuit is 0.6 V. If the current gain factor (β) is 24, then the base current is _____ μA. (Round off to the Nearest Integer)

Answer (25)

Sol. $B=\frac{I_C}{I_B}$ $\qquad I_C=\frac{0.6}{1000}=6\times10^{-4}\,A$

$24=\frac{6\times10^{-4}\,A}{x}$

$x=\frac{1}{4}\times10^{-4}\,A=25\times10^{-6}\,A$

8. A stone of mass 20 g is projected from a rubber catapult of length 0.1 m and area of cross section 10^{-6} m^2 stretched by an amount 0.04 m. The velocity of the projected stone is _____ m/s.

(Young's modulus of rubber = 0.5 × 10^9 N/m^2)

Answer (20)

Sol. $\frac{1}{2}y(\text{strain})^2\times\text{volume}=\frac{1}{2}mv^2$

$\frac{1}{2}\times0.5\times10^9\times16\times10^{-2}\times10^{-7}=\frac{1}{2}\times20\times10^{-3}\times v^2$

$\Rightarrow\ v^2=400$

$v=20$ m/s

9. A particle of mass 9.1 × 10^{-31} kg travels in a medium with a speed of 10^6 m/s and a photon of a radiation of linear momentum 10^{-27} kg m/s travels in vacuum. The wavelength of photon is _____ times the wavelength of the particle.

Answer (910)

Sol. $\frac{\lambda_1}{\lambda_2}=\frac{p_2}{p_1}$

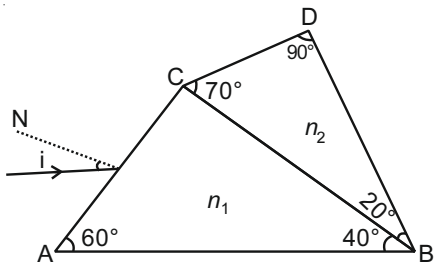
$\Rightarrow\ \lambda_2=\frac{\lambda_1 p_1}{p_2}=\frac{9.1\times10^{-31}\times10^6}{10^{-27}}\lambda_1$

$=910\lambda_1$

10. A prism of refractive index n_1 and another prism of refractive index n_2 are stuck together (as shown in the figure). n_1 and n_2 depend on λ, the wavelength of light, according to the relation

$n_1=1.2+\frac{10.8\times10^{-14}}{\lambda^2}$ and $n_2=1.45+\frac{1.8\times10^{-14}}{\lambda^2}$

The wavelength for which rays incident at any angle on the interface BC pass through without bending at that interface will be _____ nm.



Answer (600)

Sol. For no deviation $n_1 = n_2$

$1.2+\frac{10.8\times10^{-14}}{\lambda^2}=1.45+\frac{1.8\times10^{-14}}{\lambda^2}$

$0.25=\frac{9\times10^{-14}}{\lambda^2}$

$\Rightarrow\ \lambda=\frac{3}{5}\times10^{-6}=6\times10^{-7}$ m

$=600$ nm

PART–B : CHEMISTRY

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The product obtained from the electrolytic oxidation of acidified sulphate solutions, is

   (1) HO_3SOOSO_3H

   (2) HO_2SOSO_2H

   (3) HSO_4^-

   (4) HO_3SOSO_3H

Answer (1)

Sol. Anode :

$$2H_2SO_4 \longrightarrow H_2S_2O_8 + 2H^+ + 2e^-$$

$$2H_2O \longrightarrow O_2 + 4H^+ + 4e^-$$

Cathode :

$$e^- + H^+ \longrightarrow \frac{1}{2}H_2$$

Main product of electrolysis of conc. H_2SO_4 is HO_3SOOSO_3H ($H_2S_2O_8$)

2. Given below are two statements :

   Statement I : Rutherford's gold foil experiment cannot explain the line spectrum of hydrogen atom.

   Statement II : Bohr's model of hydrogen atom contradicts Heisenberg's uncertainty principle.

   In the light of the above statement, choose the **most appropriate** answer from the options given below :

   (1) Both **statement I** and **statement II** are false.

   (2) **Statement I** is true but **statement II** is false.

   (3) **Statement I** is false but **statement II** is true.

   (4) Both **statement I** and **statement II** are true.

Answer (4)

Sol. One of the drawback of Rutherford model is that, it says nothing about the electronic structure of atom. It cannot explain the line spectra of hydrogen atom.

Since uncertainty principle rules out existence of definite paths or trajectories of electrons and other similar particles. So Bohr's model contradicts H.U.P.

3. Given below are two statements : One is labelled as **Assertion A** and the other is labelled as **Reason R.**

   Assertion A : Lithium halides are some what covalent in nature.

   Reason R : Lithium possess high polarisation capability

   In the light of the above statements, choose the **most appropriate** answer from the options given below

   (1) **A** is false but **R** is true

   (2) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

   (3) **A** is true but **R** is false

   (4) Both **A** and **R** are true and **R** is the correct explanation of **A**

Answer (4)

Sol. The anomalous behaviour of lithium is due to the : (i) exceptionally small size of its atom and ion, and (ii) high polarising power.

Both LiCl and $MgCl_2$ are soluble in ethanol.

4. The parameters of the unit cell of a substance are a = 2.5, b = 3.0, c = 4.0, $\alpha = 90°$, $\beta = 120°$, $\gamma = 90°$. The crystal system of the substance is

   (1) Monoclinic (2) Hexagonal

   (3) Orthorhombic (4) Triclinic

Answer (1)

Sol. Monoclinic $a \neq b \neq c$, $\alpha = \gamma = 90°$, $\beta \neq 90°$

5. The type of hybridisation and magnetic property of the complex $[MnCl_6]^{3-}$, respectively, are

   (1) d^2sp^3 and diamagnetic

   (2) sp^3d^2 and diamagnetic

   (3) d^2sp^3 and paramagnetic

   (4) sp^3d^2 and paramagnetic

Answer (4)

Sol. $[MnCl_6]^{3-} \Rightarrow Mn^{3+}$ with weak field ligand

$\Rightarrow$ high spin complex

Hence, it is sp^3d^2 with paramagnetic character.

6. Given below are two statements

   Statement I : Aniline is less basic than acetamide.

   Statement II : In aniline, the lone pair of electrons on nitrogen atom is delocalised over benzene ring due to resonance and hence less available to a proton

Choose the **most appropriate** option

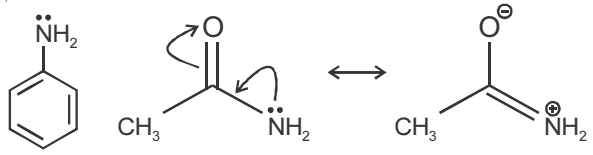
(1) **Statement I** is false but **statement II** is true.

(2) Both **statement I** and **statement II** are false.

(3) **Statement I** is true but **statement II** is false.

(4) Both **statement I** and **statement II** are true.

Answer (1)

Sol. 

As lone pair of 'N' in amide is in conjugation with carbonyl that is strong electron.

Withdrawing group, so aniline is more basic than acetamide.

In aniline, lone pair is less available for protonation.

7. For a reaction of order n, the unit of the rate constant is :

(1) $mol^{1-n} L^{2n} s^{-1}$ (2) $mol^{1-n} L^{1-n} s^{-1}$

(3) $mol^{1-n} L^{n-1} s^{-1}$ (4) $mol^{1-n} L^{1-n} s$

Answer (3)

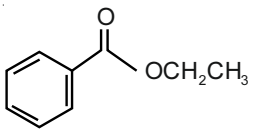
Sol. Rate = $k[A]^n$

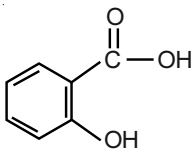
$$\frac{(mol/L)^1}{s} = k\,(mol/L)^n$$

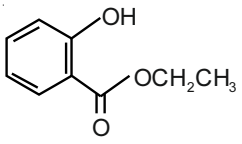
$$k = (mol/L)^{1-n}\ s^{-1}$$

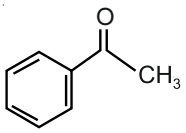
$$= mol^{1-n}\ L^{n-1}\ s^{-1}$$

8. Which one of the following compounds will give orange precipitate when treated with 2, 4-dinitrophenyl hydrazine?

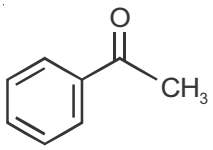
(1) 

(2) 

(3) 

(4) 

Answer (4)

Sol.  give orange ppt with 2, 4-dinitrophenyl hydrazine.

9. Match **List-I** with **List-II**

List-I	**List-II**
(a) NaOH	(i) Acidic
(b) $Be(OH)_2$	(ii) Basic
(c) $Ca(OH)_2$	(iii) Amphoteric
(d) $B(OH)_3$	
(e) $Al(OH)_3$	

Choose the **most appropriate** answer from the options given below **:**

(1) (a)-(ii), (b)-(ii), (c)-(iii), (d)-(i), (e)-(iii)

(2) (a)-(ii), (b)-(ii), (c)-(iii), (d)-(ii), (e)-(iii)

(3) (a)-(ii), (b)-(i), (c)-(ii), (d)-(iii), (e)-(iii)

(4) (a)-(ii), (b)-(iii), (c)-(ii), (d)-(i), (e)-(iii)

Answer (4)

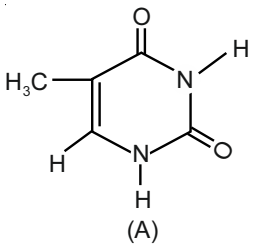
Sol. NaOH – Basic

$Ca(OH)_2$ – Basic

$Be(OH)_2$ – Amphoteric

$Al(OH)_3$ – Amphoteric

$B(OH)_3$ – Acidic

10. 
(A)

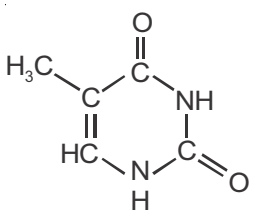
The compound 'A' is a complementary base of ___in DNA strands.

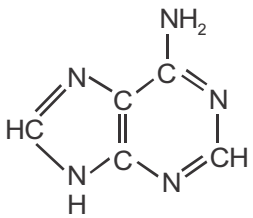
(1) Uracil (2) Guanine

(3) Adenine (4) Cytosine

Answer (3)

Sol. The given compound (A) is Thymine. It always bind with adenine in DNA.


Thymine (T)


Adenine (A)

11. Which one of the following statements is NOT correct?

(1) Eutrophication leads to increase in the oxygen level in water

(2) Eutrophication indicates that water body is polluted

(3) Eutrophication leads to anaerobic conditions

(4) The dissolved oxygen concentration below 6 ppm inhibits fish growth

Answer (1)

Sol. The lack of oxygen kills all other forms of aquatic life such as fish and plants. Fertilizers contain phosphates as additives. The addition of phosphates in water enhances algae growth. Such profuse growth of algae, covers the water surface and reduces the oxygen concentration in water. This leads to anaerobic conditions, commonly with accumulation of abnoxious decay and animal death. Thus, bloom-infested water inhibits the growth of other living organisms in the water body. This process in which nutrient enriched water bodies support a dense plant population, which kills animal life by depriving it of oxygen and results in subsequent loss of biodiversity is known as Eutrophication.

12. The statement that is INCORRECT about Ellingham diagram is:

(1) provides idea about the reaction rate.

(2) provides idea about free energy change.

(3) provides idea about changes in the phases during the reaction

(4) provides idea about reduction of metal oxide

Answer (1)

Sol. Ellingham diagram provide ideas about free energy change, phase change during the reaction and reduction of metal oxide but does not provide idea about reaction rate.

13. The oxidation states of 'P' in $H_4P_2O_7$, $H_4P_2O_5$ and $H_4P_2O_6$, respectively are

(1) 5, 4 and 3

(2) 7, 5 and 6

(3) 6, 4 and 5

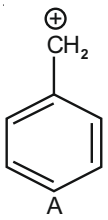
(4) 5, 3 and 4

Answer (4)

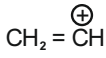
Sol.

	$H_4P_2O_7$	$H_4P_2O_5$	$H_4P_2O_6$
Oxidation state of 'P'	+5	+3	+4

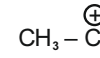
14.



A



B



C

D

The correct order of stability of given carbocation is

(1) C > A > D > B

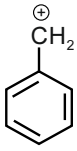
(2) D > B > C > A

(3) A > C > B > D

(4) D > B > A > C

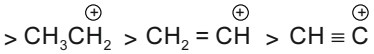
Answer (3)

Sol. Order of stability of carbocation is



A

Resonance



C

+ve charge on sp^3 hybridized carbon

B

+ve charge on sp^2 hybridized carbon

D

+ve charge on sp hybridized carbon

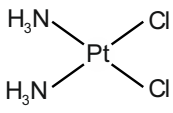
15. The number of geometrical isomers found in the metal complexes $[PtCl_2(NH_3)_2]$, $[Ni(CO)_4]$, $[Ru(H_2O)_3Cl_3]$ and $[CoCl_2(NH_3)_4]^+$ respectively, are

(1) 2, 1, 2, 1 (2) 2, 1, 2, 2

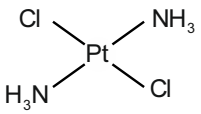
(3) 2, 0, 2, 2 (4) 1, 1, 1, 1

Answer (3)

Sol. $[Pt\,Cl_2(NH_3)_2]$

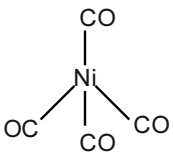


cis

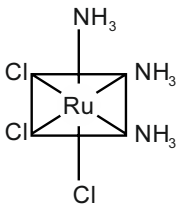


trans

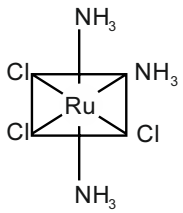
$[Ni(CO)_4]$



$[Ru(H_2O)_3\,Cl_3]$

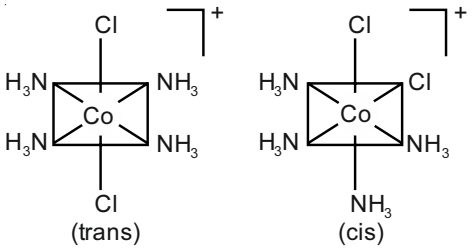


(fac)



(mer)

$[CoCl_2(NH_3)_4]^+$



16. Staggered and eclipsed conformers of ethane are

(1) Rotamers

(2) Mirror images

(3) Polymers

(4) Enantiomers

Answer (1)

Sol. Staggered and eclipsed conformers are formed because of C–C bond rotation. So they are known as Rotamers.

17. Presence of which reagent will affect the reversibility of the following reaction, and change it to a irreversible reaction

$$CH_4 + I_2 \underset{\text{Reversible}}{\overset{h\nu}{\rightleftharpoons}} CH_3 - I + HI$$

(1) Dilute HNO_2

(2) Liquid NH_3

(3) HOCl

(4) Concentrated HIO_3

Answer (4)

Sol. Iodination is very slow and a reversible reaction. It can be carried out in the presence of oxidizing agents like HIO_3 or HNO_3.

$$CH_4 + I_2 \rightleftharpoons CH_3I + HI$$

$$HIO_3 + 5HI \rightarrow 3I_2 + 3H_2O$$

18. Which one among the following chemical tests is used to distinguish monosaccharide from disaccharide?

(1) Seliwanoff's test

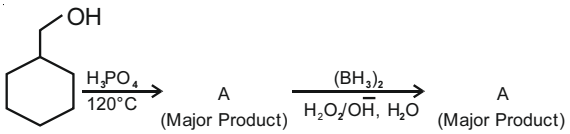
(2) Iodine test

(3) Tollen's test

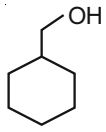
(4) Barfoed test

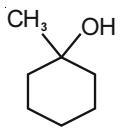
Answer (4)

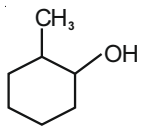
Sol. Barfoed's is a chemical test used to detect presence of monosaccharides from disaccharides.

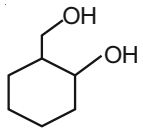
19. 

Consider the above reaction and identify the Product P

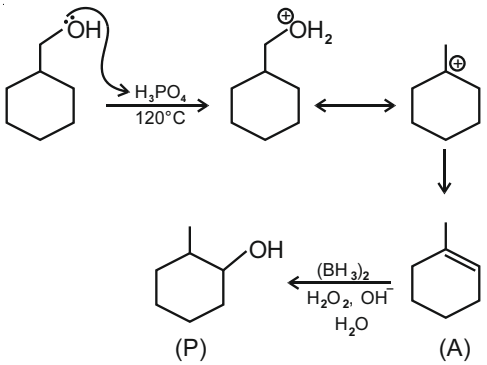
(1) 

(2) 

(3) 

(4) 

Answer (3)

Sol. 

20. Match List-I with List-II

List-I (Drug)	List-II (Class of Drug)
(a) Furacin	(i) Antibiotic
(b) Arsphenamine	(ii) Tranquilizers
(c) Dimetone	(iii) Antiseptic
(d) Valium	(iv) Synthetic antihistamines

Choose the most appropriate match

(1) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

(2) (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)

(3) (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)

(4) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

Answer (4)

Sol. Correct match are

(a) Furacin – Antiseptic

(b) Arsphenamine – Antibiotic

(c) Dimetone – Synthetic antihistamine

(d) Valium – Tranquilizers

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. $PCl_5 \rightleftharpoons PCl_3 + Cl_2 \qquad K_c = 1.844$

   3.0 moles of PCl_5 is introduced in a 1 L closed reaction vessel at 380 K. The number of moles of PCl_5 at equilibrium is _____ × 10^{-3}. (Round off to the Nearest Integer)

Answer (1400)

Sol.

	$PCl_5 \rightleftharpoons$	PCl_3 +	Cl_2
t = 0	3	0	0
$t = t_{eq}$	3 – x	x	x

$$\frac{x^2}{3-x} = 1.844$$

$x^2 = 1.844 \times 3 - 1.844x$

$x^2 + 1.844x - 5.532 = 0$

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-1.844 \pm \sqrt{(1.844)^2 + 4(5.532)}}{2(1)}$$

x = 1.60

Moles of PCl_5 at equilibrium = 3 – x = 3 – 1.6 = 1.4

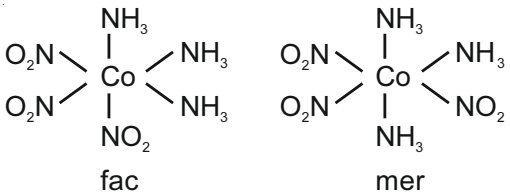
or 1400 × 10^{-3} mol

2. The number of geometrical isomers possible in triamminetrinitrocobalt (III) is X and in trioxalatochromate (III) is Y. Then the value of X + Y is _____.

Answer (2)

Sol. Triamminetrinitrocobalt (III) $[Co(NH_3)_3(NO_2)_3]$

Trioxalatochromate (III) $[Co(ox)_3]^{3-}$



fac

mer

x = 2

y = 0

3. 1.46 g of a biopolymer dissolved in a 100 mL water at 300 K exerted an osmotic pressure of 2.42 × 10^{-3} bar.

   The molar mass of the biopolymer is _____ × 10^4 g mol^{-1}. (Round off to the Nearest Integer)

   [Use : R = 0.083 L bar mol^{-1} K^{-1}]

Answer (15)

Sol. $\pi = iCRT$

$$2.42 \times 10^{-3} = \frac{1 \times 1.46 \times 1000 \times 0.083 \times 300}{M \times 100}$$

M = 150223 g mol^{-1}

M = 15.0223 × 10^4 $gmol^{-1}$

4. The difference between bond orders of CO and $NO^{\oplus}$ is $\frac{x}{2}$ where x = _____. (Round off to the Nearest Integer)

Answer (00)

Sol. Bond order of CO = 3

Bond order of NO^+ = 3

Difference = 0

5. For water at 100°C and 1 bar,

   $\Delta_{vap}H - \Delta_{vap}U$ = _____ × 10^2 J mol^{-1}. (Round off to the Nearest Integer)

   [Use : R = 8.31 J mol^{-1} K^{-1}]

   [Assume volume of $H_2O(l)$ is much smaller than volume of $H_2O(g)$. Assume $H_2O(g)$ can be treated as an ideal gas]

Answer (31)

Sol. $\Delta H - \Delta U = \Delta ngRT$

$= 1 \times 8.31 \times 373$

$= 30.99 \times 10^2$ J mol^{-1}

$\simeq 31 \times 10^2$ J mol^{-1}

6. An organic compound is subjected to chlorination to get compound A using 5.0 g of chlorine. When 0.5 g of compound A is reacted with $AgNO_3$ [Carius Method], the percentage of chlorine in compound A is _____ when it forms 0.3849 g of AgCl. (Round off to the Nearest Integer)

   (Atomic masses of Ag and Cl are 107.87 and 35.5 respectively)

Answer (19)

Sol. Number of moles of AgCl = $\frac{0.3849}{143.37}$

$= 2.684 \times 10^{-3}$ mol

% of chlorine in the compound A is

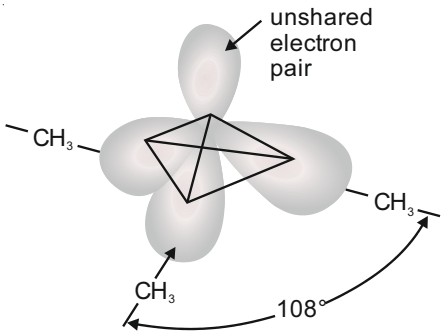
$$= \frac{2.684 \times 10^{-3} \times 35.5}{0.5} \times 100$$

$= 19.0564\%$

7. In gaseous triethyl amine the "– C – N – C –" bond angle is _____ degree.

Answer (108)

Sol.



Pyramidal shape of trimethylamine

8. The density of NaOH solution is 1.2 g cm^{-3}. The molality of this solution is _____ m.

(Round off to the Nearest Integer)

[Use : Atomic masses : Na : 23.0 u, O : 16.0 u H : 1.0 u Density of H_2O : 1.0 g cm^{-3}]

Answer (5)

Sol. Given, density of water = 1 g cm^{-3}

density of NaOH solution = 1.2 g cm^{-3}

mass of 1 L solution = 1200 g

mass of 1 L solvent = 1000 g

mass of solute = 200 g

$$\text{molality} = \frac{200 \times 1000}{40 \times 1000} = 5m$$

9. CO_2 gas adsorbs on charcoal following Freundlich adsorption isotherm. For a given amount of charcoal, the mass of CO_2 adsorbed becomes 64 times when the pressure of CO_2 is doubled. the value of n in the Freundlich isotherm equation is _____ $\times 10^{-2}$. (Round off to the Nearest integer)

Answer (17)

Sol. $\frac{x}{m} = kp^{\frac{1}{n}}$ ...(i)

When pressure in doubled,

$64\frac{x}{m} = k(2p)^{\frac{1}{n}}$ ...(ii)

$$\frac{(ii)}{(i)} \Rightarrow 64 = \frac{(2p)^{\frac{1}{n}}}{(p)^{\frac{1}{n}}} = (2)^{\frac{1}{n}}$$

$$64 = (2)^{\frac{1}{n}}$$

$$\frac{1}{n} = 6$$

$$n = \frac{1}{6} = 16.67 \times 10^{-2}$$

$$= 17 \times 10^{-2}$$

10. The conductivity of a weak acid HA of concentration 0.001 mol L^{-1} is 2.0×10^{-5} S cm^{-1}.

If $\Lambda_m^\circ(HA) = 190$ S cm^2 mol^{-1}, the ionization constant (K_a) of HA is equal to _____ $\times 10^{-6}$. (Round off to the Nearest Integer)

Answer (12)

Sol. C = 0.001 mol L^{-1}

$K = 2 \times 10^{-5}$ S cm^{-1}

$\wedge_m^\infty$ (HA) = 190 S cm^2 mol^{-1}

$$\wedge_m = \frac{K}{C} \times 1000$$

$$= \frac{2 \times 10^{-5} \times 10^3}{0.001}$$

$\wedge_m = 20$

$$\alpha = \frac{\wedge_m}{\wedge_m^\infty} = \frac{20}{190}$$

$$K_a = \frac{C\alpha^2}{1-\alpha} = \frac{10^{-3}\left(\frac{20}{190}\right)^2}{1-\frac{20}{190}}$$

$= 1.2383 \times 10^{-5}$

$= 12.38 \times 10^{-6}$

PART–C : MATHEMATICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Let the plane passing through the point (–1, 0, –2) and perpendicular to each of the planes $2x + y - z = 2$ and $x - y - z = 3$ be $ax + by + cz + 8 = 0$. Then the value of a + b + c is equal to :

(1) 3 (2) 5

(3) 8 (4) 4

Answer (4)

Sol. Let plane $\equiv A(x + 1) + B(y) + C(z + 2) = 0$

$$\therefore \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & -1 & -1 \end{vmatrix} = A\hat{i} + B\hat{j} + C\hat{k}$$

$$\Rightarrow 2\hat{i} - \hat{j}(-1) + \hat{k}(-3) = A\hat{i} + B\hat{j} + C\hat{k}$$

$\therefore$ A = –2, B = 1, C = –3

$\therefore$ required plane is

$-2x - 2 + y - 3z - 6 = 0$

$\Rightarrow 2x - y + 3z + 8 = 0$

$\therefore$ a + b + c = 4

2. The value of $\lim\limits_{n\to\infty} \frac{1}{n}\sum\limits_{j=1}^{n} \frac{(2j-1)+8n}{(2j-1)+4n}$ is equal to :

(1) $1+2\log_e\left(\frac{3}{2}\right)$ (2) $2-\log_e\left(\frac{2}{3}\right)$

(3) $3+2\log_e\left(\frac{2}{3}\right)$ (4) $5+\log_e\left(\frac{3}{2}\right)$

Answer (1)

Sol. $\lim\limits_{n\to\infty} \frac{1}{n}\sum\limits_{j=1}^{n} \frac{\frac{2j-1}{n}+8}{\frac{2j-1}{n}+4}$

$$\Rightarrow \frac{1}{2}\int_0^2 \left(1+\frac{4}{x+8}\right)dx$$

$$\Rightarrow \frac{1}{2}\left(2+4\ln\frac{3}{2}\right) = 1+2\ln\left(\frac{3}{2}\right)$$

3. Let

$A = \{(x, y) \in \mathbf{R} \times \mathbf{R} \mid 2x^2 + 2y^2 - 2x - 2y = 1\}$,

$B = \{(x, y) \in \mathbf{R} \times \mathbf{R} \mid 4x^2 + 4y^2 - 16y + 7 = 0\}$ and

$C = \{(x, y) \in \mathbf{R} \times \mathbf{R} \mid x^2 + y^2 - 4x - 2y + 5 \le r^2\}$.

Then the minimum value of |r| such that $A \cup B \subseteq C$ is equal to :

(1) $\frac{2+\sqrt{10}}{2}$ (2) $\frac{3+2\sqrt{5}}{2}$

(3) $1+\sqrt{5}$ (4) $\frac{3+\sqrt{10}}{2}$

Answer (2)

Sol. A $\equiv$ circle of centre $\left(\frac{1}{2}, \frac{1}{2}\right)$ and radius 1

B $\equiv$ circle of centre (0, 2) and radius $\frac{3}{2}$

C is circular disc of centre (2, 1) and radius *r*

for C to be superset of $A \cup B$

Distance of centre of C from farthest points on A and B both shall be less than radius of C i.e.

$$\sqrt{\left(\frac{3}{2}\right)^2+\left(\frac{1}{2}\right)^2}+1 \le r \text{ and } \sqrt{2^2+1^2}+\frac{3}{2} \le r$$

$$r \ge \frac{3+2\sqrt{5}}{2}$$

4. The compound statement $(P \vee Q) \wedge (\sim P) \Rightarrow Q$ is equivalent to :

(1) $P \vee Q$

(2) $\sim(P \Rightarrow Q) \Leftrightarrow P \wedge \sim Q$

(3) $P \wedge \sim Q$

(4) $\sim(P \Rightarrow Q)$

Answer (2)

Sol. $(P \vee Q) \wedge (\sim P) \rightarrow Q$

$= \sim(P \vee Q) \vee P \vee Q$

$= \sim(P \vee Q) \vee (P \vee Q) \Rightarrow$ It is a tautology.

Only option (2) is a tautology because

$\sim(P \rightarrow Q) = \sim(\sim P \vee Q) = P \wedge \sim Q$

5. The value of the definite integral

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{\left(1+e^{x\cos x}\right)\left(\sin^4 x+\cos^4 x\right)}$$

is equal to :

(1) $-\frac{\pi}{4}$ (2) $\frac{\pi}{2\sqrt{2}}$

(3) $-\frac{\pi}{2}$ (4) $\frac{\pi}{\sqrt{2}}$

Answer (2)

Sol. $I=\int_{-\pi/4}^{\pi/4} \frac{dx}{\left(1+e^{x\cos x}\right)\left(\sin^4 x+\cos^4 x\right)}$

applying $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$

$$I=\int_{-\pi/4}^{\pi/4} \frac{dx}{\left(1+e^{-x\cos x}\right)\left(\sin^4 x+\cos^4 x\right)}$$

$$2I=\int_{-\pi/4}^{\pi/4} \frac{dx}{\sin^4 x+\cos^4 x}=\int_{-\pi/4}^{\pi/4} \frac{\sec^2 x\left(1+\tan^2 x\right)dx}{\tan^4 x+1}$$

Put tan$x$ = t we get

$$2I=\int_{-1}^{1}\left(\frac{1+t^2}{1+t^4}\right)dt=2\int_0^1 \frac{1+t^2}{1+t^4}\,dt$$

Put $t-\frac{1}{t}=R$

$$I=\int_{-\infty}^{0} \frac{dk}{k^2+2}$$

$$\Rightarrow I=\frac{1}{\sqrt{2}}\tan^{-1}\frac{k}{\sqrt{2}}\Big|_{-\infty}^{0}=\frac{1}{\sqrt{2}}\left(0+\frac{\pi}{2}\right)$$

$$=\frac{\pi}{2\sqrt{2}}$$

6. If the coefficients of x^7 in $\left(x^2+\frac{1}{bx}\right)^{11}$ and x^{-7} in $\left(x-\frac{1}{bx^2}\right)^{11}$, $b\neq 0$, are equal, then the value of b is equal to

(1) 1 (2) –2

(3) –1 (4) 2

Answer (1)

Sol. General term of $\left(x^2+\frac{1}{bx}\right)^{11}$

$$T_{r+1}={}^{11}C_r\left(x^2\right)^{11-r}\left(\frac{1}{bx}\right)^r=\frac{{}^{11}C_r}{b^r}x^{22-3r}$$

Coeff. of $x^7=\frac{{}^{11}C_5}{b^5}$

Similarly general term of $\left(x-\frac{1}{bx^2}\right)^{11}$

$$T_{r+1}={}^{11}C_r(x)^{11-r}\left(-\frac{1}{bx^2}\right)^r=\frac{{}^{11}C_r}{(-b)^r}x^{11-2r}$$

Coeff. of $x^{-7}=\frac{{}^{11}C_6}{b^6}$

$$\Rightarrow b=\frac{{}^{11}C_6}{{}^{11}C_5}=1$$

7. Let $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=-\hat{i}+2\hat{j}+3\hat{k}$. Then the vector product $(\vec{a}+\vec{b})\times\left(\left(\vec{a}\times\left((\vec{a}-\vec{b})\times\vec{b}\right)\right)\times\vec{b}\right)$ is equal to

(1) $7\left(34\hat{i}-5\hat{j}+3\hat{k}\right)$ (2) $7\left(30\hat{i}-5\hat{j}+7\hat{k}\right)$

(3) $5\left(30\hat{i}-5\hat{j}+7\hat{k}\right)$ (4) $5\left(34\hat{i}-5\hat{j}+3\hat{k}\right)$

Answer (1)

Sol. $(\vec{a}-\vec{b})\times\vec{b}=\vec{a}\times\vec{b}$

$\therefore$ Given expression is

$$(\vec{a}+\vec{b})\times\left(\vec{a}\times\left(\vec{a}\times\vec{b}\right)\times\vec{b}\right)$$

$$\Rightarrow (\vec{a}+\vec{b})\times\left(\vec{a}\times(\vec{b}\cdot\vec{a})\vec{b}-(\vec{b}\cdot\vec{b})\vec{a}\right)$$

$$\Rightarrow \left((\vec{a}+\vec{b})\times(\vec{a}\times\vec{b})\right)\left(\vec{b}\cdot\vec{a}\right)$$

$$\Rightarrow \left(\vec{a}\times(\vec{a}\times\vec{b})+\vec{b}\times(\vec{a}\times\vec{b})\right)(\vec{b}\cdot\vec{a})$$

$$\Rightarrow \left((\vec{a}\cdot\vec{b})\vec{a}-(\vec{a}\cdot\vec{a})\vec{b}+(\vec{b}\cdot\vec{b})\vec{a}-(\vec{b}\cdot\vec{a})\vec{b}\right)(\vec{b}\cdot\vec{a})$$

Put $\vec{a}\cdot\vec{b}=7,\ \vec{a}\cdot\vec{a}=6,\ \vec{b}\cdot\vec{b}=14$ we get

$\Rightarrow \left(7\vec{a}-6\vec{b}+14\vec{a}-7\vec{b}\right)7$

$\Rightarrow 7\left(21\vec{a}-13\vec{b}\right)$

$\Rightarrow 7\left(21\hat{i}+21\hat{j}+42\hat{k}+13\hat{i}-26\hat{j}-39\hat{k}\right)$

$\Rightarrow 7\left(34\hat{i}-5\hat{j}+3\hat{k}\right)$

8. If the mean and variance of the following data :
6, 10, 7, 13, a, 12, b, 12

are 9 are $\frac{37}{4}$ respectively, then $(a-b)^2$ is equal to

(1) 32 (2) 12
(3) 24 (4) 16

Answer (4)

Sol. 6 + 10 + 7 + 13 + 12 + 12 + (a + b) = 72

$\Rightarrow$ a + b = 12

and $\frac{a^2+b^2+36+100+49+169+144+144}{8}=\frac{37}{4}$

$a^2 + b^2 + 642 - 648 = 74$

$a^2 + b^2 = 80$

$\therefore$ $(a + b)^2 = a^2 + b^2 + 2ab \Rightarrow 2ab = 64$

$(a - b)^2 = a^2 + b^2 - 2ab = 16$

9. Let α, β be two roots of the equation $x^2+(20)^{\frac{1}{4}}x+(5)^{\frac{1}{2}}=0$. Then $\alpha^8 + \beta^8$ is equal to

(1) 160 (2) 10
(3) 50 (4) 100

Answer (3)

Sol. $x^2+(20)^{\frac{1}{4}}x+(5)^{\frac{1}{2}}=0$

$\therefore$ $\alpha+\beta=-(20)^{\frac{1}{4}},\alpha\cdot\beta=(5)^{\frac{1}{2}}$

$\alpha^8 + \beta^8 = (\alpha^4 + \beta^4)^2 - 2\alpha^4\beta^4$

$=\left\{\left(\alpha^2+\beta^2\right)^2-2\alpha^2\beta^2\right\}^2-2\alpha^4\beta^4$

$=\left[\left\{(\alpha+\beta)^2-2\alpha\beta\right\}^2-2\alpha^2\beta^2\right]^2-2\alpha^4\beta^4$

$=\left[\left\{20^{\frac{1}{2}}-2.5^{\frac{1}{2}}\right\}^2-2.5\right]^2-2.5^2$

$= (0 - 10)^2 - 50$

$= 50$

10. A ray of light through (2, 1) is reflected at a point P on the *y*-axis and then passes through the point (5, 3). If this reflected ray is the directrix of an ellipse with eccentricity $\frac{1}{3}$ and the distance of the nearer focus from this directrix is $\frac{8}{\sqrt{53}}$, then the equation of the other directrix can be

(1) $2x - 7y + 29 = 0$ or $2x - 7y - 7 = 0$
(2) $11x + 7y + 8 = 0$ or $11x + 7y - 15 = 0$
(3) $2x - 7y - 39 = 0$ or $2x - 7y - 7 = 0$
(4) $11x - 7y - 8 = 0$ or $11x + 7y + 15 = 0$

Answer (1)

Sol. Image of (2, 1) w.r.t. *y* axis is (–2, 1)

$\therefore$ equation of reflected ray is

$y-1=\frac{3-1}{5+2}(x+2)$

$\therefore$ $2x - 7y + 11 = 0$ ...(i)

$\because$ $\frac{a}{e}-ae=\frac{8}{\sqrt{53}} \Rightarrow a=\frac{3}{\sqrt{53}}$

Now $\frac{2a}{e}-2\cdot\frac{3}{\sqrt{53}}\times=\frac{18}{\sqrt{53}}$

The equation of other directrix is : $2x - 7y + k = 0$

$\therefore$ $\left|\frac{k-11}{\sqrt{53}}\right|=\frac{18}{\sqrt{53}} \Rightarrow |k-11|=18$

$\therefore$ $k = 29$ or -7

$\therefore$ equation of directrix may be: $2x - 7y + 29 = 0$ or $2x - 7y - 7 = 0$

11. Let $f : \mathbf{R} \to \mathbf{R}$ be a function such that $f(2) = 4$ and $f'(2) = 1$. Then, the value of $\lim\limits_{x\to2}\frac{x^2f(2)-4f(x)}{x-2}$ is equal to

(1) 16 (2) 8
(3) 4 (4) 12

Answer (4)

Sol. $\lim\limits_{x\to2}\frac{x^2f(2)-4f(x)}{x-2}\left[\frac{0}{0}\right]$

$=\lim\limits_{x\to2}2x\cdot f(2)-4f'(x)$

$= 4f(2) - 4\cdot f'(2)$

$= 4 \times 4 - 4 \times 1$

$= 12$

12. If $\sin\theta + \cos\theta = \frac{1}{2}$, then $16(\sin(2\theta) + \cos(4\theta) + \sin(6\theta))$ is equal to

(1) 23 (2) –23

(3) 27 (4) –27

Answer (2)

Sol. $\because \quad \sin\theta + \cos\theta = \frac{1}{2}$

$\therefore \quad \sin 2\theta = \frac{1}{4} - 1 = -\frac{3}{4}$

$\therefore \quad 16(\sin2\theta + \cos4\theta + \sin6\theta)$

$= 16\{\sin2\theta + 1 - 2\sin^2 2\theta + 3\sin2\theta - 4\sin^3 2\theta\}$

$= 16\left\{-\frac{4\times3}{4} + 1 - 2\cdot\frac{9}{16} - 4\times-\frac{27}{64}\right\}$

$= 16\left\{-2 - \frac{9}{8} + \frac{27}{16}\right\}$

$= -23$

13. Let C be the set of all complex numbers. Let

$S_1 = \{z \in C : |z - 3 - 2i|^2 = 8\}$,

$S_2 = \{z \in C : |\text{Re}(z)| \geq 5\}$ and

$S_3 = \{z \in C : |z - \bar{z}| \geq 8\}$.

Then the number of elements in $S_1 \cap S_2 \cap S_3$ is equal to

(1) 0 (2) 1

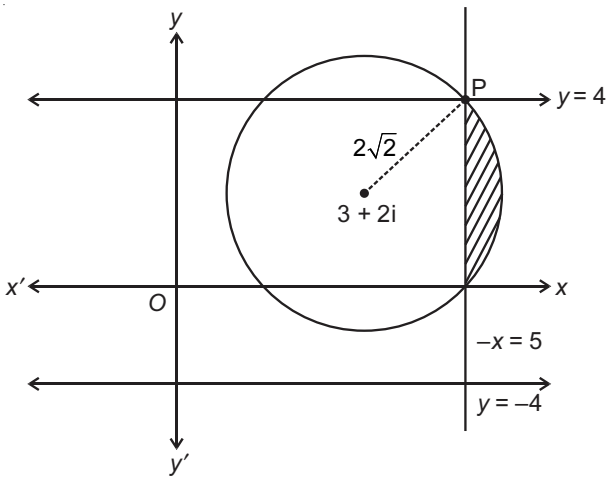
(3) 2 (4) Infinite

Answer (2)

Sol. $\because$ S_1 be a circle of centre 3 + 2i and radius $2\sqrt{2}$

S_3 is half plane with real z more than S

and S_3 is plane with $y \in (-\infty, -4] \cup [4, \infty)$



$\therefore$ Only one point P is the solution.

14. Let $y = y(x)$ be solution of the differential equation

$\log_e\left(\frac{dy}{dx}\right) = 3x + 4y$, with y(0) = 0. If

$y\left(-\frac{2}{3}\log_e 2\right) = \alpha\log_e 2$, then the value of α is

equal to

(1) $-\frac{1}{2}$

(2) $-\frac{1}{4}$

(3) $\frac{1}{4}$

(4) 2

Answer (2)

Sol. $\because \quad \ln\left(\frac{dy}{dx}\right) = 3x + 4y$

$\Rightarrow \quad \frac{dy}{dx} = e^{3x+4y}$

$\Rightarrow \quad e^{-4y}dy = e^{3x}dx$

$\Rightarrow \quad \int e^{-4y}dy = \int e^{3x}dx$

$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + C$

$\because \quad y(0) = 0$

$\Rightarrow \quad C = -\frac{7}{12}$

$\therefore \quad e^{-4y} = \frac{7}{3} + \frac{e^{3x}}{3}$

$e^{4y} = \frac{3}{7 - 4e^{3x}}$

$y = \frac{1}{4}\ln\left(\frac{3}{7 - 4e^{3x}}\right)$

$\therefore \quad y = \left(-\frac{2}{3}\ln2\right) = \frac{1}{4}\ln\left(\frac{3}{6}\right) = -\frac{1}{4}\ln2$

$\therefore \quad \alpha = -\frac{1}{4}$

15. The probability that a randomly selected 2-digit number belongs to the set $\{n \in N: (2^n - 2)$ is a multiple of 3$\}$ is equal to

(1) $\frac{1}{2}$

(2) $\frac{2}{3}$

(3) $\frac{1}{3}$

(4) $\frac{1}{6}$

Answer (1)

Sol. The given set = $\{n \in N : 2^n - 2$ is a multiple of 3$\}$

= {0, 6, 30, 62, 126, ...}

There are only 2, 2 digit numbers out of which only one is divisible by 3

$\therefore$ Required Probability = $\frac{1}{2}$.

16. Let $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$. If $A^{-1} = \alpha I + \beta A$, $\alpha, \beta \in \mathbf{R}$, I is 2×2 identity matrix, then $4(\alpha - \beta)$ is

(1) $\frac{8}{3}$ (2) 5

(3) 4 (4) 2

Answer (3)

Sol. $\because$ $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$

Then $A^{-1} = \frac{1}{6}\begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2/3 & -1/3 \\ 1/6 & 1/6 \end{bmatrix}$

$\therefore$ $\alpha I + \beta A = \begin{bmatrix} \alpha+\beta & 2\beta \\ -\beta & \alpha+4\beta \end{bmatrix}$

$\therefore$ $\beta = -\frac{1}{6}$ and $\alpha = \frac{5}{6}$

$\therefore$ $4(\alpha - \beta) = 4\left(\frac{5}{6} + \frac{1}{6}\right) = 4$

17. Let P and Q be two distinct points on a circle which has center at C(2, 3) and which passes through origin O. If OC is perpendicular to both the ling segments CP and CQ, then the set {P, Q} is equal to:

(1) $\left\{\left(2+2\sqrt{2}, 3+\sqrt{5}\right), \left(2-2\sqrt{2}, 3-\sqrt{5}\right)\right\}$

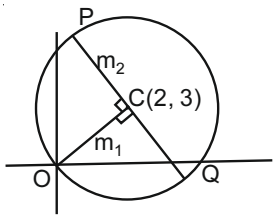
(2) {(4, 0), (0, 6)}

(3) {(–1, 5), (5, 1)}

(4) $\left\{\left(2+2\sqrt{2}, 3-\sqrt{5}\right), \left(2-2\sqrt{2}, 3+\sqrt{5}\right)\right\}$

Answer (3)

Sol. PQ is a straight line and PQ is a diameter



$m_1 = \frac{3}{2}$

$m_2 = \frac{-2}{3} = \tan\theta$

$\sin\theta = \frac{2}{\sqrt{13}}, \cos\theta = \frac{-3}{\sqrt{13}}$

$P(2 + r\cos\theta, 3 + r\sin\theta), r = \sqrt{13}$

$Q(2 + r\cos\theta, 3 + r\sin\theta), r = -\sqrt{13}$

$P \equiv (-1, 5)$, $Q \equiv (5, 1)$

18. Let $f : \left(-\frac{\pi}{4}, \frac{\pi}{4}\right) \to \mathbf{R}$ be defined as

$$f(x) = \begin{cases} (1+|\sin x|)^{\frac{3a}{|\sin x|}}, & -\frac{\pi}{4} < x < 0 \\ b, & x = 0 \\ e^{\cot 4x / \cot 2x}, & 0 < x < \frac{\pi}{4} \end{cases}$$

If f is continuous at $x = 0$, then the value of $6a + b^2$ is equal to

(1) 1 + e (2) 1 – e

(3) e (4) e – 1

Answer (1)

Sol. LHL = $f(0)$ = RHL

$$e^{3a} = b = e^{\lim\limits_{x\to 0}\frac{\tan 2x}{2x}\times\frac{4x}{\tan 4x}\times\frac{1}{2}}$$

$$e^{3a} = b = e^{\frac{1}{2}}$$

$6a + b^2 = 1 + e$

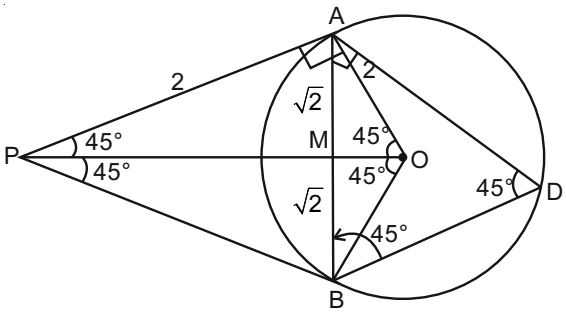
19. Two tangents are drawn from the point P(–1, 1) to the circle $x^2 + y^2 – 2x – 6y + 6 = 0$. If these tangents touch the circle at points A and B, and if D is a point on the circle such that length of the segments AB and AD are equal, then the area of the triangle ABD is equal to

(1) 2 (2) $(3\sqrt{2}+2)$

(3) 4 (4) $3(\sqrt{2}-1)$

Answer (3)

Sol.



$PA = \sqrt{1+1+2-6+6} = 2$

$MA = PA\sin 45° = \sqrt{2}$

$AB = 2\sqrt{2}$

$AD = 2\sqrt{2}$

$[ABD] = \frac{1}{2}\times(2\sqrt{2}\times 2\sqrt{2}) = 4$

20. If the area of the bounded region

$$R = \left\{(x,y) : \max\{0, \log_e x\} \le y \le 2^x, \frac{1}{2} \le x \le 2\right\}$$

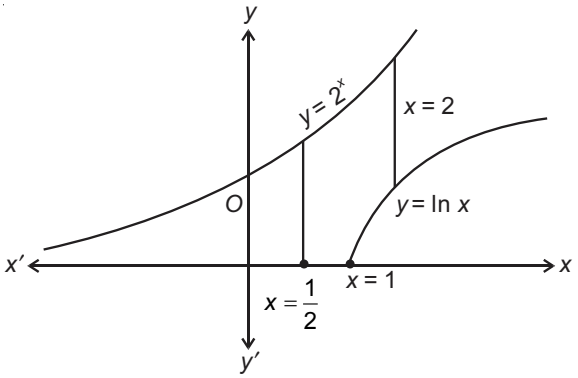
is, $\alpha(\log_e 2)^{-1} + \beta(\log_e 2) + \gamma$, then the value of $(\alpha + \beta – 2\gamma)^2$ is equal to

(1) 2 (2) 1

(3) 8 (4) 4

Answer (1)

Sol.



$$A = \int_{\frac{1}{2}}^{2} 2^x dx - \int_{1}^{2} \ln x\, dx$$

$$= \left[\frac{2^x}{\ln 2}\right]_{\frac{1}{2}}^{2} - [x\ln x - x]_1^2$$

$$= \frac{4-\sqrt{2}}{\ln 2} - 2\ln 2 + 1$$

$(\alpha+\beta-2\gamma)^2 = \left((4-\sqrt{2})+(-2)-2(1)\right)^2$

= 2

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Let $f : [0, 3] \to \mathbf{R}$ be defined by

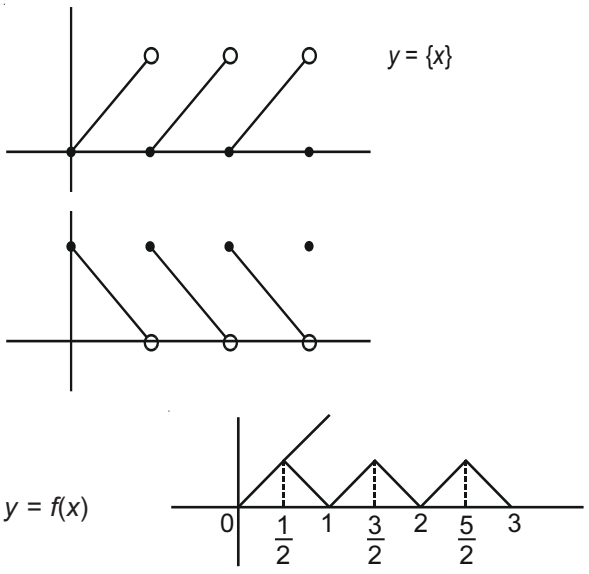
$f(x) = \min\{x – [x], 1 + [x] – x\}$

where $[x]$ is the greatest integer less than or equal to x.

Let P denote the set containing all $x \in [0, 3]$ where f is discontinuous, and Q denote the set containing all $x \in (0, 3)$ where f is not differentiable. Then the sum of number of elements in P and Q is equal to _____.

Answer (5)

Sol. $f(x) = \min\{\{x\}, 1 - \{x\}\}$



Cont. everywhere & non diff at $x = \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}$

2. For real numbers α and β consider the following system of linear equations :

$x + y - z = 2$, $x + 2y + \alpha z = 1$, $2x - y + z = \beta$.

If the system has infinite solutions, then $\alpha + \beta$ is eual to _____.

Answer (5)

Sol. $\Delta = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 2 & \alpha \\ 2 & -1 & 1 \end{vmatrix} = 0 \Rightarrow \alpha = -2$

$\Delta_2 = \begin{vmatrix} 2 & 1 & -1 \\ 1 & 2 & -2 \\ \beta & -1 & 1 \end{vmatrix} = 0 \Rightarrow \beta = 7$

$\Delta_3 = 0 \Rightarrow \beta = 7$

$\alpha + \beta = 5$

3. If $\log_3 2$, $\log_3(2^x - 5)$, $\log_3\left(2^x - \frac{7}{2}\right)$ are in an arithmetic progression, then the value of x is equal to _____.

Answer (03.00)

Sol. $\because$ $\log_3 2, \log_3(2^x - 5), \log_3\left(2^x - \frac{7}{2}\right)$ are in A.P.

$\therefore$ $2, 2^x - 5, 2^x - \frac{7}{2}$ are in G.P.

$(2^x - 5)^2 = 2.\left(2^x - \frac{7}{2}\right)$

$2^{2x} - 10.2^x + 25 = 2.2^x - 7$

$2^{2x} - 12.2^x + 32 = 0$

$(2^x - 4)(2^x - 8) = 0$

$\therefore$ $x = 2$ or 3

But $x = 2$ is not acceptable

$\therefore$ $x = 3$

4. Let a plane P pass through the point (3, 7, –7) and contain the line, $\frac{x-2}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$. If distance of the plane P from the origin is d, then d^2 is equal to _____,

Answer (3)

Sol. Equation of plane through point (3, 7, –7) and containing line $\frac{x-2}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ is

$\begin{vmatrix} x-2 & y-3 & z+2 \\ 3-2 & 7-3 & -7+2 \\ -3 & 2 & 1 \end{vmatrix} = 0$

$\begin{vmatrix} x-2 & y-3 & z+2 \\ 1 & 4 & -5 \\ -3 & 2 & 1 \end{vmatrix} = 0$

$x - y + z + 3 = 0$

$\therefore$ Distance from origin $= d = \left|\frac{3}{\sqrt{1^2 + 1^2 + 1}}\right|$

$\therefore$ $d^2 = 3$

5. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b}$ and $\vec{c} = \hat{j} - \hat{k}$ be three vectors such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \cdot \vec{b} = 1$. If the length of projection vector of the vector $\vec{b}$ on the vector $\vec{a} \times \vec{c}$ is l, then the value of $3l^2$ is equal to _____.

Answer (2)

Sol. $\because$ $\vec{a} \times \vec{b} = \vec{c}$

$\Rightarrow$ $(\vec{a} \times \vec{b}) \cdot \vec{c} = |\vec{c}|^2 = 2 = [\vec{a}\ \vec{b}\ \vec{c}]$

$\because$ $l = \left|\frac{\vec{b} \cdot (\vec{a} \times \vec{c})}{|\vec{a} \times \vec{c}|}\right|$ ($\because$ $\vec{a}$ and $\vec{c}$ are perpendicular)

$\Rightarrow$ $l = \frac{|[\vec{a}\ \vec{b}\ \vec{c}]|}{|\vec{a}||\vec{c}|} = \frac{2}{\sqrt{3}\sqrt{2}} = \sqrt{\frac{2}{3}} \Rightarrow 3l^2 = 2$

6. Let the domain of the function

$f(x) = \log_4(\log_5(\log_3(18x - x^2 - 77)))$ be (a, b).

Then the value of the integral

$\int_a^b \frac{\sin^3 x}{\left(\sin^3 x + \sin^3(a+b-x)\right)}dx$ is equal to _____.

Answer (1)

Sol. $\because \quad 18x - x^2 - 77 > 3 \Rightarrow x^2 - 18x + 80 < 0$

$\Rightarrow x \in (8, 10)$

$a = 8$ and $b = 10$

Now, $I = \int_a^b \frac{\sin^3 x}{\sin^3 x + \sin^3(a+b-x)}dx$

So, $I = \int_a^b \frac{\sin^3(a+b-x)}{\sin^3(a+b-x) + \sin^3 x}dx$

hence $2I = \int_a^b dx = b - a$

$\Rightarrow \quad I = 1$

7. Let $F : [3, 5] \to \mathbf{R}$ be a twice differentiable function on (3, 5) such that $F(x) = e^{-x}\int_3^x (3t^2 + 2t + 4F'(t))dt$.

If $F'(4) = \frac{\alpha e^{\beta} - 224}{(e^{\beta} - 4)^2}$, then $\alpha + \beta$ is equal to _____.

Answer (16)

Sol. $e^x \cdot F(x) = \int_3^x (3t^2 + 2t + 4F'(t))dt, F(3) = 0$

Differentiating w.r.t. x

$e^xF(x) + e^xF'(x) = 3x^2 + 2x + 4F'(x)$

$\Rightarrow \quad F'(x) + \left(\frac{e^x}{e^x - 4}\right)F(x) = \frac{3x^2 + 2x}{e^x - 4} \qquad \ldots(1)$

I.F. $= e^x - 4$

$F(x).(e^x - 4) = \int (3x^2 + 2x)dx + c$

$\Rightarrow \quad F(x) = \frac{x^3 + x^2 + c}{e^x - 4} \quad (\because F(3) = 0 \Rightarrow c = -36)$

$\Rightarrow \quad F(x) = \frac{x^3 + x^2 - 36}{e^x - 4} \Rightarrow F(4) = \frac{44}{e^4 - 4}$

Form (1)

$F'(4) + \left(\frac{e^4}{e^4 - 4}\right)F(4) = \frac{56}{e^4 - 4}$

$\Rightarrow \quad F'(4) = \frac{56}{e^4 - 4} - \frac{44e^4}{(e^4 - 4)^2}$

$\Rightarrow \quad F'(4) = \frac{12e^4 - 224}{(e^4 - 4)^2}$

Clearly $\alpha = 12$, $\beta = 4$

8. Let

$f(x) = \begin{vmatrix} \sin^2 x & -2 + \cos^2 x & \cos 2x \\ 2 + \sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & 1 + \cos 2x \end{vmatrix}, x \in [0, \pi].$

Then the maximum value of $f(x)$ is equal to _____.

Answer (6)

Sol. $f(x) = \begin{vmatrix} \sin^2 x & -2 + \cos^2 x & \cos 2x \\ 2 & 2 & 0 \\ 0 & 2 & 1 \end{vmatrix} \begin{matrix} \\ R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$

$\Rightarrow \quad f(x) = -2(-2\cos 2x) + (2\sin^2 x + 4 - 2\cos^2 x)$

$= 4\cos 2x + 4 - 2\cos 2x = 4 + 2\cos 2x$

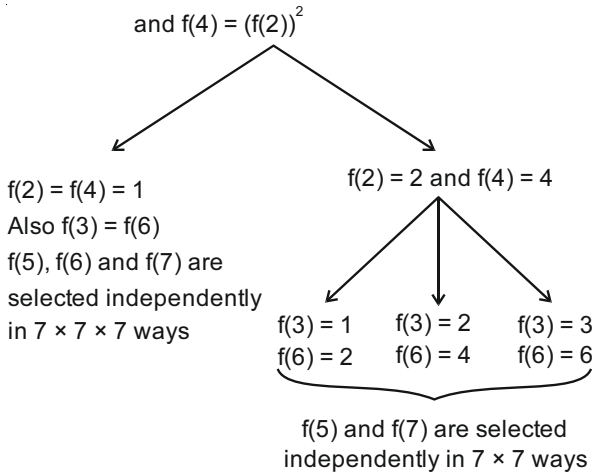
$\Rightarrow \quad f(x)_{max} = 6$

9. Let S = {1, 2, 3, 4, 5, 6, 7}. Then the number of possible functions $f : S \to S$ such that $f(m \cdot n) = f(m) \cdot f(n)$ for every $m, n \in S$ and $m \cdot n \in S$ is equal to _____.

Answer (490)

Sol. $\because \quad f(m : n) = f(m) \cdot f(n)$

Clearly f(1) = 1



Total number of ways = $7^3 + 3.7^2 = 490$

10. If $y = y(x)$, $y \in \left[0, \frac{\pi}{2}\right)$ is the solution of the differential equation

$$\sec y \frac{dy}{dx} - \sin(x+y) - \sin(x-y) = 0, \text{ with } y(0) = 0,$$

then $5y'\left(\frac{\pi}{2}\right)$ is equal to _____.

Answer (2)

Sol. $\sec y \frac{dy}{dx} = 2\sin x \cdot \cos y$

$\Rightarrow \sec^2 y dy = 2\sin x dx$

$\Rightarrow \tan y = -2\cos x + c$

put $x = 0$ and $y = 0$

$\Rightarrow c = 2$

put $x = \frac{\pi}{2}$ then $y = \tan^{-1}2$

Now, $\frac{dy}{dx} = 2\sin x \cdot \cos^2 y$

put $x = \frac{\pi}{2}$, $y = \tan^{-1}2$

$\Rightarrow y'\left(\frac{\pi}{2}\right) = 2 \cdot \left(\frac{1}{5}\right)$

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