24/01/2023
Evening

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Answers & Solutions

Time : 3 hrs. *for* **M.M. : 300**

JEE (Main)-2023 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

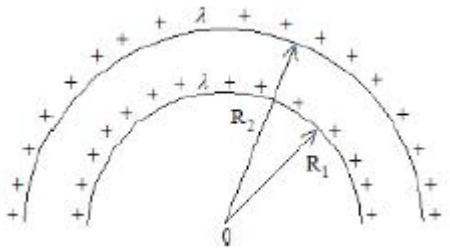
PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. The electric potential at the centre of two concentric half rings of radii R_1 and R_2, having same linear charge density λ is:



(1) $\frac{\lambda}{4\varepsilon_0}$ (2) $\frac{\lambda}{2\varepsilon_0}$

(3) $\frac{2\lambda}{\varepsilon_0}$ (4) $\frac{\lambda}{\varepsilon_0}$

Answer (2)

Sol. $V_1 = \frac{1}{4\pi\varepsilon_0} \times \frac{\lambda(\pi R_1)}{R_1}$

$V_2 = \frac{1}{4\pi\varepsilon_0} \times \frac{\lambda(\pi R_2)}{R_2}$

$V_{net} = V_1 + V_2$

$= 2 \times \frac{1}{4\pi\varepsilon_0} \pi\lambda$

$= \frac{\lambda}{2\varepsilon_0}$

2. A body of mass 200 g is tied to a spring of spring constant 12.5 N/m, while the other end of spring is fixed at point O. If the body moves about O in a circular path on a smooth horizontal surface with constant angular speed 5 rad/s. Then the ratio of extension in the spring to its natural length will be:

(1) 1 : 1

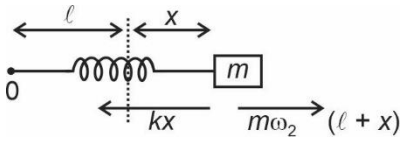
(2) 1 : 2

(3) 2 : 3

(4) 2 : 5

Answer (3)

Sol.



$\because \quad kx = m\omega^2 (\ell + x)$

$12.5(x) = \frac{1}{5}(5)^2(\ell + x)$

$\Rightarrow \quad \frac{5}{2}x = \ell + x$

$\Rightarrow \quad \frac{3}{2}x = \ell$

$\Rightarrow \quad \frac{x}{\ell} = \frac{2}{3}$

3. The electric field and magnetic field components of an electromagnetic wave going through vacuum is described by

$E_x = E_o \sin(kz - \omega t)$

$B_y = B_o \sin(kz - \omega t)$

Then the correct relation between E_o and B_o is given by

(1) $\omega E_o = kB_o$

(2) $E_o = kB_o$

(3) $kE_o = \omega B_o$

(4) $E_oB_o = \omega k$

Answer (3)

Sol. $E_x = E_0\sin(kz - \omega t)$

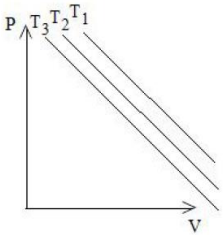
$B_y = B_0\sin(kz - \omega t)$

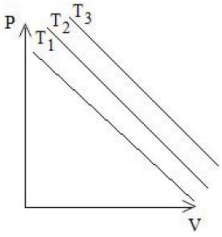
$\because$ Velocity $= \frac{E_0}{B_0}$

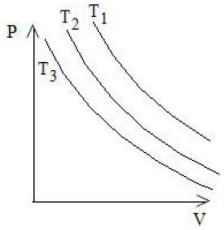
$\frac{\omega}{k} = \frac{E_0}{B_0}$

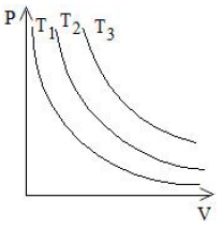
$\boxed{\omega B_0 = kE_0}$

4. In an Isothermal change, the change in pressure and volume of a gas can be represented for three different temperature; $T_3 > T_2 > T_1$ as:

(1) 

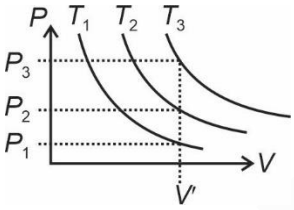
(2) 

(3) 

(4) 

Answer (4)

Sol. Correct graph is



Because at constant volume V'

$P_3 > P_2 > P_1$

$\therefore \quad T_3 > T_2 > T_1$

5. Given below are two statements:

Statement I: Acceleration due to earth's gravity decreases as you go 'up' or 'down' from earth's surface.

Statement II: Acceleration due to earth's gravity is same at a height 'h' and depth 'd' from earth's surface, if h = d.

In the light of above statements, choose the ***most appropriate*** answer from the options given below.

(1) Both Statement I and II are correct

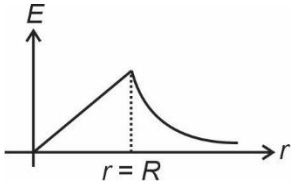
(2) Statement I is correct but statement II is incorrect

(3) Both Statement I and Statement II are incorrect

(4) Statement I is incorrect but statement II is correct

Answer (2)

Sol. Statement I is correct as



Statement II is incorrect as E_{above} and E_{below} the surface have different relation with height and depth respectively.

6. Match List I with List II

List I		List II	
A.	AM Broadcast	I.	88-108 MHz
B.	FM Broadcast	II.	540-1600 kHz
C.	Television	III.	3.7-4.2 GHz
D.	Satellite Communication	IV.	54 MHz-890 MHz

Choose the correct answer from the options given below:

(1) A-I, B-III, C-II, D-IV (2) A-IV, B-III, C-I, D-II

(3) A-II, B-III, C-I, D-IV (4) A-II, B-I, C-IV, D-III

Answer (4)

Sol. AM band is in kHz A-II

FM is in 88-108 MHz B-I

TV frequency in 54-890 MHz C-IV

Satellite Communication is in GHz D-III

7. If two vectors $\vec{P} = \hat{i} + 2m\hat{j} + m\hat{k}$ and $\vec{Q} = 4\hat{i} + 2\hat{j} + m\hat{k}$ are perpendicular to each other. Then, the value of m will be :

(1) –1 (2) 2

(3) 3 (4) 1

Answer (2)

Sol. $\vec{P}$ & $\vec{Q}$ are perpendicular

$\therefore \ \vec{P} \cdot \vec{Q} = 0$

$\Rightarrow 4 - 4m + m^2 = 0$

$= \ m = 2$

8. A photon is emitted in transition from $n = 4$ to $n = 1$ level in hydrogen atom. The corresponding wavelength for this transition is

(given, $h = 4 \times 10^{-15}$ eVs) :

(1) 941 nm (2) 99.3 nm

(3) 94.1 nm (4) 974 nm

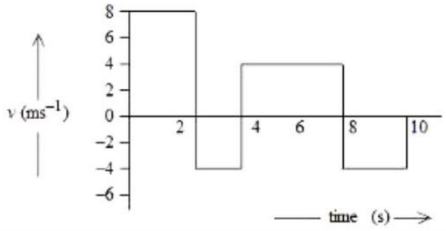
Answer (3)

Sol. $\frac{hc}{\lambda} = +13.6\,eV\left[\frac{1}{1} - \frac{1}{4^2}\right]$

$\Rightarrow \frac{4\times10^{-15}\times3\times10^{-8}}{\lambda} = 13.6\left[\frac{15}{16}\right]$

$\lambda = 94.1$ nm

9. The velocity-time graph of a body moving in a straight line is shown in figure.



The ratio of displacement to distance travelled by the body in time 0 to 10 s is :

(1) 1 : 4 (2) 1 : 2

(3) 1 : 3 (4) 1 : 1

Answer (3)

Sol. From *v-t* graph

Displacement = Area under curve considering sign also.

Distance = Area under curve considering only magnitude

Distance = 48

Displacement = 16 m

Displacement : distance = 1 : 3

10. Let γ_1 be the ratio of molar specific heat at constant pressure and molar specific heat at constant volume of a monoatomic gas and γ_2 be the similar ratio of diatomic gas. Considering the diatomic gas molecule as a rigid rotator, the ratio, $\frac{\gamma_1}{\gamma_2}$ is :

(1) $\frac{27}{35}$ (2) $\frac{35}{27}$

(3) $\frac{25}{21}$ (4) $\frac{21}{25}$

Answer (3)

Sol. $\gamma_1 = \left.\frac{C_P}{C_V}\right|_{\text{mono-atomic gas}} = \frac{5}{3}$

$\gamma_2 = \left.\frac{C_P}{C_V}\right|_{\text{di-atomic gas}} = \frac{7}{5}$

$\frac{\gamma_1}{\gamma_2} = \frac{25}{21}$

11. A long solenoid is formed by winding 70 turns cm^{-1}. If 2.0 A current flows, then the magnetic field produced inside the solenoid is _____ $\left(\mu_0 = 4\pi\times10^{-7}\,TmA^{-1}\right)$

(1) 88×10^{-4} T (2) 352×10^{-4} T

(3) 1232×10^{-4} T (4) 176×10^{-4} T

Answer (4)

Sol. Number of turns per meter = 7000 turns per m

$i = 2$ A

$B = \mu_0 ni = 4\pi \times 10^{-7} \times 7000 \times 2$

$= 56\pi \times 10^{-4}$T

$= 56\times\frac{22}{7}\times10^{-4}$T

$= 176 \times 10^{-4}$T

12. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : Steel is used in the construction of buildings and bridges.

Reason R : Steel is more elastic and its elastic limit is high.

In the light of above statements, choose the most appropriate answer from the options given below.

(1) Both **A** and **R** are correct and **R** is the correct explanation of **A**

(2) **A** is correct but **R** is not correct

(3) Both **A** and **R** are correct but **R** is not the correct explanation of **A**

(4) **A** is not correct but **R** is correct

Answer (1)

Sol. Steel is more elastic and has high elastic limit.

13. If the distance of the earth from Sun is 1.5×10^6 km. Then the distance of an imaginary planet from Sun, if its period of revolution is 2.83 years is:

(1) 6×10^6 km (2) 3×10^7 km

(3) 3×10^6 km (4) 6×10^7 km

Answer (3)

Sol. $T \propto r^{\frac{3}{2}}$

$\frac{T_e}{T_p} = \left(\frac{r_e}{r_p}\right)^{\frac{3}{2}}$

$\Rightarrow \left(\frac{1\,\text{year}}{2.83\,\text{year}}\right)^{\frac{2}{3}} = \left(\frac{1.5\times10^6\ \text{km}}{r_p}\right)$

$\Rightarrow \frac{1}{2} = \frac{1.5\times10^6\ \text{km}}{r_p}$

$r_p = 3 \times 10^6$ km

14. The frequency (ν) of an oscillating liquid drop may depend upon radius (r) of the drop, density (ρ) of liquid and the surface tension (s) of the liquid as : $\nu = r^a \rho^b s^c$. The values of a, b and c respectively are

(1) $\left(-\frac{3}{2},\frac{1}{2},\frac{1}{2}\right)$ (2) $\left(\frac{3}{2},-\frac{1}{2},\frac{1}{2}\right)$

(3) $\left(-\frac{3}{2},-\frac{1}{2},\frac{1}{2}\right)$ (4) $\left(\frac{3}{2},\frac{1}{2},-\frac{1}{2}\right)$

Answer (3)

Sol. $[\nu] = [T^{-1}]$

$[r] = L$ $\quad [s] = \left[\frac{MLT^{-2}}{L}\right]$

$[\rho] = \left[\frac{M}{L^3}\right] = \left[ML^{-3}\right]$

$\Rightarrow \nu = r^a \rho^b s^c$

$\Rightarrow T^{-1} = L^a M^b L^{-3b} M^c T^{-2c}$

$\Rightarrow T^{-1} = M^{(b+c)} L^{(a-3b)} T^{-2c}$

$-2c = -1 \Rightarrow c = \frac{1}{2}$

$b + c = 0$

$\Rightarrow b = -\frac{1}{2}$

$a - 3b = 0 \Rightarrow 3b = a \Rightarrow a = -\frac{3}{2}$

$(a, b, c) = \left(-\frac{3}{2},-\frac{1}{2},\frac{1}{2}\right)$

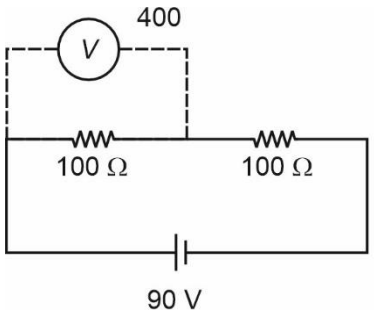
15. A cell of emf 90 V is connected across series combination of two resistors each of 100 Ω resistance. A voltmeter of resistance 400 Ω is used to measure the potential difference across each resistor. The reading of the voltmeter will be :

(1) 45 V (2) 40 V

(3) 80 V (4) 90 V

Answer (2)

Sol.



R_{eq} = 180 Ω

$I = \frac{90}{180} = \left(\frac{1}{2}A\right)$

Reading of voltmeter = 90 – 50 = 40 V

16. When a beam of white light is allowed to pass through convex lens parallel to principal axis, the different colours of light converge at different point on the principle axis after refraction. This is called

(1) Spherical aberration

(2) Chromatic aberration

(3) Polarisation

(4) Scattering

Answer (2)

Sol. The phenomena is known as chromatic aberration.

17. An α-particle, a proton and an electron have the same kinetic energy. Which one of the following is correct in case of their de-Broglie wavelength?

(1) $\lambda_\alpha > \lambda_p < \lambda_e$ (2) $\lambda_\alpha = \lambda_p = \lambda_e$

(3) $\lambda_\alpha > \lambda_p > \lambda_e$ (4) $\lambda_\alpha < \lambda_p < \lambda_e$

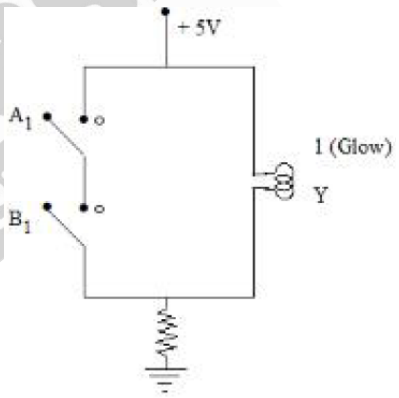
Answer (4)

Sol. $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mk}}$

So, $\lambda \propto \frac{1}{\sqrt{m}}$

So, $\lambda_e > \lambda_p > \lambda_\alpha$

18.



The logic gate equivalent to the given circuit diagram is

(1) NAND (2) NOR

(3) AND (4) OR

Answer (1)

Sol. The truth table for the circuit will be given as below

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

The above truth table is of NAND Gate.

19. Given below are two statements: One is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A: A pendulum clock when taken to Mount Everest becomes fast.

Reason R: The value of g (acceleration due to gravity) is less at Mount Everest than its value on the surface of earth.

In the light of the above statements, choose the ***most appropriate*** answer from the options given below.

(1) A is not correct but R is correct

(2) A is correct but R is not correct

(3) Both A and R are correct but R is NOT the correct explanation of A

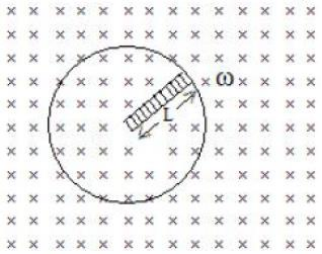
(4) Both A and R are correct and R is the correct explanation of A

Answer (1)

Sol. When we go on the Mount Everest the value of gravitational acceleration decreases $\left(g=\frac{g_0}{\left(1+\frac{h}{R_e}\right)^2}\right)$.

Therefore, the time period of oscillation $\left(T=2\pi\sqrt{\frac{l}{g}}\right)$ increases and the pendulum clock becomes slow thus the assertion is wrong but reason is correct.

20. A metallic rod of length 'L' is rotated with an angular speed of 'ω' normal to a uniform magnetic field 'B' about an axis passing through one end of rod as shown in figure. The induced emf will be



(1) $\frac{1}{2}B^2L^2\omega$ (2) $\frac{1}{2}BL^2\omega$

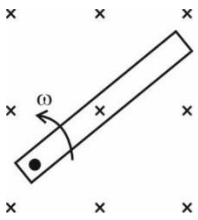
(3) $\frac{1}{4}BL^2\omega$ (4) $\frac{1}{4}B^2L\omega$

Answer (2)

Sol. Velocity of centre of rod $v=\frac{\omega L}{2}$

So, emf $=B\cdot vL$

$=\frac{B\omega L^2}{2}$



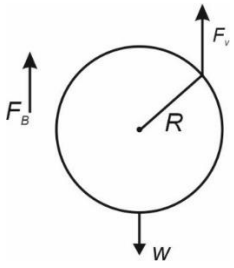
SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. A spherical ball of radius 1 mm and density 10.5 g/cc is dropped in glycerine of coefficient of viscosity 9.8 poise and density 1.5 g/cc. Viscous force on the ball when it attains constant velocity is 3696×10^{-x} N. The value of x is (Given, g = 9.8 m/s^2 and $\pi=\frac{22}{7}$)

Answer (7)

Sol. At state of terminal speed, net force on the ball is zero



$\therefore\quad F_v = w - F_B$

$=\left(\frac{4}{3}\pi R^3\rho_b g\right)-\left(\frac{4}{3}\pi R^3\rho_l g\right)$

$=\frac{4}{3}\pi R^3\left(\rho_b-\rho_l\right)g$

$=\frac{4}{3}\times\frac{22}{7}\times(10^{-3})^3[9\times10^3]\times9.8$

$= 3696\times10^{-7}$

$\therefore\quad \boxed{x=7}$

22. A parallel plate capacitor with air between the plate has a capacitance of 15 pF. The separation between the plate becomes twice and the space between them is filled with a medium of dielectric constant 3.5. Then the capacitance becomes $\frac{x}{4}$ pF . The value of x is _____

Answer (105)

Sol. Initially

$$\frac{\varepsilon_0 A}{d} = 15\times10^{-12}\text{F}$$

Finally

$$\frac{3.5\,\varepsilon_0 A}{2d} = \frac{x}{4}\times10^{-12}\text{F}$$

$$\therefore \quad \frac{3.5}{2}\times15 = \frac{x}{4}$$

$$\Rightarrow x = \frac{3.5\times15\times4}{2} = 105$$

23. A mass *m* attached to free end of a spring executes SHM with a period of 1 s. If the mass is increased by 3 kg the period of oscillation increases by one second, the value of mass *m* is _____ kg

Answer (1)

Sol. $\because \quad 2\pi\sqrt{\frac{m}{k}} = 1 \qquad \ldots(1)$

Finally

$$2\pi\sqrt{\frac{m+3}{k}} = 1+1 = 2 \qquad \ldots(2)$$

Equation $\frac{(1)}{(2)}$ gives

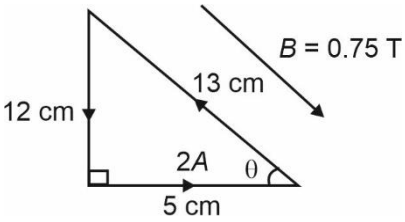
$$\sqrt{\frac{m}{m+3}} = \frac{1}{2}$$

$$\therefore \quad \boxed{m = 1\text{kg}}$$

24. A single turn current loop in the shape of a right angle triangle with sides 5 cm, 12 cm, 13 cm is carrying a current of 2 A. The loop is in a uniform magnetic field of magnitude 0.75 T whose direction is parallel to the current in the 13 cm side of the loop. The magnitude of the magnetic force on the 5 cm side will be $\frac{x}{130}$N. The value of x is _____

Answer (9)

Sol.



Force on 5 cm side = $I\ell B\sin\theta$

$$= 2\times\frac{5}{100}\times0.75\times\frac{12}{13}$$

$$= \frac{9}{130}$$

$\therefore \quad x = 9$

25. A uniform solid cylinder with radius *R* and length *L* has moment of inertia I_1, about the axis of the cylinder. A concentric solid cylinder of radius $R' = \frac{R}{2}$ and length $L' = \frac{L}{2}$ is carved out of the original cylinder. If I_2 is the moment of inertia of the carved out portion of the cylinder then $\frac{I_1}{I_2}$=_____.

(Both I_1 and I_2 are about the axis of the cylinder)

Answer (32)

Sol. $I_1 = \frac{(\rho\pi R^2 L)R^2}{2}$ (ρ : density of cylinder)

$$I_2 = \frac{\left[\rho\pi\left(\frac{R}{2}\right)^2\frac{L}{2}\right]\left(\frac{R}{2}\right)^2}{2}$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{32}{1}$$

26. A convex lens of refractive index 1.5 and focal length 18 cm in air is immersed in water. The change in focal length of the lens will be _____ cm.

(Given refractive index of water = $\frac{4}{3}$).

Answer (54)

Sol. From lens makers formula

$$\frac{1}{f} = \left(\frac{\mu_{\text{lens}}}{\mu_{\text{mrdium}}} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

when in air

$$\frac{1}{18} = \left(\frac{1.5}{1} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \qquad \ldots(1)$$

μ_{lense} = 1.5, μ_{air} = 1.

when in water

$$\frac{1}{f}=\left(\frac{1.5}{4/3}-1\right)\left[\frac{1}{R_1}-\frac{1}{R_2}\right] \qquad \ldots(2)$$

from (1) & (2)

$f = 72$

Change in focal length = 72 – 18

= 54

27. A body of mass 1 kg begins to move under the action of a time dependent force $\vec{F}=(t\hat{i}+3t^2\hat{j})$N, where $\hat{i}$ and $\hat{j}$ are the unit vectors along x and y axis. The power developed by above force, at the time t = 2 s, will be _____ W.

Answer (100)

Sol. $\vec{F}=t\hat{i}+3t^2\hat{j}$

$$\vec{a}=\frac{\vec{F}}{m}=t\hat{i}+3t^2\hat{j} \qquad (m = 1)$$

$$\vec{v}=\int_0^2\vec{a}\,dt=2\hat{i}+8\hat{j} \qquad (\vec{v} \text{ at } t=2\text{s})$$

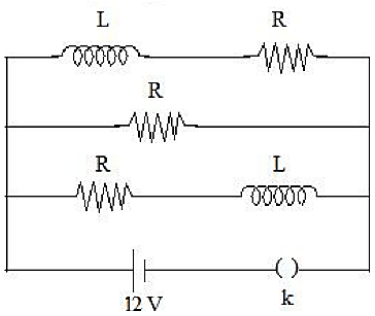
$P = \vec{F}\cdot\vec{v}=\vec{F}(t=2\text{s})\cdot\vec{v}(t=2\text{s})$

$=(2\hat{i}+12\hat{j})\cdot(2\hat{i}+8\hat{j})$

= 4 + 96

= 100 W.

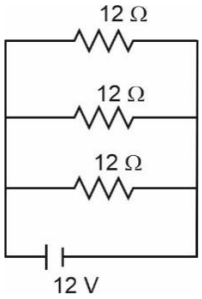
28. Three identical resistors with resistance R = 12 Ω and two identical inductors with self-inductance L = 5 mH are connected to an ideal battery with emf of 12 V as shown in figure. The current through the battery long after the switch has been closed will be _____ A.



Answer (03)

Sol. After long time, inductors are shorted.

Effective circuit becomes



Current through battery = $\frac{V}{R_{eq}}=\frac{12\text{V}}{4\Omega}=\boxed{3\text{A}}$

where R_{eq} = 3 resistors in parallel.

29. The energy released per fission of nucleus of ^{240}X is 200 MeV. The energy released if all the atoms in 120 g of pure ^{240}X undergo fission is _____ $\times 10^{25}$ MeV.

(Given $N_A = 6 \times 10^{23}$)

Answer (6)

Sol. 120 g of ^{240}X will have $\frac{1}{2}$ mole of X

Number of atom of $X=\frac{1}{2}\times N_A=3\times10^{23}$ atom

Energy released = $3 \times 10^{23} \times 200$ MeV

$= 6 \times 10^{25}$ MeV

30. If a copper wire is stretched to increase its length by 20%. The percentage increase in resistance of the wire is _____ %.

Answer (44)

Sol. let ℓ_0 be its initial length and A_0 be initial area.

Considering volume to be conserved

Vol. = $\ell_0 A_0 = (1.2\ell_0)A$

$$A_{final}=\frac{A_0}{1.2}$$

$$R_{in}=\frac{\rho\ell_0}{A_0}$$

$$R_{final}=\frac{\rho 1.2\ell_0}{\frac{A_0}{1.2}}=\frac{\rho\ell_0}{A_0}(1.2)^2$$

= R_{in} (1.44)

Hence increase = 44%

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

31. In which of the following reactions the hydrogen peroxide acts as a reducing agent?

(1) $HOCl + H_2O_2 \rightarrow H_3O^+ + Cl^- + O_2$

(2) $PbS + 4H_2O_2 \rightarrow PbSO_4 + 4H_2O$

(3) $2Fe^{2+} + H_2O_2 \rightarrow 2Fe^{3+} + 2OH^-$

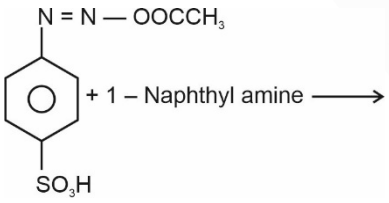
(4) $Mn^{2+} + H_2O_2 \rightarrow Mn^{4+} + 2OH^-$

Answer (1)

Sol. $HOCl + H_2O_2 \longrightarrow H_3O^{\oplus} + Cl^{\ominus} + O_2$

In this reaction H_2O_2 is acting as a reducing agent as Cl is undergoing a change in oxidation state from +1 to – 1.

32. Choose the correct colour of the product for the following reaction.

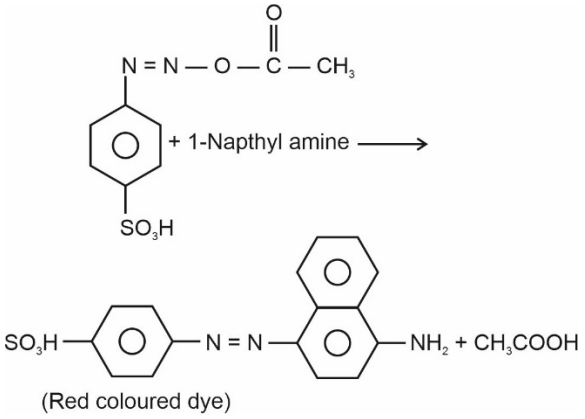


(1) Blue (2) Red

(3) Yellow (4) White

Answer (2)

Sol.



33. Which one amongst the following are good oxidizing agents?

A. Sm^{2+}

B. Ce^{2+}

C. Ce^{4+}

D. Tb^{4+}

Choose the most appropriate answer from the options given below.

(1) C only (2) A and B only

(3) D only (4) C and D only

Answer (4)

Sol. Ce^{+4} and Tb^{+4} are strong oxidising agents as the common oxidation state of Lanthanides is (+3).

34. Given below are two statements:

Statement-I : Pure Aniline and other arylamines are usually colourless.

Statement-II : Arylamines get coloured on storage due to atmospheric reduction.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Statement-I is correct but Statement-II is incorrect

(2) Both Statement-I and Statement-II are incorrect

(3) Statement-I is incorrect but Statement-II is correct

(4) Both Statement-I and Statement-II are correct

Answer (1)

Sol. Both Statement-I and Statement-II is incorrect as arylamines get coloured due to atmospheric oxidation.

35. The metal which is extracted by oxidation and subsequent reduction from its ore is

(1) Al (2) Cu

(3) Fe (4) Ag

Answer (4)

Sol. Ag is first extracted by oxidation and then subsequent reduction is carried out to obtain

$$4Ag + 8NaCN + 2H_2O + O_2 \xrightarrow{\text{Oxidation}} 4Na\left[Ag(CN)_2\right] + 4NaOH$$

$$2Na\left[Ag(CN)_2\right] + Zn \xrightarrow{\text{reduction}} Na_2\left[Zn(CN)_4\right] + 2Ag$$

36. Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : Benzene is more stable than hypothetical cyclohexatriene.

Reason R : The delocalized π electron cloud is attracted more strongly by nuclei of carbon atoms.

In the light of the above statements, choose the correct answer from the options given below.

(1) Both A and R are correct but R is NOT the correct explanation of A

(2) Both A and R are correct and R is the correct explanation of A

(3) A is true but R is false

(4) A is false but R is true

Answer (2)

Sol. Benzene is more stable than hypothetical cyclohexatriene due to resonance.

So, 1st statement is correct.

As the delocalised π-electron cloud is attracted more strongly by the nuclei of carbon atoms, therefore benzene is resonance stabilized. It is also aromatic in character.

Hence, the correct answer is (2)

37. Given below are two statements, one is labelled as **Assertion A :** and the other is labelled as **Reason R**.

Assertion A : Beryllium has less negative value of reduction potential compared to the other alkaline earth metals.

Reason R: Beryllium has large hydration energy due to small size of Be^{2+} but relatively large value of atomization enthalpy.

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) Both A and R are correct but R is NOT the correct explanation of A

(2) A is correct but R is not correct

(3) Both A and R are correct and R is the correct explanation of A

(4) A is not correct but R is correct

Answer (3)

Sol. 1st statement is correct as Be has least negative value of reduction potential among alkaline earth metals.

2nd statement is also correct.

The reducing nature is indeed less due to high atomisation enthalpy and ionisation enthalpy while having large hydration enthalpy of Be^{+2}. Correct answer is (3)

38. Which of the following cannot be explained by crystal field theory?

(1) Stability of metal complexes

(2) The order of spectrochemical series

(3) Magnetic properties of transition metal complexes

(4) Colour of metal complexes

Answer (2)

Sol. CFT does not explain the order of spectrochemical series because as per CFT, anionic ligands should exert greatest splitting effect. However, they lie on lower end of the spectrochemical series.

39. What is the number of unpaired electron(s) in the highest occupied molecular orbital of the following species: N_2 ; N_2^+ ; O_2 ; O_2^+ ?

(1) 0, 1, 0, 1 (2) 0, 1, 2, 1

(3) 2, 1, 2, 1 (4) 2, 1, 0, 1

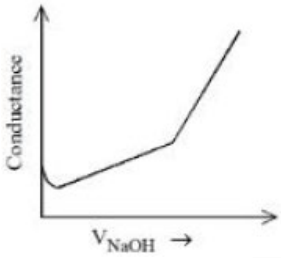
Answer (2)

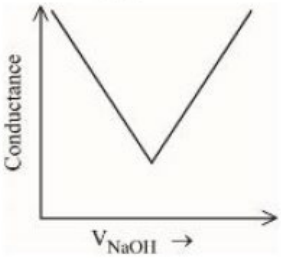
Sol.

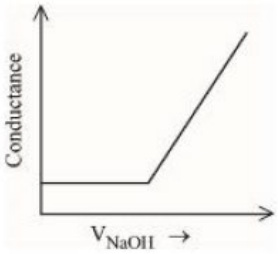
Molecule	No. of unpaired electron in highest occupied molecular orbital
N_2	0
$N_2^{\oplus}$	1
O_2	2
$O_2^{\oplus}$	1

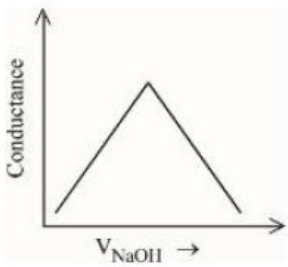
Correct answer is (2)

40. Choose the correct representation of conductometric titration of benzoic acid vs sodium hydroxide.

(1) 

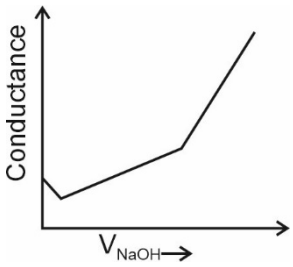
(2) 

(3) 

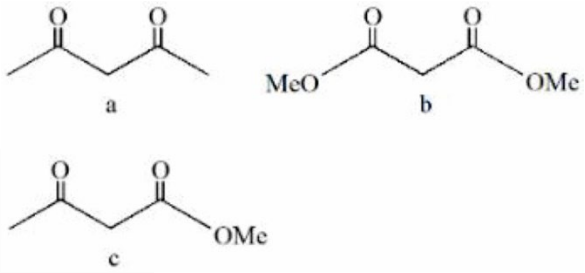
(4) 

Answer (1)

Sol. Correct graph is: →



41. Which will undergo deprotonation most readily in basic medium



(1) c only (2) a only

(3) Both a and c (4) b only

Answer (2)

Sol. (a) Since most readily is asked, deprotonation will be easily possible for (a).

$\therefore$ The correct answer is (2)

In (b) and (c), tendency for deprotonation is less due to cross conjugation.

42. Identify the correct statements about alkali metals.

A. The order of standard reduction potential ($M^+ | M$) for alkali metal ions is Na > Rb > Li.

B. CsI is highly soluble in water.

C. Lithium carbonate is highly stable to heat.

D. Potassium dissolved in concentrated liquid ammonia is blue in colour and paramagnetic.

E. All the alkali metal hydrides are ionic solids.

Choose the correct answer from the options given below.

(1) A, B and E only (2) A, B, D only

(3) C and E only (4) A and E only

Answer (4)

Sol. A. The order given is correct

B. CsI is less soluble in water due to less hydration enthalpy.

C. $Li_2CO_3 \xrightarrow{\Delta} Li_2O + CO_2$

D. In concentrated liquid ammonia, solution becomes diamagnetic

E. Alkali metal hydrides are ionic solids.

The correct answer is (A and E) only.

43. The hybridization and magnetic behaviour of cobalt ion in $[Co(NH_3)_6]^{3+}$ complex, respectively is

(1) d^2sp^3 and paramagnetic

(2) sp^3d^2 and diamagnetic

(3) sp^3d^2 and paramagnetic

(4) d^2sp^3 and diamagnetic

Answer (4)

Sol. $[Co(NH_3)_6]^{3+}$ is diamagnetic with d^2sp^3 hybridisation of Co^{+3}.

This is because NH_3 is a strong field ligand and forces electrons to pair up in a d^6 configuration.

44. Correct statement is:

(1) An average human being consumes nearly 15 times more air than food

(2) An average human being consumes more food than air

(3) An average human being consumes 100 times more air than food

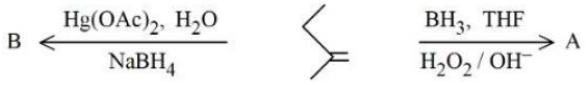
(4) An average human being consumes equal amount of food and air

Answer (1)

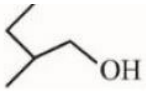
Sol. An average human being consumes 15 times more air than food.

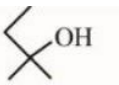
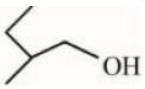
The correct answer is (1).

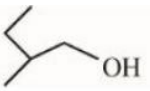
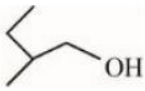
45. Find out the major products from the following reactions.



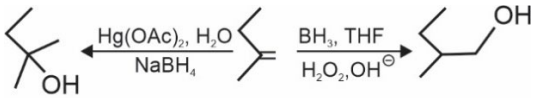
(1) A =  , B = 

(2) A =  , B = 

(3) A =  , B = 

(4) A =  , B = 

Answer (2)

Sol. 

(B) (A)

The correct answer is (2).

46. Match List I with List II.

	LIST I Type		LIST II Name
A.	Antifertility drug	I.	Norethindrone
B.	Tranquilizer	II.	Meprobomate
C.	Antihistamine	III.	Seldane
D.	Antibiotic	IV	Ampicillin

Choose the correct answer from the options given below:

(1) A-II, B-I, C-III, D-IV (2) A-I, B-II, C-III, D-IV

(3) A-I, B-III, C-II, D-IV (4) A-IV, B-III, C-II, D-I

Answer (2)

Sol. Correct match is

A. Antifertility drug (I) Norethindrone

B. Tranquilizer (II) Meprobomate

C. Antihistamine (III) Seldane

D. Antibiotic (IV) Ampicillin

47. $K_2Cr_2O_7$ paper acidified with dilute H_2SO_4 turns green when exposed to

(1) Hydrogen sulphide (2) Carbon dioxide

(3) Sulphur dioxide (4) Sulphur trioxide

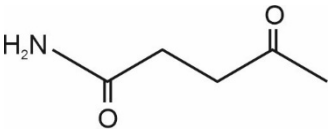
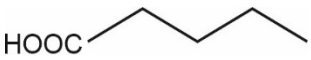
Answer (3)

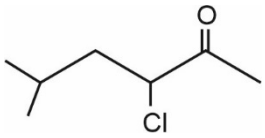
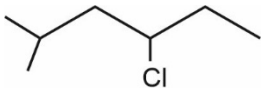
Sol. SO_2 gets oxidised in presence of $K_2Cr_2O_7$ and it converts to Cr^{+3} in presence of dil. H_2SO_4.

Similarly, H_2S can also get oxidized to sulphur.

However, most appropriate is (3).

48. Given below are two statements:

Statement I:  under Clemmensen reduction conditions will give 

Statement II:  under Wolff-Kishner reduction condition will give 

In the light of the above statements, choose the correct answer from the options given below:

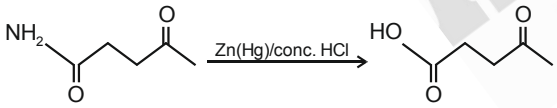
(1) Statement I is true but Statement II is false

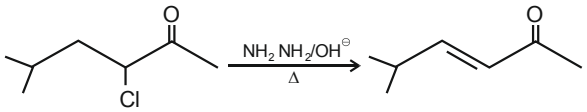
(2) Both Statement I and Statement II are false

(3) Both Statement I and Statement II are true

(4) Statement I is false but Statement II is true

Answer (1)

Sol. 



∴ Statement I is true but statement II is false.

49. The number of s-electrons present in an ion with 55 protons in its unipositive state is

(1) 10 (2) 8

(3) 9 (4) 12

Answer (1)

Sol. 55 protons are present in $Cs^{\oplus}$

∴ Number of s-electrons = 10

50. A student has studied the decomposition of a gas AB_3 at 25°C. He obtained the following data.

p (mm Hg)	50	100	200	400
relative $t_{1/2}$ (s)	4	2	1	0.5

The order of the reaction is

(1) 1 (2) 0 (Zero)

(3) 2 (4) 0.5

Answer (3)

Sol. $AB_3(g) \longrightarrow A(g) + 3B(g)$ or $\frac{3}{2}B_2(g)$

As decomposition reaction of $AB_3(g)$ is not given, we assume that p(mm Hg) is for $AB_3(g)$ only.

∴ $t_{1/2} \propto (p)^{1-n}$

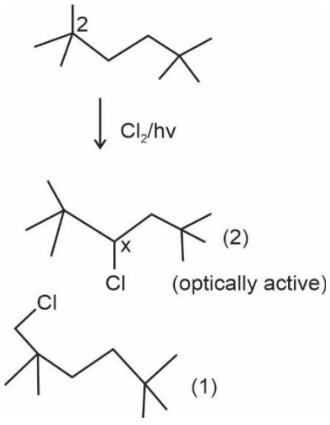
∴ Order of reaction is 2 as $t_{1/2} \propto \frac{1}{p}$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse andw the on-screen virtual numeric keypad in the place designated to enter the answer.

51. Maximum number of isomeric monochloro derivatives which can be obtained from 2,2,5, 5-tetramethylhexane by chlorination is_____

Answer (3)

Sol. 

Total isomers = 3

(considering stereoisomers)

52. Total number of tripeptides possible by mixing of valine and proline is_____

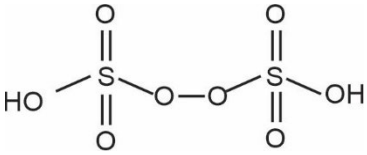
Answer (8)

Sol. Considering only linear tripeptides, total number of tripeptides are 8 or 2^3.

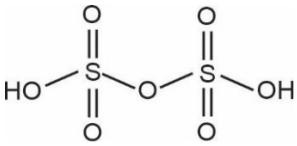
53. Sum of π - bonds present in peroxodisulphuric acid and pyrosulphuric acid is _____

Answer (8)

Sol.



($H_2S_2O_8$) (peroxodi sulphuric acid)



($H_2S_2O_7$) (pyro sulphuric acid)

Number of π-bonds = 8

54. The total pressure observed by mixing two liquids A and B is 350 mm Hg when their mole fractions are 0.7 and 0.3 respectively.

The total pressure becomes 410 mm Hg if the mole fractions are changed to 0.2 and 0.8 respectively for A and B. The vapour pressure of pure A is _____mm Hg. (Nearest integer). Consider the liquids and solutions behave ideally.

Answer (314)

Sol. $350 = P_A^\circ(0.7) + P_B^\circ(0.3)$

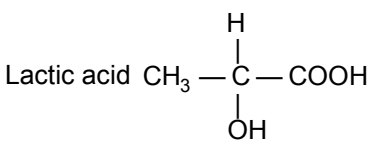
$410 = P_A^\circ(0.2) + P_B^\circ(0.8)$

$-21700 = -P_B^\circ(50)$

$P_B^\circ = 434 \text{ mm Hg}$

$P_A^\circ = 314 \text{ mm Hg}$

55. If the pKa of lactic acid is 5, then the pH of 0.005 M calcium lactate solution at 25° C is _____× 10^{-1} (Nearest integer)

Lactic acid 

Answer (85)

Sol. $pH = 7 + \frac{1}{2}(pKa + \log c)$

$= 7 + \frac{1}{2}(5 - 2)$

= 7 + 1.5

= 8.5

56. The number of statement/s which are the characteristics of physisorption is _____

A. It is highly specific in nature

B. Enthalpy of adsorption is high

C. It decreases with increases in temperature

D. It results into unimolecular layer

E. No activation energy in needed

Answer (2)

Sol. A. It is non-specific

B. It is low

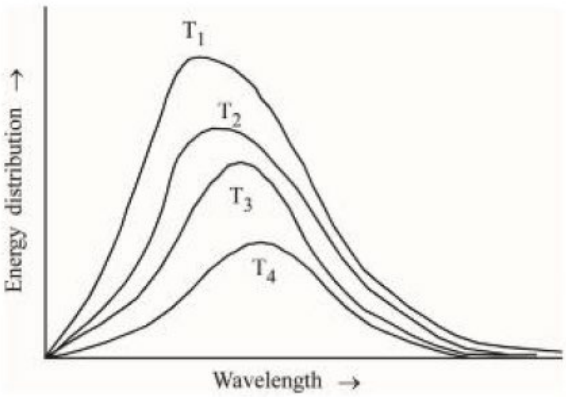
C. Extent of adsorption decreases with increase of temperature

D. It results in multimolecular layer

E. No activation energy is needed

No. of correct statements = 2

57. Following figure shows spectrum of an ideal black body at four different temperatures. The number of **correct** statement/s from the following is _____



A. $T_4 > T_3 > T_2 > T_1$

B. The black body consists of particles performing simple harmonic motion.

C. The peak of the spectrum shifts to shorter wavelength as temperature increases.

D. $\frac{T_1}{\nu_1} = \frac{T_2}{\nu_2} = \frac{T_3}{\nu_3} \neq$ constant

E. The given spectrum could be explained using quantisation of energy.

Answer (2)

Sol. A. $T_1 > T_2 > T_3 > T_4$

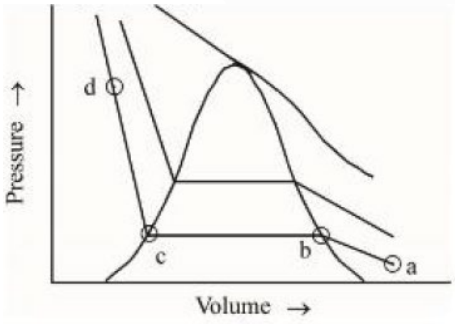
B. It is incorrect as particles do not undergo simple harmonic motion.

C. It is correct

D. It is incorrect

E. It is correct

58. The number of statement/s, which are correct with respect to the compression of carbon dioxide from point (a) in the Andrews isotherm from the following is _____



A. Carbon dioxide remains as a gas upto point (b)

B. Liquid carbon dioxide appears at point (c)

C. Liquid and gaseous carbon dioxide coexist between points (b) and (c)

D. As the volume decreases from (b) to (c), the amount of liquid decreases

Answer (2)

Sol. A. It is correct

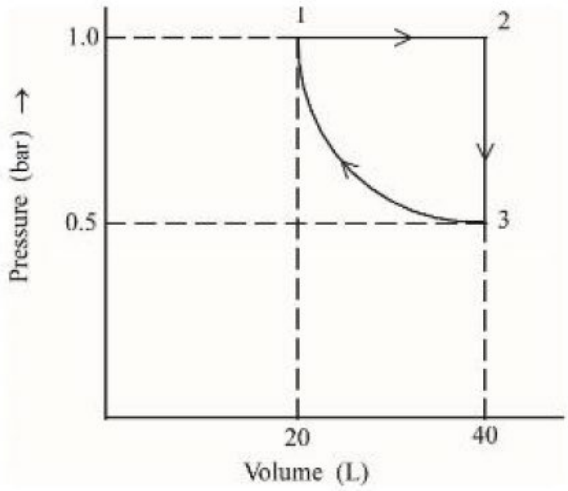
B. It is incorrect as it appears at point (b)

C. It is also correct

D. It is incorrect.

Number of correct statements = 2

59. One mole of an ideal monoatomic gas is subjected to changes as shown in the graph. The magnitude of the work done (by the system or on the system) is_____ J (nearest integer)



Given: log 2 = 0.3

ln 10 = 2.3

Answer (620)

Sol. $W_{3\to1} = -\left(20\log\frac{20}{40}\right)\times 2.3$

$= +20 \times 0.3 \times 100 \times 2.3$ J

$= 1.38$ kJ

$W_{2\to3} = 0$

$W_{1\to2} = -1 \times 20 \times 100 = -2$ kJ

$W_{1\to2} + W_{2\to3} + W_{3\to1} = 1.38 - 2 = -0.62$ kJ

$|W| = 620$ J

60. The number of units, which are used to express concentration of solutions from the following is_____

Mass percent, Mole, Mole fraction, Molarity, ppm, Molality

Answer (5)

Sol. Mass percent, mole fraction, molarity, ppm & molality are used to express concentration. So, number of units = 5

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

61. The set of all values of a for which $\lim\limits_{x\to a}\left([x-5]-[2x+2]\right)=0$, where $[\propto]$ denotes the greatest integer less than or equal to $\propto$ is equal to

(1) [–7.5, –6.5) (2) (–7.5, –6.5]

(3) [–7.5, –6.5] (4) (–7.5, –6.5)

Answer (4)

Sol. $\lim\limits_{x\to a}\left([x-5]-[2x+2]\right)=0$

$\Rightarrow [x-5]=[2x+2]$

$\Rightarrow [x]-5=[2x]+2$

$\Rightarrow [x]=[2x]+7$ …(i)

if $x\in Z$ we have

$x=-7$

also $2x\in Z$ if x is of form $z\pm\frac{1}{2}$

Hence, if $x\in(-7.5,-7)$ eq. (1) become

$-8=-15+7\Rightarrow 7=7$

Similarly, if $x\in(-7,-6.5)$ in eq. (1)

$-7=-14+7\Rightarrow 7=7$

At $x=-6.5$ in eq. (1)

$-7=-13+7\Rightarrow -14\neq -13$ not possible

At $x=-7.5$ in eq. (1)

$-8=-15+7\Rightarrow 8=8$

But $x\to a$ $a\neq -6.5$ or -7.5

$\therefore\ a\in(-7.5,-6.5)$

62. Let p and q be two statements. Then $\sim(p\wedge(p\Rightarrow\sim q))$ is equivalent to

(1) $(\sim p)\vee q$ (2) $p\vee((\sim p)\wedge q)$

(3) $p\vee(p\wedge q)$ (4) $p\vee(p\wedge(\sim q))$

Answer (1)

Sol. Making truth table $\left(E\equiv\sim\left(p\wedge(p\Rightarrow\sim q)\right)\right.$

p	q	$\sim p$	$\sim q$	$p\vee q$	$p\wedge q$	$p\Rightarrow\sim q$	$p\wedge(p\Rightarrow\sim q)$	E
T	T	F	F	T	T	F	F	T
T	F	F	T	T	F	T	T	F
F	T	T	F	T	F	T	F	T
F	F	T	T	F	F	T	F	T

&

$\sim p\vee q$	$p\vee(\sim p\wedge q)$	$p\vee(p\wedge q)$	$p\vee(p\wedge\sim q)$
T	T	T	T
F	T	T	T
T	T	F	F
T	F	F	F

$\therefore\ \sim\left(p\wedge(p\Rightarrow\sim q)\right)$ is equivalent to $\sim p\vee q$

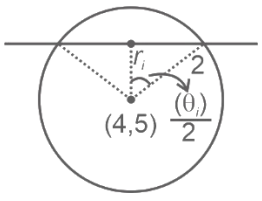
63. The locus of the mid points of the chords of the circle C_1 : $(x-4)^2+(y-5)^2=4$ which subtend an angle θ_i at the centre of the circle C_1, is a circle of radius r_i. If $\theta_1=\frac{\pi}{3}$, $\theta_3=\frac{2\pi}{3}$ and $r_1^2=r_2^2+r_3^2$, then θ_2 is equal to

(1) $\frac{3\pi}{4}$ (2) $\frac{\pi}{4}$

(3) $\frac{\pi}{6}$ (4) $\frac{\pi}{2}$

Answer (4)

Sol.



$\therefore\cos\left(\frac{\theta_1}{2}\right)=\frac{r_i}{2}\Rightarrow r_i=2\cos\left(\frac{\theta_i}{2}\right)$

Given $r_1^2 = r_2^2 + r_3^3$

$$\Rightarrow \left(\cos\left(\frac{\theta_1}{2}\right)\right)^2 = \left(\cos\left(\frac{\theta_2}{2}\right)\right)^2 + \left(\cos\left(\frac{\theta_3}{2}\right)\right)^2$$

$$\Rightarrow \frac{3}{4} = \cos^2\left(\frac{\theta_2}{2}\right) + \frac{1}{4}$$

$$\Rightarrow \cos^2\left(\frac{\theta_2}{2}\right) = \frac{1}{2}$$

$$\Rightarrow \frac{\theta_2}{2} = \frac{\pi}{4}$$

$$\Rightarrow \theta_2 = \frac{\pi}{2}$$

64. If $f(x) = \frac{2^{2x}}{2^{2x}+2}$, $x \in \mathbb{R}$, then $f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + ... + f\left(\frac{2022}{2023}\right)$ is equal to

(1) 1010 (2) 2011

(3) 1011 (4) 2010

Answer (3)

Sol. $f(x) = \frac{2^{2x}}{2^{2x}+2}$, and $f(1-x) = \frac{2^{2(1-x)}}{2^{2(1-x)}+2}$

$\therefore$ $f(x) + f(1-x) = 1$

$$\sum_{K=1}^{2022} f\left(\frac{K}{2022}\right) = \underbrace{f\left(\frac{1}{2022}\right) + f\left(\frac{2022}{2023}\right)}_{1} + \underbrace{f\left(\frac{2}{2022}\right) + f\left(\frac{2021}{2022}\right) + ...}_{1} \rightarrow 1011 \text{ Pairs}$$

$= 1011$

65. If the system of equations

$x + 2y + 3z = 3$

$4x + 3y - 4z = 4$

$8x + 4y - \lambda z = 9 + \mu$

has infinitely many solutions, then the ordered pair (λ, μ) is equal to :

(1) $\left(-\frac{72}{5}, \frac{21}{5}\right)$ (2) $\left(\frac{72}{5}, -\frac{21}{5}\right)$

(3) $\left(\frac{72}{5}, \frac{21}{5}\right)$ (4) $\left(-\frac{72}{5}, -\frac{21}{5}\right)$

Answer (2)

Sol. $D = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 3 & -4 \\ 8 & 4 & -\lambda \end{vmatrix} = 0 \quad \Rightarrow \quad \lambda = \frac{72}{5}$

$$D_z = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 3 & 4 \\ 8 & 4 & 9+\mu \end{vmatrix} = 0$$

$$\Rightarrow \quad \mu = \frac{-21}{5}$$

66. Let the plane containing the line of intersection of the planes $P1: x + (\lambda + 4)y + z = 1$ and $P2 : 2x + y + z = 2$ pass through the points (0, 1, 0) and (1, 0, 1). Then the distance of the point $(2\lambda, \lambda, -\lambda)$ from the plane $P2$ is

(1) $4\sqrt{6}$ (2) $3\sqrt{6}$

(3) $5\sqrt{6}$ (4) $2\sqrt{6}$

Answer (2)

Sol. Equation of plane :

$$(x + (\lambda + 4)y + z - 1) + k(2x + y + z - 2) = 0$$

Passes through (0, 1, 0) and (1, 0, 1)

$$\Rightarrow \lambda + 4 - 1 + k(-1) = 0$$

$$\boxed{\lambda - k = -3} \qquad ...(i)$$

& $(1 + 0 + 0) + k(1) = 0$

$k = -1 \Rightarrow \lambda = -4$

P_2: $2x + y + z = 2$, Point $(-8, -4, 4)$

$$\text{Distance} = \left|\frac{-16 - 4 + 4 - 2}{\sqrt{6}}\right| = 3\sqrt{6} \text{ units}$$

67. If $\left({}^{30}C_1\right)^2 + 2\left({}^{30}C_2\right)^2 + 3\left({}^{30}C_3\right)^2 + ... + 30\left({}^{30}C_{30}\right)^2 = \frac{\alpha 60!}{(30!)^2}$ then α is equal to :

(1) 60

(2) 30

(3) 15

(4) 10

Answer (3)

Sol. $\sum_{r=1}^{30} r\cdot\left({}^{30}C_r\right)^2 = \sum_{r=1}^{30} 30\cdot{}^{29}C_{r-1}\cdot{}^{30}C_r$

$$= \sum_{r=1}^{30} 30\cdot{}^{29}C_{r-1}\cdot{}^{30}C_{30-r}$$

$$= 30\cdot{}^{59}C_{30}$$

$$= 30\cdot\frac{59!}{30!\cdot 29!}\cdot\frac{30}{30}$$

$$= \frac{15\cdot 60!}{(30!)^2}$$

68. If the foot of the perpendicular drawn from (1, 9, 7) to the line passing through the point (3, 2, 1) and parallel to the planes $x + 2y + z = 0$ and $3y - z = 3$ is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to

(1) –1 (2) 1

(3) 3 (4) 5

Answer (4)

Sol. Direction of line

$$\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{vmatrix}$$

$$= \hat{i}(-5) - \hat{j}(-1) + \hat{k}(3)$$

$$= -5\hat{i} + \hat{j} + 3\hat{k}$$

Equation of line

$$\frac{x-3}{-5} = \frac{y-2}{1} = \frac{z-1}{3}$$

Let foot of perpendicular be = $(-5k + 3, k + 2, 3k + 1)$

$\Rightarrow (-5k+2)(-5) + (k-7)(1) + (3k-6)(3) = 0$

Or $25k - 10 + k - 7 + 9k - 18 = 0$

Or $k = 1$

$\alpha + \beta + \gamma = -k + 6 = 5$

69. Let A be a 3×3 matrix such that $|\text{adj (adj (adj } A))| = 12^4$. Then $|A^{-1} \text{ adj } A|$ is equal to

(1) 12 (2) $2\sqrt{3}$

(3) $\sqrt{6}$ (4) 1

Answer (2)

Sol. $|A|^{(n-1)^3} = 12^4$

$$|A|^8 = 12^4$$

$$|A| = \sqrt{12}$$

$$\left|A^{-1}\,\text{adj}A\right| = \left|A^{-1}\right|\cdot|A|^2$$

$$= |A|$$

70. The value of $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ is

(1) $\frac{1}{2}\left(\sqrt{3}+i\right)$ (2) $-\frac{1}{2}\left(1-i\sqrt{3}\right)$

(3) $\frac{1}{2}\left(1-i\sqrt{3}\right)$ (4) $-\frac{1}{2}\left(\sqrt{3}-i\right)$

Answer (4)

Sol. $z = \left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$

$$1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9} = 1+\cos\frac{5\pi}{18}+i\sin\frac{5\pi}{18}$$

$$= 1+2\cos^2\frac{5\pi}{36}-1+2i\sin\frac{5\pi}{36}\cos\frac{5\pi}{36}$$

$$= 2\cos\frac{5\pi}{36}\left(\cos\frac{5\pi}{36}+i\sin\frac{5\pi}{36}\right) = 2\cos\frac{5\pi}{36}e^{i\frac{5\pi}{36}}$$

$$\Rightarrow z = \left(\frac{2\cos\left(\frac{5\pi}{36}\right)e^{i\frac{5\pi}{36}}}{2\cos\left(\frac{5\pi}{36}\right)e^{-i\frac{5\pi}{36}}}\right)^3 = e^{i\frac{5\pi}{6}}$$

$$z = -\frac{\sqrt{3}}{2}+\frac{1}{2}i = \frac{1}{2}\left(i-\sqrt{3}\right) = -\frac{1}{2}\left(\sqrt{3}-i\right)$$

71. The number of square matrices of order 5 with entries from the set {0, 1}, such that the sum of all the elements in each row is 1 and the sum of all the elements in each column is also 1, is

(1) 120 (2) 225

(3) 150 (4) 125

Answer (1)

Sol. $\begin{bmatrix} - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \end{bmatrix}$

$\because$ In every row and every column there would be exactly one 1 and four zeroes.

Number of matrices = ${}^5C_1 \cdot {}^4C_1 \cdot {}^3C_1 \cdot {}^2C_1 \cdot {}^1C_1 =$ 120

Option (1) is correct.

72. $\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}}dx$ is equal to

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{2}$

(3) $\frac{\pi}{3}$ (4) 2π

Answer (4)

Sol. $I = \int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}}dx = \left[48 \cdot \sin^{-1}\left(\frac{2x}{3}\right) \cdot \frac{1}{2}\right]_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}}$

$= 24\left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right]$

$= 24\left[\frac{\pi}{3} - \frac{\pi}{4}\right] = 24 \cdot \frac{\pi}{12} = 2\pi$

Option (4) is correct.

73. The number of real solutions of the equation $3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0$, is

(1) 3 (2) 0

(3) 2 (4) 4

Answer (2)

Sol. $3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0$

$3\left[\left(x + \frac{1}{x}\right)^2 - 2\right] - 2\left(x + \frac{1}{x}\right) + 5 = 0$

Put $x + \frac{1}{x} = t \Rightarrow t \in (-\infty, -2] \cup [2, \infty)$

$3t^2 - 2t - 1 = 0$

$3t^2 - 3t + t - 1 = 0$

$\Rightarrow 3t(t-1) + 1(t-1) = 0 \Rightarrow t = 1, = -\frac{1}{3}$

$\Rightarrow t = 1, -\frac{1}{3}$

$\because t \in (-\infty, -2] \cup [2, \infty)$

No real value of $t \Rightarrow$ no real value of x.

Option (2) is correct.

74. Let $\vec{\alpha} = 4\hat{i} + 3\hat{j} + 5\hat{k}$ and $\vec{\beta} = \hat{i} + 2\hat{j} - 4\hat{k}$. Let $\vec{\beta}_1$ be parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ be perpendicular to $\vec{\alpha}$. If $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, then the value of $5\vec{\beta}_2 \cdot (\hat{i} + \hat{j} + \hat{k})$ is

(1) 7 (2) 9

(3) 6 (4) 11

Answer (1)

Sol. $\vec{\beta}_1 = \lambda(4\hat{i} + 3\hat{j} + 5\hat{k}) = \lambda\vec{\alpha}, \vec{\beta}_2 \cdot \vec{\alpha} = 0$

$\Rightarrow \vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2 = \lambda\vec{\alpha} + \vec{\beta}_2$

$\vec{\beta} \cdot \vec{\alpha} = \cdot\lambda |\vec{\alpha}|^2 + 0 \Rightarrow \lambda = \frac{-10}{50} = -\frac{1}{5}$

$\vec{\beta} = -\frac{1}{5}\vec{\alpha} + \vec{\beta}_2$

$5\vec{\beta}_2 = \vec{\alpha} + 5\vec{\beta} = (9\hat{i} + 13\hat{j} - 15\hat{k})$

$5\vec{\beta}_2 \cdot (\hat{i} + \hat{j} + \hat{k}) = 9 + 13 - 15 = 7$

Option (1) is correct.

75. Let $f(x)$ be a function such that $f(x + y) = f(x) \cdot f(y)$ for all $x, y \in \mathbb{N}$. If $f(1) = 3$ and $\sum_{k=1}^{n} f(k) = 3279$, then the value of n is

(1) 8 (2) 9

(3) 6 (4) 7

Answer (4)

Sol. $f(x + y) = f(x) \cdot f(y)$

$\Rightarrow f(x) = a^x$

$\Rightarrow f(1) = 3$

$\Rightarrow f(x) = 3^x$

$\sum_{k=1}^{n} f(x)$

$\Rightarrow \frac{3(3^n-1)}{2} = 3279$

$\Rightarrow 3^n = 2187$

$\Rightarrow n = 7$

76. The number of integers, greater than 7000 that can be formed, using the digits 3, 5, 6, 7, 8 without repetition, is

(1) 48 (2) 168

(3) 220 (4) 120

Answer (2)

Sol. $\underline{7}\_\_\_\_$

Number of integers = 4 × 3 × 2 = 24

$\underline{8}\_\_\_\_$

4 × 3 × 2 = 24

$\therefore$ 24 + 24 = 48

$\_\_\_\_\_$

5 digit integers = 5! = 120

$\therefore$ 120 + 48 = 168

77. If $f(x) = x^3 - x^2 f'(1) + xf''(2) - f'''(3), x \in R$, then

(1) $2f(0) - f(1) + f(3) = f(2)$

(2) $f(3) - f(2) = f(1)$

(3) $3f(1) + f(2) = f(3)$

(4) $f(1) + f(2) + f(3) = f(0)$

Answer (1)

Sol. $f(x) = x^3 - x^2 f'(1) + xf''(2) - f'''(3)$ ...(i)

$f'(x) = 3x^2 - 2xf'(1) + f''(2)$ ...(ii)

$f''(x) = 6x - 2f'(1)$ ...(iii)

$f'''(x) = 6$

$\Rightarrow f'''(3) = 6$

from (iii) : $f''(2) = 12 - 2f'(1)$ ...(iv)

from (ii) : $f'(1) = 3 - 2f'(1) + f''(2)$

$\Rightarrow f''(2) = 3f'(1) - 3$ ...(v)

from (iv) and (v)

$f''(2) = 6, f'(1) = 3$

$f(x) = x^3 - 3x^2 + 6x - 6$

$f(0) = -6, f(1) = -2, f(2) = 2, f(3) = 12$

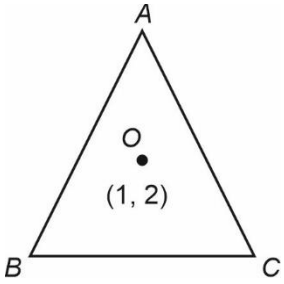
78. The equations of the sides AB and AC of a triangle ABC are $(\lambda + 1)x + \lambda y = 4$ and $\lambda x + (1 - \lambda)y + \lambda = 0$ respectively. Its vertex A is on the y-axis and its orthocentre is (1, 2). The length of the tangent from the point C to the part of the parabola $y^2 = 6x$ in the first quadrant is :

(1) $2\sqrt{2}$ (2) $\sqrt{6}$

(3) 2 (4) 4

Answer (1)

Sol. Let point A be $\left(0, \frac{4}{\lambda}\right)$ or $\left(0, \frac{-\lambda}{1-\lambda}\right)$



$\frac{4}{\lambda} = \frac{-\lambda}{1-\lambda}$

$\Rightarrow \lambda = 2$

$\therefore A(0, 2)$

Now,

$AB : 3x + 2y = 4$

$AC : 2x - y = -2$

$M_{AB} = -\frac{3}{2} \Rightarrow M_{OC} = \frac{2}{3}$

Let $C(h, 2h + 2)$

$M_{OC} = \frac{2h}{h-1}$

$\therefore \frac{2}{3} = \frac{2h}{h-1} \Rightarrow h = -\frac{1}{2}$

$\therefore C\left(\frac{-1}{2}, 1\right)$

Now, $ty = x + \frac{3}{2}t^2$ is tangent passing through $C\left(-\frac{1}{2}, 1\right)$

$\Rightarrow t = 1 \Rightarrow P_{OC}$ is $\left(\frac{3}{2}, 3\right)$

$\therefore$ Length = $2\sqrt{2}$

79. Let $y = y(x)$ be the solution of the differential equation $(x^2 - 3y^2)dx + 3xy\,dy = 0, y(1) = 1$. Then $6y^2(e)$ is equal to

(1) $3e^2$ (2) $\frac{3}{2}e^2$

(3) e^2 (4) $2e^2$

Answer (4)

Sol. $\left(x^2-3y^2\right)dx+3xy\,dy=0, y(1)=1$

$\Rightarrow \quad 3xy\dfrac{dy}{dx}-3y^2=-x^2$

$\Rightarrow \quad y\dfrac{dy}{dx}-\dfrac{y^2}{x}=\dfrac{-x}{3}$

$\Rightarrow \quad 2y\dfrac{dy}{dx}-\dfrac{2y^2}{x}=\dfrac{-2x}{3}$

Let $y^2=t \Rightarrow 2y\dfrac{dy}{dx}=\dfrac{dt}{dx}$

$\dfrac{dt}{dx}-\dfrac{2t}{x}=\dfrac{-2x}{3}$

$IF=\dfrac{1}{x^2}$

$\therefore \quad \dfrac{y^2}{x^2}=\dfrac{-2}{3}\ln|x|+c$

$x=1, y=1 \Rightarrow c=1$

$\Rightarrow \quad y^2=\dfrac{-2}{3}x^2\ln|x|+x^2$

Now, $x=e$

$y^2=\dfrac{-2e^2}{3}+e^2=\dfrac{e^2}{3}$

$\therefore \quad 6y^2=2e^2$

80. Let the six numbers $a_1, a_2, a_3, a_4, a_5, a_6$ be in A.P. and $a_1+a_3=10$. If the mean of these six numbers is $\dfrac{19}{2}$ and their variance is σ^2, then $8\sigma^2$ is equal to:

(1) 200 (2) 105

(3) 220 (4) 210

Answer (4)

Sol. Given : $a, a_2, a_3, a_4, a_5, a_6$ _____ A.P.

and $a_1+a_3=10$

Let the common difference be d

then, $2a_1+2d=10$

$\therefore \quad a_1+d=5 \qquad \ldots(i)$

and $\dfrac{a_1+a_2+....+a_6}{6}=\dfrac{19}{2}$

$\Rightarrow \quad 2a_1+5d=19 \qquad \ldots(ii)$

By (i) and (ii)

$\boxed{d=3}$ and $a_1=2$

$\therefore \quad a_2=5, a_3=8, a_4=11, a_5=14, a_6=17$

$\therefore \quad$ Variance $=\dfrac{233}{2}-\dfrac{361}{4}=\dfrac{105}{4}=\sigma^2$

$\therefore \quad 8\sigma^2=210$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

81. Let f be a differentiable function defined on $\left[0,\dfrac{\pi}{2}\right]$ such that $f(x)>0$ and

$f(x)+\int_0^x f(t)\sqrt{1-(\log_e f(t))^2}\,dt=e, \quad \forall x\in\left[0,\dfrac{\pi}{2}\right]$.

Then $\left(6\log_e f\left(\dfrac{\pi}{6}\right)\right)^2$ is equal to _____.

Answer (27)

Sol. $f(x)+\int_0^x f(t)\sqrt{1-(\log_e f(t))^2}\,dt=e \qquad \ldots(1)$

So, $f(0)=e$

Now differentiate w.r. to x

$f'(x)+f(x)\sqrt{1-(\log_e f(x)^2}=0$

$\dfrac{f'(x)}{f(x)\sqrt{1-(\log_e f(x))^2}}=-1$

Let $\log_e f(x)=t$

$\therefore \quad \int\dfrac{dt}{\sqrt{1-t^2}}=-x+c$

$\Rightarrow \quad \sin^{-1}t=-x+c$

Now $f(0)=e \Rightarrow \boxed{t=1}$ So, $\boxed{c=\dfrac{\pi}{2}}$

$\therefore \quad t=\sin\left(\dfrac{\pi}{2}-x\right)=\cos x \qquad \left(\because x\in\left[0,\dfrac{\pi}{2}\right]\right)$

$\therefore \quad \left(6\log_e f\left(\dfrac{\pi}{6}\right)\right)^2=27$

82. If $\frac{1^3+2^3+3^3+\ldots \text{ up to } n \text{ terms}}{1.3+2.5+3.7+\ldots \text{ up to } n \text{ terms}}=\frac{9}{5}$, then the value of n is

Answer (05)

Sol. Given $\frac{1^3+2^3+3^3+\ldots \text{up to } n \text{ terms}}{1.3+2.5+3.7+\ldots \text{ up to } n \text{ terms}}=\frac{9}{5}$ ...(1)

Now

Let $S = 1.3 + 2.5 + 3.7 + \ldots$

$T_n = n.(2n + 1)$

$\therefore \quad S=\frac{2n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}$

$\Rightarrow \quad \frac{\left(\frac{n(n+1)}{2}\right)^2}{n(n+1)\left[\frac{2n+1}{3}+\frac{1}{2}\right]}=\frac{9}{5}$

$\Rightarrow \quad 5n^2 - 19n - 30 = 0$

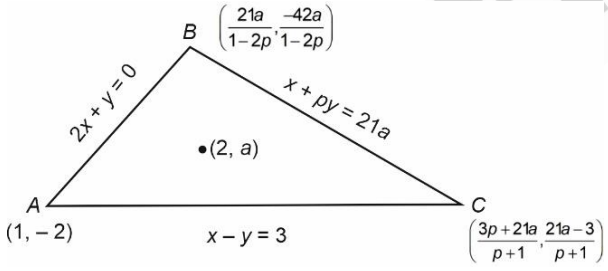
$\Rightarrow \quad (5n + 6)(n - 5) = 0$

$\therefore \quad \boxed{n=5}$

83. The equations of the sides AB, BC and CA of a triangle ABC are : $2x + y = 0$, $x + py = 21a$, $(a \neq 0)$ and $x - y = 3$ respectively. Let $P(2, a)$ be the centroid of ΔABC. Then $(BC)^2$ is equal to

Answer (122)

Sol.



$\because \quad \frac{21a}{1-2p}+1+\frac{3p+21a}{p+1}=6$

$\therefore \quad 4p^2 - 21ap + 8p + 42a - 5 = 0$ ...(1)

And $\frac{-42a}{1-2p}-2+\frac{21a-3}{p+1}=3a$

$\therefore \quad 4p^2 - 81ap + 6ap^2 - 24a + 8p - 5 = 0$ ...(2)

From equation (1) –equation (2) we get;

$60ap + 66a - 6ap^2 = 0$

$\because \quad a \neq 0 \Rightarrow p^2 - 10p - 11 = 0$

$p = -1$ or $11 \Rightarrow p = 11$.

When $p = 11$ then $a = 3$

Coordinate of $B = (-3, 6)$

And coordinate of $C = (8, 5)$

$\therefore \quad BC^2 = 122$

84. The minimum number of elements that must be added to the relation $R = \{(a, b), (b, c), (b, d)\}$ on the set $\{a, b, c, d\}$ so that it is an equivalence relation, is _____.

Answer (13)

Sol. $R = \{(a, b)(b, c)(b, d)\}$

$S : \{a, b, c, d\}$

Adding (a, a), (b, b), (c, c), (d, d) make reflexive.

Adding (b, a), (c, b), (d, b) make Symmetric

And adding (a, d), (a, c) to make transitive

Further (d, a) & (c, a) to be added to make Symmetricity.

Further (c, d) & (d, c) also be added.

So total 13 elements to be added to make equivalence.

85. Let $S = \{\theta \in [0, 2\pi) : \tan(\pi\cos\theta) + \tan(\pi \sin\theta) = 0\}$. Then $\sum_{\theta \in S} \sin^2\left(\theta+\frac{\pi}{4}\right)$ is equal to _____.

Answer (02)

Sol. $S=\{\theta \in [0, \pi) : \tan(\pi\cos\theta)+\tan(\pi\sin\theta)=0\}$

$\tan(\pi\cos\theta)+\tan(\pi\sin\theta)=0$

$\tan(\pi\cos\theta)=\tan(-\pi\sin\theta)$

$\pi\cos\theta = n\pi - \pi\sin\theta \qquad n \in I$

$\sin\theta+\cos\theta=n$

$\therefore \quad \sin\theta+\cos\theta=\{-1, 0, 1\}$

$\therefore \quad \theta=0, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{7\pi}{4}, \pi, \frac{3\pi}{2}$

Now $\sum_{\theta \in S} \sin^2\left(\theta+\frac{\pi}{4}\right)$

$=\sin^2\left(\frac{\pi}{4}\right)+\sin^2\left(\frac{\pi}{2}+\frac{\pi}{4}\right)+\sin^2\left(\frac{3\pi}{4}+\frac{\pi}{4}\right)$

$+\sin^2\left(\frac{7\pi}{4}+\frac{\pi}{4}\right)+\sin^2\left(\pi+\frac{\pi}{4}\right)+\sin^2\left(\frac{3\pi}{2}+\frac{\pi}{4}\right)$

$=\frac{1}{2}+\frac{1}{2}+0+0+\frac{1}{2}+\frac{1}{2}$

$= 2$

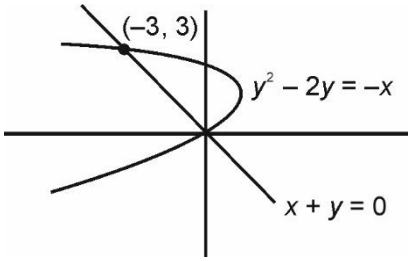
86. If the area of the region bounded by the curves $y^2 - 2y = -x$, $x + y = 0$ is A, then 8 A is equal to _____.

Answer (36)

Sol. Area enclosed by

$y^2 - 2y = -x$

$x + y = 0$



$$\text{Area} = \int_0^3 (2y - y^2) - (-y)dy$$

$$= \int_0^3 (3y - y^2)dy$$

$$= \frac{3y^2}{2} - \frac{y^3}{3}\Big|_0^3$$

$$= \frac{27}{2} - 9$$

$$= \frac{27-18}{2} = \frac{9}{2} = A$$

$$8A = \frac{9}{2} \times 8 = 36 \text{ sq. units}$$

87. If the shortest between the lines

$$\frac{x+\sqrt{6}}{2} = \frac{y-\sqrt{6}}{3} = \frac{z-\sqrt{6}}{4} \text{ and}$$

$$\frac{x-\lambda}{3} = \frac{y-2\sqrt{6}}{4} = \frac{z+2\sqrt{6}}{5}$$

is 6, then the square of sum of all possible values of λ is

Answer (384)

Sol. Shortest distance between

$$\frac{x+\sqrt{6}}{2} = \frac{y-\sqrt{6}}{3} = \frac{z-\sqrt{6}}{4} \text{ and}$$

$$\frac{x-\lambda}{3} = \frac{y-2\sqrt{6}}{4} = \frac{z+2\sqrt{6}}{5} \text{ is 6}$$

$$\vec{b_1} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\vec{b_2} = 3\hat{i} + 4\hat{j} + 5\hat{k}$$

$$\vec{a_2} - \vec{a_1} = \left(\lambda + \sqrt{6}\right)\hat{i} + \sqrt{6}\hat{j} - 3\sqrt{6}\,\hat{k}$$

$$d = \left|\frac{(\vec{a}_2 - \vec{a}_1)\cdot \vec{b_1} \times \vec{b_2}}{|\vec{b_1} \times \vec{b_2}|}\right| = 6$$

$$\left|\frac{-\lambda - \sqrt{6} + 2\sqrt{6} + 3\sqrt{6}}{\sqrt{6}}\right| = 6$$

$$|-\lambda + 4\sqrt{6}| = 6\sqrt{6}$$

$$-\lambda + 4\sqrt{6} = \pm 6\sqrt{6}$$

$$-\lambda + 4\sqrt{6} = 6\sqrt{6} \quad \Big| \quad -\lambda + 4\sqrt{6} = -6\sqrt{6}$$

$$\lambda_1 = -2\sqrt{6} \quad \Big| \quad \lambda_2 = 10\sqrt{6}$$

$$(\lambda_1 + \lambda_2)^2 = \left(8\sqrt{6}\right)^2$$

$$= 384$$

88. Let the sum of the coefficients of the first three terms in the expansion of $\left(x - \frac{3}{x^2}\right)^n$, $x \neq 0$. $n \in \mathbb{N}$, be 376. Then the coefficient of x^4 is _____.

Answer (405)

Sol. $S = 1 - 3n + \frac{9n(n-1)}{2} = 376$

$3n^2 - 5n - 250 = 0$

$n = 10, \frac{-25}{3}$ (Rejected)

$$T_{r+1} = {}^nC_r \cdot x^{n-r}\left(\frac{-3}{x^2}\right)^r$$

$$= {}^nC_r \;\; x^{n-3r}(-3)^r$$

$$= {}^{10}C_r \;\; x^{10-3r}(-3)^r$$

Here $r = 2$

Required coefficient = ${}^{10}C_2\,(-3)^2$

$= 45 \times 9$

$= 405$

89. Three urns A, B and C contain 4 red, 6 black; 5 red, 5 black, and λ red, 4 black balls respectively. One of the urns is selected at random and a ball is drawn. If the ball drawn is red and the probability that it is drawn from urn C is 0.4 then the square of the length of the side of the largest equilateral triangle, inscribed in the parabola $y^2 = \lambda x$ with one vertex at the vertex of the parabola, is

Answer (432)

Sol. E_1 : Ball is drawn from urn A ($4R + 6B$)

E_2 : Ball is drawn from urn B ($5R + 5B$)

E_3 : Ball is drawn from urn C ($\lambda R + 4B$)

$A \rightarrow$ Ball drawn is red.

Required probability $= P\left(\frac{E_3}{A}\right)$

$$= \frac{\frac{1}{3}\times\frac{\lambda}{\lambda+4}}{\frac{1}{3}\times\frac{4}{10}+\frac{1}{3}\times\frac{5}{10}+\frac{1}{3}\times\frac{\lambda}{\lambda+4}} = \frac{2}{5}$$

$$\Rightarrow \quad = \frac{10\lambda}{19\lambda+36} = \frac{2}{5}$$

$\Rightarrow \quad \lambda = 6$

Parabola: $y^2 = 6x = 4ax$

Let length of side $= l$

Point $\left(\frac{\sqrt{3}}{2}l, \frac{l}{2}\right)$ lies on parabola

$$\frac{l^2}{4} = 4a\left(\frac{\sqrt{3}}{2}l\right)$$

$$\Rightarrow \quad l = 8a\sqrt{3}$$

$$l = 12\sqrt{3}$$

$$l^2 = 432$$

90. Let

$\vec{a} = \hat{i} + 2\hat{j} + \lambda\hat{k}, \vec{b} = 3\hat{i} - 5\hat{j} - \lambda\hat{k}, \vec{a}\cdot\vec{c} = 7, 2\vec{b}\cdot\vec{c} + 43 = 0,$

$\vec{a}\times\vec{c} = \vec{b}\times\vec{c}$. Then $\left|\vec{a}\cdot\vec{b}\right|$ is equal to

Answer (08)

Sol. $\vec{a} = \hat{i} + 2\hat{j} + \lambda\hat{k}$

$\vec{b} = 3\hat{i} - 5\hat{j} - \lambda\hat{k}$

$\vec{a}\times\vec{c} = \vec{b}\times\vec{c}$

$\Rightarrow \quad \left(\vec{a} - \vec{b}\right)\times\vec{c} = 0$

$\Rightarrow \quad \vec{c} \parallel \vec{a} - \vec{b}$

$\Rightarrow \quad \vec{c} = \propto (\vec{a} - \vec{b})$

$\therefore \quad \vec{c} = \alpha\,(-2\hat{i} + 7\hat{j} + 2\lambda\hat{k})$

$$\vec{a}\cdot\vec{c} = \alpha\left(12 + 2\lambda^2\right) = 7 \qquad \ldots(i)$$

$$\vec{b}\cdot\vec{c} = \alpha\left(-41 - 2\lambda^2\right) = \frac{-43}{2} \qquad \ldots(ii)$$

(i) and (ii)

$$\Rightarrow \quad \frac{12 + 2\lambda^2}{41 + 2\lambda^2} = \frac{14}{43}$$

$\Rightarrow \quad \lambda^2 = 1$

$$\left|\vec{a}\cdot\vec{b}\right| = \left|3 - 10 - \lambda^2\right| = 8$$

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