

25/01/2023

Morning



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Answers & Solutions

Time : 3 hrs.

for

M.M. : 300

JEE (Main)-2023 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

- (1) The test is of **3 hours** duration.
- (2) The Test Booklet consists of 90 questions. The maximum marks are 300.
- (3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.
 - (i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **-1 mark** for wrong answer.
 - (ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **-1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. A message signal of frequency 5 kHz is used to modulate a carrier signal of frequency 2 MHz. The bandwidth for amplitude modulation is:

- (1) 5 kHz (2) 2.5 kHz
 (3) 10 kHz (4) 20 kHz

Answer (3)

Sol. Frequency of modulating wave = 5 kHz

$$\begin{aligned} \text{Bandwidth} &= \text{Twice the frequency of modulating signal} \\ &= 2 \times 5 \text{ kHz} \\ &= 10 \text{ kHz} \end{aligned}$$

2. Electron beam used in an electron microscope, when accelerated by a voltage of 20 kV, has a de-Broglie wavelength of λ_0 . If the voltage is increased to 40 kV, then the de-Broglie wavelength associated with the electron beam would be:

- (1) $9\lambda_0$ (2) $\frac{\lambda_0}{2}$
 (3) $\frac{\lambda_0}{\sqrt{2}}$ (4) $3\lambda_0$

Answer (3)

Sol. $\lambda_0 = \frac{h}{\sqrt{2m[e(20 \times 10^3)]}}$

$$\lambda_{\text{new}} = \frac{h}{\sqrt{2m[e(40 \times 10^3)]}} = \frac{\lambda_0}{\sqrt{2}}$$

3. The root mean square velocity of molecules of gas is

- (1) Proportional to square root of temperature (\sqrt{T})
 (2) Inversely proportional to square root of temperature ($\sqrt{\frac{1}{T}}$)
 (3) Proportional to temperature (T)
 (4) Proportional to square of temperature (T^2)

Answer (1)

Sol. $\therefore v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

$\therefore v_{\text{rms}} \propto \sqrt{T}$

4. In Young's double slits experiment, the position of 5th bright fringe from the central maximum is 5 cm. The distance between slits and screen is 1 m and wavelength of used monochromatic light is 600 nm. The separation between the slits is:

- (1) 12 μm
 (2) 60 μm
 (3) 48 μm
 (4) 36 μm

Answer (2)

Sol. $y_5 = 5 \text{ cm}$, $D = 1 \text{ m}$, $\lambda = 600 \text{ nm}$

$\therefore \frac{5\lambda D}{d} = \frac{5}{100}$

$$\begin{aligned} \therefore d &= \frac{5 \times 600 \times 10^{-9} \times 1 \times 100}{5} \\ &= 6 \times 10^{-5} \text{ m} \\ &= 60 \mu\text{m} \end{aligned}$$

5. Match **List I** with **List II**

List I	List II
A. Surface tension	I. $\text{kg m}^{-1}\text{s}^{-1}$
B. Pressure	II. kg ms^{-1}
C. Viscosity	III. $\text{kg m}^{-1}\text{s}^{-2}$
D. Impulse	IV. kg s^{-2}

Choose the correct answer from the options given below:

- (1) A-III, B-IV, C-I, D-II
 (2) A-II, B-I, C-III, D-IV
 (3) A-IV, B-III, C-I, D-II
 (4) A-IV, B-III, C-II, D-I

Answer (3)

- Sol.** (A) Surface tension : kg s^{-2} (IV)
 (B) Pressure : $\text{kg m}^{-1}\text{s}^{-2}$ (III)
 (C) Viscosity : $\text{kg m}^{-1}\text{s}^{-1}$ (I)
 (D) Impulse : kg ms^{-1} (II)

6. A bowl filled with very hot soup cools from 98°C to 86°C in 2 minutes when the room temperature is 22°C. How long it will take to cool from 75°C to 69°C?

- (1) 2 minutes (2) 1.4 minutes
(3) 0.5 minute (4) 1 minute

Answer (2)

Sol. From Newton's law of cooling.

$$\frac{dT}{dt} = -k(T - T_s)$$

Case I : $dT = 12^\circ\text{C}$, $dt = 2$ min

$$\frac{12}{2} = -k[92 - 22] = -k \cdot 70 \quad \dots(1)$$

Case II : $dT = 6^\circ\text{C}$

$$\frac{6}{dt} = -k[72 - 22] = -k \cdot 50 \quad \dots(2)$$

From (1) and (2)

$$dt = 1.4 \text{ min}$$

7. T is the time period of simple pendulum on the earth's surface. Its time period becomes xT when taken to a height R (equal to earth's radius) above the earth's surface. Then, the value of x will be:

- (1) 4 (2) $\frac{1}{2}$
(3) 2 (4) $\frac{1}{4}$

Answer (3)

Sol. $T = 2\pi\sqrt{\frac{l}{g}}$

g = acceleration due to gravity

On earth's surface $g = \frac{Gm}{R^2}$

On height R , $g_R = \frac{Gm}{4R^2}$

$$g_R = \frac{g}{4}$$

Time period at height $R = 2\pi\sqrt{\frac{l}{g_R}}$

$$= 2T$$

8. A Carnot engine with efficiency 50% takes heat from a source at 600 K. In order to increase the efficiency to 70%, keeping the temperature of sink same, the new temperature of the source will be :

- (1) 360 K (2) 300 K
(3) 900 K (4) 1000 K

Answer (4)

Sol. $\eta = 1 - \frac{T_{\text{sink}}}{T_{\text{source}}}$

50% efficiency $\Rightarrow \frac{1}{2} = 1 - \frac{T_{\text{sink}}}{T_{\text{source}}}$

$$\frac{1}{2} = 1 - \frac{T_{\text{sink}}}{600} \Rightarrow T_{\text{sink}} = 300$$

Now, 70% efficiency $\Rightarrow \frac{7}{10} = 1 - \frac{T_{\text{sink}}}{T_{\text{source}}}$

$$\frac{300}{T_{\text{source}}} = \frac{3}{10}$$

$$T_{\text{source}} = 1000 \text{ K}$$

9. The ratio of the density of oxygen nucleus ($^{16}_8\text{O}$) and helium nucleus (^4_2He) is

- (1) 2 : 1 (2) 8 : 1
(3) 1 : 1 (4) 4 : 1

Answer (3)

Sol. Nuclear density is constant.

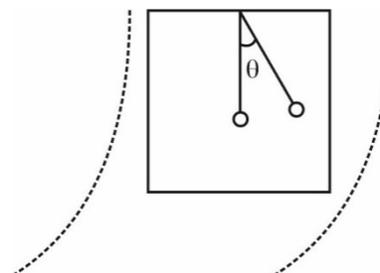
$$\frac{\rho_{\text{oxygen}}}{\rho_{\text{helium}}} = 1$$

10. A car is moving with a constant speed of 20 m/s in a circular horizontal track of radius 40 m. A bob is suspended from the roof of the car by a massless string. The angle made by the string with the vertical will be : (Take $g = 10 \text{ m/s}^2$)

- (1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$
(3) $\frac{\pi}{6}$ (4) $\frac{\pi}{4}$

Answer (4)

Sol.



In car's frame, FBD of bob

14. A uniform metallic wire carries a current 2 A. When 3.4 V battery is connected across it. The mass of uniform metallic wire is 8.92×10^{-3} kg, density is 8.92×10^3 kg/m³ and resistivity is $1.7 \times 10^{-8} \Omega -m$. The length of wire is:

- (1) $l = 100$ m
- (2) $l = 6.8$ m
- (3) $l = 10$ m
- (4) $l = 5$ m

Answer (3)

Sol. $m = 8.92 \times 10^{-3}$ kg

Density = 8.92×10^3 kg/m³

$$\text{Volume} = \frac{8.92 \times 10^{-3}}{8.92 \times 10^3} = (10^{-6})m^3$$

$$\text{Resistance} = \frac{3.4}{2} = 1.7 \Omega = \left(\frac{\rho l}{A}\right)$$

$$1.7 = \frac{\rho l^2}{(Al)}$$

$$\Rightarrow 1.7 = \frac{1.7 \times 10^{-8} \times l^2}{10^{-6}}$$

$$l^2 = 100$$

$$l = 10 \text{ m}$$

15. In an LC oscillator, if values of inductance and capacitance become twice and eight times, respectively, then the resonant frequency of oscillator becomes x times its initial resonant frequency ω_0 . The value of x is:

- (1) $\frac{1}{16}$
- (2) $\frac{1}{4}$
- (3) 4
- (4) 16

Answer (2)

Sol. $\omega_0 = \frac{1}{\sqrt{LC}}$

If inductance becomes $2L$ and capacitance becomes $8C$

$$\omega = \frac{1}{\sqrt{2L \times 8C}} = \frac{1}{4\sqrt{LC}}$$

$$\omega = \left(\frac{\omega_0}{4}\right)$$

16. A solenoid of 1200 turns is wound uniformly in a single layer on a glass tube 2 m long and 0.2 m in diameter. The magnetic intensity at the center of the solenoid when a current of 2 A flows through it is:

- (1) $A m^{-1}$
- (2) $2.4 \times 10^{-3} A m^{-1}$
- (3) $1.2 \times 10^3 A m^{-1}$
- (4) $2.4 \times 10^3 A m^{-1}$

Answer (3)

Sol. Number of turns per unit length = $\frac{1200}{2} = 600$

So, Magnetic Intensity $H = nI$

$$= 600 \times 2 A m^{-1}$$

$$= 1200 A m^{-1}$$

17. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R

Assertion A: Photodiodes are used in forward bias usually for measuring the light intensity.

Reason R: For a p-n junction diode, at applied voltage V the current in the forward bias is more than the current in the reverse bias for $|V_z| > \pm V \geq |V_0|$ where V_0 is the threshold voltage and V_z is the breakdown voltage.

In the light of the above statements, choose the **correct** answer from the options given below

- (1) A is false but R is true
- (2) Both A and R are true and R is correct explanation A
- (3) Both A and R are true but R is NOT the correct explanation A
- (4) A is true but R is false

Answer (1)

Sol. Photodiodes are used in reverse bias therefore the assertion is incorrect.

18. Assume that the earth is a solid sphere of uniform density and a tunnel is dug along its diameter throughout the earth. It is found that when a particle is released in this tunnel, it executes a simple harmonic motion. The mass of the particle is 100 g. The time period of the motion of the particle will be (approximately)

(Take $g = 10 \text{ m s}^{-2}$, radius of earth = 6400 km)

- (1) 1 hour 40 minutes
- (2) 12 hours
- (3) 24 hours
- (4) 1 hour 24 minutes

Answer (4)

Sol. Gravitational acceleration at a distance of r from centre of earth is given by

$$g' = \frac{g}{R} r$$

Where R is the radius of earth

$$\text{So, } \frac{d^2 r}{dt^2} = -\frac{g}{R} r$$

$$\Rightarrow T = 2\pi \sqrt{\frac{R}{g}} = 2\pi \sqrt{\frac{6400000}{10}}$$

$$= 2\pi \times 800 \text{ sec}$$

$$= 5024 \text{ sec}$$

$$= 1 \text{ hour } 24 \text{ minutes (approx.)}$$

19. A parallel plate capacitor has plate area 40 cm^2 and plates separation 2 mm . The space between the plates is filled with a dielectric medium of a thickness 1 mm and dielectric constant 5 . The capacitance of the system is :

(1) $\frac{3}{10} \epsilon_0 F$

(2) $\frac{10}{3} \epsilon_0 F$

(3) $10\epsilon_0 F$

(4) $24\epsilon_0 F$

Answer (2)

Sol. $c = \frac{\epsilon_0 A}{(d-t) + \frac{t}{K}}$

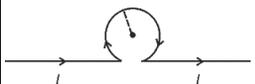
$$= \frac{K \epsilon_0 A}{Kd - t + (K-1)t}$$

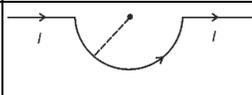
$$= \frac{5 \epsilon_0 \times 40 \times 10^{-4}}{5 \times 2 \times 10^{-3} - 1 \times 10^{-3} (5-1)}$$

$$= \frac{20 \epsilon_0}{6}$$

$$= \frac{10 \epsilon_0}{3}$$

20. Match List I with List II

List I (Current configuration)	List II (Magnitude of Magnetic Field at point O)
A. 	I. $B_0 = \frac{\mu_0 I}{4\pi r} [\pi + 2]$

B. 	II. $B_0 = \frac{\mu_0 I}{4 r}$
C. 	III. $B_0 = \frac{\mu_0 I}{2\pi r} [\pi - 1]$
D. 	IV. $B_0 = \frac{\mu_0 I}{4\pi r} [\pi + 1]$

Choose the correct answer from the options given below :

(1) A-III, B-IV, C-I, D-II (2) A-I, B-III, C-IV, D-II

(3) A-III, B-I, C-IV, D-II (4) A-II, B-I, C-IV, D-III

Answer (3)

Sol. A $\rightarrow B_0 = \frac{-\mu_0 I}{4\pi r} + \frac{\mu_0 I}{2r} - \frac{\mu_0 I}{4\pi r}$

$$B_0 = \frac{\mu_0 I}{2\pi r} (\pi - 1) \quad \text{A} \rightarrow \text{III}$$

B $\rightarrow B_0 = \frac{\mu_0 I}{4\pi r} + \frac{\mu_0 I}{4r} + \frac{\mu_0 I}{4\pi r}$

$$B_0 = \frac{\mu_0 I}{4\pi r} (\pi + 2) \quad \text{B} \rightarrow \text{I}$$

C $\rightarrow B_0 = \frac{\mu_0 I}{4\pi r} + \frac{\mu_0 I}{4r} + 0$

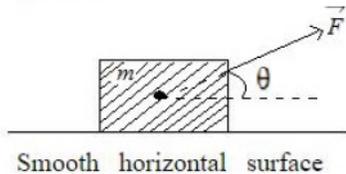
$$B_0 = \frac{\mu_0 I}{4\pi r} (\pi + 1) \quad \text{C} \rightarrow \text{IV}$$

D. $\rightarrow B_0 = \frac{\mu_0 I}{4r} \quad \text{D} \rightarrow \text{II}$

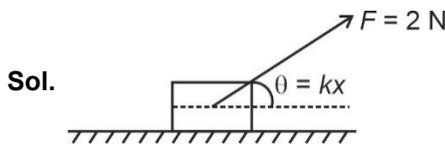
SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. An object of mass 'm' initially at rest on a smooth horizontal plane starts moving under the action of force $F = 2\text{N}$. In the process of its linear motion, the angle θ (as shown in figure) between the direction of force and horizontal varies as $\theta = kx$, where k is a constant and x is the distance covered by the object from its initial position. The expression of kinetic energy of the object will be $E = \frac{n}{k} \sin\theta$, the value of n is _____.



Answer (2)



Sol.

$$\text{Work done} = \Delta \text{K.E}$$

$$\therefore \int F \cdot dx = \frac{1}{2}mv^2 = E$$

$$\therefore E = \int_0^x 2 \cos(kx) dx$$

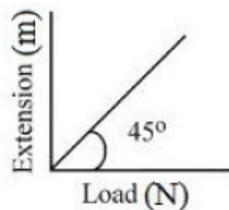
$$E = \frac{2}{k} [\sin kx]_0^x$$

$$= \frac{2}{k} \sin kx$$

$$= \frac{2 \sin \theta}{k}$$

22. As shown in the figure, in an experiment to determine Young's modulus of a wire, the extension-load curve is plotted. The curve is a straight line passing through the origin and makes an angle of 45° with the load axis. The length of wire is 62.8 cm and its diameter is 4 mm. The Young's modulus is found to be $x \times 10^4 \text{ Nm}^{-2}$.

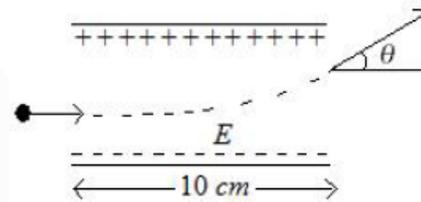
The value of x is _____.



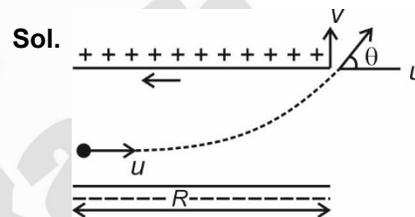
Answer (5)

$$\begin{aligned} \text{Sol. } Y &= \frac{F}{\Delta l} \times \left(\frac{l(4)}{\pi d^2} \right) \\ &= (\text{slope}) \frac{(62.8 \times 10^{-2})}{\pi(4 \times 10^{-3})^2} \\ &= (1) \times 5 \times 10^4 \text{ N/m}^2 \end{aligned}$$

23. A uniform electric field of 10 N/C is created between two parallel charged plates (as shown in figure). An electron enters the field symmetrically between the plates with a kinetic energy 0.5 eV. The length of each plate is 10 cm. The angle (θ) of deviation of the path of electron as it comes out of the field is _____ (in degree).



Answer (45)



Sol.

Let R is the range and T be the time of motion inside the plate.

$$\therefore R = vT$$

$$\text{and, } \tan \theta = \frac{v}{u}$$

$$= \left(\frac{eE}{m} \right) T$$

$$= \frac{eE \left(\frac{R}{u} \right)}{u}$$

$$= \frac{eER}{mu^2}$$

$$= \frac{eER}{2(\text{K.E.})}$$

$$= \frac{(e) \times (10) \times (10 \times 10^{-2})}{2 \times (0.5 \text{ eV})}$$

$$= 1$$

$$\therefore \tan \theta = 1$$

$$\theta = 45^\circ$$

24. An LCR series circuit of capacitance 62.5 nF and resistance of 50 Ω, is connected to an A.C. source of frequency 2.0 kHz. For maximum value of amplitude of current in circuit, the value of inductance is _____ mH.

(Take $\pi^2 = 10$)

Answer (100)

Sol. ∴ For maximum amplitude of current, circuit should be at resonance.

$$\therefore X_L = X_C$$

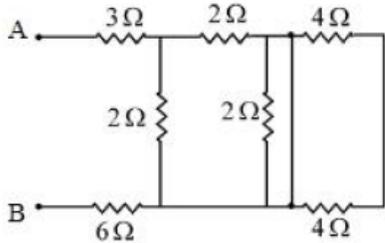
$$\omega L = \frac{1}{\omega C}$$

$$L = \frac{1}{\omega^2 C}$$

$$= \frac{1}{(2\pi \times 2 \times 10^3)^2 \times 62.5 \times 10^{-9}}$$

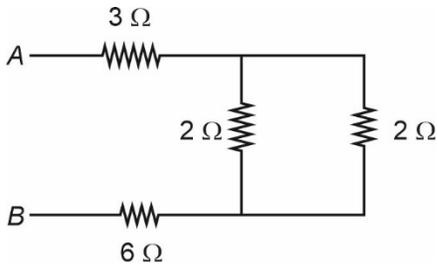
$$= 100 \text{ mH}$$

25. In the given circuit, the equivalent resistance between the terminal A and B is _____ Ω.



Answer (10)

Sol. Equivalent circuit can be redrawn as

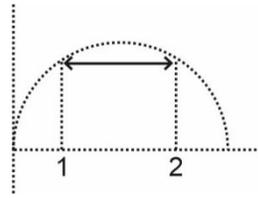


$$\therefore R_{AB} = 10 \Omega$$

26. The distance between two consecutive points with phase difference of 60° in a wave of frequency 500 Hz is 6.0 m. The velocity with which wave is traveling is _____ km/s

Answer (18)

Sol.



$$\Delta x = \frac{\lambda}{2\pi} \times \left(\frac{\pi}{3}\right) = \left(\frac{\lambda}{6}\right)$$

$$\Rightarrow \frac{\lambda}{6} = 6 \text{ m}$$

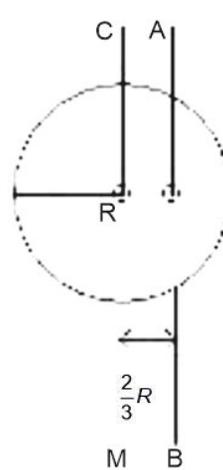
$$\lambda = 36 \text{ m}$$

$$U = f\lambda = 500 \text{ Hz} \times 36$$

$$= 18000 \text{ m/s}$$

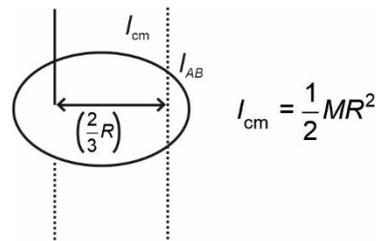
$$= 18 \text{ km/s}$$

27. I_{CM} is the moment of inertia of a circular disc about an axis (CM) passing through its center and perpendicular to the plane of disc. I_{AB} is its moment of inertia about an axis AB perpendicular to plane and parallel to axis CM at a distance $\frac{2}{3}R$ from center. Where R is the radius of the disc. The ratio of I_{AB} and I_{CM} is $x : 9$. The value of x is _____.



Answer (17)

Sol.



$$I_{cm} = \frac{1}{2} MR^2$$

$$I_{AB} = I_{cm} + M \times \left(\frac{2}{3}R\right)^2$$

$$= \frac{1}{2}MR^2 + \frac{4}{9}MR^2$$

$$= \frac{(9+8)MR^2}{18} = \left(\frac{17}{18}\right)MR^2$$

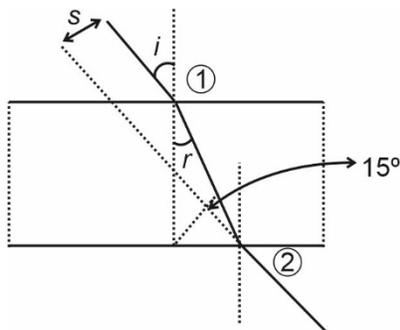
$$\frac{I_{AB}}{I_{cm}} = \frac{17/18}{1/2} = \left(\frac{17}{9}\right)$$

Value of $x = 17$

28. A ray of light is incident from air on a glass plate having thickness $\sqrt{3}$ cm and refractive index $\sqrt{2}$. The angle of incidence of a ray is equal to the critical angle for glass-air interface. The lateral displacement of the ray when it passes through the plate is _____ $\times 10^{-2}$ cm. (given $\sin 15^\circ = 0.26$)

Answer (52)

Sol.



$$\sin i = \frac{1}{\sqrt{2}} = 45^\circ$$

\Rightarrow at point (1)

$$\mu \sin r = \sin i = \frac{1}{\sqrt{2}}$$

$$\sin r = \frac{1}{2} \quad \Rightarrow \quad \boxed{r = 30^\circ}$$

Lateral displacement

$$= \frac{t}{\cos r} \sin(15^\circ) = \frac{\sqrt{3}}{\left(\frac{\sqrt{3}}{2}\right)} \times 0.26$$

$$= 2 \times 0.26$$

$$= 0.52 \text{ cm}$$

$$= 52 \times 10^{-2} \text{ cm}$$

29. If $\vec{P} = 3\hat{i} + \sqrt{3}\hat{j} + 2\hat{k}$ and $\vec{Q} = 4\hat{i} + \sqrt{3}\hat{j} + 2.5\hat{k}$ then, the unit vector in the direction of $\vec{P} \times \vec{Q}$ is $\frac{1}{x}(\sqrt{3}\hat{i} + \hat{j} - 2\sqrt{3}\hat{k})$. The value of x is

Answer (4)

Sol. $\vec{P} = 3\hat{i} + \sqrt{3}\hat{j} + 2\hat{k}$

$$\vec{Q} = 4\hat{i} + \sqrt{3}\hat{j} + 2.5\hat{k}$$

$$\vec{P} \times \vec{Q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & \sqrt{3} & 2 \\ 4 & \sqrt{3} & 2.5 \end{vmatrix}$$

$$= \hat{i}\left(\frac{\sqrt{3}}{2}\right) - \hat{j}\left(-\frac{1}{2}\right) + \hat{k}(-\sqrt{3})$$

$$= \frac{\sqrt{3}}{2}\hat{i} + \frac{\hat{j}}{2} - \sqrt{3}\hat{k}$$

$$|\vec{P} \times \vec{Q}| = \sqrt{\frac{3}{4} + \frac{1}{4} + 3} = 2$$

Unit vector along $\vec{P} \times \vec{Q} = \frac{1}{4}(\sqrt{3}\hat{i} + \hat{j} - 2\sqrt{3}\hat{k})$

$$x = 4$$

30. The wavelength of the radiation emitted is λ_0 when an electron jumps from the second excited state to the first excited state of hydrogen atom. If the electron jumps from the third excited state to the second orbit of the hydrogen atom, the wavelength of the radiation emitted will be $\frac{20}{x}\lambda_0$. The value of x is _____.

Answer (27)

Sol. Transition, $n = 3$ to $n = 2$

$$\frac{1}{\lambda_0} = R\left(\frac{1}{4} - \frac{1}{9}\right) = \left(\frac{5R}{36}\right) \quad \dots(1)$$

For transition from, $n = 4$ to $n = 2$

$$\frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{16}\right) = \left(\frac{3}{16}R\right) \quad \dots(2)$$

Taking ratio of (1) and (2)

$$\frac{\lambda}{\lambda_0} = \frac{5}{36} \times \frac{16}{3} = \left(\frac{20}{27}\right)$$

$$\boxed{\lambda = \frac{20}{27}\lambda_0}$$

$$x = 27$$

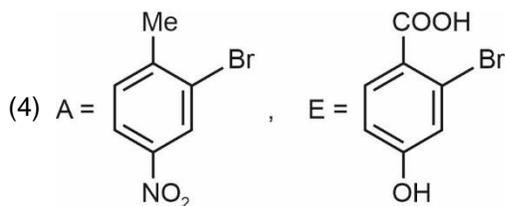
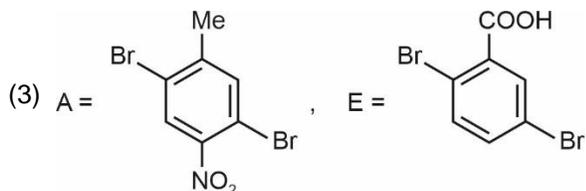
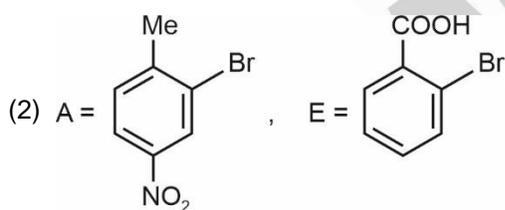
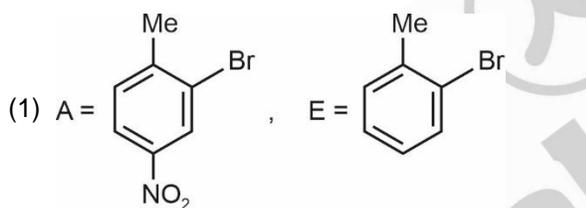
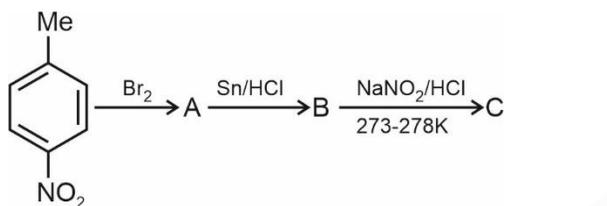
CHEMISTRY

SECTION - A

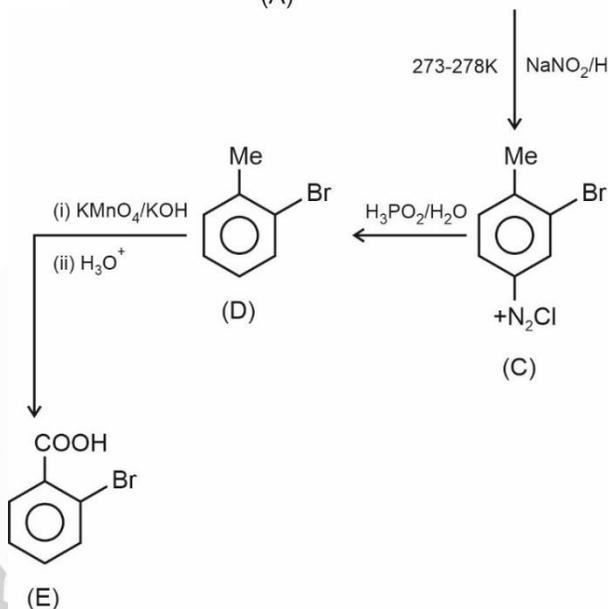
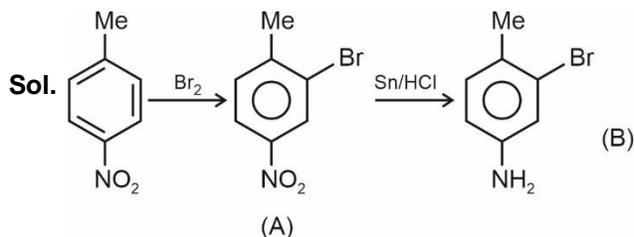
Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

31. Identify the product formed (A and E)



Answer (2)



32. A cubic solid is made up of two elements X and Y. Atoms of X are present on every alternate corner and one at the center of cube. Y is at $\frac{1}{3}$ rd of the total faces. The empirical formula of the compound is

- (1) $XY_{2.5}$ (2) $X_{1.5}Y_2$
(3) $X_2Y_{1.5}$ (4) $X_{2.5}Y$

Answer (No answer is correct)

Sol. Number of X particles = $4 \times \frac{1}{8} + 1 = 1.5$

Number of Y particles = $6 \times \frac{1}{3} \times \frac{1}{2} = 1$

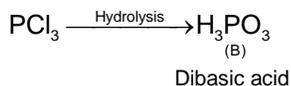
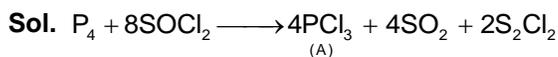
\therefore Empirical formula = $X_{1.5}Y_1 = X_3Y_2$

No answer is correct

33. Reaction of thionyl chloride with white phosphorus forms a compound [A], which on hydrolysis gives [B], a dibasic acid. [A] and [B] are respectively

- (1) $POCl_3$ and H_3PO_4 (2) PCl_3 and H_3PO_3
(3) PCl_5 and H_3PO_4 (4) P_4O_6 and H_3PO_3

Answer (2)



34. '25 volume' hydrogen peroxide means

- (1) 1 L marketed solution contains 250 g of H_2O_2 .
- (2) 100 mL marketed solution contains 25 g of H_2O_2 .
- (3) 1 L marketed solution contains 25 g of H_2O_2 .
- (4) 1 L marketed solution contains 75 g of H_2O_2 .

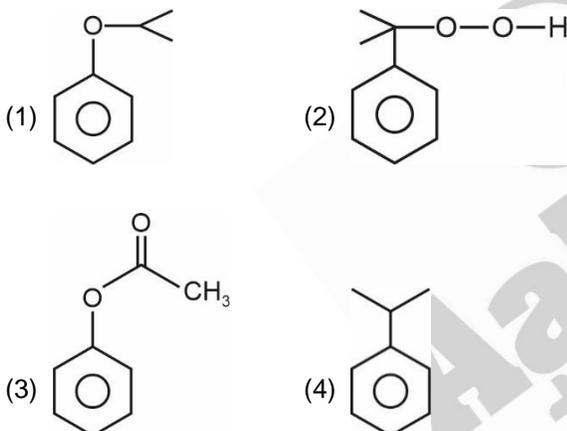
Answer (4)

Sol. Molarity of H_2O_2 solⁿ = $\frac{\text{volume strength}}{11.2}$

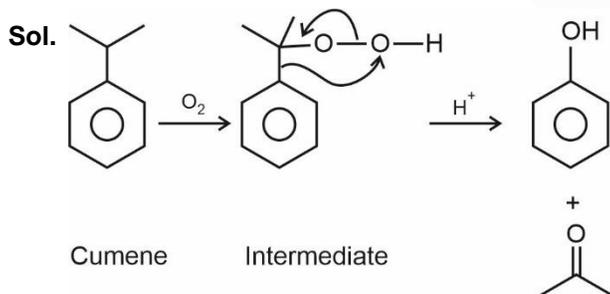
= $\frac{25}{11.2} = 2.23$ M

∴ amount of H_2O_2 in one litre = $2.23 \times 34 = 75$ gm

35. In the cumene to phenol preparation in presence of air, the intermediate is



Answer (2)



36. The radius of the 2nd orbit of Li^{2+} is x. The expected radius of the 3rd orbit of Be^{3+} is

- (1) $\frac{9}{4}x$
- (2) $\frac{16}{27}x$
- (3) $\frac{27}{16}x$
- (4) $\frac{4}{9}x$

Answer (3)

Sol. $r_{Li^{2+}} = r_0 \times \frac{2^2}{3} = x \Rightarrow r_0 = \frac{3x}{4}$

$r_{Be^{3+}} = r_0 \times \frac{3^2}{4}$

$r_{Be^{3+}} = \frac{3x}{4} \times \frac{3^2}{4} = \frac{27x}{16}$

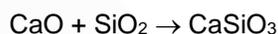
37. Which one of the following reactions does **not** occur during extraction of copper?

- (1) $CaO + SiO_2 \rightarrow CaSiO_3$
- (2) $FeO + SiO_2 \rightarrow FeSiO_3$
- (3) $2Cu_2S + 3O_2 \rightarrow 2Cu_2O + 2SO_2$
- (4) $2FeS + 3O_2 \rightarrow 2FeO + 2SO_2$

Answer (1)

Sol. In the extraction of copper FeO is removed as slag $FeSiO_3$

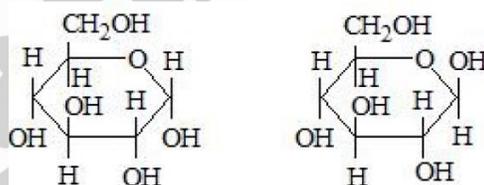
Hence the reaction



does not occur during extraction of copper

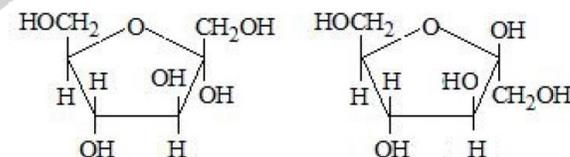
38. Match items of Row I with those of Row II.

Row I :



A

B



C

D

Row II :

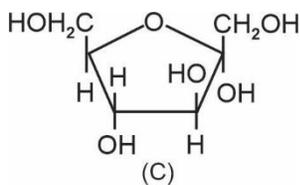
- (i) α -D-(–)-Fructofuranose
- (ii) β -D-(–)-Fructofuranose
- (iii) α -D-(–) Glucopyranose,
- (iv) β -D-(–)-Glucopyranose

Correct match is

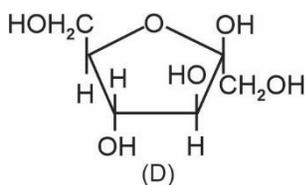
- (1) A \rightarrow iii, B \rightarrow iv, C \rightarrow i, D \rightarrow ii
- (2) A \rightarrow i, B \rightarrow ii, C \rightarrow iii, D \rightarrow iv
- (3) A \rightarrow iii, B \rightarrow iv, C \rightarrow ii, D \rightarrow i
- (4) A \rightarrow iv, B \rightarrow iii, C \rightarrow i, D \rightarrow ii

Answer (1)

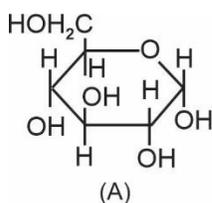
Sol. (i) α -D-(-)-Fructofuranose –



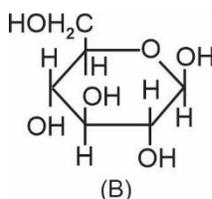
(ii) β -D-(-)-Fructofuranose –



(iii) α -D-(-)-Glucopyranose –



(iv) β -D-(-)-Glucopyranose –



39. Which of the following statements is incorrect for antibiotics?

- (1) An antibiotic should promote the growth or survival of microorganisms.
- (2) An antibiotic is a synthetic substance produced as a structural analogue of naturally occurring antibiotic.
- (3) An antibiotic should be effective in low concentrations.
- (4) An antibiotic must be a product of metabolism.

Answer (1)

Sol. An antibiotic inhibit the growth or survival of microorganism.

Except (1) all the statement are correct

40. Match List I with List II

LIST I Elements		LIST II Colour imparted to the flame	
A.	K	I.	Brick Red
B.	Ca	II.	Violet
C.	Sr	III.	Apple Green
D.	Ba	IV.	Crimson Red

Choose the correct answer from the options given below :

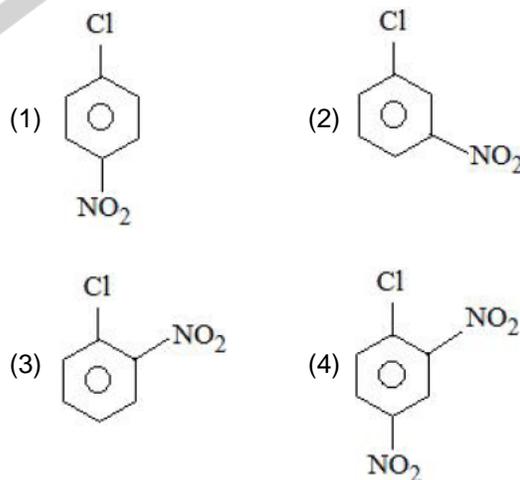
- (1) A-II, B-I, C-IV, D-III (2) A-IV, B-III, C-II, D-I
 (3) A-II, B-IV, C-I, D-III (4) A-II, B-I, C-III, D-IV

Answer (1)

Sol.

	Element	Colour imparted to the flame
(A)	K	Violet
(B)	Ca	Brick red
(C)	Sr	Crimson red
(D)	Ba	Apple green

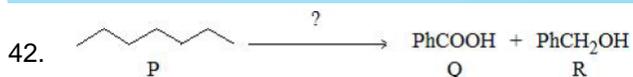
41. The compound which will have the lowest rate towards nucleophilic aromatic substitution on treatment with OH^- is



Answer (2)

Sol. Aryl halides having E.W.G at O-or P-position have greater rate than the m-isomers towards nucleophilic aromatic substitution.

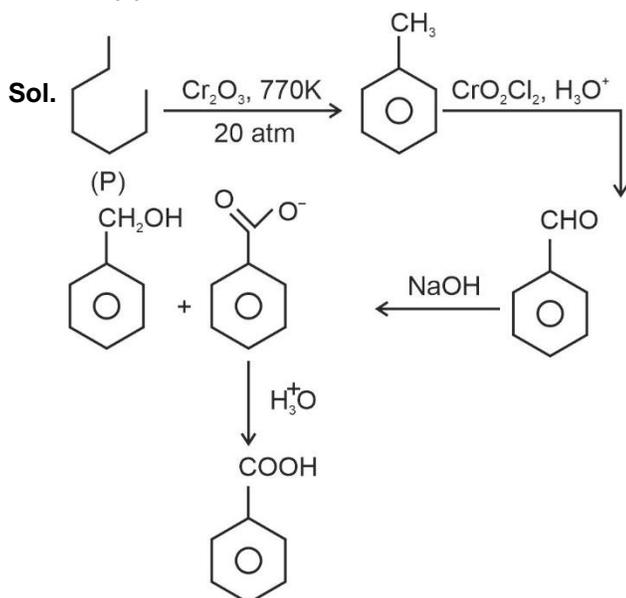
Hence the correct answer is (2)



The correct sequence of reagents for the preparation of Q and R is:

- (1) (i) $\text{KMnO}_4, \text{OH}^-$; (ii) $\text{Mo}_2\text{O}_3, \Delta$; (iii) NaOH ; (iv) H_3O^+
- (2) (i) $\text{Cr}_2\text{O}_3, 770 \text{ K}, 20 \text{ atm}$; (ii) $\text{CrO}_2\text{Cl}_2, \text{H}_3\text{O}^+$; (iii) NaOH ; (iv) H_3O^+
- (3) (i) $\text{CrO}_2\text{Cl}_2, \text{H}_3\text{O}^+$; (ii) $\text{Cr}_2\text{O}_3, 770 \text{ K}, 20 \text{ atm}$; (iii) NaOH ; (iv) H_3O^+
- (4) (i) $\text{Mo}_2\text{O}_3, \Delta$; (ii) $\text{Mo}_2\text{O}_3, \Delta$; (iii) NaOH ; (iv) H_3O^+

Answer (2)



43. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**:

Assertion A: Acetal/Ketal is stable in basic medium.

Reason R: The high leaving tendency of alkoxide ion gives the stability to acetal/ketal in basic medium.

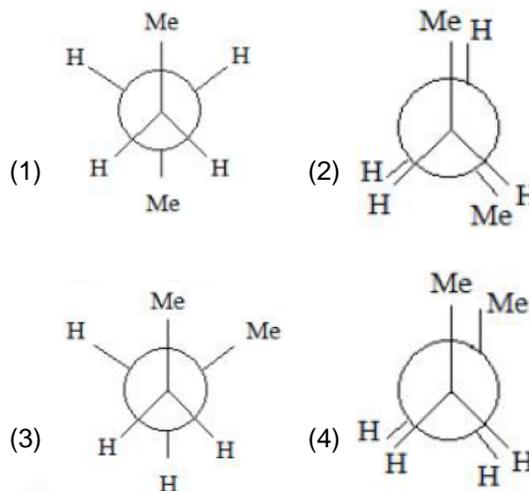
In the light of the above statements, choose the correct answer from the options given below:

- (1) Both A and R are true but R is NOT the correct explanation of A
- (2) Both A and R are true and R is the correct explanation of A
- (3) A is false but R is true
- (4) A is true but R is false

Answer (4)

Sol. Acetal/Ketal are known to be quite stable under basic conditions but readily hydrolyse to the corresponding carbonyl compound (aldehyde/ketone) and alcohol under acidic condition

44. Which of the following conformations will be the most stable?



Answer (1)

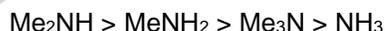
Sol. Correct stability order of butane is Anti > Gauche > Partially eclipsed > Fully eclipsed
 Hence the correct answer is (1)

45. The correct order in aqueous medium of basic strength in case of methyl substituted amines is:

- (1) $\text{Me}_2\text{NH} > \text{MeNH}_2 > \text{Me}_3\text{N} > \text{NH}_3$
- (2) $\text{Me}_3\text{N} > \text{Me}_2\text{NH} > \text{MeNH}_2 > \text{NH}_3$
- (3) $\text{NH}_3 > \text{Me}_3\text{N} > \text{MeNH}_2 > \text{Me}_2\text{NH}$
- (4) $\text{Me}_2\text{NH} > \text{Me}_3\text{N} > \text{MeNH}_2 > \text{NH}_3$

Answer (1)

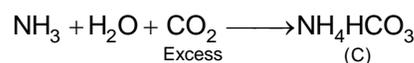
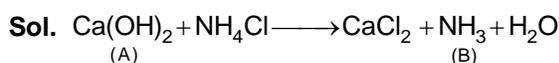
Sol. The correct order of basic strength in aqueous medium is



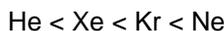
46. Compound A reacts with NH_4Cl and forms a compound B. Compound B reacts with H_2O and excess of CO_2 to form compound C which on passing through or reaction with saturated NaCl solution forms sodium hydrogen carbonate. Compound A, B and C, are respectively

- (1) $\text{Ca(OH)}_2, \text{NH}_4^{\oplus}, (\text{NH}_4)_2\text{CO}_3$
- (2) $\text{Ca(OH)}_2, \text{NH}_3, \text{NH}_4\text{HCO}_3$
- (3) $\text{CaCl}_2, \text{NH}_4^{\oplus}, (\text{NH}_4)_2\text{CO}_3$
- (4) $\text{CaCl}_2, \text{NH}_3, \text{NH}_4\text{HCO}_3$

Answer (2)



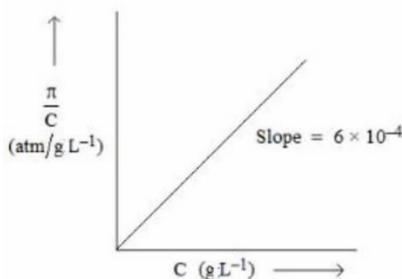
Hence, correct order of positive electron gain enthalpy is



SECTION - B

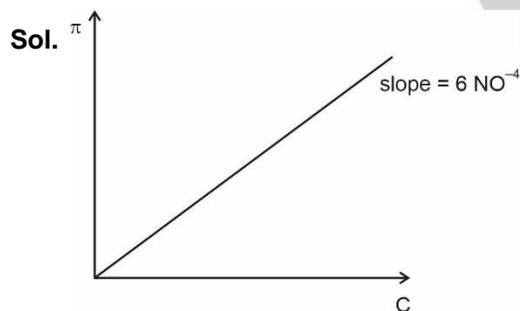
Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse andw the on-screen virtual numeric keypad in the place designated to enter the answer.

51. The osmotic pressure of solutions of PVC in cyclohexanone at 300 K are plotted on the graph. The molar mass of PVC is _____ g mol⁻¹ (Nearest integer)



(Given : R = 0.083 L atm K⁻¹ mol⁻¹)

Answer (41500)



$$\pi = CRT$$

$$\pi = \frac{\text{mole}}{\text{volume}} \times RT$$

$$\pi = \frac{\text{mole}}{\text{volume}} \times \frac{m}{mw} \times RT$$

$$\pi = \frac{\text{mass}}{\text{volume}} \times \frac{RT}{mw}$$

$$\pi(\text{atm}) = \frac{RT}{mw} \times C(\text{gm lit}^{-1})$$

$$\text{slope} = \frac{RT}{mw} = 6 \times 10^{-4}$$

$$mw = 41500$$

52. The density of a monobasic strong acid (Molar mass 24.2 g/mol) is 1.21 kg/L. The volume of its solution required for the complete neutralization of 25 mL of 0.24 M NaOH is _____ × 10⁻² mL (Nearest integer)

Answer (12)

Sol. m.eq of NaOH = m.eq of monobasic acid

$$25 \times 0.24 \times 1 = 1 \times V \times \text{molarity}$$

$$\text{Molarity} = \frac{1.21 \times 10^3}{24.2} = 50 \text{ M}$$

$$\therefore V = \frac{25 \times 0.24}{50} = 0.12 \text{ mL}$$

$$= 12 \times 10^{-2} \text{ mL}$$

53. A litre of buffer solution contains 0.1 mole of each of NH₃ and NH₄Cl. On the addition of 0.02 mole of HCl by dissolving gaseous HCl, the pH of the solution is found to be _____ × 10⁻³ (Nearest integer)

[Given : pK_b(NH₃) = 4.745

$$\log 2 = 0.301$$

$$\log 3 = 0.477$$

$$T = 298 \text{ K}]$$

Answer (9079)



At initial 0.1 0 0.1

At time t 0.1 - 0.02 0.1 + 0.02

$$\text{pOH} = \text{pK}_b + \log \left[\frac{0.1 + 0.02}{0.1 - 0.02} \right]$$

$$= 4.745 + \log \left(\frac{3}{2} \right) = 4.745 + [0.477 - 0.301]$$

$$= 4.745 + 0.176$$

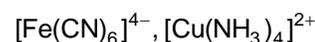
$$\text{pOH} = 4.921$$

$$\text{pH} = 14 - \text{pOH}$$

$$= 14 - 4.921 = 9.079$$

$$\text{pH} = 9079 \times 10^{-3}$$

54. The number of paramagnetic species from the following is _____.



Answer (04)

Sol. Species Magnetic property

$[\text{Ni}(\text{CN})_4]^{2-}$	Diamagnetic
$[\text{Ni}(\text{CO})_4]$	Diamagnetic
$[\text{NiCl}_4]^{2-}$	Paramagnetic
$[\text{Fe}(\text{CN})_6]^{4-}$	Diamagnetic
$[\text{Fe}(\text{CN})_6]^{3-}$	Paramagnetic
$\text{Fe}(\text{H}_2\text{O})_6^{2+}$	Paramagnetic
$[\text{Cu}(\text{NH}_3)_4]^{2+}$	Paramagnetic

55. In sulphur estimation, 0.471 g of an organic compound gave 1.4439 g of barium sulphate. The percentage of sulphur in the compound is _____ (Nearest Integer)

(Given: Atomic mass Ba: 137 u, S: 32 u, O: 16 u)

Answer (42)

Sol. $\text{S}\% = \frac{32}{233} \times \frac{1.4439}{0.471} \times 100 = 42\%$

56. For the first order reaction $\text{A} \rightarrow \text{B}$, the half life is 30 min. The time taken for 75% completion of the reaction is _____ min. (Nearest integer)

Given : $\log 2 = 0.3010$

$\log 3 = 0.4771$

$\log 5 = 0.6989$

Answer (60)

Sol. Time taken for 75% completion

$= 2 \times t_{1/2}$

$= 2 \times 30$

$= 60 \text{ min}$

57. How many of the following metal ions have similar value of spin only magnetic moment in gaseous state? _____

(Given : Atomic number : V, 23; Cr, 24; Fe, 26; Ni, 28)

V^{3+} , Cr^{3+} , Fe^{2+} , Ni^{3+}

Answer (02)

Sol. Ion Spin only magnetic moment

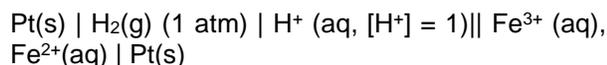
$\text{V}^{3+} \quad \sqrt{8}$

$\text{Cr}^{3+} \quad \sqrt{15}$

$\text{Fe}^{2+} \quad \sqrt{24}$

$\text{Ni}^{3+} \quad \sqrt{15}$

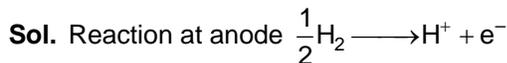
58. Consider the cell



Given $E_{\text{Fe}^{3+}/\text{Fe}^{2+}}^{\circ} = 0.771 \text{ V}$ and $E_{\text{H}^+/\text{H}_2}^{\circ} = 0 \text{ V}$,
 $T = 298 \text{ K}$

If the potential of the cell is 0.712 V, the ratio of concentration of Fe^{2+} to Fe^{3+} is _____ (Nearest integer)

Answer (10)



$$E_{\text{cell}} = E_{\text{cell}}^{\circ} - \frac{0.0591}{1} \log \left[\frac{[\text{H}^+][\text{Fe}^{2+}]}{[\text{Fe}^{3+}][\text{pH}_2]^{1/2}} \right]$$

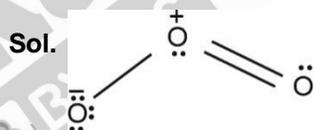
$$0.712 = 0.771 - \frac{0.0591}{1} \log \left(\frac{[\text{Fe}^{2+}]}{[\text{Fe}^{3+}]} \right)$$

$$-0.059 = -0.0591 \log \left(\frac{[\text{Fe}^{2+}]}{[\text{Fe}^{3+}]} \right)$$

$$\therefore \frac{[\text{Fe}^{2+}]}{[\text{Fe}^{3+}]} = 10^1 = 10$$

59. The total number of lone pairs of electrons on oxygen atoms of ozone is _____

Answer (06)



60. An athlete is given 100 g of glucose ($\text{C}_6\text{H}_{12}\text{O}_6$) for energy. This is equivalent to 1800 kJ of energy. The 50% of this energy gained is utilized by the athlete for sports activities at the event. In order to avoid storage of energy, the weight of extra water he would need to perspire is _____ g (Nearest integer)

Assume that there is no other way of consuming stored energy.

Given : The enthalpy of evaporation of water is 45 kJ mol⁻¹

Molar mass of C, H & O are 12, 1 and 16 g mol⁻¹

Answer (360)

Sol. wt of extra water he would need to perspire

$$= \frac{1800}{2} \times \frac{18}{45}$$

$$= 20 \times 18 = 360 \text{ gm}$$

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

61. The mean and variance of the marks obtained by the students in a test are 10 and 4 respectively. Later, the marks of one of the students is increased from 8 to 12. If the new mean of the marks is 10.2, then their new variance is equal to :

- (1) 4.08
- (2) 3.92
- (3) 3.96
- (4) 4.04

Answer (3)

Sol. $\bar{x} = 10$ & $\sigma^2 = 4$, No. of students = N (let)

$$\therefore \frac{\sum x_i}{N} = 10 \text{ \& \ } \frac{\sum x_i^2}{N} - (10)^2 = 4$$

Now if one of x_i is changed from 8 to 12 we have

$$\text{New mean } \frac{\sum x_i + 4}{N} = 10 + \frac{4}{N} = 10.2$$

$$\Rightarrow N = 20$$

$$\text{and } \sigma_{\text{new}}^2 = \frac{\sum x_i^2 - (8)^2 + (12)^2}{20} - (10.2)^2$$

$$= \frac{\sum x_i^2}{20} + \frac{144 - 64}{20} - (10.2)^2$$

$$= 104 + 4 - (10.2)^2$$

$$= 108 - 104.04 = 3.96$$

62. The statement $(p \wedge (\sim q)) \Rightarrow (p \Rightarrow (\sim q))$ is

- (1) a contradiction
- (2) equivalent to $p \vee q$
- (3) equivalent to $(\sim p) \vee (\sim q)$
- (4) a tautology

Answer (4)

Sol. Making truth table (Let $(p \wedge \sim q) \Rightarrow (p \Rightarrow \sim q) = E$)

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$p \Rightarrow \sim q$	E
T	T	F	F	F	F	T
T	F	F	T	T	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

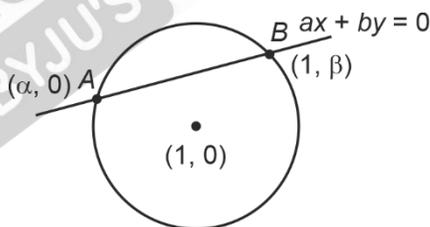
$\therefore E$ is a tautology

63. The points of intersection of the line $ax + by = 0$, ($a \neq b$) and the circle $x^2 + y^2 - 2x = 0$ are $A(\alpha, 0)$ and $B(1, \beta)$. The image of the circle with AB as a diameter in the line $x + y + 2 = 0$ is:

- (1) $x^2 + y^2 + 5x + 5y + 12 = 0$
- (2) $x^2 + y^2 + 3x + 5y + 8 = 0$
- (3) $x^2 + y^2 - 5x - 5y + 12 = 0$
- (4) $x^2 + y^2 + 3x + 3y + 4 = 0$

Answer (1)

Sol.



As A and B satisfy both line and circle we have

$$\alpha = 0 \Rightarrow A(0, 0) \text{ and } \beta = 1 \text{ i.e. } B(1, 1)$$

Centre of circle as AB diameter is $\left(\frac{1}{2}, \frac{1}{2}\right)$ and

$$\text{radius} = \frac{1}{\sqrt{2}}$$

\therefore For image of $\left(\frac{1}{2}, \frac{1}{2}\right)$ in $x + y + z = 0$ we get

$$\frac{x - \frac{1}{2}}{1} = \frac{y - \frac{1}{2}}{1} = \frac{-2(3)}{2}$$

$$\Rightarrow \text{Image} \left(-\frac{5}{2}, -\frac{5}{2} \right)$$

∴ Equation of required circle

$$\left(x + \frac{5}{2}\right)^2 + \left(y + \frac{5}{2}\right)^2 = \frac{1}{2}$$

$$\Rightarrow x^2 + y^2 + 5x + 5y + \frac{50}{4} - \frac{1}{2} = 0$$

$$\Rightarrow x^2 + y^2 + 5x + 5y + 12 = 0$$

64. Consider the lines L_1 and L_2 given by

$$L_1: \frac{x-1}{2} = \frac{y-3}{1} = \frac{z-2}{2}$$

$$L_2: \frac{x-2}{1} = \frac{y-2}{2} = \frac{z-3}{3}$$

A line L_3 having direction ratios, 1, -1, -2, intersects L_1 and L_2 at the points P and Q respectively. Then the length of line segment PQ is

- (1) $3\sqrt{2}$ (2) $2\sqrt{6}$
(3) $4\sqrt{3}$ (4) 4

Answer (2)

Sol. Let,

$$P \equiv (2\lambda + 1, \lambda + 3, 2\lambda + 2) \text{ and } Q(\mu + 2, 2\mu + 2, 3\mu + 3)$$

$$\text{d.r.'s of } PQ \equiv \langle 2\lambda - \mu - 1, \lambda - 2\mu + 1, 2\lambda - 3\mu - 1 \rangle$$

$$\therefore \frac{2\lambda - \mu - 1}{1} = \frac{\lambda - 2\mu - 1}{-1} = \frac{2\lambda - 3\mu - 1}{-2}$$

$$\therefore -2\lambda + \mu + 1 = \lambda - 2\mu + 1 \text{ and } -2\lambda + 4\mu - 2 = -2\lambda + 3\mu + 1$$

$$\Rightarrow 3\lambda - 3\mu = 0 \text{ and } \mu = 3$$

$$\therefore \lambda = \pm 3 \text{ and } \mu = 3$$

$$\therefore P \equiv (7, 6, 8) \text{ and } Q(5, 8, 12)$$

$$\therefore |PO| = \sqrt{2^2 + 2^2 + 4^2} = \sqrt{24} = 2\sqrt{6}$$

65. Let $f : (0, 1) \rightarrow \mathbb{R}$ be a function defined by

$$f(x) = \frac{1}{1 - e^{-x}}, \text{ and } g(x) = (f(-x) - f(x)). \text{ Consider}$$

two statements

- (I) g is an increasing function in $(0, 1)$
(II) g is one-one in $(0, 1)$

Then,

- (1) Only (I) is true
(2) Both (I) and (II) are true
(3) Only (II) is true
(4) Neither (I) nor (II) is true

Answer (2)

Sol. $g(x) = f(-x) - f(x)$

$$= \frac{1}{1 - e^x} - \frac{1}{1 - e^{-x}}$$

$$= \frac{1}{1 - e^x} - \frac{e^x}{e^x - 1}$$

$$= \frac{1 + e^x}{1 - e^x}$$

$$g'(x) = \frac{(1 - e^x)e^x - (1 + e^x)(-e^x)}{(1 - e^x)^2}$$

$$= \frac{e^x - 2e^{2x} + e^x + 2e^x}{(1 - e^x)^2} > 0$$

So both statements are correct

66. Let S_1 and S_2 be respectively the sets of $a \in \mathbb{R} - \{0\}$

for which the system of linear equations

$$ax + 2ay - 3az = 1$$

$$(2a + 1)x + (2a + 3)y + (a + 1)z = 2$$

$$(3a + 5)x + (a + 5)y + (a + 2)z = 3$$

has unique solution and infinitely many solutions. Then

(1) $S_1 = \Phi$ and $S_2 = \mathbb{R} - \{0\}$

(2) $S_1 = \mathbb{R} - \{0\}$ and $S_2 = \Phi$

(3) S_1 is an infinite set and $n(S_2) = 2$

(4) $n(S_1) = 2$ and S_2 is an infinite set

Answer (2)

Sol. Given system of equations

$$ax + 2ay - 3az = 1$$

$$(2a + 1)x + (2a + 3)y + (a + 1)z = 2$$

$$(3a + 5)x + (a + 5)y + (a + 2)z = 3$$

$$\text{Let } A = \begin{vmatrix} a & 2a & -3a \\ 2a+1 & 2a+3 & a+1 \\ 3a+5 & a+5 & a+2 \end{vmatrix}$$

$$= a \begin{vmatrix} 1 & 0 & 0 \\ 2a+1 & 1-2a & 7a+4 \\ 3a+5 & -5a-5 & 10a+17 \end{vmatrix}$$

$$= a(15a^2 + 31a + 37)$$

Now $A = 0$

$$\Rightarrow \boxed{a = 0}$$

So, $S_1 = \mathbb{R} - \{0\}$ and at $a = 0$

System has infinite solution but $a \in \mathbb{R} - \{0\}$

$$\therefore S_2 = \Phi$$

67. The minimum value of the function $f(x) = \int_0^2 e^{|x-t|} dt$ is

- (1) $e(e-1)$
- (2) $2e-1$
- (3) $2(e-1)$
- (4) 2

Answer (3)

Sol. $f(x) = \int_0^2 e^{|x-t|} dt$

For $x > 2$

$$f(x) = \int_0^2 e^{x-t} dt = e^x (1 - e^{-2})$$

For $x < 0$

$$f(x) = \int_0^2 e^{t-x} dt = e^{-x} (e^2 - 1)$$

For $x \in [0, 2]$

$$f(x) = \int_0^x e^{x-t} dt + \int_x^2 e^{t-x} dt$$

$$= e^{2-x} + e^x - 2$$

For $x > 2$

$$f(x)|_{\min} = e^2 - 1$$

For $x < 0$

$$f(x)|_{\min} = e^2 - 1$$

For $x \in [0, 2]$

$$f(x)|_{\min} = 2(e-1)$$

68. Let $y(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)(1+x^{16})$.

Then $y' - y''$ at $x = -1$ is equal to :

- (1) 496
- (2) 944
- (3) 976
- (4) 464

Answer (1)

Sol. $y = \frac{1-x^{32}}{1-x} = 1+x+x^2+x^3+\dots+x^{31}$

$$y' = 1+2x+3x^2+\dots+31x^{30}$$

$$y'(-1) = 1-2+3-4+\dots+31 = 16$$

$$y''(x) = 2+6x+12x^2+\dots+31.30x^{29}$$

$$y''(-1) = 2-6+12-\dots-31.30 = 480$$

$$y''(-1) - y'(-1) = -496$$

69. Let $x = 2$ be a local minima of the function $f(x) = 2x^4 - 18x^2 + 8x + 12$, $x \in (-4, 4)$. If M is local maximum value of the function f in $(-4, 4)$, then $M =$

- (1) $12\sqrt{6} - \frac{33}{2}$
- (2) $12\sqrt{6} - \frac{31}{2}$
- (3) $18\sqrt{6} - \frac{31}{2}$
- (4) $18\sqrt{6} - \frac{33}{2}$

Answer (1)

Sol. $f(x) = 8x^3 - 36x + 8$

$$= 4(2x^3 - 9x + 2)$$

$$= 4(x-2)(2x^2 + 4x - 1)$$

$$= 4(x-2) \left(x - \frac{-2+\sqrt{6}}{2} \right) \left(x - \frac{-2-\sqrt{6}}{2} \right)$$

Local maxima occurs at $x = \frac{-2+\sqrt{6}}{2} = x_0$

$$f(x_0) = 12\sqrt{6} - \frac{33}{2}$$

70. The value of

$$\lim_{n \rightarrow \infty} \frac{1+2-3+4+5-6+\dots+(3n-2)+(3n-1)-3n}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$$

is :

- (1) $3(\sqrt{2}+1)$
- (2) $\frac{3}{2}(\sqrt{2}+1)$
- (3) $\frac{\sqrt{2}+1}{2}$
- (4) $\frac{3}{2\sqrt{2}}$

Answer (2)

Sol. $I = \lim_{n \rightarrow \infty} \frac{(1+2+3+\dots+3n) - 2(3+6+9+\dots+3n)}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$

$$= \lim_{n \rightarrow \infty} \frac{\frac{3n(3n+1)}{2} - 6 \frac{n(n+1)}{2}}{\left(\sqrt{2n^4+4n+3} - \sqrt{n^4+5n+4} \right)}$$

$$= \lim_{n \rightarrow \infty} \frac{3n(n-1) \left[\sqrt{2n^4+4n+3} + \sqrt{n^4+5n+4} \right]}{2 \left[(2n^4+4n-3) - (n^4+5n+4) \right]}$$

$$= \lim_{n \rightarrow \infty} \frac{3 \cdot 1 \left(1 - \frac{1}{n} \right) \left[\sqrt{2 + \frac{4}{n^3} + \frac{3}{n^4}} + \sqrt{1 + \frac{5}{n^3} + \frac{4}{n^4}} \right]}{2 \left[1 - \frac{1}{n^3} - \frac{7}{n^4} \right]}$$

$$= \frac{3(\sqrt{2}+1)}{2}$$

Sol. $\frac{dy}{dx} = \frac{y}{x}(1 + xy^2(1 + \log_e x)), \quad y(1) = 3$

$$\Rightarrow \frac{1}{y^3} \frac{dy}{dx} - \frac{1}{x} \cdot \frac{1}{y^2} = (1 + \ln x)$$

$$-\frac{1}{y^2} = t \Rightarrow \frac{2}{y^3} \frac{dy}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \frac{1}{2} \frac{dt}{dx} + \frac{t}{x} = 1 + \ln x$$

$$\Rightarrow \frac{dt}{dx} + \frac{2t}{x} = 2(1 + \ln x)$$

IF = x^2

$$t \cdot x^2 = \int (1 + \ln x) x^2 dx$$

$$\Rightarrow -\frac{1}{y^2} \cdot x^2 = 2 \left[\frac{x^3}{3} (1 + \ln x) - \frac{x^3}{9} \right] + c$$

$y(1) = 3$

$$\Rightarrow c = -\frac{5}{9}$$

$$\therefore \frac{x^2}{y^2} = -2 \left[\frac{x^3}{3} (1 + \ln x) - \frac{x^3}{9} \right] + \frac{5}{9}$$

$$\Rightarrow \frac{y^2}{9} = \frac{x^2}{5 - 2x^3(2 + \ln x^3)}$$

79. Let $z_1 = 2 + 3i$ and $z_2 = 3 + 4i$. The set

$$S = \{z \in \mathbb{C} : |z - z_1|^2 - |z - z_2|^2 = |z_1 - z_2|^2\}$$

represents a

- (1) straight line with the sum of its intercepts on the coordinate axes equals -18
- (2) hyperbola with eccentricity 2
- (3) straight line with the sum of its intercepts on the coordinate axes equals 14
- (4) hyperbola with the length of the transverse axis 7

Answer (3)

Sol. $|z - z_1|^2 - |z - z_2|^2 = |z_1 - z_2|^2$

$$\Rightarrow (x - 2)^2 + (y - 3)^2 - (x - 3)^2 - (y - 4)^2 = 1 + 1$$

$$\Rightarrow -4x + 4 + 9 - 6y - 9 + 6x - 16 + 8y = 2$$

$$\Rightarrow 2x + 2y = 14$$

$$\Rightarrow x + y = 7$$

80. The vector $\vec{a} = -\hat{i} + 2\hat{j} + \hat{k}$ is rotated through a right angle, passing through the y -axis in its way and the resulting vector is \vec{b} . Then the projection of $3\vec{a} + \sqrt{2}\vec{b}$ on $\vec{c} = 5\hat{i} + 4\hat{j} + 3\hat{k}$ is

- (1) $\sqrt{6}$
- (2) $2\sqrt{3}$
- (3) 1
- (4) $3\sqrt{2}$

Answer (4)

Sol. Let $\vec{b} = \mu\vec{a} + \lambda\hat{j}$

Now $\vec{b} \cdot \vec{a} = 0$

$$\Rightarrow (\mu\vec{a} + \lambda\hat{j}) \cdot \vec{a} = 0$$

$$\Rightarrow \mu|\vec{a}|^2 + 2\lambda = 0 \Rightarrow 6\mu + 2\lambda = 0 \dots(i)$$

$$\Rightarrow \vec{b} = \lambda(\vec{a} - 3\hat{j}) = \lambda(-\hat{i} - \hat{j} + \hat{k})$$

$$\Rightarrow |\vec{b}| = |\vec{a}| \Rightarrow \lambda = \pm\sqrt{2}$$

$$\therefore \vec{b} = -\sqrt{2}(-\hat{i} - \hat{j} + \hat{k})$$

$$\therefore 3\vec{a} + \sqrt{2}\vec{b} = 3(-\hat{i} + 2\hat{j} + \hat{k}) - 2(-\hat{i} - \hat{j} + \hat{k}) = -\hat{i} + 8\hat{j} + \hat{k}$$

$$\therefore \text{projection } 3\sqrt{2}$$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33 , -00.30 , 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

81. Let the equation of the plane passing through the line $x - 2y - z - 5 = 0 = x + y + 3z - 5$ and parallel to the line $x + y + 2z - 7 = 0 = 2x + 3y + z - 2$ be $ax + by + cz = 65$. Then the distance of the point (a, b, c) from the plane $2x + 2y - z + 16 = 0$ is

Answer (09)

Sol. Let the equation of the plane is

$$(x - 2y - z - 5) + \lambda(x + y + 3z - 5) = 0 \dots(i)$$

\therefore it's parallel to the line

$$x + y + 2z - 7 = 0 = 2x + 3y + z - 2$$

So, vector along the line $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ 2 & 3 & 1 \end{vmatrix}$

$= -5\hat{i} + 3\hat{j} + \hat{k}$

∴ Plane is parallel to line

∴ $-5(1 + \lambda) + 3(-2 + \lambda) + 1(-1 + 3\lambda) = 0$

$\lambda = 12$

So, by (i)

$13x + 10y + 35z = 65$

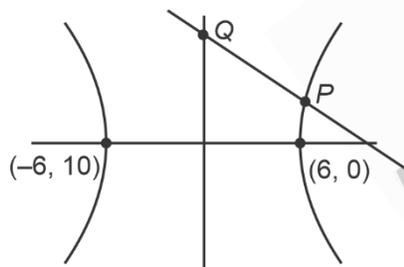
∴ $a = 13, b = 10, c = 35$

and $d = \frac{26 + 20 - 35 + 16}{\sqrt{9}} = 9$

82. The vertices of a hyperbola H are $(\pm 6, 0)$ and its eccentricity is $\frac{\sqrt{5}}{2}$. Let N be the normal to H at point in the first quadrant and parallel to the line $\sqrt{2}x + y = 2\sqrt{2}$. If d is the length of the line segment of N between H and the y -axis then d^2 is equal to _____.

Answer (216)

Sol.



$a = 6, e = \frac{\sqrt{5}}{2}$

∴ $\frac{5}{4} = 1 + \frac{b^2}{36} \Rightarrow b^2 = 36 \times \frac{1}{4} = 9$

∴ $H: \frac{x^2}{36} - \frac{y^2}{9} = 1$

$P(6\sec\theta, 3\tan\theta)$

Slope of tangent at $P = \frac{6\sec\theta}{4 \times 3\tan\theta}$

So, $\frac{1}{2\sin\theta} \times -\sqrt{2} = -1 \Rightarrow \sin\theta = \frac{1}{\sqrt{2}}$

$Q = 45^\circ$ (for first quad)

∴ $P \equiv (6\sqrt{2}, 3)$ and $N: \sqrt{2}x + y = 15$

∴ $Q(0, 15)$ Now, $PQ^2 = 72 + 144 = 216$

83. Let

$s = \left\{ \alpha : \log_2(9^{2\alpha-4} + 13) - \log_2\left(\frac{5}{2} \cdot 3^{2\alpha-4} + 1\right) = 2 \right\}$.

Then the maximum value of β for which the

equation $x^2 - 2\left(\sum_{\alpha \in S} \alpha\right)^2 x + \sum_{\alpha \in S} (\alpha + 1)^2 \beta = 0$ has

real roots, is _____.

Answer (25)

Sol. $S = \left\{ \alpha : \log_2(9^{2\alpha-4} + 13) - \log_2\left(\frac{5}{2} \cdot 3^{2\alpha-4} + 1\right) = 2 \right\}$

So,

$\frac{9^{2\alpha-4} + 13}{\frac{5}{2} \cdot 3^{2\alpha-4} + 1} = 4 \Rightarrow 9^{2\alpha-4} + 13 = 103 \cdot 3^{2\alpha-4} + 4$

Let $3^{2\alpha-4} = t$ then $t^2 - 10t + 9 = 0$

$(t - 9)(t - 1) = 0$

∴ $3^{2\alpha-4} = 3^2$ or $3^{2\alpha-4} = 3^0$

∴ $\alpha = 3, 2$

Now equation

$x^2 - 50x + 25\beta = 0$

$D \geq 0 \Rightarrow (50)^2 - 4 \times 25\beta \geq 0$

$\beta \leq 25$

∴ Max. $\beta = 25$

84. For some $a, b, c \in N$, let $f(x) = ax - 3$ and $g(x) = x^b$

+ $c, x \in R$. If $(fog)^{-1}(x) = \left(\frac{x-7}{2}\right)^{\frac{1}{3}}$, then $(fog)(ac)$ + $(gof)(b)$ is equal to _____.

Answer (2039)

Sol. $f(x) = ax - 3$

$g(x) = x^b + c$

$(fog)^{-1} = \left(\frac{x-7}{2}\right)^{\frac{1}{3}}$

$(fog)^{-1}(x) = \left(\frac{x+3-ca}{a}\right)^{\frac{1}{b}} = \left(\frac{x-7}{2}\right)^{\frac{1}{3}}$

$\Rightarrow a = 2, b = 3, c = 5$

$fog(ac) + gof(b)$

∴ $f(x) = 2x - 3$

$g(x) = x^3 + 5$

$fog(10) + gof(3)$

$= 2007 + 32$

$= 2039$

85. Let x and y be distinct integers where $1 \leq x \leq 25$ and $1 \leq y \leq 25$. Then, the number of ways of choosing x and y , such that $x + y$ is divisible by 5, is _____.

Answer (120)

Sol. Type	Numbers
$5k$	5, 10, 15, 20, 25
$5k + 1$	1, 6, 11, 16, 21
$5k + 2$	2, 7, 12, 17, 22
$5k + 3$	3, 8, 13, 18, 23
$5k + 4$	4, 9, 14, 19, 24

To select x and y .

Case I : 1 of $(5k + 1)$ and 1 of $(5k + 4) = 5 \times 5 = 25$

Case II : 1 of $(5k + 2)$ and 1 of $(5k + 3) = 5 \times 5 = 25$

Case III : Both of type $5k$ (both cannot be same) = $5 \times 4 = 20$

Total = 120

86. The constant term in the expansion of

$$\left(2x + \frac{1}{x^7} + 3x^2\right)^5 \text{ is } \underline{\hspace{2cm}}.$$

Answer (1080)

Sol. Constant term in the expansion of

$$\left(2x + \frac{1}{x^7} + 3x^2\right)^5$$

$$\frac{1}{x^{35}} (2x^8 + 1 + 3x^9)^5$$

$$\frac{1}{x^{35}} (1 + x^8(3x+2))^5$$

Term independent of $x =$ coefficient of x^{35} in

$${}^5C_4 (x^8(3x+2))^4$$

$$= {}^5C_4 \text{ coefficient of } x^3 \text{ in } (2+3x)^4$$

$$= {}^5C_4 \times {}^4C_3 (2)^1 (3)^3$$

$$= 5 \times 4 \times 2 \times 27$$

$$= 1080$$

87. Let $S = \{1, 2, 3, 5, 7, 10, 11\}$. The number of non-empty subsets of S that have the sum of all elements a multiple of 3, is

Answer (43)

Sol. Out of the given numbers one is $(3k)$ type and 3 of $(3k + 1)$ type and remaining 3 are $(3k + 2)$ type

Number of subsets of 1 element = 1

(1 of $3k$ type)

Number of subsets of 2 elements

1 of $(3k + 1)$ type + 1 of $(3k + 2)$ type = 9

Number of subsets of 3 elements

1 of $3k$ type + 1 of $(3k + 1)$ type + 1 of $(3k + 2)$ type = 9

3 of $(3k + 1)$ type = 1

3 of $(3k + 2)$ type = 1

Number of subsets of 4 elements

1 of $3k$ type + 3 of $(3k + 1)$ type = 1

1 of $3k$ type + 3 of $(3k + 2)$ type = 1

2 of $(3k + 1)$ type + 2 of $(3k + 2)$ type = 9

Number of subsets of 5 elements

1 of $3k + 2$ of $(3k + 1)$ type + 2 of $(3k + 2)$ type = 9

Number of subsets of 6 elements

3 of $(3k + 1)$ type + 3 of $(3k + 2)$ type = 1

The set itself = 1

Total = 43

88. If the sum of all the solutions of

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \frac{\pi}{3}, -1 < x < 1, x \neq 0,$$

is $\alpha - \frac{4}{\sqrt{3}}$, then α is equal to _____.

Answer (02)

Sol. Case-I

$-1 < x < 0$

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$$

$$\tan^{-1}\frac{2x}{1-x^2} = \frac{-\pi}{3}$$

$$2\tan^{-1}x = \frac{-\pi}{3}$$

$$\tan^{-1}x = \frac{-\pi}{6}$$

$$x = \frac{-1}{\sqrt{3}}$$

Case-II

$0 < x < 1$

$$\tan^{-1}\frac{2x}{1-x^2} + \tan^{-1}\frac{2x}{1-x^2} = \frac{\pi}{3}$$

$$\tan^{-1} \frac{2x}{1-x^2} = \frac{\pi}{6}$$

$$2 \tan^{-1} x = \frac{\pi}{6}$$

$$\tan^{-1} x = \frac{\pi}{12}$$

$$x = 2 - \sqrt{3}$$

$$\text{Sum} = \frac{-1}{\sqrt{3}} + 2 - \sqrt{3} = 2 - \frac{4}{\sqrt{3}}$$

$$\Rightarrow \alpha = 2$$

89. Let A_1, A_2, A_3 be the three A.P. with the same common difference d and having their first terms as $A, A + 1, A + 2$, respectively. Let a, b, c be the 7th, 9th, 17th terms of A_1, A_2, A_3 , respectively such that

$$\begin{vmatrix} a & 7 & 1 \\ 2b & 17 & 1 \\ c & 17 & 1 \end{vmatrix} + 70 = 0.$$

If $a = 29$, then the sum of first 20 terms of an AP whose first term is $c - a - b$ and common difference is $\frac{d}{12}$, is equal to _____.

Answer (495)

Sol. $a = A + 6d$

$$b = A + 8d + 1$$

$$c = A + 16d + 2$$

$$\begin{vmatrix} a & 7 & 1 \\ 2b & 17 & 1 \\ c & 17 & 1 \end{vmatrix} = -70$$

$$\Rightarrow \begin{vmatrix} A+6d & 7 & 1 \\ 2A+16d+2 & 17 & 1 \\ A+16d+2 & 17 & 1 \end{vmatrix} = -70$$

$$R_3 \rightarrow R_3 - R_2, \quad R_2 \rightarrow R_2 - R_1$$

$$\Rightarrow \begin{vmatrix} A+6d & 7 & 1 \\ A+10d+2 & 10 & 0 \\ -A & 0 & 0 \end{vmatrix} = -70$$

$$\Rightarrow A = -7$$

$$a = A + 6d = 29 \Rightarrow d = 6$$

$$b = -7 + 48 + 1 = 42$$

$$c = -7 + 96 + 2 = 91$$

$$c - a - b = 91 - 29 - 42 = 20$$

$$\text{Sum} = \frac{20}{2} \left[2 \times 20 + 19 \times \frac{6}{12} \right] = 10 \left[40 + \frac{19}{2} \right] = 495$$

90. In the area enclosed by the parabolas $P_1 : 2y = 5x^2$ and $P_2 : x^2 - y + 6 = 0$ is equal to the area enclosed by P_1 and $y = ax, a > 0$, then a^3 is equal to _____.

Answer (600)

Sol. $x^2 + 6 = \frac{5}{2}x^2 \Rightarrow x = \pm 2$

Area between P_1 and P_2 [Say A_1]

$$\begin{aligned} &= \int_{-2}^2 (x^2 + 6) - \frac{5}{2}x^2 dx \\ &= 2 \int_0^2 \left(6 - \frac{3}{2}x^2 \right) dx = 2 \left[6x - \frac{x^3}{2} \right]_0^2 = 16 \end{aligned}$$

$$ax = \frac{5}{2}x^2 \Rightarrow x = 0, \frac{2a}{5}$$

Area between P_1 and $y = ax$ [Say A_2]

$$\begin{aligned} &= \int_0^{\frac{2a}{5}} ax - \frac{5}{2}x^2 dx \\ &= \frac{ax^2}{2} - \frac{5}{6}x^3 \Big|_0^{\frac{2a}{5}} = \frac{2a^3}{75} \end{aligned}$$

$$A_1 = A_2 \Rightarrow \frac{2a^3}{75} = 16$$

$$a^3 = 600$$

