29/01/2023
Evening

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Answers & Solutions

Time : 3 hrs. *for* **M.M. : 300**

JEE (Main)-2023 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. The time period of a satellite of earth is 24 hours. If the separation between the earth and the satellite is decreased to one fourth of the previous value, then its new time period will become.

(1) 6 hours
(2) 4 hours
(3) 3 hours
(4) 12 hours

Answer (3)

Sol. $\because \quad T^2 \propto R^3$

$$\therefore \quad \frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$$

$$\frac{24^2}{T_2^2} = \frac{R_1^3}{\left(\frac{R_1}{4}\right)^3}$$

$$\frac{24^2}{T_2^2} = 4^3$$

$$T_2 = \frac{24}{2^3}$$

= 3 hours

2. A force acts for 20 s on a body of mass 20 kg, starting from rest, after which the force ceases and then body describes 50 m in the next 10 s. The value of force will be:

(1) 10 N
(2) 5 N
(3) 20 N
(4) 40 N

Answer (2)

Sol. m = 20 kg

t = 20 sec.

Acceleration $= \frac{F}{20}$ m/s^2

$\therefore \quad v = u + at$

$$v = 0 + \left(\frac{F}{20}\right)(20)$$

$= F$ ms^{-1}

Now for next 10 sec.

$S = ut$

$50 = F(10)$

$\boxed{F = 5}$

3. The ratio of de-Broglie wavelength of an α particle and a proton accelerated from rest by the same potential is $\frac{1}{\sqrt{m}}$, the value of m is

(1) 16
(2) 2
(3) 8
(4) 4

Answer (3)

Sol. $$\frac{\lambda_\alpha}{\lambda_{\text{proton}}} = \frac{1}{\sqrt{2(4m)(2eV)}} \times \frac{\sqrt{2(m)(e)V}}{1}$$

$$= \frac{1}{\sqrt{8}}$$

$m = 8$

4. At 300 K, the rms speed of oxygen molecule is $\sqrt{\frac{\alpha+5}{\alpha}}$ times to that of its average speed in the gas. Then, the value of α will be

(used $\pi = \frac{22}{7}$)

(1) 28
(2) 27
(3) 32
(4) 24

Answer (1)

Sol. $$v_{\text{rms}} = \sqrt{\frac{\alpha+5}{\alpha}} v_{\text{avg}}$$

$$\sqrt{\frac{3RT}{m}} = \sqrt{\frac{\alpha+5}{5}} \sqrt{\frac{8RT}{\pi m}}$$

$$\frac{3 \times \pi}{8} = \frac{\alpha+5}{\alpha}$$

$$\frac{33}{28} = \frac{\alpha+5}{\alpha}$$

$\boxed{\alpha = 28}$

5. Heat energy of 184 kJ is given to ice of mass 600 g at –12°C. Specific heat of ice is 2222.3 J kg^{-1} °C^{-1} and latent heat of ice in 336 kJ/kg^{-1}

A. Final temperature of system will be 0°C.

B. Final temperature of the system will be greater than 0°C.

C. The final system will have a mixture of ice and water in the ratio of 5 : 1.

D. The final system will have a mixture of ice and water in the ratio of 1 : 5.

E. The final system will have water only.

Choose the correct answer from the options given below:

(1) A and C only (2) A and D only

(3) B and D only (4) A and E only

Answer (2)

Sol. Heat required to raise the temperature of ice to 0°C is

$$= \frac{60}{1000}(2222.3)(12)$$

= 16000.5 J

≈ 16 kJ

Heat required to melt ice completely

$$= \left(\frac{600}{1000}\right)(336)\ \text{kJ}$$

= 201.6 kJ

Energy left = (184 – 16) = 168 kJ

$\therefore$ Partial ice will melt

$\therefore$ $168 = \left(m_{\text{ice melted}}\right)336$

0.5 kg = ($m_{\text{ice melted}}$)

$\therefore$ $m_{\text{ice}} : m_{\text{water}} = 1 : 5$

6. With the help of potentiometer, we can determine the value of emf of a given cell. The sensitivity of the potentiometer is

(A) Directly proportional to the length of the potentiometer wire

(B) Directly proportional to the potential gradient of the wire

(C) inversely proportional to the potential gradient of the wire

(D) inversely proportional to the length of the potentiometer wire

Choose the correct option for the above statements:

(1) C only

(2) B and D only

(3) A and C only

(4) A only

Answer (3)

Sol. Sensitivity $\propto \dfrac{1}{k(\text{potential gradient})}$

$\propto$ length

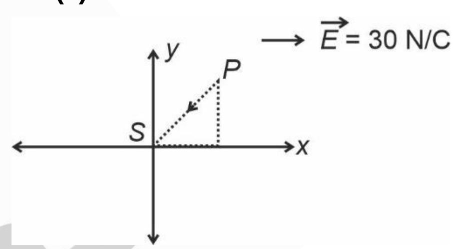
7. A point charge 2×10^{-2} C is moved from P to S in a uniform electric field of 30 NC^{-1} directed along positive x-axis. If coordinates of P and S are (1, 2, 0) m and (0, 0, 0) m respectively, the work done by electric field will be

(1) –1200 mJ (2) 600 mJ

(3) –600 mJ (4) 1200 mJ

Answer (3)

Sol.



$$w = \int F \cdot ds$$

$$\vec{F} = q\vec{E} = 2 \times 10^{-2} \times 30\hat{i} = 0.6\text{N}\hat{i}$$

$$w = \vec{F} \cdot \vec{d} = (0.6\hat{i}) \cdot (-\hat{i}, -2\hat{j})$$

= – 0.6 J

= – 600 mJ

8. A square loop of area 25 cm^2 has a resistance of 10 Ω. The loop is placed in uniform magnetic field of magnitude 40.0 T. The plane of loop is perpendicular to the magnetic field. The work done in pulling the loop out of the magnetic field slowly and uniformly in 1.0 sec, will be

(1) 2.5×10^{-3} J

(2) 1.0×10^{-4} J

(3) 5×10^{-3} J

(4) 1.0×10^{-3} J

Answer (4)

Sol. From energy conservation.

Work done to pull the loop out

= Energy lost is resistance

Emf in the loop $= \frac{d\phi}{dt} = \frac{B \times A}{t} = \frac{40 \times 25 \times 10^{-4}}{1\,s}$

$= 0.1$ V

Energy lost $= \frac{emf^2}{R} = \frac{(0.1)^2}{10} = 10^{-3}$ J

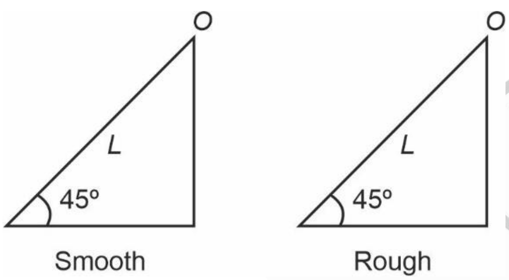
9. The time taken by an object to slide down 45° rough inclined plane is n times as it takes to slide down a perfectly smooth 45° incline plane. The coefficient of kinetic friction between the object and the incline plane is:

(1) $1-\frac{1}{n^2}$ (2) $\sqrt{1-\frac{1}{n^2}}$

(3) $\sqrt{\frac{1}{1-n^2}}$ (4) $1+\frac{1}{n^2}$

Answer (1)

Sol.



Smooth case:

$$a = g\sin 45^\circ = \frac{9}{\sqrt{2}}$$

$$t_1 = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2L}{g/\sqrt{2}}} = \sqrt{\frac{2\sqrt{2}L}{g}} \quad \text{....(1)}$$

Rough case:

$$a = g\sin 45^\circ - \mu g\cos 45^\circ$$

$$= \frac{g}{\sqrt{2}}(1-\mu)$$

$$t_2 = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2\sqrt{2}L}{g(1-\mu)}} \quad \text{....(2)}$$

From (1) to (2) and $t_1 = \frac{t_2}{n}$ we have

$$\sqrt{\frac{2\sqrt{2}L}{g}} = \frac{1}{n}\sqrt{\frac{2\sqrt{2}L}{g(1-\mu)}} \Rightarrow \boxed{\mu = 1-\frac{1}{n^2}}$$

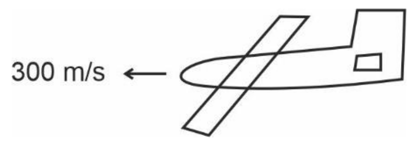
10. A fully loaded boeing aircraft has a mass of 5.4×10^5 kg. Its total wing area is 500 m². It is in level flight with a speed of 1080 km/h. If the density of air ρ is 1.2 kg m⁻³, the fractional increase in the speed of the air on the upper surface of the wing relative to the lower surface in percentage will be. (g = 10 m/s²)

(1) 16 (2) 6

(3) 10 (4) 8

Answer (3)

Sol.



Velocity of aircraft = 1080 km/h

= 300 m/s

Now, weight of aircraft $= \Delta PA$

$$\Delta P = \frac{5.4 \times 10^5 \times g}{500} = 10800 \text{ Pa}$$

From Bernoulli's principle

$$\Delta P = \frac{1}{2}\rho\left[V_{upper}^2 - V_{lower}^2\right]$$

$$10800 = \frac{1}{2} \times 1.2 \times V_{lower}^2\left[\left(\frac{V_{upper}}{V_{lower}}\right)^2 - 1\right]$$

$$\left(\frac{V_{upper}}{V_{lower}}\right)^2 = 1 + \frac{10800 \times 2}{1.2 \times (300)^2} = 1.2$$

$$\frac{V_{upper}}{V_{lower}} = 1.096$$

$\Rightarrow$ Fractional increases = 9.6%.

11. Given below are two statements:

Statement I : Electromagnetic waves are not deflected by electric and magnetic field.

Statement II : The amplitude of electric field and the magnetic field in electromagnetic waves are related to each other as $E_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}B_0$.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both statement I and statement II are false

(2) Both statement I and statement II are true

(3) Statement I is true but statement II is false

(4) Statement I is false but statement II is true

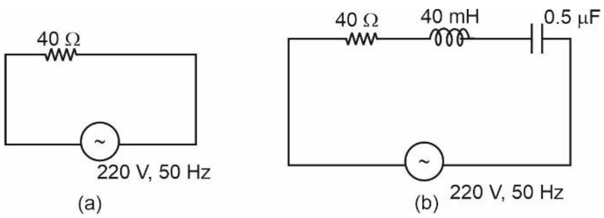
Answer (3)

Sol. Statement I is correct as photon do not carry any charge, hence cannot feel force from either fields.

Statement II is wrong as $E_0 = cB_0$

$$E_0 = \frac{B_0}{\sqrt{\mu_0 \varepsilon_0}}$$

12. For the given figures, choose the correct options:



(1) The rms current in circuit (b) can be larger than that in (a)

(2) At resonance, current in (b) is less than that in (a)

(3) The rms current in figure (a) is always equal to that in figure (b)

(4) The rms current in circuit (b) can never be larger than that in (a)

Answer (4)

Sol. For (a), $i = \frac{V}{R} = \frac{220}{40} = 5.5\text{ A}$

for (b), $X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi 50 \times 0.5 \times 10^{-6}} = \frac{10^6}{50\pi}\ \Omega$

$X_L = 2\pi fL = 2\pi \times 50 \times 50 \times 10^{-3} = 50\pi\ \Omega$

$X_C > X_L$, hence impedance is greater than 40 Ω.

$$i_{\text{rms}} = \frac{220}{Z}$$

$$\therefore\ i_{\text{rms}}\big|_b < i_{\text{rms}}\big|_a$$

13. Identify the correct statements from the following:

A. Work done by a man in lifting a bucket out of a well by means of rope tied to the bucket is negative.

B. Work done by gravitational force in lifting a bucket out of a well by a rope tied to the bucket is negative.

C. Work done by friction on a body sliding down an inclined plane is positive.

D. Word done by an applied force on a body moving on a rough horizontal plane with uniform velocity is zero.

E. Work done by the air resistance on an oscillating pendulum is negative.

Choose the correct answer from the options given below:

(1) A and C only
(2) B and E only
(3) B, D and E only
(4) B and D only

Answer (2)

Sol. B. Work done by gravitation will be negative if something is lifted upward.

D. Work done by air resistance is negative.

14. Substance A has atomic mass number 16 and half life of 1 day. Another substance B has atomic mass number 32 and half life of $\frac{1}{2}$ day. If both A and B simultaneously start undergo radio activity at the same time with initial mass 320 g each, how many total atoms of A and B combined would be left after 2 days.

(1) 6.76×10^{24}

(2) 3.38×10^{24}

(3) 6.76×10^{23}

(4) 1.69×10^{24}

Answer (2)

Sol. $n_A = 20$ moles

$n_B = 10$ moles

$N = N_0 e^{-\lambda t}$

$$N_A = (20\text{ N})e^{-\left(\frac{\ln 2}{1}\times 2\right)}$$

$$= \frac{20\text{ N}}{4} = 5\text{ N} \qquad \text{(N = Avogadro's Number)}$$

$$N_B = 10\,N\,e^{-4\ln 2}$$

$$= \left(\frac{10\text{ N}}{16}\right)$$

$$N_A + N_B = 5\text{ N} + \frac{10\text{ N}}{16} = \left(\frac{90\text{ N}}{16}\right) = 3.38 \times 10^{24}$$

15. A scientist is observing a bacteria through a compound microscope. For better analysis and to improve its resolving power he should. (Select the best option)

(1) Decrease the diameter of the objective lens

(2) Decrease the focal length of the eye piece

(3) Increase the refractive index of the medium between the object and objective lens

(4) Increase the wavelength of the light

Answer (3)

Sol. Resolving power of microscope = $\left(\dfrac{2n\sin\theta}{\lambda}\right)$

$n\sin\theta$ = Numerical aperture

n is the refractive index of medium.

16. The modulation index for an A.M. wave having maximum and minimum peak-to-peak voltages of 14 mV and 6 mV respectively is

(1) 1.4

(2) 0.6

(3) 0.4

(4) 0.2

Answer (3)

Sol. $A_c + A_m = 14$ mV

$A_c - A_m = 6$ mV

$\Rightarrow 2A_c = 20$ mV

$A_c = 10$ mV

$A_m = 4$ mV

Modulation Index = $\dfrac{A_m}{A_c} = \left(\dfrac{4}{10}\right) = 0.4$

17. The equation of a circle is given by $x^2 + y^2 = a^2$, where a is the radius. If the equation is modified to change the origin other than (0, 0), then find out the correct dimensions of A and B in a new equation $(x - At)^2 + \left(y - \dfrac{t}{B}\right)^2 = a^2$. The dimension of t is given as $[T^{-1}]$.

(1) A = $[LT]$, B = $[L^{-1}T^{-1}]$

(2) A = $[L^{-1}T]$, B = $[LT^{-1}]$

(3) A = $[L^{-1}T^{-1}]$, B = $[LT]$

(4) A = $[L^{-1}T^{-1}]$, B = $[LT^{-1}]$

Answer (1)

Sol. $[At] = [x] = [L]$

$$[A] = \frac{[x]}{[t]} = [LT]$$

$$\left[\frac{t}{B}\right] = [y] = [L]$$

$$\Rightarrow [B] = \left[\frac{t}{L}\right] = \left[L^{-1}T^{-1}\right]$$

18. An object moves at a constant speed along a circular path in a horizontal plane with centre at the origin. When the object is at $x = +2$ m, its velocity is $-4\hat{j}$ m/s. The object's velocity (v) and acceleration (a) at $x = -2$ m will be

(1) $v = 4\hat{i}$ m/s, $a = 8\hat{j}$ m/s^2

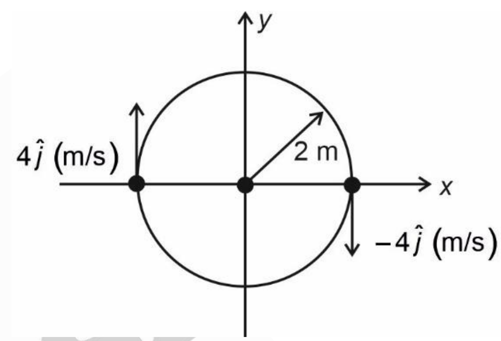
(2) $v = 4\hat{j}$ m/s, $a = 8\hat{i}$ m/s^2

(3) $v = -4\hat{i}$ m/s, $a = -8\hat{j}$ m/s^2

(4) $v = -4\hat{j}$ m/s, $a = 8\hat{i}$ m/s^2

Answer (2)

Sol.

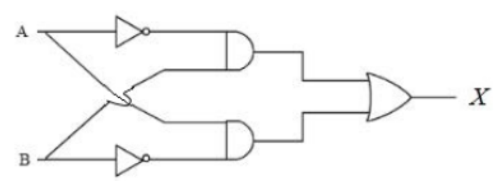


$\vec{v} = 4\hat{j}$ (m/s)

$a = \dfrac{v^2}{R} = \dfrac{16}{2} = 8$ m/s^2

$\vec{a} = 8\left(\text{m/s}^2\right)\left(\hat{i}\right)$

19. For the given logic gates combination, the correct truth table will be



(1)

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

(2)

A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

(3)

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

(4)

A	B	X
0	0	1
0	1	0
1	0	1
1	1	0

Answer (1)

Sol. As per the circuit,

$X = A'B + AB'$

If: $A = 0, B = 0 \Rightarrow X = 0$

$A = 0, B = 1 \Rightarrow X = 1$

$A = 1, B = 0 \Rightarrow X = 1$

$A = 1, B = 1 \Rightarrow X = 0$

20. The electric current in a circular coil of four turns produces a magnetic induction 32 T at its centre. The coil is unwound and is rewound into a circular coil of single turn, the magnetic induction at the centre of the coil by the same current will be

(1) 4 T (2) 16 T

(3) 8 T (4) 2 T

Answer (4)

Sol. By given information

$$32 = 4 \times \frac{\mu_0 i}{2r} \quad \ldots(i)$$

Also, $r' = 4r$ ...(ii)

$$\text{and } B' = 1 \times \frac{\mu_0 i}{2r'} \quad \ldots(iii)$$

$$\Rightarrow B' = \frac{\mu_0 i}{2(4r)} = \frac{\mu_0 i}{8r} = \frac{1}{8} \times 16 = 2\text{ T}$$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. When two resistances R_1 and R_1 connected in series and introduced into the left gap of a meter bridge and a resistance of 10 Ω is introduced into the right gap, a null point is found at 60 cm from left side. When R_1 and R_2 are connected in parallel and introduced into the left gap, a resistance of 3 Ω is introduced into the right gap to get null point at 40 cm from left end. The product of R_1R_2 is _______ Ω^2

Answer (30)

Sol. As per given information

$$\frac{R_1+R_2}{10} = \frac{0.6}{0.4} \quad \ldots(1)$$

$$\& \ \frac{\frac{R_1R_2}{R_1+R_2}}{3} = \frac{0.4}{0.6} \quad \ldots(2)$$

$$\Rightarrow \left.\begin{matrix} R_1+R_2 = 15 \\ \& \ R_1R_2 = 30 \end{matrix}\right] \Rightarrow R_1R_2 = 30\ \Omega^2$$

22. For a charged spherical ball, electrostatic potential inside the ball varies with r as $V = 2ar^2 + b$.

Here a and b are constant and r is the distance from the center. The volume charge density inside the ball is $-\lambda a\varepsilon$. The value of λ is _____.

ε = permittivity of the medium

Answer (12)

Sol. $V = 2ar^2 + b$

$$\Rightarrow E = -\frac{dV}{dr} = -4ar$$

$$\Rightarrow \frac{1}{4\pi\varepsilon}\frac{Q}{r^2} = -4ar$$

$$\Rightarrow \frac{Q}{\frac{4}{3}\pi r^3} = 3 \times \varepsilon \times (-4a) = -12a\varepsilon$$

$\Rightarrow \lambda = 12$

23. A car is moving on a circular path of radius 600 m such that the magnitudes of the tangential acceleration and centripetal acceleration are equal. The time taken by the car to complete first quarter of revolution, if it is moving with an initial speed of 54 km/hr is $t(1 - e^{-\pi/2})$ s. The value of t is_____.

Answer (40)

Sol. $$\frac{dv}{dt} = \frac{v^2}{R} \Rightarrow \frac{v^2}{R} = v\frac{dv}{ds}$$

$$\Rightarrow \frac{dv}{v} = \frac{ds}{R} \Rightarrow \ln v\Big|_{15}^{v} = \frac{s}{R}$$

$$\Rightarrow v = 15e^{\Delta/R} = \frac{ds}{dt} \Rightarrow dt = \frac{1}{15}e^{-\Delta/R}ds$$

$$\Delta t = \frac{R}{15}[1 - e^{-\Delta/R}]$$

$$= 40[1 - e^{-\pi/2}] \text{ seconds}$$

$\Rightarrow$ $t = 40$

24. Unpolarised light is incident on the boundary between two dielectric media, whose dielectric constants are 2.8 (medium – 1) and 6.8 (medium – 2), respectively . To satisfy the condition, so that the reflected and refracted rays are perpendicular to each other, the angle of incidence should be $\tan^{-1}\left(1+\frac{10}{\theta}\right)^{\frac{1}{2}}$ the value of θ is ____.

(Given for dielectric media, $\mu_r = 1$)

Answer (7)

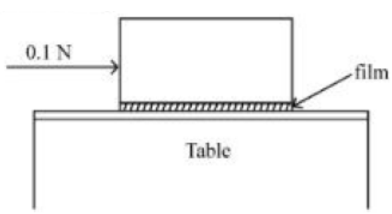
Sol We know that

$$\tan\theta_0 = \frac{\mu_2}{\mu_1}$$

$$\tan\theta_0 = \sqrt{\frac{6.8}{2.8}} = \sqrt{\frac{17}{7}}$$

$$\theta_0 = \tan^{-1}\sqrt{1+\frac{10}{7}} \Rightarrow \theta = 7$$

25. A metal block of base area 0.20 m^2 is placed on a table as shown in figure. A liquid film of thickness 0.25 mm is inserted between the block and the table. The block is pushed by a horizontal force of 0.1 N and moves with a constant speed. If the viscosity of the liquid is 5.0×10^{-3} Pl, the speed of block is ______ $\times 10^{-3}$ m/s



Answer (25)

Sol. As the block moves with constant speed, the horizontal force is balanced by viscous force thus

$$F = \eta A \frac{\Delta v}{\Delta z}$$

$$0.1 = 5\times10^{-3} \times 0.2 \times \frac{v}{.25\times10^{-3}}$$

$\Rightarrow$ $v = 25 \times 10^{-3}$ m/s

26. A particle of mass 250 g executes a simple harmonic motion under a periodic force F = (– 25 x) N. The particle attains a maximum speed of 4 m/s during its oscillation. The amplitude of the motion is _______ cm.

Answer (40)

Sol. $F = -25x$

$$.250\frac{d^2x}{dt^2} = -25x$$

$$\frac{d^2x}{dt^2} = -100x$$

$\Rightarrow \omega = 10$ rad/sec

& $\omega A = v_{max}$

10 A = 4

$\Rightarrow$ A = 0.4 m

= 40 cm

27. In an experiment of measuring the refractive index of a glass slab using travelling microscope in physics lab, a student measures real thickness of the glass slab as 5.25 mm and apparent thickness of the glass slab as 5.00 mm. Travelling microscope has 20 divisions in one cm on main scale and 50 divisions on vernier scale is equal to 49 divisions on main scale. The estimated uncertainty in the measurement of refractive index of the slab is $\frac{x}{10}\times10^{-3}$, where x is __________.

Answer (41)

Sol. $\mu = \dfrac{\text{real depth } (l_1)}{\text{apparent depth}(l_2)}$

$$= \frac{5.25}{5} = 1.05$$

$$\frac{d\mu}{\mu} = \frac{dl_1}{l_1} + \frac{dl_2}{l_2}$$

$$d\mu = \left(\frac{dl_1}{l_1} + \frac{dl_2}{l_2}\right)\mu$$

$$= \left(\frac{0.01}{5.25} + \frac{0.01}{5.00}\right) \times 1.05$$

$$= \frac{41}{10} \times 10^{-3}$$

so $x = 41$

28. A null point is found at 200 cm in potentiometer when cell in secondary circuit is shunted by 5Ω. When a resistance of 15Ω is used for shunting, null point moves to 300 cm. The internal resistance of the cell is ________ Ω.

Answer (05)

Sol. Let the emf is E and internal resistance is r of this secondary cell so

$$\frac{RE}{r+R} \propto l$$

so $\frac{R_1E}{r+R_1} \propto l_1$

& $\frac{R_2E}{r+R_2} \propto l_2$

$$\Rightarrow \frac{R_1(r+R_2)}{R_2(r+R_1)} = \frac{l_1}{l_2}$$

OR $\frac{5(r+15)}{15(r+5)} = \frac{200}{300}$

$\Rightarrow r = 5\ \Omega$

29. An inductor of inductance 2 μH is connected in series with a resistance, a variable capacitor and an AC source of frequency 7 kHz. The value of capacitance for which maximum current is drawn into the circuit is $\frac{1}{x}F$, where the value of x is _______.

(Take $\pi = \frac{22}{7}$)

Answer (3872)

Sol. Current drawn is maximum when circuit is in resonance.

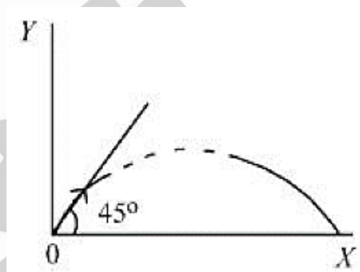
$$\omega = \frac{1}{\sqrt{LC}}$$

$$2\pi(7000) = \frac{1}{\sqrt{2\times10^{-6}C}}$$

$$\Rightarrow C = \frac{1}{3872}F$$

30. A particle of mass 100 g is projected at time t = 0 with a speed 20 ms^{-1} at an angle 45° to the horizontal as given in the figure. The magnitude of the angular momentum of the particle about the starting point at time t = 2 s is found to be $\sqrt{K}$ kg m^2/s. The value of K is ________.

(Take g = 10 ms^{-2})



Answer (800)

Sol. Horizontal displacement $x = v\cos\theta t$

$$= 10\sqrt{2}t$$

So torque of weight about point of projection is

$$\tau = mgx\cdot(-\hat{k})$$

$$\frac{d\vec{L}}{dt} = mgx(-\hat{k})$$

$$\int_0^{\vec{L}} d\vec{L} = 0.1\times10\times10\sqrt{2}\int_0^2 t\,dt(-\hat{k})$$

$$\vec{L} = -20\sqrt{2}\hat{k}$$

$$|\vec{L}| = 20\sqrt{2} = \sqrt{800}\ \text{kg m}^2/\text{s}$$

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

31. Match **List I** with **List II**.

	List I		List II
A.	Elastomeric polymer	I.	Urea formaldehyde resin
B.	Fibre polymer	II.	Polystyrene
C.	Thermosetting polymer	III.	Polyester
D.	Thermoplastic polymer	IV.	Neoprene

Choose the correct answer from the options given below?

(1) A-IV, B-III, C-I, D-II

(2) A-II, B-III, C-I, D-IV

(3) A-IV, B-I, C-III, D-II

(4) A-II, B-I, C-IV, D-III

Answer (1)

Sol. A. Elastomeric fibre (IV) Neoprene

B. Fibre polymer (III) Polyester

C. Thermosetting polymer (I) Urea Formaldehyde Resin

D. Thermoplastic polymer (II) Polystyrene

32. A solution of CrO_5 in amyl alcohol has a ______ colour.

(1) Blue (2) Yellow

(3) Green (4) Orange-Red

Answer (1)

Sol. CrO_5 is blue in colour in amyl alcohol.

33. The concentration of dissolved oxygen in water for growth of fish should be more than X ppm and Biochemical Oxygen Demand in clean water should be less than Y ppm. X and Y in ppm are, respectively.

(1) X 4, Y 15 (2) X 6, Y 12

(3) X 4, Y 8 (4) X 6, Y 5

Answer (4)

Sol. For high growth, dissolved oxygen should be less than 6 ppm and clean water has dissolved oxygen less than 5 ppm.

34. Correct order of spin only magnetic moment of the following complex ions is

(Given At.no. Fe:26, Co:27)

(1) $[Co(C_2O_4)_3]^{3-} > [CoF_6]^{3-} > [FeF_6]^{3-}$

(2) $[FeF_6]^{3-} > [Co(C_2O_4)_3]^{3-} > [CoF_6]^{3-}$

(3) $[FeF_6]^{3-} > [CoF_6]^{3-} > [Co(C_2O_4)_3]^{3-}$

(4) $[CoF_6]^{3-} > [FeF_6]^{3-} > [Co(C_2O_4)_3]^{3-}$

Answer (3)

Sol. $[FeF_6]^{3-} \longrightarrow$ 5 unpaired electrons

$[Co(C_2O_4)_3]^{3-} \longrightarrow$ 0 unpaired electron

$[CoF_6]^{3-} \longrightarrow$ 4 unpaired electrons

So, correct answer is : (3)

35. According to MO theory the bond orders for O_2^{2-}, CO and NO^+ respectively, are

(1) 1, 2 and 3 (2) 1, 3 and 2

(3) 2, 3 and 3 (4) 1, 3 and 3

Answer (4)

Sol.

Species	B.O.
O_2^{-2}	1
CO	3
$NO^{\oplus}$	3

36. Match **List-I** with **List-II.**

	List-I		**List-II**
A.	van't Hoff factor, i	I.	Cryoscopic constant
B.	k_f	II.	Isotonic solutions
C.	Solutions with same osmotic pressure	III.	$\frac{\text{Normal molar mass}}{\text{Abnormal molar mass}}$
D.	Azeotropes	IV.	Solutions with same composition of vapour above it

Choose the correct answer from the options given below.

(1) A-III, B-I, C-II, D-IV

(2) A-III, B-I, C-IV, D-II

(3) A-III, B-II, C-I, D-IV

(4) A-I, B-III, C-II, D-IV

Answer (1)

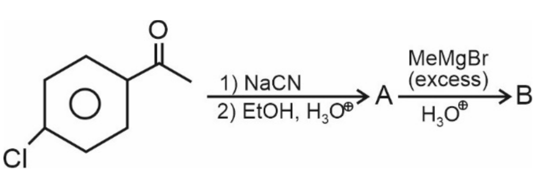
Sol. A. van't Hoff factor — III. $\frac{\text{Normal molar mass}}{\text{Abnormal molar mass}}$

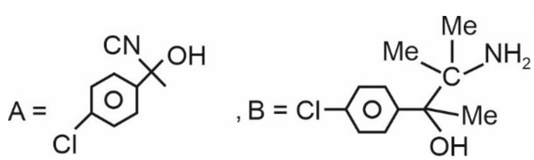
B. k_f — I. Cryoscopic constant

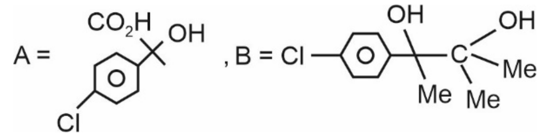
C. Solutions with same osmotic pressure — II. Isotonic solutions

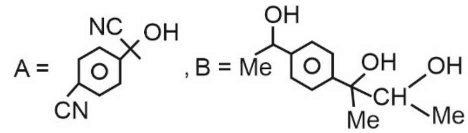
D. Azeotropes — IV. Solutions with same composition of vapour above it

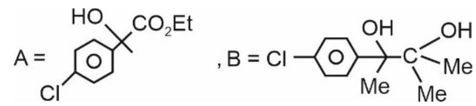
37. Find out the major products from the following reaction sequence.



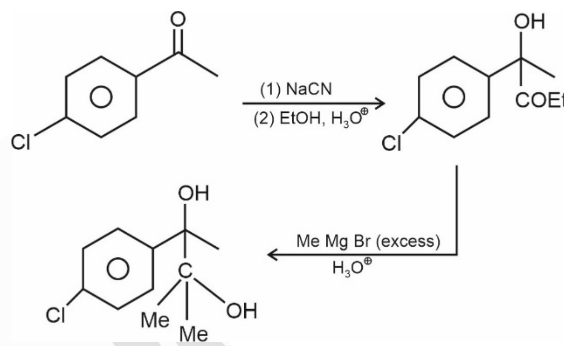
(1) 

(2) 

(3) 

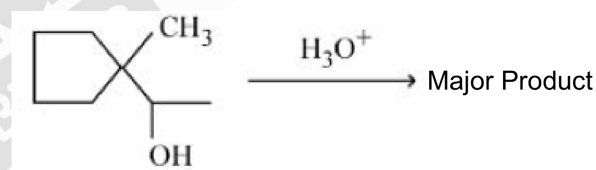
(4) 

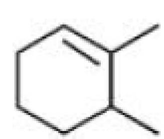
Answer (4)

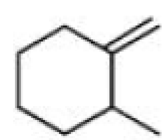
Sol. 

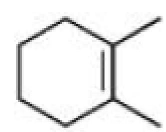
Correct answer is (4)

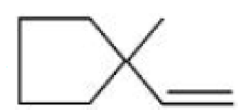
38. Find out the major product for the following reaction.



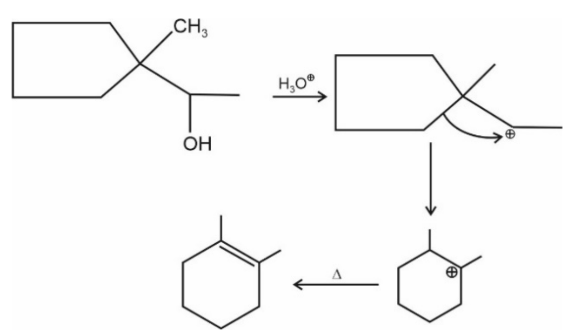
(1) 

(2) 

(3) 

(4) 

Answer (3)

Sol. 

39. The one giving maximum number of isomeric alkenes on dehydrohalogenation reaction is (excluding rearrangement)

(1) 1-Bromo-2-methylbutane

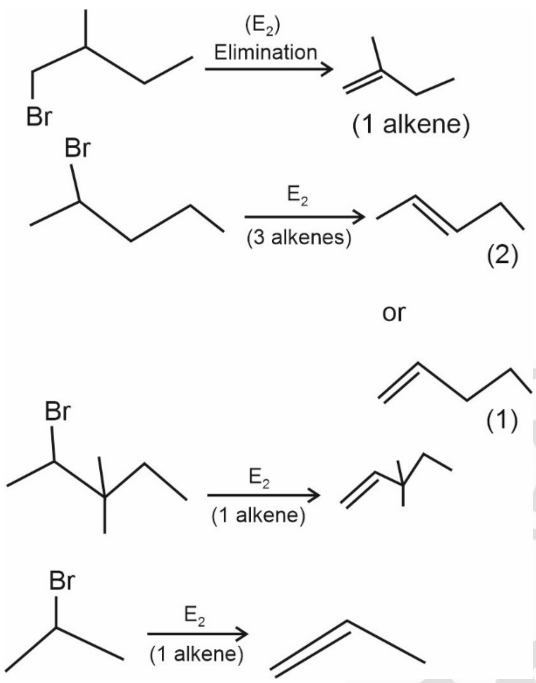
(2) 2-Bromopentane

(3) 2-Bromo-3, 3-dimethylpentane

(4) 2-Bromopropane

Answer (2)

Sol.



40. Given below are two statements:

Statement I : The decrease in first ionization enthalpy from B to Al is much larger than that from Al to Ga.

Statement-II : The d orbitals Ga are in completely filled.

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) Statement I is correct but statement II is incorrect

(2) Statement I is incorrect but statement II is correct

(3) Both the statements I and II are correct

(4) Both the statements I and II are incorrect

Answer (3)

Sol. Statement I : It is correct as decrease in first ionisation enthalpy is much larger from B to Al.

Statement II : It is also correct

41. An indicator 'X' is used for studying the effect of variation in concentration of iodide on the rate of reaction of iodide ion with H_2O_2 at room temp. The indicator 'X' forms blue colored complex with compound 'A' present in the solution. The indicator 'X' and compound 'A' respectively are

(1) Starch and H_2O_2

(2) Starch and iodine

(3) Methyl orange and H_2O_2

(4) Methyl orange and iodine

Answer (2)

Sol. Starch forms blue coloured complex with iodine.

42. Given below are two statements:

Statement I : Nickel is being used as the catalyst for producing syn gas and edible fats.

Statement II : Silicon forms both electron rich and electron deficient hydrides.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both the statements I and II are incorrect

(2) Statement I is incorrect but statement II is correct

(3) Both the statements I and II are correct

(4) Statement I is correct but statement II is incorrect

Answer (4)

Sol. Statement I is correct.

Statement II is incorrect as Si forms electron precise hydride.

43. A doctor prescribed the drug Equanil to a patient. The patient was likely to have symptoms of which disease?

(1) Hyperacidity

(2) Depression and hypertension

(3) Stomach ulcers

(4) Anxiety and stress

Answer (2)

Sol. Equanil is for depression and hypertension.

44. When a hydrocarbon A undergoes combustion in the presence of air, it requires 9.5 equivalents of oxygen and produces 3 equivalents of water. What is the molecular formula of A?

(1) C_9H_9 (2) C_8H_6

(3) C_6H_6 (4) C_9H_6

Answer (2)

Sol. $C_xH_y + \left(x+\frac{y}{4}\right)O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O$

$\frac{y}{2} = 3$

$y = 6$

$x + \frac{y}{4} = \frac{19}{2}$

$x = \frac{19}{2} - \frac{3}{2} = 8$

So, formula is C_8H_6.

45. Match **List I** and **List II**.

	List I		List II
A.	Osmosis	I.	Solvent molecules pass through semi permeable membrane towards solvent side.
B.	Reverse osmosis	II.	Movement of charged colloidal particles under the influence of applied electric potential towards oppositely charged electrodes.
C.	Electro osmosis	III.	Solvent molecules pass through semi permeable membrane towards solution side.
D.	Electrophoresis	IV.	Dispersion medium moves in an electric field.

Choose the correct answer from the options given below :

(1) A-III, B-I, C-IV, D-II (2) A-I, B-III, C-IV, D-II

(3) A-III, B-I, C-II, D-IV (4) A-I, B-III, C-II, D-IV

Answer (1)

Sol. Correct match is:

A.	Osmosis	III.	Solvent molecules pass through semi permeable membrane towards solution side.
B.	Reverse osmosis	I.	Solvent molecules pass to solvent side through SPM.
C.	Electro osmosis	IV.	Dispersion medium moves in an electric field.
D.	Electrophoresis	II.	Movement of colloidal particle towards oppositely charged electrodes.

46. Which of the following relations are correct?

(A) $\Delta U = q + p\Delta V$ (B) $\Delta G = \Delta H - T\Delta S$

(C) $\Delta S = \frac{q_{rev}}{T}$ (D) $\Delta H = \Delta U - \Delta nRT$

Choose the most appropriate answer from the options given below:

(1) B and C Only

(2) C and D Only

(3) A and B Only

(4) B and D Only

Answer (1)

Sol. (A) $\Delta U = q - p\Delta V$

(B) $\Delta G = \Delta H - T\Delta S$

(C) $\Delta S = \frac{q_{rev}}{T}$

(D) $\Delta H = \Delta U + (\Delta nRT)$

Hence, (B) and (C) relations are correct.

47. The major component of which of the following ore is sulphide based mineral?

(1) Sphalerite (2) Calamine

(3) Malachite (4) Siderite

Answer (1)

Sol. Sphalerite $\rightarrow$ ZnS

Calamine $\rightarrow ZnCO_3$

Malachite $\rightarrow CuCO_3 \cdot Cu(H)_2$

Siderite $\rightarrow FeCO_3$

48. Reaction of propanamide with Br_2/KOH(aq) produces :

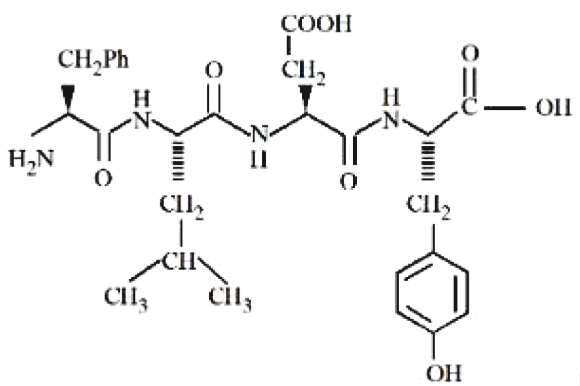
(1) Propanenitrile (2) Ethylnitrile

(3) Ethylamine (4) Propylamine

Answer (3)

Sol. Propanamide $\xrightarrow{Br_2/KOH}$ Ethylamine

49. Following tetrapeptide can be represented as



(F, L, D, Y, I, Q, P are one letter codes for amino acids)

(1) YQLF (2) FLDY

(3) PLDY (4) FIQY

Answer (2)

Sol. The tetrapeptide code is

FLDY

F $\rightarrow$ Phenyl alanine

L $\rightarrow$ Leucine

D $\rightarrow$ Aspartic acid

Y $\rightarrow$ Tyrosine

50. The set of correct statements is:

(i) Manganese exhibits +7 oxidation state in its oxide.

(ii) Ruthenium and Osmium exhibit +8 oxidation in their oxides.

(iii) Sc shows +4 oxidation state which is oxidizing in nature.

(iv) Cr shows oxidising nature in +6 oxidation state.

(1) (ii) and (iii) (2) (i), (ii) and (iv)

(3) (ii), (iii) and (iv) (4) (i) and (iii)

Answer (2)

Sol. (i) $Mn_2O_7 \rightarrow$ Mn in (+7) oxidation state

(ii) It is also correct

(iii) Sc only shows +3 oxidation state

(iv) Cr^{+6} is oxidising in nature

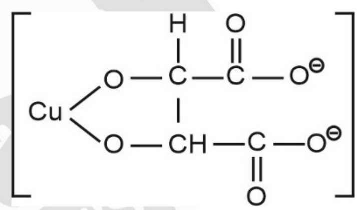
SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse andw the on-screen virtual numeric keypad in the place designated to enter the answer.

51. The denticity of the ligand present in the Fehling's reagent is ______.

Answer (4)

Sol. The denticity is 2 in Fehling' reagent.



However, the overall maximum denticity can be 4 with other metal ion.

52. On heating, $LiNO_3$ gives how many compounds among the following? _______

Li_2O, N_2, O_2, $LiNO_2$, NO_2

Answer (3)

Sol. $LiNO_3 \xrightarrow{\Delta} Li_2O + NO_2 + O_2$

53. The equilibrium constant for the reaction

$Zn(s) + Sn^{2+}(aq) \rightleftharpoons Zn^{2+}(aq) + Sn(s)$ is 1×10^{20} at 298 K. The magnitude of standard electrode potential of Sn/Sn^{2+} if $E^{o}_{Zn^{2+}/Zn} = -0.76$ V is _______ $\times 10^{-2}$ V.

(Nearest integer).

Given : $\dfrac{2.303\,RT}{F} = 0.059$ V

Answer (17)

Sol. $E^{\circ}_{cell} = \dfrac{2.303\,RT}{2\,F}\log k$

$$E^\circ_{cell} = \frac{0.059}{2}\log(10^{20})$$

$$E^\circ_{Zn^{2+}/Zn} + 0.76 = 0.59$$

$$E^\circ_{Zn^{2+}/Zn} = 0.59 - 0.76$$

$$E^\circ_{Zn/Zn^{2+}} = 0.17\text{ V}$$

54. A metal M forms hexagonal close-packed structure. The total number of voids in 0.02 mol of it is _______ × 10^{21} (Nearest integer).

(Given N_A = 6.02 × 10^{23})

Answer (36)

Sol. Number of voids = 0.02 × 3 × 6.02 × 10^{23}

≃ 36 × 10^{21}

55. When 0.01 mol of an organic compound containing 60% carbon was burnt completely, 4.4 g of CO_2 was produced. The molar mass of compound is ________ g mol^{-1} (Nearest integer).

Answer (200)

Sol. Number of moles of CO_2 = $\frac{4.4}{44} = 0.1$

∴ Number of moles of C in 1 mole of compound = 10

∴ 120 = $\frac{60}{100} \times (x)$ [where x is molar mass of OC]

Molar mass = 200 g mol^{-1}

56. Total number of acidic oxides among

N_2O_3, NO_2, N_2O, Cl_2O_7, SO_2, CO, CaO, Na_2O and NO is _______.

Answer (4)

Sol. N_2O_3, NO_2, Cl_2O_7, SO_2 are acidic in nature

57. For conversion of compounds A $\to$ B, the rate constant of the reaction was found to be 4.6 × 10–5 L mol^{-1} s^{-1}. The order of the reaction is_________.

Answer (2)

Sol. As unit is L mol^{-1} s^{-1}, order of the reaction is 2.

58. Assume that the radius of the first Bohr orbit of hydrogen atom is 0.6 Å. The radius of the third Bohr orbit of He^+ is ________ picometer. (Nearest Integer)

Answer (270)

Sol. Radius of 3rd Bohr orbit of He^{+3}

$= 0.6 \times \frac{(3)^2}{2}$

= 0.3 × 9

= 2.7 Å

= 270 × 10^{-12} pm

59. At 298 K

$$N_2(g) + 3H_2(g) \rightleftharpoons 2NH3(g), K_1 = 4 \times 10^5$$

$$N_2(g) + O_2(g) \rightleftharpoons 2NO(g), K_2 = 1.6 \times 10^{12}$$

$$H_2(g) + \frac{1}{2}O_2(g) \rightleftharpoons H_2O(g), K_3 = 1.0 \times 10^{-13}$$

Based on above equilibria, the equilibrium constant of the reaction,

$$2NH_3(g) + \frac{5}{2}O_2(g) \rightleftharpoons 2NO(g) + 3H_2O(g)$$

is________ × 10^{-33} (Nearest integer).

Answer (4)

Sol. $2NH_3(g)\frac{5}{2}O_2(g) \rightleftharpoons 2NO(g) + 3H_2O(g)$

Clearly, $K_{eq} = \frac{1}{k_1} \times k_2 \times k_3^3$

$= \frac{1.6 \times 10^{12} \times 10^{-39}}{4 \times 10^5}$

= 0.4 × 10^{-32}

= 4 × 10^{-33}

60. The volume of HCl, containing 73 g L^{-1}, required to completely neutralise NaOH obtained by reacting 0.69 g of metallic sodium with water, is ________ mL. (Nearest Integer) (Given: molar Masses of Na, Cl, O, H, are 23, 35.5, 16 and 1 g mol^{-1} respectively)

Answer (15)

Sol. $\underset{0.69\,g}{Na} + H_2O \to \underset{0.03\,moles}{NaOH} + \frac{1}{2}H_2$

= 0.03 moles

∴ 0.03 = 2 × V

$V = \frac{0.03}{2}L$

= 15 mL

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

61. The area of the region

$$A=\left\{(x,\ y):|\cos x-\sin x|\le y\le \sin x,\ 0\le x\le \frac{\pi}{2}\right\}$$

is

(1) $\sqrt{5}-2\sqrt{2}+1$

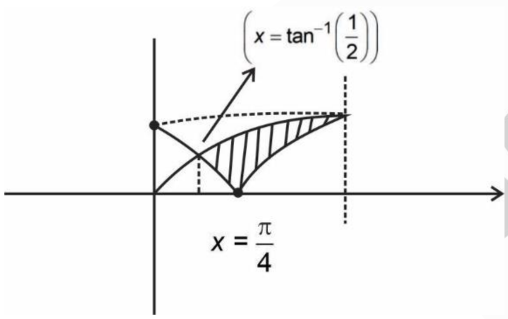
(2) $\frac{3}{\sqrt{5}}-\frac{3}{\sqrt{2}}+1$

(3) $\sqrt{5}+2\sqrt{2}-4.5$

(4) $1-\frac{3}{\sqrt{2}}+\frac{4}{\sqrt{5}}$

Answer (1)

Sol.



$$\text{Area}=\int_{\tan^{-1}\frac{1}{2}}^{\frac{\pi}{4}}(\sin x-(\cos x-\sin x))dx+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(\sin x-(\sin x-\cos x))dx$$

$$=\int_{\tan^{-1}\frac{1}{2}}^{\frac{\pi}{4}}(2\sin x-\cos x)dx+(\sin x)_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$=2\cos x-\sin x\Big|_{\tan^{-1}\frac{1}{2}}^{\frac{\pi}{4}}+\left(1-\frac{1}{\sqrt{2}}\right)$$

$$=\frac{3}{\sqrt{2}}+\frac{1}{\sqrt{5}}+\frac{1}{\sqrt{5}}+1-\frac{1}{\sqrt{2}}$$

$$=\sqrt{5}-2\sqrt{2}+1$$

62. Let K be the sum of the coefficients of the odd powers of x in the expansion of $(1 + x)^{99}$. Let a be the middle term in the expansion of $\left(2+\frac{1}{\sqrt{2}}\right)^{200}$. If $\frac{{}^{200}C_{99}K}{a}=\frac{2^l m}{n}$, where m and n are odd numbers, then the ordered pair (l, n) is equal to

(1) (51, 101)
(2) (51, 99)
(3) (50, 101)
(4) (50, 51)

Answer (3)

Sol. $K = 2^{98}$

$a={}^{200}C_{100}\ 2^{50}$

$$\therefore\ \frac{{}^{200}C_{99}\cdot 2^{98}}{{}^{200}C_{100}\cdot 2^{50}}=\frac{2^l m}{n}$$

$$\Rightarrow\ \frac{100}{101}\cdot 2^{48}=\frac{2^l m}{n}$$

$$\Rightarrow\ \frac{25}{101}\cdot 2^{50}=\frac{2^l m}{n}$$

$\therefore\ l = 50,\ m = 25,\ n = 101$

63. If the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z+3}{1}$ and $\frac{x-a}{2}=\frac{y+2}{3}=\frac{z-3}{1}$ intersect at the point P, then the distance of the point P from the plane $z = a$ is

(1) 28
(2) 16
(3) 10
(4) 22

Answer (1)

Sol. $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z+3}{1}=\lambda$ (say)

& $\frac{x-a}{2}=\frac{y+2}{3}=\frac{z-3}{1}=\mu$ (say)

$\therefore\ \lambda + 1 = 2\mu + a$ ...(i)

$2\lambda + 2 = 3\mu + 2$ ...(ii)

$\lambda - 3 = \mu + 3$ ...(iii)

By (i) & (ii)

$\Rightarrow\ 3\mu - 2 = 4\mu + 2a + 2$

$\Rightarrow$ $\mu = -2(1 + a)$ & $\lambda = 5 - 3a$

Put λ & μ in (iii) we get

$a = -9$

$\mu = 16$

$\lambda = 22$

$\therefore$ Point of intersection $\equiv (23, 46, 19)$

Distance from $z = -9$ is 28

64. If the tangent at a point P on the parabola $y^2 = 3x$ is parallel to the line $x + 2y = 1$ and the tangents at the points Q and R on the ellipse $\frac{x^2}{4}+\frac{y^2}{1}=1$ are perpendicular to the line $x - y = 2$, then the area of the triangle PQR is

(1) $5\sqrt{3}$ (2) $3\sqrt{5}$

(3) $\frac{9}{\sqrt{5}}$ (4) $\frac{3}{2}\sqrt{5}$

Answer (2)

Sol. $P \equiv \left(\frac{A}{m^2}, \frac{2A}{m}\right)$ where $\left(A = \frac{3}{4}, m = \frac{-1}{2}\right)$

& $Q, R = \left(\mp\frac{a^2m_1}{a^2m_1^2+b^2}, \frac{\mp\cdot b^2}{\sqrt{a^2m_1^2+b^2}}\right)$

Where $a^2 = 4$, $b^2 = 1$ and $m_1 = 1$

$\therefore$ $P \equiv (3, -3)$

$Q \equiv \left(\frac{-4}{\sqrt{5}}, \frac{-1}{\sqrt{5}}\right)$ & $R\left(\frac{4}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 3 & -3 & 1 \\ \frac{-4}{\sqrt{5}} & \frac{-1}{\sqrt{5}} & 1 \\ \frac{4}{\sqrt{5}} & \frac{1}{\sqrt{5}} & 1 \end{vmatrix} = \frac{1}{10}\begin{vmatrix} 3 & -3 & 1 \\ -4 & -1 & \sqrt{5} \\ 0 & 0 & 2\sqrt{5} \end{vmatrix}$$

$$= \frac{2\sqrt{5}}{10}(-15) = 3\sqrt{5}$$

65. The statement $B \Rightarrow ((\sim A) \vee B)$ is equivalent to

(1) $A \Rightarrow (A \Leftrightarrow B)$

(2) $B \Rightarrow ((\sim A) \Rightarrow B)$

(3) $B \Rightarrow (A \Rightarrow B)$

(4) $A \Rightarrow ((\sim A) \Rightarrow B)$

Answer (2, 3, 4)

Sol. $B \Rightarrow ((\sim A) \vee B) \equiv \sim B \vee (\sim A \vee B)$

$\equiv (\sim B \vee B) \vee (\sim A) \equiv T$

Option (2) $B \Rightarrow ((\sim A) \Rightarrow B) \equiv (\sim B) \vee (\sim A \Rightarrow B)$

$\equiv (\sim B) \vee (A \vee B) \equiv T$

Option (3) $B \Rightarrow (A \Rightarrow B) \equiv B \Rightarrow ((\sim A) \vee B)$

(same as given)

Option (4) $A \Rightarrow ((\sim A) \Rightarrow B) \equiv (\sim A) \vee (A \vee B) \equiv T$

66. The set of all value of $t \in R$, for which the matrix

$$\begin{bmatrix} e^t & e^{-t}(\sin t - 2\cos t) & e^{-1}(-2\sin t - \cos t) \\ e^t & e^{-t}(2\sin t + \cos t) & e^{-1}(\sin t - 2\cos t) \\ e^t & e^{-1}\cos t & e^{-1}\sin t \end{bmatrix}$$ is

invertible, is

(1) $\left\{(2k+1)\frac{\pi}{2}, k \in Z\right\}$ (2) R

(3) $\left\{k\pi + \frac{\pi}{4}, k \in Z\right\}$ (4) $\{k\pi, k \in Z\}$

Answer (2)

Sol. $$\begin{vmatrix} 0 & e^{-t}(-\sin t - 3\cos t) & e^{-t}(-3\sin t + \cos t) \\ 8 & e^{-t}(2\sin t) & e^{-t}(-2\cos t) \\ e^t & e^{-t}\cos t & e^{-t}\sin t \end{vmatrix} = 0$$

(If matrix is non invertible)

$-2\cos t(-\sin t - 3\cos t) - 2\sin t(\cos t - 3\sin t) = 0$

$\Rightarrow$ $6\cos^2 t + 6\sin^2 t = 0$

$t \in 0$

67. Let $S = \{w_1, w_2, ...\}$ be the sample space associated to a random experiment. Let $P(w_n) = \frac{P(w_{n-1})}{2}$, $n \geq 2$. Let $A = \{2k + 3l : k, l \in N\}$ and $B = \{w_n : n \in A\}$. Then $P(B)$ is equal to

(1) $\frac{1}{16}$ (2) $\frac{1}{32}$

(3) $\frac{3}{64}$ (4) $\frac{3}{32}$

Answer (3)

Sol. $P(w_1) + \frac{P(w_1)}{2} + \frac{P(w_1)}{2^2} + ... = 1$

$\therefore$ $P(w_1) = \frac{1}{2}$

Hence, $P(w_n)=\dfrac{1}{2^n}$

Every number except 1, 2, 3, 4, 6 is representable in the form

$2k+3l$ where $k,\ l\in N$.

$\therefore\quad P(B)=1-P(w_1)-P(w_2)$

$-P(w_3)-P(w_4)-P(w_6)$

$=\dfrac{3}{64}$

68. The value of the integral $\int_1^2\left(\dfrac{t^4+1}{t^6+1}\right)dt$ is

(1) $\tan^{-1}2-\dfrac{1}{3}\tan^{-1}8+\dfrac{\pi}{3}$

(2) $\tan^{-1}2+\dfrac{1}{3}\tan^{-1}8-\dfrac{\pi}{3}$

(3) $\tan^{-1}\dfrac{1}{2}+\dfrac{1}{3}\tan^{-1}8-\dfrac{\pi}{3}$

(4) $\tan^{-1}\dfrac{1}{2}-\dfrac{1}{3}\tan^{-1}8+\dfrac{\pi}{3}$

Answer (2)

Sol. $\int_1^2\dfrac{t^4+1}{t^6+1}dt$

$=\int_1^2\dfrac{(t^2+1)^2}{t^6+1}dt-2\int_1^2\dfrac{t^2}{t^6+1}dt$

$=\int_1^2\dfrac{t^2+1}{t^4-t^2+1}dt-2\int_1^2\dfrac{t^2}{(t^3)^2+1}dt$

$=\tan^{-1}(2t+\sqrt{3})+\tan^{-1}(2t-\sqrt{3})\Big|_1^2$

$-\dfrac{2}{3}\tan^{-1}(t^3)\Big|_1^2$

$=\tan^{-1}(4+\sqrt{3})+\tan^{-1}(4-\sqrt{3})-\tan^{-1}(2+\sqrt{3})$

$-\tan^{-1}(2+\sqrt{3})$

$-\tan^{-1}(2\sqrt{3})-\dfrac{2}{3}(\tan^{-1}8-\tan^{-1}1)$

$=\tan^{-1}2+\dfrac{1}{3}\tan^{-1}8-\dfrac{\pi}{3}$

69. Consider a function $f:\mathrm{N}\to\mathrm{R}$, satisfying

$f(1)+2f(2)+3f(3)+....+xf(x)=x(x+1)f(x);x\ge 2$

with $f(1)=1$. Then $\dfrac{1}{f(2022)}+\dfrac{1}{f(2028)}$ is equal to

(1) 8200
(2) 8400
(3) 8100
(4) 8000

Answer (3)

Sol. $f(1)+2f(2)+3f(3)+...+nf(n)=n(n+1)+(n)$ ...(i)

$n\to n+1$

$f(1)+2f(2)+...+(n+1)f(n+1)=(n+1)(n+2)$

$f(n+1)$ ...(ii)

(i) and (ii) gives

$3f(3)-2f(2)=0$

$4f(4)-3f(3)=0$

$\vdots$

$(n+1)f(n+1)-nf(n)=0$

$\Rightarrow\quad f(n+1)=\dfrac{2f(2)}{n+1}$

$f(n)=\dfrac{1}{2n}$

$\dfrac{1}{f(2022)}+\dfrac{1}{f(2028)}=8100$

70. Let $y=y(x)$ be the solution of the differential equation $x\log_e x\dfrac{dy}{dx}+y=x^2\log_e x,(x>1)$. If $y(2)=2$, then y(e) is equal to

(1) $\dfrac{1+e^2}{2}$
(2) $\dfrac{1+e^2}{4}$
(3) $\dfrac{4+e^2}{4}$
(4) $\dfrac{2+e^2}{2}$

Answer (3)

Sol. $x\ln x\dfrac{dy}{dx}+y=x^2\ln x$

$\dfrac{dy}{dx}+\dfrac{1}{x\ln x}\cdot y=x$

If $=e^{\int\frac{1}{x\ln x}dx}=e^{\int\frac{1}{t}dt}$, where $t=\ln x$

$=e^{\ln t}=t=\ln x$

$$y \cdot \ln x = \int x \ln x = \frac{x^2}{2}\ln x - \int \frac{x^2}{2} \cdot \frac{1}{x}$$

$$y \ln x = \frac{x^2}{2}\ln x - \frac{x^2}{4} + C \qquad \ldots(i)$$

$$y(2) = 2 \Rightarrow C = 1$$

Putting $x = e$ in (i),

$$y = \frac{e^2}{4} + 1 = \frac{4+e^2}{4}$$

71. The shortest distance between the lines $\frac{x-1}{2} = \frac{y+8}{-7} = \frac{z-4}{5}$ and $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-6}{-3}$ is

(1) $4\sqrt{3}$ (2) $3\sqrt{3}$

(3) $5\sqrt{3}$ (4) $2\sqrt{3}$

Answer (1)

Sol. $\vec{r}_1 = \hat{i} - 8\hat{j} + 4\hat{k}$

$\vec{r}_2 = \hat{i} + 2\hat{j} + 6\hat{k}$

$\vec{a} = 2\hat{i} - 7\hat{j} + 5\hat{k}$

$\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$

$$\text{S.D.} = \frac{\left|\vec{r}_1 - \vec{r}_2 \quad \vec{a} \quad \vec{b}\right|}{\left|\vec{a} \times \vec{b}\right|}$$

$$\left[\vec{r}_1 - \vec{r}_2 \quad \vec{a} \quad \vec{b}\right] = \begin{vmatrix} 0 & -10 & -2 \\ 2 & -7 & 5 \\ 2 & 1 & -3 \end{vmatrix}$$

$\therefore$ 10(–16) – 2(16) = –192

$$\left|\left[\vec{r}_1 - \vec{r}_2 \quad \vec{a} \quad \vec{b}\right]\right| = 192$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & 5 \\ 2 & 1 & -3 \end{vmatrix} = 16\hat{i} + 16\hat{j} + 16\hat{k}$$

$$\vec{a} \times \vec{b} = 16\sqrt{3}$$

$$\text{S.D.} = \frac{192}{16\sqrt{3}} = 4\sqrt{3}$$

72. The letters of the word OUGHT are written in all possible ways and these words are arranged as in a dictionary, in a series. Then the serial number of the word TOUGH is

(1) 84 (2) 86

(3) 89 (4) 79

Answer (3)

Sol.

G	....	24
H	....	24
O	....	24
T G	....	6
T H	....	6
T O G	....	2
T O H	....	2
T O U G H	....	1
		89

73. The value of the integral $\int_{1/2}^{2} \frac{\tan^{-1} x}{x} dx$ is equal to

(1) $\frac{\pi}{4}\log_e 2$ (2) $\pi \log_e 2$

(3) $\frac{\pi}{2}\log_e 2$ (4) $\frac{1}{2}\log_e 2$

Answer (3)

Sol. $$I = \int_{1/2}^{2} \frac{\tan^{-1} x}{x} dx \qquad \ldots(i)$$

$$x \to \frac{1}{x}$$

$$I = \int_{1/2}^{2} \frac{1}{x} \tan^{-1} \frac{1}{x} dx \qquad \ldots(ii)$$

$$2I = \int_{1/2}^{2} \frac{1}{x} \cdot \frac{\pi}{2} dx$$

$$= \frac{\pi}{2} \ln x \Big|_{1/2}^{2} = \pi \ln 2$$

$$\Rightarrow \quad I = \frac{\pi}{2} \ln 2$$

74. Let f and g be twice differentiable functions on $\mathbb{R}$ such that

$f''(x) = g''(x) + 6x$

$f'(1) = 4g'(1) - 3 = 9$

$f(2) = 3g(2) = 12$.

Then which of the following is **NOT** true?

(1) There exists $x_0 \in (1, 3/2)$ such that $f(x_0) = g(x_0)$

(2) $|f'(x) - g'(x)| < 6 \Rightarrow -1 < x < 1$

(3) $g(-2) - f(-2) = 20$

(4) If $-1 < x < 2$, then $|f(x) - g(x)| < 8$

Answer (4)

Sol. $f''(x) = g''(x) + 6x$

$\Rightarrow\ f'(x) = g'(x) + 3x^2 + C$

$f'(1) = g'(1) + 3 + C$

$\Rightarrow\ g = 3 + 3 + C \Rightarrow C = 3$

$\Rightarrow\ f'(x) = g'(x) + 3x^2 + 3$

$\Rightarrow\ f(x) = g(x) + x^2 + 3x + C'$

$x = 2$

$f(2) = g(2) + 14 + C'$

$12 = 4 + 14 + C'$

$\Rightarrow\ C' = -6$

$\Rightarrow\ f(x) = g(2) + x^3 + 3x - 6$

$f(-2) = g(-2) - 8 - 6 - 6$

$g(-2) - f(-2) = 20$

$f'(x) - g'(x) = 3x^2 + 3$

$x \in (-1, 1)$

$3x^2 + 3 \in (0, 6)$

$\Rightarrow\ f'(x) - g'(x) \in (0, 6)$

$f(x) - g(x) = x^3 + 3x - 6$

At $x = -1$

$|f(-1) - g(-1)| = 10$

$\therefore$ Option (4) is false.

75. If $\vec{a} = \hat{i} + 2\hat{k}, \vec{b} = \hat{i} + \hat{j} + \hat{k}, \vec{c} = 7\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{r} \times \vec{b} + \vec{b} \times \vec{c} = \vec{0}$ and $\vec{r} \cdot \vec{a} = 0$. Then $\vec{r} \cdot \vec{c}$ is equal to

(1) 32 (2) 30

(3) 34 (4) 36

Answer (3)

Sol. $(\vec{r} - \vec{c}) \times \vec{b} = 0$

$\vec{r} = \lambda\vec{b} + \vec{c}$

$\vec{r} \cdot \vec{a} = 0$

$\Rightarrow\ \lambda\vec{b} \cdot \vec{a} + \vec{c} \cdot \vec{a} = 0$

$\Rightarrow\ \lambda(3) + (7 + 8) = 0$

$\Rightarrow\ \lambda = -5$

$\vec{r} = 5\vec{b} + \vec{c}$

$= -5\hat{i} - 5\hat{j} - 5\hat{k} + (7\hat{i} + 3\hat{j} + 4\hat{k})$

$= 2\hat{i} - 8\hat{j} - \hat{k}$

$\therefore\ \vec{r} \cdot \vec{c} = 17 + 24 - 4 = 34$

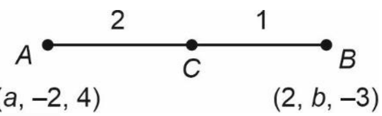
76. The plane $2x - y + z = 4$ intersects the line segment joining the points $A(a, -2, 4)$ and $B(2, b, -3)$ at the point C in the ratio 2 : 1 and the distance of the point C from the origin is $\sqrt{5}$. If $ab < 0$ and P is the point $(a - b, b, 2b - a)$ then CP^2 is equal to

(1) $\frac{73}{3}$ (2) $\frac{16}{3}$

(3) $\frac{97}{3}$ (4) $\frac{17}{3}$

Answer (4)

Sol.



$c : \left(\frac{a+4}{3}, \frac{-2+2b}{3}, \frac{4-6}{3}\right)$

$= \left(\frac{a+4}{3}, \frac{-2+2b}{3}, \frac{-2}{3}\right)$

c lies on plane $2x - y + z = y$

$2\left(\frac{a+4}{3}\right) - \left(\frac{2b-2}{3}\right) - \frac{2}{3} = 4$

$\Rightarrow\ 2a + 8 - 2b + 2 - 2 = 12$

$\Rightarrow\ a - b = 2$ ...(i)

Now, $OP = \sqrt{5}$

$\left(\frac{a+4}{3}\right)^2 + \left(\frac{2b-2}{3}\right)^2 + \frac{4}{9} = 5$ and using (i)

$a = \frac{11}{5}, 1$

$\Rightarrow\ b = \frac{1}{5}, -1$

as also $\Rightarrow a = 1, b = -1$

$\therefore\ P(2, -1, -3), c\left(\frac{5}{3}, \frac{-4}{3}, \frac{-2}{3}\right)$

$CP^2 = \frac{1}{9} + \frac{1}{9} + \frac{49}{9}$

$= \frac{17}{3}$

$\therefore$ Option (4) is correct.

77. The number of 3 digit numbers, that are divisible by either 3 or 4 but not divisible by 48, is

(1) 432 (2) 507

(3) 400 (4) 472

Answer (1)

Sol. Number divisible by 3 = 300

Number divisible by 4 = 225

Number divisible by 12 = 75

Number divisible by 48 = 18

Total required number = 300 + 225 – 75 – 18

= 432

$\therefore$ Option (1) is correct.

78. The set of all values of λ for which the equation $\cos^2 2x - 2\sin^4 x - 2\cos^2 x = \lambda$ has a real solution x, is

(1) $\left[-1, -\frac{1}{2}\right]$ (2) $\left[-\frac{3}{2}, -1\right]$

(3) $\left[-2, -\frac{3}{2}\right]$ (4) $[-2, -1]$

Answer (2)

Sol. $\cos^2 2x - 2\sin^4 x - 2\cos^2 x = \lambda$

$$\cos^2 2x - \frac{1}{2}(2\sin^2 x)^2 - (1+\cos 2x) = \lambda$$

$$\Rightarrow \cos^2 2x - \frac{1}{2}(1-\cos 2x)^2 - (1+\cos 2x)$$

$$\Rightarrow \cos^2 2x - \frac{1}{2}(1+\cos^2 2x - 2\cos 2x) - 1 - \cos 2x$$

$$\Rightarrow \frac{\cos^2 2x}{2} - \frac{3}{2}$$

$$\cos^2 2x \in [0, 1]$$

$$\frac{\cos^2 2x}{2} \in \left[0, \frac{1}{2}\right]$$

$$\frac{\cos^2 2x}{2} - \frac{3}{2} \in \left[\frac{-3}{2}, -1\right]$$

$$\therefore \quad \lambda \in \left[\frac{-3}{2}, -1\right]$$

Option (2) is correct

79. Let $\vec{a} = 4\hat{i} + 3\hat{j}$ and $\vec{b} = 3\hat{i} - 4\hat{j} + 5\hat{k}$. If $\vec{c}$ is a vector such that $\vec{c}\cdot\left(\vec{a}\times\vec{b}\right) + 25 = 0$, $\vec{c}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right) = 4$ and projection of $\vec{c}$ on $\vec{a}$ is 1, then the projection of $\vec{c}$ on $\vec{b}$ equals

(1) $\frac{1}{5}$ (2) $\frac{5}{\sqrt{2}}$

(3) $\frac{1}{\sqrt{2}}$ (4) $\frac{3}{\sqrt{2}}$

Answer (2)

Sol. $[\vec{c}\ \ \vec{a}\ \ \vec{b}] = -25$

Let $\vec{c} = l\hat{i} + n\hat{j} + n\hat{k}$

$$\begin{vmatrix} l & m & n \\ 4 & 3 & 0 \\ 3 & -4 & 5 \end{vmatrix} = -25$$

$\Rightarrow$ $3l - 4m - 5n = -5$ …(i)

$\vec{c}\cdot(\hat{i}+\hat{j}+\hat{k}) = 4$

$\Rightarrow$ $l + m + n = 4$ …(ii)

$\frac{\vec{c}\cdot\vec{a}}{|\vec{a}|} = 1 \Rightarrow \vec{c}\cdot\vec{a} = 5$

$\Rightarrow$ $4l + 3m = 5$ …(iii)

Using (i), (ii) and (iii)

$l = 2, m = -1, n = 3$

Now, $\frac{\vec{c}\cdot\vec{b}}{|\vec{b}|} = \frac{25}{5\sqrt{2}} = \frac{5}{\sqrt{2}}$

$\therefore$ Option (2) is correct.

80. Let R be a relation defined on N as $a\,R\,b$ if $2a + 3b$ is a multiple of 5, $a, b \in N$. Then R is

(1) an equivalence relation

(2) not reflexive

(3) symmetric but not transitive

(4) transitive but not symmetric

Answer (1)

Sol. $a\,R\,b$ if $2a + 3b = 5m$, $m \in I$

(1) $(a, a) \in R$ as $2a + 3a = 5a$, $a \in N$

Hence R is reflexive

(2) If $(a, b) \in R$ then $2a + 3b = 5m$

Now, $5(a + b) = 5n$

$3a + 2b + 2a + 3b = 5n$

$\therefore$ $3a + 2b = 5(n - m)$

$\therefore$ $(b, a) \in R$

$\therefore$ R is symmetric

(3) If $(a, b) \in R$ and $(b, c) \in R$ then

$2a + 3b = 5m$, $2b + 3c = 5n$

$\Rightarrow 2a + 5b + 3c = 5(m + n)$

$\Rightarrow 2a + 3c = 5(m = n - b)$

$\therefore$ $(a, c) \in R$

$\therefore$ R is transitive

Hence R is equivalence relation.

option (1) is correct.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

81. Let $\{a_k\}$ and $\{b_k\}$, $k \in \mathrm{N}$, be two G.P.s with common ratios r_1 and r_2 respectively such that $a_1 = b_1 = 4$ and $r_1 < r_2$. Let $c_k = a_k + b_k$, $k \in \mathrm{N}$. If $c_2 = 5$ and $c_3 = \frac{13}{4}$ then $\sum_{k=1}^{\infty} c_k - (12a_6 + 8b_4)$ is equal to ______.

Answer (09)

Sol. $\{a_k\}$ be *a* G.P. with $a_1 = 4$, $r = r_1$

And

$\{b_k\}$ be G.P. with $b_1 = 4$, $r = r_2$ $(r_1 < r_2)$

Now

$C_k = a_k + b_k$

$c_1 = 4 + 4 = 8$ and $c_2 = 5$

$a_2 + b_2 = 5$

$$\therefore \quad r_1 + r_2 = \frac{5}{4}$$

$$\text{and } c_3 = \frac{13}{4} \Rightarrow r_4^2 + r_2^2 = \frac{13}{16}$$

$$\therefore \quad \frac{25}{16} - 2r_1r_2 = \frac{13}{16} \Rightarrow 2r_1r_2 = \frac{3}{4}$$

$$\therefore \quad r_2 - r_1 = \sqrt{\frac{25}{16} - \frac{3}{2}} = \frac{1}{4}$$

$$\therefore \quad r_2 = \frac{3}{4}, r_1 = \frac{1}{2}$$

$$\therefore \quad a_6 = 4 \times \frac{1}{2^5} = \frac{1}{8}, b_4 = 4 \times \frac{27}{64} = \frac{27}{16}$$

$$\text{and } \sum_{K=1}^{\infty} C_K = 4\left[\frac{1}{1-\frac{1}{2}} + \frac{1}{1-\frac{3}{4}}\right] = 24$$

$$\therefore \quad \sum_{K=1}^{\infty} C_K - (12a_6 + 8b_4) = 09$$

82. the total number of 4-digit numbers whose greatest common divisor with 54 is 2, is ________.

Answer (3000)

Sol. $gcd(a, 54) = 2$ when a is a 4 digit no.

And $54 = 3 \times 3 \times 3 \times 2$

So, a = all even no. of 4 digits

– Even multiple of 3 (4 digits)

$= 4500 - 1500$

$= 3000$

83. If the equation of the normal to the curve $y = \frac{x-a}{(x+b)(x-2)}$ at the point (1, –3) is $x - 4y = 13$, then the value of $a + b$ is equal to _________.

Answer (04)

Sol. Given curve : $y = \frac{x-a}{(x+b)(x-2)}$ at $(1, -3)$

$$\therefore \quad -3 = \frac{1-a}{(1+b)(-1)} \Rightarrow 3 + 3b = 1 - a$$

$$\beta \Rightarrow a + 3b + 2 = 0$$

$$y = \frac{x-a}{(x+b)(x-2)}$$

$$\frac{dy}{dx} = \frac{(x+b)(x-2) - (x-a)[(x+b)+(x-2)]}{[(x+b)(x-2)]^2}$$

$$\text{at } (1, -3) \;\; m_T = \frac{-(1+b) - (1-a)(b)}{(1+b)^2} = -4$$

$\therefore \quad 1 + b + b - ab = 4(1 + b)^2$

$\Rightarrow \quad 1 + 2b + b(3b + 2) = 4b^2 + 4 + 8b$

$\Rightarrow \quad b^2 + 4b + 3 = 0$

$(b + 1)(b + 3) = 0$

$b = -1, a = 1$ but $1 + b \neq 0$

$b = -3, a = 7$ $\therefore \quad b \neq -1$

$\therefore \quad a + b = 04$

84. A triangle is formed by the tangents at the point (2, 2) on the curves $y^2 = 2x$ and $x^2 + y^2 = 4x$, and the line $x + y + 2 = 0$. If r is the radius of its circumcircle, then r^2 is equal to _______.

Answer (10)

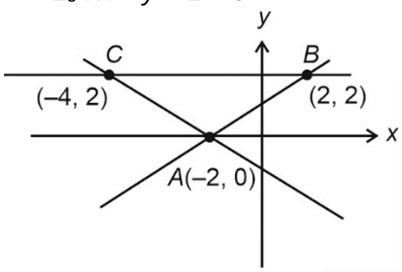
Sol. Tangent for $y^2 = 2x$ at (2, 2) is

$L_1 : 2y = x + 2$

Tangent for $x^2 + y^2 = 4x$ at (2, 2) is

$L_2 : y = 2$

$L_3 : x + y = 2 = 0$



Radius of circumcircle $= \dfrac{abc}{4\Delta}$

$$= \frac{\left(\sqrt{20}\right)(6)\left(\sqrt{8}\right)}{4 \times \frac{1}{2} \times 6 \times 2}$$

$$R = \sqrt{10}$$

$R^2 = 10$

85. Let $X = \{11, 12, 13, \ldots\ldots, 40, 41\}$ and $Y = \{61, 62, 63, \ldots\ldots, 90, 91\}$ be the two sets of observations. If $\bar{x}$ and $\bar{y}$ are their respective means and σ^2 is the variance of all the observations in $X \cup Y$, then $\left|\bar{x} + \bar{y} - \sigma^2\right|$ is equal to _____.

Answer (603)

Sol. $x = \{11, 12, 13 \ldots, 40, 41\}$

$y = \{61, 62, 63, \ldots, 90, 91\}$

$$\bar{x} = \frac{\frac{31}{2}(11+41)}{31} = \frac{1}{2} \times 52 = 26$$

$$\bar{y} = \frac{\frac{31}{2}(61+91)}{31} = \frac{1}{2} \times 152 = 76$$

$$\sigma^2 = \frac{\Sigma x_i^2 + \Sigma y_i^2}{62} - \left(\frac{\Sigma x + \Sigma y}{62}\right)^2$$

$= 705$

Now

$$\left|\bar{x} + \bar{y} - \sigma^2\right|$$

$$= |26 + 76 - 705|$$

$= 603$

86. Let $\alpha = 8 - 14t$, $A = \left\{z \in \mathbb{C} : \dfrac{\alpha z - \bar{\alpha}\bar{z}}{z^2 - (\bar{z})^2 - 112i}\right\}$ and $B = \{z \in \mathbb{C} : |z + 3i| = 4\}$. Then $\sum\limits_{z \in A \cap B} (\operatorname{Re} z - \operatorname{Im} z)$ is equal to _______.

Answer (07)

Sol. Let $z = x + iy$

and $\alpha = 8 - 14\,i$

$$\frac{\alpha z - \bar{\alpha}\,\bar{z}}{z^2 - \bar{z}^2 - 112i} = 1$$

$$\therefore \quad \frac{(16y - 28x)i}{4xy - 112i} = 1$$

$(16y - 28x + 112)i = 4xy$

$\therefore \quad z = -7i$ or 4

Now, $z = -7$i satisfy B

$B : x^2 + (y + 3)^2 = 16$

$A \cap B = (0, -7)$

$Re\, z - Im\, z = 7$

87. Let A be a symmetric matrix such that $|A| = 2$ and $\begin{bmatrix} 2 & 1 \\ 3 & \frac{3}{2} \end{bmatrix} A - \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$. If the sum of the diagonal elements of A is s, then $\dfrac{\beta s}{\alpha^2}$ is equal to _____.

Answer (05)

Sol. $A = \begin{pmatrix} a & c \\ c & b \end{pmatrix}$

$|A| = ab - c^2 = 2$ …(1)

$$\begin{pmatrix} 2 & 1 \\ 3 & \frac{3}{2} \end{pmatrix} \begin{pmatrix} a & c \\ c & b \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ \alpha & \beta \end{pmatrix}$$

$2a + c = 1$ ...(2)

$2c + b = 2$ ...(3)

$3a + \frac{3}{2}c = \alpha$ ...(4)

$3c + \frac{3}{2}b = \beta$ ...(5)

From (1), (2) and (3)

$a = \frac{3}{4} \quad b = 3 \quad c = -\frac{1}{2}$

$\Rightarrow$ Now, $\alpha = \frac{6}{4}$

$\beta = 3$

$s = \frac{15}{4}$

$$\frac{\beta s}{\alpha^2} = \frac{3 \times \frac{15}{4}}{\left(\frac{6}{4}\right)^2} = \frac{\frac{45}{4}}{\frac{9}{4}} = 5$$

88. Let $a_1 = b_1 = 1$ and $a_n = a_{n-1} + (n-1)$, $b_n = b_{n-1} + a_{n-1}$, $\forall n \geq 2$. If $S = \sum_{n=1}^{10} \frac{b_n}{2^n}$ and $T = \sum_{n=1}^{8} \frac{n}{2^{n-1}}$, then $2^7(2S - T)$ is equal to ______.

Answer (461)

Sol. $\because a_n = a_{n-1} + (n-1)$ and $a_1 = b_1 = 1$

$b_n = b_{n-1} + a_{n-1}$

$\therefore \quad b_{n+1} = 2b_n - b_{n-1} + n - 1$

n	**b_n**	**$b_n - n$**
1	1	0
2	2	0
3	4	1
4	8	4
5	15	10
6	26	20
7	42	35
8	64	56
9	93	84
10	130	120

$$\therefore \quad 2S - T = \left(\sum_{n=1}^{8} \frac{b_n - n}{2^{n-1}}\right) + \frac{b_9}{2^8} + \frac{b_{10}}{2^9}$$

$$= \frac{461}{128}$$

$\therefore \quad 2^7(2S - T) = 461$

89. A circle with centre (2, 3) and radius 4 intersects the line $x + y = 3$ at the points P and Q. If the tangents at P and Q intersect at the point $S(\alpha, \beta)$, then $4\alpha - 7\beta$ is equal to _____.

Answer (11)

Sol. The line $x + y = 3$ ...(i)

is polar of $S(\alpha, \beta)$ w.r.t. circle

$(x - 2)^2 + (y - 3)^2 = 16$

$\Rightarrow \quad x^2 + y^2 - 4x - 6y - 3 = 0$

Equation of polar is

$\alpha x + \beta y - 2(x + \alpha) - 3(4 + \beta) - 3 = 0$

$(\alpha - 2)x + (\beta - 3)y - (2\alpha + 3\beta + 3) = 0$ ...(ii)

(i) and (ii) represent the same.

$$\therefore \quad \frac{\alpha - 2}{1} = \frac{\beta - 3}{1} = \frac{2\alpha + 3\beta + 3}{3}$$

$\alpha - \beta + 1 = 0$

$\alpha - 3\beta - 9 = 0$

$\Rightarrow \quad \alpha = -6, \beta = -5$

$4\alpha - 7\beta = 11$

90. Let $\alpha_1, \alpha_2, ..., \alpha_7$ be the roots of the equation $x^7 + 3x^5 - 13x^3 - 15x = 0$ and $|\alpha_1| \geq |\alpha_2| \geq ... \geq |\alpha_7|$. Then $\alpha_1\alpha_2 - \alpha_3\alpha_4 + \alpha_5\alpha_6$ is equal to _______.

Answer (09)

Sol. $x^7 + 3x^5 - 13x^3 - 15x = 0$

$x(x^6 + 3x^4 - 13x^2 - 15) = 0$

$x = 0 \quad = \alpha_7$

Let $x^2 = t$

$t^3 + 3t^2 - 13t - 15 = 0$

$(t + 1)\ (t + 5)\ (t - 3) = 0$

$t = x^2 = -1, -5, 3$

$x = \pm i, \pm\sqrt{5}i, \pm\sqrt{3}$

$\alpha_1, \alpha_2 = \pm\sqrt{5}i, \alpha_3, \alpha_4 \pm = \sqrt{3}, \alpha_5, \alpha_6 = \pm i$

$\alpha_1\alpha_2 - \alpha_3\ \alpha_4 + \alpha_5\ \alpha_6 = 5 + 3 + 1 = 9$

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