29/01/2023
Morning

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Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2023 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. If a radioactive element having half-life of 30 min. is undergoing beta decay, the fraction of radioactive element remains undecayed after 90 min. will be

(1) $\frac{1}{8}$ (2) $\frac{1}{4}$

(3) $\frac{1}{16}$ (4) $\frac{1}{2}$

Answer (1)

Sol. t_{half} = 30 min.

In 90 min. there will be 3 half lives

Number of remaining $=\left(\frac{N_0}{2^3}\right)$

$=\frac{N_0}{8}$

$\therefore$ Fraction will be $\frac{1}{8}$

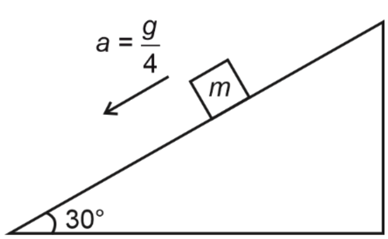
2. A block of mass m slides down the plane inclined at angle 30° with an acceleration $\frac{g}{4}$. The value of coefficient of kinetic friction will be:

(1) $\frac{2\sqrt{3}-1}{2}$ (2) $\frac{1}{2\sqrt{3}}$

(3) $\frac{2\sqrt{3}+1}{2}$ (4) $\frac{\sqrt{3}}{2}$

Answer (2)

Sol.



$\because$ $mg\sin\theta - \mu mg\cos\theta = ma$

Also $a=\frac{g}{4}$

$\therefore$ $\frac{mg}{2}-\mu mg\frac{\sqrt{3}}{2}=\frac{mg}{4}$

$\frac{mg}{4}=\mu mg\frac{\sqrt{3}}{2}$

$\mu=\frac{1}{2\sqrt{3}}$

3. Surface tension of a soap bubble is 2.0 × 10^{-2} Nm^{-1}. Work done to increase the radius of soap bubble from 3.5 cm to 7 cm will be:

Take $\left[\pi=\frac{22}{7}\right]$

(1) 5.76 × 10^{-4} J (2) 0.72 × 10^{-4} J

(3) 9.24 × 10^{-4} J (4) 18.48 × 10^{-4} J

Answer (4)

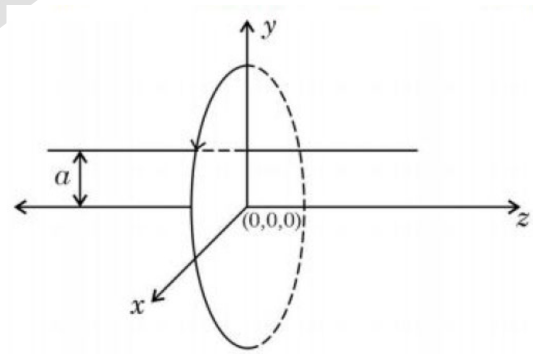
Sol. T = 2 × 10^{-2} N/m^2

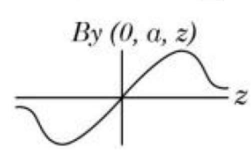
$W = T(\Delta A)$

$=2\times10^{-2}\left[2\times4\pi\left\{\left(\frac{7}{100}\right)^2-\left(\frac{3.5}{100}\right)^2\right\}\right]$

= 18.48 × 10^{-4} J

4. A single current carrying loop of wire carrying current I flowing in anticlockwise direction seen from +ve z direction and lying in xy plane is shown in figure. The plot of $\hat{j}$ component of magnetic field (By) at a distance 'a' (less than radius of the coil) and on yz plane vs z coordinate look like



(1)


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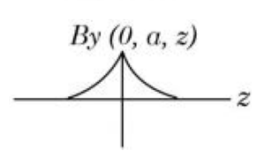

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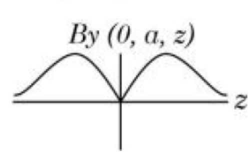

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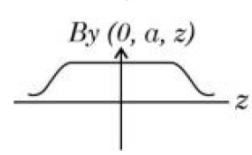
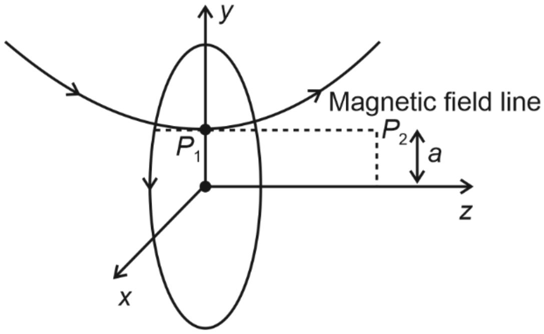


Answer (1)

Sol.



$\because$ *By* at $P_1 = 0$ [option 2 and 4 are incorrect]

By has the opposite direction for the +ve and –ve *z* axis

5. A car is moving on a horizontal curved road with radius 50 m. The approximate maximum speed of car will be, if friction between tyres and road is 0.34. [take g = 10 ms^{-2}]

(1) 22.4 ms^{-1}

(2) 13 ms^{-1}

(3) 17 ms^{-1}

(4) 3.4 ms^{-1}

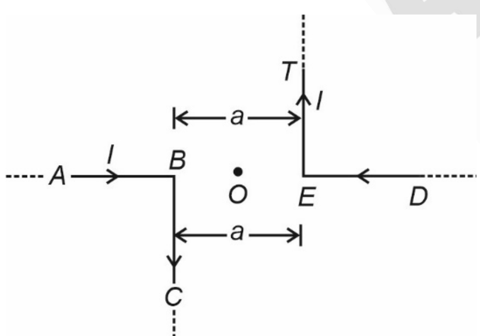
Answer (2)

Sol. $v_{max} = \sqrt{\mu g R}$

$= \sqrt{0.34 \times 10 \times 50}$

≈ 13 m/s

6. The magnitude of magnetic induction at mid point *O* due to current arrangement as shown in Fig. will be



(1) 0

(2) $\dfrac{\mu_0 I}{4\pi a}$

(3) $\dfrac{\mu_0 I}{\pi a}$

(4) $\dfrac{\mu_0 I}{2\pi a}$

Answer (3)

Sol. $B_0 = 2\left[\dfrac{\mu_0 I}{4\pi\left(\dfrac{a}{2}\right)}[\sin 0° + \sin 90°]\right]$

$= \dfrac{\mu_0 I}{\pi a}$

7. A stone is projected at angle 30° to the horizontal. The ratio of kinetic energy of the stone at point of projection to its kinetic energy at the highest point of flight will be -

(1) 4 : 3

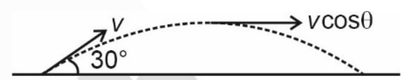
(2) 4 : 1

(3) 1 : 2

(4) 1 : 4

Answer (1)

Sol.



$KE_{in} = \dfrac{1}{2}mv^2$

$KE_{final} = \dfrac{1}{2}mv^2\cos^2 30° = \dfrac{1}{2}mv^2\left(\dfrac{\sqrt{3}}{2}\right)^2$

$\dfrac{KE_{in}}{KE_f} = \dfrac{\dfrac{1}{2}mv^2}{\dfrac{1}{2}mv^2\left(\dfrac{3}{4}\right)} = \dfrac{4}{3}$

8. Two particles of equal mass '*m*' move in a circle of radius '*r*' under the action of their mutual gravitational attraction. The speed of each particle will be :

(1) $\sqrt{\dfrac{Gm}{2r}}$

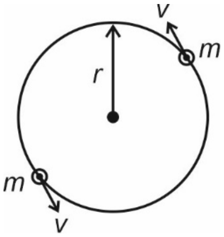
(2) $\sqrt{\dfrac{Gm}{4r}}$

(3) $\sqrt{\dfrac{4Gm}{r}}$

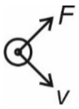
(4) $\sqrt{\dfrac{Gm}{r}}$

Answer (2)

Sol.



From one of the masses FBD



$$\frac{Gm^2}{(2r)^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{Gm}{4r}}$$

9. A bicycle tyre is filled with air having pressure of 270 kPa at 27°C. The approximate pressure of the air in the tyre when the temperature increases to 36°C is

(1) 270 kPa

(2) 278 kPa

(3) 360 kPa

(4) 262 kPa

Answer (2)

Sol. P_{in} = 270 kPa, T_{in} = 27°C

= 300 K

T_{final} = 36°C = 309 K

Hence we can consider process to be isochoric volume constant

$\therefore \quad P \propto T$

$$\frac{P_{in}}{P_f} = \frac{T_{in}}{T_f} \Rightarrow P_f = 278 \text{ kPa}$$

10. Given below are two statements: One is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : If dQ and dW represent the heat supplied to the system and the work done on the system respectively. Then according to the first law of thermodynamics $dQ = dU - dW$.

Reason R : First law of thermodynamics is based on law of conservation of energy.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both A and R are correct and R is the correct explanation of A

(2) A is correct but R is not correct

(3) A is not correct but R is correct

(4) Both A and R are correct but R is not the correct explanation of A

Answer (1)

Sol. ΔQ = heat supplied to system

ΔW = work done on the system

$\therefore \quad \Delta U = \Delta Q - \Delta W$

This comes from conservation of energy.

11. In a cuboid of dimension 2L × 2L × L, a charge q is placed at the center of the surface 'S' having area of 4 L^2. The flux through the opposite surface to 'S' is given by

(1) $\dfrac{q}{2\epsilon_0}$

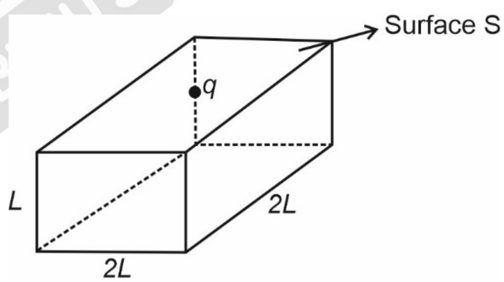
(2) $\dfrac{q}{6\epsilon_0}$

(3) $\dfrac{q}{12\epsilon_0}$

(4) $\dfrac{q}{3\epsilon_0}$

Answer (2)

Sol.



If we consider a similar box above this box then it becomes cube of side length $2L$

ϕ through a surface $= \dfrac{q}{6\varepsilon_0}$

12. Which one of the following statement is not correct in the case of light emitting diodes?

A. It is a heavily doped p-n junction.

B. It emits light only when it is forward biased.

C. It emits light only when it is reverse biased.

D. The energy of the light emitted is equal to or slightly less then the energy gap of the semiconductor used.

Choose the correct answer from the options given below:

(1) C and D (2) C

(3) B (4) A

Answer (2)

Sol. ⇒ LED is a heavily doped, forward biased *p-n*-junction diode

⇒ It will not emit light in reverse bias

⇒ Energy of emitted photon is equal to or slightly less the band gap energy of forbidden band.

13. Match List I with List II:

	List I (Physical Quantity)		List II (Dimensional Formula)
A.	Pressure gradient	I.	$[M^0L^2T^{-2}]$
B.	Energy density	II.	$[M^1L^{-1}T^{-2}]$
C.	Electric field	III.	$[M^1L^{-2}T^{-2}]$
D.	Latent heat	IV.	$[M^1L^1T^{-3}A^{-1}]$

Choose the ***correct*** answer from the options given below:

(1) A-III, B-II, C-IV, D-I (2) A-II, B-III, C-I, D-IV

(3) A-III, B-II, C-I, D-IV (4) A-II, B-III, C-IV, D-I

Answer (1)

Sol. A. $\frac{\Delta P}{\Delta x}=\left[\frac{MLT^{-2}}{L^3}\right]=[ML^{-2}T^{-2}]\ldots(III)$

B. $\frac{E}{v}=\left[\frac{ML^2T^{-2}}{L^3}\right]=[ML^{-1}T^{-2}]\ldots(II)$

C. $\frac{F}{Q}=\left[\frac{MLT^{-2}}{AT}\right]=[MLT^{-3}A^{-1}]\ldots(IV)$

D. Latent heat $=\left[\frac{ML^2T^{-2}}{M}\right]=[M^0L^2T^{-2}]\ldots(I)$

14. Which of the following are true?

A. Speed of light in vacuum is dependent on the direction of propagation.

B. Speed of light in a medium is independent of the wavelength of light.

C. The speed of light is independent of the motion of the source.

D. The speed of light in a medium is independent of intensity.

Choose the correct answer from the options given below:

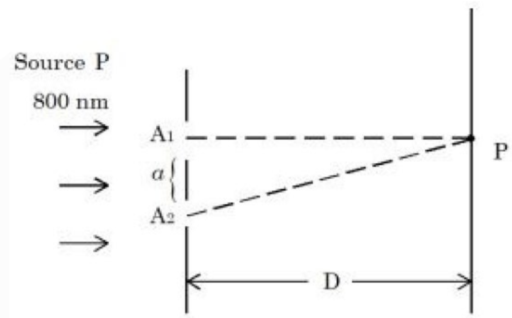
(1) C and D only (2) B and D only

(3) B and C only (4) A and C only

Answer (1)

Sol. Speed of light is independent of motion of source and Intensity.

15. In a Young's double slit experiment, two slits are illuminated with a light of wavelength 800 nm. The line joining A_1P is perpendicular to A_1A_2 as shown in the figure. If the first minimum is detected at *P*, the value of slits separation 'a' will be:



The distance of screen from slits D = 5 cm

(1) 0.4 mm

(2) 0.1 mm

(3) 0.2 mm

(4) 0.5 mm

Answer (3)

Sol. $y=\frac{(2n-1)\lambda D}{2a}=\frac{a}{2}$ for $n = 1$

$\Rightarrow \frac{\lambda D}{2a}=\left(\frac{a}{2}\right)$

$\Rightarrow \frac{800\times10^{-9}\times5\times10^{-2}}{2}=\frac{a^2}{2}$

$\Rightarrow a^2 = 4000 \times 10^{-11}$

$a=\sqrt{4\times10^{-8}} = 2 \times 10^{-4} = 0.2$ mm

16. If the height of transmitting and receiving antennas are 80 m each, the maximum line of sight distance will be:

Given: Earth's radius = 6.4×10^6 m

(1) 64 km (2) 36 km

(3) 28 km (4) 32 km

Answer (1)

Sol.

Maximum line of sight = $2\sqrt{2Rh}$

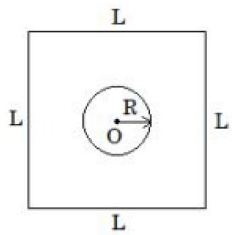
$= 2\sqrt{2 \times 6.4 \times 10^6 \times 80}$

$= 2 \times 4 \times 8 \times 10^3$

$= 64 \times 10^3$

$= 64$ km

17 . Find the mutual inductance in the arrangement, when a small circular loop of wire of radius 'R' is placed inside a large square loop of wire of side L ($L >> R$). The loops are coplanar and their centres coincide:

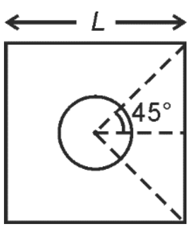


(1) $M = \frac{2\sqrt{2}\mu_0 R}{L^2}$ (2) $M = \frac{\sqrt{2}\mu_0 R}{L^2}$

(3) $M = \frac{2\sqrt{2}\mu_0 R^2}{L}$ (4) $M = \frac{\sqrt{2}\mu_0 R^2}{L}$

Answer (3)

Sol.



B at centre = $\frac{\mu_0 i}{4\pi\left(\frac{L}{2}\right)}\left(\frac{2}{\sqrt{2}}\right) \times 4$

$= \frac{\sqrt{2}\mu_0 i}{2\pi L} \times 4$

$= \left(\frac{2\sqrt{2}\mu_0 i}{\pi L}\right)$

Mutual inductance = $\frac{B \cdot A}{i}$

$= \frac{2\sqrt{2}\mu_0 i}{\pi L} \times \frac{\pi R^2}{i}$

$= \left(\frac{2\sqrt{2}\mu_0 R^2}{L}\right)$

18. Ratio of thermal energy released in two resistors R and 3R connected in parallel in an electric circuit is:

(1) 1 : 1 (2) 1 : 3

(3) 1 : 27 (4) 3 : 1

Answer (4)

Sol. For parallel connection, potential difference is same (ν)

$P_1 = \left(\frac{\nu^2}{R_1}\right)$

$P_2 = \left(\frac{\nu^2}{R_2}\right)$

$\frac{P_1}{P_2} = \frac{H_1}{H_2} = \left(\frac{R_2}{R_1}\right) = \frac{3R}{R} = (3:1)$

19. The threshold wavelength for photoelectric emission from a material is 5500 Å. Photoelectrons will be emitted, when this material is illuminated with monochromatic radiation from a

A. 75 W infra-red lamp

B. 10 W infra-red lamp

C. 75 W ultra-violet lamp

D. 10 W ultra-violet lamp

Choose the correct answer from the options given below:

(1) A and D only (2) C only

(3) C and D only (4) B and C only

Answer (3)

Sol. Wavelength of infra-red = 700 nm (minimum)

Wavelength of UV = 100 – 400 nm

Since we need $\lambda < 5000$ Å

$\Rightarrow$ Only UV would be able to emit photoelectrons.

20. A person observes two moving trains, 'A' reaching the station and 'B' leaving the station with equal speed of 30 m/s. If both trains emit sounds with frequency 300 Hz, (Speed of sound: 330 m/s) approximate difference of frequencies heard by the person will be:

(1) 33 Hz (2) 10 Hz

(3) 55 Hz (4) 80 Hz

Answer (3)

Sol. By doppler effect : $f' = f_0\left[\frac{v - v_0}{v - v_s}\right]$

$$\Rightarrow \quad f_A' = 300\left[\frac{330}{330-30}\right]\text{Hz}$$

$$= 330 \text{ Hz}$$

And $f_B' = 300\left[\frac{330}{330+30}\right]\text{Hz}$

$$= \frac{5}{6}\times 330 \text{ Hz} = 275 \text{ Hz}$$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. A certain elastic conducting material is stretched into a circular loop. It is placed with its plane perpendicular to a uniform magnetic field B = 0.8 T. When released the radius of the loop starts shrinking at a constant rate of 2 cms^{-1}. The induced emf in the loop at an instant when the radius of the loop is 10 cm will be _____ mV.

Answer (10)

Sol. $\varepsilon = \frac{-d\phi}{dt}$

$$= -\frac{d}{dt}[B\cdot \pi r^2]$$

$$= -\pi B\left[2r\frac{dr}{dt}\right]$$

$$= 2\times\pi\times 0.8\times\frac{10}{100}\times\left(\frac{-2}{100}\right)\text{ Volts}$$

$$\Rightarrow \quad \varepsilon \simeq -10.048 \text{ mV}$$

22. A 0.4 kg mass takes 8 s to reach ground when dropped from a certain height 'P' above surface of earth. The loss of potential energy in the last second of fall is _____ J.

(Take g = 10 m/s^2)

Answer (300)

Sol. $8 = \sqrt{\frac{2h}{g}}$

$\Rightarrow$ h = 320 m

Distance covered in last second

$$= \frac{1}{2}g\times 8^2 - \frac{1}{2}g\times 7^2$$

h' = 75 m

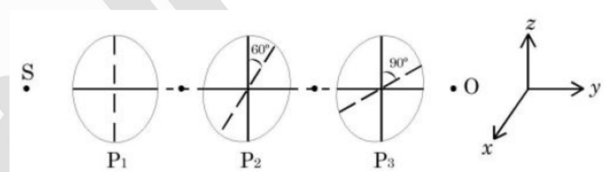
$\Rightarrow$ Loss of potential energy = mgh'

= 0.4 × 10 × 75 J

= 300 J

23. As shown in the figure, three identical polaroids P_1, P_2 and P_3 are placed one after another. The pass axis of P_2 and P_3 are inclined at angle of 60° and 90° with respect to axis of P_1. The source S has an intensity of $256\ \frac{W}{m^2}$.

The intensity of light at point O is _______ $\frac{W}{m^2}$.



Answer (24)

Sol. Using Malus' law, intensity would be

$$I = I_0\times\frac{1}{2}\times\cos^2 60^\circ\times\cos^2(90^\circ - 60^\circ)$$

$$= 256\times\frac{1}{2}\times\frac{1}{4}\times\frac{3}{4}\ \text{W/m}^2$$

$\Rightarrow$ I = 24 W/m^2

24. A point charge $q_1 = 4q_0$ is placed at origin. Another point charge $q_2 = -q_0$ is placed at x = 12 cm. Charge of proton is q_0. The proton is placed on x axis so that the electrostatic force on the proton is zero. In this situation, the position of the proton from the origin is _____ cm.

Answer (24)

Sol. $4q_0$ at Origin, $-q_0$ at 12 cm, P at $(x-12)$ cm beyond it

Field at point P = 0

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{4q_0}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_0}{(x-12)^2}$$

$\Rightarrow$ $x = 2(x - 12) \Rightarrow x$ = 24 cm

25. A solid sphere of mass 2 kg is making pure rolling on a horizontal surface with kinetic energy 2240 J. The velocity of centre of mass of the sphere will be ____ ms^{-1}.

Answer (40)

Sol. $\frac{1}{2}mv_{cm}^2+\frac{1}{2}\times\frac{2}{5}mR^2\times\frac{v_{cm}^2}{R^2}=2240\text{ J}$

$\frac{7}{10}mv_{cm}^2=2240$

$v_{cm}=\sqrt{\frac{2240\times10}{7\times2}}=40\text{ m/sec}$

26. Two simple harmonic waves having equal amplitudes of 8 cm and equal frequency of 10 Hz are moving along the same direction. The resultant amplitude is also 8 cm. The phase difference between the individual waves is _____ degree.

Answer (120)

Sol. $A_R=\sqrt{A_1^2+A_2^2+2A_1A_2\cos\phi}$

$8=\sqrt{8^2+8^2+2\times8\times8\cos\phi}$

$\Rightarrow\ \cos\phi=-\frac{1}{2}$

$\Rightarrow\ \phi = 120°$

27. In a metre bridge experiment the balance point is obtained if the gaps are closed by 2 Ω and 3 Ω. A shunt of X Ω is added to 3 Ω resistor to shift the balancing point by 22.5 cm. The value of X is _______.

Answer (2)

Sol. Case 1:

$\frac{l}{100-l}=\frac{2}{3}$

$\Rightarrow\ l = 40\text{ cm}$

as 3 Ω is shunted the balance point will shift towards 3 Ω. So, new length $l' = 22.5 + l = 62.5$

So, $\frac{62.5}{37.5}=\frac{2}{3x}(3+x)$

$\Rightarrow\ x = 2\ \Omega$

28. A radioactive element ${}^{242}_{92}X$ emits two α-particles, one electron and two positrons. The product nucleus is represented by ${}^{234}_{P}Y$. The value of P is _______.

Answer (87)

Sol. ${}^{242}_{92}X\xrightarrow{2\alpha}{}^{234}_{88}A\xrightarrow{e^-}{}^{234}_{89}B\xrightarrow{2e^+}{}^{234}_{87}Y$

So, P = 87

29. A tennis ball is dropped on to the floor from a height of 9.8 m. It rebounds to a height 5.0 m. Ball comes in contact with the floor for 0.2 s. The average acceleration during contact is_____ ms^{-2}.

(Given g =10 ms^{-2})

Answer (120)

Sol. The speed of ball just before collision with ground is $u=\sqrt{2\times gH}=\sqrt{2\times10\times9.8}=\underset{\text{(Downwards)}}{14\text{ m/sec}}$

The speed of ball just after collision is

$v=\sqrt{2gh}=\sqrt{2\times10\times5}=\underset{\text{(Upwards)}}{10\text{ m/sec}}$

So, $\vec{a}=\frac{\Delta\vec{v}}{\Delta t}$

$=\frac{10+14}{0.2}= 120\text{ m/s}^2$

30. A body cools from 60°C to 40°C in 6 minutes. If, temperature of surroundings is 10°C. Then, after the next 6 minutes, its temperature will be ______°C.

Answer (28)

Sol. $\frac{\Delta T}{\Delta t}=-k\left(T_{av}-T_0\right)$

Case 1:

$\frac{-20}{6}=-k(50-10)$

$\frac{10}{3}=40k$

$k=\frac{1}{12}$

Case 2:

$\frac{40-T}{6}=\frac{1}{12}\left(\frac{40+T}{2}-10\right)$

$80-2T=\frac{20+T}{2}$

$160 - 4T = 20 - T$

$\Rightarrow\quad T=\frac{140}{5}°\text{C}=28°\text{C}$

CHEMISTRY

SECTION – A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

31. The correct order of hydration enthalpies is

(A) K^+ (B) Rb^{+p}

(C) Mg^{2+} (D) Cs^+

(E) Ca^{2+}

Choose the correct answer from the options given below:

(1) E > C > A > B > D (2) C > E > A > D > B

(3) C > A > E > B > D (4) C > E > A > B > D

Answer (4)

Sol. Hydration enthalpy $\propto$ charge density

$\therefore$ The correct order of charge density is

$Mg^{2+} > Ca^{2+} > K^+ > Rb^+ > Cs^+$

$\therefore$ The order of hydration enthalpy

C > E > A > B > D

32. Number of cyclic tripeptides formed with 2 amino acids A and B is:

(1) 3 (2) 2

(3) 4 (4) 5

Answer (2)

Sol. The cyclic tripeptides possible with amino acids A and B will be

AAB, BBA

There are only two possibilities.

33. Which of the given compounds can enhance the efficiency of hydrogen storage tank?

(1) Li/P_4

(2) $NaNi_5$

(3) SiH_4

(4) Di-isobutylaluminium hydride

Answer (2)

Sol. Tanks of metal alloy like $NaNi_5$, $Ti\text{-}TiH_2$, $Mg\text{-}MgH_2$ are used for the storage of dihydrogen.

34. Match List I with List II.

List I	List II
Antimicrobials	Names
(A) Narrow spectrum antibiotic	(I) Furacin
(B) Antiseptic	(II) Sulphur dioxide
(C) Disinfectants	(III) Penicillin G
(D) Broad spectrum antibiotic	(IV) Chloramphenicol

Choose the correct answer from the options given below:

(1) (A)-II, (B)-I, (C)-IV, (D)-III

(2) (A)-III, (B)-I, (C)-II, (D)-IV

(3) (A)-III, (B)-I, (C)-IV, (D)-II

(4) (A)-I, (B)-II, (C)-IV, (D)-III

Answer (2)

Sol. Narrow spectrum antibiotic $\rightarrow$ Penicillin G

Antiseptic $\rightarrow$ Furacin

Disinfectants $\rightarrow$ Sulphur dioxide

Broad spectrum antibiotic $\rightarrow$ chloramphenicol

$\therefore$ Correct matching is:

A $\rightarrow$ III, B $\rightarrow$ I, C $\rightarrow$ II, D $\rightarrow$ IV

35. During the borax bead test with $CuSO_4$, a blue green colour of the bead was observed in oxidising flame due to the formation of

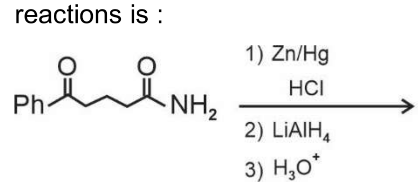
(1) CuO (2) $Cu(BO_2)_2$

(3) Cu_3B_2 (4) Cu

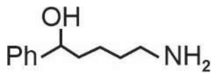
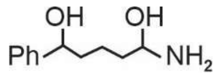
Answer (2)

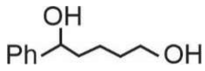
Sol. When borax is heated in a Bunsen burner flame with CuO on loop of platinum wire, a blue coloured $Cu(BO_2)_2$ bead is formed

36. The major product 'P' for the following sequence of reactions is :

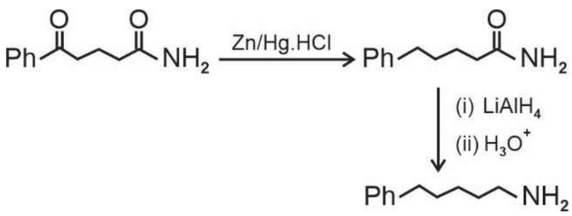


'P'
major product

(1)  (2) 

(3)  (4) 

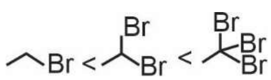
Answer (4)

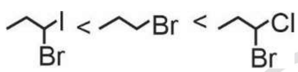
Sol. 

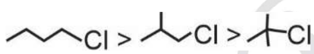
37. Identify the correct order for the given property for following compounds.

(A) Boiling Point : 

(B) Density : 

(C) Boiling Point : 

(D) Density : 

(E) Boiling Point : 

Choose the correct answer from the option given below :

(1) (A), (C) and (E) only (2) (A), (B) and (E) only

(3) (A), (C) and (D) only (4) (B), (C) and (D) only

Answer (1)

Sol. → As mass of the compound increases then their boiling point will also increase. Therefore ‘A’, ‘C’ are correct.

→ As branching of the compound increases then boiling point decreases. Therefore ‘E’ is correct.

Density of Et-Cl → 0.89 g/ml

Et -Br → 1.47 g/ml

Et -I → 1.94 g/ml

∴ Option ‘B’ is incorrect

* No option contains correct option

A, C and E are correct

38. “A” obtained by Ostwald’s method involving air oxidation of NH_3, upon further air oxidation produces “B”. “B” on hydration forms an oxoacid of Nitrogen along with evolution of “A”. The oxoacid also produces “A” and gives positive brown ring test.

Identify A and B, respectively.

(1) NO, NO_2 (2) N_2O_3, NO_2

(3) NO_2, N_2O_4 (4) NO_2, N_2O_5

Answer (1)

Sol. Ostwald’s process is :

$$4NH_3 + 5O_2 \longrightarrow \underset{(A)}{4NO} + 6H_2O$$

$$\underset{(A)}{2NO} + O_2 \longrightarrow \underset{(B)}{2NO_2}$$

$$4NO_2 + 2H_2O + O_2 \longrightarrow 4HNO_3$$

∴ A and B are NO and NO_2 respectively

39. The standard electrode potential (M^{3+}/M^{2+}) for V, Cr, Mn & Co are –0.26 V, –0.41 V, +1.57 V and +1.97 V, respectively. The metal ions which can liberate H_2 from a dilute acid are

(1) V^{2+} and Cr^{2+}

(2) V^{2+} and Mn^{2+}

(3) Cr^{2+} and Co^{2+}

(4) Mn^{2+} and Co^{2+}

Answer (1)

Sol. E° of H^+/H_2 is zero

∴ The metals having less reduction potential can produce H_2 gas with dilute acid.

∴ V and Cr metal can produce H_2 gas

40. Match List I with List II

List I	List II
Reaction	Reagents
(A) Hoffmann Degradation	(I) Conc.KOH, Δ
(B) Clemenson reduction	(II) $CHCl_3$, $NaOH/H_3O^{\oplus}$
(C) Cannizaro reaction	(III) Br_2, NaOH
(D) Reimer-Tiemann Reaction	(IV) Zn-Hg/HCl

Choose the correct answer from the option given below :

(1) (A)-III, (B) -IV, (C) – II, (D)-I

(2) (A)-III, (B) -IV, (C) –I, (D)-II

(3) (A)-II, (B) -IV, (C) –I, (D)-III

(4) (A)-II, (B) -I, (C) – III, (D)-IV

Answer (2)

Sol.

	Reaction		Reagents
(A)	Hoffmann degradation	$\longrightarrow$	Br_2, NaOH
(B)	Clemenson reduction	$\longrightarrow$	Zn-Hg/HCl
(C)	Cannizaro-reaction	$\longrightarrow$	Conc. KOH, Δ
(D)	Reimer-Tiemann reaction	$\longrightarrow$	$CHCl_3$, NaOH/H_3O^+

$\therefore$ Correct match is :

(A)-III, (B) -IV, (C) -I, (D)-II

41. The magnetic behavior of Li_2O, Na_2O_2 and KO_2, respectively, are
    (1) Paramagnetic, paramagnetic and diamagnetic
    (2) Diamagnetic, diamagnetic and paramagnetic
    (3) Paramagnetic, diamagnetic and paramagnetic
    (4) Diamagnetic, paramagnetic and diamagnetic

Answer (2)

Sol. $Li_2O \rightarrow$ diamagnetic

$Na_2O_2 \rightarrow$ diamagnetic

$KO_2 \rightarrow$ paramagnetic (as O_2^- is para magnetic)

42. The bond dissociation energy is highest for
    (1) Cl_2 (2) I_2
    (3) F_2 (4) Br_2

Answer (1)

Sol. Bond dissociation energy of Cl_2 is highest among the halogen.

43. The shortest wavelength of hydrogen atom in Lyman series is λ. The longest wavelength in Balmer series of He^+ is
    (1) $\frac{5}{9\lambda}$ (2) $\frac{5\lambda}{9}$
    (3) $\frac{36\lambda}{5}$ (4) $\frac{9\lambda}{5}$

Answer (4)

Sol. $\frac{hc}{\lambda} = 13.6$ (i)

For longest wavelength in Balmer series transition will be $3 \rightarrow 2$

$$\therefore \Delta E = 13.6 \times 2^2 \times \left(\frac{1}{4} - \frac{1}{9}\right)$$

$$= 13.6 \times 4 \times \frac{5}{4 \times 9}$$

$$\frac{hc}{\lambda^1} = 13.6 \times \frac{5}{9}$$

$$\therefore \lambda^1 = \frac{9}{5}\lambda$$

44. The reaction representing the Mond process for metal refining is __________.
    (1) $Zr + 2I_2 \xrightarrow{\Delta} ZrI_4$
    (2) $Ni + 4CO \xrightarrow{\Delta} Ni(CO)_4$
    (3) $ZnO + C \xrightarrow{\Delta} Zn + CO$
    (4) $2K[Au(CN)_2] + Zn \xrightarrow{\Delta} K_2[Zn(CN)_4] + 2Au$

Answer (2)

Sol. Mond process is need for the purification of Ni metal

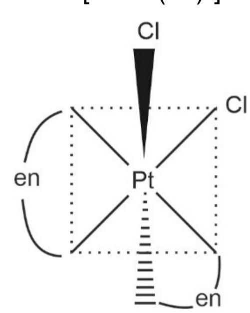
$$\therefore \underset{(\text{impure})}{Ni} + 4CO \xrightarrow{330-350\text{ k}} Ni(CO)_4$$

$$Ni(CO)_4 \xrightarrow{450-470\text{ k}} \underset{(\text{pure})}{Ni} + 4CO$$

45. Chiral complex from the following is
    Here en = ethylene diamine
    (1) trans – $[Co(NH_3)_4Cl_2]^+$
    (2) cis – $[PtCl_2(en)_2]^{2+}$
    (3) cis – $[PtCl_2(NH_3)_2]$
    (4) trans – $[PtCl_2(en)_2]^{2+}$

Answer (2)

Sol. Cis – $[Pt\ Cl_2(en)_2]^{2+}$



has no any element of symmetry so it is optically active.

46. Compound that will give positive Lassaigne's test for both nitrogen and halogen is

(1) $CH_3NH_2 \cdot HCl$ (2) $NH_2OH \cdot HCl$

(3) NH_4Cl (4) $N_2H_4 \cdot HCl$

Answer (1)

Sol. $CH_3NH_2 \cdot HCl$ will give positive Lassaigne's test for both nitrogen and halogen.

47. Which of the following salt solutions would coagulate the colloid solution formed when $FeCl_3$ is added to NaOH solution, at the fastest rate?

(1) 10 mL of 0.1 mol dm^{-3} $Ca_3(PO_4)_2$

(2) 10 mL of 0.2 mol dm^{-3} $AlCl_3$

(3) 10 mL of 0.1 mol dm^{-3} Na_2SO_4

(4) 10 mL of 0.15 mol dm^{-3} $CaCl_2$

Answer (2)

Sol. In the coagulation of a negative sol, the flocculating power is in the order.

$\therefore$ $Al^{3+} > Ba^{2+} > Na^{+}$ and $FeCl_3$ with NaOH forms a negative sol.

$\therefore$ $AlCl_3$ coagulate it most.

48. The increasing order of pK_a for the following phenols is

(A) 2,4-Dinitrophenol

(B) 4-Nitrophenol

(C) 2,4,5-Trimethylphenol

(D) Phenol

(E) 3-Chlorophenol

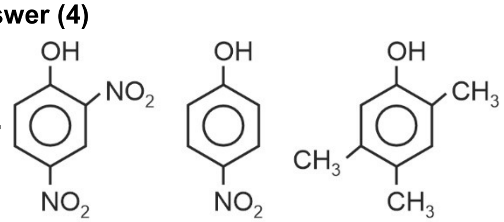
Choose the correct answer from the option given below :

(1) (C), (E), (D), (B), (A) (2) (C), (D), (E), (B), (A)

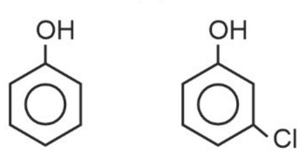
(3) (A), (E), (B), (D), (C) (4) (A), (B), (E), (D), (C)

Answer (4)

Sol.



(A) (B) (C)



(D) (E)

Their acidic order is A > B > E > D > C.

$\therefore$ Their pK_a value is A < B < E < D < C.

49. Correct statement about smog is

(1) NO_2 is present in classical smog

(2) Classical smog also has high concentration of oxidizing agents

(3) Photochemical smog has high concentration of oxidizing agents

(4) Both NO_2 and SO_2 are present in classical smog

Answer (3)

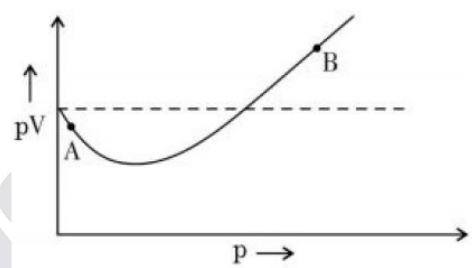
Sol.
- Classical smog occurs in cool humid climate.

  It is a mixture of smoke, fog and sulphur dioxide.

  Chemically, it is a reducing mixture and so it is also called as reducing smog.

- Photochemical smog has high concentration of oxidising agents.

50. For 1 mol of gas, the plot of pV vs. p is shown below. p is the pressure and V is the volume of the gas.



What is the value of compressibility factor at point A?

(1) $1-\frac{a}{RTV}$ (2) $1+\frac{b}{V}$

(3) $1-\frac{b}{V}$ (4) $1+\frac{a}{RTV}$

Answer (1)

Sol. At point 'A', 'a' is considerable and 'b' is negligible.

$$\therefore \left(p+\frac{a}{V^2}\right)V = RT$$

$$pV+\frac{a}{V} = RT$$

$$Z = 1-\frac{a}{VRT}$$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

51. Millimoles of calcium hydroxide required to produce 100 mL of the aqueous solution of pH 12 is $x \times 10^{-1}$. The value of x is ________ (Nearest integer).

Answer (05.00)

Sol. pH = 12

$\therefore$ $[OH^-] = 10^{-2}$

$$\frac{\text{milli mole of } Ca(OH)_2 \times 2}{100} = 10^{-2}$$

$\therefore$ milli moles of $Ca(OH)_2$ required = 5×10^{-1}

52. Solid Lead nitrate is dissolved in 1 litre of water. The solution was found to boil at 100.15°C. When 0.2 mol of NaCl is added to the resulting solution, it was observed that the solution froze at –0.8°C. The solubility product of $PbCl_2$ formed is ________ $\times 10^{-6}$ at 298 K. (Nearest integer)

Given : $K_b = 0.5$ K kg mol^{-1} and $K_f = 1.8$ K kg mol^{-1}. Assume molality to be equal to molarity in all cases.

Answer (13.00)

Sol. $0.15 = 3 \times 0.5 \times M$

$M_{Pb(NO_3)_2} = 0.1$ molar

$$Pb(NO_3)_2 + 2NaCl \longrightarrow PbCl_2 + 2NaNO_3$$

$\Delta T_f = iK_f \cdot m$

$0.8 = (0.4 + 3s)\,1.8$

$s = 0.0148$

$\therefore$ solubility product $= 4s^3$

$= 4 \times (0.0148)^3$

$\approx 13 \times 10^{-6}$

53. Water decomposes at 2300 K

$$H_2O(g) \rightarrow H_2(g) + \frac{1}{2}O_2(g)$$

The percent of water decomposing at 2300 K and 1 bar is ________ (Nearest integer).

Equilibrium constant for the reaction is 2×10^{-3} at 2300 K.

Answer (02)

Sol. $H_2O \longrightarrow H_2 + \frac{1}{2}O_2$

$$K_p = \frac{P_T^{\frac{3}{2}}\left(1+\frac{\alpha}{2}\right)\alpha^{\frac{3}{2}}}{2^{\frac{1}{2}}P_T\left(1+\frac{\alpha}{2}\right)^{\frac{3}{2}}(1-\alpha)}$$

$$2 \times 10^{-3} = \frac{\alpha^{\frac{3}{2}}}{2^{\frac{1}{2}}}$$ [as α <<1 and let P_T = 1]

$$\alpha^{\frac{3}{2}} = 2^{\frac{3}{2}} \times 10^{-3}$$

$$\approx 2 \times 10^{-2}$$

$\therefore$ % of water decomposition = 2%

54. The number of molecules or ions from the following, which do not have odd number of electrons are ______.

(A) NO_2 (B) ICl_4^-

(C) BrF_3 (D) ClO_2

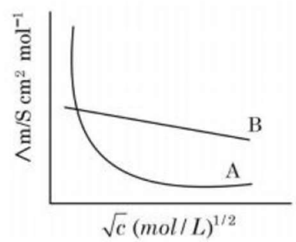
(E) NO_2^+ (F) NO

Answer (03.00)

Sol. The odd electronic species are :

NO_2, ClO_2, NO

55. Following figure shows dependence of molar conductance of two electrolytes on concentration. Λ_m^o is the limiting molar conductivity.



The number of **incorrect** statement(s) from the following is ________

(A) Λ_m^o for electrolyte A is obtained by extrapolation.

(B) For electrolyte B, Λ_m vs $\sqrt{c}$ graph is a straight line with intercept equal to Λ_m^o

(C) At infinite dilution, the value of degree of dissociation approaches zero for electrolyte B.

(D) Λ_m^o for any electrolyte A or B can be calculated using λ° for individual ions.

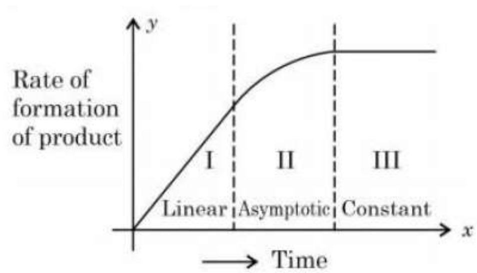
Answer (02.00)

Sol. (A) Λ_m^o for 'A' cannot be obtained by extra polation.

(C) At infinite dilution, value of degree of dissociation approaches one.

$\therefore$ A and C are incorrect

56. For certain chemical reaction $X \rightarrow Y$, the rate of formation of product is plotted against the time as shown in the figure. The number of **correct** statement/s from the following is______



(A) Over all order of this reaction is one

(B) Order of this reaction can't be determined

(C) In region I and III, the reaction is of first and zero order respectively

(D) In region-II, the reaction is of first order

(E) In region-II, the order of reaction is in the range of 0.1 to 0.9.

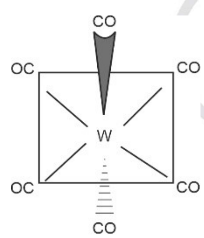
Answer (02.00)

Sol. Ether the order w.r.t. reaction is negative in region I and II or the order of the reaction depends only on the concentration of product. So by that reasoning if order of the reaction depends only on the concentration of product then statement C and E are correct.

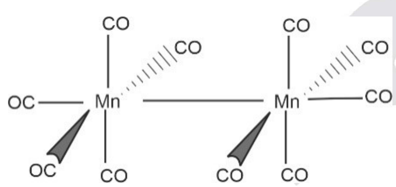
57. The sum of bridging carbonyls in $W(CO)_6$ and $Mn_2(CO)_{10}$ is______.

Answer (00.00)

Sol. $W(CO)_6$

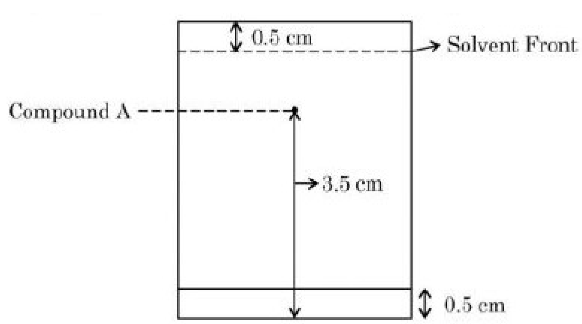


$Mn_2(CO)_{10}$



therefor there are no any bridging carbonyl are present.

58. Following chromatogram was developed by adsorption of compound 'A' on a 6 cm TLC glass plate. Retardation factor of the compound 'A' is______ × 10^{-1}.



Answer (06.00)

Sol. Retardation factor = $\frac{3}{5}$ = 0.6 = 6 × 10^{-1}

59. Consider the following reaction approaching equilibrium at 27°C and 1 atm pressure

$$A + B \underset{k_r = 10^2}{\overset{k_f = 10^3}{\rightleftharpoons}} C + D$$

The standard Gibb's energy change ($\Delta_r G°$) at 27°C is (–)______ kJ mol^{-1}

(Nearest integer).

(Given : R = 8.3 J K^{-1} mol^{-1} and In 10 = 2.3)

Answer (06.00)

Sol. $\Delta G = -RT\ln(k_{eq})$

$$k_{(eq)} = \frac{k_f}{k_r} = 10$$

$\Delta G = -8.3 \times 300 \times \ln(10)$

$= -8.3 \times 300 \times 2.3$

$= -5.727$ kJ/mol

≈ -6 kJ / mol

60. 17 mg of a hydrocarbon (M.F. $C_{10}H_{16}$) takes up 8.40 mL of the H_2 gas measured at 0°C and 760 mm of Hg. Ozonolysis of the same hydrocarbon yields

$$CH_3-\underset{\underset{O}{\|}}{C}-CH_3,\quad H-\underset{\underset{O}{\|}}{C}-H,$$

$$H-\underset{\underset{O}{\|}}{C}-CH_2-CH_2-\underset{\underset{O}{\|}}{C}-\underset{\underset{O}{\|}}{C}-H$$

The number of double bond/s present in the hydrocarbon is________

Answer (03.00)

Sol. $C_{10}H_{16} \quad + \quad H_2 \longrightarrow$

$$\frac{17 \times 10^{-3}}{136} \qquad \frac{1 \times 8.4 \times 10^{-3}}{0.082 \times 273}$$

0.125 milli moles   0.375 milli moles

0.125 milli moles of $C_{10}H_{16}$ required 0.375 milli moles of H_2 therefore there are total 3 π-bonds.

$\therefore$ DOU will also be equal to 3

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

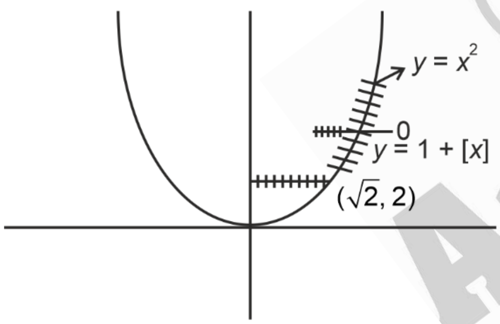
61. Let [x] denote the greatest integer $\le x$. Consider the function $f(x) = \max\{x^2, 1+[x]\}$. Then the value of the integral $\int_0^2 f(x)dx$ is

(1) $\frac{1+5\sqrt{2}}{3}$ (2) $\frac{5+4\sqrt{2}}{3}$

(3) $\frac{8+4\sqrt{2}}{3}$ (4) $\frac{4+5\sqrt{2}}{3}$

Answer (2)

Sol.



$\therefore$ $f(x) = 1 \quad x \in [0, 1)$

$2 \quad x \in [1, \sqrt{2})$

$x^2 \quad x \in [\sqrt{2}, 2]$

$$\therefore \int_0^2 f(x)dx = \int_0^1 1dx + \int_1^{\sqrt{2}} 2dx + \int_{\sqrt{2}}^2 x^2dx$$

$$= 1 + 2\sqrt{2} - 2 + \left.\frac{x^3}{3}\right|_{\sqrt{2}}^2$$

$$= 2\sqrt{2} - 1 + \frac{8}{3} - \frac{2\sqrt{2}}{3}$$

$$= \frac{5+4\sqrt{2}}{3}$$

62. Let the tangents at the points $A(4, -11)$ and $B(8, -5)$ on the circle $x^2 + y^2 - 3x + 10y - 15 = 0$, intersect at the point C. Then the radius of the circle, whose centre is C and the line joining A and B is its tangent, is equal to

(1) $\sqrt{13}$

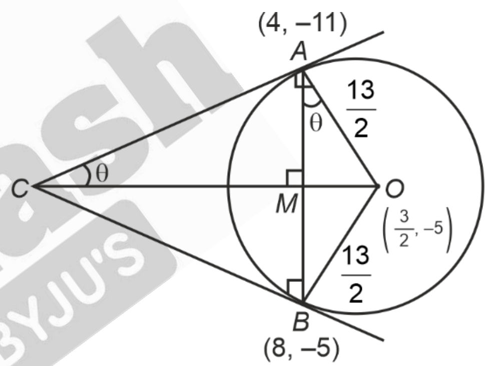
(2) $\frac{3\sqrt{3}}{4}$

(3) $\frac{2\sqrt{13}}{3}$

(4) $2\sqrt{13}$

Answer (3)

Sol.



$$OA = OB = \frac{13}{2}$$

$\because$ $AB = 2\sqrt{13}$ then $AM = \sqrt{13}$

In ΔAMO : $\angle OAM = \theta = \angle ACO$

$$\therefore \quad OC = \frac{13}{2\sin\theta}$$

$$\because \quad \sin\theta = \frac{3\sqrt{13}}{13} = \frac{3}{\sqrt{13}}$$

$$\therefore \quad OC = \frac{13\sqrt{13}}{6}$$

and $OM = \frac{3\sqrt{13}}{2}$

$$\therefore \quad CM = OC - OM = \frac{2\sqrt{13}}{3}$$

63. Let $A = \left\{(x, y) \in \mathbb{R}^2 : y \geq 0,\ 2x \leq y \leq \sqrt{4-(x-1)^2}\right\}$

and

$$B = \left\{(x, y) \in \mathbb{R} \times \mathbb{R} : 0 \leq y \leq \min\left\{2x,\ \sqrt{4-(x-1)^2}\right\}\right\}.$$

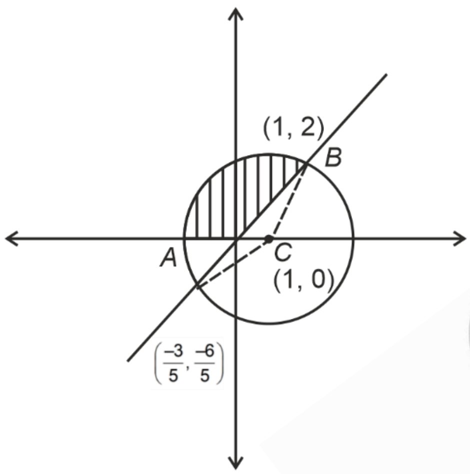
Then the ratio of the area of A to the area of B is

(1) $\dfrac{\pi}{\pi-1}$ (2) $\dfrac{\pi+1}{\pi-1}$

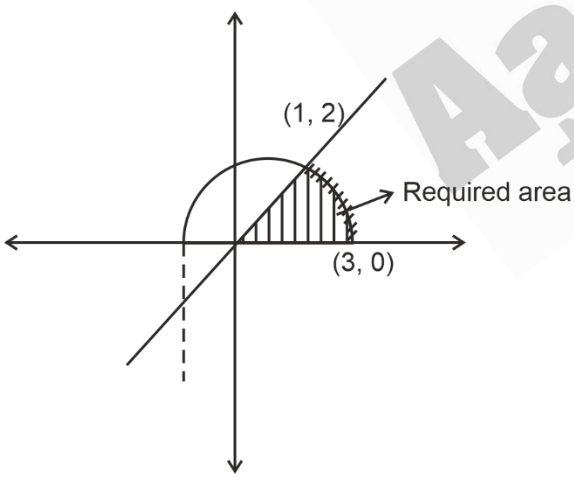
(3) $\dfrac{\pi-1}{\pi+1}$ (4) $\dfrac{\pi}{\pi+1}$

Answer (3)

Sol. For A:



For B:



Area (A) + Area $(B) = 2\pi$

$$\text{Area } B = \int_0^1 2x\,dx + \int_1^3 \sqrt{4-(x-1)^2}\,dx$$

$$= 1 + \frac{\pi 4}{4} = \pi + 1$$

Area $A = \pi - 1$

$\therefore$ Required ratio $= \dfrac{\pi-1}{\pi+1}$

64. Let α and β be real numbers. Consider a 3×3 matrix A such that $A^2 = 3A + \alpha I$. If $A^4 = 21A + \beta I$, then

(1) $\alpha = 1$ (2) $\alpha = 4$

(3) $\beta = 8$ (4) $\beta = -8$

Answer (4)

Sol. $A^4 = A^2A^2$

$= (3A + \alpha I)(3A + \alpha I)$

$= 9A^2 + 6\alpha A + \alpha^2$

$= 9(3A + \alpha I) + 6\alpha A + \alpha^2$

$= (27 + 6\alpha)A + (9\alpha + \alpha^2) = 21A + \beta I$

$\Rightarrow \alpha = -1, \beta = -8$

65. Let $f(\theta) = 3\left(\sin^4\left(\frac{3\pi}{2} - \theta\right) + \sin^4(3\pi + \theta)\right) - 2(1 - \sin^2 2\theta)$

and $S = \left\{\theta \in [0, \pi] : f'(\theta) = -\frac{\sqrt{3}}{2}\right\}$. If $4\beta = \sum_{\theta \in S} \theta$,

then $f(\beta)$ is equal to

(1) $\dfrac{9}{8}$ (2) $\dfrac{3}{2}$

(3) $\dfrac{5}{4}$ (4) $\dfrac{11}{8}$

Answer (3)

Sol. $f(\theta) = 3(\cos^4\theta + \sin^4\theta) - 2(1 - \sin^2 2\theta)$

$f'(\theta) = 12(\cos^3\theta(-\sin\theta) + \sin^3\theta\cos\theta) + 4\sin 4\theta$

$= 12(\cos\theta\sin\theta(-\cos 2\theta)) + 4\sin 4\theta$

$= -6\sin 2\theta\cos 2\theta + 4\sin 4\theta$

$= -3\sin 4\theta + 4\sin 4\theta$

$= \sin 4\theta$

$$\sin 4\theta = -\frac{\sqrt{3}}{2}$$

$$4\theta = \left\{\frac{3\pi}{2} - \frac{\pi}{3}, \frac{3\pi}{2} + \frac{\pi}{3}, \frac{7\pi}{2} - \frac{\pi}{3}, \frac{7\pi}{2} + \frac{\pi}{3}\right\}$$

$$\sum \theta = \frac{5\pi}{2} \quad \Rightarrow \beta = \frac{5\pi}{8}$$

$f(\theta) = 3(1 - 2\sin^2\theta + \cos^2\theta) - 2\cos^2 2\theta$

$$= 3\left(1 - \frac{1}{2}\sin^2\frac{5\pi}{4}\right) - 2\cos^2\frac{5\pi}{4}$$

$$= 3\left(1 - \frac{1}{4}\right) - 2\cdot\frac{1}{2} = \frac{5}{4}$$

66. Consider the following system of equations

$\alpha x + 2y + z = 1$

$2\alpha x + 3y + z = 1$

$3x + ay + 2z = b$

For some $\alpha, \beta \in \mathbb{R}$. then which of the following is NOT correct?

(1) It has no solution if $\alpha = -1$ and $\beta \neq 2$

(2) It has a solution for all $\alpha \neq -1$ and $\beta = 2$

(3) It has no solution for $\alpha = 3$ and for all $\beta \neq 2$

(4) It has no solution for $\alpha = -1$ and for all $\beta \in \mathbb{R}$.

Answer (3)

Sol. $\begin{vmatrix} \alpha & 2 & 1 \\ 2\alpha & 3 & 1 \\ 3 & \alpha & 2 \end{vmatrix} = 0$

$\alpha(6 - \alpha) - 2(4\alpha - 3) + 1(2\alpha^2 - 9) = 0$

$\Rightarrow 6\alpha - \alpha^2 - 8\alpha + 6 + 2\alpha^2 - 9 = 0$

$\Rightarrow \alpha^2 - 2\alpha - 3 = 0$

OR $\alpha = 3, -1$

For $\alpha = 3$, $\beta = 2 \Rightarrow$ Infinite solution

For $\alpha = -1$, $\beta = 2 \Rightarrow$ Infinite solution

For $\alpha = -1, \beta \neq 2 \Rightarrow$ no solution

67. Let $f(x) = x + \dfrac{a}{\pi^2 - 4}\sin x + \dfrac{b}{\pi^2 - 4}\cos x, x \in \mathbb{R}$ be a function which satisfies $f(x) = x + \int\limits_0^{\pi/2} \sin(x + y) f(y)dy$. Then $(a + b)$ is equal to

(1) $-\pi(\pi + 2)$ (2) $-\pi(\pi - 2)$

(3) $-2\pi(\pi + 2)$ (4) $-2\pi(\pi - 2)$

Answer (3)

Sol. $\because \quad f(x) = x + \dfrac{a}{x^2 - 4}\sin x + \dfrac{b}{x^2 - 4}\cos x, x \in R$

And $f(x) = x + \int_0^{\pi/2} \sin(x + y).f(y)dy$

$\Rightarrow \quad f(x) = x + \left(\int_0^{\pi/2} f(y).\cos y dy\right)\sin x$

$+ \left(\int_0^{\pi/2} f(y)\sin y dy\right)\cos x$

$\therefore \quad \dfrac{a}{\pi^2 - 4} = \int_0^{\pi/2} \cos y\left(y + \dfrac{a}{\pi^2 - 4}\sin y + \dfrac{b}{\pi^2 - 4}\cos y\right)dy$

$$\frac{a}{\pi^2 - 4} = \frac{\pi}{2} - 1 + \frac{a}{2(\pi^2 - 4)} + \frac{b\pi}{4(\pi^2 - 4)}$$

$\therefore \quad 2a - b\pi = 2(\pi + 2)(\pi - 2)^2 \quad$ ...(i)

and $\dfrac{b}{\pi^2 - 4} = \int_0^{\pi/2}\left(y + \dfrac{a}{\pi^2 - 4}\sin y + \dfrac{b}{\pi^2 - 4}\cos y\right)\sin y\, dy$

$$\frac{b}{\pi^2 - 4} = 1 + \frac{a\pi}{4(\pi^2 - 4)} + \frac{b}{2(\pi^2 - 4)}$$

$\therefore \quad a\pi - 2b = -4(\pi^2 - 4) \quad$ ...(ii)

Equation (i) – equation (ii)

$(2 - \pi)(a + b) = 2(\pi^2 - 4)(\pi - 2 + 2)$

$\therefore \quad a + b = -2\pi(\pi + 2)$

68. Fifteen football players of a club-team are given 15 T-shirts with their names written on the backside. If the players pick up the T-shirts randomly, then the probability that at least 3 players pick the correct T-shirt is

(1) $\dfrac{1}{6}$

(2) $\dfrac{5}{36}$

(3) $\dfrac{2}{15}$

(4) $\dfrac{5}{24}$

Answer (*)

69. If p, q and r are three propositions, then which of the following combination of truth values of p, q and r makes the logical expression $\{(p \vee q) \wedge ((\sim p) \vee r)\} \rightarrow ((\sim q) \vee r)$ false ?

(1) $p = F, q = T, r = F$

(2) $p = T, q = F, r = T$

(3) $p = T, q = T, r = F$

(4) $p = T, q = F, r = F$

Answer (1)

Sol. $\{(p \vee q) \wedge ((\sim p) \vee r)\} \rightarrow ((\sim q) \vee r)$

Is false when

$\{(p \vee q) \wedge ((\sim p) \vee r)\}$ T and $\sim q \vee r = F$

So, $(p \vee q) = \text{T}$ and $\sim p \vee r = T$ and

$\sim q = F$ and $r = \text{F}$

So, $q = T$, $r = F$, and $\sim p = T$

$\therefore \quad p = F$

$\therefore \quad p = F, q = T, r = F$

70. Let $x = 2$ be a root of the equation $x^2 + px + q = 0$ and

$$f(x) - \begin{cases} \dfrac{1-\cos\left(x^2 - 4px + q^2 + 8q + 16\right)}{(x-2p)^4}, & x \neq 2p \\ 0, & x = 2p \end{cases}$$

Then $\lim\limits_{x \to 2p^+} [f(x)]$,

where $[\cdot]$ denotes greatest integer function, is

(1) 1 (2) 2

(3) 0 (4) –1

Answer (3)

Sol. $4 + 2p + q = 0$ …(i) $\Rightarrow 4p^2 = q^2 + 8q + 16$

For $\lim\limits_{x \to 2p^+} f(x)$ put $x = 2p + h$

$$\Rightarrow \lim_{h \to 0} \frac{\left(1-\cos\left((2p+h)^2 - 4p(2p+h) + q^2 + 8q + 16\right)\right)}{h^4}$$

$$\Rightarrow \lim_{h \to 0} \left(\frac{1-\cos\left(h^2 - 4p^2 + q^2 + 8q + 16\right)}{h^4}\right)$$

$$\Rightarrow \lim_{h \to 0} \frac{1-\cos h^2}{h^4} = \frac{1}{2}$$

$$\therefore \left[\lim_{x \to 2p^+} f(x)\right] = 0$$

71. Let B and C be the two points on the line $y + x = 0$ such that B and C are symmetric with respect to the origin. Suppose A is a point on $y - 2x = 2$ such that ΔABC is an equilateral triangle. Then, the area of the ΔABC is

(1) $3\sqrt{3}$ (2) $2\sqrt{3}$

(3) $\dfrac{10}{\sqrt{3}}$ (4) $\dfrac{8}{\sqrt{3}}$

Answer (4)

Sol. Origin (O) is mid-point of BC ($x + y = 0$).

A lies on perpendicular bisector of BC, which is $x - y = 0$

A is point of intersection of $x - y = 0$ and $y - 2x = 2$

$\therefore \quad A \equiv (-2, -2)$

Let $h = AO = \dfrac{-2-2}{\sqrt{1^2 + 1^2}} = 2\sqrt{2}$

Area $= \dfrac{h^2}{\sqrt{3}} = \dfrac{8}{\sqrt{3}}$

72. If the vectors $\vec{a} = \lambda\hat{i} + \mu\hat{j} + 4\hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} + 3\hat{j} + \hat{k}$ are coplanar and the projection of $\vec{a}$ on the vector $\vec{b}$ is $\sqrt{54}$ units, then the sum of all possible values of $\lambda + \mu$ is equal to

(1) 24

(2) 0

(3) 6

(4) 18

Answer (1)

Sol. $\vec{a} = \lambda\hat{i} + \mu\hat{j} + 4\hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} - 2\hat{k}$, $\vec{c} = 2\hat{i} + 3\hat{j} + \hat{k}$

Now, $\vec{a} \cdot \vec{b} = \sqrt{54} \Rightarrow \dfrac{-2\lambda + 4\mu - 8}{\sqrt{24}} = \sqrt{54}$

$\Rightarrow -2\lambda + 4\mu - 8 = 36$

$\Rightarrow 2\mu - \lambda = 22$ …(i)

and $\begin{vmatrix} \lambda & \mu & 4 \\ -2 & 4 & -2 \\ 2 & 3 & 1 \end{vmatrix} = 0$

$10\lambda - 2\mu - 56 = 0$ …(ii)

By (i) & (ii) $\lambda = \dfrac{78}{9}, \mu = \dfrac{138}{9}$

$\therefore \quad \mu + \lambda = 24$

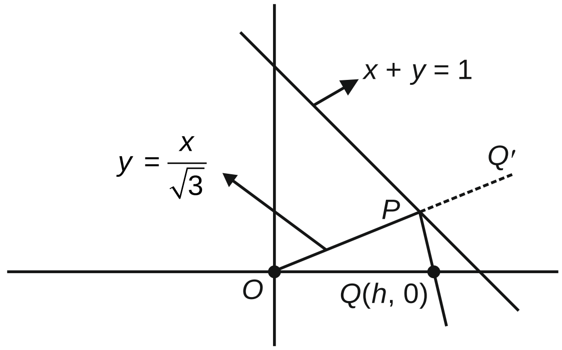
73. A light ray emits from the origin making an angle 30° with the positive x-axis. After getting reflected by the line $x + y = 1$, if this ray intersects x-axis at Q, then the abscissa of Q is

(1) $\dfrac{2}{3-\sqrt{3}}$ (2) $\dfrac{2}{3+\sqrt{3}}$

(3) $\dfrac{\sqrt{3}}{2\left(\sqrt{3}+1\right)}$ (4) $\dfrac{2}{\left(\sqrt{3}-1\right)}$

Answer (2)

Sol.



Let $Q(h, O)$

$\because$ OP reflected by $x + y = 1$.

So, image of Q lies on $y = \frac{x}{\sqrt{3}}$

$\therefore \quad \frac{x-h}{1} = \frac{y}{1} = \frac{-2(h-1)}{2}$

$\therefore \quad x = 1, y = 1 - h$

It lies on $y = \frac{x}{\sqrt{3}}$

$\therefore \quad 1 - h = \frac{1}{\sqrt{3}}$

$\therefore \quad h = 1 - \frac{1}{\sqrt{3}} = \frac{\sqrt{3}-1}{\sqrt{3}} = \frac{2}{3+\sqrt{3}}$

Option (2) is correct.

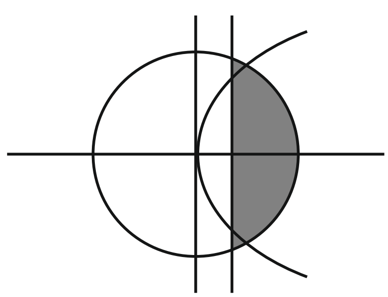
74. Let Δ be the area of the region $\left\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \le 21,\ y^2 \le 4x,\ x \ge 1\right\}$. Then $\frac{1}{2}\left(\Delta - 21\sin^{-1}\frac{2}{\sqrt{7}}\right)$ is equal to

(1) $\sqrt{3} - \frac{4}{3}$ (2) $2\sqrt{3} - \frac{1}{3}$

(3) $\sqrt{3} - \frac{2}{3}$ (4) $2\sqrt{3} - \frac{2}{3}$

Answer (1)

Sol.



Required area $= 2\left[\int_1^3 2\sqrt{x}\,dx + \int_3^{\sqrt{21}} \sqrt{(21-x^2)}\,dx\right]$

$= 2\left(\left[2\left(\frac{x^{3/2}}{3/2}\right)\right]_1^3 + \left[\frac{x}{2}\sqrt{21-x^2} + \frac{21}{2}\sin^{-1}\left(\frac{x}{\sqrt{21}}\right)\right]_3^{\sqrt{21}}\right)$

$= 2\sqrt{3} + \frac{21\pi}{2} - \frac{8}{3} - 21\sin^{-1}\sqrt{\frac{3}{7}} = \Delta$

$\therefore \frac{1}{2}\left(\Delta - 21\sin^{-1}\left(\frac{2}{\sqrt{7}}\right)\right) = \frac{\sqrt{3}-4}{3}$

Option (1) is correct.

75. The domain of $f(x) = \frac{\log_{(x+1)}(x-2)}{e^{2\log_e x} - (2x+3)}, x \in \mathbb{R}$ is

(1) $\mathbb{R} - \{-1, 3\}$ (2) $(-1, \infty) - \{3\}$

(3) $\mathbb{R} - \{3\}$ (4) $(2, \infty) - \{3\}$

Answer (4)

Sol. $f(x) = \frac{\log_{x+1}(x-2)}{e^{2\ln x} - (2x+3)}$

(i) $x - 2 > 0 \Rightarrow x > 2$

(ii) $x + 1 > 0 \Rightarrow x > -1$ and $x \ne -1$

(iii) $x > 0$

(iv) $x^2 - 2x - 3 \ne 0$

$\Rightarrow (x - 3)(x + 1) \ne 0$

$\Rightarrow x \ne -1, 3$

(i) $\cap$ (ii) $\cap$ (iii) $\cap$ (iv)

$x \in (2, \infty) - \{3\}$

76. Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $f(x) = \frac{x^2+2x+1}{x^2+1}$. Then

(1) $f(x)$ is many-one in $(-\infty, -1)$

(2) $f(x)$ is one-one in $[1, \infty)$ but not in $(-\infty, \infty)$

(3) $f(x)$ is many-one in $(1, \infty)$

(4) $f(x)$ is one-one in $(-\infty, \infty)$

Answer (2)

Sol. $f(x) = \frac{x^2+2x+1}{x^2+1}$, where $f : \mathbb{R} \to \mathbb{R}$

$= \frac{(x+1)^2}{x^2+1} \ge 0$

$$f'(x)=\frac{(x^2+1)(2x+2)-(x^2-2x+1)(2x)}{(x^2+1)^2}$$

$$=\frac{2(x+1)(x^2+1)-(x+1)^2(2x)}{(x^2+1)^2}$$

$\Rightarrow$ $2(x + 1)(x^2 + 1 - (x + 1)x) = 0$

$\Rightarrow$ $2(x + 1)(x - 1) = 0$

$\Rightarrow$ $x = 1, -1 \Rightarrow$ points of minima and maxima

77. Let $y = f(x)$ be the solution of the differential equation $y(x + 1)dx - x^2dy = 0$, $y(1) = e$. Then $\lim_{x\to 0} f(x)$ is equal to

(1) $\frac{1}{e}$ (2) 0

(3) $\frac{1}{e^2}$ (4) e^2

Answer (2)

Sol. $y(x + 1)dx = x^2dy$

$\Rightarrow$ $\left(\frac{x+1}{x^2}\right)dx=\frac{dy}{y}$

$\Rightarrow$ $\ln x-\frac{1}{x}=\ln y+c$

$x = 1, y = e$

$\Rightarrow$ $c = -2$

$\Rightarrow$ $\ln y=\ln x-\frac{1}{x}+2$

$y= xe^{2-\frac{1}{x}}$

$\lim_{x\to 0^+} y=0\times e^{-\infty}=0$

78. For two non-zero complex numbers z_1 and z_2, if $Re(z_1z_2) = 0$ and $Re(z_1 + z_2)$, then which of the following are possible?

A. $\text{Im}(z_1) > 0$ and $\text{Im}(z_2) > 0$

B. $\text{Im}(z_1) < 0$ and $\text{Im}(z_2) > 0$

C. $\text{Im}(z_1) > 0$ and $\text{Im}(z_2) < 0$

D. $\text{Im}(z_1) < 0$ and $\text{Im}(z_2) < 0$

Choose the correct answer from the options given below

(1) B and C (2) B and D

(3) A and B (4) A and C

Answer (1)

Sol. Let $z_1 = x_1 + iy_1$

$z_2 = x_2 + iy_2$

$\Rightarrow$ $x_1x_2 - y_1y_2 = 0$ …(i)

$(x_1 + x_2) = 0$ …(ii)

$x_1^2+y_1y_2=0$

$\Rightarrow$ $y_1y_2=-x_1^2$

$\Rightarrow$ y_1 and y_2 have opposite signs.

79. Three rotten apples are mixed accidently with seven good apples and four apples are drawn one by one without replacement. Let the random variable X denote the number of rotten apples. If μ and σ^2 represent mean and variance of X, respectively, then $10(\mu^2 + \sigma^2)$ is equal to

(1) 25

(2) 250

(3) 30

(4) 20

Answer (4)

Sol.

x_i	0	1	2	3
p_i	$\frac{35}{210}=\frac{1}{6}$	$\frac{105}{210}=\frac{1}{2}$	$\frac{3\times 21}{210}=\frac{3}{10}$	$\frac{7}{210}=\frac{1}{30}$

$$\mu=\sum p_ix_i=\frac{1}{2}+\frac{6}{10}+\frac{21}{210}$$

$$=\frac{1}{2}+\frac{3}{5}+\frac{1}{10}$$

$$=\frac{6}{5}$$

$$\sigma^2=\sum p_ix_1^2-\mu^2$$

$$=\left(\frac{1}{2}+\frac{4.3}{10}+9\cdot\frac{1}{30}\right)-\left(\frac{6}{5}\right)^2$$

$$=\left(\frac{1}{2}+\frac{6}{5}+\frac{3}{10}\right)-\frac{36}{25}$$

$$=\frac{14}{25}$$

Now, $10(\mu^2 + \sigma^2)$

$= 20$

80. Let $\lambda \neq 0$ be a real number. Let α, β be the roots of the equation $14x^2 - 31x + 3\lambda = 0$ and α, γ be the roots of the equation $35x^2 - 53x + 4\lambda = 0$. Then $\frac{3\alpha}{\beta}$ and $\frac{4\alpha}{\gamma}$ are the roots of the equation

(1) $7x^2 + 245x - 250 = 0$

(2) $49x^2 + 245x + 250 = 0$

(3) $7x^2 - 245x + 250 = 0$

(4) $49x^2 - 245x + 250 = 0$

Answer (4)

Sol. $35x^2 - 53x + 4\lambda = 0$ ...(i)

$(14x^2 - 31x + 3\lambda = 0) \times 2.5$ ...(ii)

(i) and (ii) gives

$$x = \frac{\lambda}{7} = \alpha$$

$$\alpha\beta = \frac{3\lambda}{14} \Rightarrow \beta = \frac{3\lambda}{14} \cdot \frac{7}{\lambda} = \frac{3}{2}$$

$$\alpha\gamma = \frac{4\lambda}{35} \Rightarrow \gamma = \frac{4}{35} \cdot 7 = \frac{4}{5}$$

$$\alpha + \beta = \frac{31}{14} \Rightarrow \alpha = \frac{5}{7}$$

$$\frac{3\alpha}{\beta} = \frac{10}{7}, \frac{4\alpha}{\gamma} = \frac{20}{7} \cdot \frac{5}{4} = \frac{25}{7}$$

Equation formed will be

$$x^2 - 5x + \frac{250}{49} = 0$$

$49x^2 - 245x + 250 = 0$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

81. Let a_1, a_2, a_3, ... be a *G.P* of increasing positive numbers. If the product of fourth and sixth terms is 9 and the sum of fifth and seventh terms is 24, then $a_1a_9 + a_2a_4a_9 + a_5 + a_7$ is equal to __________.

Answer (60)

Sol. Let r be the common ratio of the *G.P*

$\therefore$ $a_1r^3 \times a_1r^5 = 9$

$a_1^2r^8 = 9 \Rightarrow a_1r^4 = 3$

And

$a_1(r^4 + r^6) = 24$

$\Rightarrow$ $3(1 + r^2) = 24$

$\therefore$ $r^2 = 7$ and $a_1 = \frac{3}{49}$

Now

$a_1a_9 + a_2a_4a_9 + a_5 + a_7$

$$= a_1^2r^8 + a_1^3r^{12} + 24$$

$$= 24 + \frac{9}{7^4} \times 7^4 + \frac{27}{7^6} \cdot 7^6 = 60$$

82. Five digit numbers are formed using the digits 1, 2, 3, 5, 7 with repetitions and are written in descending order with serial number. For example, the number 77777 has serial number 1. Then the serial number of 35337 is _________.

Answer (1436)

Sol. Given digits 1, 2, 3, 5, 7

and number 35337

7 _ _ _ _ = 5^4 = 625

5 _ _ _ _ = 5^4 = 625

37 _ _ _ = 5^3 = 125

357 _ _ = 5^2 = 25

355 _ _ = 5^2 = 25

3537 _ = 5

3535 _ = 5

35337 = 1

$\therefore$ Serial no. = 1436

83. Let the equation of the plane P containing the line $x + 10 = \frac{8 - y}{2} = z$ be $ax + by + 3z = 2(a + b)$ and the distance of the plane P from the point (1, 27, 7) be c. Then $a^2 + b^2 + c^2$ is equal to ____________.

Answer (355)

Sol. Equation of the line:

$$x + 10 = \frac{8 - y}{2} = z$$

and plane P : $ax + by + 3z = 2(a + b)$

$\because$ line lies in P

$\therefore \quad -10a + 8b = 2a + 2b$

$12a = 6b \quad \Rightarrow \quad \boxed{b = 2a}$

and

$a - 2b + 3 = 0$

So, $\boxed{a = 1}$ $\boxed{b = 2}$

Now distance of (1, 27, 7) from $P = c$

$$\Rightarrow \quad \frac{1 + 54 + 21 - 6}{\sqrt{14}} = c$$

$$\therefore \quad a^2 + b^2 + c^2 = 1 + 4 + \frac{4900}{14}$$

$$= 355$$

84. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function that satisfies the relation $f(x + y) = f(x) + f(y) - 1$, $\forall x, y \in \mathbb{R}$. If $f'(0) = 2$, then $|f(-2)|$ is equal to ___________.

Answer (03)

Sol. $f(x + y) = f(x) + f(y) - 1 \qquad f(0) = 1$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x) + f(h) - 1 - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{f(h) - 1}{h}$$

$f'(x) = f'(0)$

$f'(x) = 2$

$f(x) = 2x + c$

$\because \quad f(0) = 1$

$1 = c$

$\Rightarrow \quad c = 1$

$f(x) = 2x + 1$

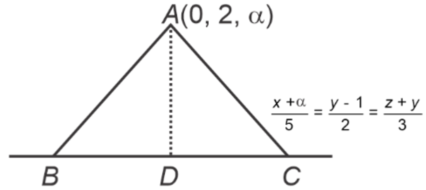
$|f(-2)| = |2(+2) - 1|$

$= 3$

85. Let the co-ordinates of one vertex of ΔABC be $A(0, 2, \alpha)$ and the other two vertices lie on the line $\frac{x + \alpha}{5} = \frac{y - 1}{2} = \frac{z + 4}{3}$. For $\alpha \in Z$, if the area of ΔABC is 21 sq. units and the line segment BC has length $2\sqrt{21}$ units, then α^2 is equal to ___________.

Answer (09)

Sol.



Let coordinate of $D = (5k - \alpha, 2k + 1, 3k - 4)$

$\therefore \quad D.R^s.$ of $AD = \langle 5k - \alpha, 2k - 1, 3k - 4 - \alpha \rangle$

$\therefore \quad 5(5k - \alpha) + 2(2k - 1) + 3(3k - 4 - \alpha) = 0$

$\therefore \quad 19k - 4\alpha - 7 = 0 \qquad \ldots$(i)

and $\frac{1}{2} \times 2\sqrt{21} \times AD = 21$

$\therefore \quad AD = \sqrt{21}$

$\therefore \quad (5k - \alpha)^2 + (2k - 1)^2 + (3k - 4 - \alpha)^2 = 21$

$\therefore \quad 19k^2 - 8k\alpha + \alpha^2 - 14k + 4\alpha = 2 \ldots$(ii)

from eq. (i) and (ii) : $\alpha = 3$

$\therefore \quad \alpha^2 = 9$

86. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero non-coplanar vectors. Let the position vectors of four points A, B, C and D be $\vec{a} - \vec{b} + \vec{c}, \lambda\vec{a} - 3\vec{b} + 4\vec{c}, -\vec{a} + 2\vec{b} - 3\vec{c}$ and $2\vec{a} - 4\vec{b} + 6\vec{c}$ respectively. If $\overrightarrow{AB}, \overrightarrow{AC}$ and $\overrightarrow{AD}$ are coplanar, then λ is equal to ____.

Answer (2)

Sol. $\overrightarrow{AB} = (\lambda - 1)\vec{a} + (-2)\vec{b} + 3\vec{c}$

$\overrightarrow{AC} = -2\vec{a} + 3\vec{b} - 4\vec{c}$

$\overrightarrow{AD} = -\vec{a} - 3\vec{b} + 5\vec{c}$

$\because \quad \overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$ are co-planar

$$\begin{vmatrix} \lambda - 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix} [\vec{a}\ \vec{b}\ \vec{c}\,] = 0$$

$$\Rightarrow \begin{vmatrix} \lambda - 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix} = 0$$

$3(\lambda - 1) - 2(6) + 3(6 - 3) = 0$

$3(\lambda - 1) - 12 + 9 = 0$

$3(\lambda - 1) = 3$

$\lambda = 2$

87. Let the coefficients of three consecutive terms in the binomial expansion of $(1 + 2x)^n$ be the ratio 2 : 5 : 8. Then the coefficient of the term, which is in the middle of these three terms, is _______.

Answer (1120)

Sol. $\dfrac{{}^nC_r\ 2^r}{{}^nC_{r+1}\ 2^{r+1}} = \dfrac{2}{5}$

$\dfrac{r+1}{n-\gamma} = \dfrac{4}{5}$ ...(i)

$\dfrac{{}^nC_{r+1}\ 2^{r+1}}{{}^nC_{r+2}\ 2^{r+2}} = \dfrac{5}{8}$

$\dfrac{r+2}{n-r-1} = \dfrac{5}{4}$ ...(ii)

Solving (i) and (ii)

$r = 3$, $n = 8$

Middle term = ${}^nC_{r+1}\,(2)^{r+1}$

$= {}^8C_4\ (2)^4$

$= 1120$

88. Suppose f is a function satisfying $f(x + y) + f(x) + f(y)$ for all x, $y \in \mathbb{N}$ and $f(1) = \dfrac{1}{5}$. If $\displaystyle\sum_{n=1}^{m} \frac{f(n)}{n(n+1)(n+2)} = \frac{1}{12}$, then m is equal to ________.

Answer (10)

Sol. $f(x + y) = f(x) + f(y)$

$\Rightarrow$ $f(x) = kx$

$f(1) = \dfrac{1}{5} \Rightarrow k = \dfrac{1}{5}$

$\therefore$ $f(x) = \dfrac{1}{5}x$

$$\sum_{n=1}^{m} \frac{f(n)}{n(n+1)(n+2)} = \frac{1}{5}\sum_{n=1}^{m} \frac{n}{n(n+1)(n+2)} = \frac{1}{12}$$

$$\Rightarrow \frac{1}{5}\sum_{n=1}^{m} \frac{1}{(n+1)(n+2)} = \frac{1}{12}$$

$$\Rightarrow \frac{1}{5}\left[\frac{1}{2} - \frac{1}{m+2}\right] = \frac{1}{12}$$

$\Rightarrow$ $m = 10$

89. If all the six digit numbers $x_1\ x_2\ x_3\ x_4\ x_5\ x_6$ with $0 < x_1 < x_2 < x_3 < x_4 < x_5 < x_6$ are arranged in the increasing order, then the sum of the digits in the 72[th] number is _____________.

Answer (32)

Sol. 1 ....$\rightarrow$ ${}^8C_5 = 56$

23 ....$\rightarrow$ ${}^6C_4 = \dfrac{15}{71}$

72[th] number = 245678

Sum = 32

90. If the co-efficient of x^9 in $\left(\alpha x^3 + \dfrac{1}{\beta x}\right)^{11}$ and the co-efficient of x^{-9} in $\left(\alpha x - \dfrac{1}{\beta x^3}\right)^{11}$ are equal, then $(\alpha\beta)^2$ is equal to ________.

Answer (01)

Sol. $T_{r_1+1} = {}^{11}C_{r_1}\left(\alpha x^3\right)^{11-r_1}(\beta x)^{-r_1}$

$= {}^{11}C_{r_1}\ \alpha^{11-r_1}\ \beta^{-r_1}\ x^{33-4r_1}$

$33 - 4r_1 = 9 \Rightarrow r_1 = 6$

$T_{r_2+1} = {}^{11}C_{r_2}\ (\alpha x)^{11-r_2}\ (-1)^{r_2}\ \left(\beta x^3\right)^{-r_2}$

$= (-1)^{r_2}\ {}^{11}C_{r_2}\ \alpha^{11-r_2}\ \beta^{-r_2}\ x^{11-4r_2}$

$11 - 4r_2 = -9 \Rightarrow r_2 = 5$

Equating the coefficients

${}^{11}C_6\ \alpha^5\beta^{-6} = {}^{11}C_5\ \alpha^6\ \beta^{-5}$

$\Rightarrow$ $\alpha\beta = 1$

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