30/01/2023
Morning

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Answers & Solutions

Time : 3 hrs. M.M. : 300

for

JEE (Main)-2023 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Choose the correct relationship between Poisson ratio (σ), bulk modulus (K) and modulus of rigidity (η) of a given solid object

(1) $\sigma = \frac{6K - 2\eta}{3K - 2\eta}$ (2) $\sigma = \frac{6K + 2\eta}{3K - 2\eta}$

(3) $\sigma = \frac{3K - 2\eta}{6K + 2\eta}$ (4) $\sigma = \frac{3K + 2\eta}{6K + 2\eta}$

Answer (3)

Sol. Poisson ratio (σ), bulk modulus (*K*) and modulus of rigidity (η) are related by

$\because\ 2\eta(1 + \sigma) = 3K(1 - 2\sigma)$

$2\eta + 2\eta\sigma = 3K - 6K\sigma$

$\sigma = \frac{3K - 2\eta}{2\eta + 6K}$

2. A small object at rest, absorbs a light pulse of power 20 mW and duration 300 ns. Assuming speed of light as 3 × 10^8 m/s, the momentum of the object becomes equal to

(1) 2 × 10^{-17} kg m/s (2) 3 × 10^{-17} kg m/s

(3) 1 × 10^{-17} kg m/s (4) 0.5 × 10^{-17} kg m/s

Answer (1)

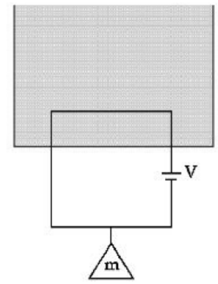
Sol. Assuming the small object as photon.

Momentum $(p) = \frac{E}{C}$

$= \frac{20 \times 10^{-3} \times 300 \times 10^{-9}}{3 \times 10^{8}}$

= 2 × 10^{-17} kg m/s

3. A massless square loop, of wire of resistance 10 Ω, supporting a mass of 1 g, hangs vertically with one of its sides in a uniform magnetic field of 10^3 G, directed outwards in the shaded region. A dc voltage *V* is applied to the loop. For what value of *V*, the magnetic force will exactly balance the weight of the supporting mass of 1 g? (If sides of the loop = 10 cm, *g* = 10 m/s^2)



(1) 1 V (2) 10 V

(3) $\frac{1}{10}$ V (4) 100 V

Answer (2)

Sol. For balancing of force

$\therefore\ F_{loop}$ = weight

$\left(\frac{V}{R}\right) IB = mg$

$\left(\frac{V}{10}\right) \times \frac{10}{100} \times (10^3 \times 10^{-4}) = \left(\frac{1}{1000}\right) \times 10$

V = 10 volts

4. The magnetic moment associated with two closely wound circular coils *A* and *B* of radius r_A = 10 cm and r_B = 20 cm respectively are equal if: (where N_A, I_A and N_B, I_B are number of turn and current of *A* and B respectively)

(1) $N_AI_A = 4N_BI_B$ (2) $2N_AI_A = N_BI_B$

(3) $N_A = 2N_B$ (4) $4N_AI_A = N_BI_B$

Answer (1)

Sol. $M_A = M_B$

$I_A N_A (\pi r_A^2) = I_B N_B (\pi r_B^2)$

$I_A N_A = 4 I_B N_B$

5. A ball of mass 200 g rests on a vertical post of height 20 m. A bullet of mass 10 g, travelling in horizontal direction, hits the centre of the ball. After collision both travels independently. The ball hits the ground at a distance 30 m and bullet at a distance of 120 m from the foot of the post. The value of initial velocity of the bullet will be (if g = 10 m/s^2)

(1) 60 m/s (2) 120 m/s

(3) 400 m/s (4) 360 m/s

Answer (4)

Sol. $\because$ Time of flight of each ball and bullet

$$= \sqrt{\frac{2H}{g}} = \sqrt{\frac{2\times 20}{10}} = 2\text{ s}$$

$\Rightarrow$ By applying linear momentum conservation

$$10u + 200(0) = 200\left(\frac{30}{2}\right) + 10\left(\frac{120}{2}\right)$$

u = 360 m/s

6. Two isolated metallic solid spheres of radii R and $2R$ are charged such that both have same charge density σ. The spheres are then connected by a thin conducting wire. If the new charge density of the bigger sphere is σ'. The ratio $\frac{\sigma'}{\sigma}$ is

(1) $\frac{5}{6}$ (2) $\frac{4}{3}$

(3) $\frac{5}{3}$ (4) $\frac{9}{4}$

Answer (1)

Sol. $\sigma = \frac{Q_1}{4\pi R^2} = \frac{Q_2}{4\pi(2R)^2}$

Now $Q_2' = Q_{\text{total}}\left[\frac{R_2}{R_1+R_2}\right]$

$$= (Q_1 + Q_2)\left[\frac{2R}{3R}\right]$$

$$= \sigma(20\pi R^2)\frac{2}{3}$$

$$\therefore \quad \sigma_2' = \frac{Q_2'}{4\pi(2R)^2} = \frac{\sigma(20\pi R^2)\frac{2}{3}}{16\pi R^2}$$

$$= \frac{5}{4}\times\frac{2}{3}\sigma$$

$$= \frac{5}{6}\sigma$$

7. The pressure (P) and temperature (T) relationship of an ideal gas obeys the equation PT^2 = constant. The volume expansion coefficient of the gas will be

(1) $\frac{3}{T}$ (2) $\frac{3}{T^2}$

(3) $\frac{3}{T^3}$ (4) $3T^2$

Answer (1)

Sol. PT^2 = constant

From $PV = nRT \Rightarrow \frac{T^3}{V}$ = constant

$T^3 \propto V$ ...(1)

$3T^2dT \propto dV$ ...(2)

From (1) and (2)

$$\frac{3dT}{T} = \frac{dV}{V}$$

$$\therefore \quad \gamma = \frac{1}{V}\frac{dV}{dT} = \frac{3}{T}$$

8. Heat is given to an ideal gas in an isothermal process.

A. Internal energy of the gas will decrease.

B. Internal energy of the gas will increase.

C. Internal energy of the gas will not change.

D. The gas will do positive work.

E. The gas will do negative work.

Choose the correct answer from the options given below:

(1) C and E only (2) C and D only

(3) A and E only (4) B and D only

Answer (2)

Sol. Isothermal process $\Delta T = 0$

$$\Delta U = \frac{f}{2}nR\Delta T$$

$\Delta U = 0$

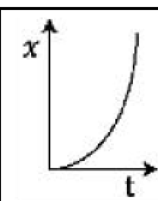
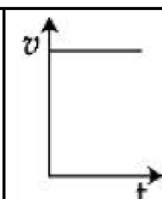
No change in internal energy

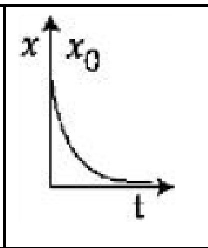
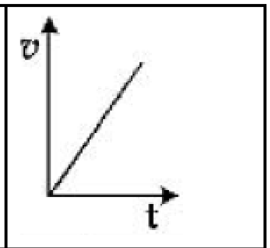
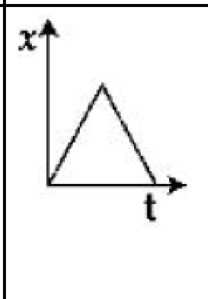
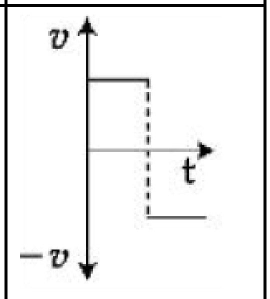
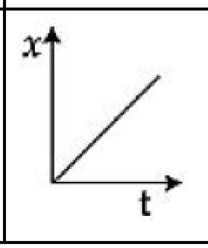
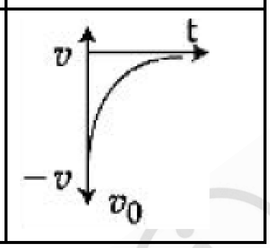
$\Delta Q = \Delta W$ (1st law)

ΔQ = +ve

ΔW = +ve

9. Match Column-I with Column-II :

Column-I (*x-t* graphs)		Column-II (*v-t* graphs)	
A.		I.	

B.		II.	
C.		III.	
D.		IV.	

Choose the correct answer from the options given below :

(1) A-I, B-III, C-IV, D-II

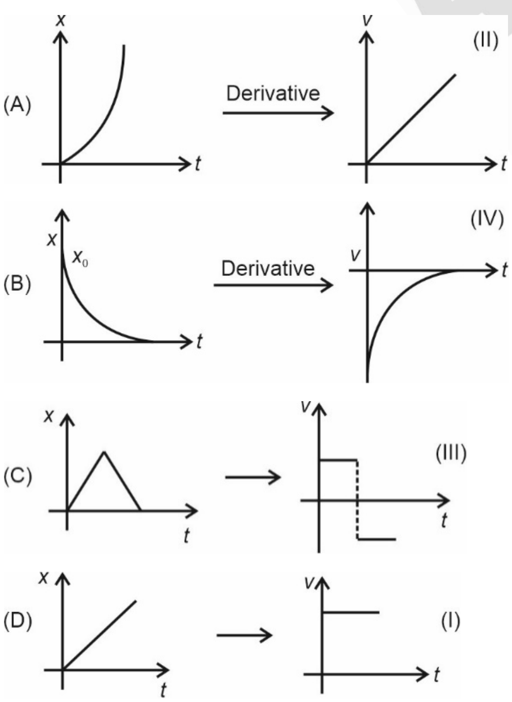
(2) A-II, B-III, C-IV, D-I

(3) A-I, B-II, C-III, D-IV

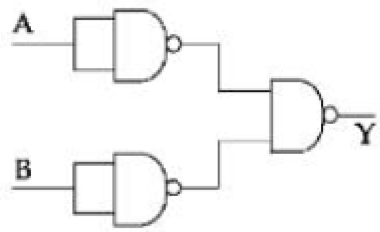
(4) A-II, B-IV, C-III, D-I

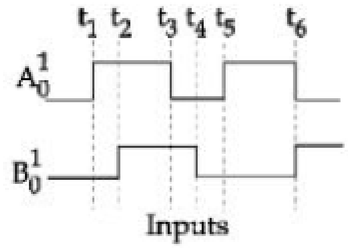
Answer (4)

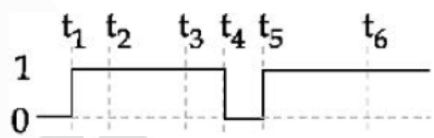
Sol.

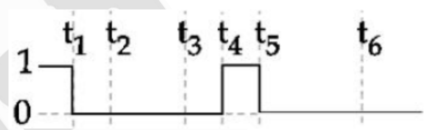


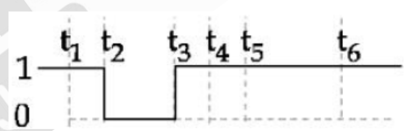
10. The output waveform of the given logical circuit for the following inputs A and B as shown below, is

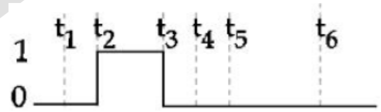




(1) 

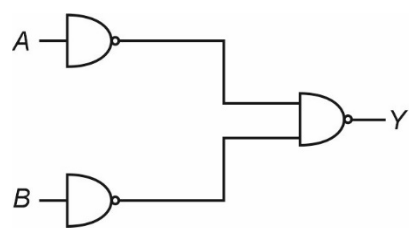
(2) 

(3) 

(4) 

Answer (1)

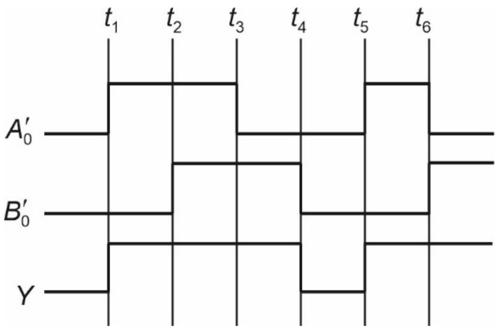
Sol.



Truth table

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

OR Gate



11. The charge flowing in a conductor changes with time as $Q(t) = \alpha t - \beta t^2 + \gamma t^3$. Where α, β and γ are constants. Minimum value of current is

(1) $\alpha - \frac{\gamma^2}{3\beta}$ (2) $\alpha - \frac{\beta^2}{3\gamma}$

(3) $\alpha - \frac{3\beta^2}{\gamma}$ (4) $\beta - \frac{\alpha^2}{3\gamma}$

Answer (2)

Sol. $Q(t) = \alpha t - \beta t^2 + \gamma t^3$

$i(t) = \alpha - 2\beta t + 3\gamma t^2$

$\frac{di}{dt} = -2\beta + 6\gamma t = 0$ (for max/min of i)

at $t = \frac{\beta}{3r}$ (i is minimum as i is an upward parabola)

$i\left(\frac{\beta}{3\gamma}\right) = \alpha - 2\beta\left(\frac{\beta}{3\gamma}\right) + \frac{3\gamma\beta^2}{9\gamma^2}$

$= \alpha \frac{-\beta^2}{3\gamma}$

12. The height of liquid column raised in a capillary tube of certain radius when dipped in liquid A vertically is, 5 cm. If the tube is dipped in a similar manner in another liquid B of surface tension and density double the values of liquid A, the height of liquid column raised in liquid B would be _______m.

(1) 0.20 (2) 0.05

(3) 0.5 (4) 0.10

Answer (2)

Sol. height of capillary rise $= \frac{2s\cos\theta}{\rho g R}$

When in A $5\ cm = \frac{2s_A\cos\theta}{\rho_A g R}$

When in B $h = \frac{2s_B\cos\theta}{\rho_B g R}$

$s_B = 2s_A$ and $\rho_B = 2\rho_A$

$h = \frac{2\times 2s_A \times \cos\theta}{2\rho_A g R} = 5$ cm

13. A person has been using spectacles of power –1.0 dioptre for distant vision and a separate reading glass of power 2.0 dioptres. What is the least distance of distinct vision for this person

(1) 50 cm (2) 10 cm

(3) 30 cm (4) 40 cm

Answer (1)

Sol. u = 25 cm

$f = \frac{1}{2}\text{m} = 50$ cm

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\Rightarrow \frac{1}{v} + \frac{1}{25} = -\frac{1}{50}$

$\frac{1}{v} = -\frac{1}{50}$

$\Rightarrow u = -50$ cm

14. Speed of an electron in Bohr's 7th orbit for Hydrogen atom is 3.6×10^6 m/s. The corresponding speed of the electron in 3rd orbit, in m/s is

(1) (7.5×10^6) (2) (1.08×10^6)

(3) (8.4×10^6) (4) (3.6×10^6)

Answer (3)

Sol. $v \alpha \frac{z}{n}$

$\frac{v_1}{v_2} = \left(\frac{n_2}{n_1}\right)$

$\Rightarrow \frac{3.6\times 10^6}{v_2} = \frac{3}{7}$

$\Rightarrow v_2 = \frac{7}{3}\times 3.6\times 10^6$ m/s

$= 8.4 \times 10^6$ m/s

15. Electric field in a certain region is given by $\vec{E} = \left(\frac{A}{x^2}\hat{i} + \frac{B}{y^2}\hat{j}\right)$. The SI unit of A and B are

(1) Nm^3C; Nm^2C

(2) Nm^2C; Nm^3C

(3) Nm^2C^{-1}; Nm^2C^{-1}

(4) Nm^3C^{-1}; Nm^2C^{-1}

Answer (3)

Sol. $\vec{E} = \left(\frac{A}{x^2}\hat{i} + \frac{B}{y^3}\hat{j}\right)$

$$\left[\frac{A}{x^2}\right] = [E] = \left[\frac{F}{q}\right] = \left[\frac{N}{C}\right] = NC^{-1}$$

$[A] = (Nm^2C^{-1})$

$[B] = Nm^3C^{-1}$

16. A sinusoidal carrier voltage is amplitude modulated. The resultant amplitude modulated wave has maximum and minimum amplitude of 120 V and 80 V respectively. The amplitude of each sideband is

(1) 10 V (2) 15 V
(3) 20 V (4) 5 V

Answer (1)

Sol. Amplitude of each side band $= \frac{A_{messsage}}{2}$

$A_{carrier} + A_{message} = 120$ …(1)

$A_{carrier} - A_{message} = 80$ …(2)

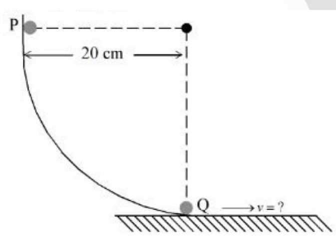
From (1) and (2)

$A_{message} = 20$ V

$\therefore$ Amplitude of each side band = 10 V

17. As per the given figure, a small ball P slides down the quadrant of a circle and hits the other ball Q of equal mass which is initially at rest. Neglecting the effect of friction and assume the collision to be elastic, the velocity of ball Q after collision will be

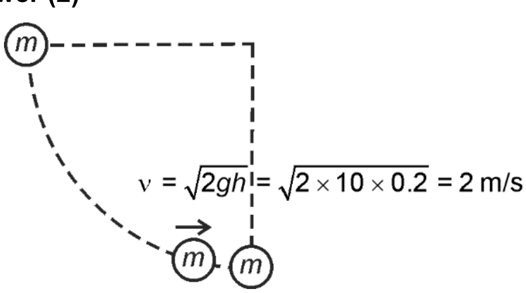
($g = 10$ m/s^2)



(1) 0.25 m/s (2) 2 m/s
(3) 0 (4) 4 m/s

Answer (2)

Sol.



Mass is same and elastic collision, so speed gets exchanged, v = 2 m/s

18. If the gravitational field in the space is given as $\left(-\frac{K}{r^2}\right)$. Taking the reference point to be at r = 2 cm with gravitational potential V = 10 J/kg. Find the gravitational potential at r = 3 cm in SI unit

(Given, that K = 6 Jcm/kg)

(1) 10

(2) 12

(3) 11

(4) 9

Answer (3)

Sol. $E = -\frac{K}{r^2}$

$$\Delta V = -\int_{r=2\text{ cm}}^{3\text{ cm}} E \cdot dr$$

$$= \int_2^3 \frac{k}{r^2} dr$$

$$= \left[-\frac{K}{r}\right]_2^3 = \left(\frac{K}{6}\right) = \frac{6}{6} = 1 \text{ J/kg}$$

$V_f - V_i = 1$

$\Rightarrow$ $V_f - 10 = 1$

$V_f = 11$ J/kg

19. In a series LR circuit with $X_L = R$, power factor is P_1. If a capacitor of capacitance C with $X_C = X_L$ is added to the circuit the power factor becomes P_2. The ratio of P_1 to P_2 will be:

(1) 1 : 2

(2) 1 : 3

(3) 1 : $\sqrt{2}$

(4) 1 : 1

Answer (3)

Sol. $X_L = R$

$$\Rightarrow \quad P_1 = \frac{R}{\sqrt{X_L^2 + R^2}} = \frac{1}{\sqrt{2}}$$

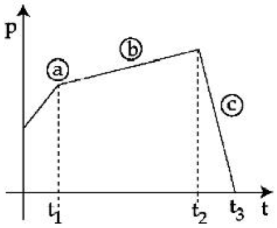
Now, $X_L = X_C = R$

$$\Rightarrow \quad P_2 = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = 1$$

$$\Rightarrow \quad \frac{P_1}{P_2} = \frac{1}{\sqrt{2}}$$

20. The figure represents the momentum time (p-t) curve for a particle moving along an axis under the influence of the force. Identify the regions on the graph where the magnitude of the force is maximum and minimum respectively?

If $(t_3 - t_2) < t_1$



(1) a and b (2) c and a

(3) c and b (4) b and c

Answer (3)

Sol. $F = \frac{dp}{dt}$

$\Rightarrow \quad |F| = \left|\frac{dp}{dt}\right| = |\text{slope of } p - t \text{ curve}|$

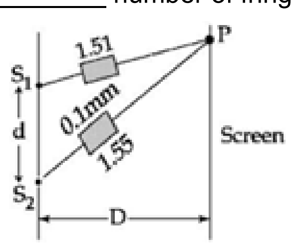
As we can see from graph,

$|F_c|$ is maximum and $|F_b|$ is minimum.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. In Young's double slit experiment, two slits S_1 and S_2 are 'd' distance apart and the separation from slits to screen is D (as shown in figure). Now if two transparent slabs of equal thickness 0.1 mm but refractive index 1.51 and 1.55 are introduced in the path of beam (λ = 4000 Å) from S_1 and S_2 respectively. The central bright fringe spot will shift by ______________ number of fringes.



Answer (10)

Sol. Path difference introduced by two slabs = $(\mu_2 - \mu_1)t$

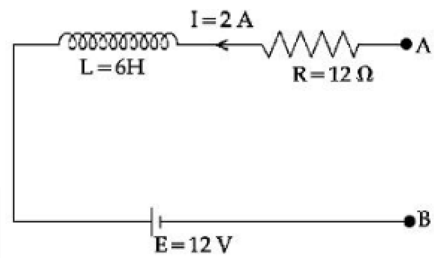
$\Rightarrow \quad \text{Number of shifts } = \frac{(\mu_2 - \mu_1)t}{\lambda}$

$= \frac{0.04 \times 0.1 \text{ mm}}{4000 \text{ Å}}$

$= \frac{4 \times 10^{-2} \times 10^{-4}}{4 \times 10^{-7}}$

= 10

22.



As per the given figure, if $\frac{dI}{dt} = -1$ A/s then the value of V_{AB} at this instant will be ______V.

Answer (30)

Sol. From the circuit :

$V_A - iR - \frac{Ldi}{dt} - 12 = V_B$

$\Rightarrow \quad V_A - V_B = 2 \times 12 + 6(-1) + 12$ volts

= 30 volts

23. A horse rider covers half the distance with 5 m/s speed. The remaining part of the distance was travelled with speed 10 m/s for half the time and with speed 15 m/s for other half of the time. The mean speed of the rider averaged over the whole time of motion is $\frac{x}{7}$ m/s. The value of x is _______.

Answer (50)

Sol. Let S total distance

$\Rightarrow \quad t_1 = \frac{\frac{S}{2}}{5} \qquad \ldots(1)$

Also, $\frac{S}{2} = \frac{10t_2}{2} + \frac{15t_2}{2}$

$\Rightarrow \quad t_2 = \frac{S}{25} \qquad \ldots(2)$

$\Rightarrow \quad \text{Mean speed } = \frac{S}{t_1 + t_2}$

$= \frac{S}{\frac{S}{10} + \frac{S}{25}} = \frac{250}{35} \text{ m/s } = \frac{50}{7} \text{ m/s}$

24. In an experiment for estimating the value of focal length of converging mirror, image of an object placed at 40 cm from the pole of the mirror is formed at distance 120 cm from the pole of the mirror. These distances are measured with a modified scale in which there are 20 small divisions in 1 cm. The value of error in measurement of focal length of the mirror is $\frac{1}{K}$ cm. The value of x is ________.

Answer (32)

Sol. $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ ...(1)

$\Rightarrow -\frac{1}{f^2}df=-\frac{1}{v^2}dv-\frac{1}{u^2}du$

$\Rightarrow \frac{df}{f^2}=\frac{dv}{v^2}+\frac{du}{u^2}$ ...(2)

From (1) : $-\frac{1}{120}-\frac{1}{40}=\frac{1}{f} \Rightarrow f=-30$ cm

Also, least count $=\frac{1\text{ cm}}{20}=0.05$ cm

$\Rightarrow df=\left[\frac{0.05}{120^2}+\frac{0.05}{40^2}\right]\times 30^2$

$=0.05\left[\frac{1}{16}+\frac{9}{16}\right]=\frac{5}{8}\times\frac{5}{100}=\frac{1}{32}$ cm

$\Rightarrow k=32$

25. In a screw gauge, there are 100 divisions on the circular scale and the main scale moves by 0.5 mm on a complete rotation of the circular scale. The zero of circular scale lies 6 divisions below the line of graduation when two studs are brought in contact with each other. When a wire is placed between the studs, 4 linear scale divisions are clearly visible while 46th division the circular scale coincide with the reference line. The diameter of the wire is ________$\times 10^{-2}$ mm.

Answer (220)

Sol. Least count of screw gauge $=\frac{0.5}{100}$ mm $=\frac{1}{200}$ mm

Zero error of screw gauge $=+\frac{6}{200}$ mm $=+\frac{3}{100}$

$=0.03$ mm

Reading of screw gauge $=4\times 0.5+\frac{46}{200}$ mm

$=2+\frac{23}{100}$ mm = 2.23 mm

So diameter of wire = 2.23 mm – 0.03 mm

= 2.20 mm

= 220 × 10^{-2} mm

26. A capacitor of capacitance 900 μF is charged by a 100 V battery. The capacitor is disconnected from the battery and connected to another uncharged identical capacitor such that one plate of uncharged capacitor connected to positive plate and another plate of uncharged capacitor connected to negative plate of the charged capacitor. The loss of energy in this process is measured as x × 10^{-2} J. The value of x is

Answer (225)

Sol. $U_i=\frac{1}{2}CV^2=\frac{1}{2}\times 900\times 10^{-6}\times 100^2=4.5$ J

As the other capacitor is identical therefore charge is equally divided and potential difference across the capacitors becomes half. So

$U_f=\frac{1}{2}2C\left(\frac{V}{2}\right)^2=\frac{1}{2}\times 2\times 900\times 10^{-6}\left(\frac{100}{2}\right)^2$

$=\frac{9}{4}\text{ J}=2.25$ J

So, loss in energy $\Delta U_{\text{loss}}=U_{\text{i}}-U_{\text{f}}$

= 2.25 J

= 225 × 10^{-2} J

27. A thin uniform of length 2 m, cross sectional area 'A' and density 'd' is rotated about an axis passing through the centre and perpendicular to its length with angular velocity ω. If value of ω in terms of its rotational kinetic energy E is $\sqrt{\frac{\alpha E}{Ad}}$ then value of α is _______.

Answer (3)

Sol. Kinetic energy of rod $E=\frac{1}{2}\frac{ml^2}{12}\omega^2$

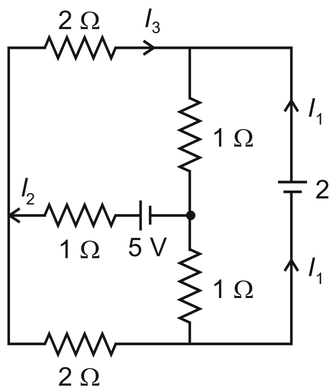
or $\omega=\sqrt{\frac{24E}{ml^2}}=\sqrt{\frac{24E}{d\times A\times l^3}}$

$\Rightarrow \omega=\sqrt{\frac{24E}{dA2^3}}$

$=\sqrt{\frac{3E}{Ad}}$

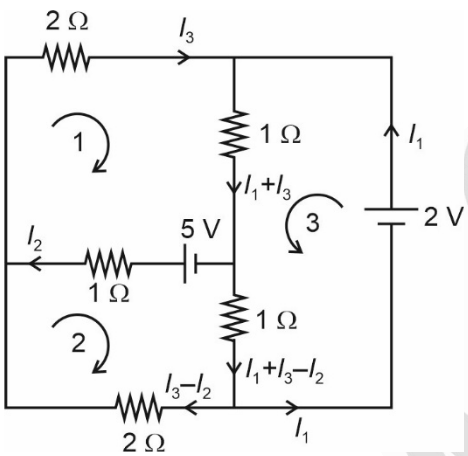
So, α = 3

28. In the following circuit, the magnitude of current I_1, is ________ A.



Answer (01.50)

Sol. The indicated diagram shows current flow diagram loops for writing Kirchhoff's law are also indicated, writing the equation



$2I_3 + I_1 + I_3 + I_2 = 5$

or $I_1 + I_2 + 3I_3 = 5$ ...(1)

$I_2 - 5 = 2(I_3 - I_2) + (I_1 + I_3 - I_2)$

or $I_1 - 4I_2 + 3I_3 = -5$ ...(2)

$(I_1 + I_3) + (I_1 + I_3 - I_2) = 2$

or $2I_1 - I_2 + 2I_3 = 2$ ...(3)

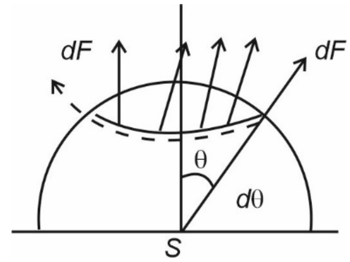
on solving $I_1 = \frac{3}{2}$A , I_2= 2, $I_3 = \frac{1}{2}$A

= 01.50

29. A point source of light is placed at the centre of curvature of a hemispherical surface. The source emits a power of 24 W. The radius of curvature of hemisphere is 10 cm and the inner surface is completely reflecting. The force on the hemisphere due to the light falling on it is ________ × 10^{-8} N.

Answer (4)

Sol.



$dA = 2\pi R \sin\theta R d\theta$

$= 2\pi R^2 \sin\theta d\theta$

So, $dF = 2\frac{IdA}{C}$

$= \frac{2 \times 24}{4\pi R^2} \times \frac{2\pi R^2 \sin\theta d\theta}{C}$

$dF = \frac{24}{C}\sin\theta d\theta$

This dF force will be radially outward so the component of this force in vertical direction is

$dF_v = dF\cos\theta$

$\int_0^{F_v} dF_v = \frac{24}{C}\int_0^{\pi/2}\sin\theta\cos\theta d\theta$

$= \frac{24}{2C} = \frac{24}{2 \times 3 \times 10^8} = 4 \times 10^{-8}$ N

30. The general displacement of a simple harmonic oscillator is x = Asinωt. Let T be its time period. The slope of its potential energy (U) – time (t) curve will be maximum when $t = \frac{T}{\beta}$. The value of β is

Answer (8)

Sol. $U = \frac{1}{2}m\omega^2 A^2 \sin^2 \omega t$

So, $\frac{dU}{dt} = \frac{m\omega^3 A^2}{2}\sin 2\omega t$

This value will be maximum when

$\sin 2\omega t = 1$

or $2\omega t = \frac{\pi}{2}$

$2 \times \frac{2\pi}{T} t = \frac{\pi}{2}$

$\Rightarrow t = \frac{T}{8}$

So β = 8

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

31. During the qualitative analysis of SO_3^{2-} using dilute H_2SO_4, SO_2 gas is evolved which turns $K_2Cr_2O_7$ solution (acidified with dilute H_2SO_4) :

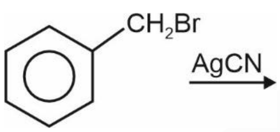
(1) red (2) black

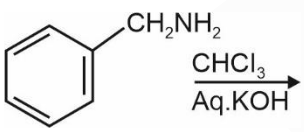
(3) blue (4) green

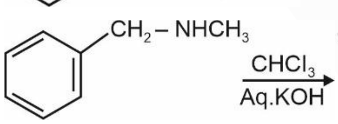
Answer (4)

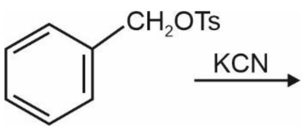
Sol. $SO_2 + Cr_2O_7^{2-} \longrightarrow \underset{\text{(green)}}{Cr^{3+}} + SO_4^{2-}$

32. Benzyl isocyanide can be obtained by:

A. 

B. 

C. 

D. 

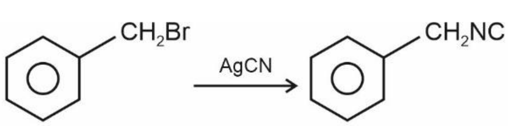
Choose the **correct** answer from the options given below :

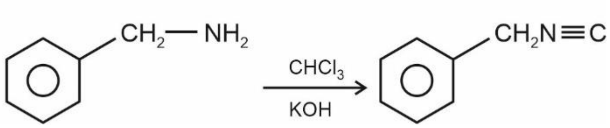
(1) A and D (2) Only B

(3) A and B (4) B and C

Answer (3)

Sol.

(A) 

(B) 

33. In the wet tests for identification of various cations by precipitation, which transition element cation doesn't belong to group IV in qualitative inorganic analysis?

(1) Co^{2+} (2) Zn^{2+}

(3) Ni^{2+} (4) Fe^{3+}

Answer (4)

Sol. Fe^{3+} belongs to IIIrd group

34. Amongst the following compounds, which one is an antacid?

(1) Meprobamate (2) Brompheniramine

(3) Ranitidine (4) Terfenadine

Answer (3)

Sol. Ranitidine is not an antacid.

35. The alkaline earth metal sulphate(s) which are readily soluble in water is/are:

A. $BeSO_4$

B. $MgSO_4$

C. $CaSO_4$

D. $SrSO_4$

E. $BaSO_4$

Choose the **correct** answer from the options given below:

(1) B only (2) A and B

(3) B and C (4) A only

Answer (2)

Sol. $BeSO_4$ and $MgSO_4$ are readily soluble in water.

36. Formation of photochemical smog involves the following reaction in which A, B and C are respectively.

i. $NO_2 \xrightarrow{h\nu} A + B$

ii. $B + O_2 \rightarrow C$

iii. $A + C \rightarrow NO_2 + O_2$

Choose the correct answer from the options given below:

(1) O, N_2O and NO (2) NO, O and O_3

(3) N, O_2 and O_3 (4) O, NO and NO_3^-

Answer (2)

Sol. i) $NO_2 \xrightarrow{h\nu} \underset{(A)}{NO} + \underset{(B)}{O}$

ii) $\underset{(B)}{O} + O_2 \longrightarrow \underset{(C)}{O_3}$

iii) $\underset{(A)}{NO} + \underset{(C)}{O_3} \longrightarrow NO_2 + O_2$

37. Lithium aluminium hydride can be prepared from the reaction of

(1) LiH and $Al(OH)_3$ (2) LiCl and Al_2H_6

(3) LiCl, Al and H_2 (4) LiH and Al_2Cl_6

Answer (4)

Sol. $8LiH + Al_2Cl_6 \rightarrow 2LiAlH_4 + 6LiCl$

38. For OF_2 molecule consider the following:

A. Number of lone pairs on oxygen is 2.

B. FOF angle is less than 104.5°.

C. Oxidation state of O is –2.

D. Molecule is bent ‘V’ shaped

E. Molecular geometry is linear.

Correct options are:

(1) C, D, E only (2) B, E, A only

(3) A, C, D only (4) A, B, D only

Answer (4)

Sol.

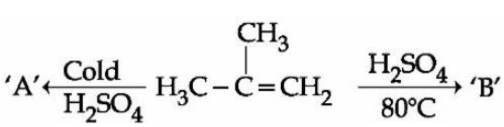
A : No. of lone pairs on oxygen = 2

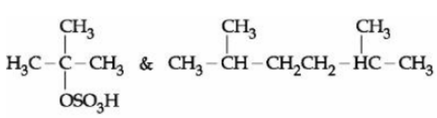
B : 

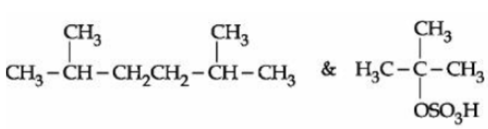
(θ < Bond angle in H_2O (104.5°))

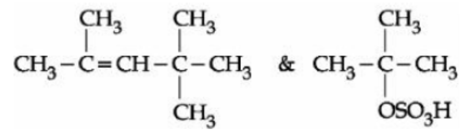
D : molecule is bent “v” shaped

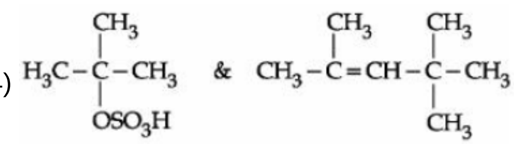
39. The major products ‘A’ and ‘B’, respectively, are



(1) 

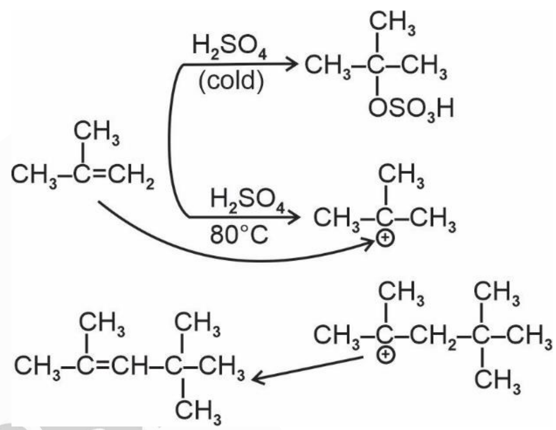
(2) 

(3) 

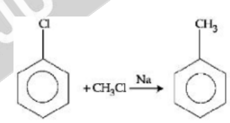
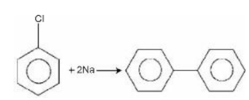
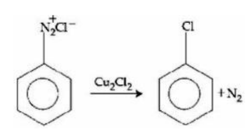
(4) 

Answer (4)

Sol.



40. Match **List I** with **List II**

List I	List II
A. 	I. Fittig reaction
B. 	II. Wurtz Fittig reaction
C. 	III. Finkelstein reaction
D. $C_2H_5Cl + NaI \rightarrow C_2H_5I + NaCl$	IV. Sandmeyer reaction

Choose the correct answer from the options given below:

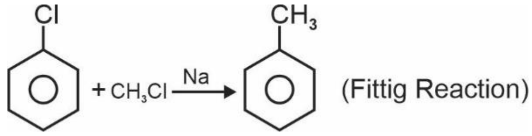
(1) A - II, B - I, C - III, D - IV

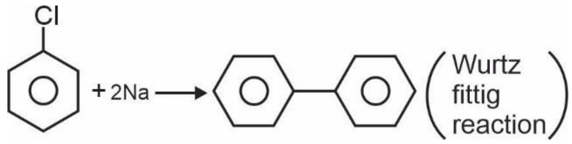
(2) A - II, B - I, C - IV, D - III

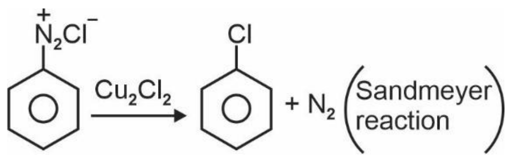
(3) A - IV, B - II, C - III, D - I

(4) A - III, B - II, C - IV, D - I

Answer (2)

Sol. (A) 

(B) 

(C) 

(D) $C_2H_5Cl + NaI \longrightarrow C_2H_5I + NaCl$

(Finkelstein reaction)

41. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : In expensive scientific instruments, silica gel is kept in watch-glasses or in semipermeable membrane bags.

Reason (R) : Silica gel adsorbs moisture from air via adsorption, thus protects the instrument from water corrosion (rusting) and / or prevents malfunctioning.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both **(A)** and **(R)** are true but **(R)** is **not** the correct explanation of **(A)**

(2) **(A)** is false but **(R)** is true

(3) **(A)** is true but **(R)** is false

(4) Both **(A)** and **(R)** are true but **(R)** is the correct explanation of **(A)**

Answer (4)

Sol. Assertion is correct and Reason is correct explanation of Assertion.

Silica gel adsorbs moisture and thus protects the instrument from water corrosion (rusting) and prevents malfunctioning

42. To inhibit the growth of tumours, identify the compounds used from the following :

(A) EDTA

(B) Coordination Compounds of Pt

(C) D - Penicillamine

(D) Cis - Platin

Choose the correct answer from the option given below :

(1) C and D only (2) B and D only

(3) A and B only (4) A and C only

Answer (2)

Sol. Cis-platin is $[Pt(NH_3)_2Cl_2]$; cis platin and other complexes of pt are used to inhibit the growth of tumours.

43. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : Ketoses give Seliwonoff's test faster than Aldoses.

Reason (R) : Ketoses undergo β-elimination followed by formation of furfural.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) **(A)** is true but **(R)** is false

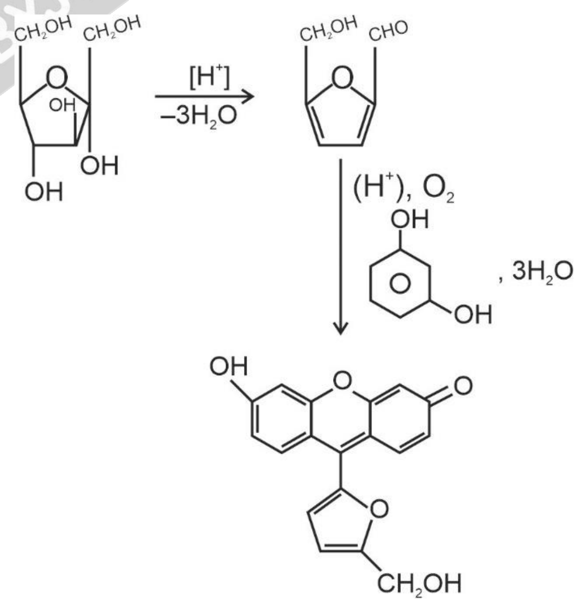
(2) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

(3) **(A)** is false but **(R)** is true

(4) Both **(A)** and **(R)** are true but **(R)** is **not** the correct explanation of **(A)**

Answer (1)

Sol.

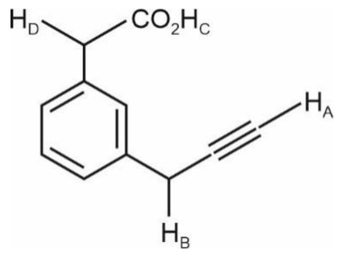


This test relies on the principle that, when heated, ketoses are more rapidly dehydrated than Aldoses.

Ketose $\rightarrow$ Red color formed immediately

Aldose $\rightarrow$ light pink color formed slowly

44. What is the correct order of acidity of the protons marked A–D in the given compounds?

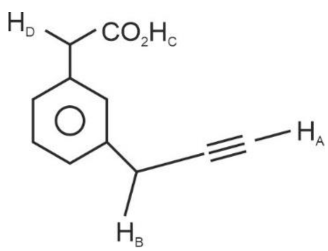


(1) $H_C > H_A > H_D > H_B$ (2) $H_D > H_C > H_B > H_A$

(3) $H_C > H_D > H_B > H_A$ (4) $H_C > H_D > H_A > H_B$

Answer (4)

Sol.



$H_C > H_D > H_A > H_B$

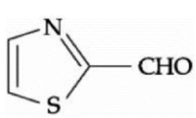
H_C is hydrogen of carboxylic acid

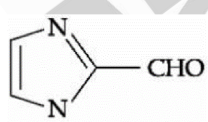
H_D removal will lead to stable carbanion.

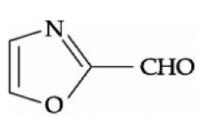
45. Which of the following compounds would give the following set of qualitative analysis?

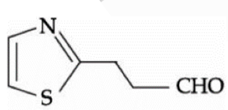
(i) Fehling's Test : Positive

(ii) Na fusion extract upon treatment with sodium nitroprusside gives a blood red colour but not prussian blue.

(1) 

(2) 

(3) 

(4) 

Answer (4)

Sol. Fehling solution is not given by aromatic aldehydes.

1, 2, 3 are aromatic aldehydes

46. In the extraction of copper, its sulphide ore is heated in a reverberatory furnace after mixing with silica to

(1) Decrease the temperature needed for roasting of Cu_2S

(2) Remove calcium as $CaSiO_3$

(3) Separate CuO as $CuSiO_3$

(4) Remove FeO as $FeSiO_3$

Answer (4)

Sol. $\underset{\text{Basic}}{FeO} + \underset{\text{Acidic}}{SiO_2} \longrightarrow \underset{\text{(Slag)}}{FeSiO_3}$

47. Match **List I** with **List II**

	List-I (Atomic number)		**List-II (Block of periodic table)**
A.	37	I.	p-block
B.	78	II.	d-block
C.	52	III.	f-block
D.	65	IV.	s-block

Choose the **correct** answer from the options given below

(1) A-II, B-IV, C-I, D-III (2) A-IV, B-III, C-II, D-I

(3) A-IV, B-II, C-I, D-III (4) A-I, B-III, C-IV, D-II

Answer (3)

Sol.

37 –	*s*-Block
78 –	*d*-Block
52 –	*p*-Block
65 –	*f*-Block

48. Which of the following is correct order of ligand field strength?

(1) $NH_3 < en < CO < S^{2-} < C_2O_4^{2-}$

(2) $S^{2-} < NH_3 < en < CO < C_2O_4^{2-}$

(3) $S^{2-} < C_2O_4^{2-} < NH_3 < en < CO$

(4) $CO < en < NH_3 < C_2O_4^{2-} < S^{2-}$

Answer (3)

Sol. Ligand field strength

$S^{2-} < C_2O_4^{2-} < NH_3 < en < CO$

49. Match **List I** with **List II**

	List-I (Molecules/Ions)		**List-II (No. of lone pairs of e⁻ on central atom)**
A.	IF_7	I.	Three
B.	ICl_4^-	II.	One
C.	XeF_6	III.	Two
D.	XeF_2	IV.	Zero

Choose the **correct** answer from the options given below

(1) A-II, B-III, C-IV, D-I (2) A-IV, B-I, C-II, D-III

(3) A-II, B-I, C-IV, D-III (4) A-IV, B-III, C-II, D-I

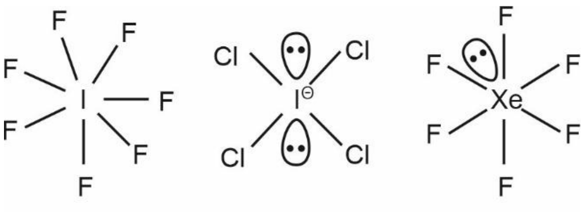
Answer (4)

Sol. (A) IF_7 – 0 lone pairs

(B) ICl_4^- – 2 lone pairs

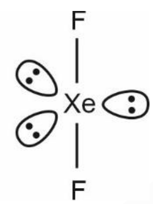
(C) XeF_6 – 1 lone pair

(D) XeF_2 – 3 lone pairs



(IF_7) (ICl_4^-) (XeF_6)

(XeF_2)



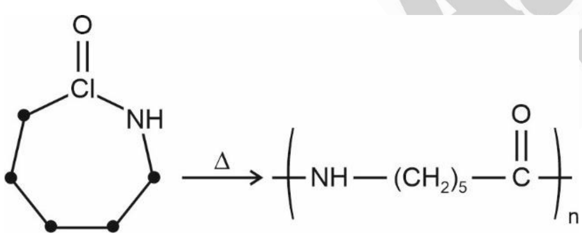
50. Caprolactam when heated at high temperature in presence of water, gives

(1) Nylon 6, 6 (2) Nylon 6

(3) Dacron (4) Teflon

Answer (2)

Sol. Caprolactum $\xrightarrow{\Delta}$ Nylon – 6



SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse andw the on-screen virtual numeric keypad in the place designated to enter the answer.

51. A solution containing 2 g of a non-volatile solute in 20 g of water boils at 373.52 K. The molecular mass of the solute is ________ g mol^{-1}. (Nearest integer)

Given, water boils at 373 K, K_b for water = 0.52 K kg mol^{-1}

Answer (100)

Sol. $\Delta T_b = K_b.m$

(0.52) = (0.52) (m)

$$m = 1 = \frac{2\,(1000)}{(mw)\,(20)}$$

mw = 100

52. A 300 mL bottle of soft drink has 0.2 M CO_2 dissolved in it. Assuming CO_2 behaves as an ideal gas, the volume of the dissolved CO_2 at STP is ________ mL. (Nearest integer)

Given: At STP, molar volume of an ideal gas is 22.7 L mol^{-1}

Answer (1362)

Sol. Moles = 0.3 × 0.2

Volume at STP = 0.3 × 0.2 × 22.7

= 1.362 litre

= 1362 mL

53. The energy of one mole of photons of radiation of frequency 2 × 10^{12} Hz in J mol^{-1} is _______. (Nearest integer)

[Given: h = 6.626 × 10^{-34} Js

N_A = 6.022 × 10^{23} mol^{-1}]

Answer (798)

Sol. E = nhν

= (6.022 × 10^{23}) (6.626 × 10^{-34}) × (2 × 10^{12})

= 798.03 J

≈ 798 J

54. Some amount of dichloromethane (CH_2Cl_2) is added to 671.141 mL of chloroform ($CHCl_3$) to prepare 2.6 × 10^{-3} M solution of CH_2Cl_2 (DCM). The concentration of DCM is _______ ppm (by mass).

Given:

Atomic mass : C = 12

H = 1

Cl = 35.5

Density of $CHCl_3$ = 1.49 g cm^{-3}

Answer (148)

Sol. Mass of $CHCl_3$ = 671.141 × 1.49

= 1000 gm

$$2.6 \times 10^{-3} = \frac{\text{moles of } CH_2Cl_2}{0.671141}$$

$\Rightarrow$ moles of CH_2Cl_2 = 1.74496×10^{-3}

mass of CH_2Cl_2 = 148.32×10^{-3} gm

Composition of CH_2Cl_2 $= \dfrac{148.32\times10^{-3}}{1000}\times10^{6}$

$= 148.32$ ppm

≈ 148

55. Consider the cell

$Pt_{(s)}$ | H_2 (g, 1 atm) | H^+ (aq, 1 M) || Fe^{3+}(aq), Fe^{2+}(aq) | Pt(s)

When the potential of the cell is 0.712 V at 298 K, the ratio $[Fe^{2+}] / [Fe^{3+}]$ is ________.

(Nearest integer)

Given: $Fe^{3+} + e^- \rightleftharpoons Fe^{2+}$, $E^{\ominus}_{Fe^{3+}, Fe^{2+} | Pt} = 0.771$

$$\frac{2.303\,RT}{F} = 0.06\text{ V}$$

Answer (10)

Sol. Anode $H_2 \rightarrow 2H^+ + 2e^-$

Cathode $(Fe^{3+} + e^- \rightarrow Fe^{2+}) \times 2$

$\boxed{H_2 + 2Fe^{3+} \rightarrow 2H^+ + 2Fe^{2+}}$

$$E_{cell} = E^{\circ}_{cell} - \frac{0.059}{2}\log\left(\frac{Fe^{2+}}{Fe^{3+}}\right)^2$$

$$0.712 = 0.771 - 0.059\log\frac{Fe^{2+}}{Fe^{3+}}$$

$$-0.059 = -0.059\log\frac{Fe^{2+}}{Fe^{3+}}$$

$$\frac{[Fe^{2+}]}{[Fe^{3+}]} = 10$$

56. If compound A reacts with B following first order kinetics with rate constant 2.011×10^{-3} s^{-1}. The time taken by A (in seconds) to reduce from 7 g to 2 g will be __________. (Nearest Integer)

[log 5 = 0.698, log 7 = 0.845, log 2 = 0.301]

Answer (623)

Sol. $t = \dfrac{2.303}{k}\log\dfrac{C_0}{C_t}$

$$= \frac{2.303}{2.011\times10^{-3}}\log\frac{7}{2}$$

$$= \frac{2.303\times10^{3}}{2.011}(.845 - .301)$$

$= 622.99$

≈ 623 sec.

57. When 2 litre of ideal gas expands isothermally into vacuum to a total volume of 6 litre, the change in internal energy is _________ J. (Nearest integer)

Answer (0)

Sol. For isothermal process of an ideal gas; $\Delta E = 0$

58. The number of electrons involved in the reduction of permanganate of manganese dioxide in acidic medium is _________.

Answer (3)

Sol. $3e^- + 4H^+ + MnO_4^- \longrightarrow MnO_2 + 2H_2O$

59. 600 mL of 0.01 M HCl is mixed with 400 mL of 0.01 M H_2SO_4. The pH of the mixture is _________ $\times 10^{-2}$. (Nearest integer)

[Given log 2 = 0.30
log 3 = 0.48
log 5 = 0.69
log 7 = 0.84
log 11 = 1.04]

Answer (186)

Sol. $[H^+] = \dfrac{6+8}{1000} = 14\times10^{-3}$

pH $= 3 - \log 14$

$= 3 - .3 - .84$

$= 1.86 = 186 \times 10^{-2}$

60. A trisubstituted compound 'A', $C_{10}H_{12}O_2$ gives neutral $FeCl_3$ test positive. Treatment of compound 'A' with NaOH and CH_3Br gives $C_{11}H_{14}O_2$, with hydroiodic acid gives methyl iodide and with hot conc. NaOH gives a compound B, $C_{10}H_{12}O_2$. Compound 'A' also decolorises alkaline $KMnO_4$. The number of π bond/s present in the compound 'A' is _________.

Answer (4)

Sol. A : $C_{10}H_{12}O_2$

DU of A = $\dfrac{22-12}{2} = 5$

1 DU is due to Ring (Benzene ring)

4 π-bonds will be there

(3 π-bonds in ring and 1 π-bond outside ring) as it decolorises alkaline $KMnO_4$.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

61. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors and $\hat{n}$ is a unit vector perpendicular to $\vec{c}$ such that $\vec{a} = \alpha\vec{b} - \hat{n}, \ (\alpha \neq 0)$ and $\vec{b}\cdot\vec{c} = 12,$ then $\left|\vec{c}\times(\vec{a}\times\vec{b})\right|$ is equal to

(1) 9 (2) 6

(3) 12 (4) 15

Answer (3)

Sol. $\hat{n} = \alpha\vec{b} - \vec{a}$

$$\vec{c}\times(\vec{a}\times\vec{b}) = (\vec{c}\cdot\vec{b})\vec{a} - (\vec{c}\cdot\vec{a})\vec{b}$$

$$= 12\vec{a} - \left(\vec{c}\cdot(\alpha\vec{b} - \hat{n})\right)\vec{b}$$

$$= 12\vec{a} - (12\alpha - 0)\vec{b}$$

$$= 12(\vec{a} - \alpha\vec{b})$$

$\therefore \quad \left|\vec{c}\times(\vec{a}\times\vec{b})\right| = 12$

62. If the coefficient of x^{15} in the expansion of $\left(ax^3 + \frac{1}{bx^{1/3}}\right)^{15}$ is equal to the coefficient of x^{-15} in the expansion of $\left(ax^{1/3} - \frac{1}{bx^3}\right)^{15}$, where a and b are positive real numbers, then for each such ordered pair (a, b)

(1) $ab = 1$ (2) $a = b$

(3) $a = 3b$ (4) $ab = 3$

Answer (1)

Sol. For $\left(ax^3 + \frac{1}{bx^{\frac{1}{3}}}\right)$

$$T_{r+1} = {}^{15}C_r\,(ax^3)^{15-r}\left(\frac{1}{bx^{\frac{1}{3}}}\right)^1$$

$\therefore$ For $x^{15} \to 3(15-r) - \frac{r}{3} = 15$

$\Rightarrow 30 = \frac{10r}{3} \Rightarrow r = 9$

Similarly, for $\left(ax^{\frac{1}{3}} - \frac{1}{bx^3}\right)^{15}$

$$T_{r+1} = {}^{15}C_r\left(ax^{\frac{1}{3}}\right)^{15-r}\left(-\frac{1}{bx^3}\right)^2$$

$\therefore$ For $x^{-15} \to \frac{15-r}{3} - 3r = -15 \Rightarrow r = 6$

$\therefore \quad {}^{15}C_9\frac{a^6}{b^9} = {}^{15}C_6\frac{a^9}{b^6} \Rightarrow ab = 1$

63. The number of points on the curve $y = 54x^5 - 135x^4 - 70x^3 + 180x^2 + 210x$ at which the normal lines are parallel to $x + 90y + 2 = 0$ is

(1) 4 (2) 3

(3) 2 (4) 0

Answer (1)

Sol. $y' = 270x^4 - 540x^3 - 210x^2 + 360x + 210$

Slope of normal $= -\frac{1}{90}$

$\therefore$ Slope of tangent = 90

$\therefore$ Number of normal will be number of solutions of $270x^4 - 540x^3 - 210x^2 + 360x + 210 = 90$

$\Rightarrow 9x^4 - 18x^3 - 7x^2 + 12x + 4 = 0$

$\therefore \quad x = 1, 2, -\frac{1}{3}, -\frac{2}{3}$ are roots

64. The line l_1 passes through the point (2, 6, 2) and is perpendicular to the plane $2x + y - 2z = 10$. Then the shortest distance between the line l_1 and the line $\frac{x+1}{2} = \frac{y+4}{-3} = \frac{z}{2}$ is

(1) $\frac{19}{3}$ (2) $\frac{13}{3}$

(3) 9 (4) 7

Answer (3)

Sol. Equation of $l_1 = \frac{x-2}{2} = \frac{y-6}{1} = \frac{z-2}{-2}$

Shortest distance with $\frac{x+1}{2} = \frac{y+4}{-3} = \frac{z}{2}$ is

$$\text{S.d} = \left| \frac{\begin{vmatrix} 3 & 10 & 2 \\ 2 & 1 & -2 \\ 2 & -3 & 2 \end{vmatrix}}{\left|-4\hat{i} - 8\hat{j} - 8\hat{k}\right|} \right| = \left| \frac{(-12) - 10(8) + 2(-8)}{12} \right|$$

= 9 units

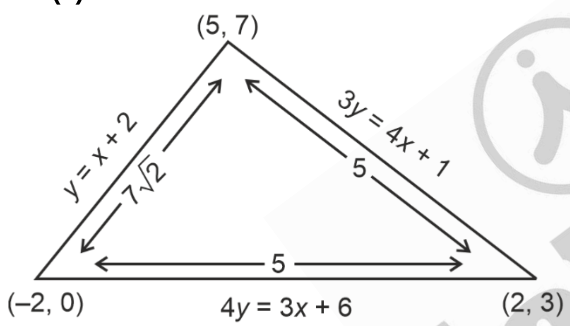
65. Let $y = x + 2$, $4y = 3x + 6$ and $3y = 4x + 1$ be three tangent lines to the circle $(x - h)^2 + (y - k)^2 = r^2$. Then $h + k$ is equal to

(1) $5\left(1+\sqrt{2}\right)$ (2) 5

(3) 6 (4) $5\sqrt{2}$

Answer (2)

Sol.



$$(h, k) = \left(\frac{5.5 + 5(-2) + 14\sqrt{2}}{10 + 7\sqrt{2}}, \frac{35 + 21\sqrt{2}}{10 + 7\sqrt{2}} \right)$$

$$h + k = \frac{50 + 35\sqrt{2}}{10 + 7\sqrt{2}} = 5$$

66. Among the statements :

($S1$) $((p \vee q) \Rightarrow r) \Leftrightarrow (p \Rightarrow r)$

($S2$) $((p \vee q) \Rightarrow r) \Leftrightarrow ((p \Rightarrow r) \vee (q \Rightarrow r))$

(1) only ($S1$) is a tautology

(2) only ($S2$) is a tautology

(3) neither ($S1$) nor ($S2$) is a tautology

(4) both ($S1$) and ($S2$) are tautologies

Answer (3)

Sol. $S_1 : \left((p \vee q) \Rightarrow r\right) \Leftrightarrow (p \Rightarrow r)$

$S_2 : \left((p \vee q) \Rightarrow r\right) \Leftrightarrow \left((p \Rightarrow r) \vee (q \Rightarrow r)\right)$

In S_1 : If p = F, q = T, r = F then S_1 is false

In S_2 : if P = T, q = F, r = F then S_2 is false

∴ Neither $S1$ nor $S2$ is a tautology

67. The coefficient of x^{301} in $(1+x)^{500} + x(1+x)^{499} + x^2(1+x)^{498} + ... + x^{500}$ is

(1) ${}^{501}C_{302}$ (2) ${}^{501}C_{200}$

(3) ${}^{500}C_{300}$ (4) ${}^{500}C_{301}$

Answer (2)

Sol. ${}^{500}C_{301} + {}^{499}C_{300} + {}^{498}C_{299} + ... + {}^{199}C_0$

$= {}^{500}C_{199} + {}^{499}C_{199} + {}^{498}C_{199} + ... + {}^{199}C_{89}$

$= {}^{501}C_{200}$

68. The minimum number of elements that must be added to the relation $R = \{a, b), (b, c)\}$ on the set $\{a, b, c\}$ so that it becomes symmetric and transitive is

(1) 7 (2) 3

(3) 5 (4) 4

Answer (1)

Sol. For symmetric $(b, a), (c, b) \in R$

For transitive $(a, c) \in R$

$\Rightarrow (c, a) \in R$

$\therefore (a, b), (b, a) \in R$

$\Rightarrow (a, a) \in R$

$(b, c), (c, b) \in R$

$\Rightarrow (b, b) \in R, (c, c) \in R$

7 elements must be added

69. If $P(h, k)$ be a point on the parabola $x = 4y^2$, which is nearest to the point $Q(0,33)$, then the distance of P from the directrix of the parabola $y^2 = 4(x + y)$ is equal to

(1) 4 (2) 6

(3) 8 (4) 2

Answer (2)

Sol. Equation of normal

$$y = -tx + 2 \cdot \frac{1}{16}t + \frac{1}{16}t^3$$

$$33 = \frac{t}{8} + \frac{t^3}{16}$$

$\Rightarrow$ $t^3 + 2t = 528$

$t = 8$

$(at^2, 2at) = (4, 1)$

Distance from $x = -2$

70. If [t] denotes the greatest integer $\le t$, then the value of $\frac{3(e-1)}{e}\int_1^2 x^2 e^{[x]+[x^3]}dx$ is

(1) $e^7 - 1$ (2) $e^8 - 1$

(3) $e^9 - e$ (4) $e^8 - e$

Answer (4)

Sol. $I = \frac{3(e-1)}{e}\int_1^2 x^2 e^{[x]+[x^3]}dx$

$= \frac{3(e-1)}{e}\int_1^2 x^2 e^{1+[x^3]}dx \quad (\because [x]=1 \text{ when } x\in(12))$

$= 3(e-1)\int_1^2 x^2 e^{[x^3]}dx$

Let $x^3 = t$

$I = (e-1)\int_1^8 e^{[t]}dt$

$= (e^{-1})(e^1 + e^2 + e^3 + ... + e^7)$

$= (e-1)e\frac{(e^7-1)}{e-1}$

$= e^8 - e$

71. Let a unit vector $\widehat{OP}$ makes angles α, β, γ with the positive directions of the co-ordinate axes OX, OY, OZ respectively, where $\beta \in \left(0, \frac{\pi}{2}\right)$. If $\widehat{OP}$ is perpendicular to the plane through points (1, 2, 3), (2, 3, 4) and (1, 5, 7), then which one of the following is true?

(1) $\alpha \in \left(0, \frac{\pi}{2}\right)$ and $\gamma \in \left(0, \frac{\pi}{2}\right)$

(2) $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\gamma \in \left(0, \frac{\pi}{2}\right)$

(3) $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\gamma \in \left(\frac{\pi}{2}, \pi\right)$

(4) $\alpha \in \left(0, \frac{\pi}{2}\right)$ and $\gamma \in \left(\frac{\pi}{2}, \pi\right)$

Answer (3)

Sol. Let $A \equiv (1, 2, 3)$, $B \equiv (2, 3, 4)$, $C \equiv (1, 5, 7)$

$$\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 0 & 3 & 4 \end{vmatrix}$$

$= \hat{i} - 4\hat{j} + 3\hat{k}$

$\widehat{OP} = \frac{\pm(\hat{i} - 4\hat{j} + 3\hat{k})}{\sqrt{26}}$

Since $\cos\beta > 0$, take – sign

$\widehat{OP} = \frac{\hat{i} - 4\hat{j} + 3\hat{k}}{\sqrt{26}}$

$\Rightarrow$ $\cos\alpha < 0$, $\cos\gamma < 0$

$\alpha, \gamma \in \left(\frac{\pi}{2}, \pi\right)$

72. Suppose $f : \mathbb{R} \to (0, \infty)$ be a differentiable function such that $5f(x+y) = f(x)\cdot f(y), \forall x, y \in \mathbb{R}$. If $f(3) = 320$, then $\sum_{n=0}^{5} f(n)$ is equal to

(1) 6825 (2) 6525

(3) 6875 (4) 6575

Answer (1)

Sol. $5f(x + y) = f(x)\cdot f(y)$

$5f(3) = f(1)\cdot f(2)$

$5f(2) = (f(1))^2$ $f(10) = 5$

$f(1) = 20$

$\Rightarrow f(1)\cdot\frac{(f(1))^2}{5} = 1600$

$\sum_{n=0}^{5} f(n) = f(0) + 20 + 80 + 320 + 1280 + 5120$

$= 1750 + 5120$

$= 6825$

73. Let $A = \begin{pmatrix} m & n \\ p & q \end{pmatrix}$, $d = |A| \ne 0$ and $|A - d(Adj\ A)| = 0$.

Then

(1) $(1+d)^2 = m^2 + q^2$ (2) $(1+d)^2 = (m+q)^2$

(3) $1 + d^2 = (m+q)^2$ (4) $1 + d^2 = m^2 + q^2$

Answer (2)

Sol. $\left|A-d\begin{pmatrix} q & -n \\ -p & m \end{pmatrix}\right|=0$

$\begin{vmatrix} m-qd & n(1+d) \\ p(1+d) & q-md \end{vmatrix}=0$

$(m-qd)(q-md)=np(1+d)^2$

$mq-(q^2+m^2)d+qmd^2=np(1+d^2)+2npd$

$d^2(mq-np)+1(mq-np)=(2np+m^2+q^2)d$

$(d^2+1)(mq-np)=(2np+m+a)d$

$d^2+1=2np+m^2+q^2$

$2d=2mq-2np$

$\Rightarrow\ (1+d)^2=(m+q)^2$

74. If $\tan15^\circ+\frac{1}{\tan75^\circ}+\frac{1}{\tan105^\circ}+\tan195^\circ=2a$, then the value of $\left(a+\frac{1}{a}\right)$ is

(1) 4
(2) 2
(3) $4-2\sqrt{3}$
(4) $5-\frac{3}{2}\sqrt{3}$

Answer (1)

Sol. tan15º + tan15º – tan15º + tan15º

= 2tan15º

$=2\left(2-\sqrt{3}\right)=2a\ \Rightarrow a=2-\sqrt{3}$

$\therefore\frac{1}{a}+a\Rightarrow\left(2+\sqrt{3}\right)+\left(2-\sqrt{3}\right)=4$

75. Let the system of linear equations

$x+y+kz=2$

$2x+3y-z=1$

$3x+4y+2z=k$

have infinitely many solutions. Then the system

$(k+1)x+2(k-1)y=7$

$(2k+1)x+(k+5)y=10$

has :

(1) Unique solution satisfying x + y = 1
(2) Unique solution satisfying x – y = 1
(3) Infinitely many solutions
(4) No solution

Answer (1)

Sol. $x+y+kz=2$ ...(1)

$2x+3y-z=1$ ...(2)

$3x+4y+2z=k$ ...(3)

(1) + (2)

$3x+4y+z(k-1)=3$

Comparing with (3)

$k=3$

Now, $4x+5y=7$ $\Rightarrow 3x+3y=3$

$7x+8y=10$

as $\frac{4}{7}\neq\frac{5}{8}$

$\therefore$ unique solution satisfying $x+y=1$

76. If the solution of the equation $\log_{\cos x}\cot x+4\log_{\sin x}\tan x=1$, $x\in\left(0,\frac{\pi}{2}\right)$, is $\sin^{-1}\left(\frac{\alpha+\sqrt{\beta}}{2}\right)$, where α, β are integers, then α + β is equal to

(1) 6
(2) 5
(3) 4
(4) 3

Answer (3)

Sol. $\log_{\cos x}\cot x+4\log_{\sin x}\tan x=1$

$\Rightarrow\ \log_{\cos x}\cot x-4\log_{\sin x}\cot x=1$

$\Rightarrow\ 1-\log_{\cos x}\sin x-4-4\log_{\sin x}\cos x=1$

Let $\log_{\cos x}\sin x=t$

$t+\frac{4}{t}=4$

$\Rightarrow\ t=2$

$\sin x=\cos^2 x$

$\Rightarrow\ \sin x=1-\sin^2 x$

$\Rightarrow\ \sin^2 x+\sin x^{-1}=0$

$\Rightarrow\ \sin x=\frac{-1\pm\sqrt{5}}{2}$

as $x\in\left(0,\frac{\pi}{2}\right)$

$\sin x=\frac{\sqrt{5}-1}{2}$

$x=\sin^{-1}\left(\frac{-1+\sqrt{5}}{2}\right)$

$\Rightarrow\ \alpha=-1,\ \beta=5$

$\alpha+\beta=4$

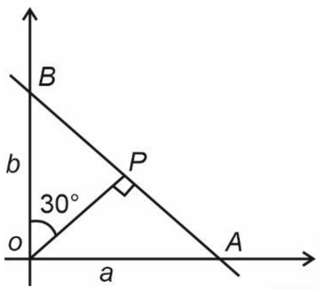
77. A straight line cuts off the intercepts $OA = a$ and $OB = b$ on the positive direction of x-axis and y-axis respectively. If the perpendicular from origin O to this line makes an angle of $\frac{\pi}{6}$ with positive direction of y-axis and the area of ΔOAB is $\frac{98}{3}\sqrt{3}$, then $a^2 - b^2$ is equal to

(1) 196 (2) $\frac{196}{3}$

(3) $\frac{392}{3}$ (4) 98

Answer (3)

Sol. $\frac{1}{2}ab = \frac{98\sqrt{3}}{3}$



$\Rightarrow \sqrt{3}ab = 196$ ...(i)

$OP = OB \cos 30° = OA \cos 60°$

$\Rightarrow \frac{b\sqrt{3}}{2} = \frac{a}{2}$

$\Rightarrow \sqrt{3}b = a$ ...(ii)

By (i) and (ii)

$a^2 = 196$

$a = 14$

$b^2 = \frac{a^2}{3}$

$a^2 - b^2 = \frac{2a^2}{3} = \frac{392}{3}$

78. If $a_n = \frac{-2}{4n^2 - 16n + 5}$, then $a_1 + a_2 + \ldots + a_{25}$ is equal to

(1) $\frac{49}{138}$ (2) $\frac{52}{147}$

(3) $\frac{51}{144}$ (4) $\frac{50}{141}$

Answer (4)

Sol. $\sum_{i=1}^{25} a_i = \sum \frac{-2}{4n^2 - 16n + 15} = \sum \frac{-2}{(2n-5)(2n-3)}$

$= \sum_{i=1}^{25}\left(\frac{1}{2n-3} - \frac{1}{2n-5}\right)$

$= \left[\left(\frac{1}{-1} - \frac{1}{-3}\right) + \left(\frac{1}{1} - \frac{1}{-1}\right) + \left(\frac{1}{3} - \frac{1}{1}\right) \ldots\ldots\right.$

$= \frac{1}{2(25)-3} + \frac{1}{3} = \frac{50}{141}$

79. Let the solution curve $y = y(x)$ of the differential equation

$$\frac{dy}{dx} - \frac{3x^5 \tan^{-1}(x^3)}{(1+x^6)^{3/2}} y = 2x \exp\left\{\frac{x^3 - \tan^{-1} x^3}{\sqrt{(1+x^6)}}\right\}$$

pass through the origin. Then $y(1)$ is equal to

(1) $\exp\left(\frac{1-\pi}{4\sqrt{2}}\right)$ (2) $\exp\left(\frac{4-\pi}{4\sqrt{2}}\right)$

(3) $\exp\left(\frac{4+\pi}{4\sqrt{2}}\right)$ (4) $\exp\left(\frac{\pi-4}{4\sqrt{2}}\right)$

Answer (2)

Sol. $\frac{dy}{dx} - \frac{3x^5 \tan^{-1}(x^3)}{(1+x^6)^{\frac{3}{2}}} y = 2x \exp\left\{\frac{x^3 - \tan^{-1} x^3}{\sqrt{1+x^6}}\right\}$

$$IF = e^{-\int \frac{3x^5 \tan^{-1}(x^3)}{(1+x^6)^{\frac{3}{2}}} dx}$$

Let $\tan^{-1} x^3 = t \Rightarrow \frac{3x^2}{1+x^6} dx = dt$

$\Rightarrow IF = e^{-\int \frac{\tan t}{\sec t} \cdot t\, dt} = e^{-\int \sin t \cdot t dt} = e^{t \cos t - \sin t}$

$\Rightarrow IF = e^{\frac{\tan^{-1}(x^3)}{\sqrt{1+x^6}} - \frac{x^3}{\sqrt{1+x^6}}}$

$\therefore$ Solution is

$$y \cdot e^{\frac{\tan^{-1} x^3}{\sqrt{1+x^6}} - \frac{x^3}{\sqrt{1+x^6}}} = \int 2x\, dx + c$$

$$\Rightarrow y \cdot e^{\frac{\tan^{-1} x^3 - x^3}{\sqrt{1+x^6}}} = x^2 + c$$

$y(0) = 0 \Rightarrow c = 0$

$x = 1$

$$y \cdot e^{\frac{\frac{\pi}{4}-1}{\sqrt{2}}} = 1$$

$$\Rightarrow \quad y = e^{\frac{1-\frac{\pi}{4}}{\sqrt{2}}}$$

$$\Rightarrow \quad y = e^{\frac{4-\pi}{4\sqrt{2}}}$$

80. If an unbiased die, marked with –2, –1, 0, 1, 2, 3 on its faces, is thrown five times, then the probability that product of the outcomes is positive is:

(1) $\frac{881}{2592}$

(2) $\frac{440}{2592}$

(3) $\frac{27}{288}$

(4) $\frac{521}{2592}$

Answer (4)

Sol. $^5C_0 \times 3^5 = 243$

$^5C_2 \times 2^2 \times 3^3 = 1080$

$^5C_4 \times 2^4 \cdot 3 = 240$

$\therefore$ required probability

$$= \frac{243 + 1080 + 240}{6 \times 6 \times 6 \times 6 \times 6} = \frac{521}{2592}$$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

81. Let $S = \{1, 2, 3, 4, 5, 6\}$. Then the number of one-one functions $f : S \rightarrow P(S)$, where $P(S)$ denote the power set of S, such that $f(n) \subset f(m)$ where $n < m$ is ________.

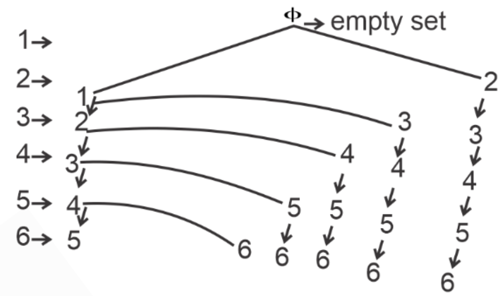
Answer (3240)

Sol. $\because \quad S = \{1, 2, 3, 4, 5, 6\}$

and $P(S) = \{\phi, \{1\}, \{2\}, \ldots, \{1, 2, 3, 4, 5, 6\}\}$

$f(n)$ corresponding a set having m elements which belongs to $P(S)$, should be a subset of $f(n + 1)$, so $f(n + 1)$ should be a subset of $P(S)$ having at least $m + 1$ elements.

Now if $f(1)$ has one element then $f(2)$ has 3, $f(3)$ has 3 and so on and $f(6)$ has 6 elements. Total number of possible functions = 6! = 720 …(1) if $f(1)$ has no elements (*i.e.* null set ϕ) then



Each index number represents the number of elements in respective rows

Taking every series of arrow and counting number of such possible functions (sets)

$$= {}^6C_2 \times {}^4C_1 \times {}^3C_1 \times {}^2C_1 + {}^6C_1 \times {}^5C_2 \times {}^3C_1 \times {}^2C_1$$

$$+ {}^6C_1 \times {}^5C_1 \times {}^4C_2 \times {}^2C_1 + {}^6C_1 \times {}^5C_1 \times {}^4C_1 \times {}^3C_2$$

$$+ {}^6C_1 \times {}^5C_1 \times {}^4C_1 \times {}^3C_1 \times {}^2C_2 + {}^6C_1 \times {}^5C_1 \times {}^4C_1 \times {}^3C_1 \times {}^2C_1$$

= 2520 …(2)

From (1) and (2) : Total number of functions

= 2520 + 720

= 3240

82. Number of 4-digit numbers (the repeation of digits is allowed) which are made using the digits 1, 2, 3 and 5, and are divisible by 15, is equal to ________

Answer (21)

Sol. Digits 1, 2, 3, 5 and number should be divisible by 15 (i.e., divisible by both 3 and 5)

So,

…(5)

Case-I: $5 \rightarrow 1, 2 \rightarrow 1, 1 \rightarrow 2 = \frac{3!}{2!} = 3$

$5 \rightarrow 1, 3 \rightarrow 1, 2 \rightarrow 2 = \frac{3!}{2!} = 3$

$5 \rightarrow 1, 3 \rightarrow 2, 1 \rightarrow 1 = \frac{3!}{2!} = 3$

Case-II: $5 \rightarrow 2, 3 \rightarrow 1, 2 \rightarrow 1 = 3! = 6$

$5 \rightarrow 2, 1 \rightarrow 2 = \frac{3!}{2!} = 3$

Case-III: $5 \rightarrow 3, 3 \rightarrow 1 = \frac{3!}{2!} = 3$

$\therefore$ Total no. = 21

83. If $\lambda_1 < \lambda_2$ are two values of λ such that the angle between the planes $P_1 : \vec{r}\left(3\hat{i} - 5\hat{j} + \hat{k}\right) = 7$ and $P_2 : \vec{r}\left(\lambda\hat{i} + \hat{j} - 3\hat{k}\right) = 9$ is $\sin^{-1}\left(\frac{2\sqrt{6}}{5}\right)$, then the square of the length of perpendicular from the point $(38\lambda_1, 10\lambda_2, 2)$ to the plane P_1 is _______.

Answer (315)

Sol. $P_1 : \vec{r}\cdot\left(3\hat{i} - 5\hat{j} + \hat{k}\right) = 7$

$P_2 : \vec{r}\cdot\left(\lambda\hat{i} + \hat{j} - 3\hat{k}\right) = 9$

Let angle between P_1 and P_2 is θ

Then $\cos\theta = \frac{3\lambda - 5 - 3}{\sqrt{35}\sqrt{\lambda^2 + 10}}$

But $\sin\theta = \frac{2\sqrt{6}}{5}$

$\therefore \quad \frac{(3\lambda - 8)^2}{35\left(\lambda^2 + 10\right)} = 1 - \frac{24}{25}$

$\Rightarrow \quad 5(9\lambda^2 + 64 - 48\lambda) = 7\lambda^2 + 70$

$\Rightarrow \quad 38\lambda^2 - 240\lambda + 250 = 0$

$\Rightarrow \quad 19\lambda^2 - 120\lambda + 125 = 0$

$\Rightarrow \quad (19\lambda - 25)(\lambda - 5) = 0$

$\therefore \quad \lambda_1 = \frac{25}{19}, \lambda_2 = 5$

So, point (50, 50, 2)

$\therefore \quad d = \frac{|150 - 250 + 2 - 7|}{\sqrt{35}} = 315$

84. Let $\sum_{n=0}^{\infty} \frac{n^3\left((2n)!\right) + (2n-1)(n!)}{(n!)\left((2n)!\right)} = ae + \frac{b}{e} + c$, where $a, b, c \in \mathbb{Z}$ and $e = \sum_{n=0}^{\infty} \frac{1}{n!}$ Then $a^2 - b + c$ is equal to _______.

Answer (26)

Sol. $\sum_{n=0}^{\infty} \frac{n^3(2n!) + (2n-1)(n!)}{n!\cdot(2n)!}$

$= \sum_{n=0}^{\infty} \frac{n^3}{n!} + \frac{2n-1}{2n!}$

$= \sum_{n=0}^{\infty} \frac{3}{(n-2)!} + \frac{1}{(n-3)!} + \frac{1}{(n-1)!} + \frac{1}{(2n-1)!} - \frac{1}{(2n)!}$

$= 3e + e + e - \frac{1}{e}$

$= 5e - \frac{1}{e}$

$\therefore \quad a = 5, b = -1, c = 0$

$\therefore \quad a^2 - b + c = 26$

85. Let $z = 1 + i$ and $z_1 = \frac{1 + i\bar{z}}{\bar{z}(1 - z) + \frac{1}{z}}$. Then $\frac{12}{\pi}\arg(z_1)$ is equal to _______.

Answer (09)

Sol. $z = 1 + i$

$z_1 = \frac{1 + i\bar{z}}{\bar{z}(1 - z) + \frac{1}{z}}$

$= \frac{z(1 + i\bar{z})}{|z|^2(1 - z) + 1}$

$= \frac{(1 + i)(1 + i(1 - i))}{2(1 - 1 - i) + 1}$

$z_1 = 1 - i$

$\arg z_1 = \tan^{-1}\left(\frac{-1}{1}\right) = \frac{3\pi}{4}$

$\frac{12}{\pi}\arg(z_1) = \frac{3\pi}{4}\cdot\frac{12}{\pi}$

$= 9$

86. Let $f^1(x) = \frac{3x + 2}{2x + 3}, x \in R - \left\{\frac{-3}{2}\right\}$

For $n \geq 2$, define $f^n(x) = f^1 of^{n-1}(x)$.

If $f^5(x) = \frac{ax + b}{bx + a}$, gcd$(a, b) = 1$, then $a + b$ is equal to _______.

Answer (3125)

Sol. $f'(x) = \frac{3x + 2}{2x + 3} x \in R - \left\{-\frac{3}{2}\right\}$

$f^5(x) = f_o f_o f_o f_o f(x)$

$f_o f(x) = \frac{13x + 12}{12x + 13}$

$f_o f_o f_o f_o f(x) = \frac{1563x + 1562}{1562x + 1563}$

$\equiv \frac{ax + b}{bx + a}$

$\therefore \quad a = 1563, b = 1562$

$= 3125$

87. $\lim_{x\to 0} \frac{48}{x^4}\int_0^x \frac{t^3}{t^6+1}dt$ is equal to _________.

Answer (12)

Sol. $\underset{x\to 0}{\text{limit}} \frac{48}{x^4}\int_0^x \frac{t^3}{t^6+1}dt.$

$\underset{x\to 0}{\text{limit}} \frac{48}{4x^3}\cdot\left(\frac{x^3}{x^6+1}\right)$

$\underset{x\to 0}{\text{limit}} \frac{12}{x^6+1} = 12$

88. The mean and variance of 7 observations are 8 and 16 respectively. If one observation 14 is omitted and a and b are respectively mean and variance of remaining 6 observation, then $a + 3b - 5$ is equal to _______.

Answer (37)

Sol. $\sum x_i = 7\times 8 = 56$

$\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 = 16$

$\frac{\sum x_i^2}{7} - 64 = 16$

$\sum x_i^2 = 560$

When 14 is omitted

$\sum x_i = 56 - 14 = 42$

New mean = $a = \frac{\sum x_i}{6} = 7$

$\sum x_i^2 = 560 - 196 = 364$

new variance, $b = \frac{\sum x_i^2}{6} - \left(\frac{\sum x_i}{6}\right)^2$

$= \frac{364}{6} - 49 = \frac{35}{3}$

$3b = 35$

$a + 3b - 5 = 7 + 35 - 5 = 37$

89. If the equation of the plane passing through the point 1, 1, 2) and perpendicular to the line $x - 3y + 2z - 1 = 0 = 4x - y + z$ is $Ax + By + Cz = 1$, then $140(C - B + A)$ is equal to ______.

Answer (15)

Sol. Line of intersection of the planes $x - 3y + 2z - 1 = 0$ and $4x - y + z = 0$ is normal $(\vec{n})$ to the required plane.

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 4 & -1 & 1 \end{vmatrix} = -\hat{i} + 7\hat{j} + 11\hat{k}$$

Equation of plane is

$-x + 7y + 11z = \lambda$

It passes through (1, 1, 2)

$\therefore\ \lambda = 28$

So, the plane is

$-x + 7y + 11z = 28$

$\Rightarrow \frac{-1}{28}x + \frac{7}{28}y + \frac{11}{28}z = 1$

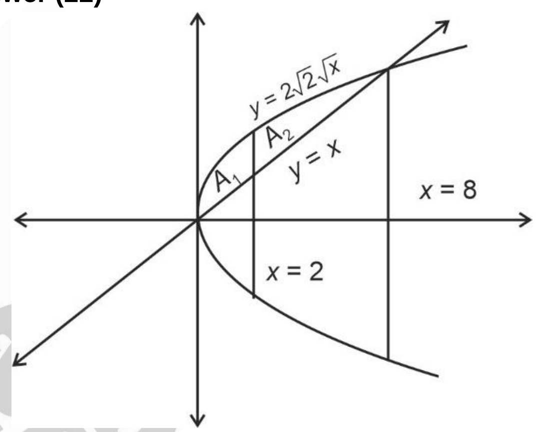
$A = \frac{-1}{28}, B = \frac{7}{28}, C = \frac{11}{28}$

$140(C - B + A) = 15$

90. Let α be the area of the larger region bounded by the curve $y^2 = 8x$ and the line $y = x$ and $x = 2$, which lies in the first quadrant. Then the value of 3α is equal to ________.

Answer (22)

Sol.



$A_1 = \int_0^2 2\sqrt{2}\sqrt{x} - x dx$

$= 2\sqrt{2}\times\frac{2}{3}\cdot x^{\frac{3}{2}} - \frac{x^2}{2}\Big]_0^2$

$= \frac{4\sqrt{2}}{3}\times 2\sqrt{2} - 2$

$= \frac{16}{3} - 2 = \frac{10}{3}$

$A_2 = \int_2^8 2\sqrt{2}\sqrt{x} - x\, dx = \frac{4\sqrt{2}}{3}\cdot x^{\frac{3}{2}} - \frac{x^2}{2}\Big]_2^8$

$= \frac{4\sqrt{2}}{3}\left(16\sqrt{2} - 2\sqrt{2}\right) - 30$

$= \frac{112}{3} - 30 = \frac{22}{3}$

$A_2 > A_1 \Rightarrow 3\alpha = 22$

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