31/01/2023
Evening

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Answers & Solutions

Time : 3 hrs. **for** M.M. : 300

JEE (Main)-2023 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. For a solid rod, the Young's modulus of elasticity is 3.2×10^{11} Nm^{-2} and density is 8×10^3 kg m^{-3}. The velocity of longitudinal wave in the rod will be

(1) 18.96×10^3 ms^{-1} (2) 3.65×10^3 ms^{-1}

(3) 145.75×10^3 ms^{-1} (4) 6.32×10^3 ms^{-1}

Answer (4)

Sol. $v = \sqrt{\dfrac{Y}{\rho}} = \sqrt{\dfrac{3.2\times10^{11}}{8\times10^3}}$

$= 2\times10^3\sqrt{10}$

$= 6.32 \times 10^3$ m/s

2. A microscope is focused on an object at the bottom of a bucket. If liquid with refractive index $\dfrac{5}{3}$ is poured inside the bucket, then microscope have to be raised by 30 cm to focus the object again. The height of the liquid in the bucket is

(1) 12 cm (2) 18 cm

(3) 75 cm (4) 50 cm

Answer (3)

Sol. Shift $= \left(d - \dfrac{d}{\mu}\right) = 30$ cm

$d\left[1-\dfrac{1}{\frac{5}{3}}\right] = 30$

$d = \dfrac{30\times5}{2} = 75$ cm

3. Match List I with List II:

	List I		List II
A.	Microwaves	I.	Physiotherapy
B.	UV rays	II.	Treatment of cancer
C.	Infra-red light	III.	Lasik eye surgery
D.	X-ray	IV.	Aircraft navigation

Choose the correct answer from the options given below:

(1) A-IV, B-I, C-II, D-III (2) A-III, B-II, C-I, D-IV

(3) A-II, B-IV, C-III, D-I (4) A-IV, B-III, C-I, D-II

Answer (4)

Sol. (Theoretical)

A. Microwave $\rightarrow$ IV B. UV rays $\rightarrow$ III

C. Infra-red $\rightarrow$ I D. X-ray $\rightarrow$ II

4. A body is moving with constant speed, in a circle of radius 10 m. The body completes one revolution in 4 s. At the end of 3rd second, the displacement of body (in m) from its starting point is:

(1) 30 (2) 15π

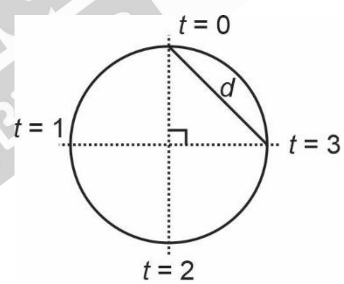
(3) 5π (4) $10\sqrt{2}$

Answer (4)

Sol. $r = 10$ m

$T = 4$ sec

$d = \sqrt{2}(10)$ m



5. If the two metals A and B are exposed to radiation of wavelength 350 nm. The work functions of metals A and B are 4.8 eV and 2.2 eV. Then choose the correct option.

(1) Metals B will not emit photo-electrons

(2) Both metals A and B will not emit photo-electrons

(3) Both metals A and B will emit photo-electrons

(4) Metals A will not emit photo-electrons

Answer (4)

Sol. $\phi = \dfrac{hc}{\lambda} = \dfrac{1240}{350}$ eV $= 3.54$ eV

$\therefore$ Only metal B will emit photoelectron.

6. A body of mass 10 kg is moving with an initial speed of 20 m/s. The body stops after 5 s due to friction between body and the floor. The value of the coefficient of friction is (Take acceleration due to gravity g = ms^{-2})

(1) 0.2 (2) 0.4

(3) 0.3 (4) 0.5

Answer (2)

Sol. $a = -\mu g$

$\because\ v = u + at$

$0 = 20 + (-\mu \times 10) \times 5$

$50\mu = 20$

$\mu = \frac{2}{5} = 0.4$

7. A hypothetical gas expands adiabatically such that its volume changes from 08 litres to 27 litres. If the ratio of final pressure of the gas to initial pressure of the gas is $\frac{16}{81}$. Then the ratio of $\frac{C_p}{C_v}$ will be

(1) $\frac{4}{3}$ (2) $\frac{1}{2}$

(3) $\frac{3}{2}$ (4) $\frac{3}{1}$

Answer (1)

Sol. Let γ be the ratio of $\frac{C_p}{C_v}$

Then for adiabatic process

PV^{γ} = Constant

$$\frac{P_i}{P_f} = \left(\frac{V_f}{V_i}\right)^{\gamma}$$

$$\frac{81}{16} = \left(\frac{27}{8}\right)^{\gamma}$$

$$\gamma = \frac{4}{3}$$

8. An alternating voltage source V = 260 sin (628t) is connected across a pure inductor of 5 mH. Inductive reactance in the circuit is

(1) 0.318 Ω (2) 6.28 Ω

(3) 0.5 Ω (4) 3.14 Ω

Answer (4)

Sol. $X_L = L\,\omega$

= 5 mH × 628

= 3.14 Ω

9. Under the same load, wire A having length 5.0 m and cross-section 2.5 × 10^{-5} m^2 stretches uniformly by the same amount as another wire B of length 6.0 m and a cross-section of 3.0 × 10^{-5} m^2 stretches. The ratio of the Young's modulus of wire A to that of wire B will be

(1) 1 : 4 (2) 1 : 2

(3) 1 : 10 (4) 1 : 1

Answer (4)

Sol. $\Delta\ell = \frac{F\ell}{SY}$

F is same for both wire and $\Delta\ell$ is also same

$$\frac{\Delta\ell}{F} = \frac{\ell}{SY} \Rightarrow \frac{\ell_A}{S_A Y_A} = \frac{\ell_B}{S_B Y_B}$$

$$\Rightarrow \frac{5}{2.5 \times Y_A} = \frac{6}{3 \times Y_B}$$

$$\Rightarrow \frac{Y_A}{Y_B} = 1$$

10. Considering a group of positive charges, which of the following statements is correct?

(1) Net potential of the system cannot be zero at a point but net electric field can be zero at that point.

(2) Both the net potential and the net field can be zero at a point.

(3) Net potential of the system at a point can be zero but net electric field can't be zero at that point.

(4) Both the net potential and the net electric field cannot be zero at a point.

Answer (1)

Sol. $V = \frac{\sum KQ_i}{r_i}$

Here, Q_i & r_i are positive

$\therefore\ V > 0$

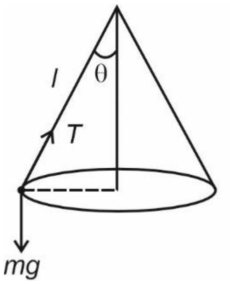
11. A stone of mass 1 kg is tied to end of a massless string of length 1m. If the breaking tension of the string is 400 N, then maximum linear velocity, the stone can have without breaking the string, while rotating in horizontal plane, is:

(1) 40 ms^{-1} (2) 10 ms^{-1}

(3) 20 ms^{-1} (4) 400 ms^{-1}

Answer (3)

Sol.



$T\cos\theta = mg$

$$T\sin\theta = \frac{mv^2}{l^2\sin\theta}$$

$$\cos\theta = \frac{mg}{T} \quad ...(1)$$

$$\sin 2\theta = \frac{mv^2}{Tl^2} \quad ...(2)$$

From (1) & (2)

$$l = \left(\frac{mg}{T}\right)^2 + \frac{mv^2}{Tl^2}$$

$$l = \left(\frac{10}{400}\right)^2 + \frac{v^2}{400}$$

$v^2 = 399.78$

$v = 20$ m/s

12. The number of turns of the coil of a moving coil galvanometer is increased in order to increase current sensitivity by 50%. The percentage change in voltage sensitivity of the galvanometer will be:

(1) 0% (2) 100%

(3) 75% (4) 50%

Answer (1)

Sol. Current sensitivity = Voltage sensitivity × R

Current sensitivity is made 1.5 times.

R also increase 1.5 times.

$$\text{Hence voltage sensitivity} = \frac{1.5 \times \text{current sensitivity}}{1.5 \times R}$$

= no change

13. Given below are two statements:

Statement I: In a typical transistor, all three regions emitter, base and collector have same doping level.

Statement II: In a transistor, collector is the thickest and base is the thinnest segment.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) **Statement I** is correct but **Statement II** is incorrect

(2) Both **Statement I** and **Statement II** are incorrect

(3) **Statement I** is incorrect but **Statement II** is correct

(4) Both **Statement I** and **Statement II** are correct

Answer (3)

Sol. In transistor, emitter collector and base have different doping levels and collector is the thickest while base is thinnest segment.

14. The radius of electron's second stationary orbit in Bohr's atom is R. The radius of 3rd orbit will be

(1) $\frac{R}{3}$ (2) $2.25R$

(3) $9R$ (4) $3R$

Answer (2)

Sol. $r \propto \frac{n^2}{Z}$

$$\frac{r_{2nd}}{r_{3rd}} = \left(\frac{n_2}{n_3}\right)^2$$

$$\Rightarrow \frac{R}{r_{3rd}} = \left(\frac{2}{3}\right)^2$$

$$\Rightarrow r_{3rd} = \frac{9}{4}R$$

$= 2.25R$

15. A long conducting wire having a current I flowing through it, is bent into a circular coil of N turns. Then it is bent into a circular coil of n turns. The magnetic field is calculated at the centre of coils in both the cases. The ratio of the magnetic field in first case to that of second case is:

(1) $n^2 : N^2$ (2) $N : n$

(3) $N^2 : n^2$ (4) $n : N$

Answer (3)

Sol. $l = (2\pi r)n$

$$r \propto \left(\frac{l}{n}\right)$$

$$B = n\left(\frac{\mu_0 i}{2r}\right) \propto \left(\frac{\mu_0 i}{2L}\right)n^2$$

$$\frac{B_1}{B_2} = \left(\frac{N^2}{n^2}\right)$$

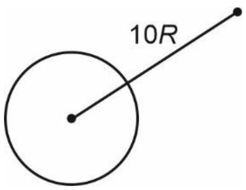
16. A body weight W, is projected vertically upwards from earth's surface to reach a height above the earth which is equal to nine times the radius of earth. The weight of the body at that height will be:

(1) $\frac{W}{91}$ (2) $\frac{W}{3}$

(3) $\frac{W}{100}$ (4) $\frac{W}{9}$

Answer (3)

Sol.



$$g' = \frac{GM}{(10R)^2} = \left(\frac{g}{100}\right)$$

$$W' = \left(\frac{W}{100}\right)$$

17. Given below are two statements :

Statement I : For transmitting a signal, size of antenna (l) should be comparable to wavelength of signal (at least $l = \frac{\lambda}{4}$ in dimension)

Statement II : In amplitude modulation, amplitude of carrier wave remains constant (unchanged).

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) Both Statement I and Statement II are correct

(2) Statement I is incorrect but Statement II is correct

(3) Both Statement I and Statement II are incorrect

(4) Statement I is correct but Statement II is incorrect

Answer (4)

Sol. • In amplitude modulation frequency of carrier wave remains unchanged.

• Minimum size of antenna should be $\frac{1}{4}$th of wavelength.

18. The H amount of thermal energy is developed by a resistor in 10 s when a current of 4 A is passed through it. If the current is increased to 16 A, the thermal energy developed by the resistor in 10 s will be:

(1) H (2) 16H

(3) 4H (4) $\frac{H}{4}$

Answer (2)

Sol. $H \propto i^2$ for t = constant

$$\frac{H}{H'} = \left(\frac{4}{16}\right)^2$$

$H' = 16H$

19. Match List I with List II

	List I		List II
A.	Angular momentum	I.	$[ML^2T^{-2}]$
B.	Torque	II.	$[ML^{-2}T^{-2}]$
C.	Stress	III.	$[ML^2T^{-1}]$
D.	Pressure gradient	IV.	$[ML^{-1}T^{-2}]$

(1) A - I, B - IV, C - III, D - II

(2) A - II, B - III, C - IV, D - I

(3) A - IV, B- II, C - I, D - III

(4) A - III, B - I, C - IV, D - II

Answer (4)

Sol. $\vec{L} = \vec{r} \times \vec{p} \Rightarrow [L] = [M^0L^1T^0]\,[M^1L^1T^{-1}]$

$= [M^1L^2T^{-1}]$

$\vec{\tau} = \vec{r} \times \vec{F} \Rightarrow [\tau] = [L^1]\,[MLT^{-2}]$

$= [ML^2T^{-2}]$

Stress $\equiv$ Pressure = $\frac{F}{A} \Rightarrow$ [Stress] = $[ML^{-1}T^{-2}]$

Pressure Gradient = $\frac{dP}{dx} \Rightarrow$ [Pressure Gradient]

$= [ML^{-2}T^{-2}]$

20. Heat energy of 735 J is given to a diatomic gas allowing the gas to expand at constant pressure. Each gas molecule rotates around an internal axis but do not oscillate. The increase in the internal energy of the gas will be:

(1) 572 J (2) 525 J

(3) 441 J (4) 735 J

Answer (3)

Sol. $\Delta Q = nC_P\Delta T = 735$ J

$$\Rightarrow \frac{5nR\Delta T}{2} = 735 \text{ J}$$

$$\Delta U = nC_V\Delta T = \frac{3}{2}(nR\Delta T) = \frac{3}{2} \times \frac{2}{5} \times 735$$

$= 441$ J

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. A water heater of power 2000 W is used to heat water. The specific heat capacity of water is 4200 J kg^{-1} K^{-1}. The efficiency of heater is 70%. Time required to heat 2 kg of water from 10°C to 60°C is_____________ s.

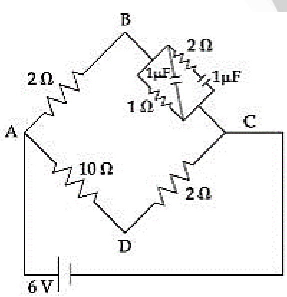
(Assume that the specific heat capacity of water remains constant over the temperature range of the water).

Answer (300)

Sol. $\eta \times P \times \Delta t = M \times s \times \Delta T$

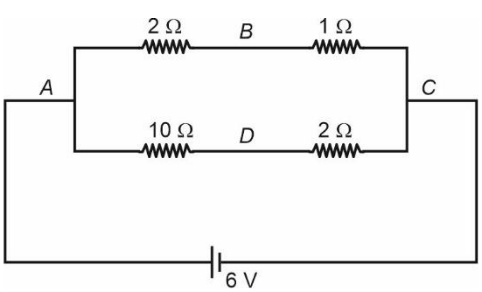
$$\Rightarrow \quad \Delta t = \frac{2 \times 4200 \times (60-10)}{0.7 \times 2000} \text{ s} = 300 \text{ s}$$

22. For the given circuit, in the steady state, $|V_B - V_D| =$ ___________________ V.



Answer (1)

Sol. In steady state, capacitor behaves as an open circuit. Circuit is:



$$\Rightarrow \quad i_{AB} = \frac{6}{3} = 2\text{A} \qquad \& \qquad i_{AD} = \frac{6}{12} = 0.5 \text{ A}$$

$\Rightarrow \quad V_B + 2 \times 2 - 10 \times 0.5 = V_D$

$\Rightarrow \quad V_B - V_D = 1$ volt

23. A ball is dropped from a height of 20 m. If the coefficient of restitution for the collision between ball and floor is 0.5, after hitting the floor, the ball rebounds to a height of ______________ m.

Answer (5)

Sol. We know $h' = e^2h$

$h' = (0.5)^2 \times 20$ m = 5 m

24. Two bodies are projected from ground with same speeds 40 ms^{-1} at two different angles with respect to horizontal. The bodies were found to have same range. If one of the body was projected at an angle of 60°, with horizontal then sum of the maximum heights, attained by the two projectiles, is ___________m. (Given g = 10 ms^{-2})

Answer (80)

Sol. Since range is same.

$\Rightarrow \quad \theta_1 + \theta_2 = 90°$

$\Rightarrow \quad \theta_2 = 30°$

$$\Rightarrow \quad (H_{\max})_1 + (H_{\max})_2 = \frac{U^2 \sin^2 \theta_1}{2g} + \frac{U^2 \sin^2 \theta_2}{2g}$$

$$= \frac{40^2}{20}\left(\frac{1}{4} + \frac{3}{4}\right) = 80 \text{ m}$$

25. Two parallel plate capacitors C_1 and C_2 each having capacitance of 10 μF are individually charged by a 100 V D.C. source. Capacitor C_1 is kept connected to the source and a dielectric slab is inserted between it plates. Capacitor C_2 is disconnected from the source and then a dielectric slab is inserted in it. Afterwards the capacitor C_1 is also disconnected from the source and the two capacitors are finally connected in parallel combination. The common potential of the combination will be __________ V.

(Assuming Dielectric constant = 10)

Answer (55)

Sol. Charge on C_1 = KCE

And charge on C_2 = CE

When they are connected in parallel charge will be equally divided so charge on one capacitor is

$$q = \frac{K+1}{2}CV$$

So $V = \frac{q}{KC} = \frac{K+1}{2K} = 55\text{ V}$

26. If the binding energy of ground state electron in a hydrogen atom is 13.6 eV, then, the energy required to remove the electron from the second excited state of Li^{2+} will be $x \times 10^{-1}$ eV. The value of x is __________.

Answer (136)

Sol. $E_H = 13.6$

$$E_{Li^{2+}} = 13.6\frac{Z^2}{n^2} = 13.6 \times \frac{9}{9} = 13.6\text{ eV}$$

= 136×10^{-1} eV

27. Two discs of same mass and different radii are made of different materials such that their thickness are 1 cm and 0.5 cm respectively. The densities of materials are in the ratio 3 : 5. The moment of inertia of these discs respectively about their diameters will be in the ratio of $\frac{x}{6}$. The value of x is ________.

Answer (05)

Sol. $m = \rho\pi R^2 t$

so $R^2 = \frac{m}{\rho\pi t}$

$$I = \frac{mR^2}{4} = \frac{m^2}{4\rho\pi t}$$

so $\frac{I_1}{I_2} = \frac{\rho_2 t_2}{\rho_1 t_1} = \frac{5}{3} \times \frac{0.5}{1} = \frac{5}{6}$

so $x = 5$

28. A series LCR circuit consists of R = 80 Ω, X_L = 100 Ω, and X_C = 40 Ω. The input voltage is 2500 cos(100 πt) V. The amplitude of current, in the circuit, is __________ A.

Answer (25)

Sol. $\omega = 100\pi$

so $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{80^2 + (100-40)^2}$

= 100 Ω

$$i_0 = \frac{V_0}{Z} = \frac{2500}{100}\text{A} = 25\text{ A}$$

29. The displacement equations of two interfering waves are given by $y_1 = 10\sin\left(\omega t + \frac{\pi}{3}\right)$ cm, $y_2 = 5\left[\sin\omega t + \sqrt{3}\cos\omega t\right]$ cm respectively. The amplitude of the resultant wave is __________ cm.

Answer (20)

Sol. $y_2 = 5(\sin\omega t + \sqrt{3}\cos\omega t)$

$= 10\sin\left(\omega t + \frac{\pi}{3}\right)$

Thus the phase difference between the waves is 0.

so A = $A_1 + A_2$ = 20 cm

30. Two light waves of wavelengths 800 and 600 nm are used in Young's double slit experiment to obtain interference fringes on a screen placed 7 m away from plane of slits. If the two slits are separated by 0.35 mm, then shortest distance from the central bright maximum to the point where the bright fringes of the two wavelength coincide will be __________ mm.

Answer (48)

Sol. $\omega_1 = \frac{\lambda_1 D}{d}$ & $\omega_2 = \frac{\lambda_2 D}{d}$

ω_1 = 16 mm & ω_2 = 12 mm

so LCM (ω_1, ω_2) = 48 mm

so at 48 mm distance both bright fringes will be found.

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

31. The normal rain water is slightly acidic and its pH value is 5.6 because of which one of the following?

(1) $4NO_2 + O_2 + 2H_2O \rightarrow 4HNO_3$

(2) $CO_2 + H_2O \rightarrow H_2CO_3$

(3) $N_2O_5 + H_2O \rightarrow 2HNO_3$

(4) $2SO_2 + O_2 + 2H_2O \rightarrow 2H_2SO_4$

Answer (2)

Sol. Normally rain water has a pH of 5.6 due to presence of H^+ ions formed by the reaction of rain water with carbon dioxide present in the atmosphere.

$$H_2O(l) + CO_2(g) \rightleftharpoons H_2CO_3(g)$$

Hence correct answer is (2)

32. Given below are two statements:

Statement I : Upon heating a borax bead dipped in cupric sulphate in a luminous flame, the colour of the bead becomes green.

Statement II: The green colour observed is due to the formation of copper(I) metaborate.

In the light of the above statements, choose the **most appropriate** answer from the options given below.

(1) **Statement I** is false but **Statement II** is true

(2) **Statement I** is true but **Statement II** is false

(3) Both **Statement I** and **Statement II** are false

(4) Both **Statement I** and **Statement II** are true

Answer (3)

Sol. Both statements are incorrect.

33. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A): The first ionization enthalpy of 3d series elements is more than that of group 2 metals.

Reason (R): In 3d series of elements successive filling of d-orbitals takes place.

In the light of the above statements, choose the **correct** answer from the options given below.

(1) **(A)** is true but **(R)** is false

(2) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

(3) **(A)** is false but **(R)** is true

(4) Both **(A)** and **(R)** are true but **(R)** is **not** the correct explanation of **(A)**

Answer (2)

Sol. The first ionization energy of 3d series elements is more than that of group 2 metals because in 3d series of elements successive filling of d-orbitals takes place.

34. The element playing significant role in neuromuscular function and interneuronal transmission is

(1) Li (2) Mg

(3) Ca (4) Be

Answer (3)

Sol. Ca plays significant role in muscular function and interneuronal transmission.

35. Arrange the following orbitals in decreasing order of energy.

A. n = 3, l = 0, m = 0

B. n = 4, l = 0, m = 0

C. n = 3, l = 1, m = 0

D. n = 3, l = 2, m = 1

The correct option for the order is

(1) A > C > B > D (2) B > D > C > A

(3) D > B > A > C (4) D > B > C > A

Answer (4)

Sol. (A) n = 3, l = 0, m = 0 ; 3s

(B) n = 4, l = 0, m = 0 ; 4s

(C) n = 3, l = 1, m = 1; 3p

(D) n = 3, l = 2, m = 1 ; 3d

Correct order of energy will be (D) > (B) > (C) > (A)

36. Evaluate the following statements for their correctness.

A. The elevation in boiling point temperature of water will be same for 0.1 M NaCl and 0.1 M urea.

B. Azeotropic mixture boil without change in their composition.

C. Osmosis always takes place from hypertonic to hypotonic solution.

D. The density of 32% H_2SO_4 solution having molarity 4.09 M is approximately 1.26 g mL^{-1}.

E. A negatively charged sol is obtained when KI solution is added to silver nitrate solution.

Choose the correct answer from the options given below.

(1) A and C only
(2) B, D and E only
(3) A, B and D only
(4) B and D only

Answer (4)

Sol. Elevation in boiling point temperature of water will be higher for 0.1 M NaCl as compared to 0.1 M urea.

Azeotropic mixtures boil without change in their composition

Osmosis always takes place from hypotonic (low concentration of solute) solution to hypertonic (high concentration of solute) solution.

A negative charged sol is obtained when KI solution is added to silver nitrate solution whereas positive charged sol is obtained when $AgNO_3$ solution is added to KI solution.

Let the mass of H_2SO_4 (32%) is 100.

$\therefore$ wt of H_2SO_4 = 32

Moles of H_2SO_4 = $\frac{32}{98}$

Now, $4.09 = \frac{32}{98 \times V} \Rightarrow V = 79$ ml

Density = $\frac{100}{79} = 1.265$

Hence, correct answer is (4) B and D only

37. When a hydrocarbon A undergoes complete combustion it require 11 equivalents of oxygen and produces 4 equivalents of water. What is the molecular formula of A?

(1) $C_{11}H_8$
(2) C_9H_8
(3) $C_{11}H_4$
(4) C_5H_8

Answer (2)

Sol. $C_xH_y + \left(x + \frac{y}{4}\right)O_2 \longrightarrow xCO_2 + \frac{y}{2}H_2O$

$x + \frac{y}{4} = 11$

$\frac{y}{2} = 4$

$\therefore$ y = 8

x = 9

$\therefore$ C_9H_8 will be the formula of hydrocarbon A.

38. The Lewis acid character of boron tri halides follows the order

(1) $BBr_3 > BI_3 > BCl_3 > BF_3$

(2) $BCl_3 > BF_3 > BBr_3 > BI_3$

(3) $BI_3 > BBr_3 > BCl_3 > BF_3$

(4) $BF_3 > BCl_3 > BBr_3 > BI_3$

Answer (3)

Sol. Correct order of Lewis acidity is

$BI_3 > BBr_3 > BCl_3 > BF_3$

39. A hydrocarbon 'X' with formula C_6H_8 uses two moles H_2 on catalytic hydrogenation of its one mole. On ozonolysis, 'X' yields two moles of methane dicarbaldehyde. The hydrocarbon 'X' is

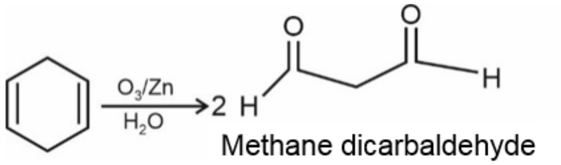
(1) hexa –1, 3, 5–triene

(2) cyclohexa – 1, 3 – diene

(3) cyclohexa – 1, 4 – diene

(4) 1 – methylcyclopenta – 1, 4 – diene

Answer (3)

Sol. 

Hence, correct answer is (3)

40. In the following halogenated organic compounds the one with maximum number of chlorine atoms in its structure is

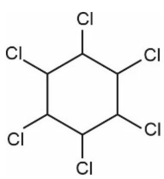
(1) Chloropicrin (2) Freon – 12

(3) Gammaxene (4) Chloral

Answer (3)

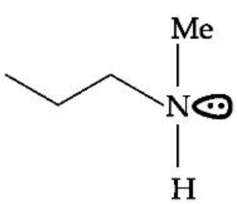
Sol. Chloropicrin — $Cl_3C – NO_2$

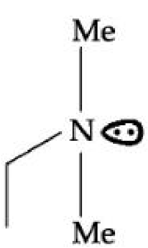
Freon-12 — CF_2Cl_2

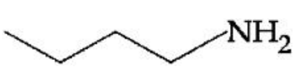
Gammaxene — 

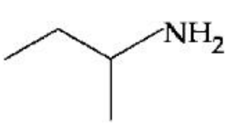
Chloral — $Cl_3C–CHO$

41. An organic compound [A] ($C_4H_{11}N$), shows optical activity and gives N_2 gas on treatment with HNO_2. The compound [A] reacts with $PhSO_2Cl$ producing a compound which is soluble in KOH. The structure of A is :

(1) 

(2) 

(3) 

(4) 

Answer (4)

Sol. Only primary amines react with $PhSO_2Cl$ to produce a compounds which are soluble in KOH. Option (3) and (4) are primary amines but the given compound is also optically active.

Hence the correct answer is (4).

42. Which of the following elements have half-filled f-orbitals in their ground state?

(Given : atomic number Sm = 62; Eu = 63; Tb = 65; Gd = 64, Pm = 61)

(A) Sm (B) Eu

(C) Tb (D) Gd

(E) Pm

Choose the **correct** answer from the options given below:

(1) A and E only (2) B and D only

(3) C and D only (4) A and B only

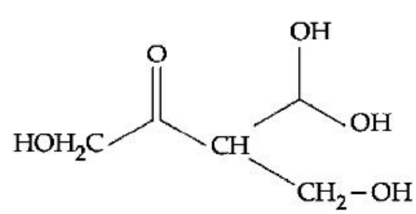
Answer (2)

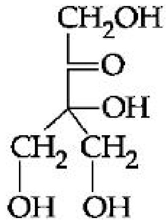
Sol.

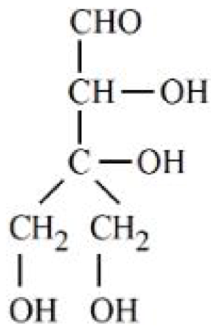
Element	Electronic configuration
(A) Sm	$[Xe]4f^6 6s^2$
(B) Eu	$[Xe]4f^7 6s^2$
(C) Tb	$[Xe]4f^9 6s^2$
(D) Gd	$[Xe]4f^7 5d^1 6s^2$
(E) Pm	$[Xe]4f^5 6s^2$

43. Compound A, $C_5H_{10}O_5$, given a tetraacetate with AC_2O and oxidation of A with $Br_2 – H_2O$ gives an acid, $C_5H_{10}O_6$. Reduction of A with HI gives isopentane. The possible structure of A is :

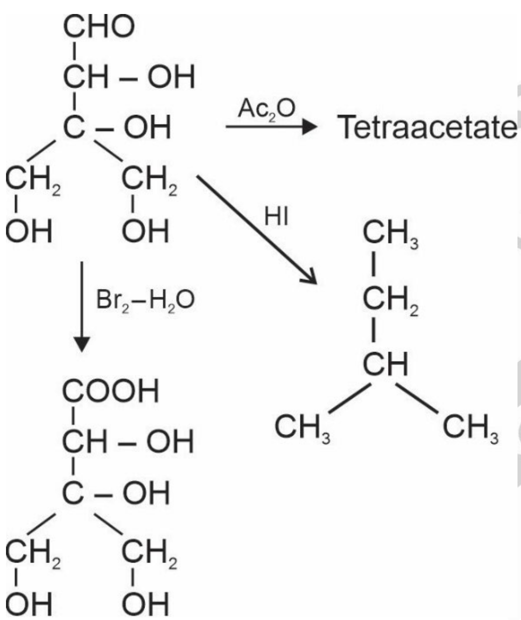
(1) CHO
|
$(CHOH)_3$
|
CH_2OH

(2) 

(3) 

(4) 

Answer (4)

Sol. 

Correct answer is (4)

44. Which of the following compounds are not used as disinfectants?

(A) Chloroxylenol (B) Bithional

(C) Veronal (D) Prontosil

(E) Terpineol

Choose the **correct** answer from the options given below:

(1) A, B (2) C, D

(3) A, B, E (4) B, D, E

Answer (2)

Sol. (A) Chloroxylenol

(B) Bithional

(E) Terpineol

are used as disinfectants.

45. Incorrect statement for the use of indicators in acid-base titration is :

(1) Phenolphthalein may be used for a strong acid vs strong base titration.

(2) Phenolphthalein is a suitable indicator for a weak acid vs strong base titration.

(3) Methyl orange may be used for a weak acid vs weak base titration.

(4) Methyl orange is a suitable indicator for a strong acid vs weak base titration.

Answer (3)

Sol. There is no suitable indicator that can be used in the titration of weak acid and weak base.

Hence correct answer (3).

46. Given below are two statements

Statement I: H_2O_2 is used in the synthesis of Cephalosporin

Statement II: H_2O_2 is used for the restoration of aerobic conditions to sewage wastes.

In the light of the above statements, choose the **most appropriate** answer from the options given below

(1) **Statement I** is incorrect but **Statement II** is correct

(2) Both **Statement I** and **Statement II** are incorrect

(3) **Statement I** is correct but **Statement II** is incorrect

(4) Both **Statement I** and **Statement II** are correct

Answer (4)

Sol. H_2O_2 is used in the synthesis of hydroquinone, tartaric acid and certain food products and pharmaceuticals (cephalosporin).

Nowadays it is also used in environmental (green) chemistry for example in pollution control treatment of domestic and industrial effluents, oxidation of cyanides restoration of aerobic condition to sewage waste.

Hence both statements are correct.

47. Which one of the following statements is incorrect?

(1) Boron and Indium can be purified by zone refining method

(2) van Arkel method is used to purify tungsten

(3) The malleable iron is prepared from cast iron by oxidising impurities in a reverberatory furnace

(4) Cast iron is obtained by melting pig iron with scrap iron and coke using hot air blast

Answer (2)

Sol. Van Arkel method is used to purify Zirconium or Titanium. Rest all statements are correct.

Hence correct answer is option (2).

48. In Dumas method for the estimation of N_2, the sample is heated with copper oxide and the gas evolved is passed over

(1) Pd (2) Copper gauze

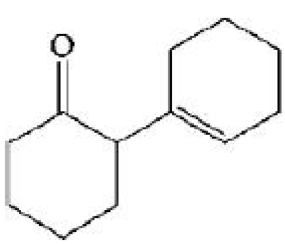
(3) Ni (4) Copper oxide

Answer (2)

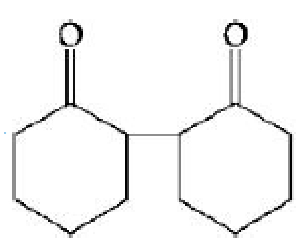
Sol. In dumas method for the estimation of N_2, the sample is heated with copper oxide and the gas evolved is passed over copper gauze.

49. Cyclohexylamine when treated with nitrous acid yields (P). On treating (P) with PCC results in (Q). When (Q) is heated with dil. NaOH we get (R). The final product (R) is

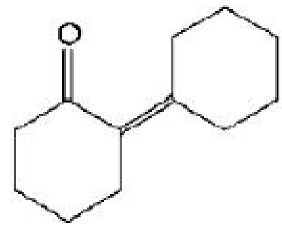
(1)



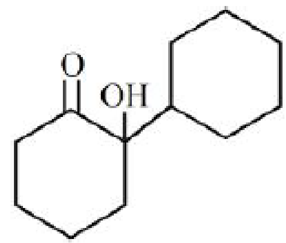
(2)



(3)

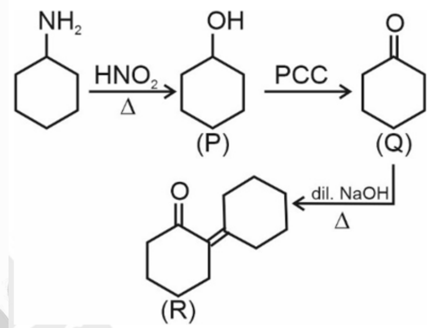


(4)



Answer (3)

Sol.



50. Match **List I** with **List II**

	LIST-I		LIST-II
A.	Physisorption	I.	Single Layer Adsorption
B.	Chemisorption	II.	20 – 40 kJ mol^{-1}
C.	$N_2(g) + 3H_2(g) \xrightarrow{Fe(s)} 2NH_3(g)$	III.	Chromatography
D.	Analytical Application or Adsorption	IV.	Heterogeneous catalysis

Choose the **correct** answer from the options given below

(1) A – II, B – I, C – IV, D – III

(2) A – IV, B – II, C – III, D – I

(3) A – II, B – III, C – I, D – IV

(4) A – III, B – IV, C – I, D – II

Answer (1)

Sol. A - (II), B - (I), C - (IV), D - (III),

	List I		List II
A.	Physisorption	(II)	20 – 40 kJ mol^{-1}
B.	Chemisorption	(I)	Single Layer Adsorptions
C.	$N_2(g) + 3H_2(g) \xrightarrow{Fe(s)} 2NH_3(g)$	(IV)	Heterogeneous catalysis
D.	Analytical Application or Adsorption	(III)	Chromatography

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse andw the on-screen virtual numeric keypad in the place designated to enter the answer.

51. The rate constant for a first order reaction is 20 min^{-1}. The time required for the initial concentration of the reactant to reduce to its $\frac{1}{32}$ level is __________ × 10^{-2} min. (Nearest integer)

(Given: ln 10 = 2.303

log 2 = 0.3010)

Answer (17)

Sol. K = 20 min^{-1}

$$t_{1/2} = \frac{0.6932}{20} = \frac{\ln 2}{20}$$

Required time = $n \times t_{1/2}$

n will be 5

Required time = $\frac{5 \times 0.6932}{20}$

= 0.173 min

= 17.3 × 10^{-2} min

52. The resistivity of a 0.8 M solution of an electrolyte is 5 × 10^{-3} Ω cm. Its molar conductivity is _____ × 10^4 Ω^{-1} cm^2 mol^{-1}. (Nearest integer)

Answer (25)

Sol. Molar conductivity = $\frac{k \times 1000}{C}$

$$= \frac{\frac{1}{5 \times 10^{-3}} \times 1000}{0.8}$$

$$= \frac{10^6}{4} = 0.25 \times 10^6$$

= 25 × 10^4 $\Omega^{-1}cm^2mol^{-1}$

53. If the CFSE of $[Ti(H_2O)_6]^{3+}$ is –96.0 kJ/mol, this complex will absorb maximum at wavelength __________ nm. (Nearest integer)

Assume Planck's constant (h) = 6.4 × 10^{-34} Js, Speed of light (c) = 3.0 × 10^8 m/s and Avogadro's Constant (N_A) = 6 × 10^{23}/mol.

Answer (480)

Sol. $[Ti(H_2O)_6]^{3+}$, CFSE $= -0.4\Delta_0$

= –96.0 kJ mol^{-1}

$$\therefore \Delta_0 = \frac{-96.0}{-0.4}$$

$$\therefore \Delta_0 = 240 \text{ kJ mol}^{-1}$$

$$\lambda = \frac{64 \times 10^{-34} \times 3 \times 10^8 \times 6 \times 10^{23}}{240 \times 10^3}$$

$$= \frac{6.4 \times 3}{240} \times 10^{-29} \times 6 \times 10^{23}$$

$$= 480 \times 10^{-9} m$$

= 480 nm

54. Assume carbon burns according to following equation:

$$2C_{(s)} + O_{2(g)} \rightarrow 2CO(g)$$

When 12 g carbon is burnt in 48 g of oxygen, the volume of carbon monoxide produced is __________ × 10^{-1} L at STP [nearest integer]

[Given : Assume CO as ideal gas, Mass of C is 12 g mol^{-1}, Mass of O is 16 g mol^{-1} and molar volume of an ideal gas at STP is 22.7 L mol^{-1})

Answer (227)

$2C(s)+O_2(g) \longrightarrow 2CO(g)$

Sol. $\frac{12}{12}=1\,\text{mole}$ $\qquad$ $1\,\text{mol}$

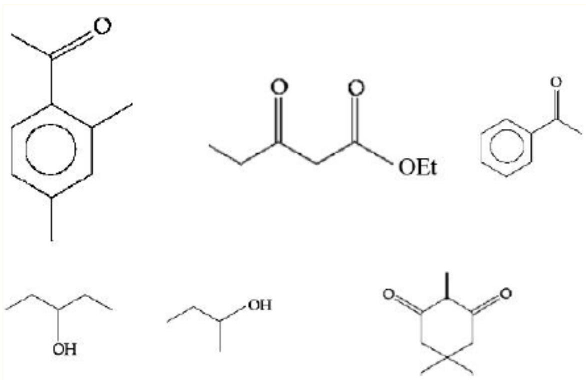
Given that molar volume at STP is 22.7 L

Hence 22.7 L of CO(g) will be formed

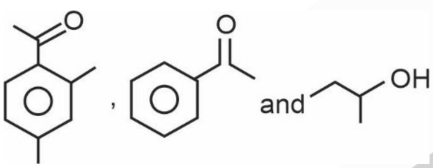
Required volume will be 22.7 × 10 × 10^{-1}

= 227 × 10^{-1}L

55. The number of molecules which gives haloform test among the following molecules is ___________.



Answer (03)

Sol. 

will give positive haloform test

56. The number of alkali metal(s), from Li, K, Cs, Rb having ionization enthalpy greater than 400 kJ mol^{-1} and forming stable superoxide is ___________.

Answer (02)

Sol.

Ionization enthalpy / $kJmol^{-1}$	Li	K	Rb	Cs
	520	419	403	376

Li does not form superoxide.

Hence the correct answer is 02.

57. A sample of a metal oxide has formula $M_{0.83}O_{1.00}$. The metal M can exist in two oxidation states +2 and +3. In the sample of $M_{0.83}O_{1.00}$, the percentage of metal ions existing in +2 oxidation state is ___________ %. (Nearest integer).

Answer (59)

Sol. $\%M^{3+}=\frac{0.34}{0.83}\times 100 \;=40.96\approx 41\%$

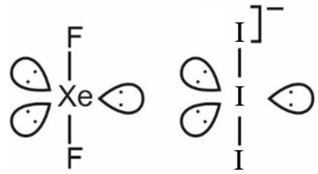
$\% M^{2+}$ = 100 – 41 = 59%

Hence correct answer is 59

58. Amongst the following, the number of species having the linear shape is ___________.

XeF_2, I_3^+, C_3O_2, I_3^-, CO_2, SO_2, $BeCl_2$ and $BCl_2^{\ominus}$

Answer (05) I



O=C=C=C=O, O=C=O, Cl—Be—Cl

59. Enthalpies of formation of $CCl_4(g)$, $H_2O(g)$, $CO_2(g)$ and HCl(g) are –105, –242, –349 and –92 kJ mol^{-1} respectively. The magnitude of enthalpy of the reaction given below is ___________ kJ mol^{-1}. (Nearest integer)

$CCl_4(g)+2H_2O(g) \rightarrow CO_2(g)+4HCl(g)$

Answer (173)

Sol. $CCl_4(g) + 2H_2O(g) \rightarrow CO_2(g) + 4HCl(g)$

Enthalpy of above reaction

$= \Delta H_f(CO_2(g)) + 4\Delta H_f(HCl(g)) - \Delta H_f(CCl_4) - 2\Delta H_f(H_2O)$

= –394 – 4 × 92 + 105 + 2 × 242

= – 394 – 368 + 105 + 484

= –173 kJ mol^{-1}

Hence the magnitude of this will be 173 kJ mol^{-1}.

60. At 298 K, the solubility of silver chloride in water is 1.434 × 10^{-3} g L^{-1}. The value of –log K_{sp} for silver chloride is ___________.

(Given mass of Ag is 107.9 g mol^{-1} and mass of Cl is 35.5 g mol^{-1})

Answer (10)

Sol. $AgCl \rightleftharpoons Ag^+ + Cl^-$

$K_{sp}=\left(Ag^+\right)\left(Cl^-\right)=S\times S=S^2$

$S=\sqrt{K_{sp}}$

$S=\frac{1.434\times 10^{-3}}{143.4}=10^{-5}$

$K_{sp} = S^2 = 10^{-10}$

$\Rightarrow -\log(K_{sp}) = -\log(10^{-10}) = 10$

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

61. Let the mean and standard deviation of marks of class A of 100 students be respectively 40 and α (>0), and the mean and standard deviation of marks of class B of n students be respectively 55 and 30 – α. If the mean and variance of the marks of the combined class of 100 + n students are respectively 50 and 350, then the sum of variances of classes A and B is :

(1) 450 (2) 650

(3) 900 (4) 500

Answer (4)

Sol. Let mean of class $A = \overline{x}_A$

& Mean of class $B = \overline{x}_B$

$\therefore \quad \frac{\sum x_A}{100} = 40 \quad$ ...(i)

& $\frac{\sum x_A^2}{100} - 40^2 = \alpha^2 \quad$ ...(ii)

also $\frac{\sum x_B}{n} = 55 \quad$ ...(iii)

& $\frac{\sum x_B^2}{n} - 55^2 = (30-\alpha)^2$ ...(iv)

and $\frac{\sum x_A + \sum x_B}{100+n} = 50 \quad$ ...(v)

$\frac{\sum x_A^2 + \sum x_B^2}{100+n} - 50^2 = 350$ ...(vi)

By equation (i), (ii) & (iii)

$4000 + 55n = 50(100 + n)$

$\Rightarrow 5n = 1000 \qquad \Rightarrow n = 200$

also by (ii), (iii) & (iv)

$(\alpha^2 + 40^2)100 + (55^2 + (30 - \alpha)^2)200 = (50^2 + 350)300$

$\Rightarrow 3\alpha^2 - 120\alpha + (40^2 + 2 \times 55^2) - 3(50^2 + 350) = 0$

$\Rightarrow 3\alpha^2 - 120\alpha + 900 = 0$

$\Rightarrow \alpha^2 - 40\alpha + 300 = 0$

$\Rightarrow \alpha = 10$ or 30 (rejected)

Sum of variances = $10^2 + 20^2 = 500$

62. The foot of perpendicular form the origin O to a plane P which meets the co-ordinate axes at the points A, B, C is (2, a, 4), a $\in$ N. If the volume of the tetrahedron OABC is 144 unit^3, then which of the following points is **NOT** on P?

(1) (2, 2, 4) (2) (3, 0, 4)

(3) (0, 6, 3) (4) (0, 4, 4)

Answer (2)

Sol. As (2, a, 4) is foot of perpendicular

equation of plane

$2(x - 2) + a(y - a) + 4(z - 4) = 0$

Clearly for (3, 0, 4) $a \notin N$.

63. Let $y = y(x)$ be the solution of the differential equation $(3y^2 - 5x^2)y\,dx + 2x(x^2 - y^2)dy = 0$ such that $y(1) = 1$. Then $|(y(2))^3 - 12y(2)|$ is equal to :

(1) $16\sqrt{2}$ (2) 32

(3) $32\sqrt{2}$ (4) 64

Answer (3)

Sol. $\frac{dy}{dx} = \frac{(5x^2 - 3y^2)y}{(x^2 - y^2)2x} \qquad y(1) = 1$

$\Rightarrow v = 1$

Put $y = vx$

$v + x\frac{dv}{dx} = \frac{v(5-3v^2)}{2(1-v^2)}$

$\Rightarrow x\frac{dv}{dx} = \frac{5v - 3v^3 - 2v + 2v^3}{2(1-v^2)}$

$\Rightarrow \frac{2(v^2-1)dv}{v^3 - 3v} = \frac{dx}{x}$

$\Rightarrow \frac{2}{3}\ln\left|v^3 - 3v\right| = \ln x + c$

$\downarrow y(1) = 1$

$\Rightarrow \quad \frac{2}{3}\ln 2 = c$

$\therefore \quad \frac{2}{3}\ln\left(\frac{\left(\left(\frac{y}{x}\right)^{-3} - \frac{3y}{x}\right)}{2}\right) = \ln x$

$\downarrow y(2)$

$\Rightarrow \quad \left(\frac{y^3}{8} - \frac{3y}{2}\right) = 2.2^{\frac{3}{2}}$

$\Rightarrow \quad y^3(2) - 12y(2) = 32\sqrt{2}$

64. Let (a, b) $\subset$ (0, 2π) be the largest interval for which $\sin^{-1}(\sin\theta) - \cos^{-1}(\sin\theta) > 0$, $\theta \in (0, 2\pi)$, holds. If $\alpha x^2 + \beta x + \sin^{-1}(x^2 - 6x + 10) + \cos^{-1}(x^2 - 6x + 10) = 0$ and $\alpha - \beta = b - a$, then α is equal to :

(1) $\frac{\pi}{16}$ (2) $\frac{\pi}{8}$

(3) $\frac{\pi}{48}$ (4) $\frac{\pi}{12}$

Answer (4)

Sol. $\sin^{-1}(\sin\theta) > \cos^{-1}\sin\theta$

For $Q \rightarrow \left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

$\therefore \quad \alpha - \beta = \frac{\pi}{2}$ ...(i)

also $\alpha x^2 + \beta x + \frac{\pi}{2} = 0$ & $x = 3$ only

$\therefore \quad 9\alpha + 3\beta = -\frac{\pi}{2}$ ...(ii)

$12\alpha = \pi \Rightarrow \alpha = \frac{\pi}{12}$

65. $\lim\limits_{x\to\infty} \frac{\left(\sqrt{3x+1} + \sqrt{3x-1}\right)^6 + \left(\sqrt{3x+1} - \sqrt{3x-1}\right)^6}{\left(x + \sqrt{x^2-1}\right)^6 + \left(x - \sqrt{x^2-1}\right)^6} x^3$

(1) is equal to 9

(2) does not exist

(3) is equal to $\frac{27}{2}$

(4) is equal to 27

Answer (3)

Sol. $\lim\limits_{x\to\infty} \frac{2\left({}^6C_0\left(\sqrt{3x+1}\right)^6 + {}^6C_2 \times \left(\sqrt{3x+1}\right)^4 + {}^6C_4\left(\sqrt{3x+1}\right)^2 + {}^6C_6\right)x^3}{2\left({}^6C_0 x^6 + {}^6C_2 x^4 + {}^6C_4 x^2 + {}^6C_6\right)}$

$= \frac{3^3}{1} = 27$

66. The equation $e^{4x} + 8e^{3x} + 13e^{2x} - 8e^x + 1 = 0$, $x \in R$ has :

(1) two solutions and only one of them is negative

(2) two solutions and both are negative

(3) four solutions two of which are negative

(4) no solution

Answer (2)

Sol. $\left(e^{2x} + e^{-2x}\right) + 8\left(e^x - e^{-x}\right) + 13 = 0$

$e^x - e^{-x} = t$

$\left(t^2 + 2\right) + 8t + 13 = 0$

$t = -5, \; -3$

$e^x - e^{-x} = -5$	$e^x - e^{-x} = -3$
One negative Root	One negative Root

67. Let P be the plane, passing through the point $(1, -1, -5)$ and perpendicular to the line joining the points $(4, 1, -3)$ and (2, 4, 3). Then the distance of P from the point $(3, -2, 2)$ is

(1) 4

(2) 7

(3) 5

(4) 6

Answer (3)

Sol. $\vec{n} = 2\hat{i} - 3\hat{j} - 6\hat{k}$

Equation of plane is

$2x - 3y - 6z = 35$

Or $\quad 2x - 3y - 6z - 35 = 0$

Distance from (3, –2, 2)

$= \left|\frac{6 + 6 - 12 - 35}{7}\right|$

= 5 units

68. The complex number $z=\dfrac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$ is equal to :

(1) $\sqrt{2}\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$

(2) $\cos\frac{\pi}{12}-i\sin\frac{\pi}{12}$

(3) $\sqrt{2}\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$

(4) $\sqrt{2}i\left(\cos\frac{5\pi}{12}-i\sin\frac{5\pi}{12}\right)$

Answer (1)

Sol. $z=\dfrac{\sqrt{2}e^{\left(3\frac{\pi}{4}\right)i}}{e^{i\frac{\pi}{3}}}$

$=\sqrt{2}e^{i\left(\frac{5\pi}{12}\right)}$

69. Let the plane $P: 8x+a_1y+a_2z+12=0$ be parallel to the line $L: \dfrac{x+2}{2}=\dfrac{y-3}{3}=\dfrac{z+4}{5}$. If the intercept of P on the y-axis is 1, then the distance between P and L is:

(1) $\sqrt{\frac{7}{2}}$

(2) $\frac{6}{\sqrt{14}}$

(3) $\sqrt{\frac{2}{7}}$

(4) $\sqrt{14}$

Answer (4)

Sol. $16+3\alpha_1+5\alpha_2=0$

At *y*-axis $x=z=0$

$\alpha_1 y+12=0$

$\boxed{\alpha_1=-12}$

$\boxed{\alpha_2=4}$

Equation of plane is

$8x-12y+4z+12=0$

Or $2x-3y+z+3=0$

Distance from (–2, 3, –4)

$=\left|\dfrac{-4-9-4+3}{\sqrt{14}}\right|=\sqrt{14}$

70. Let H be the hyperbola, whose foci are $(1\pm\sqrt{2},0)$ and eccentricity is $\sqrt{2}$. Then the length of its latus rectum is ________.

(1) 3

(2) $\frac{3}{2}$

(3) $\frac{5}{2}$

(4) 2

Answer (4)

Sol. $2ae=\left(1+\sqrt{2}\right)-\left(1-\sqrt{2}\right)=2\sqrt{2}$

$2\times a\times\sqrt{2}=2\sqrt{2}$

$a=1$

$b^2=a^2\left(e^2-1\right)=i\left(2-1\right)=1$

$\text{LR}=\dfrac{2b^2}{a}=2$

71. If a point $P(\alpha, \beta, \gamma)$ satisfying

$(\alpha\beta\gamma)\begin{pmatrix}2&10&8\\9&3&8\\8&4&8\end{pmatrix}=(0\ 0\ 0)$

lies on the plane $2x+4y+3z=5$, then $6\alpha+9\beta+7\gamma$ is equal to

(1) $\frac{11}{5}$

(2) 11

(3) $\frac{5}{4}$

(4) –1

Answer (2)

Sol. $[\alpha\quad\beta\quad\gamma]\begin{bmatrix}2&10&8\\9&3&8\\8&4&8\end{bmatrix}=[0\quad 0\quad 0]$

$2\alpha+9\beta+8\gamma=0$ …(i)

$10\alpha+3\beta+4\gamma=0$ …(ii)

$\alpha+\beta+\gamma=0$ …(iii)

$\begin{vmatrix}2&10&8\\9&3&8\\8&4&8\end{vmatrix}=0$

$\therefore$ Above system of equations has infinitely many Solutions

(ii) – 4(iii) $\Rightarrow\ \beta=6\alpha$ …(iv)

(iii) $\Rightarrow\gamma=-7\alpha$ …(v)

(α, β, γ) lies on $2x + 4y + 3z = 5$

$\therefore \quad 2\alpha + 4\beta + 3\gamma = 5 \quad \ldots$(vi)

Using (iv) and (v) in (vi);

$\alpha = 1, \beta = 6, \gamma = -7$

$\therefore \quad 6\alpha + 9\beta + 7\gamma = 11$

72. Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = 5\hat{i} - 3\hat{j} + 3\hat{k}$ be three vectors if $\vec{r}$ is a vector such that, $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r} \cdot \vec{a} = 0$, then $25|\vec{r}|^2$ is equal to

(1) 560 (2) 449

(3) 336 (4) 339

Answer (4)

Sol. $\vec{r} - \vec{c} = \lambda\vec{b}$

$\vec{r} = \vec{c} + \lambda\vec{b}$

$\vec{r} \cdot \vec{a} = 0$

$\vec{c} \cdot \vec{a} + \lambda(\vec{b} \cdot \vec{a}) = 0$

$8 + \lambda(5) = 0$

$\lambda = \frac{-8}{5}$

$\vec{r} = \vec{c} - \frac{8}{5}\vec{b}$

$5\vec{r} = 5\vec{c} - 8\vec{b}$

$= 17\hat{i} - 7\hat{j} - \hat{k}$

$25|\vec{r}|^2 = 339$

73. The set of all values of a^2 for which the line $x + y = 0$ bisects two distinct chords drawn from a point $P\left(\frac{1+a}{2}, \frac{1-a}{2}\right)$ on the circle $2x^2 + 2y^2 - (1 + a)x - (1 - a)y = 0$ is equal to

(1) $(8, \infty)$ (2) $(4, \infty)$

(3) $(0, 4]$ (4) $(2, 12]$

Answer (1)

Sol. If $(k, -k)$ is mid-point

Equation of chord :

$$2xk + 2y(-k) - \frac{1+a}{2}(x+k) - \frac{(1-a)}{2}(y-k)$$

$$= 4k^2 - (1+a)k - (1-a)(-k)$$

or $x\left(2k - \frac{1+a}{2}\right) + y\left(-2k - \frac{1-a}{2}\right)$

$$= 4k^2 - \left(\frac{1+a}{2}\right)k - (-k)\left(\frac{1-a}{2}\right)$$

As it passes through $\left(\frac{1+a}{2}, \frac{1-a}{2}\right)$

$$\frac{1+a}{2}\left(2k - \frac{1+a}{2}\right) + \frac{1-a}{2}\left(-2k - \frac{1-a}{2}\right)$$

$$= 4k^2 - \frac{(1+a)}{2}k + k\frac{(1-a)}{2}$$

So, quadratic in k should have $D > 0$.

$\boxed{a > 8}$

74. Among the relations

$S = \left\{(a,b) : a, b \in \mathbb{R} - \{0\}, 2 + \frac{a}{b} > 0\right\}$ and

$T = \left\{(a,b) : a, b \in \mathbb{R}, a^2 - b^2 \in \mathbb{Z}\right\}$,

(1) S is transitive but T is not

(2) neither S nor T is transitive

(3) both S and T are symmetric

(4) T is symmetric but S is not

Answer (4)

Sol. For S

If $2 + \frac{a}{b} > 0$ or $\frac{a}{b} > -2$

$\Rightarrow \frac{b}{a} > -2 \qquad \therefore$ not symmetric

For T

$a^2 - b^2 \in I \Rightarrow b^2 - a^2 \in I \ \forall a, b \in \mathbb{R}$

$\therefore$ T is symmetric but S is not.

75. The number of values of $r \in \{p, q, \sim p, \sim q\}$ for which $((p \wedge q) \Rightarrow (r \vee q)) \wedge ((p \wedge r) \Rightarrow q)$ is a tautology, is

(1) 4 (2) 1

(3) 2 (4) 3

Answer (3)

Sol. $((p \wedge q) \Rightarrow (r \vee q)) \wedge ((p \wedge r) \Rightarrow q) \equiv T$ (given)

$\equiv ((\sim p \vee \sim q) \vee (r \vee q)) \wedge (\sim p \vee \sim r \vee q)$

$\equiv ((\sim p \vee r) \vee (\sim q \vee q)) \wedge (\sim p \vee \sim r \vee q)$

$\equiv \sim p \vee \sim r \vee q$

For above statement to be tautology

r can be $\sim p$ or q

$\therefore$ Two values of r are possible.

76. If $\phi(x) = \frac{1}{\sqrt{x}} \int_{\frac{\pi}{4}}^{x} (4\sqrt{2}\sin t - 3\phi'(t)dt, x > 0$, then $\phi'\left(\frac{\pi}{4}\right)$ is equal to

(1) $\frac{8}{6+\sqrt{\pi}}$ (2) $\frac{4}{6+\sqrt{\pi}}$

(3) $\frac{4}{6-\sqrt{\pi}}$ (4) $\frac{8}{\sqrt{\pi}}$

Answer (1)

Sol. $\phi(x) = \frac{1}{\sqrt{x}} \int_{\pi/4}^{x} \left(4\sqrt{2}\sin t - 3\phi'(t)\right) dt$

$\Rightarrow \phi'(x) = \frac{-1}{2x^{3/2}} \int_{\pi/4}^{x} \left(4\sqrt{2}\sin t - 3\phi'(t)\right) dt + \frac{1}{\sqrt{x}}\left(4\sqrt{2}\sin(x) - 3\phi'(x)\right)$

$x = \frac{\pi}{4}$

$\phi'\left(\frac{\pi}{4}\right) = \frac{-1}{2\left(\frac{\pi}{4}\right)^{3/2}} \times 0 + \sqrt{\frac{4}{\pi}}\left(4\sqrt{2} \times \frac{1}{\sqrt{2}} - 3\phi'\left(\frac{\pi}{4}\right)\right)$

$\Rightarrow \phi'\left(\frac{\pi}{4}\right)\left[1 + \frac{6}{\sqrt{\pi}}\right] = \frac{2}{\sqrt{\pi}} \times 4$

$\Rightarrow \phi'\left(\frac{\pi}{4}\right) = \frac{8}{6+\sqrt{x}}$

Option (1) is correct.

77. Let a_1, a_2, a_3, …. be an A.P. If a_7 = 3, the product a_1a_4 is minimum and the sum of its first n terms is zero, then $n! - 4a_n(n + 2)$ is equal to :

(1) $\frac{381}{4}$ (2) 24

(3) 9 (4) $\frac{33}{4}$

Answer (2)

Sol. $a_7 = 3 \Rightarrow a + 6d = 3 \Rightarrow a = 3 - 6d$

$a_1 \cdot a_4 = a(a + 3d)$

$\Rightarrow (3 - 6d)(3 - 6d + 3d)$

$\Rightarrow 3(1 - 2d)3(1 - d)$

$\Rightarrow 9(2d^2 - 3d + 1)$

Let $f(d) = 2d^2 - 3d + 1$

$f'(d) = 4d - 3 \Rightarrow d = \frac{3}{4}$

$\therefore a = 3 - 6\cdot\frac{3}{4} = 3 - \frac{9}{2} = -\frac{3}{2}$

$S_n = 0$

$\frac{n}{2}(29 + (n-1)d) = 0$

$\Rightarrow 2\cdot\left(-\frac{3}{2}\right) + (n-1)\left(\frac{3}{4}\right) = 0$

$\Rightarrow 3 = \frac{3}{4}(n-1)$

$\Rightarrow n = 5$

Now, $n! - 4.a_{n(n+2)}$

$= 5! - 4\cdot a_{35}$

$= 120 - 4\left(-\frac{3}{2} + 34 \cdot \frac{3}{4}\right)$

$= 120 - (-6 + 102)$

$= 120 - (96)$

$= 24$

78. The absolute minimum value, of the function $f(x) = |x^2 - x + 1| + [x^2 - x + 1]$, where $[t]$ denotes the greatest integer function, in the interval [–1. 2], is

(1) $\frac{3}{4}$ (2) $\frac{3}{2}$

(3) $\frac{1}{4}$ (4) $\frac{5}{4}$

Answer (1)

Sol. $x^2 - x + 1 > 0$

$\Rightarrow f(x) = (x^2 - x + 1) + [x^2 - x + 1]$

Now,

$x^2 - x + 1$ attains its minimum value at $x = \frac{1}{2}$

and $\min\left[x^2 - x + 1\right] = 0$ as $x^2 - x + 1 > 0$

$\Rightarrow f(x)$ attains it min at $x = \frac{1}{2}$

$\therefore f\left(\frac{1}{2}\right) = \frac{3}{4} + 0 = \frac{3}{4}$

Option (1) is correct.

79. Let $\alpha > 0$. If $\int_0^{\alpha} \frac{x}{\sqrt{x+\alpha}-\sqrt{x}}dx = \frac{16+20\sqrt{2}}{15}$, then α is equal to:

(1) 2 (2) 4

(3) $\sqrt{2}$ (4) $2\sqrt{2}$

Answer (3)

Sol. $I = \int_0^{\alpha} \frac{x}{\sqrt{x+\alpha}-\sqrt{x}}dx, \alpha > 0$

$$= \frac{1}{\alpha}\int_0^{\alpha} x\left(\sqrt{x+\alpha}+\sqrt{x}\right)dx$$

$$= \frac{1}{\alpha}\left\{\int_0^{\alpha} x\sqrt{x+\alpha}\,dx + \int_0^{\alpha} x^{3/2}\,dx\right\}$$

$$= \frac{2\alpha^{3/2}}{15}\left(2^{3/2}+2\right)+\frac{2\alpha^{3/2}}{5}$$

$$= \frac{2\alpha^{3/2}\left(2^{3/2}+5\right)}{15}$$

When $\alpha = \sqrt{2}$ then $\int_0^{\alpha} \frac{x}{\sqrt{x+\alpha}-\sqrt{x}}dx = \frac{16+20\sqrt{2}}{15}$

$\therefore \quad \alpha = \sqrt{2}$

80. Let $f : \mathbb{R}-\{2,6\} \to \mathbb{R}$ be real valued function defined as $f(x) = \frac{x^2+2x+1}{x^2-8x+12}$. Then range of f is

(1) $\left(-\infty,-\frac{21}{4}\right] \cup [0,\infty)$ (2) $\left(-\infty,-\frac{21}{4}\right) \cup (0,\infty)$

(3) $\left(-\infty,-\frac{21}{4}\right] \cup [1,\infty)$ (4) $\left(-\infty,-\frac{21}{4}\right] \cup \left[\frac{21}{4},\infty\right)$

Answer (1)

Sol. $y = \frac{x^2+2x+1}{x^2-8x+12}$

$\Rightarrow \quad (y-1)x^2-(8y+2)x+12y-1=0$

Let $y \neq 1$, then $D \geq 0$

$4(4y+1)^2-4(y-1)(12y-1) \geq 0$

$\Rightarrow \quad 16y^2+1+8y-(12y^2-13y+1) \geq 0$

$\Rightarrow \quad 4y^2+21y \geq 0$

$\Rightarrow \quad y \in \left(-\infty,-\frac{21}{4}\right) \cup [0,\infty)-\{1\}$

for $y = 1$,

$-8x + 12 = 2x + 1$

$x = \frac{11}{10} \quad \therefore \; I \in R$

$\therefore \quad$ Range $= \left(-\infty,-\frac{21}{4}\right] \cup [0,\infty)$

$\therefore \quad$ option (1) is correct.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

81. If the constant term in the binomial expansion of $\left(\frac{x^{\frac{5}{2}}}{2}-\frac{4}{x^l}\right)^9$ is –84 and the coefficient of x^{-3l} is $2^{\alpha}\beta$, where $\beta < 0$ is an odd number, then $|\alpha l - \beta|$ is equal to ________.

Answer (98)

Sol. Given binomial expansion of $\left(\frac{x^{\frac{5}{2}}}{2}-\frac{4}{x^l}\right)^9$

$$T_{r+1} = {}^9C_r\left(\frac{x^{\frac{5}{2}}}{2}\right)^{9-r}\left(\frac{-4}{x^l}\right)^r$$

$$= {}^9C_r\, x^{\frac{45-5r}{2}-lr}\cdot 2^{r-9}\cdot r^r\cdot(-1)^r$$

Now constant term = –84

So, $\frac{45-5r}{2} = lr \Rightarrow 2lr+5r = 45$

and ${}^9C_r\cdot 2^{3r-9}(-1)^r = -84$

So, $\boxed{r=3}$ and $l = 5$

Now for x^{-15} $\frac{45-5r}{2}-5r=-15$

$45 - 15r = -30$

$\boxed{r=5}$

$\therefore$ Coefficient $=-{}^9C_5 2^6=-63.2^7$

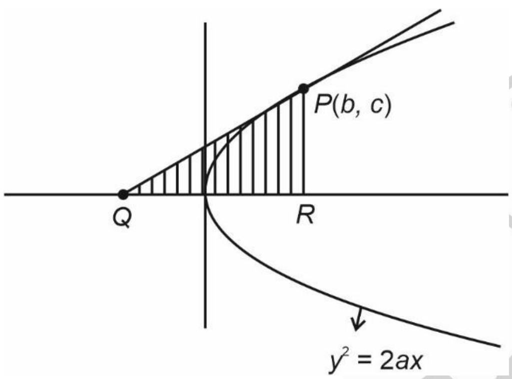
$\therefore$ $\alpha = 7, \beta = -63$

and $|\alpha l - \beta| = |7 \times 5 + 63| = 98$

82. Let S be the set of all $a \in \mathbb{N}$ such that the area of the triangle formed by the tangent at the point $P(b, c)$, $b, c \in \mathbb{N}$, on the parabola $y^2 = 2ax$ the lines $x = b$, $y = 0$ is 16 unit^2, then $\sum_{a \in S} a$ is equal to_______.

Answer (146)

Sol.



Tangent at $P(b, c)$:

$yc = a(x + b)$

for Q : $y = 0$ $\boxed{x=-b}$

$\therefore$ Area of shaded region = 16

$\frac{1}{2}\times 2b\times c=16$

$bc = 16$ and $c^2 = 2ab$

$\therefore$ $2a=\frac{c^2.c}{16}$ $\frac{16\times16\times16}{32}$

$a=\frac{c^3}{32}, 1\le C\le 16$ and divisor of 16

$\therefore$ $a = 2, 16, 128$

$\therefore$ $\Sigma a = 146$

83. The sum $1^2 - 2{\cdot}3^2 + 3{\cdot}5^2 - 4{\cdot}7^2 + 5{\cdot}9^2 - \ldots + 15{\cdot}29^2$ is _______.

Answer (6592)

Sol. $S = 1^2 - 2.3^2 + 3.5^2 - 4.7^2 + \ldots\ldots + 15.29^2$

$=\sum_{r=1}^{8}(2r-1)(4r-3)^2-\sum_{r=1}^{7}2r(4r-1)^2$

$=\sum_{r=1}^{8}32r^3-64r^2+32r-9-2-\sum_{r=1}^{7}16r^3-8r^2+r$

$= 32 \times 36^2 - 64 \times 204 + 1152 - 72$

$- 2(16 \times 28^2 - 1120 + 28)$

$= 6592$

84. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $|\vec{a}|=\sqrt{31}$, $4|\vec{b}|=|\vec{c}|=2$ and $2(\vec{a}\times\vec{b})=2(\vec{c}\times\vec{a})$. If the angle between $\vec{b}$ and $\vec{c}$ is $\frac{2\pi}{3}$, then $\left(\frac{\vec{a}\times\vec{c}}{\vec{a}\cdot\vec{b}}\right)^2$ is equal to _______.

Answer (03)

Sol. $\vec{a}\times(2\vec{b}+3\vec{c})=0$

$\vec{a}=\lambda(2\vec{b}+3\vec{c})$

$|\vec{a}|^2=\lambda^2(4|b|^2+9|c|^2+12\vec{b}\cdot\vec{c})$

$31=31\lambda^2$

$\lambda=\pm1$

$\vec{a}=\pm(2\vec{b}+3\vec{c})$

$\frac{|\vec{a}\times\vec{c}|}{|\vec{a}\cdot\vec{b}|}=\frac{2|\vec{b}\times\vec{c}|}{2\vec{b}\cdot\vec{b}+3\vec{c}\cdot\vec{b}}$

$|\vec{b}\times\vec{c}|^2=\frac{1}{4}\cdot4-\left(1-\frac{1}{2}\right)^2$

$=\frac{3}{4}$

$\therefore$ $\frac{|\vec{a}\times\vec{c}|}{|\vec{a}\cdot\vec{b}|}=\frac{\sqrt{3}}{2\cdot\frac{1}{4}-\frac{3}{2}}=\frac{\sqrt{3}}{-1}$

$\left(\frac{|\vec{a}\times\vec{c}|}{|\vec{a}\cdot\vec{b}|}\right)^2=3$

85. Let A be a $n \times n$ matrix such that $|A| = 2$. If the determinant of the matrix Adj(2·Adj($2A^{-1}$))· is 2^{84}, then n is equal to _______.

Answer (05)

Sol. $\because \quad \left|adj\left(2\cdot adj(2A^{-1})\right)\right| = 2^{84}$

$\Rightarrow \quad 2^{n\cdot(n-1)}\left|adj\left(2A^{-1}\right)\right|^{(n-1)} = 2^{84}$

$\Rightarrow \quad 2^{n(n-1)}\left|2A^{-1}\right|^{(n-1)^2} = 2^{84}$

$\Rightarrow \quad 2^{n(n-1)}\cdot 2^{n(n-1)^2}\cdot\dfrac{1}{|A|^{(n-1)^2}} = 2^{84}$

$\Rightarrow \quad 2^{n(n-1)+n(n-1)^2-(n-1)^2} = 2^{84} \quad \{\because |4| = 2\}$

$\therefore \quad n(n-1) + (n-1)^3 = 84$

$\therefore \quad n = 5$

86. The coefficient of x^{-6}, in the expansion of $\left(\dfrac{4x}{5}+\dfrac{5}{2x^2}\right)^9$, is ________.

Answer (5040)

Sol. Coeff of x^{-6} in $\left(\dfrac{4x}{5}+\dfrac{5}{2x^2}\right)^9$

$T_{r+1} = {}^9C_r\left(\dfrac{4x}{5}\right)^{9-r}\left(\dfrac{5}{2x^2}\right)^r$

$9 - 3r = -6$

$r = 5$

Coeff of $x^{-6} = {}^9C_5\left(\dfrac{4}{5}\right)^4\left(\dfrac{5}{2}\right)^5$

$= 5040$

87. Let $A = \left[a_{ij}\right], a_{ij} \in \mathbb{Z} \cap [0,4], 1 \le i, j \le 2$. The number of matrices A such that the sum of all entries is a prime number $p \in (2,13)$ is ______.

Answer (204)

Sol. $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{13} & a_{14} \end{bmatrix}$

Such that $\Sigma a_{ii} = 3, 5, 7$ or 11

Then for sum 3, the possible entries are (0, 0, 0, 3), (0, 0, 1, 2), (0, 1, 1, 1).

Then total number of possible matrices

$= 4 + 12 + 4$

$= 20$

For sum 5 the possible entries are (0, 0, 1, 4), (0, 0, 2, 3), (0, 1, 2, 2), (0, 1, 1, 3) and (1, 1, 1, 2).

$\therefore$ Total possible matrices = 12+12+12+12+4 = 52

For sum 7 the possible entries are (0, 0, 3, 4), (0, 2, 2, 3), (0, 1, 2, 4), (0, 1, 3, 3), (1, 2, 2, 2), (1, 1, 2, 3) and (1, 1, 1, 4).

$\therefore$ Total possible matrices = 80

For sum 11 the possible entries are (0, 3, 4, 4), (1, 2, 4, 4), (2, 3, 3, 3), (2, 2, 3, 4).

$\therefore$ Total number of matrices = 52

$\therefore$ Total required matrices = 20 + 52 + 80 + 52

= 204

88. Let the area of the region $\left\{(x,y) : |2x-1| \le y \le \left|x^2 - x\right|, 0 \le x \le 1\right\}$ be A. Then $(6A + 11)^2$ is equal to ______.

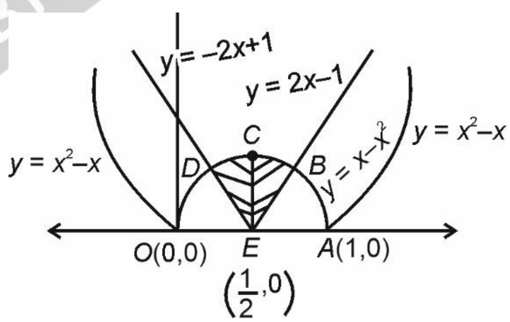
Answer (125)

Sol. For B,

$x - x^2 = 2x - 1$

$x^2 + x - 1 = 0$

$x = \dfrac{-1+\sqrt{5}}{2}$



Area = 2(area of BCE)

$A = 2\displaystyle\int_{\frac{1}{2}}^{\frac{\sqrt{5}-1}{2}}\left(x - x^2\right) - (2x-1)\,dx$

$= 2\displaystyle\int_{\frac{1}{2}}^{\frac{\sqrt{5}-1}{2}} 1 - x - x^2\,dx = 2\left(x - \dfrac{x^2}{2} - \dfrac{x^3}{3}\right)\Bigg]_{\frac{1}{2}}^{\frac{\sqrt{5}-1}{2}}$

$$= 2\left\{\left(\frac{\sqrt{5}-1}{2}-\frac{1}{2}\right)-\left\{\left(\frac{\sqrt{5}-1}{2}\right)^2-\frac{1}{4}\right\}\frac{1}{2}\right.$$

$$\left.-\left[\left(\frac{\sqrt{5}-1}{2}\right)^3-\frac{1}{8}\right]\frac{1}{3}\right\}$$

$$A = \frac{-11+5\sqrt{5}}{6}$$

$\Rightarrow$ $(6A + 11)^2 = 125$

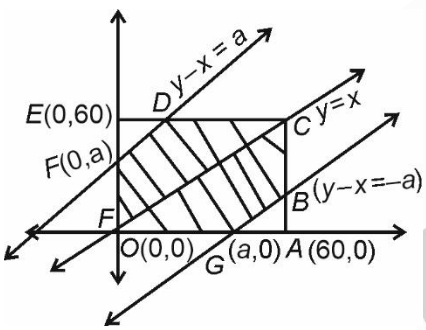
89. Let A be the event that the absolute difference between two randomly chosen real numbers in the sample space [0, 60] is less than or equal to a. If $P(A) = \frac{11}{36}$, then a is equal to _______.

Answer (10)

Sol. Let two numbers be x and y

$|y - x| < a$ (where $a > 0$)

$-a < y - x < a$



$$\text{Required probability} = \frac{\text{Area of shaded region}}{\text{Total area}}$$

$$\frac{[ABG]+[DEF]}{[OGAB\ CDEF]} = 1 - \frac{11}{36}$$

$$\frac{2[ABG]}{3600} = \frac{25}{36}$$

$[ABG] = 1250$

$$\frac{1}{2}(60-a)^2 = 1250$$

$(60 - a)^2 = 2500$

$a = 10, 110$ (Rejected)

$a = 10$

90. If ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 11 : 21$, then $n^2 + n + 15$ is equal to

Answer (45)

Sol. $$\frac{\frac{(2n+1)!}{(n+2)!}}{\frac{(2n-1)!}{(n-1)!}} = \frac{11}{21}$$

$$\frac{(2n+1)2n}{(n+2)(n+1)n} = \frac{11}{21}$$

$84n + 42 = 11(n^2 + 3n + 2)$

$11n^2 - 51n - 20 = 0$

$(n - 5)(11n + 4) = 0$

$$n = 5, \frac{-4}{11} \text{ (Rejected)}$$

$n^2 + n + 15 = 45$

❑ ❑ ❑