31/01/2023
Morning

Corporate Office : Aakash Tower, 8, Pusa Road, New Delhi-110005 | Ph.: 011-47623456

Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2023 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

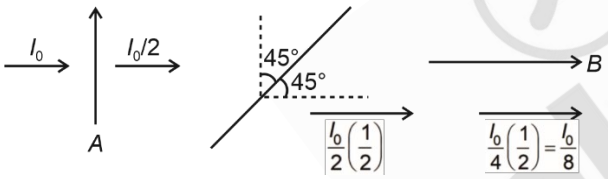
1. Two polaroids *A* and *B* are placed in such a way that the pass-axis of polaroids are perpendicular to each other. Now, another polaroid *C* is placed between *A* and *B* bisecting angle between them. If intensity of unpolarized light is I_0 then intensity of transmitted light after passing through polaroid *B* will be

(1) $\frac{I_0}{8}$ (2) Zero

(3) $\frac{I_0}{4}$ (4) $\frac{I_0}{2}$

Answer (1)

Sol.



2. A bar magnet with a magnetic moment 5.0 Am^2 is placed in parallel position relative to a magnetic field of 0.4 T. The amount of required work done in turning the magnet from parallel to antiparallel position relative to the field direction is

(1) Zero (2) 1 J

(3) 4 J (4) 2 J

Answer (3)

Sol. $W = -MB(\cos\theta_2 - \cos\theta_1)$

$= -0.4 \times 5[\cos180° - \cos0]$

$= 4$ J

3. The maximum potential energy of a block executing simple harmonic motion is 25 J. *A* is amplitude of oscillation. At $\frac{A}{2}$, the kinetic energy of the block is

(1) 18.75 J (2) 12.5 J

(3) 37.5 J (4) 9.75 J

Answer (1)

Sol. $E_{Total} = U_{max} = 25$ J

$K.E_{A/2} + U_{A/2} = 25$

$$K.E_{A/2} + \left(\frac{1}{2}KA^2\right)\frac{1}{4} = 25$$

$$K.E_{A/2} = 25\left[1-\frac{1}{4}\right]$$

$$= \frac{3}{4}\times 25 = \frac{75}{4}\text{ J}$$

$= 18.75$ J

4. A rod with circular cross-section area 2 cm^2 and length 40 cm is wound uniformly with 400 turns of an insulated wire. If a current of 0.4 A flows in the wire windings, the total magnetic flux produced inside windings is $4\pi \times 10^{-6}$ Wb. The relative permeability of the rod is

(Given: Permeability of vacuum $\mu_0 = 4\pi\times10^{-7}$ N A^{-2})

(1) 125 (2) $\frac{32}{5}$

(3) 12.5 (4) $\frac{5}{16}$

Answer (4)

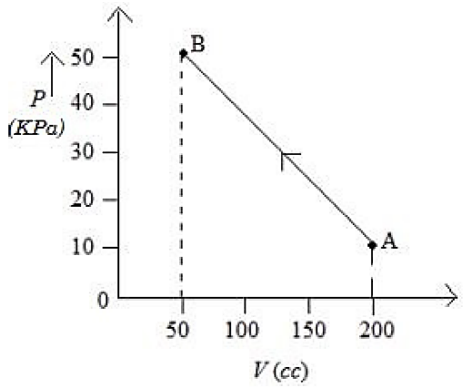
Sol. $\phi = NBA$

$(4\pi \times 10^{-6}) = 400[\mu_r\mu_0 ni](2 \times 10^{-4})$

$$(4\pi\times10^{-6}) = 400\left[\mu_r \times 4\pi\times10^{-7}\times\frac{400}{0.4}\times0.4\right]\times2\times10^{-4}$$

$$\mu_r = \frac{5}{16}$$

5. The pressure of a gas changes linearly with volume from *A* to *B* as shown in figure. If no heat is supplied to or extracted from the gas then change in the internal energy of the gas will be



(1) 4.5 J (2) Zero

(3) 6 J (4) –4.5 J

Answer (1)

Sol. $\because \quad \Delta Q = 0$

$$\Delta U = -W$$

$$= -\left[-\frac{1}{2}\times(50+10)\times10^{3}\times150\times10^{-6}\right]$$

$$= 4.5 \text{ J}$$

6. 100 balls each of mass m moving with speed v simultaneously strike a wall normally and reflected back with same speed, in time t s. The total force exerted by the balls on the wall is

(1) $\frac{200mv}{t}$ (2) $200mvt$

(3) $\frac{mv}{100\ t}$ (4) $\frac{100\ mv}{t}$

Answer (1)

Sol. Total force exerted = $\frac{\Delta P}{\Delta t}$

$$= \frac{100(m)(2v)}{t}$$

$$= \frac{200\ mv}{t}$$

7. The effect of increase in temperature on the number of electrons in conduction band (n_e) and resistance of a semiconductor will be as

(1) Both n_e and resistance decrease

(2) n_e increases, resistance decreases

(3) n_e decreases, resistance increases

(4) Both n_e and resistance increase

Answer (2)

Sol. As temperature increases n_e increases, this results in increase in conductance

$\therefore$ T increases, n_e increases and R decreases

8. A free neutron decays into a proton but a free proton does not decay into neutron. This is because

(1) neutron has larger rest mass than proton

(2) neutron is a composite particle made of a proton and an electron

(3) proton is a charged particle

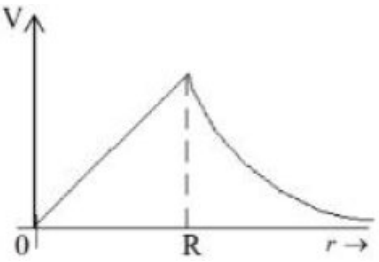
(4) neutron is an uncharged particle

Answer (1)

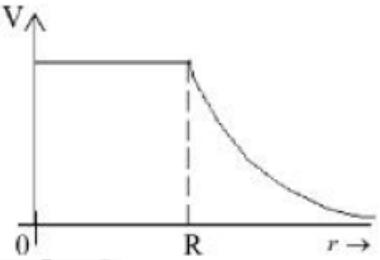
Sol. Rest mass of proton > Rest mass of neutron.

9. Which of the following correctly represents the variation of electric potential(V) of a charged spherical conductor of radius (R) with radial distance (r) from the center?

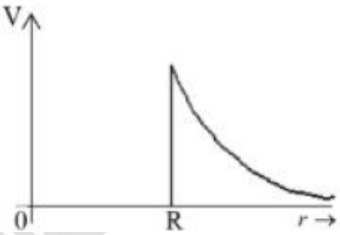
(1)



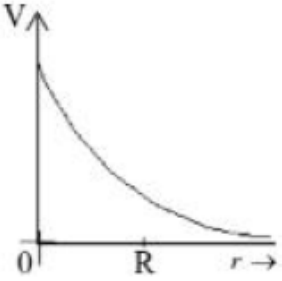
(2)



(3)

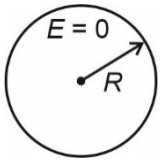


(4)



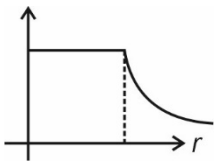
Answer (2)

Sol.



Electric field inside a conductor = 0

Hence V = constant



10. The amplitude of 15 sin (1000π t) is modulated by 10 sin (4π t) signal. The amplitude modulated signal contains frequency(ies) of

A. 500 Hz

B. 2 Hz

C. 250 Hz

D. 498 Hz

E. 502 Hz

Choose the correct answer from the options given below

(1) B only (2) A and B only

(3) A, D and E only (4) A only

Answer (3)

Sol. Frequencies in AM are

$f_c, f_c + f_m, f_c - f_m$

$f_c = 500, f_m = 2$

500, 498 and 502 are present

11. If a source of electromagnetic radiation having power 15kW produces 10^{16} photons per second, the radiation belongs to a part of spectrum is.

(Take Planck constant h = 6 × 10^{-34}Js)

(1) Gamma rays (2) Ultraviolet rays

(3) Micro waves (4) Radio waves

Answer (1)

Sol. huv = 15kW

$$huv = \frac{15\times10^3}{6\times10^{-34}\times10^{16}} = 2.5\times10^{21}\text{ Hz}$$

gamma rays

12. At a certain depth "d" below surface of earth, value of acceleration due to gravity becomes four times that of tis value at a height 3R above earth surface. Where R is Radius of earth (Take R = 6400 km). The depth d is equal to

(1) 4800 km (2) 5260 km

(3) 2560 km (4) 640 km

Answer (1)

Sol. $g_d = \frac{GM}{R^3}(R-d)$ (depth variation)

$g_h = \frac{GM}{(R+h)^2}$ (h above surface)

$g_d = 4\ g_h$

$$\frac{GM}{R^3}(R-d) = 4\frac{GM}{(R+3R)^2}$$

$$R-d = \frac{R}{4}$$

$$d = \frac{3R}{4}$$

d = 4800 km

13. Spherical insulating ball and a spherical metallic ball of same size and mass are dropped from the same height. Choose the correct statement out of the following {Assume negligible air friction}

(1) Time taken by them to reach the earth's surface will be independent of the properties of their materials

(2) Insulating ball will reach the earth's surface earlier than the metal ball

(3) Metal ball will reach the earth's surface earlier than the insulating ball

(4) Both will reach the earth's surface simultaneously.

Answer (2)

Sol. Consider magnetic force of earth, induced eddy current will develop inside the conducting sphere which retards the conducting sphere.

14. The initial speed of a projectile fired from ground is u. At the highest point during its motion, the speed of projectile is $\frac{\sqrt{3}}{2}u$ The time of filight of the projectile is

(1) $\frac{\sqrt{3u}}{g}$ (2) $\frac{u}{g}$

(3) $\frac{2u}{g}$ (4) $\frac{u}{2g}$

Answer (2)

Sol. $u\cos\theta = \frac{\sqrt{3}}{2}u$

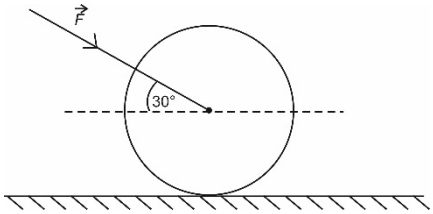
$$\cos\theta = \frac{\sqrt{3}}{2}$$

$$\boxed{\theta = 30^\circ}$$

Time of flight $= \frac{2u\sin\theta}{g} = \left(\frac{u}{g}\right)$

15. As shown in figure, a 70 kg garden roller is pushed with a force of $\vec{F} = 200$ N at an angle of 30° with horizontal. The normal reaction on the roller is

(Given g = 10 m s^{-2})



(1) 600 N

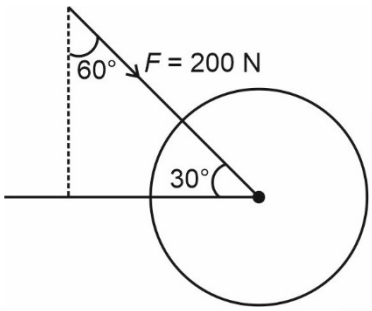
(2) 800 N

(3) $200\sqrt{3}$ N

(4) $800\sqrt{2}$ N

Answer (2)

Sol.



Normal reaction = 70 g + Fcos60

= 700 + 100

= 800 N

16. If 1000 droplets of water of surface tension 0.07 N/m, having same radius 1 mm each, combine to from a single drop. In the process the released surface energy is-

$\left(\text{Take } \pi = \frac{22}{7}\right)$

(1) 7.92 × 10^{-6} J

(2) 7.92 × 10^{-4} J

(3) 9.68 × 10^{-4} J

(4) 8.8 × 10^{-5} J

Answer (2)

Sol. Radius of bigger drop = 10r = R

(r = radius of smaller droplet)

$\Delta E = 1000 \times 4\pi r^2 \times T - 4\pi R^2 T$

$= 4\pi T\,[1000 \times r^2 - 100r^2]$

$= 3600\,\pi r^2 T$

$= 3600 \times \frac{22}{7} \times 1 \times 10^{-6} \times \frac{7}{100}$

$= 22 \times 36 \times 10^{-6}$

$= 792 \times 10^{-6}$ J

$= 7.92 \times 10^{-4}$ J

17. The drift velocity of electrons for a conductor connected in an electrical circuit is V_d. The conductor in now replaced by another conductor with same material and same length but double the area of cross section. The applied voltage remains same. The new drift velocity of electrons will be

(1) V_d

(2) $2V_d$

(3) $\frac{V_d}{4}$

(4) $\frac{V_d}{2}$

Answer (1)

Sol. $i = nAV_d e$

$i_1 = \left(\frac{V}{R}\right)$

$i_2 = \left(\frac{2V}{R}\right)$

So, $\frac{i_1}{i_2} = \frac{1}{2} = \frac{(AV_d)_1}{(AV_d)_2} = \frac{V_d}{(V_d)_2} \times \left(\frac{1}{2}\right)$

$\frac{1}{2} \times \frac{V_d}{(V_d)_2} = \frac{1}{2}$

$\Rightarrow (V_d)_2 = V_d$

18. If R, X_L and X_C represent resistance, inductive reactance and capacitive reactance. Then which of the following is dimensionless

(1) $R\frac{X_L}{X_C}$ (2) $R\,X_L\,X_C$

(3) $\frac{R}{\sqrt{X_L X_C}}$ (4) $\frac{R}{X_L X_C}$

Answer (3)

Sol. R = Resistance

$[X_L] = [R]$

$[X_C] = [R]$

So, $\frac{R}{\sqrt{X_L X_C}}$ is dimensionless.

19. Given below are two statements: One is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : The beam of electrons show wave nature and exhibit interference and diffraction.

Reason R : Davisson Germer Experimentally verified the wave nature of electrons.

In the light of the above statements, choose the ***most appropriate*** answer from the options given below:

(1) A is not correct but R is correct

(2) Both A and R are correct and R is the correct explanation of A

(3) Both A and R are correct but R is not the correct explanation of A

(4) A is correct but R is not correct

Answer (2)

Sol. Beam of electrons show wave nature and exhibit interference and diffraction as shown by Davisson Germer experiment.

20. The correct relation between $\gamma = \frac{C_p}{C_v}$ and temperature T is

(1) $\gamma \alpha T°$

(2) $\gamma \alpha \frac{1}{\sqrt{T}}$

(3) $\gamma \alpha \frac{1}{T}$

(4) $\gamma \alpha T$

Answer (1)

Sol. $\gamma = \frac{C_P}{C_V}$

At low temperature (T), γ is independent of T.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. In a medium the speed of light wave decreases to 0.2 times to its speed in free space. The ratio of relative permittivity to the refractive index of the medium is x : 1. The value of x is _____.

(Given speed of light in free space =3 × 10^8 ms^{-1} and for the given medium $\mu_1 = 1$)

Answer (5)

Sol. We know that $v = \frac{c}{n} = \frac{c}{\sqrt{\varepsilon_r}}$

Putting the values:

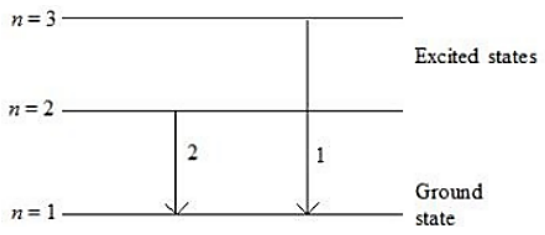
$0.2c = \frac{c}{\sqrt{\varepsilon_r}}$

$\Rightarrow \sqrt{\varepsilon_r} = 5$

$\Rightarrow$ Required ratio $= \frac{\varepsilon_r}{n} = \frac{\varepsilon_r}{\sqrt{\varepsilon_r}} = \sqrt{\varepsilon_r} = 5$

$\Rightarrow$ x = 5

22. For hydrogen atom, λ_1 and λ_2 are the wavelengths corresponding to the transitions 1 and 2 respectively as shown in figure. The ratio of λ_1 and λ_2 is $\frac{x}{32}$. The value of x is _____.



Answer (27)

Sol. $\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$\Rightarrow \frac{1}{\lambda_1} = R\left[1 - \frac{1}{9}\right]$

& $\frac{1}{\lambda_2} = R\left[1 - \frac{1}{4}\right]$

$\Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{R \cdot \frac{3}{4}}{R \cdot \frac{8}{9}} = \frac{27}{32}$

$\Rightarrow x = 27$

23. The speed of a swimmer is 4 km h^{-1} in still water. If the swimmer makes his strokes normal to the flow of river of width 1 km, he reaches a point 750 m down the stream on the opposite bank.

The speed of the river water is _____ km h^{-1}

Answer (3)

Sol. Let speed of river water = v_0

$\Rightarrow$ drift $= v_0 \times \Delta t$ ...(1)

& $\Delta t = \frac{1}{4}h$ ...(2)

$\Rightarrow 0.75 = v_0 \times \frac{1}{4}$

$\Rightarrow v_0 = 3$ km/hr

24. Two identical cells, when connected either in parallel or in series gives same current in an external resistance 5 Ω. The internal resistance of each cell will be _____ Ω.

Answer (5)

Sol. $\varepsilon_{series} = \varepsilon_1 + \varepsilon_2 = 2\varepsilon$

$r_{series} = r_1 + r_2 = 2r$

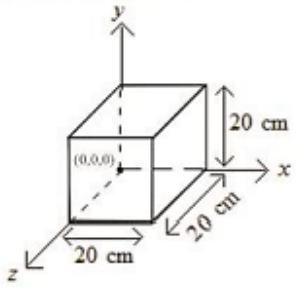
$\varepsilon_{parallel} = \frac{\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}} = \varepsilon$

& $r_{parallel} = \frac{r}{2}$

$\Rightarrow \frac{2\varepsilon}{2r + 5} = \frac{\varepsilon}{\frac{r}{2} + 5}$

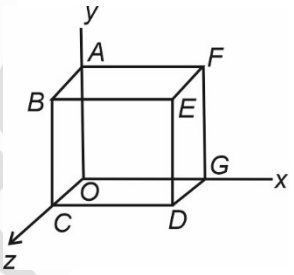
$\Rightarrow r + 10 = 2r + 5 \Rightarrow r = 5\ \Omega$

25. Expression for an electric field is given by $\vec{E} = 4000x^2\hat{i}\frac{V}{m}$. The electric flux through the cube of side 20 cm when placed in electric field (as shown in the figure) is _____ V cm.



Answer (640)

Sol. The flux will be only from *DEFG* surface as on the surface *OABC* field is 0 and for rest of the surface, area vector is perpendicular to field.



So $\phi = EA$

$= 4000 \times (\cdot 2)^2 \times \cdot 2 \times \cdot 2$

$= \frac{32}{5}$ Vm

$= \frac{32}{5} \times 100$ V cm

= 640 V cm

26. An inductor of 0.5 mH, a capacitor of 20 μF and resistance of 20 Ω are connected in series with a 220 V ac source. If the current is in phase with the emf, the amplitude of current of the circuit is $\sqrt{x}$ A. The value of x is

Answer (242)

Sol. As the current is in lase with emf the circuit is in resonance so

$i_{rms} = \frac{V_{rms}}{R} = \frac{220}{20} = 11$ A

as $i_0 = \sqrt{2}i_{rms} = \sqrt{2} \times 11 = \sqrt{242}$

27. A lift of mass M = 500 kg is descending with speed of 2 ms^{-1}. Its supporting cable begins to slip thus allowing it to fall with a constant acceleration of 2 ms^{-2}. The kinetic energy of the lift at the end of fall through to a distance of 6 m will be _____ kJ.

Answer (7)

Sol. u = 2m/s

a = 2m/s^2

s = 6m

v = ?

$v^2 = u^2 + 2as$

$v^2 = 4 + 2 \times 2 \times 6$

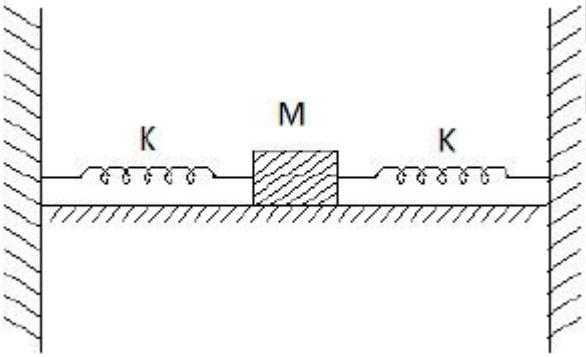
$= 28$

So $KE = \frac{1}{2}mv^2 = \frac{1}{2} \times 500 \times 28\,J$

= 7000 J

= 7 kJ

28. In the figure given below, a block of mass M = 490 g placed on a frictionless table is connected with two springs having same spring constant (K = 2 N m^{-1}). If the block is horizontally displaced through 'X' m then the number of complete oscillations it will make in 14π seconds will be _____.



Answer (20)

Sol. $k_{net} = k_1 + k_2 = 4$ N/m

$T = 2\pi\sqrt{\frac{m}{k_{net}}} = 2\pi\sqrt{\frac{0.49}{4}}$

$= \frac{2\pi \times .7}{2}$

$= \frac{7\pi}{10}$

So number of oscillation completed.

$n = \frac{t}{T} = \frac{14\pi}{7\pi/10} = 20$

29. A solid sphere of mass 1 kg rolls without slipping on a plane surface. Its kinetic energy is 7×10^{-3} J. The speed of the centre of mass of the sphere is _____ cm s^{-1}.

Answer (10)

Sol. $K = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}\frac{2}{5}mR^2\frac{v_{cm}^2}{R^2}$

$7 \times 10^{-3} = \frac{7}{10} \times 1 \times v_{cm}^2$

v_{cm} = 0.1 m/sec

= 10 cm/sec

30. A thin rod having a length of 1 m and area of cross-section 3×10^{-6} m^2 is suspended vertically from one end. The rod is cooled from 210°C to 160°C. After cooling, a mass M is attached at the lower end of the rod such that the length of rod again becomes 1 m. Young's modulus and coefficient of linear expansion of the rod are 2×10^{11} N m^{-2} and 2×10^{-5} K^{-1}, respectively. The value of M is _____ kg. (Take g = 10 m s^{-2})

Answer (60)

Sol. Stress $= \frac{T}{AY} = \frac{\Delta l}{l}$

and $\Delta l = l\alpha\Delta T$

or $\frac{\Delta l}{l} = \alpha\Delta T$

so $\frac{T}{AY} = \alpha\Delta T$

$\frac{M \times 10}{2 \times 10^{11} \times 3 \times 10^{-6}} = 2 \times 10^{-5} \times 50$

$M = \frac{2 \times 10^{-5} \times 50 \times 3 \times 10^{-6} \times 2 \times 10^{11}}{10}$

= 60 kg

CHEMISTRY

SECTION – A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

31. Cobalt chloride when dissolved in water forms pink coloured complex X which has octahedral geometry. This solution on treating with conc. HCl forms deep blue complex, Y which has a Z geometry. X, Y and Z, respectively, are

(1) X = $[Co(H_2O)_6]^{3+}$, Y = $[CoCl_6]^{3-}$, Z = Octahedral

(2) X = $[Co(H_2O)_4Cl_2]^{+}$, Y = $[CoCl_4]^{2-}$, Z = Tetrahedral

(3) X = $[Co(H_2O)_6]^{2+}$, Y = $[CoCl_6]^{3-}$, Z = Octahedral

(4) X = $[Co(H_2O)_6]^{2+}$, Y = $[CoCl_4]^{2-}$, Z = Tetrahedral

Answer (4)

Sol. $CoCl_2 \xrightarrow{H_2O} [Co(H_2O)_6]^{2+}$ (X) (Octahedral)

$\downarrow$ Conc HCl

$[CoCl_4]^{2-}$ (Y) Tetrahedral (Z)

Hence correct answer is option (4)

32. The correct order of basicity of oxides of vanadium is

(1) $V_2O_3 > V_2O_5 > V_2O_4$ (2) $V_2O_4 > V_2O_3 > V_2O_5$

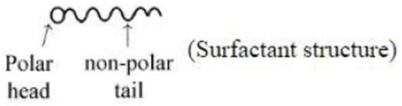
(3) $V_2O_3 > V_2O_4 > V_2O_5$ (4) $V_2O_5 > V_2O_4 > V_2O_3$

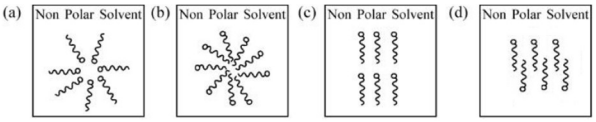
Answer (3)

Sol. $V_2O_3 > V_2O_4 > V_2O_5$

As positive oxidation state increases acidic nature increases and basic nature decreases.

33. Adding surfactants in non polar solvent, the micelles structure will look like



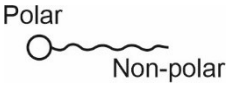


(1) c (2) a

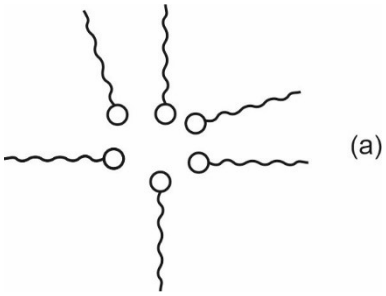
(3) d (4) b

Answer (2)

Sol.



In non-polar solvent non-polar part will point out



Non-polar part will interact with non-polar solvent.

34. Which one of the following statements is correct for electrolysis of brine solution?

(1) Cl_2 is formed at cathode

(2) H_2 is formed at anode

(3) O_2 is formed at cathode

(4) OH^- is formed at cathode

Answer (4)

Sol. During electrolysis of Brine

$2NaCl \rightarrow Na^+ + Cl^-$

$2H_2O \rightarrow 2H^+ + 2OH^-$

Cathode $2H^+ + 2e \rightarrow H_2$

Anode $2Cl^- \rightarrow Cl_2 + 2e.$

At cathode H_2 is liberated

At anode Cl_2 is formed.

35. When Cu^{2+} ion is treated with KI, a white precipitate, X appears in solution. The solution is titrated with sodium thiosulphate, the compound Y is formed. X and Y respectively are

(1) X = CuI_2 Y = $Na_2S_2O_3$

(2) X = CuI_2 Y = $Na_2S_4O_6$

(3) X = Cu_2I_2 Y = $Na_2S_4O_5$

(4) X = Cu_2I_2 Y = $Na_2S_4O_6$

Answer (4)

Sol. $2Cu^{2+} + 4KI \longrightarrow \underset{\text{White ppt.}}{Cu_2I_2} + I_2$

$I_2 + Na_2S_2O_3 \longrightarrow 2NaI + Na_2S_4O_6$

X = Cu_2I_2

Y = $Na_2S_4O_6$

36. Which transition in the hydrogen spectrum would have the same wavelength as the Balmer type transition from n = 4 to n = 2 of He^+ spectrum

(1) n = 2 to n = 1 (2) n = 3 to n = 4

(3) n = 1 to n = 2 (4) n = 1 to n = 3

Answer (1)

Sol. $\bar{\nu}_{He^+} = \frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]z^2$

$= R\left[\frac{1}{(2)^2} - \frac{1}{(4)^2}\right]4$

$= R\left[\frac{1}{1} - \frac{1}{4}\right]$

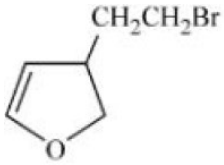
$= \frac{3}{4}R$

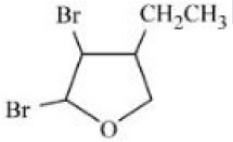
$\bar{\nu}_{2\to1} = \frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

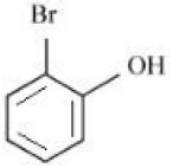
$= R\left[\frac{1}{1} - \frac{1}{(2)^2}\right]$

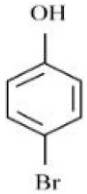
$= \frac{3}{4}R$

37. An organic compound 'A' with empirical formula C_6H_6O gives sooty flame on burning. Its reaction with bromine solution in low polarity solvent results in high yield of B. B is

(1) 

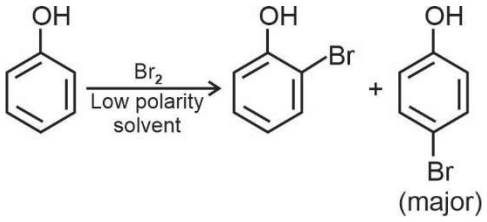
(2) 

(3) 

(4) 

Answer (4)

Sol.



38. Match **List I** with **List II**

	List I		List II
A.	XeF_4	I.	See-saw
B.	SF_4	II.	Square planar
C.	NH_4^+	III.	Bent T-shaped
D.	BrF_3	IV.	Tetrahedral

Choose the correct answer from the options given below:

(1) A - IV, B - III, C - II, D - I

(2) A - II, B - I, C - III, D - IV

(3) A - II, B - I, C - IV, D III

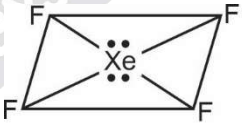
(4) A - IV, B - I, C - II, D - III

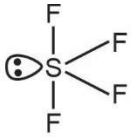
Answer (3)

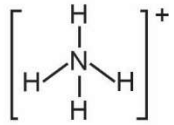
Sol.

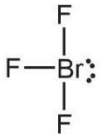
A-II B-I C-IV D-III

				Hybridisation
XeF_4	–	Square planar	–	sp^3d^2
SF_4	–	See Saw	–	sp^3d
NH_4^+	–	Tetrahedral	–	sp^3
BrF_3	–	Bent-T-shape	–	sp^3d









39. Consider the following reaction

$$\text{Propanal} + \text{Methanal} \xrightarrow[\substack{\text{(ii) }\Delta \\ \text{(iii) NaCN} \\ \text{(iv) } H_3O^+}]{\text{(i) dil.NaOH}} \underset{(C_5H_8O_3)}{\text{Product B}}$$

The correct statement for product B is. It is

(1) racemic mixture and gives a gas with saturated $NaHCO_3$ solution

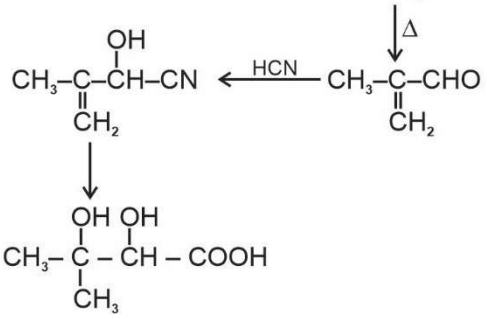
(2) optically active alcohol and is neutral

(3) optically active and adds one mole of bromine

(4) racemic mixture and is neutral

Answer (1)

Sol. $CH_3–CH_2CHO + HCHO \xrightarrow{dil.base} CH_3–\underset{\underset{CH_2OH}{|}}{CH}–CHO$



Racemic mixture effervescence with $NaHCO_3$

40. H_2O_2 acts as a reducing agent in

(1) $2Fe^{2+} + 2H^+ + H_2O_2 \rightarrow 2Fe^{3+} + 2H_2O$

(2) $Mn^{2+} + 2\,H_2O_2 \rightarrow MnO_2 + 2H_2O$

(3) $Na_2S + 4H_2O_2 \rightarrow Na_2SO_4 + 4H_2O$

(4) $2NaOCl + H_2O_2 \rightarrow 2\,NaCl + H_2O + O_2$

Answer (4)

Sol. H_2O_2 act as a reducing agent

$$\overset{+1}{2Na}\overset{-2}{O}\overset{+1}{Cl} + H_2O_2 \longrightarrow 2\,NaCl + H_2O + O_2$$

Cl from (+1) state changes to Cl^{-1}

41. Identify X, Y and Z in the following reaction. (Equation not balanced)

$$ClO^{\bullet} + NO_2 \rightarrow \underline{X} \xrightarrow{H_2O} \underline{Y} + \underline{Z}$$

(1) X = $ClONO_2$, Y = HOCl, Z = NO_2

(2) X = $ClONO_2$, Y = HOCl, Z = HNO_3

(3) X = $ClNO_3$, Y = Cl_2, Z = NO_2

(4) X = $ClNO_2$, Y = HCl, Z = HNO_3

Answer (2)

Sol. $ClO + NO_2 \longrightarrow \underset{(X)}{ClONO_2}$

$$ClONO_2 \xrightarrow{H_2O} \underset{(Y)}{HOCl} + \underset{(Z)}{HNO_3}$$

42. The correct increasing order of the ionic radii is

(1) $K^+ < S^{2-} < Ca^{2+} < Cl^-$ (2) $Cl^- < Ca^{2+} < K^+ < S^{2-}$

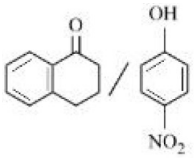
(3) $Ca^{2+} < K^+ < Cl^- < S^{2-}$ (4) $S^{2-} < Cl^- < Ca^{2+} < K^+$

Answer (3)

Sol. Given ions are isoelectronic more is nuclear charge per electron smaller is size

	Ca^{+2}	<	K^+	<	Cl^-	<	S^{2-}
p	20		19		17		16
e	18		18		18		18

43. Match items of columsn I and II

Column I (Mixture of compounds)		Column II (Separation Technique)	
(A)	H_2O / CH_2Cl_2	(i)	Crystallization
(B)		(ii)	Differential solvent extraction
(C)	Kerosene / Naphthalene	(iii)	Column chromatography
(D)	$C_6H_{12}O_6$ / NaCl	(iv)	Fractional Distillation

Correct match is

(1) A-(ii), B-(iii), C-(iv), D-(i)

(2) A-(ii), B-(iv), C-(i), D-(iii)

(3) A-(i), B-(iii), C-(ii), D-(iv)

(4) A-(iii), B-(iv), C-(ii), D-(i)

Answer (1)

Sol. Water and dichloromethane can be separated by differential extraction.

Which $C_6H_{12}O_6$ and NaCl can be separated by crystallization.

44. Choose the correct set of reagents for the following conversion.

trans (Ph – CH = CH – CH_3) →

cis (Ph – CH = CH – CH_3)

(1) Br_2, alc•KOH, $NaNH_2$, H_2 Lindlar Catalyst

(2) Br_2, aq•KOH, $NaNH_2$, Na (Liq NH_3)

(3) Br_2, alc•KOH, $NaNH_2$, Na (Liq NH_3)

(4) Br_2, aq•KOH, $NaNH_2$, H_2 Lindlar Catalyst

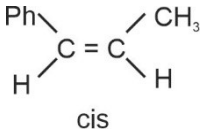
Answer (1)

Sol. $Ph–\underset{Trans}{CH}=CH–CH_3 \longrightarrow Ph–\underset{cis}{CH}=CH–CH_3$

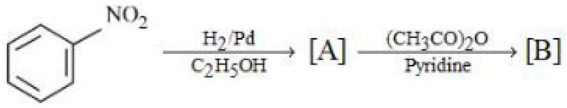
$$Ph–\underset{Trans}{CH}=CH–CH_3 \xrightarrow{Br_2}$$

$$Ph–\underset{\underset{Br}{|}}{CH}–\underset{\underset{Br}{|}}{CH}–CH_3 \xrightarrow{alc.KOH/NaNH_2}$$

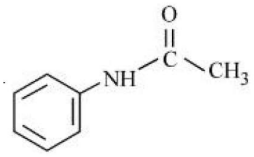
$$Ph–C\equiv C–CH_3 \xrightarrow{Lindale's\ catalyst}$$

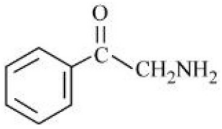


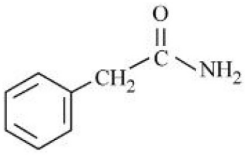
cis

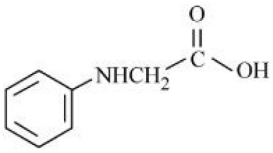
45. 

Consider the above reaction and identify the product B.

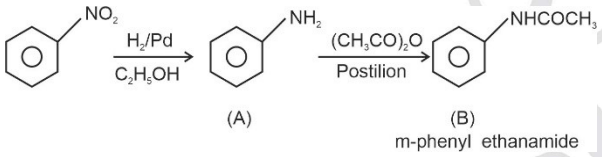
(1) 

(2) 

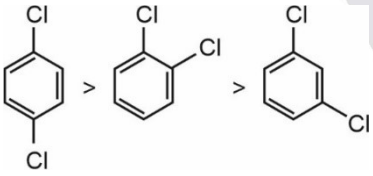
(3) 

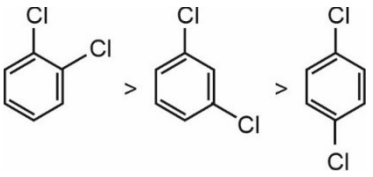
(4) 

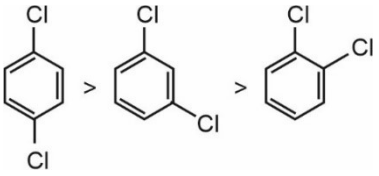
Answer (1)

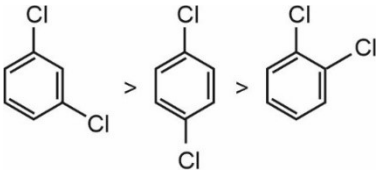
Sol. 

46. The correct order of melting points of dichlorobenzenes is

(1) 

(2) 

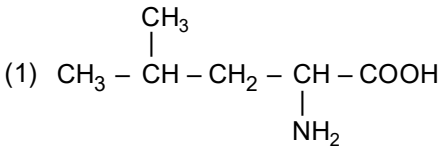
(3) 

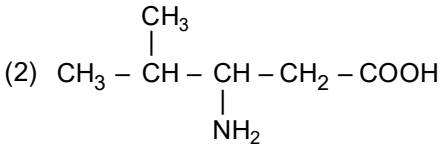
(4) 

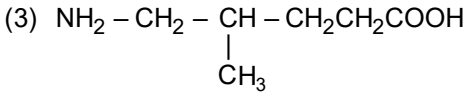
Answer (1)

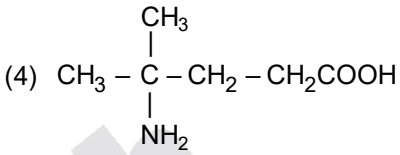
Sol. Out of o, m, p-dichlorobenzene para isomer has maximum melting point due to symmetrical nature.

47. A protein 'X' with molecular weight of 70,000 u, on hydrolysis gives amino acids. One of these amino acid is

(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. Protein upon hydrolysis gives α-amino acids. Only option (1) contains α-amino acid. Hence the correct answer is (1).

48. Nd^{2+} = _____

(1) $4f^3$ (2) $4f^4 6s^2$

(3) $4f^4$ (4) $4f^2 6s^2$

Answer (3)

Sol. Neodymium Nd = $4f^4\ 6s^2$

$Nd^{2+} = 4f^4$.

49. Which of the following artificial sweeteners has the highest sweetness value in comparison to cane sugar?

(1) Sucralose (2) Aspartame

(3) Saccharin (4) Alitame

Answer (4)

Sol. Highest sweetness value is of Alitame

Sucralose = 600

Aspartame = 100

Saccharin = 550

Alitame = 2000

50. The methods NOT involved in concentration of ore are

A. Liquation B. Leaching

C. Electrolysis D. Hydraulic washing

E. Froth floatation

Choose the correct answer from the options given below

(1) B, D and C only (2) B, D and E only

(3) C, D and E only (4) A and C only

Answer (4)

Sol. (A) and (C) only

Liquation is used for purification of metal.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

51. On complete combustion, 0.492 g of an organic compound gave 0.792 g of CO_2. The % of carbon in the organic compound is _____ (Nearest integer)

Answer (44)

Sol. Percentage of C $= \frac{W_{CO_2}}{W_{org.comp}} \times \frac{12}{44} \times 100$

$= \frac{0.792}{0.492} \times \frac{12}{44} \times 100$

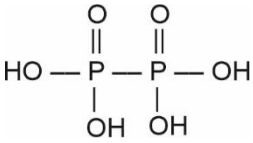
$= 43.90$

52. The oxidation state of phosphorus in hypophosphoric acid is + _____.

Answer (4)

Sol. Hypophosphoric acid $H_4P_2O_6$

Oxidation state is +4



53. For reaction : $SO_2(g) + \frac{1}{2}O_2(g) \rightleftharpoons SO_3(g)$

$K_p = 2\times10^{12}$ at 27°C and 1 atm pressure. The K_c for the same reaction is _____ × 10^{13}. (Nearest integer)

(Given R = 0.082 L atm K^{-1} mol^{-1})

Answer (1)

Sol. $SO_{2(g)} + \frac{1}{2}O_{2(g)} \rightleftharpoons SO_{3(g)}$

$K_p = K_c (RT)^{\Delta n}$

$2 \times 10^{12} = K_c (0.082 \times 300)^{-½}$

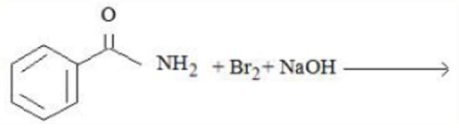
$K_c = 2 \times 10^{12} \times (0.082 \times 300)^{½}$

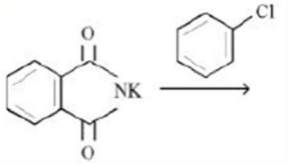
$= 9.9 \times 10^{12}$

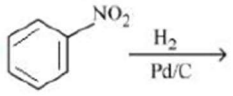
$= 0.99 \times 10^{13}$

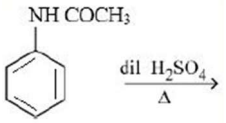
$\approx 1 \times 10^{13}$

54. How many of the transformations given below would result in aromatic amines?

(1) 

(2) 

(3) 

(4) 

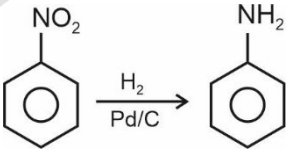
Answer (3)

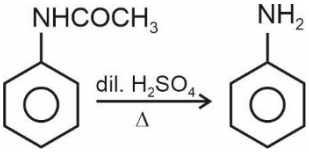
Sol. 1, 3, 4 will give Aniline.

Gabriel phthalimide synthesis cannot be used to prepare Aniline.

(1) $C_6H_5CONH_2 \xrightarrow{Br_2/NaOH} C_6H_5NH_2$

Hoffmann Bromamide synthesis

(3) 

(4) 

55. The enthalpy change for the conversion of $\frac{1}{2}Cl_2(g)$ to Cl^- (aq) is (–) _____ kJ mol^{-1} (Nearest integer)

Given : $\Delta_{dis}H^{\ominus}_{Cl_2(g)} = 240$ kJ mol^{-1},

$\Delta_{eg}H^{\ominus}_{Cl(g)} = -350$ kJ mol^{-1},

$\Delta_{hyd}H^{\ominus}_{Cl^-_{(g)}} = -380$ kJ mol^{-1}

Answer (610)

Sol. $\frac{1}{2}Cl_2(g) \longrightarrow Cl^-_{(aq)}$ $\quad \Delta H = ?$

$$\Delta H = \frac{1}{2}\Delta_{diss}H^\circ_{Cl_2} + \Delta_{eg}\Delta H^\circ_{Cl(g)} + \Delta_{hyd}H^\circ_{Cl^-_{(g)}}$$

$$= \frac{1}{2}\times 240 + (-350) + (-380)$$

$$= -610 \text{ kJ mol}^-$$

56. The total pressure of a mixture of non-reacting gases X (0.6 g) and Y (0.45 g) in a vessel is 740 mm of Hg. The partial pressure of the gas X is _____ mm of Hg. (Nearest integer)

(Given : molar mass X = 20 and Y = 45 g mol^{-1})

Answer (555)

Sol. P_{Total} = 740 mm of Hg

P_X = mole fraction of [X] P_{Total}

$$n_X = \frac{0.6}{20} = 0.03$$

$$n_Y = \frac{0.45}{45} = 0.01$$

Mole fraction of X = $\frac{0.03}{0.01+0.03} = \frac{3}{4}$

Partial pressure of X = $\frac{3}{4}\times 740$

= 555 mm of Hg

57. The logarithm of equilibrium constant for the reaction $Pd^{2+} + 4Cl^- \rightleftharpoons PdCl_4^{2-}$ is _____. (Nearest integer)

Given : $\frac{2.303RT}{F} = 0.06V$

$Pd^{2+}_{(aq)} + 2e^- \rightleftharpoons Pd(s) \quad E^\ominus = 0.83V$

$PdCl_4^{2-}(aq) + 2e^- \rightleftharpoons Pd(s) + 4Cl^-(aq) \quad E^\ominus = 0.65V$

Answer (6)

Sol. $Pd^{2+} + 4Cl^- \rightleftharpoons [PdCl_4]^{2-}$

$$E^o = (0.83) - (0.65) = 0.18 \text{ V}$$

$$0 = 0.18 - \frac{0.06}{2}\log k_{eq}$$

$$0.18 = 0.03 \log k_{eq}$$

$$\boxed{\log k_{eq} = 6}$$

58. A→B

The rate constants of the above reaction at 200 K and 300 K are 0.03 min^{-1} and 0.05 min^{-1} respectively. The activation energy for the reaction is _____ J (Nearest integer)

(Given : ln 10 = 2.3

R = 8.3 J K^{-1} mol^{-1}

log 5 = 0.70

log 3 = 0.48

log 2 = 0.30)

Answer (2520)

Sol. $$\log\frac{k_2}{k_1} = \frac{E_a}{2.3\times 8.3}\left(\frac{1}{200} - \frac{1}{300}\right)$$

$$\log\frac{0.05}{0.03} = \frac{E_a}{2.3\times 8.3}\left(\frac{1}{600}\right)$$

$$(0.70 - 0.48) = \frac{E_a}{2.3\times 8.3}\times\frac{1}{600}$$

$$\Rightarrow 0.22 = \frac{E_a}{2.3\times 8.3}\times\frac{1}{600}$$

$$E_a = 2.3\times 8.3\times 600\times 0.22$$

$$= 2519.88$$

$$\approx 2520 \text{ J}$$

59. At 27°C, a solution containing 2.5 g of solute in 250.0 mL of solution exerts an osmotic pressure of 400 Pa. The molar mass of the solute is _____ g mol^{-1}. (Nearest integer)

(Given : R = 0.083 L bar K^{-1} mol^{-1})

Answer (62250)

Sol. $$400 = \frac{2.5}{mw}\times 4\times(.083\times 10^5)\times 300$$

$$mw = \frac{10\times 0.083\times 3}{4}\times 10^5$$

$$= 62250$$

60. Zinc reacts with hydrochloric acid to give hydrogen and zinc chloride. The volume of hydrogen gas produced at STP from the reaction of 11.5 g of zinc with excess HCl is _____ L. (Nearest integer)

(Given : Molar mass of Zn is 65.4g mol^{-1} and Molar volume of H_2 at STP = 22.7 L)

Answer (4)

Sol. $Zn + 2HCl \longrightarrow ZnCl_2 + H_2$

$$n_{Zn} = \frac{11.5}{65.4} = 0.176$$

$$V_{H_2} = 0.176\times 22.7 = 3.99 \text{ litre}$$

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

61. Let the shortest distance between the line $L: \frac{x-5}{-2}=\frac{y-\lambda}{1}=\frac{z+\lambda}{1}$, $\lambda \geq 0$ and $L_1 : x + 1 = y - 1 = 4 - z$ be $2\sqrt{6}$. If (α, β, γ) lies on L, then which of the following is **NOT** possible?

(1) $\alpha + 2\gamma = 24$ (2) $2\alpha - \gamma = 9$

(3) $2\alpha + \gamma = 7$ (4) $\alpha - 2\gamma = 19$

Answer (1)

Sol. $\frac{x-5}{-2}=\frac{y-\lambda}{0}=\frac{z+\lambda}{1}$, $\lambda \geq 0$

$\frac{x+1}{1}=\frac{y-1}{1}=\frac{z-4}{-1}$

$\vec{a}_1 = 5\hat{i} + \lambda\hat{j} - \lambda\hat{k}, \vec{a}_2 = -\hat{i} + \hat{j} + 4\hat{k}$

$\vec{a}_1 - \vec{a}_2 = 6\hat{i} + (\lambda - 1)\hat{j} - (\lambda + 4)\hat{k}$

$\vec{b}_1 = -2\hat{i} + \hat{k}, \vec{b}_2 = \hat{i} + \hat{j} - \hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$= -\hat{i} - \hat{j} - 2\hat{k}$

$(\vec{a}_1 - \vec{a}_2) \cdot \vec{b}_1 \times \vec{b}_2 = -6 + 1 - \lambda + 2\lambda + 8 = \lambda + 3$

and $|\vec{b}_1 \times \vec{b}_2| = \sqrt{6}$

$\because \quad \frac{|\lambda + 3|}{\sqrt{6}} = 2\sqrt{6}$

$\therefore \quad \lambda = 9, \because \lambda \geq 0$

$\therefore \quad L: \frac{x-5}{-2}=\frac{y-9}{0}=\frac{z+9}{1}=k$

$\therefore \quad \alpha = -2k + 5, \beta = 9, \gamma = k - 9$

Here k is real then

$\alpha + 2\gamma = -13 \neq 24$.

But all other are in terms of k hence possible.

Correct option is (1).

62. Let $\alpha \in (0, 1)$ and $\beta = \log_e (1 - \alpha)$.

Let $P_n(x) = x + \frac{x^2}{2} + \frac{x^3}{3} + ... \frac{x^n}{n}$, $x \in (0,1)$.

Then the integral $\int_0^{\alpha} \frac{t^{50}}{1-t} dt$ is equal to

(1) $-(\beta + P_{50}(\alpha))$

(2) $\beta + P_{50}(\alpha)$

(3) $P_{50}(\alpha) - \beta$

(4) $\beta + P_{50}(\alpha)$

Answer (1)

Sol. $\int_0^{\alpha} \frac{t^{50}}{1-t} dt = -\int_0^{\alpha} \left(\frac{1-t^{50}}{1-t} - \frac{1}{1-t} \right) dt$

$= -\left(\int_0^{\alpha} 1 + t + t^2 + ... + t^{49} \right) dt + \ln|1-t|_0^{\alpha}$

$= -\left(\alpha + \frac{\alpha^2}{2} + \frac{\alpha^3}{3} + ... + \frac{\alpha^{50}}{50} \right) + \ln(1-\alpha)$

$= -\beta - P_{50}(\alpha)$

63. (S1) $(p \Rightarrow q) \vee (p \wedge (\sim q))$ is a tautology

(S2) $((\sim p) \Rightarrow (\sim q) \wedge ((\sim p) \vee q)$ is a contradication.

Then

(1) both (S1) and (S2) are wrong

(2) both (S1) and (S2) are correct

(3) only (S1) is correct

(4) only (S2) is correct

Answer (3)

Sol. S1

p	q	$\sim q$	$p \to q$	$p \wedge (\sim q)$	$(p \to q) \vee p \wedge (\sim q)$
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	T	F	T
F	F	T	T	F	T

$\therefore$ S1 is correct

S2

p	q	$\sim p$	$\sim q$	$\sim p \to \sim q$	$\sim p \vee q$	(S2)
T	T	F	F	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	T	T	T

$\therefore$ S2 is incorrect

Option (3) is correct.

64. A bag contains 6 balls. Two balls are drawn from it at random and both are found to be black. The probability that the bag contains at least 5 black balls is

(1) $\frac{3}{7}$ (2) $\frac{5}{6}$

(3) $\frac{2}{7}$ (4) $\frac{5}{7}$

Answer (4)

Sol. Let $E_i \to$ Bag have at least i black balls

E $\to$ 2 balls are drawn & both black

$$\therefore \quad P\left(\frac{E_5 \text{ or } E_6}{E}\right) = \frac{P\left(\frac{E}{E_5}\right)+P\left(\frac{E}{E_6}\right)}{\sum_{i=1}^{6} P\left(\frac{E}{E_i}\right)}$$

$$= \frac{\frac{{}^5C_2}{{}^6C_2}+\frac{{}^6C_2}{{}^6C_2}}{0+\frac{{}^2C_2}{{}^6C_2}+\frac{{}^3C_2}{{}^6C_2}+\frac{{}^4C_2}{{}^6C_2}+\frac{{}^5C_2}{{}^6C_2}+\frac{{}^6C_2}{{}^6C_2}}$$

$$= \frac{10+15}{1+3+6+10+15} = \frac{25}{35} = \frac{5}{7}$$

65. Let R be a relation on N × N defined by (a, b) R (c, d) if and only if $ad(b - c) = bc(a - d)$. Then R is

(1) symmetric and transitive but not reflexive

(2) reflexive and symmetric but not transitive

(3) symmetric but neither reflexive nor transitive

(4) transitive but neither reflexive nor symmetric

Answer (3)

Sol. (a, b) R $(c, d) \Rightarrow ad(b - c) = bc(a - d)$

For Reflexive

(a, b) R $(a, b) \Rightarrow ab(b - a) = ba(a - b)$

So not reflexive

For symmetric

(c, d) R $(a, b) \Rightarrow cb(d - a) = ad(c - b)$

OR $ad(b - c) = bc(a - d)$

So symmetric

For transitive

(a, b) R $(c, d) \Rightarrow ad(b - c) = bc(a - d)$

(c, d) R $(e, f) \Rightarrow cf(d - e) = de(c - f)$

So $adcf(b - c)(d - e) = bcde(c - d)(c - f)$

$af(b - c)(d - e) = be(a - d)(c - f)$

$\Rightarrow$ Not transitive

66. Let $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$, and $\vec{b}$ and $\vec{c}$ be two nonzero vectors such that $|\vec{a} + \vec{b} + \vec{c}| = |\vec{a} + \vec{b} - \vec{c}|$ and $\vec{b}.\vec{c} = 0$. Consider the following two statements.

(A) $|\vec{a} + \lambda\vec{c}| \geq |\vec{a}|$ for all $\lambda \in \mathbb{R}$

(B) $\vec{a}$ and $\vec{c}$ are always parallel.

Then

(1) Neither (A) nor (B) is correct

(2) Both (A) and (B) are correct

(3) Only (B) is correct

(4) Only (A) is correct

Answer (3)

Sol. $|\vec{a} + \vec{b} + \vec{c}| = |\vec{a} + \vec{b} - \vec{c}|$

$$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2\left(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\,\vec{a}\right)$$

$$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2\left(\vec{a}\cdot\vec{b} - \vec{b}\cdot\vec{c} - \vec{c}\cdot\vec{a}\right)$$

$\Rightarrow \quad \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = 0 \Rightarrow \vec{c}\cdot\vec{a} = 0$

$$|\vec{a} + \lambda\vec{c}|^2 = |\vec{a}|^2 + \lambda^2|\vec{c}|^2 + 0 \geq |\vec{a}|^2$$

So A is correct

B is incorrect

67. A wire of lenth 20 m to be cut into two pieces. A piece of length l_1 is bent to make a square of area A_1 and the other piece of length l_2 is made into a circle of area A_2. If $2A_1 + 3A_2$ is minimum then $(\pi l_1) : l_2$ is equal to

(1) 1 : 6 (2) 3 : 1

(3) 6 : 1 (4) 4 : 1

Answer (3)

Sol. $l_1 = 20 - x,\ l_2 = x$

$$2A_1 + 3A_2 = 2\left(\frac{20-x}{4}\right)^2 + 3\pi\left(\frac{x}{2\pi}\right)^2$$

$$f(x) = \frac{(20-x)^2}{8} + \frac{3x^2}{4\pi}$$

$$f'(x_0) = \frac{1}{8}2(20-x)(-1) + \frac{3}{4\pi}\cdot 2x\Big|_{x_0} = 0$$

$$0 = -\frac{1}{4}(20-x_0) + \frac{6x_0}{4\pi}$$

$$\Rightarrow \quad \frac{20-x_0}{4} = \frac{6x_0}{4\pi}$$

$$\pi(20-x_0) = 6x_0$$

$$20\pi = (6+\pi)x_0$$

$$x_0 = \frac{20\pi}{\pi+6}$$

$$\frac{\pi l_1}{l_2} = \pi\left(\frac{20-x_0}{x_0}\right) = \pi\left(\frac{\pi+6}{\pi} - 1\right)$$

= 6

68. Let

$$y = f(x) = \sin^3\left(\frac{\pi}{3}\cos\left(\frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{\frac{3}{2}}\right)\right)$$

Then at x =1,

(1) $2y' + 3\pi^2 y = 0$ (2) $2y' + \sqrt{3}\pi^2 y = 0$

(3) $2y' + 3\pi^2 y = 0$ (4) $\sqrt{2}y' - 3\pi^2 y = 0$

Answer (1)

Sol. $$f(x) = \sin^3\left(\frac{\pi}{3}\cos\left(\frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{3/2}\right)\right)$$

$$f'(x) = 3\sin^2\left(\frac{\pi}{3}\cos\left(\frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{3/2}\right)\right)$$

$$\cos\left(\frac{\pi}{3}\cos\left(\frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{3/2}\right)\right)$$

$$\frac{\pi}{3}\left(-\sin\left(\frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{3/2}\right)\right)$$

$$\frac{\pi}{3\sqrt{2}}\frac{3}{2}\left(-4x^3+5x^3+1\right)^{1/2}\left(-12x^2+10x\right)$$

$$f'(1) = \frac{3\pi^2}{16}$$

$$f(1) = \sin^3\left(\frac{\pi}{3}\cos\left(\frac{\pi}{3\sqrt{2}}2\sqrt{2}\right)\right)$$

$$= \sin^3\left(-\frac{\pi}{6}\right) = \frac{-1}{8}$$

$$\therefore \quad 2f'(1) + 3\pi^2 f(1) = 0$$

69. Let $y = f(x)$ represent a parabola with focus $\left(-\frac{1}{2}, 0\right)$ and directrix $y = -\frac{1}{2}$.

Then

$$S = \left\{x \in \mathbb{R} : \tan^{-1}\left(\sqrt{f(x)}\right) + \sin^{-1}\left(\sqrt{f(x)+1}\right) = \frac{\pi}{2}\right\}:$$

(1) Is an empty set

(2) Contains exactly one element

(3) Is an infinite set

(4) Contains exactly two elements

Answer (4)

Sol. Equation of parabola

$$k^2 + \left(h + \frac{1}{2}\right)^2 = \left|k + \frac{1}{2}\right|^2$$

$$k^2 + h^2 + h + \frac{1}{4} = k^2 + \frac{1}{4} + k$$

$$\boxed{y = x^2 + x}$$

$$\tan^{-1}\sqrt{x^2+x} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$$

$$\tan^{-1}\sqrt{x^2+x} = \cos^{-1}\sqrt{x^2+x+1}$$

$$\sqrt{x^2+x+1} = \frac{1}{\sqrt{x^2+x+1}}$$

$x = 0, -1$

70. If the domain of the function $f(x) = \frac{[x]}{1+x^2}$, where [x] is greatest integer $\le x$, is $[2,6)$, then its range is

(1) $\left(\frac{5}{37}, \frac{2}{5}\right]$

(2) $\left(\frac{5}{37}, \frac{2}{5}\right] - \left\{\frac{9}{29}, \frac{27}{109}, \frac{18}{89}, \frac{9}{53}\right\}$

(3) $\left(\frac{5}{26}, \frac{2}{5}\right]$

(4) $\left(\frac{5}{26}, \frac{2}{5}\right] - \left\{\frac{9}{29}, \frac{27}{109}, \frac{18}{89}, \frac{9}{53}\right\}$

Answer (1)

Sol. $f(x)=\frac{k}{1+x^2}$ is a decreasing function

where k > 0

$\therefore \quad x\in[2,3)\Rightarrow f(x)=\frac{2}{1+x^2}\in\left(\frac{2}{10},\frac{2}{5}\right]=R_1$

$x\in[3,4)\Rightarrow f(x)=\frac{3}{1+x^2}\in\left(\frac{3}{17},\frac{3}{10}\right]=R_2$

$x\in[4,5)\Rightarrow f(x)=\frac{4}{1+x^2}\in\left(\frac{4}{26},\frac{4}{17}\right]=R_3$

$x\in[5,6)\Rightarrow f(x)=\frac{5}{1+x^2}\in\left(\frac{5}{37},\frac{5}{26}\right]=R_4$

Range = $R_1\cup R_2\cup R_3\cup R_4$

$=\left(\frac{5}{37},\frac{2}{5}\right]$

71. The number of real roots of the equation $\sqrt{x^2-4x+3}+\sqrt{x^2-9}=\sqrt{4x^2-14x+6}$, is

(1) 1 (2) 0

(3) 2 (4) 3

Answer (1)

Sol. Common domain of functions is $(-\infty,\ -3]\cup[3,\ \infty)$

$\sqrt{x^2-4x+3}+\sqrt{x^2-9}=\sqrt{4x^2-14x+6}$

$\sqrt{x-3}\left(\sqrt{x-1}+\sqrt{x+3}\right)=\sqrt{x-3}\sqrt{4x-2}$

$\sqrt{x-3}=0\ \Rightarrow x=3$

Or $\sqrt{x-1}+\sqrt{x+3}=\sqrt{4x-2}$

On squaring,

$x-1+x+3+2\sqrt{(x-1)(x+3)}=4x-2$

$2\sqrt{x^2+2x-3}=2x-4$

$4\left(x^2+2x-3\right)=4x^2-16x+16$

$x=\frac{7}{6}\notin(-\infty,\ -3]\cup[3,\ \infty)$

$\therefore$ Only 1 solution

72. The value of $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\frac{(2+3\sin x)}{\sin x(1+\cos x)}dx$ is equal to

(1) $\frac{10}{3}-\sqrt{3}-\log_e\sqrt{3}$ (2) $-2+3\sqrt{3}+\log_e\sqrt{3}$

(3) $\frac{7}{2}-\sqrt{3}-\log_e\sqrt{3}$ (4) $\frac{10}{3}-\sqrt{3}+\log_e\sqrt{3}$

Answer (4)

Sol. $\underbrace{\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\frac{2\sin x}{\sin^2 x(1+\cos x)}dx}_{I_1}+\underbrace{\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\frac{3}{1+\cos x}dx}_{I_2}$

$\cos x = t$

$\int_{\frac{1}{2}}^{0}\frac{-2dt}{(1-t^2)(1+t)}+\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\frac{3}{2}\sec^2\frac{x}{2}dx$

$2\int_0^{\frac{1}{2}}\frac{dt}{\left(1-t^2\right)(1+t)}+3\tan\frac{x}{2}\Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}}$

$=\ln\sqrt{3}-\sqrt{3}+\frac{10}{3}$

73. For the system of linear equations

$x + y + z = 6$

$\alpha x + \beta y + 7z = 3$

$x + 2y + 3z = 14$,

which of the following is **NOT** true?

(1) If $\alpha = \beta$ and $\alpha \neq 7$, then the system has a unique solution

(2) If $\alpha = \beta = 7$, then the system has no solution

(3) There is a unique point (α, β) on the line $x + 2y + 18 = 0$ for which the system has infinitely many solutions

(4) For every point $(\alpha, \beta) \neq (7, 7)$ on the line $x - 2y + 7 = 0$, the system has infinitely many solutions

Answer (4)

Sol. $\Delta=\begin{vmatrix}1 & 1 & 1\\ \alpha & \beta & 7\\ 1 & 2 & 3\end{vmatrix}$

$= 1(3\beta - 14) - 1\ (3\alpha - 7) + 1\ (2\alpha - \beta)$

$= 3\beta - 14 + 7 - 3\alpha + 2\alpha - \beta$

$= 2\beta - \alpha - 7$

So, for $\alpha = \beta \neq 7$, $\Delta \neq 0$ so unique solution

$\alpha = \beta = 7$, equation (i) & (ii) represent 2 parallel planes so no solution.

If $\alpha - 2\beta + 7 = 0$, but $(\alpha, \beta) \neq (7, 7)$, then no solution.

74. Let a differentiable function f satisfy

$f(x)+\int_3^x \frac{f(t)}{t}dt=\sqrt{x+1},\ x\geq 3$. Then $12f(8)$ is equal to

(1) 1 (2) 34

(3) 17 (4) 19

Answer (3)

Sol. Differentiating both sides we get

$$f'(x)+\frac{f(x)}{x}=\frac{1}{2\sqrt{x+1}}$$

$$\Rightarrow \frac{dy}{dx}+\frac{y}{x}=\frac{1}{2\sqrt{x+1}}$$

$\Rightarrow$ IF $= x$

$$\Rightarrow yx=\frac{1}{2}\int\frac{x}{\sqrt{x+1}}dx+c$$

$$\Rightarrow yx=\frac{1}{2}\left(\frac{(x+1)^{\frac{3}{2}}}{\frac{3}{2}}-2(x+1)^{\frac{1}{2}}\right)+c$$

$$xy=\frac{1}{3}(x+1)^{\frac{3}{2}}-(x+1)^{\frac{1}{2}}+c$$

$f(3) = 2$

So, $x = 3$, $y = 2$

$$\Rightarrow c=\frac{16}{3}$$

Now, $x = 8$

$$8f(8)=\frac{27}{3}-3+\frac{16}{3}=\frac{34}{3}$$

$$12f(8)=\frac{34}{3}\times\frac{12}{8}=17$$

Option (3) is correct.

75. Let $A=\begin{pmatrix}1&0&0\\0&4&-1\\0&12&-3\end{pmatrix}$. Then the sum of the diagonal elements of the matrix $(A + I)^{11}$ is equal to

(1) 2050 (2) 4097

(3) 6144 (4) 4094

Answer (2)

Sol. $$A^2=\begin{bmatrix}1&0&0\\0&4&-1\\0&12&-3\end{bmatrix}\begin{bmatrix}1&0&0\\0&4&-1\\0&12&-3\end{bmatrix}$$

$$=\begin{bmatrix}1&0&0\\0&4&-1\\0&12&-3\end{bmatrix}=A$$

Now,

$$(A+I)^{11}={}^{11}C_0A^{11}+{}^{11}C_1A^{10}+\ldots{}^{11}C_{11}I$$

$$=A\left({}^{11}C_0+{}^{11}C_1\ldots{}^{11}C_{10}\right)+I$$

$$=A(2^{11}-1)+I$$

Trace of

$$(A+I)^{11}=2^{11}+4(2^{11}-1)+1-3(2^{11}-1)+1$$

$$=2\times2^{11}-4+3+2$$

$$=2^{12}+1$$

$= 4097$

76. Let a circle C_1 be obtained on rolling the circle $x^2 + y^2 - 4x - 6y + 11 = 0$ upwards 4 units on the tangent T to it at the point (3,2). Let C_2 be the image of C_1 in T. Let A and B be the centers of circles C_1 and C_2 respectively, and M and N be respectively the feet of perpendiculars drawn from A and B on the x-axis. Then the area of the trapezium AMNB is:

(1) $2\left(2+\sqrt{2}\right)$ (2) $3+2\sqrt{2}$

(3) $4\left(1+\sqrt{2}\right)$ (4) $2\left(1+\sqrt{2}\right)$

Answer (3)

Sol. Given circle is $x^2 + y^2 - 4x - 6y + 11 = 0$, centre (2, 3)

Tangent at (3, 2) is $x - y = 1$

After rolling up by 4 units centre of C_1 is

$$A\equiv\left(2+\frac{4}{\sqrt{2}},3+\frac{4}{\sqrt{2}}\right)$$

$$\Rightarrow A=\left(2+2\sqrt{2},3+2\sqrt{2}\right)$$

B is the image of A in $x - y = 1$

$$\frac{x-\left(2+2\sqrt{2}\right)}{1}=\frac{y-\left(3+2\sqrt{2}\right)}{-1}=\frac{-2(-2)}{2}=2$$

$$\Rightarrow x=4+2\sqrt{2},\ y=1+2\sqrt{2}$$

Area of $AMNB$

$$=\frac{1}{2}\left(4+4\sqrt{2}\right)\left(4+2\sqrt{2}-\left(2+2\sqrt{2}\right)\right)$$

$$=4\left(1+\sqrt{2}\right)$$

77. If $\sin^{-1}\frac{\alpha}{17}+\cos^{-1}\frac{4}{5}-\tan^{-1}\frac{77}{36}=0$, $0<\alpha<13$, then $\sin^{-1}(\sin\alpha)+\cos^{-1}(\cos\alpha)$ is equal to

(1) π (2) $16-5\pi$

(3) 16 (4) 0

Answer (1)

Sol. $\sin^{-1}\left(\frac{\alpha}{17}\right)=-\cos^{4}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{77}{36}\right)$

Let $\cos^{-1}\left(\frac{4}{5}\right)=p$ and $\tan^{-1}\left(\frac{77}{36}\right)=q$

$\Rightarrow \sin\left(\sin^{-1}\frac{\alpha}{17}\right)=\sin(q-p)$

$=\sin q\cdot\cos p-\cos q\cdot\sin p$

$\Rightarrow \frac{\alpha}{17}=\frac{77}{85}\cdot\frac{4}{5}-\frac{36}{85}\cdot\frac{3}{5}$

$\Rightarrow \alpha=\frac{200}{25}=8$

$\sin^{-1}\sin 8+\cos^{-1}\cos 8$

$\Rightarrow -8+3\pi+8-2\pi$

$=\pi$

78. If the sum and product of four positive consecutive terms of a G.P., are 126 and 1296, respectively, then the sum of common ratio of all such GPs is

(1) 14 (2) 7

(3) 3 (4) $\frac{9}{2}$

Answer (3)

Sol. Let the terms be $\frac{a}{r^3},\frac{a}{r},ar,ar^3$

$\frac{a}{r^3}\cdot\frac{a}{r}\cdot ar\cdot ar^3=1296$

$\Rightarrow a=6$

Now, $\frac{a}{r^3}+\frac{a}{r}+ar+ar^3=126$

$\Rightarrow \frac{1}{r^3}+\frac{1}{r}+r+r^3=21$

$\Rightarrow \left(r+\frac{1}{r}\right)\left(\left(r+\frac{1}{r}\right)^2-3\right)+\left(r+\frac{1}{r}\right)=21$

Let $r+\frac{1}{r}=t$

$t^3-3t+t=21$

$\Rightarrow t^3-2t-21=0$

$\Rightarrow (t-3)(t^2+3t+7)=0$

$\Rightarrow t=3$

$r+\frac{1}{r}=3$

$\Rightarrow r^2-3r+1=0$

$\Rightarrow r_1+r_2=3$

79. For all $z\in C$ on the curve $C_1:|z|=4$, let the locus of the point $z+\frac{1}{z}$ be the curve C_2. Then:

(1) the curve C_1 lies inside C_2

(2) the curve C_2 lies inside C_1

(3) the curves C_1 and C_2 intersect at 4 points

(4) the curves C_1 and C_2 intersect at 2 points

Answer (3)

Sol. Let $z=4e^{i\theta}$

$\Rightarrow z+\frac{1}{z}=4e^{i\theta}+\frac{1}{4}e^{-i\theta}$

$\Rightarrow x+iy=\frac{17}{4}\cos\theta+i\frac{15}{4}\sin\theta$

$\Rightarrow x=\frac{17}{4}\cos\theta,\quad y=\frac{15}{4}\sin\theta$

$\Rightarrow \frac{x^2}{\left(\frac{17}{4}\right)^2}+\frac{y^2}{\left(\frac{15}{4}\right)^2}=1$

Which is an ellipse whose $a>r$.

80. If the maximum distance of normal to the ellipse $\frac{x^2}{4}+\frac{y^2}{b^2}=1, b<2$, from the origin is 1, then the eccentricity of the ellipse is:

(1) $\frac{\sqrt{3}}{2}$

(2) $\frac{1}{2}$

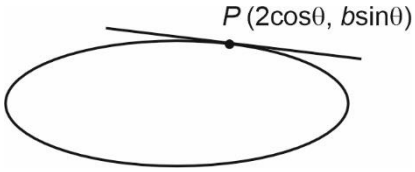
(3) $\frac{\sqrt{3}}{4}$

(4) $\frac{1}{\sqrt{2}}$

Answer (1)

Sol. $\frac{x^2}{4}+\frac{y^2}{b^2}=1, \quad b<2$

Equation of normal at P :



$2\sec\theta x - b\,\text{cosec}\theta = 4 - b^2$ …(i)

Distance from (0, 0)

$$d=\left|\frac{b^2-4}{\sqrt{4\sec^2\theta+b^2\text{cosec}^2\theta}}\right|$$

$$d=\left|\frac{b^2-4}{\sqrt{4+b^2+4\tan^2\theta+b^2\cot^2\theta}}\right|$$

Now, $d_{max} = 1$

$$\therefore \quad \frac{4-b^2}{\sqrt{b^2+4+4b}}=1$$

$\Rightarrow \quad 4 - b^2 = (b + 2) \Rightarrow b^2 + b - 2 = 0$

$\Rightarrow (b + 2)(b - 1) = 0$

$\Rightarrow b = 1$

$$\therefore \quad e=\sqrt{1-\frac{1}{4}}=\frac{\sqrt{3}}{2}$$

$\therefore$ option (1) is correct.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

81. Let θ be the angle between the planes $P_1 : \vec{r}\cdot\left(\hat{i}+\hat{j}+2\hat{k}\right)=9$ and $P_2 : \vec{r}\cdot\left(2\hat{i}-\hat{j}+\hat{k}\right)=15$.

Let L be the line that meets P_2 at the point (4, –2, 5) and makes an angle θ with the normal of P_2. If α is the angle between L and P_2, then $(\tan^2\theta)(\cot^2\alpha)$ is equal to _____.

Answer (09)

Sol. $P_1 : \vec{r}\cdot\left(\hat{i}+\hat{j}+2\hat{k}\right)=9$

$P_2 : \vec{r}\cdot\left(2\hat{i}-\hat{j}+\hat{k}\right)=15$

then $\cos\theta=\frac{3}{\sqrt{6}\cdot\sqrt{6}}=\frac{1}{2}$

$\therefore \quad \boxed{\theta=\frac{\pi}{3}}$ Now, $\alpha=\frac{\pi}{2}-\theta$

$\therefore \quad \tan^2\theta\cdot\cot^2\alpha = \tan^4\theta$

$$=\left(\sqrt{3}\right)^4=9$$

82. The remainder on dividing 5^{99} by 11 is _____.

Answer (09)

Sol. $5 \equiv 5 \pmod{11}$

$5^2 \equiv 3 \pmod{11}$

$5^4 \equiv -2 \pmod{11}$

$5^5 \equiv 1 \pmod{11}$

$5^{99} \equiv -2 \pmod{11}$

$\therefore$ remainder = 9

83. Number of 4-digit numbers that are less than or equal to 2800 and either divisible by 3 or 11, is equal to _____.

Answer (710)

Sol. Numbers which are divisible by 3 (4 digit) and less than or equal to 2800

$$=\frac{2799-1002}{3}+1=600$$

Numbers which are divisible by 11 (4 digit) and less than or equal to 2800

$$=\frac{2794-1001}{11}+1=164$$

Numbers which are divisible by 33 (4 digit) and less than or equal to 2800

$$=\frac{2772-1023}{33}+1=54$$

$\therefore$ Total no. = 710

84. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}|=\sqrt{14}, |\vec{b}|=\sqrt{6}$ and $|\vec{a}\times\vec{b}|=\sqrt{48}$. Then $\left(\vec{a}\cdot\vec{b}\right)^2$ is equal to _____.

Answer (36)

Sol. $|\vec{a}| = \sqrt{14}, |\vec{b}| = \sqrt{6}$ and $|\vec{a} \times \vec{b}| = \sqrt{48}$

$$|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2$$

$$48 + (\vec{a} \cdot \vec{b})^2 = 6 \times 14$$

$$(\vec{a} \cdot \vec{b})^2 = 84 - 48$$

$$= 36$$

85. If the variance of the frequency distribution

x_i	2	3	4	5	6	7	8
Frequency f_i	3	6	16	α	9	5	6

is 3, then α is equal to _____.

Answer (05.00)

Sol. $$3 = \frac{3.2^2 + 6.3^2 + 16.4^2 + \alpha.5^2 + 9.6^2 + 5.7^2 + 6.8^2}{45 + \alpha} - \left(\frac{225 + 5\alpha}{45 + \alpha}\right)^2$$

$$3 = \frac{12 + 54 + 256 + 25\alpha + 324 + 245 + 384}{45 + \alpha} - 25$$

$28(45 + \alpha) = 1275 + 25\alpha$

OR $1260 + 28\alpha = 1275 + 25\alpha$

$\Rightarrow \alpha = 5$

86. Let $\alpha > 0$, be the smallest number such that the expansion of $\left(x^{\frac{2}{3}} + \frac{2}{x^3}\right)^{30}$ has a term $\beta x^{-\alpha}, \beta \in \mathbb{N}$.

Then α is equal to _____.

Answer (2.00)

Sol. $\because \left(x^{\frac{2}{3}} + \frac{2}{x^3}\right)^{30} = \sum_{r=0}^{30} {}^{30}C_r \left(x^{\frac{2}{3}}\right)^{30-r} \cdot \left(\frac{2}{x^3}\right)^r$

Here $\frac{60 - 2r}{3} - 3r \in$ integer.

$\because$ β is always a natural number.

$\therefore$ $r = 6$

Thus $\alpha = 2$

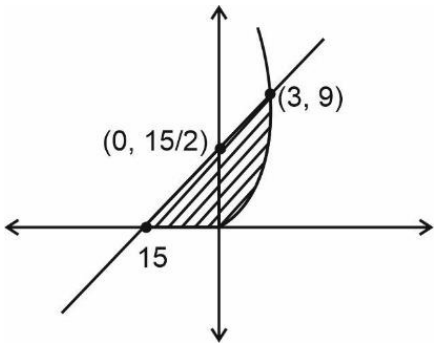
87. Let for $x \in \mathbb{R}$,

$$f(x) = \frac{x + |x|}{2} \text{ and } g(x) = \begin{cases} x, & x < 0 \\ x^2, & x \geq 0 \end{cases}.$$

Then area bounded by the curve $y = (fog)(x)$ and the line $y = 0$, $2y - x = 15$ is equal to _____.

Answer (72)

Sol. $f_0 g(x) = \begin{cases} 0 & x < 0 \\ x^2 & x \geq 0 \end{cases}$



$$\text{Area} = \frac{1}{2} \times 15 \times \frac{15}{2} + \int_0^3 \left(\frac{x + 15}{2} - x^2\right) dx$$

$$\frac{225}{4} + \frac{99}{4} - 9$$

$$\frac{324}{4} - 9$$

$81 - 9$

$= 72$

88. Let 5 digit numbers be constructed using the digits 0, 2, 3, 4, 7, 9 with repetition allowed, and are arranged in ascending order with serial numbers. Then the serial number of the number 42923 is _____.

Answer (2997)

Sol.

2	_	_	_	_	$\rightarrow$	$6^4 = 1296$
3	_	_	_	_	$\rightarrow$	$6^4 = 1296$
4	0	_	_	_	$\rightarrow$	$6^3 = 216$
4	2	_	_	_		
4	3	_	_	_		
4	4	_	_	_		
4	7	_	_	_		$\rightarrow 5 \times 6^2 = 180$

4	2	9	0	–	$\rightarrow$	6
4	2	9	2	0	$\rightarrow$	1
4	2	9	2	2	$\rightarrow$	1
4	2	9	2	3	$\rightarrow$	1
						2997

89. Let $a_1, a_2, \ldots, a_n$ be in A.P. If $a_5 = 2a_7$ and $a_{11} = 18$, then

$$12\left(\frac{1}{\sqrt{a_{10}} + \sqrt{a_{11}}} + \frac{1}{\sqrt{a_{11}} + \sqrt{a_{12}}} + \ldots + \frac{1}{\sqrt{a_{17}} + \sqrt{a_{18}}}\right)$$

is equal to _____.

Answer (08)

Sol. $a_{11} = 18$

$a + 10d = 18$ …(i)

$a_5 = 2a_7$

$a + 4d = 2(a + 6d)$

$a = -8d$ …(ii)

(i) and (ii) $\Rightarrow a = -72, d = 9$.

On rationalising the denominator, given expression

$$= 12\left[\frac{\sqrt{a_{10}} - \sqrt{a_{11}}}{-d} + \frac{\sqrt{a_{11}} - \sqrt{a_{12}}}{-d} + \ldots + \frac{\sqrt{a_{17}} - \sqrt{a_{18}}}{-d}\right]$$

$$= 12\left[\frac{\sqrt{a_{10}} - \sqrt{a_{18}}}{-d}\right]$$

$$= 12\left[\frac{\sqrt{a_{11} - d} - \sqrt{a_{11} + 7d}}{-d}\right]$$

$$= 12\left[\frac{\sqrt{18 - 9} - \sqrt{18 + 63}}{-9}\right]$$

$$= 12 \times \frac{2}{3} = 8$$

90. Let the line $L : \frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{1}$ intersect the plane $2x + y + 3z = 16$ at the point P. Let the point Q be the foot of perpendicular from the point $R(1, -1, -3)$ on the line L. If α is the area of the triangle PQR then α^2 is equal to _____.

Answer (180)

Sol. $L : \frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{1} = r$ (say)

Let $P \equiv (2r_1 + 1, -r_1, r_1 + 3)$

P lies on $2x + y + 3z = 16$

$\therefore$ $2(2r_1 + 1) + (-r_1 - 1) + 3(r_1 + 3) = 16$

$r_1 = 1$

$P \equiv (3, -2, 4)$

$R \equiv (1, -1, -3)$

Let $Q \equiv (2r_2 + 1, -r_2 - 1, r_2 + 3)$

DRs of $QR \equiv (2r_2 - r_2\ r_2 + 6)$

DRs of $L \equiv (2, -1, 1)$

$QR \perp L \Rightarrow 4r_2 + r_2 + r_2 + 6 = 0$

$r_2 = -1$

$Q \equiv (-1, 0, 2)$

$$\overrightarrow{QP} \times \overrightarrow{RP} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -2 & 2 \\ 2 & -1 & 7 \end{vmatrix} = -12\hat{i} - 24\hat{j} + 0\hat{k}$$

$$\alpha = [PQR] = \frac{1}{2}\left|\overrightarrow{QP} \times \overrightarrow{RP}\right| = \frac{1}{2} \times 12\sqrt{5}$$

$= 6\sqrt{5}$

$\alpha^2 = 180$

❑ ❑ ❑